Computer algebra independent integration tests

Summer 2022 edition

3-Logarithms/59-3.2.1-f+g-x-^m-A+B-log-e-a+b-x-over-c+d-x-^n-^p

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Contents

Chapter 1

Introduction

Local contents

This report gives the result of running the computer algebra independent integration test.

The download section in the appendix contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [314]. This is test number [59].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.1 (June 29, 2022) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.0.1 on windows 10.
3. Maple 2022.1 (June 1, 2022) on windows 10.
4. Maxima 5.46 (April 13, 2022) using Lisp SBCL 2.1.11.debian on Linux via sagemath 9.6.
5. Fricas 1.3.8 (June 21, 2022) based on sbcl 2.1.11.debian on Linux via sagemath 9.6.
6. Giac/Xcas 1.9.0-13 (July 3, 2022) on Linux via sagemath 9.6.
7. Sympy 1.10.1 (March 20, 2022) Using Python 3.10.4 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly from Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (314)	0.00 (0)
Mathematica	94.90 (298)	5.10 (16)
Maxima	75.80 (238)	24.20 (76)
Fricas	66.88 (210)	33.12 (104)
Mupad	63.69 (200)	36.31 (114)
Maple	61.46 (193)	38.54 (121)
Giac	59.24 (186)	40.76 (128)
Sympy	35.35 (111)	64.65 (203)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

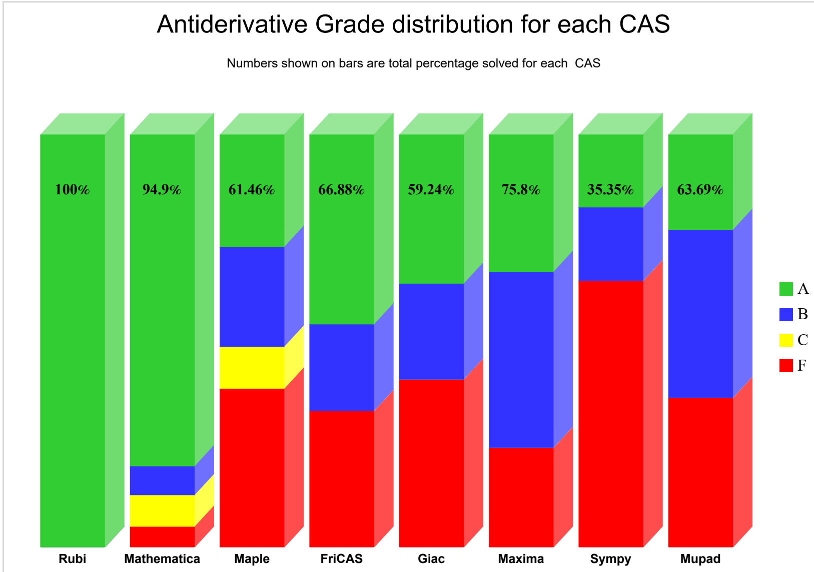
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

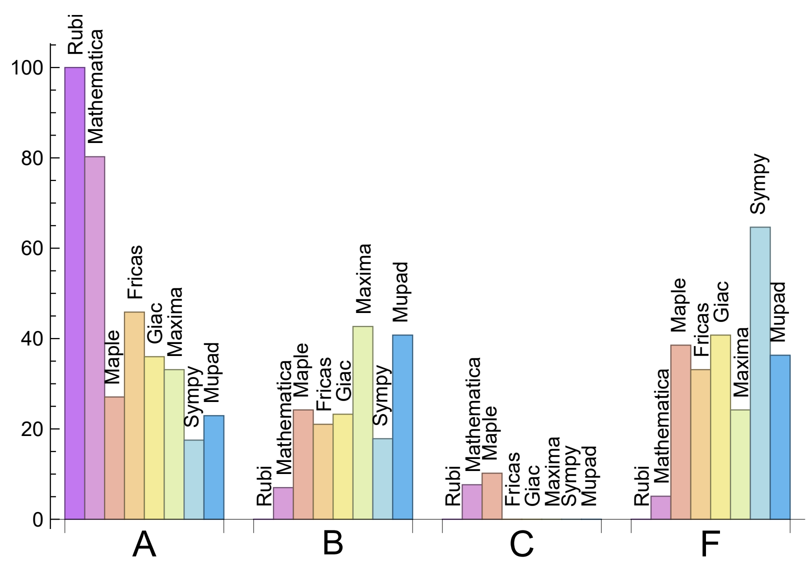
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Mathematica	80.25	7.01	7.64	5.10
Fricas	45.86	21.02	0.00	33.12
Giac	35.99	23.25	0.00	40.76
Maxima	33.12	42.68	0.00	24.20
Maple	27.07	24.20	10.19	38.54
Mupad	N/A	40.76	0.00	36.31
Sympy	17.52	17.83	0.00	64.65

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and

Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	16	100.00 %	0.00 %	0.00 %
Maple	121	100.00 %	0.00 %	0.00 %
Fricas	104	92.31 %	7.69 %	0.00 %
Giac	128	89.84 %	8.59 %	1.56 %
Maxima	76	100.00 %	0.00 %	0.00 %
Sympy	203	24.63 %	61.08 %	14.29 %
Mupad	114	100.00 %	0.00 %	0.00 %

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

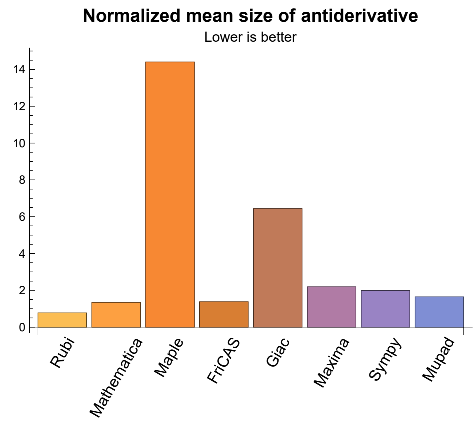
Similarly the **Normalized median** is relative to the median leaf size of the optimal.

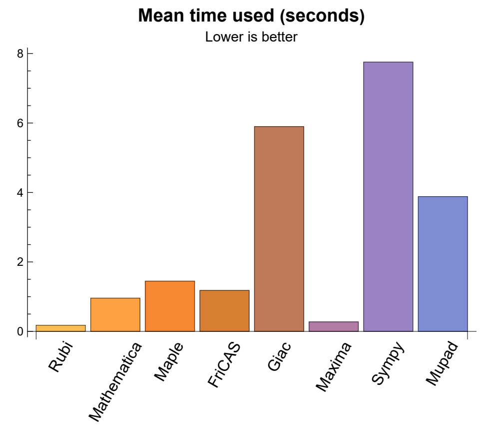
For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.18	199.75	0.77	149.00	1.00
Mathematica	0.96	373.96	1.35	144.00	0.93
Maple	1.45	5100.87	14.40	299.00	2.37
Maxima	0.28	613.99	2.19	369.00	2.37
Fricas	1.18	307.70	1.38	148.50	1.48
Sympy	7.75	350.24	1.99	150.00	2.48
Giac	5.90	1350.18	6.44	188.00	1.59
Mupad	3.88	408.51	1.64	160.00	1.54

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used from the above table.


Normalized mean size of antiderivative
Lower is better
0
2
4
6
8
10
12
14
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad


Mean time used (seconds)
Lower is better
0
2
4
6
8
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad

1.4 list of integrals that has no closed form antiderivative

$\{19, 20, 21, 24, 25, 26, 47, 48, 49, 52, 53, 54, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 109, 110, 111, 114, 115, 116, 137, 138, 139, 142, 143, 144, 191, 192, 193, 196, 197, 198, 219, 220, 221, 224, 225, 226, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292\}$

1.5 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
```

```
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.9.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

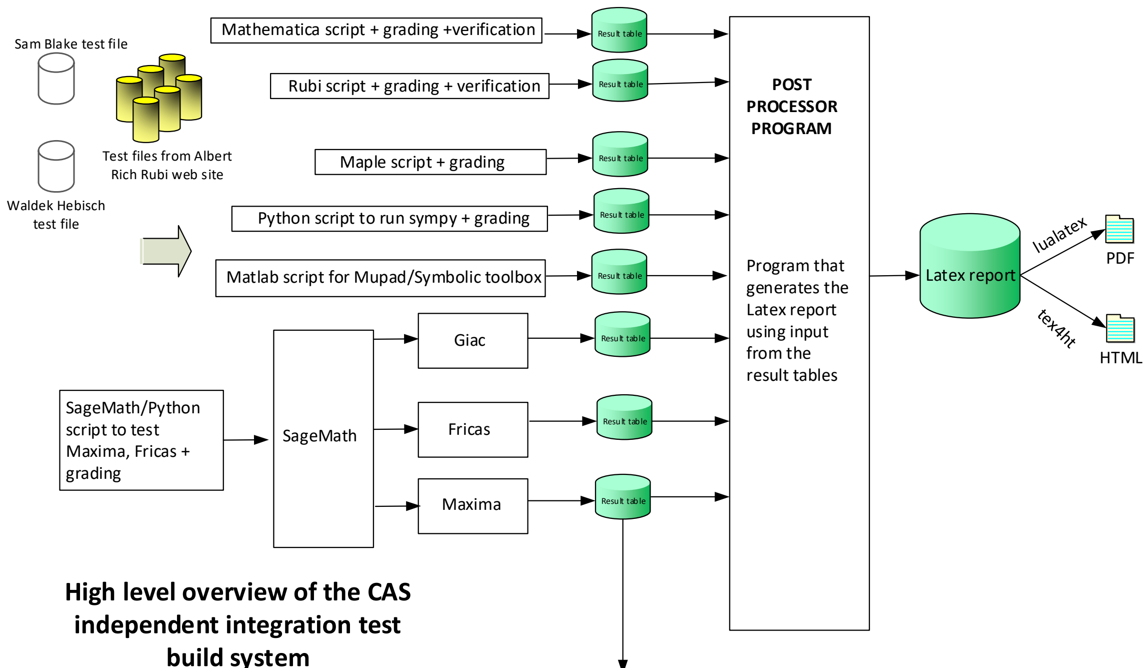
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer, the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be “A”, “B”, “C”, or “F”
13. String. Small string description of why the grade was given.
14. integer. 1 if result was verified or 0 if not verified.

*The following fields are present only in **Rubi** Table file*

15. integer. Number of steps used.
16. integer. Number of rules used.
17. integer. Integrand leaf size.
18. real number. Ratio. Field 16 over field 17
19. String of form “{n,n,..}” which is list of the rules used by Rubi
20. String. The optimal antiderivative in Mathematica syntx

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August 26, 2022

Chapter 2

detailed summary tables of results

Local contents

2.1 List of integrals sorted by grade for each CAS

Local contents

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 137, 138, 139, 142, 143, 144, 147, 148, 149, 150, 151, 152, 153, 154, 155, 160, 161, 162, 163, 169, 170, 171, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 219, 220, 221, 224, 225, 226, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 311 }

B grade: { 14, 42, 71, 72, 106, 107, 108, 156, 157, 158, 159, 164, 165, 166, 167, 168, 245, 277, 305, 306, 307, 308 }

C grade: { 15, 16, 17, 18, 43, 44, 45, 46, 102, 103, 104, 105, 133, 134, 135, 136, 187, 188, 189, 190, 215, 216, 217, 218 }

F grade: { 140, 141, 145, 146, 172, 222, 223, 227, 228, 229, 276, 309, 310, 312, 313, 314 }

2.1.3 Maple

A grade: { 19, 20, 21, 24, 25, 26, 47, 48, 49, 52, 53, 54, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 137, 138, 139, 142, 143, 144, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 206, 219, 220, 221, 224, 225, 226, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292 }

B grade: { 61, 88, 89, 90, 91, 92, 93, 94, 95, 96, 101, 102, 103, 104, 105, 119, 120, 121, 122, 123, 124, 125, 126, 127, 133, 134, 135, 136, 173, 174, 175, 176, 177, 178, 179, 180, 181, 186, 187, 188, 189, 190, 201, 202, 203, 204, 205, 207, 208, 209, 215, 216, 217, 218, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 244, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 297 }

C grade: { 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 160, 161, 162, 163, 168, 169, 170, 171, 293, 294, 295, 296, 298, 299, 300, 301, 302, 303, 304, 305 }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 22, 23, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 50, 51, 55, 56, 57, 58, 59, 60, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 97, 98, 99, 100, 128, 129, 130, 131, 132, 140, 141, 145, 146, 159, 164, 165, 166, 167, 172, 182, 183, 184, 185, 210, 211, 212, 213, 214, 222, 223, 227, 228, 229, 240, 241, 242, 243, 245, 246, 247, 248, 272, 273, 274, 275, 276, 277, 278, 279, 280, 306, 307, 308, 309, 310, 311, 312, 313, 314 }

2.1.4 Maxima

A grade: { 4, 7, 19, 20, 21, 24, 25, 26, 32, 34, 35, 47, 48, 49, 52, 53, 54, 57, 58, 59, 60, 61, 63, 64, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 91, 94, 109, 110, 111, 114, 115, 116, 137, 138, 139, 142, 143, 144, 150, 152, 153, 176, 179, 191, 192, 193, 196, 197, 198, 206, 219, 220, 221, 224, 225, 226, 230, 231, 232, 233, 234, 236, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 266, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 296, 297, 299 }

B grade: { 1, 2, 3, 6, 8, 9, 10, 11, 12, 13, 15, 16, 17, 18, 29, 30, 31, 36, 37, 38, 39, 40, 41, 43, 44, 45, 46, 65, 66, 67, 68, 69, 88, 89, 90, 93, 95, 96, 97, 98, 99, 100, 102, 103, 104, 105, 106, 107, 108, 119, 120, 121, 122, 124, 125, 126, 127, 128, 129, 130, 131, 133, 134, 135, 136, 147, 148, 149, 154, 155, 156, 157, 158, 160, 161, 162, 163, 168, 169, 170, 171, 173, 174, 175, 178, 180, 181, 182, 183, 184, 185, 187, 188, 189, 190, 201, 202, 203, 204, 207, 208, 209, 210, 211, 212, 213, 215, 216, 217, 218, 237, 238, 239, 240, 241, 242, 262, 263, 264, 265, 268, 269, 270, 271, 272, 273, 274, 294, 295, 300, 301, 302, 303, 304 }

C grade: { }

F grade: { 5, 14, 22, 23, 27, 28, 33, 42, 50, 51, 55, 56, 62, 70, 71, 72, 73, 74, 75, 92, 101, 112, 113, 117, 118, 123, 132, 140, 141, 145, 146, 151, 159, 164, 165, 166, 167, 172, 177, 186, 194, 195, 199, 200, 205, 214, 222, 223, 227, 228, 229, 235, 243, 244, 245, 246, 247, 248, 267, 275, 276, 277, 278, 279, 280, 298, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314 }

2.1.5 FriCAS

A grade: { 4, 6, 7, 15, 16, 19, 20, 21, 22, 23, 24, 25, 26, 27, 32, 34, 35, 43, 44, 47, 48, 49, 50, 51, 52, 53, 54, 55, 57, 59, 60, 61, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 90, 91, 93, 94, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 122, 124, 125, 133, 134, 135, 136, 137, 138, 139, 142, 143, 144, 150, 152, 172, 175, 176, 178, 179, 187, 188, 189, 191, 192, 193, 194, 195, 196, 197, 198, 199, 204, 206, 207, 215, 216, 217, 219, 220, 221, 224, 225, 226, 229, 230, 232, 233, 234, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 265, 266, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 296, 297, 299 }

B grade: { 1, 2, 3, 8, 9, 17, 18, 28, 29, 30, 31, 36, 37, 45, 46, 56, 58, 63, 64, 88, 89, 95, 96, 118, 119, 120, 121, 126, 127, 147, 148, 149, 153, 154, 155, 160, 161, 162, 163, 168, 169, 170, 171, 173, 174, 180, 181, 190, 200, 201, 202, 203, 208, 209, 218, 231, 236, 237, 263, 264, 268, 269, 293, 294, 295, 300 }

C grade: { }

F grade: { 5, 10, 11, 12, 13, 14, 33, 38, 39, 40, 41, 42, 62, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 92, 97, 98, 99, 100, 101, 123, 128, 129, 130, 131, 132, 140, 141, 145, 146, 151, 156, 157, 158, 159, 164, 165, 166, 167, 177, 182, 183, 184, 185, 186, 205, 210, 211, 212, 213, 214, 222, 223, 227, 228, 235, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 267, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 298, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314 }

2.1.6 Sympy

A grade: { 19, 20, 21, 24, 25, 26, 47, 48, 49, 52, 53, 54, 61, 76, 77, 78, 79, 82, 83, 84, 85, 109, 110, 111, 115, 116, 137, 138, 139, 143, 144, 191, 192, 193, 197, 198, 219, 220, 221, 225, 226, 234, 249, 250, 251, 252, 253, 254, 257, 258, 281, 282, 283, 288, 289 }

B grade: { 4, 32, 60, 88, 89, 90, 91, 93, 94, 95, 96, 102, 103, 104, 119, 120, 121, 122, 124, 125, 126, 127, 133, 134, 135, 173, 174, 175, 176, 178, 179, 180, 181, 187, 188, 189, 201, 202, 203, 204, 206, 207, 208, 209, 215, 216, 217, 230, 231, 232, 233, 262, 263, 264, 265, 266 }

C grade: { }

F grade: { 1, 2, 3, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 22, 23, 27, 28, 29, 30, 31, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 50, 51, 55, 56, 57, 58, 59, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 80, 81, 86, 87, 92, 97, 98, 99, 100, 101, 105, 106, 107, 108, 112, 113, 114, 117, 118, 123, 128, 129, 130, 131, 132, 136, 140, 141, 142, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 177, 182, 183, 184, 185, 186, 190, 194, 195, 196, 199, 200, 205, 210, 211, 212, 213, 214, 218, 222, 223, 224, 227, 228, 229, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 255, 256, 259, 260, 261, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 284, 285, 286, 287, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314 }

2.1.7 Giac

A grade: { 6, 7, 15, 16, 17, 18, 19, 20, 21, 24, 25, 26, 34, 35, 43, 44, 45, 47, 48, 49, 52, 53, 54, 55, 56, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 93, 94, 102, 103, 104, 105, 109, 110, 111, 114, 115, 116, 122, 125, 136, 137, 138, 139, 142, 143, 144, 150, 152, 153, 178, 179, 187, 188, 189, 191, 192, 193, 196, 197, 198, 199, 204, 206, 207, 218, 219, 220, 221, 224, 225, 226, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 264, 265, 266, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 296, 297, 299 }

B grade: { 1, 2, 3, 4, 8, 9, 29, 30, 31, 32, 33, 36, 37, 46, 57, 58, 59, 60, 61, 63, 64, 65, 66, 88, 89, 90, 91, 95, 96, 106, 107, 108, 119, 120, 121, 124, 126, 127, 133, 147, 148, 149, 154, 155, 173, 174, 175, 176, 177, 180, 181, 190, 200, 202, 203, 208, 209, 215, 230, 231, 232, 233, 234, 236, 237, 238, 239, 249, 269, 270, 300, 301, 302 }

C grade: { }

F grade: { 5, 10, 11, 12, 13, 14, 22, 23, 27, 28, 38, 39, 40, 41, 42, 50, 51, 62, 67, 68, 69, 70, 71, 72, 73, 74, 75, 92, 97, 98, 99, 100, 101, 112, 113, 117, 118, 123, 128, 129, 130, 131, 132, 134, 135, 140, 141, 145, 146, 151, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 182, 183, 184, 185, 186, 194, 195, 201, 205, 210, 211, 212, 213, 214, 216, 217, 222, 223, 227, 228, 229, 235, 240, 241, 242, 243, 244, 245, 246, 247, 248, 262, 263, 267, 268, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 293, 294, 295, 298, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314 }

2.1.8 Mupad

A grade: { 19, 20, 21, 24, 25, 26, 47, 48, 49, 52, 53, 54, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 109, 110, 111, 114, 115, 116, 137, 138, 139, 142, 143, 144, 191, 192, 193, 196, 197, 198, 219, 220, 221, 224, 225, 226, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292 }

B grade: { 1, 2, 3, 4, 6, 7, 8, 9, 15, 16, 17, 18, 29, 30, 31, 32, 34, 35, 36, 37, 43, 44, 45, 46, 57, 58, 59, 60, 61, 63, 64, 65, 66, 88, 89, 90, 91, 93, 94, 95, 96, 102, 103, 104, 105, 106, 107, 108, 119, 120, 121, 122, 124, 125, 126, 127, 133, 134, 135, 136, 147, 148, 149, 150, 152, 153, 154, 155, 160, 161, 162, 163, 168, 169, 170, 171, 173, 174, 175, 176, 178, 179, 180, 181, 187, 188, 189, 190, 201, 202, 203, 204, 206, 207, 208, 209, 215, 216, 217, 218, 230, 231, 232, 233, 234, 236, 237, 238, 239, 249, 262, 263, 264, 265, 266, 268, 269, 270, 271, 293, 294, 295, 296, 297, 299, 300, 301, 302 }

C grade: { }

F grade: { 5, 10, 11, 12, 13, 14, 22, 23, 27, 28, 33, 38, 39, 40, 41, 42, 50, 51, 55, 56, 62, 67, 68, 69, 70, 71, 72, 73, 74, 75, 92, 97, 98, 99, 100, 101, 112, 113, 117, 118, 123, 128, 129, 130, 131, 132, 140, 141, 145, 146, 151, 156, 157, 158, 159, 164, 165, 166, 167, 172, 177, 182, 183, 184, 185, 186, 194, 195, 199, 200, 205, 210, 211, 212, 213, 214, 222, 223, 227, 228, 229, 235, 240, 241, 242, 243, 244, 245, 246, 247, 248, 267, 272, 273, 274, 275, 276, 277, 278, 279, 280, 298, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column `N.S.` in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	146	0	681	494	0	4392	1046
N.S.	1	1.00	0.78	0.00	3.62	2.63	0.00	23.36	5.56
time (sec)	N/A	0.093	0.086	0.046	0.310	0.452	0.000	3.956	4.548

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	124	0	483	367	0	2986	588
N.S.	1	1.00	0.79	0.00	3.10	2.35	0.00	19.14	3.77
time (sec)	N/A	0.070	0.077	0.049	0.306	0.409	0.000	5.699	4.399

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	103	0	312	253	0	1836	303
N.S.	1	1.00	0.83	0.00	2.52	2.04	0.00	14.81	2.44
time (sec)	N/A	0.056	0.044	0.046	0.282	0.390	0.000	4.589	4.284

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	73	0	158	138	352	864	134
N.S.	1	1.00	0.85	0.00	1.84	1.60	4.09	10.05	1.56
time (sec)	N/A	0.035	0.029	0.075	0.281	0.473	162.558	5.758	4.073

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	101	0	0	0	0	0	-1
N.S.	1	1.00	1.20	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.145	0.039	0.123	0.000	0.000	0.000	0.000	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	79	115	0	138	94	0	85	112
N.S.	1	1.18	1.72	0.00	2.06	1.40	0.00	1.27	1.67
time (sec)	N/A	0.034	0.046	0.048	0.295	0.425	0.000	3.819	5.655

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	114	0	260	243	0	220	222
N.S.	1	1.00	0.75	0.00	1.72	1.61	0.00	1.46	1.47
time (sec)	N/A	0.087	0.110	0.049	0.308	0.394	0.000	4.703	4.522

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	145	0	433	446	0	375	349
N.S.	1	1.00	0.79	0.00	2.37	2.44	0.00	2.05	1.91
time (sec)	N/A	0.107	0.123	0.053	0.321	0.362	0.000	5.566	4.845

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	215	162	0	652	685	0	533	603
N.S.	1	1.00	0.75	0.00	3.03	3.19	0.00	2.48	2.80
time (sec)	N/A	0.129	0.156	0.054	0.394	0.382	0.000	4.477	5.121

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	396	396	535	0	2764	0	0	0	-1
N.S.	1	1.00	1.35	0.00	6.98	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.369	0.348	0.098	0.836	0.000	0.000	0.000	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	335	335	411	0	2039	0	0	0	-1
N.S.	1	1.00	1.23	0.00	6.09	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.283	0.241	0.083	0.894	0.000	0.000	0.000	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	274	274	303	0	1412	0	0	0	-1
N.S.	1	1.00	1.11	0.00	5.15	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.215	0.165	0.076	0.816	0.000	0.000	0.000	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	215	0	786	0	0	0	-1
N.S.	1	1.00	1.10	0.00	4.01	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.127	0.136	0.044	0.886	0.000	0.000	0.000	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	537	0	0	0	0	0	-1
N.S.	1	1.00	3.89	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.116	0.197	0.048	0.000	0.000	0.000	0.000	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	330	0	433	212	0	163	238
N.S.	1	1.00	2.43	0.00	3.18	1.56	0.00	1.20	1.75
time (sec)	N/A	0.067	0.383	0.053	0.308	0.407	0.000	5.033	5.590

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	288	288	463	0	864	528	0	458	506
N.S.	1	1.00	1.61	0.00	3.00	1.83	0.00	1.59	1.76
time (sec)	N/A	0.153	0.339	0.047	0.344	0.439	0.000	7.780	6.171

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	448	448	609	0	1435	946	0	810	1038
N.S.	1	1.00	1.36	0.00	3.20	2.11	0.00	1.81	2.32
time (sec)	N/A	0.229	0.470	0.051	0.482	0.415	0.000	5.631	7.693

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	615	615	776	0	2139	1438	0	1166	1769
N.S.	1	1.00	1.26	0.00	3.48	2.34	0.00	1.90	2.88
time (sec)	N/A	0.298	0.678	0.048	0.509	0.456	0.000	4.868	9.221

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	38	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.023	0.472	0.040	0.000	0.000	0.000	0.000	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	36	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.013	0.184	0.056	0.000	0.000	0.000	0.000	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	38	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.026	0.097	0.123	0.000	0.000	0.000	0.000	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	94	0	0	56	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.60	0.00	0.00	-0.01
time (sec)	N/A	0.080	0.093	0.046	0.000	0.343	0.000	0.000	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	172	0	0	137	0	0	-1
N.S.	1	1.00	0.87	0.00	0.00	0.70	0.00	0.00	-0.01
time (sec)	N/A	0.185	0.178	0.049	0.000	0.401	0.000	0.000	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	38	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.023	0.632	0.040	0.000	0.000	0.000	0.000	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	36	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.013	0.604	0.013	0.000	0.000	0.000	0.000	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	38	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.024	0.370	0.047	0.000	0.000	0.000	0.000	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	146	0	0	241	0	0	-1
N.S.	1	1.00	0.95	0.00	0.00	1.58	0.00	0.00	-0.01
time (sec)	N/A	0.103	0.126	0.049	0.000	0.341	0.000	0.000	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	314	314	254	0	0	639	0	0	-1
N.S.	1	1.00	0.81	0.00	0.00	2.04	0.00	0.00	-0.00
time (sec)	N/A	0.242	0.375	0.049	0.000	0.367	0.000	0.000	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	146	0	681	497	0	1862	1045
N.S.	1	1.00	0.78	0.00	3.62	2.64	0.00	9.90	5.56
time (sec)	N/A	0.084	0.072	0.047	0.310	0.425	0.000	4.865	4.484

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	124	0	483	370	0	1390	588
N.S.	1	1.00	0.79	0.00	3.10	2.37	0.00	8.91	3.77
time (sec)	N/A	0.067	0.067	0.045	0.361	0.394	0.000	7.027	4.367

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	101	0	312	254	0	980	303
N.S.	1	1.00	0.81	0.00	2.52	2.05	0.00	7.90	2.44
time (sec)	N/A	0.050	0.043	0.043	0.312	0.371	0.000	4.032	4.307

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	74	0	158	140	382	572	134
N.S.	1	1.00	0.86	0.00	1.84	1.63	4.44	6.65	1.56
time (sec)	N/A	0.037	0.029	0.077	0.291	0.421	163.301	5.605	4.098

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	101	0	0	0	0	558	-1
N.S.	1	1.00	1.26	0.00	0.00	0.00	0.00	6.98	-0.01
time (sec)	N/A	0.147	0.030	0.129	0.000	0.000	0.000	58.772	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	114	0	137	96	0	89	113
N.S.	1	1.00	1.12	0.00	1.34	0.94	0.00	0.87	1.11
time (sec)	N/A	0.031	0.044	0.050	0.300	0.423	0.000	2.691	4.017

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	115	0	260	244	0	203	221
N.S.	1	1.00	0.76	0.00	1.72	1.62	0.00	1.34	1.46
time (sec)	N/A	0.081	0.105	0.049	0.293	0.355	0.000	3.918	4.551

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	146	0	434	447	0	399	349
N.S.	1	1.00	0.80	0.00	2.37	2.44	0.00	2.18	1.91
time (sec)	N/A	0.101	0.120	0.051	0.307	0.353	0.000	3.907	4.723

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	215	162	0	653	687	0	676	603
N.S.	1	1.00	0.75	0.00	3.04	3.20	0.00	3.14	2.80
time (sec)	N/A	0.130	0.160	0.048	0.319	0.376	0.000	4.259	4.985

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	544	544	533	0	2693	0	0	0	-1
N.S.	1	1.00	0.98	0.00	4.95	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.479	0.337	0.089	0.817	0.000	0.000	0.000	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	454	454	409	0	1991	0	0	0	-1
N.S.	1	1.00	0.90	0.00	4.39	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.344	0.231	0.087	0.810	0.000	0.000	0.000	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	361	361	303	0	1382	0	0	0	-1
N.S.	1	1.00	0.84	0.00	3.83	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.255	0.160	0.078	0.750	0.000	0.000	0.000	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	220	216	0	788	0	0	0	-1
N.S.	1	1.00	0.98	0.00	3.58	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.160	0.138	0.046	0.729	0.000	0.000	0.000	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	537	0	0	0	0	0	-1
N.S.	1	1.00	3.92	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.085	0.188	0.050	0.000	0.000	0.000	0.000	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	331	0	431	215	0	164	237
N.S.	1	1.00	2.03	0.00	2.64	1.32	0.00	1.01	1.45
time (sec)	N/A	0.053	0.285	0.048	0.302	0.354	0.000	4.581	5.649

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	317	317	464	0	864	530	0	387	505
N.S.	1	1.00	1.46	0.00	2.73	1.67	0.00	1.22	1.59
time (sec)	N/A	0.113	0.306	0.049	0.333	0.344	0.000	5.498	5.467

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	429	429	609	0	1438	948	0	746	1040
N.S.	1	1.00	1.42	0.00	3.35	2.21	0.00	1.74	2.42
time (sec)	N/A	0.180	0.446	0.049	0.406	0.353	0.000	3.455	7.162

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	536	536	776	0	2141	1442	0	1225	1765
N.S.	1	1.00	1.45	0.00	3.99	2.69	0.00	2.29	3.29
time (sec)	N/A	0.204	0.615	0.051	0.479	0.407	0.000	6.633	9.081

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	38	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.024	0.276	0.044	0.000	0.000	0.000	0.000	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	36	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.014	0.182	0.059	0.000	0.000	0.000	0.000	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	38	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.027	0.114	0.128	0.000	0.000	0.000	0.000	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	96	0	0	56	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.58	0.00	0.00	-0.01
time (sec)	N/A	0.074	0.087	0.045	0.000	0.344	0.000	0.000	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	199	174	0	0	135	0	0	-1
N.S.	1	1.00	0.87	0.00	0.00	0.68	0.00	0.00	-0.01
time (sec)	N/A	0.152	0.172	0.049	0.000	0.354	0.000	0.000	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	38	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.024	0.632	0.038	0.000	0.000	0.000	0.000	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	36	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.013	0.628	0.013	0.000	0.000	0.000	0.000	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	38	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.026	0.377	0.049	0.000	0.000	0.000	0.000	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	180	0	0	254	0	140	-1
N.S.	1	1.00	1.17	0.00	0.00	1.65	0.00	0.91	-0.01
time (sec)	N/A	0.081	0.120	0.047	0.000	0.337	0.000	4.877	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	256	256	288	0	0	652	0	312	-1
N.S.	1	1.00	1.12	0.00	0.00	2.55	0.00	1.22	-0.00
time (sec)	N/A	0.187	0.325	0.051	0.000	0.375	0.000	2.941	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	364	364	285	0	636	664	0	11806	1433
N.S.	1	1.00	0.78	0.00	1.75	1.82	0.00	32.43	3.94
time (sec)	N/A	0.349	0.436	0.054	0.302	0.784	0.000	7.133	4.680

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	235	235	219	0	447	465	0	6660	766
N.S.	1	1.00	0.93	0.00	1.90	1.98	0.00	28.34	3.26
time (sec)	N/A	0.212	0.200	0.047	0.292	0.463	0.000	5.349	4.742

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	146	0	285	294	0	3346	371
N.S.	1	1.00	0.93	0.00	1.82	1.87	0.00	21.31	2.36
time (sec)	N/A	0.109	0.105	0.044	0.280	0.439	0.000	5.533	4.179

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	120	0	152	156	493	1189	153
N.S.	1	1.00	1.04	0.00	1.32	1.36	4.29	10.34	1.33
time (sec)	N/A	0.065	0.089	0.074	0.283	0.364	168.917	4.305	4.257

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	56	122	53	58	150	237	52
N.S.	1	1.00	1.00	2.18	0.95	1.04	2.68	4.23	0.93
time (sec)	N/A	0.022	0.009	0.055	0.277	0.368	3.341	5.521	4.005

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	122	0	0	0	0	0	-1
N.S.	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.093	0.044	0.122	0.000	0.000	0.000	0.000	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	109	0	143	271	0	455	140
N.S.	1	1.00	1.20	0.00	1.57	2.98	0.00	5.00	1.54
time (sec)	N/A	0.063	0.101	0.050	0.300	3.395	0.000	4.583	4.640

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	190	173	0	356	1094	0	2952	430
N.S.	1	1.00	0.91	0.00	1.87	5.76	0.00	15.54	2.26
time (sec)	N/A	0.149	0.371	0.050	0.310	47.836	0.000	4.901	6.199

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	283	283	264	0	853	0	0	9570	1182
N.S.	1	1.00	0.93	0.00	3.01	0.00	0.00	33.82	4.18
time (sec)	N/A	0.278	0.518	0.047	0.357	0.000	0.000	3.235	9.225

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	388	388	359	0	1762	0	0	21485	2569
N.S.	1	1.00	0.93	0.00	4.54	0.00	0.00	55.37	6.62
time (sec)	N/A	0.442	0.694	0.049	0.466	0.000	0.000	6.197	13.771

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	923	923	757	0	2534	0	0	0	-1
N.S.	1	1.00	0.82	0.00	2.75	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.167	0.689	0.082	0.834	0.000	0.000	0.000	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	565	565	506	0	1591	0	0	0	-1
N.S.	1	1.00	0.90	0.00	2.82	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.750	0.377	0.077	0.752	0.000	0.000	0.000	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	290	290	362	0	868	0	0	0	-1
N.S.	1	1.00	1.25	0.00	2.99	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.379	0.215	0.042	0.751	0.000	0.000	0.000	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	226	0	0	0	0	0	-1
N.S.	1	1.00	1.67	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.167	0.125	0.057	0.000	0.000	0.000	0.000	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	297	297	1441	0	0	0	0	0	-1
N.S.	1	1.00	4.85	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.362	0.260	0.051	0.000	0.000	0.000	0.000	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	206	418	0	0	0	0	0	-1
N.S.	1	1.00	2.03	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.124	0.340	0.055	0.000	0.000	0.000	0.000	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	389	389	615	0	0	0	0	0	-1
N.S.	1	1.00	1.58	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.551	1.039	0.048	0.000	0.000	0.000	0.000	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	747	747	918	0	0	0	0	0	-1
N.S.	1	1.00	1.23	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.140	2.114	0.049	0.000	0.000	0.000	0.000	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1208	1208	1329	0	0	0	0	0	-1
N.S.	1	1.00	1.10	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.771	5.151	0.048	0.000	0.000	0.000	0.000	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	35	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.021	0.277	0.039	0.000	0.000	0.000	0.000	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	33	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.012	0.187	0.059	0.000	0.000	0.000	0.000	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.004	0.013	0.039	0.000	0.000	0.000	0.000	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	35	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.023	0.375	0.129	0.000	0.000	0.000	0.000	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	35	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.021	0.566	0.052	0.000	0.000	0.000	0.000	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	35	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.023	6.405	0.046	0.000	0.000	0.000	0.000	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	35	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.020	0.602	0.039	0.000	0.000	0.000	0.000	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	33	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.011	0.464	0.011	0.000	0.000	0.000	0.000	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.004	0.411	0.052	0.000	0.000	0.000	0.000	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	35	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.021	0.953	0.046	0.000	0.000	0.000	0.000	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	35	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.019	1.841	0.040	0.000	0.000	0.000	0.000	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	35	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.020	20.877	0.049	0.000	0.000	0.000	0.000	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	142	9745	633	430	969	5428	1009
N.S.	1	1.00	0.79	54.14	3.52	2.39	5.38	30.16	5.61
time (sec)	N/A	0.085	0.074	0.340	0.298	0.442	4.801	4.178	4.780

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	120	5350	447	317	706	3795	566
N.S.	1	1.00	0.81	35.91	3.00	2.13	4.74	25.47	3.80
time (sec)	N/A	0.066	0.069	0.280	0.295	0.392	2.443	3.116	4.645

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	99	2512	286	221	491	2450	290
N.S.	1	1.00	0.84	21.29	2.42	1.87	4.16	20.76	2.46
time (sec)	N/A	0.051	0.040	0.271	0.298	0.368	1.572	4.058	4.480

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	69	901	148	124	253	1319	126
N.S.	1	1.00	0.85	11.12	1.83	1.53	3.12	16.28	1.56
time (sec)	N/A	0.033	0.025	0.246	0.272	0.349	1.034	2.717	4.303

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	95	345	0	0	0	0	-1
N.S.	1	1.00	1.19	4.31	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.142	0.034	0.600	0.000	0.000	0.000	0.000	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	75	105	173	134	82	233	110	104
N.S.	1	1.19	1.67	2.75	2.13	1.30	3.70	1.75	1.65
time (sec)	N/A	0.031	0.044	0.254	0.264	0.353	0.723	2.843	5.020

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	110	335	257	216	422	237	209
N.S.	1	1.00	0.76	2.33	1.78	1.50	2.93	1.65	1.45
time (sec)	N/A	0.084	0.092	0.308	0.325	0.357	1.279	2.541	5.039

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	141	503	430	405	656	382	339
N.S.	1	1.00	0.81	2.87	2.46	2.31	3.75	2.18	1.94
time (sec)	N/A	0.100	0.109	0.325	0.308	0.391	2.006	2.565	5.584

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	206	158	675	649	628	944	528	577
N.S.	1	1.00	0.77	3.28	3.15	3.05	4.58	2.56	2.80
time (sec)	N/A	0.124	0.143	0.366	0.316	0.367	2.951	3.132	6.166

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	365	365	511	0	2108	0	0	0	-1
N.S.	1	1.00	1.40	0.00	5.78	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.347	0.327	0.306	0.398	0.000	0.000	0.000	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	309	309	391	0	1539	0	0	0	-1
N.S.	1	1.00	1.27	0.00	4.98	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.262	0.224	0.244	0.385	0.000	0.000	0.000	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	253	253	287	0	1045	0	0	0	-1
N.S.	1	1.00	1.13	0.00	4.13	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.199	0.149	0.172	0.394	0.000	0.000	0.000	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	203	0	571	0	0	0	-1
N.S.	1	1.00	1.13	0.00	3.17	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.123	0.121	0.095	0.361	0.000	0.000	0.000	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	250	662	0	0	0	0	-1
N.S.	1	1.00	1.95	5.17	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.111	0.334	0.622	0.000	0.000	0.000	0.000	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	314	349	422	148	434	176	222
N.S.	1	1.00	2.49	2.77	3.35	1.17	3.44	1.40	1.76
time (sec)	N/A	0.061	0.311	0.249	0.313	0.369	1.342	3.067	5.260

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	268	268	443	689	854	365	894	424	507
N.S.	1	1.00	1.65	2.57	3.19	1.36	3.34	1.58	1.89
time (sec)	N/A	0.145	0.316	0.318	0.360	0.373	2.616	3.598	5.851

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	418	418	585	1039	1425	670	1544	709	1064
N.S.	1	1.00	1.40	2.49	3.41	1.60	3.69	1.70	2.55
time (sec)	N/A	0.213	0.447	0.402	0.448	0.384	16.427	4.065	7.405

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	575	575	748	1393	2129	1033	0	995	1881
N.S.	1	1.00	1.30	2.42	3.70	1.80	0.00	1.73	3.27
time (sec)	N/A	0.274	0.631	0.435	0.510	0.366	0.000	3.640	10.303

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	29	114	30	158	30	0	1203	25
N.S.	1	1.04	4.07	1.07	5.64	1.07	0.00	42.96	0.89
time (sec)	N/A	0.016	0.038	0.228	0.284	0.346	0.000	35.390	4.248

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	140	15	61	22	0	320	15
N.S.	1	1.00	9.33	1.00	4.07	1.47	0.00	21.33	1.00
time (sec)	N/A	0.010	0.011	0.170	0.271	0.357	0.000	7.674	4.027

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	133	17	59	22	0	322	13
N.S.	1	1.00	10.23	1.31	4.54	1.69	0.00	24.77	1.00
time (sec)	N/A	0.009	0.011	0.168	0.271	0.335	0.000	7.001	4.231

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	35	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.022	0.574	0.756	0.000	0.000	0.000	0.000	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	33	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.012	0.168	0.799	0.000	0.000	0.000	0.000	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	35	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.024	0.209	0.471	0.000	0.000	0.000	0.000	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	52	61	0	47	0	0	-1
N.S.	1	1.00	1.04	1.22	0.00	0.94	0.00	0.00	-0.02
time (sec)	N/A	0.066	0.096	8.679	0.000	0.344	0.000	0.000	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	89	117	0	122	0	0	-1
N.S.	1	1.00	0.83	1.09	0.00	1.14	0.00	0.00	-0.01
time (sec)	N/A	0.155	0.157	9.964	0.000	0.371	0.000	0.000	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	35	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.024	1.064	0.674	0.000	0.000	0.000	0.000	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	33	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.013	0.624	0.599	0.000	0.000	0.000	0.000	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	35	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.025	0.496	1.280	0.000	0.000	0.000	0.000	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	87	132	0	196	0	0	-1
N.S.	1	1.00	0.84	1.28	0.00	1.90	0.00	0.00	-0.01
time (sec)	N/A	0.089	0.126	10.084	0.000	0.375	0.000	0.000	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	212	136	258	0	542	0	0	-1
N.S.	1	1.00	0.64	1.22	0.00	2.56	0.00	0.00	-0.00
time (sec)	N/A	0.205	0.470	13.539	0.000	0.349	0.000	0.000	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	144	1958	900	452	998	496	1025
N.S.	1	1.00	0.79	10.76	4.95	2.48	5.48	2.73	5.63
time (sec)	N/A	0.075	0.066	0.335	0.326	0.433	4.437	255.273	4.990

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	122	1395	659	339	707	361	567
N.S.	1	1.00	0.81	9.24	4.36	2.25	4.68	2.39	3.75
time (sec)	N/A	0.057	0.068	0.318	0.316	0.383	2.549	77.951	4.740

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	98	928	446	241	517	252	296
N.S.	1	1.00	0.82	7.73	3.72	2.01	4.31	2.10	2.47
time (sec)	N/A	0.048	0.038	0.291	0.308	0.348	1.780	7.523	4.589

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	72	521	256	146	250	131	120
N.S.	1	1.00	0.92	6.68	3.28	1.87	3.21	1.68	1.54
time (sec)	N/A	0.031	0.027	0.236	0.335	0.359	1.014	3.198	4.389

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	88	572	0	0	0	0	-1
N.S.	1	1.00	1.06	6.89	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.142	0.028	0.507	0.000	0.000	0.000	0.000	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	77	111	145	190	108	255	188	108
N.S.	1	1.18	1.71	2.23	2.92	1.66	3.92	2.89	1.66
time (sec)	N/A	0.032	0.043	0.305	0.312	0.344	1.038	2.401	5.251

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	109	286	310	236	418	264	206
N.S.	1	1.00	0.79	2.07	2.25	1.71	3.03	1.91	1.49
time (sec)	N/A	0.070	0.091	0.371	0.290	0.371	1.390	2.888	5.140

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	140	454	483	428	677	473	341
N.S.	1	1.00	0.79	2.56	2.73	2.42	3.82	2.67	1.93
time (sec)	N/A	0.088	0.083	0.469	0.310	0.362	2.177	3.390	5.800

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	208	162	653	702	652	947	419	579
N.S.	1	1.00	0.78	3.14	3.38	3.13	4.55	2.01	2.78
time (sec)	N/A	0.109	0.130	0.542	0.322	0.363	3.064	3.492	6.511

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	377	377	523	0	2374	0	0	0	-1
N.S.	1	1.00	1.39	0.00	6.30	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.356	0.307	0.210	0.447	0.000	0.000	0.000	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	319	319	402	0	1754	0	0	0	-1
N.S.	1	1.00	1.26	0.00	5.50	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.274	0.212	0.175	0.443	0.000	0.000	0.000	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	298	0	1207	0	0	0	-1
N.S.	1	1.00	1.17	0.00	4.73	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.212	0.152	0.137	0.418	0.000	0.000	0.000	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	207	0	686	0	0	0	-1
N.S.	1	1.00	1.10	0.00	3.65	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.125	0.120	0.074	0.432	0.000	0.000	0.000	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	257	0	0	0	0	0	-1
N.S.	1	1.00	1.95	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.115	0.163	0.135	0.000	0.000	0.000	0.000	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	321	306	583	196	454	378	228
N.S.	1	1.00	2.47	2.35	4.48	1.51	3.49	2.91	1.75
time (sec)	N/A	0.062	0.290	0.463	0.333	0.352	1.417	3.977	5.971

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	272	272	451	626	1010	406	879	0	503
N.S.	1	1.00	1.66	2.30	3.71	1.49	3.23	0.00	1.85
time (sec)	N/A	0.146	0.301	0.689	0.371	0.416	2.685	0.000	5.887

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	429	429	598	1019	1584	715	1561	0	1069
N.S.	1	1.00	1.39	2.38	3.69	1.67	3.64	0.00	2.49
time (sec)	N/A	0.216	0.424	0.899	0.455	0.374	16.280	0.000	7.669

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	587	587	762	1486	2288	1080	0	874	1883
N.S.	1	1.00	1.30	2.53	3.90	1.84	0.00	1.49	3.21
time (sec)	N/A	0.270	0.592	1.174	0.565	0.415	0.000	4.724	10.546

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	37	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.023	0.091	1.041	0.000	0.000	0.000	0.000	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	35	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.013	0.075	1.046	0.000	0.000	0.000	0.000	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	37	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.024	0.051	1.032	0.000	0.000	0.000	0.000	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	94	91	0	0	0	0	0	0	-1
N.S.	1	0.97	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.079	0.055	0.626	0.000	0.000	0.000	0.000	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	152	149	0	0	0	0	0	0	-1
N.S.	1	0.98	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.173	0.058	1.182	0.000	0.000	0.000	0.000	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	37	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.022	0.251	0.613	0.000	0.000	0.000	0.000	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	35	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.012	0.205	0.516	0.000	0.000	0.000	0.000	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	37	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.024	0.102	0.477	0.000	0.000	0.000	0.000	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	150	147	0	0	0	0	0	0	-1
N.S.	1	0.98	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.102	0.129	0.627	0.000	0.000	0.000	0.000	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	266	263	0	0	0	0	0	0	-1
N.S.	1	0.99	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.214	0.221	1.141	0.000	0.000	0.000	0.000	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	338	2372	681	503	0	497	936
N.S.	1	1.00	1.98	13.87	3.98	2.94	0.00	2.91	5.47
time (sec)	N/A	0.064	0.250	0.455	0.318	0.367	0.000	39.240	4.564

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	250	1838	475	370	0	355	520
N.S.	1	1.00	1.76	12.94	3.35	2.61	0.00	2.50	3.66
time (sec)	N/A	0.046	0.158	0.378	0.332	0.350	0.000	8.902	4.486

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	171	1323	300	248	0	235	262
N.S.	1	1.00	1.51	11.71	2.65	2.19	0.00	2.08	2.32
time (sec)	N/A	0.038	0.109	0.328	0.330	0.354	0.000	4.556	4.239

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	100	817	158	143	0	127	127
N.S.	1	1.00	1.19	9.73	1.88	1.70	0.00	1.51	1.51
time (sec)	N/A	0.027	0.065	0.202	0.295	0.350	0.000	3.976	4.283

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	95	523	0	0	0	0	-1
N.S.	1	1.00	1.20	6.62	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.139	0.061	0.504	0.000	0.000	0.000	0.000	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	89	823	119	98	0	108	97
N.S.	1	1.00	0.92	8.48	1.23	1.01	0.00	1.11	1.00
time (sec)	N/A	0.046	0.068	0.253	0.286	0.364	0.000	3.638	4.909

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	132	1379	235	274	0	239	192
N.S.	1	1.00	0.96	10.07	1.72	2.00	0.00	1.74	1.40
time (sec)	N/A	0.054	0.095	0.323	0.313	0.354	0.000	5.138	4.663

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	167	1976	404	504	0	448	317
N.S.	1	1.00	1.01	11.90	2.43	3.04	0.00	2.70	1.91
time (sec)	N/A	0.069	0.141	0.328	0.309	0.367	0.000	4.209	4.912

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	160	2583	621	772	0	710	555
N.S.	1	1.00	0.82	13.25	3.18	3.96	0.00	3.64	2.85
time (sec)	N/A	0.082	0.195	0.390	0.363	0.368	0.000	4.647	5.334

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	B	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	322	322	1709	26938	1809	0	0	0	-1
N.S.	1	1.00	5.31	83.66	5.62	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.293	0.697	2.598	0.821	0.000	0.000	0.000	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	B	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	263	263	1149	19970	1244	0	0	0	-1
N.S.	1	1.00	4.37	75.93	4.73	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.231	0.479	2.094	0.881	0.000	0.000	0.000	0.000

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	B	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	656	10210	753	0	0	0	-1
N.S.	1	1.00	3.36	52.36	3.86	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.137	0.348	1.366	0.808	0.000	0.000	0.000	0.000

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	443	0	0	0	0	0	-1
N.S.	1	1.00	3.38	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.124	0.134	0.075	0.000	0.000	0.000	0.000	0.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	236	10098	442	296	0	0	200
N.S.	1	1.00	1.83	78.28	3.43	2.29	0.00	0.00	1.55
time (sec)	N/A	0.079	0.201	1.334	0.309	0.381	0.000	0.000	5.269

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	274	274	332	17300	885	774	0	0	444
N.S.	1	1.00	1.21	63.14	3.23	2.82	0.00	0.00	1.62
time (sec)	N/A	0.166	0.286	2.179	0.345	0.372	0.000	0.000	5.316

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	427	427	432	25057	1469	1371	0	0	911
N.S.	1	1.00	1.01	58.68	3.44	3.21	0.00	0.00	2.13
time (sec)	N/A	0.240	0.404	3.634	0.415	0.406	0.000	0.000	6.842

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	587	587	1011	33370	2187	2062	0	0	1579
N.S.	1	1.00	1.72	56.85	3.73	3.51	0.00	0.00	2.69
time (sec)	N/A	0.306	0.531	4.320	0.516	0.400	0.000	0.000	9.607

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	809	809	9054	0	0	0	0	0	-1
N.S.	1	1.00	11.19	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.745	5.483	0.133	0.000	0.000	0.000	0.000	0.000

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	614	614	5668	0	0	0	0	0	-1
N.S.	1	1.00	9.23	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.500	1.926	0.108	0.000	0.000	0.000	0.000	0.000

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	376	376	3813	0	0	0	0	0	-1
N.S.	1	1.00	10.14	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.264	1.392	0.066	0.000	0.000	0.000	0.000	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	2513	0	0	0	0	0	-1
N.S.	1	1.00	13.51	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.155	0.442	0.088	0.000	0.000	0.000	0.000	0.000

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	B	B	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	524	69354	1092	673	0	0	474
N.S.	1	1.00	2.85	376.92	5.93	3.66	0.00	0.00	2.58
time (sec)	N/A	0.105	0.467	14.076	0.365	0.418	0.000	0.000	6.060

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	390	390	693	120138	2176	1725	0	0	966
N.S.	1	1.00	1.78	308.05	5.58	4.42	0.00	0.00	2.48
time (sec)	N/A	0.222	0.692	21.279	0.460	0.431	0.000	0.000	8.988

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	611	611	1003	175812	3508	3044	0	0	2069
N.S.	1	1.00	1.64	287.74	5.74	4.98	0.00	0.00	3.39
time (sec)	N/A	0.315	0.893	30.044	0.625	0.518	0.000	0.000	10.731

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	830	830	1370	236754	5098	4585	0	0	2500
N.S.	1	1.00	1.65	285.25	6.14	5.52	0.00	0.00	3.01
time (sec)	N/A	0.392	1.309	38.204	0.849	0.783	0.000	0.000	11.290

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	A	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	0	0	0	56	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.58	0.00	0.00	-0.01
time (sec)	N/A	0.102	0.061	0.068	0.000	0.376	0.000	0.000	0.000

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	142	4273	629	432	969	5960	1008
N.S.	1	1.00	0.79	23.74	3.49	2.40	5.38	33.11	5.60
time (sec)	N/A	0.085	0.071	0.421	0.294	0.426	4.376	3.409	4.863

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	120	2730	444	319	706	4137	566
N.S.	1	1.00	0.81	18.32	2.98	2.14	4.74	27.77	3.80
time (sec)	N/A	0.068	0.065	0.445	0.293	0.413	2.532	4.572	4.692

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	99	1535	284	222	491	2640	290
N.S.	1	1.00	0.84	13.01	2.41	1.88	4.16	22.37	2.46
time (sec)	N/A	0.053	0.038	0.400	0.287	0.355	1.668	4.408	4.572

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	69	681	147	126	253	1395	126
N.S.	1	1.00	0.85	8.41	1.81	1.56	3.12	17.22	1.56
time (sec)	N/A	0.035	0.028	0.357	0.283	0.388	1.066	4.284	4.310

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	95	228	0	0	0	1368	-1
N.S.	1	1.00	1.17	2.81	0.00	0.00	0.00	16.89	-0.01
time (sec)	N/A	0.141	0.031	0.857	0.000	0.000	0.000	43.159	0.000

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	99	86	171	136	86	231	126	106
N.S.	1	1.55	1.34	2.67	2.12	1.34	3.61	1.97	1.66
time (sec)	N/A	0.028	0.041	0.361	0.292	0.336	1.022	3.086	5.006

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	128	341	257	220	422	254	208
N.S.	1	1.00	0.89	2.37	1.78	1.53	2.93	1.76	1.44
time (sec)	N/A	0.076	0.067	0.394	0.279	0.381	1.581	2.542	5.195

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	141	514	430	411	656	382	339
N.S.	1	1.00	0.81	2.94	2.46	2.35	3.75	2.18	1.94
time (sec)	N/A	0.095	0.099	0.411	0.300	0.340	2.012	2.931	5.875

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	206	166	688	649	636	944	511	578
N.S.	1	1.00	0.81	3.34	3.15	3.09	4.58	2.48	2.81
time (sec)	N/A	0.120	0.137	0.534	0.330	0.437	3.049	3.680	6.623

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	503	503	512	0	2101	0	0	0	-1
N.S.	1	1.00	1.02	0.00	4.18	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.462	0.334	0.240	0.395	0.000	0.000	0.000	0.000

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	420	420	392	0	1529	0	0	0	-1
N.S.	1	1.00	0.93	0.00	3.64	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.337	0.232	0.219	0.386	0.000	0.000	0.000	0.000

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	335	335	290	0	1027	0	0	0	-1
N.S.	1	1.00	0.87	0.00	3.07	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.244	0.162	0.161	0.380	0.000	0.000	0.000	0.000

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	203	0	534	0	0	0	-1
N.S.	1	1.00	1.00	0.00	2.64	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.147	0.126	0.097	0.358	0.000	0.000	0.000	0.000

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	251	474	0	0	0	0	-1
N.S.	1	1.00	1.96	3.70	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.078	0.127	0.888	0.000	0.000	0.000	0.000	0.000

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	314	346	422	152	430	188	223
N.S.	1	1.00	2.05	2.26	2.76	0.99	2.81	1.23	1.46
time (sec)	N/A	0.050	0.323	0.393	0.291	0.342	1.340	4.804	6.379

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	296	296	444	701	853	371	892	493	507
N.S.	1	1.00	1.50	2.37	2.88	1.25	3.01	1.67	1.71
time (sec)	N/A	0.109	0.296	0.387	0.335	0.359	2.913	4.606	6.002

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	399	399	585	1061	1426	678	1544	760	1064
N.S.	1	1.00	1.47	2.66	3.57	1.70	3.87	1.90	2.67
time (sec)	N/A	0.171	0.449	0.491	0.458	0.456	18.407	3.119	7.703

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	498	498	748	1422	2128	1043	0	1029	1880
N.S.	1	1.00	1.50	2.86	4.27	2.09	0.00	2.07	3.78
time (sec)	N/A	0.196	0.599	0.547	0.532	0.438	0.000	3.131	10.940

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	35	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.022	0.592	1.412	0.000	0.000	0.000	0.000	0.000

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	33	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.013	0.166	0.687	0.000	0.000	0.000	0.000	0.000

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	35	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.024	0.293	0.991	0.000	0.000	0.000	0.000	0.000

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	50	69	0	48	0	0	-1
N.S.	1	1.00	0.94	1.30	0.00	0.91	0.00	0.00	-0.02
time (sec)	N/A	0.062	0.071	4.179	0.000	0.372	0.000	0.000	0.000

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	89	126	0	120	0	0	-1
N.S.	1	1.00	0.82	1.16	0.00	1.10	0.00	0.00	-0.01
time (sec)	N/A	0.127	0.147	6.166	0.000	0.403	0.000	0.000	0.000

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	35	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.022	1.131	1.405	0.000	0.000	0.000	0.000	0.000

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	33	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.012	0.671	1.090	0.000	0.000	0.000	0.000	0.000

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	35	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.024	0.396	0.607	0.000	0.000	0.000	0.000	0.000

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	88	138	0	205	0	152	-1
N.S.	1	1.00	0.85	1.33	0.00	1.97	0.00	1.46	-0.01
time (sec)	N/A	0.069	0.107	2.720	0.000	0.353	0.000	4.462	0.000

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	135	268	0	550	0	317	-1
N.S.	1	1.00	0.85	1.69	0.00	3.46	0.00	1.99	-0.01
time (sec)	N/A	0.159	0.307	3.955	0.000	0.356	0.000	5.145	0.000

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	144	718	897	455	998	0	1024
N.S.	1	1.00	0.79	3.95	4.93	2.50	5.48	0.00	5.63
time (sec)	N/A	0.081	0.071	0.370	0.327	0.401	5.312	0.000	4.789

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	122	543	657	341	707	364	567
N.S.	1	1.00	0.81	3.60	4.35	2.26	4.68	2.41	3.75
time (sec)	N/A	0.060	0.060	0.356	0.311	0.397	3.109	68.087	4.853

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	98	393	445	243	517	248	296
N.S.	1	1.00	0.82	3.28	3.71	2.02	4.31	2.07	2.47
time (sec)	N/A	0.052	0.038	0.382	0.307	0.383	1.918	13.760	4.646

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	72	245	256	147	250	128	120
N.S.	1	1.00	0.92	3.14	3.28	1.88	3.21	1.64	1.54
time (sec)	N/A	0.033	0.028	0.266	0.290	0.462	1.439	7.089	4.380

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	87	258	0	0	0	0	-1
N.S.	1	1.00	1.05	3.11	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.148	0.028	0.390	0.000	0.000	0.000	0.000	0.000

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	89	205	190	108	253	188	108
N.S.	1	1.00	0.87	2.01	1.86	1.06	2.48	1.84	1.06
time (sec)	N/A	0.029	0.041	0.359	0.295	0.369	0.767	4.277	5.943

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	128	294	309	238	418	259	206
N.S.	1	1.00	0.92	2.12	2.22	1.71	3.01	1.86	1.48
time (sec)	N/A	0.072	0.064	0.396	0.317	0.354	1.599	2.661	5.928

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	140	422	483	430	677	473	341
N.S.	1	1.00	0.79	2.38	2.73	2.43	3.82	2.67	1.93
time (sec)	N/A	0.090	0.075	0.427	0.323	0.381	3.295	2.665	6.733

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	208	162	582	702	656	947	416	579
N.S.	1	1.00	0.78	2.80	3.38	3.15	4.55	2.00	2.78
time (sec)	N/A	0.110	0.121	0.492	0.330	0.356	3.378	3.800	7.897

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	515	515	524	0	2371	0	0	0	-1
N.S.	1	1.00	1.02	0.00	4.60	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.463	0.324	0.167	0.450	0.000	0.000	0.000	0.000

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	422	422	402	0	1749	0	0	0	-1
N.S.	1	1.00	0.95	0.00	4.14	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.333	0.224	0.186	0.440	0.000	0.000	0.000	0.000

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	343	343	298	0	1190	0	0	0	-1
N.S.	1	1.00	0.87	0.00	3.47	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.246	0.156	0.160	0.432	0.000	0.000	0.000	0.000

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	195	0	656	0	0	0	-1
N.S.	1	1.00	0.92	0.00	3.11	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.157	0.126	0.076	0.397	0.000	0.000	0.000	0.000

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	257	0	0	0	0	0	-1
N.S.	1	1.00	1.95	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.082	0.163	0.110	0.000	0.000	0.000	0.000	0.000

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	322	431	582	196	450	374	227
N.S.	1	1.00	2.05	2.75	3.71	1.25	2.87	2.38	1.45
time (sec)	N/A	0.051	0.319	0.418	0.327	0.390	2.752	3.147	6.489

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	299	299	452	639	1010	409	877	0	504
N.S.	1	1.00	1.51	2.14	3.38	1.37	2.93	0.00	1.69
time (sec)	N/A	0.106	0.310	0.565	0.370	0.340	5.220	0.000	6.707

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	407	407	598	915	1585	717	1561	0	1069
N.S.	1	1.00	1.47	2.25	3.89	1.76	3.84	0.00	2.63
time (sec)	N/A	0.159	0.432	0.669	0.456	0.372	28.992	0.000	9.083

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	501	501	762	1245	2287	1084	0	868	1882
N.S.	1	1.00	1.52	2.49	4.56	2.16	0.00	1.73	3.76
time (sec)	N/A	0.195	0.577	0.895	0.559	0.361	0.000	6.030	12.108

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	37	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.023	0.090	0.651	0.000	0.000	0.000	0.000	0.000

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	35	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.013	0.073	0.560	0.000	0.000	0.000	0.000	0.000

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	37	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.025	0.052	0.521	0.000	0.000	0.000	0.000	0.000

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.067	0.057	0.591	0.000	0.000	0.000	0.000	0.000

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.135	0.061	0.650	0.000	0.000	0.000	0.000	0.000

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	37	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.022	0.258	0.601	0.000	0.000	0.000	0.000	0.000

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	35	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.012	0.208	0.650	0.000	0.000	0.000	0.000	0.000

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	37	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.024	0.105	0.531	0.000	0.000	0.000	0.000	0.000

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.075	0.131	0.550	0.000	0.000	0.000	0.000	0.000

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	206	206	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.175	0.221	0.664	0.000	0.000	0.000	0.000	0.000

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	A	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	0	0	0	56	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.58	0.00	0.00	-0.01
time (sec)	N/A	0.102	0.046	0.013	0.000	0.357	0.000	0.000	0.000

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	355	355	279	14848	603	635	1436	19084	1392
N.S.	1	1.00	0.79	41.83	1.70	1.79	4.05	53.76	3.92
time (sec)	N/A	0.338	0.401	0.457	0.303	0.778	89.010	4.902	5.342

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	227	215	7244	423	444	998	11299	741
N.S.	1	1.00	0.95	31.91	1.86	1.96	4.40	49.78	3.26
time (sec)	N/A	0.206	0.182	0.421	0.288	0.479	15.116	5.797	4.689

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	142	2998	268	279	658	5950	356
N.S.	1	1.00	0.95	19.99	1.79	1.86	4.39	39.67	2.37
time (sec)	N/A	0.103	0.093	0.393	0.277	0.390	8.035	5.432	4.730

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	114	964	144	149	318	2355	144
N.S.	1	1.00	1.05	8.84	1.32	1.37	2.92	21.61	1.32
time (sec)	N/A	0.060	0.076	0.375	0.300	0.358	3.378	3.548	4.241

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	52	418	56	55	83	427	47
N.S.	1	1.00	1.00	8.04	1.08	1.06	1.60	8.21	0.90
time (sec)	N/A	0.019	0.007	0.369	0.284	0.352	0.868	3.265	4.118

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	115	854	0	0	0	0	-1
N.S.	1	1.00	0.82	6.10	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.090	0.042	2.148	0.000	0.000	0.000	0.000	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	105	302	140	254	0	1537	166
N.S.	1	1.00	1.21	3.47	1.61	2.92	0.00	17.67	1.91
time (sec)	N/A	0.056	0.086	0.429	0.280	3.511	0.000	5.060	5.169

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	169	2442	353	1016	0	7600	417
N.S.	1	1.00	0.92	13.34	1.93	5.55	0.00	41.53	2.28
time (sec)	N/A	0.142	0.343	0.505	0.311	44.829	0.000	5.492	7.208

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	275	275	260	10411	850	0	0	21182	1154
N.S.	1	1.00	0.95	37.86	3.09	0.00	0.00	77.03	4.20
time (sec)	N/A	0.252	0.476	0.551	0.361	0.000	0.000	6.141	10.670

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	379	379	355	29346	1759	0	0	44231	2518
N.S.	1	1.00	0.94	77.43	4.64	0.00	0.00	116.70	6.64
time (sec)	N/A	0.405	0.615	0.881	0.519	0.000	0.000	4.713	16.223

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	874	874	733	0	1973	0	0	0	-1
N.S.	1	1.00	0.84	0.00	2.26	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.102	0.639	0.350	0.405	0.000	0.000	0.000	0.000

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	532	532	486	0	1209	0	0	0	-1
N.S.	1	1.00	0.91	0.00	2.27	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.715	0.351	0.236	0.365	0.000	0.000	0.000	0.000

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	270	270	346	0	629	0	0	0	-1
N.S.	1	1.00	1.28	0.00	2.33	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.356	0.200	0.126	0.357	0.000	0.000	0.000	0.000

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	214	0	0	0	0	0	-1
N.S.	1	1.00	1.71	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.162	0.116	0.052	0.000	0.000	0.000	0.000	0.000

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	277	277	431	1395	0	0	0	0	-1
N.S.	1	1.00	1.56	5.04	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.335	0.476	2.149	0.000	0.000	0.000	0.000	0.000

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	402	0	0	0	0	0	-1
N.S.	1	1.00	2.05	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.118	0.368	0.193	0.000	0.000	0.000	0.000	0.000

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	369	369	595	0	0	0	0	0	-1
N.S.	1	1.00	1.61	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.516	0.948	0.449	0.000	0.000	0.000	0.000	0.000

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	714	714	894	0	0	0	0	0	-1
N.S.	1	1.00	1.25	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.065	1.979	0.809	0.000	0.000	0.000	0.000	0.000

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1159	1159	1301	0	0	0	0	0	-1
N.S.	1	1.00	1.12	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.667	4.823	1.446	0.000	0.000	0.000	0.000	0.000

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	30	46	32	29	20	103	28
N.S.	1	1.00	0.86	1.31	0.91	0.83	0.57	2.94	0.80
time (sec)	N/A	0.019	0.005	0.151	0.274	0.354	0.053	5.508	0.186

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	32	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.021	0.277	0.921	0.000	0.000	0.000	0.000	0.000

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.012	0.173	0.681	0.000	0.000	0.000	0.000	0.000

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.004	0.018	0.628	0.000	0.000	0.000	0.000	0.000

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	32	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.021	0.523	0.723	0.000	0.000	0.000	0.000	0.000

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	32	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.020	1.403	0.681	0.000	0.000	0.000	0.000	0.000

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	32	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.020	30.382	1.255	0.000	0.000	0.000	0.000	0.000

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	32	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.019	0.782	1.072	0.000	0.000	0.000	0.000	0.000

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.011	0.575	0.811	0.000	0.000	0.000	0.000	0.000

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.004	0.338	0.680	0.000	0.000	0.000	0.000	0.000

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	32	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.020	1.524	0.735	0.000	0.000	0.000	0.000	0.000

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	32	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.019	2.045	0.769	0.000	0.000	0.000	0.000	0.000

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	32	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.019	112.176	1.586	0.000	0.000	0.000	0.000	0.000

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	357	357	282	2907	870	658	1477	0	1403
N.S.	1	1.00	0.79	8.14	2.44	1.84	4.14	0.00	3.93
time (sec)	N/A	0.322	0.393	0.488	0.324	0.750	58.216	0.000	5.333

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	229	217	1966	635	466	998	0	743
N.S.	1	1.00	0.95	8.59	2.77	2.03	4.36	0.00	3.24
time (sec)	N/A	0.202	0.177	0.447	0.306	0.481	9.136	0.000	5.032

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	142	1210	428	299	692	279	362
N.S.	1	1.00	0.93	7.96	2.82	1.97	4.55	1.84	2.38
time (sec)	N/A	0.104	0.098	0.375	0.285	0.403	3.885	30.379	4.786

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	118	621	252	172	314	145	133
N.S.	1	1.00	1.13	5.97	2.42	1.65	3.02	1.39	1.28
time (sec)	N/A	0.054	0.076	0.363	0.286	0.361	1.609	6.866	4.500

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	233	59	78	104	83	50
N.S.	1	1.00	1.00	4.31	1.09	1.44	1.93	1.54	0.93
time (sec)	N/A	0.020	0.019	0.296	0.270	0.341	0.506	3.747	4.289

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	119	1169	0	0	0	0	-1
N.S.	1	1.00	0.83	8.12	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.091	0.041	3.534	0.000	0.000	0.000	0.000	0.000

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	108	299	195	277	0	0	191
N.S.	1	1.00	1.20	3.32	2.17	3.08	0.00	0.00	2.12
time (sec)	N/A	0.058	0.085	0.421	0.281	3.186	0.000	0.000	5.339

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	172	1024	408	1034	0	495	412
N.S.	1	1.00	0.98	5.85	2.33	5.91	0.00	2.83	2.35
time (sec)	N/A	0.133	0.345	0.687	0.309	44.968	0.000	2.538	7.448

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	277	277	263	2563	903	0	0	1391	1147
N.S.	1	1.00	0.95	9.25	3.26	0.00	0.00	5.02	4.14
time (sec)	N/A	0.248	0.468	0.882	0.358	0.000	0.000	2.354	11.577

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	F(-1)	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	381	381	358	5183	1812	0	0	0	2520
N.S.	1	1.00	0.94	13.60	4.76	0.00	0.00	0.00	6.61
time (sec)	N/A	0.401	0.605	1.243	0.460	0.000	0.000	0.000	17.435

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	869	869	746	0	2188	0	0	0	-1
N.S.	1	1.00	0.86	0.00	2.52	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.095	0.633	0.221	0.423	0.000	0.000	0.000	0.000

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	542	542	497	0	1372	0	0	0	-1
N.S.	1	1.00	0.92	0.00	2.53	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.718	0.353	0.188	0.403	0.000	0.000	0.000	0.000

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	281	281	351	0	753	0	0	0	-1
N.S.	1	1.00	1.25	0.00	2.68	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.366	0.194	0.088	0.386	0.000	0.000	0.000	0.000

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	220	0	0	0	0	0	-1
N.S.	1	1.00	1.71	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.161	0.118	0.057	0.000	0.000	0.000	0.000	0.000

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	285	285	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.341	1.276	0.131	0.000	0.000	0.000	0.000	0.000

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	409	0	0	0	0	0	-1
N.S.	1	1.00	2.04	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.122	0.372	0.204	0.000	0.000	0.000	0.000	0.000

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	381	381	603	0	0	0	0	0	-1
N.S.	1	1.00	1.58	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.533	0.926	0.413	0.000	0.000	0.000	0.000	0.000

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	724	724	909	0	0	0	0	0	-1
N.S.	1	1.00	1.26	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.076	1.958	0.670	0.000	0.000	0.000	0.000	0.000

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1154	1154	1317	0	0	0	0	0	-1
N.S.	1	1.00	1.14	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.664	4.824	1.062	0.000	0.000	0.000	0.000	0.000

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	34	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.027	0.099	0.971	0.000	0.000	0.000	0.000	0.000

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	32	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.015	0.082	0.831	0.000	0.000	0.000	0.000	0.000

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.004	0.027	0.724	0.000	0.000	0.000	0.000	0.000

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	34	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.027	0.066	0.744	0.000	0.000	0.000	0.000	0.000

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	34	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.021	0.063	0.882	0.000	0.000	0.000	0.000	0.000

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	34	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.020	0.067	0.860	0.000	0.000	0.000	0.000	0.000

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	34	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.020	0.352	0.876	0.000	0.000	0.000	0.000	0.000

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	32	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.011	0.246	0.779	0.000	0.000	0.000	0.000	0.000

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.004	0.141	0.626	0.000	0.000	0.000	0.000	0.000

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	34	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.021	0.291	0.731	0.000	0.000	0.000	0.000	0.000

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	34	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.019	0.344	0.758	0.000	0.000	0.000	0.000	0.000

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	34	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.019	0.349	0.880	0.000	0.000	0.000	0.000	0.000

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F(-2)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	365	365	452	2612	681	733	0	0	1434
N.S.	1	1.00	1.24	7.16	1.87	2.01	0.00	0.00	3.93
time (sec)	N/A	0.358	0.328	0.642	0.309	0.374	0.000	0.000	5.132

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	F(-2)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	236	236	303	2000	475	515	0	0	767
N.S.	1	1.00	1.28	8.47	2.01	2.18	0.00	0.00	3.25
time (sec)	N/A	0.221	0.218	0.581	0.286	0.350	0.000	0.000	4.761

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	F(-2)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	195	1425	300	325	0	0	372
N.S.	1	1.00	1.23	9.02	1.90	2.06	0.00	0.00	2.35
time (sec)	N/A	0.112	0.139	0.379	0.292	0.375	0.000	0.000	4.471

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	112	839	158	169	0	149	154
N.S.	1	1.00	0.97	7.23	1.36	1.46	0.00	1.28	1.33
time (sec)	N/A	0.065	0.089	0.299	0.272	0.385	0.000	11.995	4.393

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	57	123	61	54	0	55	53
N.S.	1	1.00	1.00	2.16	1.07	0.95	0.00	0.96	0.93
time (sec)	N/A	0.022	0.011	0.178	0.271	0.393	0.000	2.029	4.111

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	150	597	0	0	0	0	-1
N.S.	1	1.00	1.01	4.03	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.100	0.059	0.284	0.000	0.000	0.000	0.000	0.000

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	155	1796	153	227	0	166	141
N.S.	1	1.00	1.29	14.97	1.28	1.89	0.00	1.38	1.18
time (sec)	N/A	0.081	0.156	0.373	0.270	3.328	0.000	4.624	4.717

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	191	238	4925	382	1046	0	523	431
N.S.	1	1.00	1.25	25.79	2.00	5.48	0.00	2.74	2.26
time (sec)	N/A	0.148	0.250	0.705	0.299	45.457	0.000	2.718	6.349

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	284	284	330	9645	914	0	0	1512	1183
N.S.	1	1.00	1.16	33.96	3.22	0.00	0.00	5.32	4.17
time (sec)	N/A	0.293	0.423	0.741	0.375	0.000	0.000	3.254	9.259

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	389	389	455	16077	1888	0	0	3293	2570
N.S.	1	1.00	1.17	41.33	4.85	0.00	0.00	8.47	6.61
time (sec)	N/A	0.459	0.406	1.104	0.509	0.000	0.000	7.231	14.282

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	570	570	906	23167	1606	0	0	0	-1
N.S.	1	1.00	1.59	40.64	2.82	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.787	0.972	4.208	0.751	0.000	0.000	0.000	0.000

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	294	294	472	11007	874	0	0	0	-1
N.S.	1	1.00	1.61	37.44	2.97	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.389	0.497	2.192	0.794	0.000	0.000	0.000	0.000

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	327	4749	0	0	0	0	-1
N.S.	1	1.00	2.39	34.66	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.171	0.149	1.104	0.000	0.000	0.000	0.000	0.000

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	301	301	1082	0	0	0	0	0	-1
N.S.	1	1.00	3.59	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.377	0.236	0.112	0.000	0.000	0.000	0.000	0.000

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	208	3460	0	0	0	0	0	-1
N.S.	1	1.00	16.63	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.141	0.695	0.103	0.000	0.000	0.000	0.000	0.000

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	393	393	15406	0	0	0	0	0	-1
N.S.	1	1.00	39.20	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.567	6.171	0.104	0.000	0.000	0.000	0.000	0.000

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	875	875	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.117	2.989	0.166	0.000	0.000	0.000	0.000	0.000

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	466	466	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.543	1.605	0.061	0.000	0.000	0.000	0.000	0.000

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	372	0	0	0	0	0	-1
N.S.	1	1.00	1.83	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.116	0.333	0.025	0.000	0.000	0.000	0.000	0.000

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-2)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	425	425	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.462	0.814	0.104	0.000	0.000	0.000	0.000	0.000

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	302	302	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.189	1.824	0.116	0.000	0.000	0.000	0.000	0.000

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	629	629	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.771	3.618	0.105	0.000	0.000	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [172] had the largest ratio of [36]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	4	3	1.00	33	0.091
2	A	4	3	1.00	33	0.091
3	A	4	3	1.00	33	0.091
4	A	4	3	1.00	31	0.097
5	A	5	5	1.00	33	0.152
6	A	2	2	1.18	33	0.061
7	A	4	3	1.00	33	0.091
8	A	4	3	1.00	33	0.091
9	A	4	3	1.00	33	0.091
10	A	8	5	1.00	35	0.143
11	A	7	5	1.00	35	0.143
12	A	6	5	1.00	35	0.143
13	A	5	5	1.00	33	0.152
14	A	4	4	1.00	35	0.114
15	A	3	3	1.00	35	0.086
16	A	7	4	1.00	35	0.114
17	A	9	4	1.00	35	0.114
18	A	11	4	1.00	35	0.114
19	A	0	0	0.00	0	0.000
20	A	0	0	0.00	0	0.000
21	A	0	0	0.00	0	0.000
22	A	3	3	1.00	35	0.086
23	A	7	4	1.00	35	0.114
24	A	0	0	0.00	0	0.000
25	A	0	0	0.00	0	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
26	A	0	0	0.00	0	0.000
27	A	4	4	1.00	35	0.114
28	A	9	5	1.00	35	0.143
29	A	4	3	1.00	33	0.091
30	A	4	3	1.00	33	0.091
31	A	4	3	1.00	33	0.091
32	A	4	3	1.00	31	0.097
33	A	5	5	1.00	33	0.152
34	A	3	2	1.00	33	0.061
35	A	4	3	1.00	33	0.091
36	A	4	3	1.00	33	0.091
37	A	4	3	1.00	33	0.091
38	A	19	8	1.00	35	0.229
39	A	15	8	1.00	35	0.229
40	A	11	8	1.00	35	0.229
41	A	7	7	1.00	33	0.212
42	A	4	4	1.00	35	0.114
43	A	4	3	1.00	35	0.086
44	A	8	6	1.00	35	0.171
45	A	6	5	1.00	35	0.143
46	A	5	5	1.00	35	0.143
47	A	0	0	0.00	0	0.000
48	A	0	0	0.00	0	0.000
49	A	0	0	0.00	0	0.000
50	A	3	3	1.00	35	0.086
51	A	7	5	1.00	35	0.143
52	A	0	0	0.00	0	0.000
53	A	0	0	0.00	0	0.000
54	A	0	0	0.00	0	0.000
55	A	4	4	1.00	35	0.114
56	A	10	6	1.00	35	0.171
57	A	3	2	1.00	30	0.067
58	A	3	2	1.00	30	0.067
59	A	3	2	1.00	30	0.067
60	A	3	2	1.00	28	0.071

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
61	A	3	2	1.00	22	0.091
62	A	7	4	1.00	30	0.133
63	A	3	3	1.00	30	0.100
64	A	3	2	1.00	30	0.067
65	A	3	2	1.00	30	0.067
66	A	3	2	1.00	30	0.067
67	A	15	10	1.00	32	0.312
68	A	12	10	1.00	32	0.312
69	A	9	8	1.00	30	0.267
70	A	6	6	1.00	24	0.250
71	A	9	5	1.00	32	0.156
72	A	4	4	1.00	32	0.125
73	A	9	8	1.00	32	0.250
74	A	12	10	1.00	32	0.312
75	A	15	10	1.00	32	0.312
76	A	0	0	0.00	0	0.000
77	A	0	0	0.00	0	0.000
78	A	0	0	0.00	0	0.000
79	A	0	0	0.00	0	0.000
80	A	0	0	0.00	0	0.000
81	A	0	0	0.00	0	0.000
82	A	0	0	0.00	0	0.000
83	A	0	0	0.00	0	0.000
84	A	0	0	0.00	0	0.000
85	A	0	0	0.00	0	0.000
86	A	0	0	0.00	0	0.000
87	A	0	0	0.00	0	0.000
88	A	4	3	1.00	30	0.100
89	A	4	3	1.00	30	0.100
90	A	4	3	1.00	30	0.100
91	A	4	3	1.00	28	0.107
92	A	5	5	1.00	30	0.167
93	A	2	2	1.19	30	0.067
94	A	4	3	1.00	30	0.100
95	A	4	3	1.00	30	0.100

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
96	A	4	3	1.00	30	0.100
97	A	8	5	1.00	32	0.156
98	A	7	5	1.00	32	0.156
99	A	6	5	1.00	32	0.156
100	A	5	5	1.00	30	0.167
101	A	4	4	1.00	32	0.125
102	A	3	3	1.00	32	0.094
103	A	7	4	1.00	32	0.125
104	A	9	4	1.00	32	0.125
105	A	11	4	1.00	32	0.125
106	A	1	1	1.04	29	0.034
107	A	1	1	1.00	18	0.056
108	A	1	1	1.00	20	0.050
109	A	0	0	0.00	0	0.000
110	A	0	0	0.00	0	0.000
111	A	0	0	0.00	0	0.000
112	A	3	3	1.00	32	0.094
113	A	7	4	1.00	32	0.125
114	A	0	0	0.00	0	0.000
115	A	0	0	0.00	0	0.000
116	A	0	0	0.00	0	0.000
117	A	4	4	1.00	32	0.125
118	A	9	5	1.00	32	0.156
119	A	4	3	1.00	32	0.094
120	A	4	3	1.00	32	0.094
121	A	4	3	1.00	32	0.094
122	A	4	3	1.00	30	0.100
123	A	5	5	1.00	32	0.156
124	A	2	2	1.18	32	0.062
125	A	4	3	1.00	32	0.094
126	A	4	3	1.00	32	0.094
127	A	4	3	1.00	32	0.094
128	A	8	5	1.00	34	0.147
129	A	7	5	1.00	34	0.147
130	A	6	5	1.00	34	0.147

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
131	A	5	5	1.00	32	0.156
132	A	4	4	1.00	34	0.118
133	A	3	3	1.00	34	0.088
134	A	7	4	1.00	34	0.118
135	A	9	4	1.00	34	0.118
136	A	11	4	1.00	34	0.118
137	A	0	0	0.00	0	0.000
138	A	0	0	0.00	0	0.000
139	A	0	0	0.00	0	0.000
140	A	3	3	0.97	34	0.088
141	A	7	4	0.98	34	0.118
142	A	0	0	0.00	0	0.000
143	A	0	0	0.00	0	0.000
144	A	0	0	0.00	0	0.000
145	A	4	4	0.98	34	0.118
146	A	9	5	0.99	34	0.147
147	A	3	2	1.00	31	0.065
148	A	3	2	1.00	31	0.065
149	A	3	2	1.00	31	0.065
150	A	3	2	1.00	29	0.069
151	A	5	5	1.00	31	0.161
152	A	3	2	1.00	31	0.065
153	A	3	2	1.00	31	0.065
154	A	3	2	1.00	31	0.065
155	A	3	2	1.00	31	0.065
156	A	8	6	1.00	33	0.182
157	A	7	6	1.00	33	0.182
158	A	6	6	1.00	31	0.194
159	A	5	5	1.00	33	0.152
160	A	4	4	1.00	33	0.121
161	A	8	5	1.00	33	0.152
162	A	10	5	1.00	33	0.152
163	A	12	5	1.00	33	0.152
164	A	27	15	1.00	33	0.454
165	A	17	14	1.00	33	0.424

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
166	A	11	9	1.00	31	0.290
167	A	6	6	1.00	33	0.182
168	A	5	4	1.00	33	0.121
169	A	10	5	1.00	33	0.152
170	A	13	5	1.00	33	0.152
171	A	16	5	1.00	33	0.152
172	A	4	4	1.00	36	0.111
173	A	4	3	1.00	30	0.100
174	A	4	3	1.00	30	0.100
175	A	4	3	1.00	30	0.100
176	A	4	3	1.00	28	0.107
177	A	5	5	1.00	30	0.167
178	A	3	2	1.55	30	0.067
179	A	4	3	1.00	30	0.100
180	A	4	3	1.00	30	0.100
181	A	4	3	1.00	30	0.100
182	A	19	8	1.00	32	0.250
183	A	15	8	1.00	32	0.250
184	A	11	8	1.00	32	0.250
185	A	7	7	1.00	30	0.233
186	A	4	4	1.00	32	0.125
187	A	4	3	1.00	32	0.094
188	A	8	6	1.00	32	0.188
189	A	6	5	1.00	32	0.156
190	A	5	5	1.00	32	0.156
191	A	0	0	0.00	0	0.000
192	A	0	0	0.00	0	0.000
193	A	0	0	0.00	0	0.000
194	A	3	3	1.00	32	0.094
195	A	7	5	1.00	32	0.156
196	A	0	0	0.00	0	0.000
197	A	0	0	0.00	0	0.000
198	A	0	0	0.00	0	0.000
199	A	4	4	1.00	32	0.125
200	A	10	6	1.00	32	0.188

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
201	A	4	3	1.00	32	0.094
202	A	4	3	1.00	32	0.094
203	A	4	3	1.00	32	0.094
204	A	4	3	1.00	30	0.100
205	A	5	5	1.00	32	0.156
206	A	3	2	1.00	32	0.062
207	A	4	3	1.00	32	0.094
208	A	4	3	1.00	32	0.094
209	A	4	3	1.00	32	0.094
210	A	19	8	1.00	34	0.235
211	A	15	8	1.00	34	0.235
212	A	11	8	1.00	34	0.235
213	A	7	7	1.00	32	0.219
214	A	4	4	1.00	34	0.118
215	A	4	3	1.00	34	0.088
216	A	8	6	1.00	34	0.176
217	A	6	5	1.00	34	0.147
218	A	5	5	1.00	34	0.147
219	A	0	0	0.00	0	0.000
220	A	0	0	0.00	0	0.000
221	A	0	0	0.00	0	0.000
222	A	3	3	1.00	34	0.088
223	A	7	5	1.00	34	0.147
224	A	0	0	0.00	0	0.000
225	A	0	0	0.00	0	0.000
226	A	0	0	0.00	0	0.000
227	A	4	4	1.00	34	0.118
228	A	10	6	1.00	34	0.176
229	A	4	4	1.00	36	0.111
230	A	3	2	1.00	27	0.074
231	A	3	2	1.00	27	0.074
232	A	3	2	1.00	27	0.074
233	A	3	2	1.00	25	0.080
234	A	3	2	1.00	19	0.105
235	A	7	4	1.00	27	0.148

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
236	A	3	3	1.00	27	0.111
237	A	3	2	1.00	27	0.074
238	A	3	2	1.00	27	0.074
239	A	3	2	1.00	27	0.074
240	A	15	10	1.00	29	0.345
241	A	12	10	1.00	29	0.345
242	A	9	8	1.00	27	0.296
243	A	6	6	1.00	21	0.286
244	A	9	5	1.00	29	0.172
245	A	4	4	1.00	29	0.138
246	A	9	8	1.00	29	0.276
247	A	12	10	1.00	29	0.345
248	A	15	10	1.00	29	0.345
249	A	3	3	1.00	14	0.214
250	A	0	0	0.00	0	0.000
251	A	0	0	0.00	0	0.000
252	A	0	0	0.00	0	0.000
253	A	0	0	0.00	0	0.000
254	A	0	0	0.00	0	0.000
255	A	0	0	0.00	0	0.000
256	A	0	0	0.00	0	0.000
257	A	0	0	0.00	0	0.000
258	A	0	0	0.00	0	0.000
259	A	0	0	0.00	0	0.000
260	A	0	0	0.00	0	0.000
261	A	0	0	0.00	0	0.000
262	A	3	2	1.00	29	0.069
263	A	3	2	1.00	29	0.069
264	A	3	2	1.00	29	0.069
265	A	3	2	1.00	27	0.074
266	A	3	2	1.00	21	0.095
267	A	7	4	1.00	29	0.138
268	A	3	3	1.00	29	0.103
269	A	3	2	1.00	29	0.069
270	A	3	2	1.00	29	0.069

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
271	A	3	2	1.00	29	0.069
272	A	15	10	1.00	31	0.323
273	A	12	10	1.00	31	0.323
274	A	9	8	1.00	29	0.276
275	A	6	6	1.00	23	0.261
276	A	9	5	1.00	31	0.161
277	A	4	4	1.00	31	0.129
278	A	9	8	1.00	31	0.258
279	A	12	10	1.00	31	0.323
280	A	15	10	1.00	31	0.323
281	A	0	0	0.00	0	0.000
282	A	0	0	0.00	0	0.000
283	A	0	0	0.00	0	0.000
284	A	0	0	0.00	0	0.000
285	A	0	0	0.00	0	0.000
286	A	0	0	0.00	0	0.000
287	A	0	0	0.00	0	0.000
288	A	0	0	0.00	0	0.000
289	A	0	0	0.00	0	0.000
290	A	0	0	0.00	0	0.000
291	A	0	0	0.00	0	0.000
292	A	0	0	0.00	0	0.000
293	A	3	2	1.00	31	0.065
294	A	3	2	1.00	31	0.065
295	A	3	2	1.00	31	0.065
296	A	3	2	1.00	29	0.069
297	A	3	2	1.00	23	0.087
298	A	7	4	1.00	31	0.129
299	A	3	2	1.00	31	0.065
300	A	3	2	1.00	31	0.065
301	A	3	2	1.00	31	0.065
302	A	3	2	1.00	31	0.065
303	A	13	11	1.00	33	0.333
304	A	10	9	1.00	31	0.290
305	A	6	6	1.00	25	0.240

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
306	A	10	6	1.00	33	0.182
307	A	5	5	1.00	33	0.152
308	A	10	9	1.00	33	0.273
309	A	19	16	1.00	33	0.485
310	A	13	11	1.00	31	0.355
311	A	6	6	1.00	25	0.240
312	A	12	7	1.00	33	0.212
313	A	6	6	1.00	33	0.182
314	A	13	11	1.00	33	0.333

Chapter 3

Listing of integrals

Local contents

3.1 $\int (ag+bgx)^4 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$

Optimal. Leaf size=188

$$\frac{B(bc-ad)^4g^4nx}{5d^4}-\frac{B(bc-ad)^3g^4n(a+bx)^2}{10bd^3}+\frac{B(bc-ad)^2g^4n(a+bx)^3}{15bd^2}-\frac{B(bc-ad)g^4n(a+bx)^4}{20bd}+\frac{g^4(a+$$

```
[Out] 1/5*B*(-a*d+b*c)^4*g^4*n*x/d^4-1/10*B*(-a*d+b*c)^3*g^4*n*(b*x+a)^2/b/d^3+1/
15*B*(-a*d+b*c)^2*g^4*n*(b*x+a)^3/b/d^2-1/20*B*(-a*d+b*c)*g^4*n*(b*x+a)^4/b
/d+1/5*g^4*(b*x+a)^5*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b-1/5*B*(-a*d+b*c)^5*g
^4*n*ln(d*x+c)/b/d^5
```

Rubi [A]
time = 0.09, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {2547, 21, 45}

$$\frac{g^4(a+bx)^5\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{5b}-\frac{Bg^4n(bc-ad)^5\log(c+dx)}{5bd^5}+\frac{Bg^4nx(bc-ad)^4}{5d^4}-\frac{Bg^4n(a+bx)^2(bc-ad)^3}{10bd^3}+\frac{Bg^4n(a+bx)^3(bc-ad)^2}{15bd^2}-\frac{Bg^4n(a+bx)^4(bc-ad)}{20bd}$$

Antiderivative was successfully verified.

```
[In] Int[(a*g + b*g*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

```
[Out] (B*(b*c - a*d)^4*g^4*n*x)/(5*d^4) - (B*(b*c - a*d)^3*g^4*n*(a + b*x)^2)/(10
*b*d^3) + (B*(b*c - a*d)^2*g^4*n*(a + b*x)^3)/(15*b*d^2) - (B*(b*c - a*d)*g
^4*n*(a + b*x)^4)/(20*b*d) + (g^4*(a + b*x)^5*(A + B*Log[e*((a + b*x)/(c +
d*x))^n]))/(5*b) - (B*(b*c - a*d)^5*g^4*n*Log[c + d*x])/(5*b*d^5)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2547

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*((A +
 B*Log[e*((a + b*x)/(c + d*x))^n])/(g*(m + 1))), x] - Dist[B*n*((b*c - a*d)
```

```
/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /; FreeQ
[{a, b, c, d, e, f, g, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, -2]
```

Rubi steps

$$\begin{aligned}\int (ag+bgx)^4 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)dx &= \frac{g^4(a+bx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{5b} - \frac{(Bn)\int \frac{(bc-ad)g^5(a+}{c+dx}}{5bg}\\ &= \frac{g^4(a+bx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{5b} - \frac{(B(bc-ad)g^4n)\int}{5b}\\ &= \frac{g^4(a+bx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{5b} - \frac{(B(bc-ad)g^4n)\int}{}\\ &= \frac{B(bc-ad)^4g^4nx}{5d^4} - \frac{B(bc-ad)^3g^4n(a+bx)^2}{10bd^3} + \frac{B(bc-a}{}\end{aligned}$$

Mathematica [A]
time = 0.09, size = 146, normalized size = 0.78

$$\frac{g^4\left((a+bx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)-\frac{B(bc-ad)n\left(-12bd(bc-ad)^3x+6d^2(bc-ad)^2(a+bx)^2+4d^3(-bc+ad)(a+bx)^3+3d^4(a+bx)^4+12(bc-ad)^4\log(c+dx)\right)}{12d^5}\right)}{5b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*g + b*g*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

```
[Out] (g^4*((a + b*x)^5*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - (B*(b*c - a*d)*n
*(-12*b*d*(b*c - a*d)^3*x + 6*d^2*(b*c - a*d)^2*(a + b*x)^2 + 4*d^3*(-(b*c)
 + a*d)*(a + b*x)^3 + 3*d^4*(a + b*x)^4 + 12*(b*c - a*d)^4*Log[c + d*x]))/(
12*d^5)))/(5*b)
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int (bgx+ag)^4\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*g*x+a*g)^4*(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

```
[Out] int((b*g*x+a*g)^4*(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 681 vs. $2(177) = 354$.
time = 0.31, size = 681, normalized size = 3.62

$$\frac{1}{5} B b^{4} g^{4} x^{5} \log\left(\left(\frac{b x}{d x + c} + \frac{a}{d x + c}\right)^{n} e\right) + \frac{1}{5} A b^{4} g^{4} x^{5} + B a b^{3} g^{4} x^{4} \log\left(\left(\frac{b x}{d x + c} + \frac{a}{d x + c}\right)^{n} e\right) + A a b^{3} g^{4} x^{4} + 2 B a^{2} b^{2} g^{4} x^{3} \log\left(\left(\frac{b x}{d x + c} + \frac{a}{d x + c}\right)^{n} e\right) + 2 A a^{2} b^{2} g^{4} x^{3} + 2 B a^{3} b g^{4} x^{2} \log\left(\left(\frac{b x}{d x + c} + \frac{a}{d x + c}\right)^{n} e\right) + 2 A a^{3} b g^{4} x^{2} + \frac{1}{60} B b^{4} g^{4} n \left(\frac{12 a^{5} \log\left(b x + a\right)}{b^{5}} - \frac{12 c^{5} \log\left(d x + c\right)}{d^{5}} - \frac{3 (b^{4} c d^{3} - a b^{3} d^{4}) x^{4} - 4 (b^{4} c^{2} d^{2} - a^{2} b^{2} d^{4}) x^{3} + 6 (b^{4} c^{3} d - a^{3} b d^{4}) x^{2} - 12 (b^{4} c^{4} - a^{4} d^{4}) x}{b^{4} d^{4}}\right) - \frac{1}{6} B a b^{3} g^{4} n \left(\frac{6 a^{4} \log\left(b x + a\right)}{b^{4}} - \frac{6 c^{4} \log\left(d x + c\right)}{d^{4}} + \frac{2 (b^{3} c d^{2} - a b^{2} d^{3}) x^{3} - 3 (b^{3} c^{2} d - a^{2} b d^{3}) x^{2} + 6 (b^{3} c^{3} - a^{3} d^{3}) x}{b^{3} d^{3}}\right) + B a^{2} b^{2} g^{4} n \left(\frac{2 a^{3} \log\left(b x + a\right)}{b^{3}} - \frac{2 c^{3} \log\left(d x + c\right)}{d^{3}} - \frac{(b^{2} c d - a b d^{2}) x^{2} - 2 (b^{2} c^{2} - a^{2} d^{2}) x}{b^{2} d^{2}}\right) - 2 B a^{3} b g^{4} n \left(\frac{a^{2} \log\left(b x + a\right)}{b^{2}} - \frac{c^{2} \log\left(d x + c\right)}{d^{2}} + \frac{(b c - a d) x}{b d}\right) + B a^{4} g^{4} n \left(\frac{a \log\left(b x + a\right)}{b} - \frac{c \log\left(d x + c\right)}{d}\right) + B a^{4} g^{4} x \log\left(\left(\frac{b x}{d x + c} + \frac{a}{d x + c}\right)^{n} e\right) + A a^{4} g^{4} x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^4*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="maxi
ma")
```

```
[Out] 1/5*B*b^4*g^4*x^5*log((b*x/(d*x + c) + a/(d*x + c))^n*e) + 1/5*A*b^4*g^4*x^
5 + B*a*b^3*g^4*x^4*log((b*x/(d*x + c) + a/(d*x + c))^n*e) + A*a*b^3*g^4*x^
4 + 2*B*a^2*b^2*g^4*x^3*log((b*x/(d*x + c) + a/(d*x + c))^n*e) + 2*A*a^2*b^
2*g^4*x^3 + 2*B*a^3*b*g^4*x^2*log((b*x/(d*x + c) + a/(d*x + c))^n*e) + 2*A*
a^3*b*g^4*x^2 + 1/60*B*b^4*g^4*n*(12*a^5*log(b*x + a)/b^5 - 12*c^5*log(d*x
+ c)/d^5 - (3*(b^4*c*d^3 - a*b^3*d^4)*x^4 - 4*(b^4*c^2*d^2 - a^2*b^2*d^4)*x
^3 + 6*(b^4*c^3*d - a^3*b*d^4)*x^2 - 12*(b^4*c^4 - a^4*d^4)*x)/(b^4*d^4)) -
 1/6*B*a*b^3*g^4*n*(6*a^4*log(b*x + a)/b^4 - 6*c^4*log(d*x + c)/d^4 + (2*(b
^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^
3*d^3)*x)/(b^3*d^3)) + B*a^2*b^2*g^4*n*(2*a^3*log(b*x + a)/b^3 - 2*c^3*log(
d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2
)) - 2*B*a^3*b*g^4*n*(a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c -
a*d)*x/(b*d)) + B*a^4*g^4*n*(a*log(b*x + a)/b - c*log(d*x + c)/d) + B*a^4*g
^4*x*log((b*x/(d*x + c) + a/(d*x + c))^n*e) + A*a^4*g^4*x
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 494 vs. $2(177) = 354$.
time = 0.45, size = 494, normalized size = 2.63

$$\frac{12 (A + B) b^{5} d^{5} g^{4} x^{5} + 12 B a^{5} d^{5} g^{4} n \log\left(b x + a\right) - 12 (B b^{5} c^{5} - 5 B a b^{4} c^{4} d + 10 B a^{2} b^{3} c^{3} d^{2} - 10 B a^{3} b^{2} c^{2} d^{3} + 5 B a^{4} b c d^{4}) g^{4} n \log\left(d x + c\right) + 3 (20 (A + B) a b^{4} d^{5} g^{4} - (B b^{5} c d^{4} - B a b^{4} d^{5}) g^{4} n) x^{4} + 4 (30 (A + B) a^{2} b^{3} d^{5} g^{4} + (B b^{5} c^{2} d^{3} - 5 B a b^{4} c d^{4} + 4 B a^{2} b^{3} d^{5}) g^{4} n) x^{3} + 6 (20 (A + B) a^{3} b^{2} d^{5} g^{4} - (B b^{5} c^{3} d^{2} - 5 B a b^{4} c^{2} d^{3} + 10 B a^{2} b^{3} c d^{4} - 6 B a^{3} b^{2} d^{5}) g^{4} n) x^{2} + 12 (5 (A + B) a^{4} b d^{5} g^{4} + (B b^{5} c^{4} d - 5 B a b^{4} c^{3} d^{2} + 10 B a^{2} b^{3} c^{2} d^{3} - 10 B a^{3} b^{2} c d^{4} + 4 B a^{4} b d^{5}) g^{4} n) x + 12 (B b^{5} d^{5} g^{4} n x^{5} + 5 B a b^{4} d^{5} g^{4} n x^{4} + 10 B a^{2} b^{3} d^{5} g^{4} n x^{3} + 10 B a^{3} b^{2} d^{5} g^{4} n x^{2} + 5 B a^{4} b d^{5} g^{4} n x) \log\left(\frac{b x + a}{d x + c}\right)}{60 b d^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^4*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fric
as")
```

```
[Out] 1/60*(12*(A + B)*b^5*d^5*g^4*x^5 + 12*B*a^5*d^5*g^4*n*log(b*x + a) - 12*(B*
b^5*c^5 - 5*B*a*b^4*c^4*d + 10*B*a^2*b^3*c^3*d^2 - 10*B*a^3*b^2*c^2*d^3 + 5
*B*a^4*b*c*d^4)*g^4*n*log(d*x + c) + 3*(20*(A + B)*a*b^4*d^5*g^4 - (B*b^5*c
*d^4 - B*a*b^4*d^5)*g^4*n)*x^4 + 4*(30*(A + B)*a^2*b^3*d^5*g^4 + (B*b^5*c^2
*d^3 - 5*B*a*b^4*c*d^4 + 4*B*a^2*b^3*d^5)*g^4*n)*x^3 + 6*(20*(A + B)*a^3*b^
2*d^5*g^4 - (B*b^5*c^3*d^2 - 5*B*a*b^4*c^2*d^3 + 10*B*a^2*b^3*c*d^4 - 6*B*a
^3*b^2*d^5)*g^4*n)*x^2 + 12*(5*(A + B)*a^4*b*d^5*g^4 + (B*b^5*c^4*d - 5*B*a
*b^4*c^3*d^2 + 10*B*a^2*b^3*c^2*d^3 - 10*B*a^3*b^2*c*d^4 + 4*B*a^4*b*d^5)*g
^4*n)*x + 12*(B*b^5*d^5*g^4*n*x^5 + 5*B*a*b^4*d^5*g^4*n*x^4 + 10*B*a^2*b^3*
d^5*g^4*n*x^3 + 10*B*a^3*b^2*d^5*g^4*n*x^2 + 5*B*a^4*b*d^5*g^4*n*x)*log((b*
x + a)/(d*x + c)))/(b*d^5)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)**4*(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 4392 vs. $2(177) = 354$.
time = 3.96, size = 4392, normalized size = 23.36

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^4*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="giac
")
```

```
[Out] 1/60*(12*(B*b^10*c^6*g^4*n - 6*B*a*b^9*c^5*d*g^4*n - 5*(b*x + a)*B*b^9*c^6*
d*g^4*n/(d*x + c) + 15*B*a^2*b^8*c^4*d^2*g^4*n + 30*(b*x + a)*B*a*b^8*c^5*d
^2*g^4*n/(d*x + c) + 10*(b*x + a)^2*B*b^8*c^6*d^2*g^4*n/(d*x + c)^2 - 20*B*
a^3*b^7*c^3*d^3*g^4*n - 75*(b*x + a)*B*a^2*b^7*c^4*d^3*g^4*n/(d*x + c) - 60
*(b*x + a)^2*B*a*b^7*c^5*d^3*g^4*n/(d*x + c)^2 - 10*(b*x + a)^3*B*b^7*c^6*d
^3*g^4*n/(d*x + c)^3 + 15*B*a^4*b^6*c^2*d^4*g^4*n + 100*(b*x + a)*B*a^3*b^6
*c^3*d^4*g^4*n/(d*x + c) + 150*(b*x + a)^2*B*a^2*b^6*c^4*d^4*g^4*n/(d*x + c
)^2 + 60*(b*x + a)^3*B*a*b^6*c^5*d^4*g^4*n/(d*x + c)^3 + 5*(b*x + a)^4*B*b^
6*c^6*d^4*g^4*n/(d*x + c)^4 - 6*B*a^5*b^5*c*d^5*g^4*n - 75*(b*x + a)*B*a^4*
b^5*c^2*d^5*g^4*n/(d*x + c) - 200*(b*x + a)^2*B*a^3*b^5*c^3*d^5*g^4*n/(d*x
+ c)^2 - 150*(b*x + a)^3*B*a^2*b^5*c^4*d^5*g^4*n/(d*x + c)^3 - 30*(b*x + a)
^4*B*a*b^5*c^5*d^5*g^4*n/(d*x + c)^4 + B*a^6*b^4*d^6*g^4*n + 30*(b*x + a)*B
*a^5*b^4*c*d^6*g^4*n/(d*x + c) + 150*(b*x + a)^2*B*a^4*b^4*c^2*d^6*g^4*n/(d
*x + c)^2 + 200*(b*x + a)^3*B*a^3*b^4*c^3*d^6*g^4*n/(d*x + c)^3 + 75*(b*x +
 a)^4*B*a^2*b^4*c^4*d^6*g^4*n/(d*x + c)^4 - 5*(b*x + a)*B*a^6*b^3*d^7*g^4*n
/(d*x + c) - 60*(b*x + a)^2*B*a^5*b^3*c*d^7*g^4*n/(d*x + c)^2 - 150*(b*x +
a)^3*B*a^4*b^3*c^2*d^7*g^4*n/(d*x + c)^3 - 100*(b*x + a)^4*B*a^3*b^3*c^3*d^
7*g^4*n/(d*x + c)^4 + 10*(b*x + a)^2*B*a^6*b^2*d^8*g^4*n/(d*x + c)^2 + 60*(
b*x + a)^3*B*a^5*b^2*c*d^8*g^4*n/(d*x + c)^3 + 75*(b*x + a)^4*B*a^4*b^2*c^2
*d^8*g^4*n/(d*x + c)^4 - 10*(b*x + a)^3*B*a^6*b*d^9*g^4*n/(d*x + c)^3 - 30*
(b*x + a)^4*B*a^5*b*c*d^9*g^4*n/(d*x + c)^4 + 5*(b*x + a)^4*B*a^6*d^10*g^4*
n/(d*x + c)^4)*log((b*x + a)/(d*x + c))/(b^5*d^5 - 5*(b*x + a)*b^4*d^6/(d*x
 + c) + 10*(b*x + a)^2*b^3*d^7/(d*x + c)^2 - 10*(b*x + a)^3*b^2*d^8/(d*x +
c)^3 + 5*(b*x + a)^4*b*d^9/(d*x + c)^4 - (b*x + a)^5*d^10/(d*x + c)^5) + (2
5*B*b^10*c^6*g^4*n - 150*B*a*b^9*c^5*d*g^4*n - 113*(b*x + a)*B*b^9*c^6*d*g^
```

```
4*n/(d*x + c) + 375*B*a^2*b^8*c^4*d^2*g^4*n + 678*(b*x + a)*B*a*b^8*c^5*d^2
*g^4*n/(d*x + c) + 196*(b*x + a)^2*B*b^8*c^6*d^2*g^4*n/(d*x + c)^2 - 500*B*
a^3*b^7*c^3*d^3*g^4*n - 1695*(b*x + a)*B*a^2*b^7*c^4*d^3*g^4*n/(d*x + c) -
1176*(b*x + a)^2*B*a*b^7*c^5*d^3*g^4*n/(d*x + c)^2 - 156*(b*x + a)^3*B*b^7*
c^6*d^3*g^4*n/(d*x + c)^3 + 375*B*a^4*b^6*c^2*d^4*g^4*n + 2260*(b*x + a)*B*
a^3*b^6*c^3*d^4*g^4*n/(d*x + c) + 2940*(b*x + a)^2*B*a^2*b^6*c^4*d^4*g^4*n/
(d*x + c)^2 + 936*(b*x + a)^3*B*a*b^6*c^5*d^4*g^4*n/(d*x + c)^3 + 48*(b*x +
 a)^4*B*b^6*c^6*d^4*g^4*n/(d*x + c)^4 - 150*B*a^5*b^5*c*d^5*g^4*n - 1695*(b
*x + a)*B*a^4*b^5*c^2*d^5*g^4*n/(d*x + c) - 3920*(b*x + a)^2*B*a^3*b^5*c^3*
d^5*g^4*n/(d*x + c)^2 - 2340*(b*x + a)^3*B*a^2*b^5*c^4*d^5*g^4*n/(d*x + c)^
3 - 288*(b*x + a)^4*B*a*b^5*c^5*d^5*g^4*n/(d*x + c)^4 + 25*B*a^6*b^4*d^6*g^
4*n + 678*(b*x + a)*B*a^5*b^4*c*d^6*g^4*n/(d*x + c) + 2940*(b*x + a)^2*B*a^
4*b^4*c^2*d^6*g^4*n/(d*x + c)^2 + 3120*(b*x + a)^3*B*a^3*b^4*c^3*d^6*g^4*n/
(d*x + c)^3 + 720*(b*x + a)^4*B*a^2*b^4*c^4*d^6*g^4*n/(d*x + c)^4 - 113*(b*
x + a)*B*a^6*b^3*d^7*g^4*n/(d*x + c) - 1176*(b*x + a)^2*B*a^5*b^3*c*d^7*g^4
*n/(d*x + c)^2 - 2340*(b*x + a)^3*B*a^4*b^3*c^2*d^7*g^4*n/(d*x + c)^3 - 960
*(b*x + a)^4*B*a^3*b^3*c^3*d^7*g^4*n/(d*x + c)^4 + 196*(b*x + a)^2*B*a^6*b^
2*d^8*g^4*n/(d*x + c)^2 + 936*(b*x + a)^3*B*a^5*b^2*c*d^8*g^4*n/(d*x + c)^3
 + 720*(b*x + a)^4*B*a^4*b^2*c^2*d^8*g^4*n/(d*x + c)^4 - 156*(b*x + a)^3*B*
a^6*b*d^9*g^4*n/(d*x + c)^3 - 288*(b*x + a)^4*B*a^5*b*c*d^9*g^4*n/(d*x + c)
^4 + 48*(b*x + a)^4*B*a^6*d^10*g^4*n/(d*x + c)^4 + 12*A*b^10*c^6*g^4 + 12*B
*b^10*c^6*g^4 - 72*A*a*b^9*c^5*d*g^4 - 72*B*a*b^9*c^5*d*g^4 - 60*(b*x + a)*
A*b^9*c^6*d*g^4/(d*x + c) - 60*(b*x + a)*B*b^9*c^6*d*g^4/(d*x + c) + 180*A*
a^2*b^8*c^4*d^2*g^4 + 180*B*a^2*b^8*c^4*d^2*g^4 + 360*(b*x + a)*A*a*b^8*c^5
*d^2*g^4/(d*x + c) + 360*(b*x + a)*B*a*b^8*c^5*d^2*g^4/(d*x + c) + 120*(b*x
 + a)^2*A*b^8*c^6*d^2*g^4/(d*x + c)^2 + 120*(b*x + a)^2*B*b^8*c^6*d^2*g^4/(
d*x + c)^2 - 240*A*a^3*b^7*c^3*d^3*g^4 - 240*B*a^3*b^7*c^3*d^3*g^4 - 900*(b
*x + a)*A*a^2*b^7*c^4*d^3*g^4/(d*x + c) - 900*(b*x + a)*B*a^2*b^7*c^4*d^3*g
^4/(d*x + c) - 720*(b*x + a)^2*A*a*b^7*c^5*d^3*g^4/(d*x + c)^2 - 720*(b*x +
 a)^2*B*a*b^7*c^5*d^3*g^4/(d*x + c)^2 - 120*(b*x + a)^3*A*b^7*c^6*d^3*g^4/(
d*x + c)^3 - 120*(b*x + a)^3*B*b^7*c^6*d^3*g^4/(d*x + c)^3 + 180*A*a^4*b^6*
c^2*d^4*g^4 + 180*B*a^4*b^6*c^2*d^4*g^4 + 1200*(b*x + a)*A*a^3*b^6*c^3*d^4*
g^4/(d*x + c) + 1200*(b*x + a)*B*a^3*b^6*c^3*d^4*g^4/(d*x + c) + 1800*(b*x
+ a)^2*A*a^2*b^6*c^4*d^4*g^4/(d*x + c)^2 + 1800*(b*x + a)^2*B*a^2*b^6*c^4*d
^4*g^4/(d*x + c)^2 + 720*(b*x + a)^3*A*a*b^6*c^5*d^4*g^4/(d*x + c)^3 + 720*
(b*x + a)^3*B*a*b^6*c^5*d^4*g^4/(d*x + c)^3 + 60*(b*x + a)^4*A*b^6*c^6*d^4*
g^4/(d*x + c)^4 + 60*(b*x + a)^4*B*b^6*c^6*d^4*g^4/(d*x + c)^4 - 72*A*a^5*b
^5*c*d^5*g^4 - 72*B*a^5*b^5*c*d^5*g^4 - 900*(b*x + a)*A*a^4*b^5*c^2*d^5*g^4
/(d*x + c) - 900*(b*x + a)*B*a^4*b^5*c^2*d^5*g^4/(d*x + c) - 2400*(b*x + a)
^2*A*a^3*b^5*c^3*d^5*g^4/(d*x + c)^2 - 2400*(b*x + a)^2*B*a^3*b^5*c^3*d^5*g
^4/(d*x + c)^2 - 1800*(b*x + a)^3*A*a^2*b^5*c^4*d^5*g^4/(d*x + c)^3 - 1800*
(b*x + a)^3*B*a^2*b^5*c^4*d^5*g^4/(d*x + c)^3 -...
```

Mupad [B]

time = 4.55, size = 1046, normalized size = 5.56

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*g + b*g*x)^4*(A + B*log(e*((a + b*x)/(c + d*x))^n)),x)
```

```
[Out] x^2*(((5*a*d + 5*b*c)*((((b^3*g^4*(25*A*a*d + 5*A*b*c + B*a*d*n - B*b*c*n))
/(5*d) - (A*b^3*g^4*(5*a*d + 5*b*c))/(5*d))*(5*a*d + 5*b*c))/(5*b*d) - (a*b
^2*g^4*(10*A*a*d + 5*A*b*c + B*a*d*n - B*b*c*n))/d + (A*a*b^3*c*g^4)/d))/(1
0*b*d) - (a*c*((b^3*g^4*(25*A*a*d + 5*A*b*c + B*a*d*n - B*b*c*n))/(5*d) - (
A*b^3*g^4*(5*a*d + 5*b*c))/(5*d)))/(2*b*d) + (a^2*b*g^4*(5*A*a*d + 5*A*b*c
+ B*a*d*n - B*b*c*n))/d) - x^3*((((b^3*g^4*(25*A*a*d + 5*A*b*c + B*a*d*n -
B*b*c*n))/(5*d) - (A*b^3*g^4*(5*a*d + 5*b*c))/(5*d))*(5*a*d + 5*b*c))/(15*b
*d) - (a*b^2*g^4*(10*A*a*d + 5*A*b*c + B*a*d*n - B*b*c*n))/(3*d) + (A*a*b^3
*c*g^4)/(3*d)) + log(e*((a + b*x)/(c + d*x))^n)*((B*b^4*g^4*x^5)/5 + B*a^4*
g^4*x + 2*B*a^3*b*g^4*x^2 + B*a*b^3*g^4*x^4 + 2*B*a^2*b^2*g^4*x^3) + x*((a^
3*g^4*(5*A*a*d + 10*A*b*c + 2*B*a*d*n - 2*B*b*c*n))/d - ((5*a*d + 5*b*c)*((
(5*a*d + 5*b*c)*((((b^3*g^4*(25*A*a*d + 5*A*b*c + B*a*d*n - B*b*c*n))/(5*d)
 - (A*b^3*g^4*(5*a*d + 5*b*c))/(5*d))*(5*a*d + 5*b*c))/(5*b*d) - (a*b^2*g^4
*(10*A*a*d + 5*A*b*c + B*a*d*n - B*b*c*n))/d + (A*a*b^3*c*g^4)/d))/(5*b*d)
- (a*c*((b^3*g^4*(25*A*a*d + 5*A*b*c + B*a*d*n - B*b*c*n))/(5*d) - (A*b^3*g
^4*(5*a*d + 5*b*c))/(5*d)))/(b*d) + (2*a^2*b*g^4*(5*A*a*d + 5*A*b*c + B*a*d
*n - B*b*c*n))/d))/(5*b*d) + (a*c*((((b^3*g^4*(25*A*a*d + 5*A*b*c + B*a*d*n
 - B*b*c*n))/(5*d) - (A*b^3*g^4*(5*a*d + 5*b*c))/(5*d))*(5*a*d + 5*b*c))/(5
*b*d) - (a*b^2*g^4*(10*A*a*d + 5*A*b*c + B*a*d*n - B*b*c*n))/d + (A*a*b^3*c
*g^4)/d))/(b*d)) + x^4*((b^3*g^4*(25*A*a*d + 5*A*b*c + B*a*d*n - B*b*c*n))/
(20*d) - (A*b^3*g^4*(5*a*d + 5*b*c))/(20*d)) - (log(c + d*x)*(B*b^4*c^5*g^4
*n + 5*B*a^4*c*d^4*g^4*n - 5*B*a*b^3*c^4*d*g^4*n - 10*B*a^3*b*c^2*d^3*g^4*n
 + 10*B*a^2*b^2*c^3*d^2*g^4*n))/(5*d^5) + (A*b^4*g^4*x^5)/5 + (B*a^5*g^4*n*
log(a + b*x))/(5*b)
```

3.2 $\int (ag+bgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$

Optimal. Leaf size=156

$$-\frac{B(bc-ad)^3g^3nx}{4d^3}+\frac{B(bc-ad)^2g^3n(a+bx)^2}{8bd^2}-\frac{B(bc-ad)g^3n(a+bx)^3}{12bd}+\frac{g^3(a+bx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)\right.\right.}{4b}$$

```
[Out] -1/4*B*(-a*d+b*c)^3*g^3*n*x/d^3+1/8*B*(-a*d+b*c)^2*g^3*n*(b*x+a)^2/b/d^2-1/
12*B*(-a*d+b*c)*g^3*n*(b*x+a)^3/b/d+1/4*g^3*(b*x+a)^4*(A+B*ln(e*((b*x+a)/(d
*x+c))^n))/b+1/4*B*(-a*d+b*c)^4*g^3*n*ln(d*x+c)/b/d^4
```

Rubi [A]
time = 0.07, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {2547, 21, 45}

$$\frac{g^3(a+bx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{4b}+\frac{Bg^3n(bc-ad)^4\log(c+dx)}{4bd^4}-\frac{Bg^3nx(bc-ad)^3}{4d^3}+\frac{Bg^3n(a+bx)^2(bc-ad)^2}{8bd^2}-\frac{Bg^3n(a+bx)^3(bc-ad)}{12bd}$$

Antiderivative was successfully verified.

```
[In] Int[(a*g + b*g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

```
[Out] -1/4*(B*(b*c - a*d)^3*g^3*n*x)/d^3 + (B*(b*c - a*d)^2*g^3*n*(a + b*x)^2)/(8
*b*d^2) - (B*(b*c - a*d)*g^3*n*(a + b*x)^3)/(12*b*d) + (g^3*(a + b*x)^4*(A
+ B*Log[e*((a + b*x)/(c + d*x))^n]))/(4*b) + (B*(b*c - a*d)^4*g^3*n*Log[c +
 d*x])/(4*b*d^4)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2547

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*((A +
 B*Log[e*((a + b*x)/(c + d*x))^n])/(g*(m + 1))), x] - Dist[B*n*((b*c - a*d)
/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /; FreeQ
```

```
[{a, b, c, d, e, f, g, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, -2]
```

Rubi steps

$$\int (ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)dx=\frac{g^3(a+bx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4b}-\frac{(Bn)\int\frac{(bc-ad)g^4(a+}{c+dx}}{4bg}$$

$$=\frac{g^3(a+bx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4b}-\frac{(B(bc-ad)g^3n)\int}{4b}$$

$$=\frac{g^3(a+bx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4b}-\frac{(B(bc-ad)g^3n)\int}{}$$

$$=-\frac{B(bc-ad)^3g^3nx}{4d^3}+\frac{B(bc-ad)^2g^3n(a+bx)^2}{8bd^2}-\frac{B(bc-}{}$$

Mathematica [A]
time = 0.08, size = 124, normalized size = 0.79

$$\frac{g^3\left((a+bx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)-\frac{B(bc-ad)n\left(6bd(bc-ad)^2x+3d^2(-bc+ad)(a+bx)^2+2d^3(a+bx)^3-6(bc-ad)^3\log(c+dx)\right)}{6d^4}\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*g + b*g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

```
[Out] (g^3*((a + b*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - (B*(b*c - a*d)*n
*(6*b*d*(b*c - a*d)^2*x + 3*d^2*(-(b*c) + a*d)*(a + b*x)^2 + 2*d^3*(a + b*x
)^3 - 6*(b*c - a*d)^3*Log[c + d*x]))/(6*d^4)))/(4*b)
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int (bgx+ag)^3\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*g*x+a*g)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

```
[Out] int((b*g*x+a*g)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 483 vs. 2(147) = 294.
time = 0.31, size = 483, normalized size = 3.10

$$\frac{1}{4}Bb^3g^3x^4\log\left(\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^ne\right)+\frac{1}{4}Ab^3g^3x^4+Bab^2g^3x^3\log\left(\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^ne\right)+Aab^2g^3x^3+\frac{3}{2}Ba^2bg^3x^2\log\left(\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^ne\right)+\frac{3}{2}Aa^2bg^3x^2-\frac{1}{24}Bb^3g^3n\left(\frac{6a^4\log(bx+a)}{b^4}-\frac{6c^4\log(dx+c)}{d^4}+\frac{2(b^3cd^2-ab^2d^3)x^3-3(b^3c^2d-a^2bd^3)x^2+6(b^3c^3-a^3d^3)x}{b^3d^3}\right)+\frac{1}{2}Bab^2g^3n\left(\frac{2a^3\log(bx+a)}{b^3}-\frac{2c^3\log(dx+c)}{d^3}-\frac{(b^2cd-abd^2)x^2-2(b^2c^2-a^2d^2)x}{b^2d^2}\right)-\frac{3}{2}Ba^2bg^3n\left(\frac{a^2\log(bx+a)}{b^2}-\frac{c^2\log(dx+c)}{d^2}+\frac{(bc-ad)x}{bd}\right)+Ba^3g^3n\left(\frac{a\log(bx+a)}{b}-\frac{c\log(dx+c)}{d}\right)+Ba^3g^3x\log\left(\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^ne\right)+Aa^3g^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="maxi
ma")
```

```
[Out] 1/4*B*b^3*g^3*x^4*log((b*x/(d*x + c) + a/(d*x + c))^n*e) + 1/4*A*b^3*g^3*x^
4 + B*a*b^2*g^3*x^3*log((b*x/(d*x + c) + a/(d*x + c))^n*e) + A*a*b^2*g^3*x^
3 + 3/2*B*a^2*b*g^3*x^2*log((b*x/(d*x + c) + a/(d*x + c))^n*e) + 3/2*A*a^2*
b*g^3*x^2 - 1/24*B*b^3*g^3*n*(6*a^4*log(b*x + a)/b^4 - 6*c^4*log(d*x + c)/d
^4 + (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^
3*c^3 - a^3*d^3)*x)/(b^3*d^3)) + 1/2*B*a*b^2*g^3*n*(2*a^3*log(b*x + a)/b^3
- 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)
*x)/(b^2*d^2)) - 3/2*B*a^2*b*g^3*n*(a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)
/d^2 + (b*c - a*d)*x/(b*d)) + B*a^3*g^3*n*(a*log(b*x + a)/b - c*log(d*x + c
)/d) + B*a^3*g^3*x*log((b*x/(d*x + c) + a/(d*x + c))^n*e) + A*a^3*g^3*x
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 367 vs. $2(147) = 294$.
time = 0.41, size = 367, normalized size = 2.35

$$\frac{6\,(A+B)b^4d^4g^3x^4 + 6\,Ba^4d^4g^3n\log\left(bx+a\right) + 6\,(Bb^4c^4 - 4\,Bab^3c^3d + 6\,Ba^2b^2c^2d^2 - 4\,Ba^3bcd^3)g^3n\log\left(dx+c\right) + 2\,(12\,(A+B)ab^3d^4g^3 - (Bb^4cd^3 - Bab^3d^4)g^3n)x^3 + 3\,(12\,(A+B)a^2b^2d^4g^3 + (Bb^4c^2d^2 - 4\,Bab^3cd^3 + 3\,Ba^2b^2d^4)g^3n)x^2 + 6\,(4\,(A+B)a^3bd^4g^3 - (Bb^4c^3d - 4\,Bab^3c^2d^2 + 6\,Ba^2b^2cd^3 - 3\,Ba^3bd^4)g^3n)x + 6\,(Bb^4d^4g^3nx^4 + 4\,Bab^3d^4g^3nx^3 + 6\,Ba^2b^2d^4g^3nx^2 + 4\,Ba^3bd^4g^3nx)\log\left(\frac{bx+a}{dx+c}\right)}{24\,bd^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fric
as")
```

```
[Out] 1/24*(6*(A + B)*b^4*d^4*g^3*x^4 + 6*B*a^4*d^4*g^3*n*log(b*x + a) + 6*(B*b^4
*c^4 - 4*B*a*b^3*c^3*d + 6*B*a^2*b^2*c^2*d^2 - 4*B*a^3*b*c*d^3)*g^3*n*log(d
*x + c) + 2*(12*(A + B)*a*b^3*d^4*g^3 - (B*b^4*c*d^3 - B*a*b^3*d^4)*g^3*n)*
x^3 + 3*(12*(A + B)*a^2*b^2*d^4*g^3 + (B*b^4*c^2*d^2 - 4*B*a*b^3*c*d^3 + 3*
B*a^2*b^2*d^4)*g^3*n)*x^2 + 6*(4*(A + B)*a^3*b*d^4*g^3 - (B*b^4*c^3*d - 4*B
*a*b^3*c^2*d^2 + 6*B*a^2*b^2*c*d^3 - 3*B*a^3*b*d^4)*g^3*n)*x + 6*(B*b^4*d^4
*g^3*n*x^4 + 4*B*a*b^3*d^4*g^3*n*x^3 + 6*B*a^2*b^2*d^4*g^3*n*x^2 + 4*B*a^3*
b*d^4*g^3*n*x)*log((b*x + a)/(d*x + c)))/(b*d^4)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)**3*(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2986 vs. $2(147) = 294$.

time = 5.70, size = 2986, normalized size = 19.14

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="giac
")
```

```
[Out] -1/24*(6*(B*b^8*c^5*g^3*n - 5*B*a*b^7*c^4*d*g^3*n - 4*(b*x + a)*B*b^7*c^5*d
*g^3*n/(d*x + c) + 10*B*a^2*b^6*c^3*d^2*g^3*n + 20*(b*x + a)*B*a*b^6*c^4*d^
2*g^3*n/(d*x + c) + 6*(b*x + a)^2*B*b^6*c^5*d^2*g^3*n/(d*x + c)^2 - 10*B*a^
3*b^5*c^2*d^3*g^3*n - 40*(b*x + a)*B*a^2*b^5*c^3*d^3*g^3*n/(d*x + c) - 30*(
b*x + a)^2*B*a*b^5*c^4*d^3*g^3*n/(d*x + c)^2 - 4*(b*x + a)^3*B*b^5*c^5*d^3*
g^3*n/(d*x + c)^3 + 5*B*a^4*b^4*c*d^4*g^3*n + 40*(b*x + a)*B*a^3*b^4*c^2*d^
4*g^3*n/(d*x + c) + 60*(b*x + a)^2*B*a^2*b^4*c^3*d^4*g^3*n/(d*x + c)^2 + 20
*(b*x + a)^3*B*a*b^4*c^4*d^4*g^3*n/(d*x + c)^3 - B*a^5*b^3*d^5*g^3*n - 20*(
b*x + a)*B*a^4*b^3*c*d^5*g^3*n/(d*x + c) - 60*(b*x + a)^2*B*a^3*b^3*c^2*d^5
*g^3*n/(d*x + c)^2 - 40*(b*x + a)^3*B*a^2*b^3*c^3*d^5*g^3*n/(d*x + c)^3 + 4
*(b*x + a)*B*a^5*b^2*d^6*g^3*n/(d*x + c) + 30*(b*x + a)^2*B*a^4*b^2*c*d^6*g
^3*n/(d*x + c)^2 + 40*(b*x + a)^3*B*a^3*b^2*c^2*d^6*g^3*n/(d*x + c)^3 - 6*(
b*x + a)^2*B*a^5*b*d^7*g^3*n/(d*x + c)^2 - 20*(b*x + a)^3*B*a^4*b*c*d^7*g^3
*n/(d*x + c)^3 + 4*(b*x + a)^3*B*a^5*d^8*g^3*n/(d*x + c)^3)*log((b*x + a)/(
d*x + c))/(b^4*d^4 - 4*(b*x + a)*b^3*d^5/(d*x + c) + 6*(b*x + a)^2*b^2*d^6/
(d*x + c)^2 - 4*(b*x + a)^3*b*d^7/(d*x + c)^3 + (b*x + a)^4*d^8/(d*x + c)^4
) + (11*B*b^8*c^5*g^3*n - 55*B*a*b^7*c^4*d*g^3*n - 38*(b*x + a)*B*b^7*c^5*d
*g^3*n/(d*x + c) + 110*B*a^2*b^6*c^3*d^2*g^3*n + 190*(b*x + a)*B*a*b^6*c^4*
d^2*g^3*n/(d*x + c) + 45*(b*x + a)^2*B*b^6*c^5*d^2*g^3*n/(d*x + c)^2 - 110*
B*a^3*b^5*c^2*d^3*g^3*n - 380*(b*x + a)*B*a^2*b^5*c^3*d^3*g^3*n/(d*x + c) -
 225*(b*x + a)^2*B*a*b^5*c^4*d^3*g^3*n/(d*x + c)^2 - 18*(b*x + a)^3*B*b^5*c
^5*d^3*g^3*n/(d*x + c)^3 + 55*B*a^4*b^4*c*d^4*g^3*n + 380*(b*x + a)*B*a^3*b
^4*c^2*d^4*g^3*n/(d*x + c) + 450*(b*x + a)^2*B*a^2*b^4*c^3*d^4*g^3*n/(d*x +
 c)^2 + 90*(b*x + a)^3*B*a*b^4*c^4*d^4*g^3*n/(d*x + c)^3 - 11*B*a^5*b^3*d^5
*g^3*n - 190*(b*x + a)*B*a^4*b^3*c*d^5*g^3*n/(d*x + c) - 450*(b*x + a)^2*B*
a^3*b^3*c^2*d^5*g^3*n/(d*x + c)^2 - 180*(b*x + a)^3*B*a^2*b^3*c^3*d^5*g^3*n
/(d*x + c)^3 + 38*(b*x + a)*B*a^5*b^2*d^6*g^3*n/(d*x + c) + 225*(b*x + a)^2
*B*a^4*b^2*c*d^6*g^3*n/(d*x + c)^2 + 180*(b*x + a)^3*B*a^3*b^2*c^2*d^6*g^3*
n/(d*x + c)^3 - 45*(b*x + a)^2*B*a^5*b*d^7*g^3*n/(d*x + c)^2 - 90*(b*x + a)
^3*B*a^4*b*c*d^7*g^3*n/(d*x + c)^3 + 18*(b*x + a)^3*B*a^5*d^8*g^3*n/(d*x +
c)^3 + 6*A*b^8*c^5*g^3 + 6*B*b^8*c^5*g^3 - 30*A*a*b^7*c^4*d*g^3 - 30*B*a*b^
7*c^4*d*g^3 - 24*(b*x + a)*A*b^7*c^5*d*g^3/(d*x + c) - 24*(b*x + a)*B*b^7*c
^5*d*g^3/(d*x + c) + 60*A*a^2*b^6*c^3*d^2*g^3 + 60*B*a^2*b^6*c^3*d^2*g^3 +
120*(b*x + a)*A*a*b^6*c^4*d^2*g^3/(d*x + c) + 120*(b*x + a)*B*a*b^6*c^4*d^2
*g^3/(d*x + c) + 36*(b*x + a)^2*A*b^6*c^5*d^2*g^3/(d*x + c)^2 + 36*(b*x + a
)^2*B*b^6*c^5*d^2*g^3/(d*x + c)^2 - 60*A*a^3*b^5*c^2*d^3*g^3 - 60*B*a^3*b^5
*c^2*d^3*g^3 - 240*(b*x + a)*A*a^2*b^5*c^3*d^3*g^3/(d*x + c) - 240*(b*x + a
```

```
)*B*a^2*b^5*c^3*d^3*g^3/(d*x + c) - 180*(b*x + a)^2*A*a*b^5*c^4*d^3*g^3/(d*
x + c)^2 - 180*(b*x + a)^2*B*a*b^5*c^4*d^3*g^3/(d*x + c)^2 - 24*(b*x + a)^3
*A*b^5*c^5*d^3*g^3/(d*x + c)^3 - 24*(b*x + a)^3*B*b^5*c^5*d^3*g^3/(d*x + c)
^3 + 30*A*a^4*b^4*c*d^4*g^3 + 30*B*a^4*b^4*c*d^4*g^3 + 240*(b*x + a)*A*a^3*
b^4*c^2*d^4*g^3/(d*x + c) + 240*(b*x + a)*B*a^3*b^4*c^2*d^4*g^3/(d*x + c) +
 360*(b*x + a)^2*A*a^2*b^4*c^3*d^4*g^3/(d*x + c)^2 + 360*(b*x + a)^2*B*a^2*
b^4*c^3*d^4*g^3/(d*x + c)^2 + 120*(b*x + a)^3*A*a*b^4*c^4*d^4*g^3/(d*x + c)
^3 + 120*(b*x + a)^3*B*a*b^4*c^4*d^4*g^3/(d*x + c)^3 - 6*A*a^5*b^3*d^5*g^3
- 6*B*a^5*b^3*d^5*g^3 - 120*(b*x + a)*A*a^4*b^3*c*d^5*g^3/(d*x + c) - 120*(
b*x + a)*B*a^4*b^3*c*d^5*g^3/(d*x + c) - 360*(b*x + a)^2*A*a^3*b^3*c^2*d^5*
g^3/(d*x + c)^2 - 360*(b*x + a)^2*B*a^3*b^3*c^2*d^5*g^3/(d*x + c)^2 - 240*(
b*x + a)^3*A*a^2*b^3*c^3*d^5*g^3/(d*x + c)^3 - 240*(b*x + a)^3*B*a^2*b^3*c^
3*d^5*g^3/(d*x + c)^3 + 24*(b*x + a)*A*a^5*b^2*d^6*g^3/(d*x + c) + 24*(b*x
+ a)*B*a^5*b^2*d^6*g^3/(d*x + c) + 180*(b*x + a)^2*A*a^4*b^2*c*d^6*g^3/(d*x
 + c)^2 + 180*(b*x + a)^2*B*a^4*b^2*c*d^6*g^3/(d*x + c)^2 + 240*(b*x + a)^3
*A*a^3*b^2*c^2*d^6*g^3/(d*x + c)^3 + 240*(b*x + a)^3*B*a^3*b^2*c^2*d^6*g^3/
(d*x + c)^3 - 36*(b*x + a)^2*A*a^5*b*d^7*g^3/(d*x + c)^2 - 36*(b*x + a)^2*B
*a^5*b*d^7*g^3/(d*x + c)^2 - 120*(b*x + a)^3*A*a^4*b*c*d^7*g^3/(d*x + c)^3
- 120*(b*x + a)^3*B*a^4*b*c*d^7*g^3/(d*x + c)^3 + 24*(b*x + a)^3*A*a^5*d^8*
g^3/(d*x + c)^3 + 24*(b*x + a)^3*B*a^5*d^8*g^3/(d*x + c)^3)/(b^4*d^4 - 4*(b
*x + a)*b^3*d^5/(d*x + c) + 6*(b*x + a)^2*b^2*d^6/(d*x + c)^2 - 4*(b*x + a)
^3*b*d^7/(d*x + c)^3 + (b*x + a)^4*d^8/(d*x + c)^4) + 6*(B*b^5*c^5*g^3*n -
5*B*a*b^4*c^4*d*g^3*n + 10*B*a^2*b^3*c^3*d^2*g^3*n - 10*B*a^3*b^2*c^2*d^3*g
^3*n + 5*B*a^4*b*c*d^4*g^3*n - B*a^5*d^5*g^3*n)*log(-b + (b*x + a)*d/(d*x +
 c))/(b*d^4) - 6*(B*b^5*c^5*g^3*n - 5*B*a*b^4*c^4*d*g^3*n + 10*B*a^2*b^3*c^
3*d^2*g^3*n - 10*B*a^3*b^2*c^2*d^3*g^3*n + 5*B*a^4*b*c*d^4*g^3*n - B*a^5*d^
5*g^3*n)*log((b*x + a)/(d*x + c))/(b*d^4))*(b*c/(b*c - a*d)^2 - a*d/(b*c -
a*d)^2)
```

Mupad [B]
time = 4.40, size = 588, normalized size = 3.77

$$x^3\left(\frac{b^2g^3(16Aad+4Abc+Badn-Bbcn)}{12d}-\frac{Ab^2g^3(4ad+4bc)}{12d}\right)-x^2\left(\frac{\left(\frac{b^2g^3(16Aad+4Abc+Badn-Bbcn)}{4d}-\frac{Ab^2g^3(4ad+4bc)}{4d}\right)(4ad+4bc)}{8bd}-\frac{abg^3(6Aad+4Abc+Badn-Bbcn)}{2d}+\frac{Aab^2cg^3}{2d}\right)+\ln\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\left(Ba^3g^3x+\frac{3Ba^2bg^3x^2}{2}+Bab^2g^3x^3+\frac{Bb^3g^3x^4}{4}\right)+x\left(\frac{(4ad+4bc)\left(\frac{\left(\frac{b^2g^3(16Aad+4Abc+Badn-Bbcn)}{4d}-\frac{Ab^2g^3(4ad+4bc)}{4d}\right)(4ad+4bc)}{4bd}-\frac{abg^3(6Aad+4Abc+Badn-Bbcn)}{d}+\frac{Aab^2cg^3}{d}\right)}{4bd}+\frac{a^2g^3(8Aad+12Abc+3Badn-3Bbcn)}{2d}-\frac{ac\left(\frac{b^2g^3(16Aad+4Abc+Badn-Bbcn)}{4d}-\frac{Ab^2g^3(4ad+4bc)}{4d}\right)}{bd}\right)+\frac{\ln(c+dx)(-4Bna^3cd^3g^3+6Bna^2bc^2d^2g^3-4Bnab^2c^3dg^3+Bnb^3c^4g^3)}{4d^4}+\frac{Ab^3g^3x^4}{4}+\frac{Ba^4g^3n\ln(a+bx)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*g + b*g*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n)),x)
```

```
[Out] x^3*((b^2*g^3*(16*A*a*d + 4*A*b*c + B*a*d*n - B*b*c*n))/(12*d) - (A*b^2*g^3
*(4*a*d + 4*b*c))/(12*d)) - x^2*((((b^2*g^3*(16*A*a*d + 4*A*b*c + B*a*d*n -
 B*b*c*n))/(4*d) - (A*b^2*g^3*(4*a*d + 4*b*c))/(4*d))*(4*a*d + 4*b*c))/(8*b
*d) - (a*b*g^3*(6*A*a*d + 4*A*b*c + B*a*d*n - B*b*c*n))/(2*d) + (A*a*b^2*c*
g^3)/(2*d)) + log(e*((a + b*x)/(c + d*x))^n)*((B*b^3*g^3*x^4)/4 + B*a^3*g^3
*x + (3*B*a^2*b*g^3*x^2)/2 + B*a*b^2*g^3*x^3) + x*(((4*a*d + 4*b*c)*((((b^2
*g^3*(16*A*a*d + 4*A*b*c + B*a*d*n - B*b*c*n))/(4*d) - (A*b^2*g^3*(4*a*d +
4*b*c))/(4*d))*(4*a*d + 4*b*c))/(4*b*d) - (a*b*g^3*(6*A*a*d + 4*A*b*c + B*a
```

```
*d*n - B*b*c*n))/d + (A*a*b^2*c*g^3)/d))/(4*b*d) + (a^2*g^3*(8*A*a*d + 12*A
*b*c + 3*B*a*d*n - 3*B*b*c*n))/(2*d) - (a*c*((b^2*g^3*(16*A*a*d + 4*A*b*c +
 B*a*d*n - B*b*c*n))/(4*d) - (A*b^2*g^3*(4*a*d + 4*b*c))/(4*d)))/(b*d)) + (
log(c + d*x)*(B*b^3*c^4*g^3*n - 4*B*a^3*c*d^3*g^3*n - 4*B*a*b^2*c^3*d*g^3*n
 + 6*B*a^2*b*c^2*d^2*g^3*n))/(4*d^4) + (A*b^3*g^3*x^4)/4 + (B*a^4*g^3*n*log
(a + b*x))/(4*b)
```

3.3 $\int (ag+bgx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$

Optimal. Leaf size=124

$$\frac{B(bc-ad)^2g^2nx}{3d^2}-\frac{B(bc-ad)g^2n(a+bx)^2}{6bd}+\frac{g^2(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3b}-\frac{B(bc-ad)^3g^2n\log(c+$$

```
[Out] 1/3*B*(-a*d+b*c)^2*g^2*n*x/d^2-1/6*B*(-a*d+b*c)*g^2*n*(b*x+a)^2/b/d+1/3*g^2
*(b*x+a)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b-1/3*B*(-a*d+b*c)^3*g^2*n*ln(d*
x+c)/b/d^3
```

Rubi [A]
time = 0.06, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2547, 21, 45}

$$\frac{g^2(a+bx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3b}-\frac{Bg^2n(bc-ad)^3\log(c+dx)}{3bd^3}+\frac{Bg^2nx(bc-ad)^2}{3d^2}-\frac{Bg^2n(a+bx)^2(bc-ad)}{6bd}$$

Antiderivative was successfully verified.

```
[In] Int[(a*g + b*g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

```
[Out] (B*(b*c - a*d)^2*g^2*n*x)/(3*d^2) - (B*(b*c - a*d)*g^2*n*(a + b*x)^2)/(6*b*
d) + (g^2*(a + b*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(3*b) - (B*(b
*c - a*d)^3*g^2*n*Log[c + d*x])/(3*b*d^3)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2547

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*((A +
 B*Log[e*((a + b*x)/(c + d*x))^n])/(g*(m + 1))), x] - Dist[B*n*((b*c - a*d)
/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /; FreeQ
[{a, b, c, d, e, f, g, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
```

```
& NeQ[m, -2]
```

Rubi steps

$$\begin{aligned}\int (ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)dx &= \frac{g^2(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3b}-\frac{(Bn)\int\frac{(bc-ad)g^3(a+}{c+dx}}{3bg}\\ &= \frac{g^2(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3b}-\frac{(B(bc-ad)g^2n)\int}{3b}\\ &= \frac{g^2(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3b}-\frac{(B(bc-ad)g^2n)\int}{}\\ &= \frac{B(bc-ad)^2g^2nx}{3d^2}-\frac{B(bc-ad)g^2n(a+bx)^2}{6bd}+\frac{g^2(a+bx}{}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 103, normalized size = 0.83

$$\frac{g^2\left((a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)+\frac{B(-bc+ad)n\left(d\left(a^2d+4abdx+b^2x(-2c+dx)\right)+2(bc-ad)^2\log(c+dx)\right)}{2d^3}\right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*g + b*g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

```
[Out] (g^2*((a + b*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + (B*(-(b*c) + a*d
)*n*(d*(a^2*d + 4*a*b*d*x + b^2*x*(-2*c + d*x)) + 2*(b*c - a*d)^2*Log[c + d
*x]))/(2*d^3)))/(3*b)
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int (bgx+ag)^2\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*g*x+a*g)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

```
[Out] int((b*g*x+a*g)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 312 vs. 2(117) = 234.
time = 0.28, size = 312, normalized size = 2.52

$$\frac{1}{3}Bb^2g^2x^3\log\left(\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^ne\right)+\frac{1}{3}Ab^2g^2x^3+Babg^2x^2\log\left(\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^ne\right)+Aabg^2x^2+\frac{1}{6}Bb^2g^2n\left(\frac{2a^3\log(bx+a)}{b^3}-\frac{2c^3\log(dx+c)}{d^3}-\frac{(b^2cd-abd^2)x^2-2(b^2c^2-a^2d^2)x}{b^2d^2}\right)-Babg^2n\left(\frac{a^2\log(bx+a)}{b^2}-\frac{c^2\log(dx+c)}{d^2}+\frac{(bc-ad)x}{bd}\right)+Ba^2g^2n\left(\frac{a\log(bx+a)}{b}-\frac{c\log(dx+c)}{d}\right)+Ba^2g^2x\log\left(\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^ne\right)+Aa^2g^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="maxi
ma")
```

```
[Out] 1/3*B*b^2*g^2*x^3*log((b*x/(d*x + c) + a/(d*x + c))^n*e) + 1/3*A*b^2*g^2*x^
3 + B*a*b*g^2*x^2*log((b*x/(d*x + c) + a/(d*x + c))^n*e) + A*a*b*g^2*x^2 +
1/6*B*b^2*g^2*n*(2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*
d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2)) - B*a*b*g^2*n*(a^2*l
og(b*x + a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d)) + B*a^2*g^2*n
*(a*log(b*x + a)/b - c*log(d*x + c)/d) + B*a^2*g^2*x*log((b*x/(d*x + c) + a
/(d*x + c))^n*e) + A*a^2*g^2*x
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 253 vs. $2(117) = 234$.
time = 0.39, size = 253, normalized size = 2.04

$$\frac{2\,(A+B)b^3d^3g^2x^3+2\,Ba^3d^3g^2n\log\left(bx+a\right)-2\,(Bb^3c^3-3\,Bab^2c^2d+3\,Ba^2bcd^2)g^2n\log\left(dx+c\right)+(6\,(A+B)ab^2d^3g^2-(Bb^3cd^2-Bab^2d^3)g^2n)x^2+2\,(3\,(A+B)a^2bd^3g^2+(Bb^3c^2d-3\,Bab^2cd^2+2\,Ba^2bd^3)g^2n)x+2\,(Bb^3d^3g^2nx^3+3\,Bab^2d^3g^2nx^2+3\,Ba^2bd^3g^2nx)\log\left(\frac{bx+a}{dx+c}\right)}{6\,bd^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fric
as")
```

```
[Out] 1/6*(2*(A + B)*b^3*d^3*g^2*x^3 + 2*B*a^3*d^3*g^2*n*log(b*x + a) - 2*(B*b^3*
c^3 - 3*B*a*b^2*c^2*d + 3*B*a^2*b*c*d^2)*g^2*n*log(d*x + c) + (6*(A + B)*a*
b^2*d^3*g^2 - (B*b^3*c*d^2 - B*a*b^2*d^3)*g^2*n)*x^2 + 2*(3*(A + B)*a^2*b*d
^3*g^2 + (B*b^3*c^2*d - 3*B*a*b^2*c*d^2 + 2*B*a^2*b*d^3)*g^2*n)*x + 2*(B*b^
3*d^3*g^2*n*x^3 + 3*B*a*b^2*d^3*g^2*n*x^2 + 3*B*a^2*b*d^3*g^2*n*x)*log((b*x
 + a)/(d*x + c)))/(b*d^3)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)**2*(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1836 vs. $2(117) = 234$.
time = 4.59, size = 1836, normalized size = 14.81

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="giac
")
```

```
[Out] 1/6*(2*(B*b^6*c^4*g^2*n - 4*B*a*b^5*c^3*d*g^2*n - 3*(b*x + a)*B*b^5*c^4*d*g
^2*n/(d*x + c) + 6*B*a^2*b^4*c^2*d^2*g^2*n + 12*(b*x + a)*B*a*b^4*c^3*d^2*g
^2*n/(d*x + c) + 3*(b*x + a)^2*B*b^4*c^4*d^2*g^2*n/(d*x + c)^2 - 4*B*a^3*b^
3*c*d^3*g^2*n - 18*(b*x + a)*B*a^2*b^3*c^2*d^3*g^2*n/(d*x + c) - 12*(b*x +
a)^2*B*a*b^3*c^3*d^3*g^2*n/(d*x + c)^2 + B*a^4*b^2*d^4*g^2*n + 12*(b*x + a)
*B*a^3*b^2*c*d^4*g^2*n/(d*x + c) + 18*(b*x + a)^2*B*a^2*b^2*c^2*d^4*g^2*n/(
d*x + c)^2 - 3*(b*x + a)*B*a^4*b*d^5*g^2*n/(d*x + c) - 12*(b*x + a)^2*B*a^3
*b*c*d^5*g^2*n/(d*x + c)^2 + 3*(b*x + a)^2*B*a^4*d^6*g^2*n/(d*x + c)^2)*log
((b*x + a)/(d*x + c))/(b^3*d^3 - 3*(b*x + a)*b^2*d^4/(d*x + c) + 3*(b*x + a
)^2*b*d^5/(d*x + c)^2 - (b*x + a)^3*d^6/(d*x + c)^3) + (3*B*b^6*c^4*g^2*n -
 12*B*a*b^5*c^3*d*g^2*n - 7*(b*x + a)*B*b^5*c^4*d*g^2*n/(d*x + c) + 18*B*a^
2*b^4*c^2*d^2*g^2*n + 28*(b*x + a)*B*a*b^4*c^3*d^2*g^2*n/(d*x + c) + 4*(b*x
 + a)^2*B*b^4*c^4*d^2*g^2*n/(d*x + c)^2 - 12*B*a^3*b^3*c*d^3*g^2*n - 42*(b*
x + a)*B*a^2*b^3*c^2*d^3*g^2*n/(d*x + c) - 16*(b*x + a)^2*B*a*b^3*c^3*d^3*g
^2*n/(d*x + c)^2 + 3*B*a^4*b^2*d^4*g^2*n + 28*(b*x + a)*B*a^3*b^2*c*d^4*g^2
*n/(d*x + c) + 24*(b*x + a)^2*B*a^2*b^2*c^2*d^4*g^2*n/(d*x + c)^2 - 7*(b*x
+ a)*B*a^4*b*d^5*g^2*n/(d*x + c) - 16*(b*x + a)^2*B*a^3*b*c*d^5*g^2*n/(d*x
+ c)^2 + 4*(b*x + a)^2*B*a^4*d^6*g^2*n/(d*x + c)^2 + 2*A*b^6*c^4*g^2 + 2*B*
b^6*c^4*g^2 - 8*A*a*b^5*c^3*d*g^2 - 8*B*a*b^5*c^3*d*g^2 - 6*(b*x + a)*A*b^5
*c^4*d*g^2/(d*x + c) - 6*(b*x + a)*B*b^5*c^4*d*g^2/(d*x + c) + 12*A*a^2*b^4
*c^2*d^2*g^2 + 12*B*a^2*b^4*c^2*d^2*g^2 + 24*(b*x + a)*A*a*b^4*c^3*d^2*g^2/
(d*x + c) + 24*(b*x + a)*B*a*b^4*c^3*d^2*g^2/(d*x + c) + 6*(b*x + a)^2*A*b^
4*c^4*d^2*g^2/(d*x + c)^2 + 6*(b*x + a)^2*B*b^4*c^4*d^2*g^2/(d*x + c)^2 - 8
*A*a^3*b^3*c*d^3*g^2 - 8*B*a^3*b^3*c*d^3*g^2 - 36*(b*x + a)*A*a^2*b^3*c^2*d
^3*g^2/(d*x + c) - 36*(b*x + a)*B*a^2*b^3*c^2*d^3*g^2/(d*x + c) - 24*(b*x +
 a)^2*A*a*b^3*c^3*d^3*g^2/(d*x + c)^2 - 24*(b*x + a)^2*B*a*b^3*c^3*d^3*g^2/
(d*x + c)^2 + 2*A*a^4*b^2*d^4*g^2 + 2*B*a^4*b^2*d^4*g^2 + 24*(b*x + a)*A*a^
3*b^2*c*d^4*g^2/(d*x + c) + 24*(b*x + a)*B*a^3*b^2*c*d^4*g^2/(d*x + c) + 36
*(b*x + a)^2*A*a^2*b^2*c^2*d^4*g^2/(d*x + c)^2 + 36*(b*x + a)^2*B*a^2*b^2*c
^2*d^4*g^2/(d*x + c)^2 - 6*(b*x + a)*A*a^4*b*d^5*g^2/(d*x + c) - 6*(b*x + a
)*B*a^4*b*d^5*g^2/(d*x + c) - 24*(b*x + a)^2*A*a^3*b*c*d^5*g^2/(d*x + c)^2
- 24*(b*x + a)^2*B*a^3*b*c*d^5*g^2/(d*x + c)^2 + 6*(b*x + a)^2*A*a^4*d^6*g^
2/(d*x + c)^2 + 6*(b*x + a)^2*B*a^4*d^6*g^2/(d*x + c)^2)/(b^3*d^3 - 3*(b*x
+ a)*b^2*d^4/(d*x + c) + 3*(b*x + a)^2*b*d^5/(d*x + c)^2 - (b*x + a)^3*d^6/
(d*x + c)^3) + 2*(B*b^4*c^4*g^2*n - 4*B*a*b^3*c^3*d*g^2*n + 6*B*a^2*b^2*c^2
*d^2*g^2*n - 4*B*a^3*b*c*d^3*g^2*n + B*a^4*d^4*g^2*n)*log(b - (b*x + a)*d/(
d*x + c))/(b*d^3) - 2*(B*b^4*c^4*g^2*n - 4*B*a*b^3*c^3*d*g^2*n + 6*B*a^2*b^
2*c^2*d^2*g^2*n - 4*B*a^3*b*c*d^3*g^2*n + B*a^4*d^4*g^2*n)*log((b*x + a)/(d
*x + c))/(b*d^3))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)
```

Mupad [B]

time = 4.28, size = 303, normalized size = 2.44

$$\ln\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\left(B a^2 g^2 x + B a b g^2 x^2 + \frac{B b^2 g^2 x^3}{3}\right) - x\left(\frac{(3ad+3bc)\left(\frac{bg^2(9Aad+3Abc+Badn-Bbcn)}{3d} - \frac{Abg^2(3ad+3bc)}{3d}\right)}{3bd} - \frac{ag^2(3Aad+3Abc+Badn-Bbcn)}{d} + \frac{Aabcg^2}{d}\right) + x^2\left(\frac{bg^2(9Aad+3Abc+Badn-Bbcn)}{6d} - \frac{Abg^2(3ad+3bc)}{6d}\right) - \frac{\ln(c+dx)\left(3Bna^2cd^2g^2 - 3Bnabc^2dg^2 + Bnb^2c^3g^2\right)}{3d^3} + \frac{Ab^2g^2x^3}{3} + \frac{Ba^3g^2n\ln(a+bx)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*g + b*g*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n)),x)
```

```
[Out] log(e*((a + b*x)/(c + d*x))^n)*((B*b^2*g^2*x^3)/3 + B*a^2*g^2*x + B*a*b*g^2
*x^2) - x*(((3*a*d + 3*b*c)*((b*g^2*(9*A*a*d + 3*A*b*c + B*a*d*n - B*b*c*n)
)/(3*d) - (A*b*g^2*(3*a*d + 3*b*c))/(3*d)))/(3*b*d) - (a*g^2*(3*A*a*d + 3*A
*b*c + B*a*d*n - B*b*c*n))/d + (A*a*b*c*g^2)/d) + x^2*((b*g^2*(9*A*a*d + 3*
A*b*c + B*a*d*n - B*b*c*n))/(6*d) - (A*b*g^2*(3*a*d + 3*b*c))/(6*d)) - (log
(c + d*x)*(B*b^2*c^3*g^2*n + 3*B*a^2*c*d^2*g^2*n - 3*B*a*b*c^2*d*g^2*n))/(3
*d^3) + (A*b^2*g^2*x^3)/3 + (B*a^3*g^2*n*log(a + b*x))/(3*b)
```

3.4 $\int(ag + bgx)\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$

Optimal. Leaf size=86

$$-\frac{B(bc-ad)gnx}{2d}+\frac{g(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2b}+\frac{B(bc-ad)^2gn\log(c+dx)}{2bd^2}$$

```
[Out] -1/2*B*(-a*d+b*c)*g*n*x/d+1/2*g*(b*x+a)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b
+1/2*B*(-a*d+b*c)^2*g*n*ln(d*x+c)/b/d^2
```

Rubi [A]
time = 0.04, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {2547, 21, 45}

$$\frac{g(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2b}+\frac{Bgn(bc-ad)^2\log(c+dx)}{2bd^2}-\frac{Bgnx(bc-ad)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a*g + b*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

```
[Out] -1/2*(B*(b*c - a*d)*g*n*x)/d + (g*(a + b*x)^2*(A + B*Log[e*((a + b*x)/(c +
d*x))^n]))/(2*b) + (B*(b*c - a*d)^2*g*n*Log[c + d*x])/(2*b*d^2)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2547

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*((A +
 B*Log[e*((a + b*x)/(c + d*x))^n])/(g*(m + 1))), x] - Dist[B*n*((b*c - a*d)
/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /; FreeQ
[{a, b, c, d, e, f, g, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, -2]
```

Rubi steps

$$\begin{aligned}\int (ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)dx &= \frac{g(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2b} - \frac{(Bn)\int\frac{(bc-ad)g^2(a+b}{c+dx}}{2bg} \\ &= \frac{g(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2b} - \frac{(B(bc-ad)gn)\int}{2b} \\ &= \frac{g(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2b} - \frac{(B(bc-ad)gn)\int(}{2b} \\ &= -\frac{B(bc-ad)gnx}{2d} + \frac{g(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2b} +\end{aligned}$$

Mathematica [A]
time = 0.03, size = 73, normalized size = 0.85

$$\frac{g\left((a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)+\frac{B(-bc+ad)n(bdx+(-bc+ad)\log(c+dx))}{d^2}\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*g + b*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

```
[Out] (g*((a + b*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + (B*(-(b*c) + a*d)*
n*(b*d*x + (-(b*c) + a*d)*Log[c + d*x]))/d^2))/(2*b)
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int (bgx+ag)\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*g*x+a*g)*(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

```
[Out] int((b*g*x+a*g)*(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

Maxima [A]
time = 0.28, size = 158, normalized size = 1.84

$$\frac{1}{2}Bbgx^2\log\left(\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n e\right)+\frac{1}{2}Abgx^2-\frac{1}{2}Bbgn\left(\frac{a^2\log(bx+a)}{b^2}-\frac{c^2\log(dx+c)}{d^2}+\frac{(bc-ad)x}{bd}\right)+Bagn\left(\frac{a\log(bx+a)}{b}-\frac{c\log(dx+c)}{d}\right)+Bagx\log\left(\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n e\right)+Aagx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="maxima
")
```

```
[Out] 1/2*B*b*g*x^2*log((b*x/(d*x + c) + a/(d*x + c))^n*e) + 1/2*A*b*g*x^2 - 1/2*
B*b*g*n*(a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d))
 + B*a*g*n*(a*log(b*x + a)/b - c*log(d*x + c)/d) + B*a*g*x*log((b*x/(d*x +
c) + a/(d*x + c))^n*e) + A*a*g*x
```

Fricas [A]
time = 0.47, size = 138, normalized size = 1.60

$$\frac{(A+B)b^2d^2gx^2 + Ba^2d^2gn\log(bx+a) + (Bb^2c^2 - 2\,Babcd)gn\log(dx+c) + (2\,(A+B)abd^2g - (Bb^2cd - Babd^2)gn)x + (Bb^2d^2gnx^2 + 2\,Babd^2gnx)\log\left(\frac{bx+a}{dx+c}\right)}{2\,bd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fricas
")
```

```
[Out] 1/2*((A + B)*b^2*d^2*g*x^2 + B*a^2*d^2*g*n*log(b*x + a) + (B*b^2*c^2 - 2*B*
a*b*c*d)*g*n*log(d*x + c) + (2*(A + B)*a*b*d^2*g - (B*b^2*c*d - B*a*b*d^2)*
g*n)*x + (B*b^2*d^2*g*n*x^2 + 2*B*a*b*d^2*g*n*x)*log((b*x + a)/(d*x + c)))/
(b*d^2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 352 vs. 2(73) = 146.
time = 162.56, size = 352, normalized size = 4.09

$$\begin{cases} agx\left(A + B\log\left(e\left(\frac{a}{c}\right)^n\right)\right) & \text{for}\ b = 0 \wedge d = 0 \\ Aagx + \frac{Abgx^2}{2} + \frac{Ba^2g\log\left(e\left(\frac{a}{c} + \frac{bx}{c}\right)^n\right)}{2b} - \frac{Bagnx}{2} + Bagx\log\left(e\left(\frac{a}{c} + \frac{bx}{c}\right)^n\right) - \frac{Bbgnx^2}{4} + \frac{Bbgx^2\log\left(e\left(\frac{a}{c} + \frac{bx}{c}\right)^n\right)}{2} & \text{for}\ d = 0 \\ ag\left(Ax + \frac{Bc\log\left(e\left(\frac{a}{c+dx}\right)^n\right)}{d} + Bnx + Bx\log\left(e\left(\frac{a}{c+dx}\right)^n\right)\right) & \text{for}\ b = 0 \\ Aagx + \frac{Abgx^2}{2} + \frac{Ba^2gn\log\left(\frac{c}{d} + x\right)}{2b} + \frac{Ba^2g\log\left(e\left(\frac{a}{c+dx} + \frac{bx}{c+dx}\right)^n\right)}{2b} - \frac{Bacgn\log\left(\frac{c}{d} + x\right)}{d} + \frac{Bagnx}{2} + Bagx\log\left(e\left(\frac{a}{c+dx} + \frac{bx}{c+dx}\right)^n\right) + \frac{Bbc^2gn\log\left(\frac{c}{d} + x\right)}{2d^2} - \frac{Bbcgnx}{2d} + \frac{Bbgx^2\log\left(e\left(\frac{a}{c+dx} + \frac{bx}{c+dx}\right)^n\right)}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)*(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

```
[Out] Piecewise((a*g*x*(A + B*log(e*(a/c)**n)), Eq(b, 0) & Eq(d, 0)), (A*a*g*x +
A*b*g*x**2/2 + B*a**2*g*log(e*(a/c + b*x/c)**n)/(2*b) - B*a*g*n*x/2 + B*a*g
*x*log(e*(a/c + b*x/c)**n) - B*b*g*n*x**2/4 + B*b*g*x**2*log(e*(a/c + b*x/c
)**n)/2, Eq(d, 0)), (a*g*(A*x + B*c*log(e*(a/(c + d*x))**n)/d + B*n*x + B*x
*log(e*(a/(c + d*x))**n)), Eq(b, 0)), (A*a*g*x + A*b*g*x**2/2 + B*a**2*g*n*
log(c/d + x)/(2*b) + B*a**2*g*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(2*b)
 - B*a*c*g*n*log(c/d + x)/d + B*a*g*n*x/2 + B*a*g*x*log(e*(a/(c + d*x) + b*
x/(c + d*x))**n) + B*b*c**2*g*n*log(c/d + x)/(2*d**2) - B*b*c*g*n*x/(2*d) +
 B*b*g*x**2*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/2, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 864 vs. 2(81) = 162.
time = 5.76, size = 864, normalized size = 10.05

$$-\frac{1}{2}\left(\frac{\left(Bb^4c^3gn - 3\,Bab^3c^2dgn - \frac{2\,(bx+a)Bb^3c^3dgn}{dx+c} + 3\,Ba^2b^2cd^2gn + \frac{6\,(bx+a)Bab^2c^2d^2gn}{dx+c} - Ba^3bd^3gn - \frac{6\,(bx+a)Ba^2bcd^3gn}{dx+c} + \frac{2\,(bx+a)Ba^3d^4gn}{dx+c}\right)\log\left(\frac{bx+a}{dx+c}\right)}{b^2d^2 - \frac{2\,(bx+a)bd^3}{dx+c} + \frac{(bx+a)^2d^4}{(dx+c)^2}} + \frac{Bb^4c^3gn - 3\,Bab^3c^2dgn - \frac{(bx+a)Bb^3c^3dgn}{dx+c} + 3\,Ba^2b^2cd^2gn + \frac{3\,(bx+a)Bab^2c^2d^2gn}{dx+c} - Ba^3bd^3gn - \frac{3\,(bx+a)Ba^2bcd^3gn}{dx+c} + \frac{(bx+a)Ba^3d^4gn}{dx+c} + Ab^4c^3g + Bb^4c^3g - 3\,Aab^3c^2dg - 3\,Bab^3c^2dg - \frac{2\,(bx+a)Ab^3c^3dg}{dx+c} - \frac{2\,(bx+a)Bb^3c^3dg}{dx+c} + 3\,Aa^2b^2cd^2g + 3\,Ba^2b^2cd^2g + \frac{6\,(bx+a)Aab^2c^2d^2g}{dx+c} + \frac{6\,(bx+a)Bab^2c^2d^2g}{dx+c} - Aa^3bd^3g - Ba^3bd^3g - \frac{6\,(bx+a)Aa^2bcd^3g}{dx+c} - \frac{6\,(bx+a)Ba^2bcd^3g}{dx+c} + \frac{2\,(bx+a)Aa^3d^4g}{dx+c} + \frac{2\,(bx+a)Ba^3d^4g}{dx+c}}{b^2d^2 - \frac{2\,(bx+a)bd^3}{dx+c} + \frac{(bx+a)^2d^4}{(dx+c)^2}} + \frac{(Bb^3c^3gn - 3\,Bab^2c^2dgn + 3\,Ba^2bcd^2gn - Ba^3d^3gn)\log\left(-b + \frac{(bx+a)d}{dx+c}\right)}{bd^2} - \frac{(Bb^3c^3gn - 3\,Bab^2c^2dgn + 3\,Ba^2bcd^2gn - Ba^3d^3gn)\log\left(\frac{bx+a}{dx+c}\right)}{bd^2}\right)\left(\frac{bc}{(bc-ad)^2} - \frac{ad}{(bc-ad)^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="giac")
```

```
[Out] -1/2*((B*b^4*c^3*g*n - 3*B*a*b^3*c^2*d*g*n - 2*(b*x + a)*B*b^3*c^3*d*g*n/(d
*x + c) + 3*B*a^2*b^2*c*d^2*g*n + 6*(b*x + a)*B*a*b^2*c^2*d^2*g*n/(d*x + c)
 - B*a^3*b*d^3*g*n - 6*(b*x + a)*B*a^2*b*c*d^3*g*n/(d*x + c) + 2*(b*x + a)*
B*a^3*d^4*g*n/(d*x + c))*log((b*x + a)/(d*x + c))/(b^2*d^2 - 2*(b*x + a)*b*
d^3/(d*x + c) + (b*x + a)^2*d^4/(d*x + c)^2) + (B*b^4*c^3*g*n - 3*B*a*b^3*c
^2*d*g*n - (b*x + a)*B*b^3*c^3*d*g*n/(d*x + c) + 3*B*a^2*b^2*c*d^2*g*n + 3*
(b*x + a)*B*a*b^2*c^2*d^2*g*n/(d*x + c) - B*a^3*b*d^3*g*n - 3*(b*x + a)*B*a
^2*b*c*d^3*g*n/(d*x + c) + (b*x + a)*B*a^3*d^4*g*n/(d*x + c) + A*b^4*c^3*g
+ B*b^4*c^3*g - 3*A*a*b^3*c^2*d*g - 3*B*a*b^3*c^2*d*g - 2*(b*x + a)*A*b^3*c
^3*d*g/(d*x + c) - 2*(b*x + a)*B*b^3*c^3*d*g/(d*x + c) + 3*A*a^2*b^2*c*d^2*
g + 3*B*a^2*b^2*c*d^2*g + 6*(b*x + a)*A*a*b^2*c^2*d^2*g/(d*x + c) + 6*(b*x
+ a)*B*a*b^2*c^2*d^2*g/(d*x + c) - A*a^3*b*d^3*g - B*a^3*b*d^3*g - 6*(b*x +
 a)*A*a^2*b*c*d^3*g/(d*x + c) - 6*(b*x + a)*B*a^2*b*c*d^3*g/(d*x + c) + 2*(
b*x + a)*A*a^3*d^4*g/(d*x + c) + 2*(b*x + a)*B*a^3*d^4*g/(d*x + c))/(b^2*d^
2 - 2*(b*x + a)*b*d^3/(d*x + c) + (b*x + a)^2*d^4/(d*x + c)^2) + (B*b^3*c^3
*g*n - 3*B*a*b^2*c^2*d*g*n + 3*B*a^2*b*c*d^2*g*n - B*a^3*d^3*g*n)*log(-b +
(b*x + a)*d/(d*x + c))/(b*d^2) - (B*b^3*c^3*g*n - 3*B*a*b^2*c^2*d*g*n + 3*B
*a^2*b*c*d^2*g*n - B*a^3*d^3*g*n)*log((b*x + a)/(d*x + c))/(b*d^2))*(b*c/(b
*c - a*d)^2 - a*d/(b*c - a*d)^2)
```

Mupad [B]
time = 4.07, size = 134, normalized size = 1.56

$$x\left(\frac{g\,\left(4\,A\,a\,d+2\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n\right)}{2\,d}-\frac{A\,g\,\left(2\,a\,d+2\,b\,c\right)}{2\,d}\right)+\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\,\left(\frac{B\,b\,g\,x^2}{2}+B\,a\,g\,x\right)+\frac{\ln\left(c+d\,x\right)\,\left(B\,b\,c^2\,g\,n-2\,B\,a\,c\,d\,g\,n\right)}{2\,d^2}+\frac{A\,b\,g\,x^2}{2}+\frac{B\,a^2\,g\,n\,\ln\left(a+b\,x\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*g + b*g*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n)),x)
```

```
[Out] x*((g*(4*A*a*d + 2*A*b*c + B*a*d*n - B*b*c*n))/(2*d) - (A*g*(2*a*d + 2*b*c)
)/(2*d)) + log(e*((a + b*x)/(c + d*x))^n)*((B*b*g*x^2)/2 + B*a*g*x) + (log(
c + d*x)*(B*b*c^2*g*n - 2*B*a*c*d*g*n))/(2*d^2) + (A*b*g*x^2)/2 + (B*a^2*g*
n*log(a + b*x))/(2*b)
```

3.5 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{ag+bgx}\,dx$

Optimal. Leaf size=84

$$-\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{bg}+\frac{Bn\text{Li}_2\left(1+\frac{bc-ad}{d(a+bx)}\right)}{bg}$$

```
[Out] -ln((a*d-b*c)/d/(b*x+a))*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b/g+B*n*polylog(2,
1+(-a*d+b*c)/d/(b*x+a))/b/g
```

Rubi [A]
time = 0.14, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {2541, 2458, 2378, 2370, 2352}

$$\frac{Bn\text{PolyLog}\left(2,\frac{bc-ad}{d(a+bx)}+1\right)}{bg}-\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{bg}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(a*g + b*g*x),x]
```

```
[Out] -((Log[-((b*c - a*d)/(d*(a + b*x)))]*(A + B*Log[e*((a + b*x)/(c + d*x))^n])
)/(b*g)) + (B*n*PolyLog[2, 1 + (b*c - a*d)/(d*(a + b*x))])/(b*g)
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2370

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)/(x_))^(q_.)*(
x_)^(m_.), x_Symbol] :> Int[(e + d*x)^q*(a + b*Log[c*x^n])^p, x] /; FreeQ[{
a, b, c, d, e, m, n, p}, x] && EqQ[m, q] && IntegerQ[q]
```

Rule 2378

```
Int[((a_.) + Log[(c_.)*(x_)^(n_)]*(b_.))/((x_)*((d_) + (e_.)*(x_)^(r_.))),
x_Symbol] :> Dist[1/n, Subst[Int[(a + b*Log[c*x])/(x*(d + e*x^(r/n))), x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IntegerQ[r/n]
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
```

```
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2541

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))/((f_.) + (g_.)*(x_)), x_Symbol] :> Simp[(-Log[-(b*c - a*d)/(d*(a + b*
x))])*((A + B*Log[e*((a + b*x)/(c + d*x))^n])/g), x] + Dist[B*n*((b*c - a*d
)/g), Int[Log[-(b*c - a*d)/(d*(a + b*x))]/((a + b*x)*(c + d*x)), x], x] /;
FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[b*f -
a*g, 0]
```

Rubi steps

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{ag+bgx}\,dx = \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log(ag+bgx)}{bg} - \frac{(Bn)\int \frac{(c+dx)\left(-\frac{d(a+bx)}{(c+dx)^2}+\frac{b}{c+dx}\right)\log(ag+bg}{a+bx}}{bg}$$

$$= \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log(ag+bgx)}{bg} - \frac{(Bn)\int\left(\frac{b\log(ag+bgx)}{a+bx}-\frac{d\log(ag+bgx)}{c+dx}\right)}{bg}$$

$$= \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log(ag+bgx)}{bg} - \frac{(Bn)\int\frac{\log(ag+bgx)}{a+bx}\,dx}{g} + \frac{(Bdn)\int \frac{\log}{}}{bg}$$

$$= \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log(ag+bgx)}{bg} + \frac{Bn\log\left(\frac{b(c+dx)}{bc-ad}\right)\log(ag+bgx)}{bg} - (\ $$

$$= \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log(ag+bgx)}{bg} + \frac{Bn\log\left(\frac{b(c+dx)}{bc-ad}\right)\log(ag+bgx)}{bg} - $$

$$= -\frac{Bn\log^2(g(a+bx))}{2bg} + \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log(ag+bgx)}{bg} + \frac{Bn\log\left(\frac{b(}{b}\right.}{}$$

Mathematica [A]
time = 0.04, size = 101, normalized size = 1.20

$$\frac{\log(g(a+bx))\left(-Bn\log(g(a+bx))+2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+Bn\log\left(\frac{b(c+dx)}{bc-ad}\right)\right)\right)+2Bn\text{Li}_2\left(\frac{d(a+bx)}{-bc+ad}\right)}{2bg}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(a*g + b*g*x),x]
```

```
[Out] (Log[g*(a + b*x)]*(-(B*n*Log[g*(a + b*x)]) + 2*(A + B*Log[e*((a + b*x)/(c +
 d*x))^n] + B*n*Log[(b*(c + d*x))/(b*c - a*d)])) + 2*B*n*PolyLog[2, (d*(a +
 b*x))/(-(b*c) + a*d)])/(2*b*g)
```

Maple [F]
time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{A + B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)}{bgx + ag}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g),x)
```

```
[Out] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g),x, algorithm="maxima
")
```

```
[Out] B*((log(b*x + a)*log((b*x + a)^n) - log(b*x + a)*log((d*x + c)^n))/(b*g) +
integrate((b*d*x + b*c - (b*c*n - a*d*n)*log(b*x + a))/(b^2*d*g*x^2 + a*b*c
*g + (b^2*c*g + a*b*d*g)*x), x)) + A*log(b*g*x + a*g)/(b*g)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g),x, algorithm="fricas
")
```

```
[Out] integral((B*log(((b*x + a)/(d*x + c))^n*e) + A)/(b*g*x + a*g), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A}{a+bx}\,dx + \int \frac{B\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^{n}\right)}{a+bx}\,dx}{g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))/(b*g*x+a*g),x)
```

```
[Out] (Integral(A/(a + b*x), x) + Integral(B*log(e*(a/(c + d*x) + b*x/(c + d*x))*
*n)/(a + b*x), x))/g
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g),x, algorithm="giac")
```

```
[Out] integrate((B*log(((b*x + a)/(d*x + c))^n*e) + A)/(b*g*x + a*g), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)}{a\,g+b\,g\,x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log(e*((a + b*x)/(c + d*x))^n))/(a*g + b*g*x),x)
```

```
[Out] int((A + B*log(e*((a + b*x)/(c + d*x))^n))/(a*g + b*g*x), x)
```

3.6 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^2}\,dx$

Optimal. Leaf size=67

$$-\frac{Bn}{bg^2(a+bx)}-\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)g^2(a+bx)}$$

```
[Out] -B*n/b/g^2/(b*x+a)-(d*x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)/g^2/(
b*x+a)
```

Rubi [A]
time = 0.03, antiderivative size = 79, normalized size of antiderivative = 1.18, number of steps used = 2, number of rules used = 2, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.061$, Rules used = {2549, 2341}

$$-\frac{(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{g^2(a+bx)(bc-ad)}-\frac{Bn(c+dx)}{g^2(a+bx)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(a*g + b*g*x)^2,x]
```

```
[Out] -((B*n*(c + d*x))/((b*c - a*d)*g^2*(a + b*x))) - ((c + d*x)*(A + B*Log[e*((
a + b*x)/(c + d*x))^n]))/((b*c - a*d)*g^2*(a + b*x))
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2549

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(m +
1)*(g/b)^m, Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x], x,
(a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ
[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[b*f - a*g, 0] && (GtQ[p, 0] || LtQ
[m, -1])
```

Rubi steps

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^2}\,dx = -\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{bg^2(a+bx)} + \frac{(Bn)\int \frac{bc-ad}{g(a+bx)^2(c+dx)}\,dx}{bg}$$
$$= -\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{bg^2(a+bx)} + \frac{(B(bc-ad)n)\int \frac{1}{(a+bx)^2(c+dx)}\,dx}{bg^2}$$
$$= -\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{bg^2(a+bx)} + \frac{(B(bc-ad)n)\int \left(\frac{b}{(bc-ad)(a+bx)^2} - \frac{bd}{(bc-ad)^2(a+bx)} + \right.}{bg^2}$$
$$= -\frac{Bn}{bg^2(a+bx)} - \frac{Bdn\log(a+bx)}{b(bc-ad)g^2} - \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{bg^2(a+bx)} + \frac{Bdn\log(c+dx}{b(bc-ad)g^2}$$

Mathematica [A]
time = 0.05, size = 115, normalized size = 1.72

$$-\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{bg(ag+bgx)} + \frac{B(bc-ad)n\left(-\frac{1}{(bc-ad)(a+bx)} - \frac{d\log(a+bx)}{(bc-ad)^2} + \frac{d\log(c+dx)}{(bc-ad)^2}\right)}{bg^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(a*g + b*g*x)^2,x]
```

```
[Out] -((A + B*Log[e*((a + b*x)/(c + d*x))^n])/(b*g*(a*g + b*g*x))) + (B*(b*c - a
*d)*n*(-(1/((b*c - a*d)*(a + b*x))) - (d*Log[a + b*x])/(b*c - a*d)^2 + (d*L
og[c + d*x])/(b*c - a*d)^2))/(b*g^2)
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}{(bgx+ag)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^2,x)
```

```
[Out] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^2,x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 138 vs. 2(68) = 136.
time = 0.30, size = 138, normalized size = 2.06

$$-Bn\left(\frac{1}{b^2g^2x+abg^2} + \frac{d\log(bx+a)}{(b^2c-abd)g^2} - \frac{d\log(dx+c)}{(b^2c-abd)g^2}\right) - \frac{B\log\left(\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n e\right)}{b^2g^2x+abg^2} - \frac{A}{b^2g^2x+abg^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^2,x, algorithm="maxi
ma")
```

```
[Out] -B*n*(1/(b^2*g^2*x + a*b*g^2) + d*log(b*x + a)/((b^2*c - a*b*d)*g^2) - d*lo
g(d*x + c)/((b^2*c - a*b*d)*g^2)) - B*log((b*x/(d*x + c) + a/(d*x + c))^n*e
)/(b^2*g^2*x + a*b*g^2) - A/(b^2*g^2*x + a*b*g^2)
```

Fricas [A]
time = 0.43, size = 94, normalized size = 1.40

$$-\frac{(A+B)bc-(A+B)ad+(Bbc-Bad)n+(Bbdnx+Bbcn)\log\left(\frac{bx+a}{dx+c}\right)}{(b^3c-ab^2d)g^2x+(ab^2c-a^2bd)g^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^2,x, algorithm="fric
as")
```

```
[Out] -((A + B)*b*c - (A + B)*a*d + (B*b*c - B*a*d)*n + (B*b*d*n*x + B*b*c*n)*log
((b*x + a)/(d*x + c)))/((b^3*c - a*b^2*d)*g^2*x + (a*b^2*c - a^2*b*d)*g^2)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))/(b*g*x+a*g)**2,x)
```

```
[Out] Exception raised: NotImplementedError >> no valid subset found
```

Giac [A]
time = 3.82, size = 85, normalized size = 1.27

$$-\left(\frac{(dx+c)Bn\log\left(\frac{bx+a}{dx+c}\right)}{(bx+a)g^2}+\frac{(Bn+A+B)(dx+c)}{(bx+a)g^2}\right)\left(\frac{bc}{(bc-ad)^2}-\frac{ad}{(bc-ad)^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^2,x, algorithm="giac
")
```

```
[Out] -((d*x + c)*B*n*log((b*x + a)/(d*x + c))/((b*x + a)*g^2) + (B*n + A + B)*(d
*x + c)/((b*x + a)*g^2))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)
```

Mupad [B]
time = 5.65, size = 112, normalized size = 1.67

$$-\frac{A+B\,n}{x\,b^2\,g^2+a\,b\,g^2}-\frac{B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)}{b\,\left(a\,g^2+b\,g^2\,x\right)}-\frac{B\,d\,n\,\mathrm{atan}\left(\frac{b\,c\,2{}\mathrm{i}+b\,d\,x\,2{}\mathrm{i}}{a\,d-b\,c}+1{}\mathrm{i}\right)\,2{}\mathrm{i}}{b\,g^2\,\left(a\,d-b\,c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log(e*((a + b*x)/(c + d*x))^n))/(a*g + b*g*x)^2,x)
```

```
[Out] - (A + B*n)/(b^2*g^2*x + a*b*g^2) - (B*log(e*((a + b*x)/(c + d*x))^n))/(b*(
a*g^2 + b*g^2*x)) - (B*d*n*atan((b*c*2i + b*d*x*2i)/(a*d - b*c) + 1i)*2i)/(
b*g^2*(a*d - b*c))
```

3.7 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^3}\,dx$

Optimal. Leaf size=151

$$-\frac{Bn}{4bg^3(a+bx)^2}+\frac{Bdn}{2b(bc-ad)g^3(a+bx)}+\frac{Bd^2n\log(a+bx)}{2b(bc-ad)^2g^3}-\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{2bg^3(a+bx)^2}-\frac{Bd^2n\log(c+dx)}{2b(bc-ad)^2g^3}$$

```
[Out] -1/4*B*n/b/g^3/(b*x+a)^2+1/2*B*d*n/b/(-a*d+b*c)/g^3/(b*x+a)+1/2*B*d^2*n*ln(
b*x+a)/b/(-a*d+b*c)^2/g^3+1/2*(-A-B*ln(e*((b*x+a)/(d*x+c))^n))/b/g^3/(b*x+a
)^2-1/2*B*d^2*n*ln(d*x+c)/b/(-a*d+b*c)^2/g^3
```

Rubi [A]
time = 0.09, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {2547, 21, 46}

$$-\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{2bg^3(a+bx)^2}+\frac{Bd^2n\log(a+bx)}{2bg^3(bc-ad)^2}-\frac{Bd^2n\log(c+dx)}{2bg^3(bc-ad)^2}+\frac{Bdn}{2bg^3(a+bx)(bc-ad)}-\frac{Bn}{4bg^3(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(a*g + b*g*x)^3,x]
```

```
[Out] -1/4*(B*n)/(b*g^3*(a + b*x)^2) + (B*d*n)/(2*b*(b*c - a*d)*g^3*(a + b*x)) +
(B*d^2*n*Log[a + b*x])/(2*b*(b*c - a*d)^2*g^3) - (A + B*Log[e*((a + b*x)/(c
 + d*x))^n])/(2*b*g^3*(a + b*x)^2) - (B*d^2*n*Log[c + d*x])/(2*b*(b*c - a*d
)^2*g^3)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2547

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*((A +
```

```
 B*Log[e*((a + b*x)/(c + d*x))^n])/(g*(m + 1))), x] - Dist[B*n*((b*c - a*d)
/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /; FreeQ
[{a, b, c, d, e, f, g, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^3}\,dx &= -\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{2bg^3(a+bx)^2}+\frac{(Bn)\int\frac{bc-ad}{g^2(a+bx)^3(c+dx)}\,dx}{2bg}\\ &= -\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{2bg^3(a+bx)^2}+\frac{(B(bc-ad)n)\int\frac{1}{(a+bx)^3(c+dx)}\,dx}{2bg^3}\\ &= -\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{2bg^3(a+bx)^2}+\frac{(B(bc-ad)n)\int\left(\frac{b}{(bc-ad)(a+bx)^3}-\frac{bd}{(bc-ad)^2(a+bx)^2}+\right.}{2bg^3}\\ &= -\frac{Bn}{4bg^3(a+bx)^2}+\frac{Bdn}{2b(bc-ad)g^3(a+bx)}+\frac{Bd^2n\log(a+bx)}{2b(bc-ad)^2g^3}-\frac{A+B\log\left(\right.}{2bg^3(a+}\end{aligned}$$

Mathematica [A]
time = 0.11, size = 114, normalized size = 0.75

$$-\frac{2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)+\frac{Bn\left((bc-ad)(-3ad+b(c-2dx))-2d^2(a+bx)^2\log(a+bx)+2d^2(a+bx)^2\log(c+dx)\right)}{(bc-ad)^2}}{4bg^3(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(a*g + b*g*x)^3,x]
```

```
[Out] -1/4*(2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + (B*n*((b*c - a*d)*(-3*a*d
+ b*(c - 2*d*x)) - 2*d^2*(a + b*x)^2*Log[a + b*x] + 2*d^2*(a + b*x)^2*Log[c
 + d*x]))/(b*c - a*d)^2)/(b*g^3*(a + b*x)^2)
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}{\left(bgx+ag\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^3,x)
```

```
[Out] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^3,x)
```

Maxima [A]
time = 0.31, size = 260, normalized size = 1.72

$$\frac{1}{4}\,Bn\left(\frac{2\,bdx-bc+3\,ad}{(b^4c-ab^3d)g^3x^2+2\,(ab^3c-a^2b^2d)g^3x+(a^2b^2c-a^3bd)g^3}+\frac{2\,d^2\log\left(bx+a\right)}{(b^3c^2-2\,ab^2cd+a^2bd^2)g^3}-\frac{2\,d^2\log\left(dx+c\right)}{(b^3c^2-2\,ab^2cd+a^2bd^2)g^3}\right)-\frac{B\log\left(\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^ne\right)}{2\,(b^3g^3x^2+2\,ab^2g^3x+a^2bg^3)}-\frac{A}{2\,(b^3g^3x^2+2\,ab^2g^3x+a^2bg^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^3,x, algorithm="maxi
ma")
```

```
[Out] 1/4*B*n*((2*b*d*x - b*c + 3*a*d)/((b^4*c - a*b^3*d)*g^3*x^2 + 2*(a*b^3*c -
a^2*b^2*d)*g^3*x + (a^2*b^2*c - a^3*b*d)*g^3) + 2*d^2*log(b*x + a)/((b^3*c^
2 - 2*a*b^2*c*d + a^2*b*d^2)*g^3) - 2*d^2*log(d*x + c)/((b^3*c^2 - 2*a*b^2*
c*d + a^2*b*d^2)*g^3)) - 1/2*B*log((b*x/(d*x + c) + a/(d*x + c))^n*e)/(b^3*
g^3*x^2 + 2*a*b^2*g^3*x + a^2*b*g^3) - 1/2*A/(b^3*g^3*x^2 + 2*a*b^2*g^3*x +
 a^2*b*g^3)
```

Fricas [A]
time = 0.39, size = 243, normalized size = 1.61

$$-\frac{2\,(A+B)b^2c^2-4\,(A+B)abcd+2\,(A+B)a^2d^2-2\,(Bb^2cd-Babd^2)nx+(Bb^2c^2-4\,Babcd+3\,Ba^2d^2)n-2\,(Bb^2d^2nx^2+2\,Babd^2nx-(Bb^2c^2-2\,Babcd)n)\log\left(\frac{bx+a}{dx+c}\right)}{4\,((b^5c^2-2\,ab^4cd+a^2b^3d^2)g^3x^2+2\,(ab^4c^2-2\,a^2b^3cd+a^3b^2d^2)g^3x+(a^2b^3c^2-2\,a^3b^2cd+a^4bd^2)g^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^3,x, algorithm="fric
as")
```

```
[Out] -1/4*(2*(A + B)*b^2*c^2 - 4*(A + B)*a*b*c*d + 2*(A + B)*a^2*d^2 - 2*(B*b^2*
c*d - B*a*b*d^2)*n*x + (B*b^2*c^2 - 4*B*a*b*c*d + 3*B*a^2*d^2)*n - 2*(B*b^2
*d^2*n*x^2 + 2*B*a*b*d^2*n*x - (B*b^2*c^2 - 2*B*a*b*c*d)*n)*log((b*x + a)/(
d*x + c)))/((b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*g^3*x^2 + 2*(a*b^4*c^2 -
2*a^2*b^3*c*d + a^3*b^2*d^2)*g^3*x + (a^2*b^3*c^2 - 2*a^3*b^2*c*d + a^4*b*d
^2)*g^3)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))/(b*g*x+a*g)**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 4.70, size = 220, normalized size = 1.46

$$-\frac{1}{4}\left(\frac{2\left(Bbn-\frac{2\,(bx+a)Bdn}{dx+c}\right)\log\left(\frac{bx+a}{dx+c}\right)}{\frac{(bx+a)^2bcg^3}{(dx+c)^2}-\frac{(bx+a)^2adg^3}{(dx+c)^2}}+\frac{Bbn-\frac{4\,(bx+a)Bdn}{dx+c}+2\,Ab+2\,Bb-\frac{4\,(bx+a)Ad}{dx+c}-\frac{4\,(bx+a)Bd}{dx+c}}{\frac{(bx+a)^2bcg^3}{(dx+c)^2}-\frac{(bx+a)^2adg^3}{(dx+c)^2}}\right)\left(\frac{bc}{(bc-ad)^2}-\frac{ad}{(bc-ad)^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^3,x, algorithm="giac
")
```

```
[Out] -1/4*(2*(B*b*n - 2*(b*x + a)*B*d*n/(d*x + c))*log((b*x + a)/(d*x + c))/((b*
x + a)^2*b*c*g^3/(d*x + c)^2 - (b*x + a)^2*a*d*g^3/(d*x + c)^2) + (B*b*n -
4*(b*x + a)*B*d*n/(d*x + c) + 2*A*b + 2*B*b - 4*(b*x + a)*A*d/(d*x + c) - 4
*(b*x + a)*B*d/(d*x + c))/((b*x + a)^2*b*c*g^3/(d*x + c)^2 - (b*x + a)^2*a*
d*g^3/(d*x + c)^2))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)
```

Mupad [B]
time = 4.52, size = 222, normalized size = 1.47

$$-\frac{\frac{2\,A\,a\,d-2\,A\,b\,c+3\,B\,a\,d\,n-B\,b\,c\,n}{2\,\left(a\,d-b\,c\right)}+\frac{B\,b\,d\,n\,x}{a\,d-b\,c}}{2\,a^2\,b\,g^3+4\,a\,b^2\,g^3\,x+2\,b^3\,g^3\,x^2}-\frac{B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)}{2\,b\,\left(a^2\,g^3+2\,a\,b\,g^3\,x+b^2\,g^3\,x^2\right)}-\frac{B\,d^2\,n\,\mathrm{atanh}\left(\frac{2\,b^3\,c^2\,g^3-2\,a^2\,b\,d^2\,g^3}{2\,b\,g^3\,{\left(a\,d-b\,c\right)}^2}-\frac{2\,b\,d\,x}{a\,d-b\,c}\right)}{b\,g^3\,{\left(a\,d-b\,c\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log(e*((a + b*x)/(c + d*x))^n))/(a*g + b*g*x)^3,x)
```

```
[Out] - ((2*A*a*d - 2*A*b*c + 3*B*a*d*n - B*b*c*n)/(2*(a*d - b*c)) + (B*b*d*n*x)/
(a*d - b*c))/(2*a^2*b*g^3 + 2*b^3*g^3*x^2 + 4*a*b^2*g^3*x) - (B*log(e*((a +
 b*x)/(c + d*x))^n))/(2*b*(a^2*g^3 + b^2*g^3*x^2 + 2*a*b*g^3*x)) - (B*d^2*n
*atanh((2*b^3*c^2*g^3 - 2*a^2*b*d^2*g^3)/(2*b*g^3*(a*d - b*c)^2) - (2*b*d*x
)/(a*d - b*c)))/(b*g^3*(a*d - b*c)^2)
```

3.8 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^4}\,dx$

Optimal. Leaf size=183

$$-\frac{Bn}{9bg^4(a+bx)^3}+\frac{Bdn}{6b(bc-ad)g^4(a+bx)^2}-\frac{Bd^2n}{3b(bc-ad)^2g^4(a+bx)}-\frac{Bd^3n\log(a+bx)}{3b(bc-ad)^3g^4}-\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{3bg^4(a+bx)^3}$$

```
[Out] -1/9*B*n/b/g^4/(b*x+a)^3+1/6*B*d*n/b/(-a*d+b*c)/g^4/(b*x+a)^2-1/3*B*d^2*n/b
/(-a*d+b*c)^2/g^4/(b*x+a)-1/3*B*d^3*n*ln(b*x+a)/b/(-a*d+b*c)^3/g^4+1/3*(-A-
B*ln(e*((b*x+a)/(d*x+c))^n))/b/g^4/(b*x+a)^3+1/3*B*d^3*n*ln(d*x+c)/b/(-a*d+
b*c)^3/g^4
```

Rubi [A]
time = 0.11, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2547, 21, 46}

$$-\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{3bg^4(a+bx)^3}-\frac{Bd^3n\log(a+bx)}{3bg^4(bc-ad)^3}+\frac{Bd^3n\log(c+dx)}{3bg^4(bc-ad)^3}-\frac{Bd^2n}{3bg^4(a+bx)(bc-ad)^2}+\frac{Bdn}{6bg^4(a+bx)^2(bc-ad)}-\frac{Bn}{9bg^4(a+bx)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(a*g + b*g*x)^4,x]
```

```
[Out] -1/9*(B*n)/(b*g^4*(a + b*x)^3) + (B*d*n)/(6*b*(b*c - a*d)*g^4*(a + b*x)^2)
- (B*d^2*n)/(3*b*(b*c - a*d)^2*g^4*(a + b*x)) - (B*d^3*n*Log[a + b*x])/(3*b
*(b*c - a*d)^3*g^4) - (A + B*Log[e*((a + b*x)/(c + d*x))^n])/(3*b*g^4*(a +
b*x)^3) + (B*d^3*n*Log[c + d*x])/(3*b*(b*c - a*d)^3*g^4)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2547

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*((A +
```

```
B*Log[e*((a + b*x)/(c + d*x))^n])/(g*(m + 1))), x] - Dist[B*n*((b*c - a*d)
/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /; FreeQ
[{a, b, c, d, e, f, g, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^4}\,dx &= -\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{3bg^4(a+bx)^3}+\frac{(Bn)\int\frac{bc-ad}{g^3(a+bx)^4(c+dx)}\,dx}{3bg}\\ &= -\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{3bg^4(a+bx)^3}+\frac{(B(bc-ad)n)\int\frac{1}{(a+bx)^4(c+dx)}\,dx}{3bg^4}\\ &= -\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{3bg^4(a+bx)^3}+\frac{(B(bc-ad)n)\int\left(\frac{b}{(bc-ad)(a+bx)^4}-\frac{bd}{(bc-ad)^2(a+bx)^3}+\right.}{3b}\\ &= -\frac{Bn}{9bg^4(a+bx)^3}+\frac{Bdn}{6b(bc-ad)g^4(a+bx)^2}-\frac{Bd^2n}{3b(bc-ad)^2g^4(a+bx)}-\frac{Bd^3}{3b(}\end{aligned}$$

Mathematica [A]
time = 0.12, size = 145, normalized size = 0.79

$$-\frac{6\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)+\frac{Bn\left((bc-ad)\left(11a^2d^2+abd(-7c+15dx)+b^2\left(2c^2-3cdx+6d^2x^2\right)\right)+6d^3(a+bx)^3\log(a+bx)-6d^3(a+bx)^3\log(c+dx)\right)}{(bc-ad)^3}}{18bg^4(a+bx)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(a*g + b*g*x)^4,x]
```

```
[Out] -1/18*(6*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + (B*n*((b*c - a*d)*(11*a^2
*d^2 + a*b*d*(-7*c + 15*d*x) + b^2*(2*c^2 - 3*c*d*x + 6*d^2*x^2)) + 6*d^3*(
a + b*x)^3*Log[a + b*x] - 6*d^3*(a + b*x)^3*Log[c + d*x]))/(b*c - a*d)^3)/(
b*g^4*(a + b*x)^3)
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}{(bgx+ag)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^4,x)
```

```
[Out] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^4,x)
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 433 vs. $2(172) = 344$.
time = 0.32, size = 433, normalized size = 2.37

$$-\frac{1}{18} \, Bn \left(\frac{6 \, b^{2} d^{2} x^{2} + 2 \, b^{2} c^{2} - 7 \, abcd + 11 \, a^{2} d^{2} - 3 \, {\left(b^{2} cd - 5 \, abd^{2}\right)} x}{{\left(b^{6} c^{2} - 2 \, ab^{5} cd + a^{2} b^{4} d^{2}\right)} g^{4} x^{3} + 3 \, {\left(ab^{5} c^{2} - 2 \, a^{2} b^{4} cd + a^{3} b^{3} d^{2}\right)} g^{4} x^{2} + 3 \, {\left(a^{2} b^{4} c^{2} - 2 \, a^{3} b^{3} cd + a^{4} b^{2} d^{2}\right)} g^{4} x + {\left(a^{3} b^{3} c^{2} - 2 \, a^{4} b^{2} cd + a^{5} bd^{2}\right)} g^{4}} + \frac{6 \, d^{3} \log\left(bx + a\right)}{{\left(b^{4} c^{3} - 3 \, ab^{3} c^{2} d + 3 \, a^{2} b^{2} cd^{2} - a^{3} bd^{3}\right)} g^{4}} - \frac{6 \, d^{3} \log\left(dx + c\right)}{{\left(b^{4} c^{3} - 3 \, ab^{3} c^{2} d + 3 \, a^{2} b^{2} cd^{2} - a^{3} bd^{3}\right)} g^{4}}\right) - \frac{B \log\left({\left(\frac{bx}{dx + c} + \frac{a}{dx + c}\right)}^{n} e\right)}{3 \, {\left(b^{4} g^{4} x^{3} + 3 \, ab^{3} g^{4} x^{2} + 3 \, a^{2} b^{2} g^{4} x + a^{3} bg^{4}\right)}} - \frac{A}{3 \, {\left(b^{4} g^{4} x^{3} + 3 \, ab^{3} g^{4} x^{2} + 3 \, a^{2} b^{2} g^{4} x + a^{3} bg^{4}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^4,x, algorithm="maxi
ma")
```

```
[Out] -1/18*B*n*((6*b^2*d^2*x^2 + 2*b^2*c^2 - 7*a*b*c*d + 11*a^2*d^2 - 3*(b^2*c*d
 - 5*a*b*d^2)*x)/((b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2)*g^4*x^3 + 3*(a*b^5*
c^2 - 2*a^2*b^4*c*d + a^3*b^3*d^2)*g^4*x^2 + 3*(a^2*b^4*c^2 - 2*a^3*b^3*c*d
 + a^4*b^2*d^2)*g^4*x + (a^3*b^3*c^2 - 2*a^4*b^2*c*d + a^5*b*d^2)*g^4) + 6*
d^3*log(b*x + a)/((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*g
^4) - 6*d^3*log(d*x + c)/((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*
b*d^3)*g^4)) - 1/3*B*log((b*x/(d*x + c) + a/(d*x + c))^n*e)/(b^4*g^4*x^3 +
3*a*b^3*g^4*x^2 + 3*a^2*b^2*g^4*x + a^3*b*g^4) - 1/3*A/(b^4*g^4*x^3 + 3*a*b
^3*g^4*x^2 + 3*a^2*b^2*g^4*x + a^3*b*g^4)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 446 vs. $2(172) = 344$.
time = 0.36, size = 446, normalized size = 2.44

$$-\frac{6 \, {\left(A + B\right)} b^{3} c^{3} - 18 \, {\left(A + B\right)} ab^{2} c^{2} d + 18 \, {\left(A + B\right)} a^{2} bcd^{2} - 6 \, {\left(A + B\right)} a^{3} d^{3} + 6 \, {\left(Bb^{3} cd^{2} - Bab^{2} d^{3}\right)} nx^{2} - 3 \, {\left(Bb^{3} c^{2} d - 6 \, Bab^{2} cd^{2} + 5 \, Ba^{2} bd^{3}\right)} nx + {\left(2 \, Bb^{3} c^{3} - 9 \, Bab^{2} c^{2} d + 18 \, Ba^{2} bcd^{2} - 11 \, Ba^{3} d^{3}\right)} n + 6 \, {\left(Bb^{3} d^{3} nx^{3} + 3 \, Bab^{2} d^{3} nx^{2} + 3 \, Ba^{2} bd^{3} nx + {\left(Bb^{3} c^{3} - 3 \, Bab^{2} c^{2} d + 3 \, Ba^{2} bcd^{2}\right)} n\right)} \log\left(\frac{bx + a}{dx + c}\right)}{18 \, {\left({\left(b^{7} c^{3} - 3 \, ab^{6} c^{2} d + 3 \, a^{2} b^{5} cd^{2} - a^{3} b^{4} d^{3}\right)} g^{4} x^{3} + 3 \, {\left(ab^{6} c^{3} - 3 \, a^{2} b^{5} c^{2} d + 3 \, a^{3} b^{4} cd^{2} - a^{4} b^{3} d^{3}\right)} g^{4} x^{2} + 3 \, {\left(a^{2} b^{5} c^{3} - 3 \, a^{3} b^{4} c^{2} d + 3 \, a^{4} b^{3} cd^{2} - a^{5} b^{2} d^{3}\right)} g^{4} x + {\left(a^{3} b^{4} c^{3} - 3 \, a^{4} b^{3} c^{2} d + 3 \, a^{5} b^{2} cd^{2} - a^{6} bd^{3}\right)} g^{4}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^4,x, algorithm="fric
as")
```

```
[Out] -1/18*(6*(A + B)*b^3*c^3 - 18*(A + B)*a*b^2*c^2*d + 18*(A + B)*a^2*b*c*d^2
- 6*(A + B)*a^3*d^3 + 6*(B*b^3*c*d^2 - B*a*b^2*d^3)*n*x^2 - 3*(B*b^3*c^2*d
- 6*B*a*b^2*c*d^2 + 5*B*a^2*b*d^3)*n*x + (2*B*b^3*c^3 - 9*B*a*b^2*c^2*d + 1
8*B*a^2*b*c*d^2 - 11*B*a^3*d^3)*n + 6*(B*b^3*d^3*n*x^3 + 3*B*a*b^2*d^3*n*x^
2 + 3*B*a^2*b*d^3*n*x + (B*b^3*c^3 - 3*B*a*b^2*c^2*d + 3*B*a^2*b*c*d^2)*n)*
log((b*x + a)/(d*x + c)))/((b^7*c^3 - 3*a*b^6*c^2*d + 3*a^2*b^5*c*d^2 - a^3
*b^4*d^3)*g^4*x^3 + 3*(a*b^6*c^3 - 3*a^2*b^5*c^2*d + 3*a^3*b^4*c*d^2 - a^4*
b^3*d^3)*g^4*x^2 + 3*(a^2*b^5*c^3 - 3*a^3*b^4*c^2*d + 3*a^4*b^3*c*d^2 - a^5
*b^2*d^3)*g^4*x + (a^3*b^4*c^3 - 3*a^4*b^3*c^2*d + 3*a^5*b^2*c*d^2 - a^6*b*
d^3)*g^4)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))/(b*g*x+a*g)**4,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 375 vs. 2(172) = 344.
time = 5.57, size = 375, normalized size = 2.05

$$-\frac{1}{18}\left(\frac{6\left(Bb^2n-\frac{3(bx+a)Bbdn}{dx+c}+\frac{3(bx+a)^2Bd^2n}{(dx+c)^2}\right)\log\left(\frac{bx+a}{dx+c}\right)}{\frac{(bx+a)^3b^2c^2g^4}{(dx+c)^3}-\frac{2(bx+a)^3abcdg^4}{(dx+c)^3}+\frac{(bx+a)^3a^2d^2g^4}{(dx+c)^3}}+\frac{2Bb^2n-\frac{9(bx+a)Bbdn}{dx+c}+\frac{18(bx+a)^2Bd^2n}{(dx+c)^2}+6Ab^2+6Bb^2-\frac{18(bx+a)Abd}{dx+c}-\frac{18(bx+a)Bbd}{dx+c}+\frac{18(bx+a)^2Ad^2}{(dx+c)^2}+\frac{18(bx+a)^2Bd^2}{(dx+c)^2}}{\frac{(bx+a)^3b^2c^2g^4}{(dx+c)^3}-\frac{2(bx+a)^3abcdg^4}{(dx+c)^3}+\frac{(bx+a)^3a^2d^2g^4}{(dx+c)^3}}\right)\left(\frac{bc}{(bc-ad)^2}-\frac{ad}{(bc-ad)^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^4,x, algorithm="giac
")
```

```
[Out] -1/18*(6*(B*b^2*n - 3*(b*x + a)*B*b*d*n/(d*x + c) + 3*(b*x + a)^2*B*d^2*n/(
d*x + c)^2)*log((b*x + a)/(d*x + c))/((b*x + a)^3*b^2*c^2*g^4/(d*x + c)^3 -
 2*(b*x + a)^3*a*b*c*d*g^4/(d*x + c)^3 + (b*x + a)^3*a^2*d^2*g^4/(d*x + c)^
3) + (2*B*b^2*n - 9*(b*x + a)*B*b*d*n/(d*x + c) + 18*(b*x + a)^2*B*d^2*n/(d
*x + c)^2 + 6*A*b^2 + 6*B*b^2 - 18*(b*x + a)*A*b*d/(d*x + c) - 18*(b*x + a)
*B*b*d/(d*x + c) + 18*(b*x + a)^2*A*d^2/(d*x + c)^2 + 18*(b*x + a)^2*B*d^2/
(d*x + c)^2)/((b*x + a)^3*b^2*c^2*g^4/(d*x + c)^3 - 2*(b*x + a)^3*a*b*c*d*g
^4/(d*x + c)^3 + (b*x + a)^3*a^2*d^2*g^4/(d*x + c)^3))*(b*c/(b*c - a*d)^2 -
 a*d/(b*c - a*d)^2)
```

Mupad [B]
time = 4.85, size = 349, normalized size = 1.91

$$\frac{2Aacd}{3g^4(ad-bc)^2(a+bx)^3}-\frac{Abc^2}{3g^4(ad-bc)^2(a+bx)^3}-\frac{Aa^2d^2}{3bg^4(ad-bc)^2(a+bx)^3}-\frac{Bbc^2n}{9g^4(ad-bc)^2(a+bx)^3}-\frac{B\ln\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{3bg^4(a+bx)^3}-\frac{Bbd^2nx^2}{3g^4(ad-bc)^2(a+bx)^3}+\frac{7Bacdn}{18g^4(ad-bc)^2(a+bx)^3}-\frac{11Ba^2d^2n}{18bg^4(ad-bc)^2(a+bx)^3}-\frac{5Bad^2nx}{6g^4(ad-bc)^2(a+bx)^3}+\frac{Bbcdnx}{6g^4(ad-bc)^2(a+bx)^3}-\frac{Bd^3n\,\mathrm{atan}\left(\frac{ad\,1i+bc\,1i+bdx\,2i}{ad-bc}\right)2i}{3bg^4(ad-bc)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log(e*((a + b*x)/(c + d*x))^n))/(a*g + b*g*x)^4,x)
```

```
[Out] (2*A*a*c*d)/(3*g^4*(a*d - b*c)^2*(a + b*x)^3) - (A*b*c^2)/(3*g^4*(a*d - b*c
)^2*(a + b*x)^3) - (A*a^2*d^2)/(3*b*g^4*(a*d - b*c)^2*(a + b*x)^3) - (B*b*c
^2*n)/(9*g^4*(a*d - b*c)^2*(a + b*x)^3) - (B*d^3*n*atan((a*d*1i + b*c*1i +
b*d*x*2i)/(a*d - b*c))*2i)/(3*b*g^4*(a*d - b*c)^3) - (B*log(e*((a + b*x)/(c
 + d*x))^n))/(3*b*g^4*(a + b*x)^3) - (B*b*d^2*n*x^2)/(3*g^4*(a*d - b*c)^2*(
a + b*x)^3) + (7*B*a*c*d*n)/(18*g^4*(a*d - b*c)^2*(a + b*x)^3) - (11*B*a^2*
d^2*n)/(18*b*g^4*(a*d - b*c)^2*(a + b*x)^3) - (5*B*a*d^2*n*x)/(6*g^4*(a*d -
 b*c)^2*(a + b*x)^3) + (B*b*c*d*n*x)/(6*g^4*(a*d - b*c)^2*(a + b*x)^3)
```

3.9 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^5}\,dx$

Optimal. Leaf size=215

$$-\frac{Bn}{16bg^5(a+bx)^4}+\frac{Bdn}{12b(bc-ad)g^5(a+bx)^3}-\frac{Bd^2n}{8b(bc-ad)^2g^5(a+bx)^2}+\frac{Bd^3n}{4b(bc-ad)^3g^5(a+bx)}+\frac{Bd^4n\log(a}{4b(bc-ad}$$

```
[Out] -1/16*B*n/b/g^5/(b*x+a)^4+1/12*B*d*n/b/(-a*d+b*c)/g^5/(b*x+a)^3-1/8*B*d^2*n
/b/(-a*d+b*c)^2/g^5/(b*x+a)^2+1/4*B*d^3*n/b/(-a*d+b*c)^3/g^5/(b*x+a)+1/4*B*
d^4*n*ln(b*x+a)/b/(-a*d+b*c)^4/g^5+1/4*(-A-B*ln(e*((b*x+a)/(d*x+c))^n))/b/g
^5/(b*x+a)^4-1/4*B*d^4*n*ln(d*x+c)/b/(-a*d+b*c)^4/g^5
```

Rubi [A]

time = 0.13, antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {2547, 21, 46}

$$-\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{4bg^5(a+bx)^4}+\frac{Bd^4n\log(a+bx)}{4bg^5(bc-ad)^4}-\frac{Bd^4n\log(c+dx)}{4bg^5(bc-ad)^4}+\frac{Bd^3n}{4bg^5(a+bx)(bc-ad)^3}-\frac{Bd^2n}{8bg^5(a+bx)^2(bc-ad)^2}+\frac{Bdn}{12bg^5(a+bx)^3(bc-ad)}-\frac{Bn}{16bg^5(a+bx)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(a*g + b*g*x)^5,x]
```

```
[Out] -1/16*(B*n)/(b*g^5*(a + b*x)^4) + (B*d*n)/(12*b*(b*c - a*d)*g^5*(a + b*x)^3
) - (B*d^2*n)/(8*b*(b*c - a*d)^2*g^5*(a + b*x)^2) + (B*d^3*n)/(4*b*(b*c - a
*d)^3*g^5*(a + b*x)) + (B*d^4*n*Log[a + b*x])/(4*b*(b*c - a*d)^4*g^5) - (A
+ B*Log[e*((a + b*x)/(c + d*x))^n])/(4*b*g^5*(a + b*x)^4) - (B*d^4*n*Log[c
+ d*x])/(4*b*(b*c - a*d)^4*g^5)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2547

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*((A +
```

```
 B*Log[e*((a + b*x)/(c + d*x))^n])/(g*(m + 1))), x] - Dist[B*n*((b*c - a*d)
/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /; FreeQ
[{a, b, c, d, e, f, g, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^5}\,dx &= -\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{4bg^5(a+bx)^4}+\frac{(Bn)\int\frac{bc-ad}{g^4(a+bx)^5(c+dx)}\,dx}{4bg}\\ &= -\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{4bg^5(a+bx)^4}+\frac{(B(bc-ad)n)\int\frac{1}{(a+bx)^5(c+dx)}\,dx}{4bg^5}\\ &= -\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{4bg^5(a+bx)^4}+\frac{(B(bc-ad)n)\int\left(\frac{b}{(bc-ad)(a+bx)^5}-\frac{bd}{(bc-ad)^2(a+bx)^4}+\right.}{}\\ &= -\frac{Bn}{16bg^5(a+bx)^4}+\frac{Bdn}{12b(bc-ad)g^5(a+bx)^3}-\frac{Bd^2n}{8b(bc-ad)^2g^5(a+bx)^2}+\end{aligned}$$

Mathematica [A]
time = 0.16, size = 162, normalized size = 0.75

$$\frac{-\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)^4}+\frac{Bn\left(-\frac{3(bc-ad)^4}{(a+bx)^4}+\frac{4d(bc-ad)^3}{(a+bx)^3}-\frac{6d^2(bc-ad)^2}{(a+bx)^2}+\frac{12d^3(bc-ad)}{a+bx}+12d^4\log(a+bx)-12d^4\log(c+dx)\right)}{12(bc-ad)^4}}{4bg^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(a*g + b*g*x)^5,x]
```

```
[Out] (-((A + B*Log[e*((a + b*x)/(c + d*x))^n])/(a + b*x)^4) + (B*n*((-3*(b*c - a
*d)^4)/(a + b*x)^4 + (4*d*(b*c - a*d)^3)/(a + b*x)^3 - (6*d^2*(b*c - a*d)^2
)/(a + b*x)^2 + (12*d^3*(b*c - a*d))/(a + b*x) + 12*d^4*Log[a + b*x] - 12*d
^4*Log[c + d*x]))/(12*(b*c - a*d)^4))/(4*b*g^5)
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}{(bgx+ag)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^5,x)
```

```
[Out] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^5,x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 652 vs. $2(202) = 404$.
time = 0.39, size = 652, normalized size = 3.03

$$\frac{1}{48}\,Bn\left(\frac{12\,b^{3}d^{3}x^{3}-3\,b^{3}c^{3}+13\,ab^{2}c^{2}d-23\,a^{2}bcd^{2}+25\,a^{3}d^{3}-6\,(b^{3}cd^{2}-7\,ab^{2}d^{3})x^{2}+4\,(b^{3}c^{2}d-5\,ab^{2}cd^{2}+13\,a^{2}bd^{3})x}{(b^{8}c^{3}-3\,ab^{7}c^{2}d+3\,a^{2}b^{6}cd^{2}-a^{3}b^{5}d^{3})g^{5}x^{4}+4\,(ab^{7}c^{3}-3\,a^{2}b^{6}c^{2}d+3\,a^{3}b^{5}cd^{2}-a^{4}b^{4}d^{3})g^{5}x^{3}+6\,(a^{2}b^{6}c^{3}-3\,a^{3}b^{5}c^{2}d+3\,a^{4}b^{4}cd^{2}-a^{5}b^{3}d^{3})g^{5}x^{2}+4\,(a^{3}b^{5}c^{3}-3\,a^{4}b^{4}c^{2}d+3\,a^{5}b^{3}cd^{2}-a^{6}b^{2}d^{3})g^{5}x+(a^{4}b^{4}c^{3}-3\,a^{5}b^{3}c^{2}d+3\,a^{6}b^{2}cd^{2}-a^{7}bd^{3})g^{5}}+\frac{12\,d^{4}\log(bx+a)}{(b^{5}c^{4}-4\,ab^{4}c^{3}d+6\,a^{2}b^{3}c^{2}d^{2}-4\,a^{3}b^{2}cd^{3}+a^{4}bd^{4})g^{5}}-\frac{12\,d^{4}\log(dx+c)}{(b^{5}c^{4}-4\,ab^{4}c^{3}d+6\,a^{2}b^{3}c^{2}d^{2}-4\,a^{3}b^{2}cd^{3}+a^{4}bd^{4})g^{5}}\right)-\frac{B\log\left(\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^{n}e\right)}{4\,(b^{5}g^{5}x^{4}+4\,ab^{4}g^{5}x^{3}+6\,a^{2}b^{3}g^{5}x^{2}+4\,a^{3}b^{2}g^{5}x+a^{4}bg^{5})}-\frac{A}{4\,(b^{5}g^{5}x^{4}+4\,ab^{4}g^{5}x^{3}+6\,a^{2}b^{3}g^{5}x^{2}+4\,a^{3}b^{2}g^{5}x+a^{4}bg^{5})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^5,x, algorithm="maxi
ma")
```

```
[Out] 1/48*B*n*((12*b^3*d^3*x^3 - 3*b^3*c^3 + 13*a*b^2*c^2*d - 23*a^2*b*c*d^2 + 2
5*a^3*d^3 - 6*(b^3*c*d^2 - 7*a*b^2*d^3)*x^2 + 4*(b^3*c^2*d - 5*a*b^2*c*d^2
+ 13*a^2*b*d^3)*x)/((b^8*c^3 - 3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 - a^3*b^5*d^
3)*g^5*x^4 + 4*(a*b^7*c^3 - 3*a^2*b^6*c^2*d + 3*a^3*b^5*c*d^2 - a^4*b^4*d^3
)*g^5*x^3 + 6*(a^2*b^6*c^3 - 3*a^3*b^5*c^2*d + 3*a^4*b^4*c*d^2 - a^5*b^3*d^
3)*g^5*x^2 + 4*(a^3*b^5*c^3 - 3*a^4*b^4*c^2*d + 3*a^5*b^3*c*d^2 - a^6*b^2*d
^3)*g^5*x + (a^4*b^4*c^3 - 3*a^5*b^3*c^2*d + 3*a^6*b^2*c*d^2 - a^7*b*d^3)*g
^5) + 12*d^4*log(b*x + a)/((b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4
*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5) - 12*d^4*log(d*x + c)/((b^5*c^4 - 4*a*b^4*
c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5)) - 1/4*B*log(
(b*x/(d*x + c) + a/(d*x + c))^n*e)/(b^5*g^5*x^4 + 4*a*b^4*g^5*x^3 + 6*a^2*b
^3*g^5*x^2 + 4*a^3*b^2*g^5*x + a^4*b*g^5) - 1/4*A/(b^5*g^5*x^4 + 4*a*b^4*g^
5*x^3 + 6*a^2*b^3*g^5*x^2 + 4*a^3*b^2*g^5*x + a^4*b*g^5)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 685 vs. $2(202) = 404$.
time = 0.38, size = 685, normalized size = 3.19

$$-\frac{12\,(A+B)b^{4}c^{4}-48\,(A+B)ab^{3}c^{3}d+72\,(A+B)a^{2}b^{2}c^{2}d^{2}-48\,(A+B)a^{3}bcd^{3}+12\,(A+B)a^{4}d^{4}-12\,(Bb^{4}cd^{3}-Bab^{3}d^{4})nx^{3}+6\,(Bb^{4}c^{2}d^{2}-8\,Bab^{3}cd^{3}+7\,Ba^{2}b^{2}d^{4})nx^{2}-4\,(Bb^{4}c^{3}d-6\,Bab^{3}c^{2}d^{2}+18\,Ba^{2}b^{2}cd^{3}-13\,Ba^{3}bd^{4})nx+(3\,Bb^{4}c^{4}-16\,Bab^{3}c^{3}d+36\,Ba^{2}b^{2}c^{2}d^{2}-48\,Ba^{3}bcd^{3}+25\,Ba^{4}d^{4})n-12\,(Bb^{4}d^{4}nx^{4}+4\,Bab^{3}d^{4}nx^{3}+6\,Ba^{2}b^{2}d^{4}nx^{2}+4\,Ba^{3}bd^{4}nx-(Bb^{4}c^{4}-4\,Bab^{3}c^{3}d+6\,Ba^{2}b^{2}c^{2}d^{2}-4\,Ba^{3}bcd^{3})n)\log\left(\frac{bx+a}{dx+c}\right)}{48\,((b^{9}c^{4}-4\,ab^{8}c^{3}d+6\,a^{2}b^{7}c^{2}d^{2}-4\,a^{3}b^{6}cd^{3}+a^{4}b^{5}d^{4})g^{5}x^{4}+4\,(ab^{8}c^{4}-4\,a^{2}b^{7}c^{3}d+6\,a^{3}b^{6}c^{2}d^{2}-4\,a^{4}b^{5}cd^{3}+a^{5}b^{4}d^{4})g^{5}x^{3}+6\,(a^{2}b^{7}c^{4}-4\,a^{3}b^{6}c^{3}d+6\,a^{4}b^{5}c^{2}d^{2}-4\,a^{5}b^{4}cd^{3}+a^{6}b^{3}d^{4})g^{5}x^{2}+4\,(a^{3}b^{6}c^{4}-4\,a^{4}b^{5}c^{3}d+6\,a^{5}b^{4}c^{2}d^{2}-4\,a^{6}b^{3}cd^{3}+a^{7}b^{2}d^{4})g^{5}x+(a^{4}b^{5}c^{4}-4\,a^{5}b^{4}c^{3}d+6\,a^{6}b^{3}c^{2}d^{2}-4\,a^{7}b^{2}cd^{3}+a^{8}bd^{4})g^{5})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^5,x, algorithm="fric
as")
```

```
[Out] -1/48*(12*(A + B)*b^4*c^4 - 48*(A + B)*a*b^3*c^3*d + 72*(A + B)*a^2*b^2*c^2
*d^2 - 48*(A + B)*a^3*b*c*d^3 + 12*(A + B)*a^4*d^4 - 12*(B*b^4*c*d^3 - B*a*
b^3*d^4)*n*x^3 + 6*(B*b^4*c^2*d^2 - 8*B*a*b^3*c*d^3 + 7*B*a^2*b^2*d^4)*n*x^
2 - 4*(B*b^4*c^3*d - 6*B*a*b^3*c^2*d^2 + 18*B*a^2*b^2*c*d^3 - 13*B*a^3*b*d^
4)*n*x + (3*B*b^4*c^4 - 16*B*a*b^3*c^3*d + 36*B*a^2*b^2*c^2*d^2 - 48*B*a^3*
b*c*d^3 + 25*B*a^4*d^4)*n - 12*(B*b^4*d^4*n*x^4 + 4*B*a*b^3*d^4*n*x^3 + 6*B
*a^2*b^2*d^4*n*x^2 + 4*B*a^3*b*d^4*n*x - (B*b^4*c^4 - 4*B*a*b^3*c^3*d + 6*B
*a^2*b^2*c^2*d^2 - 4*B*a^3*b*c*d^3)*n)*log((b*x + a)/(d*x + c)))/((b^9*c^4
- 4*a*b^8*c^3*d + 6*a^2*b^7*c^2*d^2 - 4*a^3*b^6*c*d^3 + a^4*b^5*d^4)*g^5*x^
4 + 4*(a*b^8*c^4 - 4*a^2*b^7*c^3*d + 6*a^3*b^6*c^2*d^2 - 4*a^4*b^5*c*d^3 +
```

```
a^5*b^4*d^4)*g^5*x^3 + 6*(a^2*b^7*c^4 - 4*a^3*b^6*c^3*d + 6*a^4*b^5*c^2*d^2
 - 4*a^5*b^4*c*d^3 + a^6*b^3*d^4)*g^5*x^2 + 4*(a^3*b^6*c^4 - 4*a^4*b^5*c^3*
d + 6*a^5*b^4*c^2*d^2 - 4*a^6*b^3*c*d^3 + a^7*b^2*d^4)*g^5*x + (a^4*b^5*c^4
 - 4*a^5*b^4*c^3*d + 6*a^6*b^3*c^2*d^2 - 4*a^7*b^2*c*d^3 + a^8*b*d^4)*g^5)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))/(b*g*x+a*g)**5,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 533 vs. 2(202) = 404.
time = 4.48, size = 533, normalized size = 2.48

$$-\frac{1}{48}\left(\frac{12\left(Bb^3n - \frac{4(bx+a)Bb^2dn}{dx+c} + \frac{6(bx+a)^2Bbd^2n}{(dx+c)^2} - \frac{4(bx+a)^3Bd^3n}{(dx+c)^3}\right)\log\left(\frac{bx+a}{dx+c}\right)}{\frac{(bx+a)^4b^3c^3g^5}{(dx+c)^4} - \frac{3(bx+a)^4ab^2c^2dg^5}{(dx+c)^4} + \frac{3(bx+a)^4a^2bcd^2g^5}{(dx+c)^4} - \frac{(bx+a)^4a^3d^3g^5}{(dx+c)^4}} + \frac{3Bb^3n - \frac{16(bx+a)Bb^2dn}{dx+c} + \frac{36(bx+a)^2Bbd^2n}{(dx+c)^2} - \frac{48(bx+a)^3Bd^3n}{(dx+c)^3} + 12Ab^3 + 12Bb^3 - \frac{48(bx+a)Ab^2d}{dx+c} - \frac{48(bx+a)Bb^2d}{dx+c} + \frac{72(bx+a)^2Abd^2}{(dx+c)^2} + \frac{72(bx+a)^2Bbd^2}{(dx+c)^2} - \frac{48(bx+a)^3Ad^3}{(dx+c)^3} - \frac{48(bx+a)^3Bd^3}{(dx+c)^3}}{\frac{(bx+a)^4b^3c^3g^5}{(dx+c)^4} - \frac{3(bx+a)^4ab^2c^2dg^5}{(dx+c)^4} + \frac{3(bx+a)^4a^2bcd^2g^5}{(dx+c)^4} - \frac{(bx+a)^4a^3d^3g^5}{(dx+c)^4}}\right)\left(\frac{bc}{(bc-ad)^2} - \frac{ad}{(bc-ad)^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^5,x, algorithm="giac
")
```

```
[Out] -1/48*(12*(B*b^3*n - 4*(b*x + a)*B*b^2*d*n/(d*x + c) + 6*(b*x + a)^2*B*b*d^
2*n/(d*x + c)^2 - 4*(b*x + a)^3*B*d^3*n/(d*x + c)^3)*log((b*x + a)/(d*x + c
))/((b*x + a)^4*b^3*c^3*g^5/(d*x + c)^4 - 3*(b*x + a)^4*a*b^2*c^2*d*g^5/(d*
x + c)^4 + 3*(b*x + a)^4*a^2*b*c*d^2*g^5/(d*x + c)^4 - (b*x + a)^4*a^3*d^3*
g^5/(d*x + c)^4) + (3*B*b^3*n - 16*(b*x + a)*B*b^2*d*n/(d*x + c) + 36*(b*x
+ a)^2*B*b*d^2*n/(d*x + c)^2 - 48*(b*x + a)^3*B*d^3*n/(d*x + c)^3 + 12*A*b^
3 + 12*B*b^3 - 48*(b*x + a)*A*b^2*d/(d*x + c) - 48*(b*x + a)*B*b^2*d/(d*x +
 c) + 72*(b*x + a)^2*A*b*d^2/(d*x + c)^2 + 72*(b*x + a)^2*B*b*d^2/(d*x + c)
^2 - 48*(b*x + a)^3*A*d^3/(d*x + c)^3 - 48*(b*x + a)^3*B*d^3/(d*x + c)^3)/(
(b*x + a)^4*b^3*c^3*g^5/(d*x + c)^4 - 3*(b*x + a)^4*a*b^2*c^2*d*g^5/(d*x +
c)^4 + 3*(b*x + a)^4*a^2*b*c*d^2*g^5/(d*x + c)^4 - (b*x + a)^4*a^3*d^3*g^5/
(d*x + c)^4))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)
```

Mupad [**B**]
time = 5.12, size = 603, normalized size = 2.80

$$-\frac{\frac{12\,A\,a^3\,d^3-12\,A\,b^3\,c^3+25\,B\,a^3\,d^3\,n-3\,B\,b^3\,c^3\,n+36\,A\,a\,b^2\,c^2\,d-36\,A\,a^2\,b\,c\,d^2+13\,B\,a\,b^2\,c^2\,d\,n-23\,B\,a^2\,b\,c\,d^2\,n}{12\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}+\frac{d\,x\,\left(13\,B\,n\,a^2\,b\,d^2-5\,B\,n\,a\,b^2\,c\,d+B\,n\,b^3\,c^2\right)}{3\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}-\frac{d^2\,x^2\,\left(B\,b^3\,c\,n-7\,B\,a\,b^2\,d\,n\right)}{2\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}+\frac{B\,b^3\,d^3\,n\,x^3}{a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3}}{4\,a^4\,b\,g^5+16\,a^3\,b^2\,g^5\,x+24\,a^2\,b^3\,g^5\,x^2+16\,a\,b^4\,g^5\,x^3+4\,b^5\,g^5\,x^4}-\frac{B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)}{4\,b\,\left(a^4\,g^5+4\,a^3\,b\,g^5\,x+6\,a^2\,b^2\,g^5\,x^2+4\,a\,b^3\,g^5\,x^3+b^4\,g^5\,x^4\right)}-\frac{B\,d^4\,n\,\mathrm{atanh}\left(\frac{-4\,a^4\,b\,d^4\,g^5+8\,a^3\,b^2\,c\,d^3\,g^5-8\,a\,b^4\,c^3\,d\,g^5+4\,b^5\,c^4\,g^5}{4\,b\,g^5\,{\left(a\,d-b\,c\right)}^4}-\frac{2\,b\,d\,x\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}{{\left(a\,d-b\,c\right)}^4}\right)}{2\,b\,g^5\,{\left(a\,d-b\,c\right)}^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log(e*((a + b*x)/(c + d*x))^n))/(a*g + b*g*x)^5,x)
```

```
[Out] - ((12*A*a^3*d^3 - 12*A*b^3*c^3 + 25*B*a^3*d^3*n - 3*B*b^3*c^3*n + 36*A*a*b
^2*c^2*d - 36*A*a^2*b*c*d^2 + 13*B*a*b^2*c^2*d*n - 23*B*a^2*b*c*d^2*n)/(12*
(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) + (d*x*(B*b^3*c^2*n +
13*B*a^2*b*d^2*n - 5*B*a*b^2*c*d*n))/(3*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d
- 3*a^2*b*c*d^2)) - (d^2*x^2*(B*b^3*c*n - 7*B*a*b^2*d*n))/(2*(a^3*d^3 - b^3
*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) + (B*b^3*d^3*n*x^3)/(a^3*d^3 - b^3*c
^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))/(4*a^4*b*g^5 + 4*b^5*g^5*x^4 + 16*a^3*
b^2*g^5*x + 16*a*b^4*g^5*x^3 + 24*a^2*b^3*g^5*x^2) - (B*log(e*((a + b*x)/(c
 + d*x))^n))/(4*b*(a^4*g^5 + b^4*g^5*x^4 + 4*a*b^3*g^5*x^3 + 6*a^2*b^2*g^5*
x^2 + 4*a^3*b*g^5*x)) - (B*d^4*n*atanh((4*b^5*c^4*g^5 - 4*a^4*b*d^4*g^5 - 8
*a*b^4*c^3*d*g^5 + 8*a^3*b^2*c*d^3*g^5)/(4*b*g^5*(a*d - b*c)^4) - (2*b*d*x*
(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))/(a*d - b*c)^4))/(2*b*g
^5*(a*d - b*c)^4)
```

3.10 $\int (ag+bgx)^4 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 \, dx$

Optimal. Leaf size=396

$$-\frac{B(bc-ad)g^4n(a+bx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{10bd}+\frac{g^4(a+bx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{5b}+\frac{B(bc-ad)^2g^4n\cdots}{}$$

```
[Out] -1/10*B*(-a*d+b*c)*g^4*n*(b*x+a)^4*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b/d+1/5*
g^4*(b*x+a)^5*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/b+1/30*B*(-a*d+b*c)^2*g^4*n
*(b*x+a)^3*(4*A+B*n+4*B*ln(e*((b*x+a)/(d*x+c))^n))/b/d^2-1/60*B*(-a*d+b*c)^
3*g^4*n*(b*x+a)^2*(12*A+7*B*n+12*B*ln(e*((b*x+a)/(d*x+c))^n))/b/d^3+1/30*B*
(-a*d+b*c)^4*g^4*n*(b*x+a)*(12*A+13*B*n+12*B*ln(e*((b*x+a)/(d*x+c))^n))/b/d
^4+1/30*B*(-a*d+b*c)^5*g^4*n*(12*A+25*B*n+12*B*ln(e*((b*x+a)/(d*x+c))^n))*l
n((-a*d+b*c)/b/(d*x+c))/b/d^5+2/5*B^2*(-a*d+b*c)^5*g^4*n^2*polylog(2,d*(b*x
+a)/b/(d*x+c))/b/d^5
```

Rubi [A]
time = 0.37, antiderivative size = 396, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2549, 2381, 2384, 2354, 2438}

$$\frac{2B^2g^4n^2(bc-ad)^5\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{5bd^5}+\frac{Bg^4n(bc-ad)^5\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(12B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+12A+25Bn\right)}{30bd^5}+\frac{Bg^4n(a+bx)(bc-ad)^4\left(12B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+12A+13Bn\right)}{30bd^4}-\frac{Bg^4n(a+bx)^2(bc-ad)^3\left(12B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+12A+7Bn\right)}{60bd^3}+\frac{Bg^4n(a+bx)^3(bc-ad)^2\left(4B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+4A+Bn\right)}{30bd^2}-\frac{Bg^4n(a+bx)^4(bc-ad)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{10bd}+\frac{g^4(a+bx)^5\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[(a*g + b*g*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

```
[Out] -1/10*(B*(b*c - a*d)*g^4*n*(a + b*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n
]))/(b*d) + (g^4*(a + b*x)^5*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(5*b
) + (B*(b*c - a*d)^2*g^4*n*(a + b*x)^3*(4*A + B*n + 4*B*Log[e*((a + b*x)/(c
 + d*x))^n]))/(30*b*d^2) - (B*(b*c - a*d)^3*g^4*n*(a + b*x)^2*(12*A + 7*B*n
 + 12*B*Log[e*((a + b*x)/(c + d*x))^n]))/(60*b*d^3) + (B*(b*c - a*d)^4*g^4*
n*(a + b*x)*(12*A + 13*B*n + 12*B*Log[e*((a + b*x)/(c + d*x))^n]))/(30*b*d^
4) + (B*(b*c - a*d)^5*g^4*n*(12*A + 25*B*n + 12*B*Log[e*((a + b*x)/(c + d*x
))^n])*Log[(b*c - a*d)/(b*(c + d*x))])/(30*b*d^5) + (2*B^2*(b*c - a*d)^5*g^
4*n^2*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/(5*b*d^5)
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2381

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + Dist[b*n*(p/(d*(q + 1))), Int[(f*x)^
m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, d
, e, f, m, n, q}, x] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 2384

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Dist[f/(e*(q + 1)), Int[(f*x)^(m - 1)*(d + e*x)^(q + 1
)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x
] && ILtQ[q, -1] && GtQ[m, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2549

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(m +
1)*(g/b)^m, Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x], x,
(a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ
[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[b*f - a*g, 0] && (GtQ[p, 0] || LtQ
[m, -1])
```

Rubi steps

$$\int (ag+bgx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx = \frac{g^4(a+bx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{5b} - \frac{(2Bn)\int \frac{(bc-ad)}{}}{}$$

$$= \frac{g^4(a+bx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{5b} - \frac{(2B(bc-ad)g^4}{}$$

$$= \frac{g^4(a+bx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{5b} - \frac{(2B(bc-ad)g^4}{}$$

$$= \frac{g^4(a+bx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{5b} - \frac{(2B(bc-ad)g^4}{}$$

$$= \frac{2AB(bc-ad)^4g^4nx}{5d^4} - \frac{B(bc-ad)^3g^4n(a+bx)^2\left(A+\right.}{5bd^3}$$

$$= \frac{2AB(bc-ad)^4g^4nx}{5d^4} + \frac{2B^2(bc-ad)^4g^4n(a+bx)\log\left(\right.}{5bd^4}$$

$$= \frac{2AB(bc-ad)^4g^4nx}{5d^4} + \frac{2B^2(bc-ad)^4g^4n(a+bx)\log\left(\right.}{5bd^4}$$

$$= \frac{2AB(bc-ad)^4g^4nx}{5d^4} + \frac{13B^2(bc-ad)^4g^4n^2x}{30d^4} - \frac{7B^2(bc}{}$$

$$= \frac{2AB(bc-ad)^4g^4nx}{5d^4} + \frac{13B^2(bc-ad)^4g^4n^2x}{30d^4} - \frac{7B^2(bc}{}$$

$$= \frac{2AB(bc-ad)^4g^4nx}{5d^4} + \frac{13B^2(bc-ad)^4g^4n^2x}{30d^4} - \frac{7B^2(bc}{}$$

Mathematica [A]
time = 0.35, size = 535, normalized size = 1.35

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[(a*g + b*g*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

```
[Out] (g^4*((a + b*x)^5*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 + (B*(b*c - a*d)
*n*(24*A*b*d*(b*c - a*d)^3*x + 24*B*d*(b*c - a*d)^3*(a + b*x)*Log[e*((a + b
*x)/(c + d*x))^n] - 12*d^2*(b*c - a*d)^2*(a + b*x)^2*(A + B*Log[e*((a + b*x
)/(c + d*x))^n]) + 8*d^3*(b*c - a*d)*(a + b*x)^3*(A + B*Log[e*((a + b*x)/(c
 + d*x))^n]) - 6*d^4*(a + b*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - 2
```

```
4*B*(b*c - a*d)^4*n*Log[c + d*x] - 24*(b*c - a*d)^4*(A + B*Log[e*((a + b*x)
/(c + d*x))^n])*Log[c + d*x] + 4*B*(b*c - a*d)^2*n*(2*b*d*(b*c - a*d)*x - d
^2*(a + b*x)^2 - 2*(b*c - a*d)^2*Log[c + d*x]) + B*(b*c - a*d)*n*(6*b*d*(b*
c - a*d)^2*x + 3*d^2*(-(b*c) + a*d)*(a + b*x)^2 + 2*d^3*(a + b*x)^3 - 6*(b*
c - a*d)^3*Log[c + d*x]) + 12*B*(b*c - a*d)^3*n*(b*d*x + (-(b*c) + a*d)*Log
[c + d*x]) + 12*B*(b*c - a*d)^4*n*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - L
og[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(12*
d^5)))/(5*b)
```

Maple [F]
time = 0.10, size = 0, normalized size = 0.00

$$\int (bgx+ag)^4 \left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*g*x+a*g)^4*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

```
[Out] int((b*g*x+a*g)^4*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 2764 vs. $2(387) = 774$.
time = 0.84, size = 2764, normalized size = 6.98

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^4*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="ma
xima")
```

```
[Out] 2/5*A*B*b^4*g^4*x^5*log((b*x/(d*x + c) + a/(d*x + c))^n*e) + 1/5*A^2*b^4*g^
4*x^5 + 2*A*B*a*b^3*g^4*x^4*log((b*x/(d*x + c) + a/(d*x + c))^n*e) + A^2*a*
b^3*g^4*x^4 + 4*A*B*a^2*b^2*g^4*x^3*log((b*x/(d*x + c) + a/(d*x + c))^n*e)
+ 2*A^2*a^2*b^2*g^4*x^3 + 4*A*B*a^3*b*g^4*x^2*log((b*x/(d*x + c) + a/(d*x +
 c))^n*e) + 2*A^2*a^3*b*g^4*x^2 + 1/30*A*B*b^4*g^4*n*(12*a^5*log(b*x + a)/b
^5 - 12*c^5*log(d*x + c)/d^5 - (3*(b^4*c*d^3 - a*b^3*d^4)*x^4 - 4*(b^4*c^2*
d^2 - a^2*b^2*d^4)*x^3 + 6*(b^4*c^3*d - a^3*b*d^4)*x^2 - 12*(b^4*c^4 - a^4*
d^4)*x)/(b^4*d^4)) - 1/3*A*B*a*b^3*g^4*n*(6*a^4*log(b*x + a)/b^4 - 6*c^4*lo
g(d*x + c)/d^4 + (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)
*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3)) + 2*A*B*a^2*b^2*g^4*n*(2*a^3*log
(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*
c^2 - a^2*d^2)*x)/(b^2*d^2)) - 4*A*B*a^3*b*g^4*n*(a^2*log(b*x + a)/b^2 - c^
2*log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d)) + 2*A*B*a^4*g^4*n*(a*log(b*x + a)
/b - c*log(d*x + c)/d) + 2*A*B*a^4*g^4*x*log((b*x/(d*x + c) + a/(d*x + c))^
n*e) + A^2*a^4*g^4*x - 1/30*((25*n^2 + 12*n)*b^4*c^5*g^4 - (113*n^2 + 60*n)
*a*b^3*c^4*d*g^4 + 4*(49*n^2 + 30*n)*a^2*b^2*c^3*d^2*g^4 - 12*(13*n^2 + 10*
```

```
n)*a^3*b*c^2*d^3*g^4 + 12*(4*n^2 + 5*n)*a^4*c*d^4*g^4)*B^2*log(d*x + c)/d^5
 - 2/5*(b^5*c^5*g^4*n^2 - 5*a*b^4*c^4*d*g^4*n^2 + 10*a^2*b^3*c^3*d^2*g^4*n^
2 - 10*a^3*b^2*c^2*d^3*g^4*n^2 + 5*a^4*b*c*d^4*g^4*n^2 - a^5*d^5*g^4*n^2)*(
log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c
 - a*d)))*B^2/(b*d^5) + 1/60*(12*B^2*b^5*d^5*g^4*x^5 - 12*B^2*a^5*d^5*g^4*n
^2*log(b*x + a)^2 + 6*(a*b^4*d^5*g^4*(n + 10) - b^5*c*d^4*g^4*n)*B^2*x^4 +
2*((n^2 + 4*n)*b^5*c^2*d^3*g^4 - 2*(n^2 + 10*n)*a*b^4*c*d^4*g^4 + (n^2 + 16
*n + 60)*a^2*b^3*d^5*g^4)*B^2*x^3 - ((7*n^2 + 12*n)*b^5*c^3*d^2*g^4 - 3*(9*
n^2 + 20*n)*a*b^4*c^2*d^3*g^4 + 3*(11*n^2 + 40*n)*a^2*b^3*c*d^4*g^4 - (13*n
^2 + 72*n + 120)*a^3*b^2*d^5*g^4)*B^2*x^2 + 24*(b^5*c^5*g^4*n^2 - 5*a*b^4*c
^4*d*g^4*n^2 + 10*a^2*b^3*c^3*d^2*g^4*n^2 - 10*a^3*b^2*c^2*d^3*g^4*n^2 + 5*
a^4*b*c*d^4*g^4*n^2)*B^2*log(b*x + a)*log(d*x + c) - 12*(b^5*c^5*g^4*n^2 -
5*a*b^4*c^4*d*g^4*n^2 + 10*a^2*b^3*c^3*d^2*g^4*n^2 - 10*a^3*b^2*c^2*d^3*g^4
*n^2 + 5*a^4*b*c*d^4*g^4*n^2)*B^2*log(d*x + c)^2 + 2*((13*n^2 + 12*n)*b^5*c
^4*d*g^4 - (59*n^2 + 60*n)*a*b^4*c^3*d^2*g^4 + 6*(17*n^2 + 20*n)*a^2*b^3*c^
2*d^3*g^4 - (79*n^2 + 120*n)*a^3*b^2*c*d^4*g^4 + (23*n^2 + 48*n + 30)*a^4*b
*d^5*g^4)*B^2*x + 2*(12*a*b^4*c^4*d*g^4*n^2 - 54*a^2*b^3*c^3*d^2*g^4*n^2 +
94*a^3*b^2*c^2*d^3*g^4*n^2 - 77*a^4*b*c*d^4*g^4*n^2 + (25*n^2 + 12*n)*a^5*d
^5*g^4)*B^2*log(b*x + a) + 12*(B^2*b^5*d^5*g^4*x^5 + 5*B^2*a*b^4*d^5*g^4*x^
4 + 10*B^2*a^2*b^3*d^5*g^4*x^3 + 10*B^2*a^3*b^2*d^5*g^4*x^2 + 5*B^2*a^4*b*d
^5*g^4*x)*log((b*x + a)^n)^2 + 12*(B^2*b^5*d^5*g^4*x^5 + 5*B^2*a*b^4*d^5*g^
4*x^4 + 10*B^2*a^2*b^3*d^5*g^4*x^3 + 10*B^2*a^3*b^2*d^5*g^4*x^2 + 5*B^2*a^4
*b*d^5*g^4*x)*log((d*x + c)^n)^2 + 2*(12*B^2*b^5*d^5*g^4*x^5 + 12*B^2*a^5*d
^5*g^4*n*log(b*x + a) + 3*(a*b^4*d^5*g^4*(n + 20) - b^5*c*d^4*g^4*n)*B^2*x^
4 + 4*(2*a^2*b^3*d^5*g^4*(2*n + 15) + b^5*c^2*d^3*g^4*n - 5*a*b^4*c*d^4*g^4
*n)*B^2*x^3 + 6*(2*a^3*b^2*d^5*g^4*(3*n + 10) - b^5*c^3*d^2*g^4*n + 5*a*b^4
*c^2*d^3*g^4*n - 10*a^2*b^3*c*d^4*g^4*n)*B^2*x^2 + 12*(a^4*b*d^5*g^4*(4*n +
 5) + b^5*c^4*d*g^4*n - 5*a*b^4*c^3*d^2*g^4*n + 10*a^2*b^3*c^2*d^3*g^4*n -
10*a^3*b^2*c*d^4*g^4*n)*B^2*x - 12*(b^5*c^5*g^4*n - 5*a*b^4*c^4*d*g^4*n + 1
0*a^2*b^3*c^3*d^2*g^4*n - 10*a^3*b^2*c^2*d^3*g^4*n + 5*a^4*b*c*d^4*g^4*n)*B
^2*log(d*x + c))*log((b*x + a)^n) - 2*(12*B^2*b^5*d^5*g^4*x^5 + 12*B^2*a^5*
d^5*g^4*n*log(b*x + a) + 3*(a*b^4*d^5*g^4*(n + 20) - b^5*c*d^4*g^4*n)*B^2*x
^4 + 4*(2*a^2*b^3*d^5*g^4*(2*n + 15) + b^5*c^2*d^3*g^4*n - 5*a*b^4*c*d^4*g^
4*n)*B^2*x^3 + 6*(2*a^3*b^2*d^5*g^4*(3*n + 10) - b^5*c^3*d^2*g^4*n + 5*a*b^
4*c^2*d^3*g^4*n - 10*a^2*b^3*c*d^4*g^4*n)*B^2*x^2 + 12*(a^4*b*d^5*g^4*(4*n
+ 5) + b^5*c^4*d*g^4*n - 5*a*b^4*c^3*d^2*g^4*n + 10*a^2*b^3*c^2*d^3*g^4*n -
 10*a^3*b^2*c*d^4*g^4*n)*B^2*x - 12*(b^5*c^5*g^4*n - 5*a*b^4*c^4*d*g^4*n +
10*a^2*b^3*c^3*d^2*g^4*n - 10*a^3*b^2*c^2*d^3*g^4*n + 5*a^4*b*c*d^4*g^4*n)*
B^2*log(d*x + c) + 12*(B^2*b^5*d^5*g^4*x^5 + 5*B^2*a*b^4*d^5*g^4*x^4 + 10*B
^2*a^2*b^3*d^5*g^4*x^3 + 10*B^2*a^3*b^2*d^5*g^4*x^2 + 5*B^2*a^4*b*d^5*g^4*x
)*log((b*x + a)^n))*log((d*x + c)^n))/(b*d^5)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^4*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="fr
icas")
```

```
[Out] integral(A^2*b^4*g^4*x^4 + 4*A^2*a*b^3*g^4*x^3 + 6*A^2*a^2*b^2*g^4*x^2 + 4*
A^2*a^3*b*g^4*x + A^2*a^4*g^4 + (B^2*b^4*g^4*x^4 + 4*B^2*a*b^3*g^4*x^3 + 6*
B^2*a^2*b^2*g^4*x^2 + 4*B^2*a^3*b*g^4*x + B^2*a^4*g^4)*log(((b*x + a)/(d*x
+ c))^n*e)^2 + 2*(A*B*b^4*g^4*x^4 + 4*A*B*a*b^3*g^4*x^3 + 6*A*B*a^2*b^2*g^4
*x^2 + 4*A*B*a^3*b*g^4*x + A*B*a^4*g^4)*log(((b*x + a)/(d*x + c))^n*e), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)**4*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^4*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="gi
ac")
```

```
[Out] integrate((b*g*x + a*g)^4*(B*log(((b*x + a)/(d*x + c))^n*e) + A)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a\,g+b\,g\,x)^4\,{\left(A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*g + b*g*x)^4*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2,x)
```

```
[Out] int((a*g + b*g*x)^4*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2, x)
```

3.11 $\int (ag+bgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$

Optimal. Leaf size=335

$$-\frac{B(bc-ad)g^3n(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{6bd}+\frac{g^3(a+bx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4b}+\frac{B(bc-ad)^2g^3n(}{}$$

```
[Out] -1/6*B*(-a*d+b*c)*g^3*n*(b*x+a)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b/d+1/4*g
^3*(b*x+a)^4*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/b+1/12*B*(-a*d+b*c)^2*g^3*n*
(b*x+a)^2*(3*A+B*n+3*B*ln(e*((b*x+a)/(d*x+c))^n))/b/d^2-1/12*B*(-a*d+b*c)^3
*g^3*n*(b*x+a)*(6*A+5*B*n+6*B*ln(e*((b*x+a)/(d*x+c))^n))/b/d^3-1/12*B*(-a*d
+b*c)^4*g^3*n*(6*A+11*B*n+6*B*ln(e*((b*x+a)/(d*x+c))^n))*ln((-a*d+b*c)/b/(d
*x+c))/b/d^4-1/2*B^2*(-a*d+b*c)^4*g^3*n^2*polylog(2,d*(b*x+a)/b/(d*x+c))/b/
d^4
```

Rubi [A]

time = 0.28, antiderivative size = 335, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2549, 2381, 2384, 2354, 2438}

$$-\frac{B^2g^3n^2(bc-ad)^4\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{2bd^4}-\frac{Bg^3n(bc-ad)^4\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(6B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+6A+11Bn\right)}{12bd^4}-\frac{Bg^3n(a+bx)(bc-ad)^3\left(6B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+6A+5Bn\right)}{12bd^3}+\frac{Bg^3n(a+bx)^2(bc-ad)^2\left(3B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+3A+Bn\right)}{12bd^2}-\frac{Bg^3n(a+bx)^3(bc-ad)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{6bd}+\frac{g^3(a+bx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(a*g + b*g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

```
[Out] -1/6*(B*(b*c - a*d)*g^3*n*(a + b*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]
))/(b*d) + (g^3*(a + b*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(4*b)
 + (B*(b*c - a*d)^2*g^3*n*(a + b*x)^2*(3*A + B*n + 3*B*Log[e*((a + b*x)/(c
+ d*x))^n]))/(12*b*d^2) - (B*(b*c - a*d)^3*g^3*n*(a + b*x)*(6*A + 5*B*n + 6
*B*Log[e*((a + b*x)/(c + d*x))^n]))/(12*b*d^3) - (B*(b*c - a*d)^4*g^3*n*(6*
A + 11*B*n + 6*B*Log[e*((a + b*x)/(c + d*x))^n])*Log[(b*c - a*d)/(b*(c + d*
x))])/(12*b*d^4) - (B^2*(b*c - a*d)^4*g^3*n^2*PolyLog[2, (d*(a + b*x))/(b*(
c + d*x))])/(2*b*d^4)
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2381

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + Dist[b*n*(p/(d*(q + 1))), Int[(f*x)^
```

```
m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, d
, e, f, m, n, q}, x] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 2384

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Dist[f/(e*(q + 1)), Int[(f*x)^(m - 1)*(d + e*x)^(q + 1
)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x
] && ILtQ[q, -1] && GtQ[m, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2549

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(m +
1)*(g/b)^m, Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x], x,
(a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ
[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[b*f - a*g, 0] && (GtQ[p, 0] || LtQ
[m, -1])
```

Rubi steps

$$\int (ag+bgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx = \frac{g^3(a+bx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4b} - \frac{(Bn)\int \frac{(bc-ad)g^{}}{}}{}$$

$$= \frac{g^3(a+bx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4b} - \frac{(B(bc-ad)g^3 n}{}$$

$$= \frac{g^3(a+bx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4b} - \frac{(B(bc-ad)g^3 n}{}$$

$$= \frac{g^3(a+bx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4b} - \frac{(B(bc-ad)g^3 n}{}$$

$$= -\frac{AB(bc-ad)^3 g^3 nx}{2d^3} + \frac{B(bc-ad)^2 g^3 n(a+bx)^2 (A+}{4bd^2}$$

$$= -\frac{AB(bc-ad)^3 g^3 nx}{2d^3} - \frac{B^2(bc-ad)^3 g^3 n(a+bx)\log (e}{2bd^3}$$

$$= -\frac{AB(bc-ad)^3 g^3 nx}{2d^3} - \frac{B^2(bc-ad)^3 g^3 n(a+bx)\log (e}{2bd^3}$$

$$= -\frac{AB(bc-ad)^3 g^3 nx}{2d^3} - \frac{5B^2(bc-ad)^3 g^3 n^2 x}{12d^3} + \frac{B^2(bc-}{}$$

$$= -\frac{AB(bc-ad)^3 g^3 nx}{2d^3} - \frac{5B^2(bc-ad)^3 g^3 n^2 x}{12d^3} + \frac{B^2(bc-}{}$$

$$= -\frac{AB(bc-ad)^3 g^3 nx}{2d^3} - \frac{5B^2(bc-ad)^3 g^3 n^2 x}{12d^3} + \frac{B^2(bc-}{}$$

Mathematica [A]
time = 0.24, size = 411, normalized size = 1.23

$$\frac{g^3\left((a+bx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 - \frac{B(bc-ad)n\left(6Abd(bc-ad)^2x+6Bd(bc-ad)^2(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+3d^2(-bc+ad)(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)+2d^3(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)-6B(bc-ad)^3n\log(c+dx)-6(bc-ad)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log(c+dx)+B(bc-ad)n(2bd(bc-ad)x-d^2(a+bx)^2-2(bc-ad)^2\log(c+dx))+3B(bc-ad)^2n(bdx+(-bc+ad)\log(c+dx))+3B(bc-ad)^3n\left(\left(2\log\left(\frac{d(a+bx)}{-bc+ad}\right)-\log(c+dx)\right)\log(c+dx)+2\text{Li}_2\left(\frac{b(c+dx)}{bc-ad}\right)\right)\right)}{3d^4}\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*g + b*g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

```
[Out] (g^3*((a + b*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 - (B*(b*c - a*d)
*n*(6*A*b*d*(b*c - a*d)^2*x + 6*B*d*(b*c - a*d)^2*(a + b*x)*Log[e*((a + b*x
)/(c + d*x))^n] + 3*d^2*(-(b*c) + a*d)*(a + b*x)^2*(A + B*Log[e*((a + b*x)/
(c + d*x))^n]) + 2*d^3*(a + b*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) -
 6*B*(b*c - a*d)^3*n*Log[c + d*x] - 6*(b*c - a*d)^3*(A + B*Log[e*((a + b*x)
```

```
/(c + d*x))^n])*Log[c + d*x] + B*(b*c - a*d)*n*(2*b*d*(b*c - a*d)*x - d^2*(
a + b*x)^2 - 2*(b*c - a*d)^2*Log[c + d*x]) + 3*B*(b*c - a*d)^2*n*(b*d*x + (
-(b*c) + a*d)*Log[c + d*x]) + 3*B*(b*c - a*d)^3*n*((2*Log[(d*(a + b*x))/(-(
b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c
 - a*d)])))/(3*d^4)))/(4*b)
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int (bgx+ag)^3 \left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*g*x+a*g)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

```
[Out] int((b*g*x+a*g)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 2039 vs. $2(327) = 654$.
time = 0.89, size = 2039, normalized size = 6.09

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="ma
xima")
```

```
[Out] 1/2*A*B*b^3*g^3*x^4*log((b*x/(d*x + c) + a/(d*x + c))^n*e) + 1/4*A^2*b^3*g^
3*x^4 + 2*A*B*a*b^2*g^3*x^3*log((b*x/(d*x + c) + a/(d*x + c))^n*e) + A^2*a*
b^2*g^3*x^3 + 3*A*B*a^2*b*g^3*x^2*log((b*x/(d*x + c) + a/(d*x + c))^n*e) +
3/2*A^2*a^2*b*g^3*x^2 - 1/12*A*B*b^3*g^3*n*(6*a^4*log(b*x + a)/b^4 - 6*c^4*
log(d*x + c)/d^4 + (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^
3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3)) + A*B*a*b^2*g^3*n*(2*a^3*log(b
*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^
2 - a^2*d^2)*x)/(b^2*d^2)) - 3*A*B*a^2*b*g^3*n*(a^2*log(b*x + a)/b^2 - c^2*
log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d)) + 2*A*B*a^3*g^3*n*(a*log(b*x + a)/b
 - c*log(d*x + c)/d) + 2*A*B*a^3*g^3*x*log((b*x/(d*x + c) + a/(d*x + c))^n*
e) + A^2*a^3*g^3*x + 1/12*((11*n^2 + 6*n)*b^3*c^4*g^3 - 2*(19*n^2 + 12*n)*a
*b^2*c^3*d*g^3 + 9*(5*n^2 + 4*n)*a^2*b*c^2*d^2*g^3 - 6*(3*n^2 + 4*n)*a^3*c*
d^3*g^3)*B^2*log(d*x + c)/d^4 + 1/2*(b^4*c^4*g^3*n^2 - 4*a*b^3*c^3*d*g^3*n^
2 + 6*a^2*b^2*c^2*d^2*g^3*n^2 - 4*a^3*b*c*d^3*g^3*n^2 + a^4*d^4*g^3*n^2)*(l
og(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c
- a*d)))*B^2/(b*d^4) + 1/12*(3*B^2*b^4*d^4*g^3*x^4 - 3*B^2*a^4*d^4*g^3*n^2*
log(b*x + a)^2 + 2*(a*b^3*d^4*g^3*(n + 6) - b^4*c*d^3*g^3*n)*B^2*x^3 + ((n^
2 + 3*n)*b^4*c^2*d^2*g^3 - 2*(n^2 + 6*n)*a*b^3*c*d^3*g^3 + (n^2 + 9*n + 18)
*a^2*b^2*d^4*g^3)*B^2*x^2 - 6*(b^4*c^4*g^3*n^2 - 4*a*b^3*c^3*d*g^3*n^2 + 6*
```

```
a^2*b^2*c^2*d^2*g^3*n^2 - 4*a^3*b*c*d^3*g^3*n^2)*B^2*log(b*x + a)*log(d*x +
 c) + 3*(b^4*c^4*g^3*n^2 - 4*a*b^3*c^3*d*g^3*n^2 + 6*a^2*b^2*c^2*d^2*g^3*n^
2 - 4*a^3*b*c*d^3*g^3*n^2)*B^2*log(d*x + c)^2 - ((5*n^2 + 6*n)*b^4*c^3*d*g^
3 - (17*n^2 + 24*n)*a*b^3*c^2*d^2*g^3 + (19*n^2 + 36*n)*a^2*b^2*c*d^3*g^3 -
 (7*n^2 + 18*n + 12)*a^3*b*d^4*g^3)*B^2*x - (6*a*b^3*c^3*d*g^3*n^2 - 21*a^2
*b^2*c^2*d^2*g^3*n^2 + 26*a^3*b*c*d^3*g^3*n^2 - (11*n^2 + 6*n)*a^4*d^4*g^3)
*B^2*log(b*x + a) + 3*(B^2*b^4*d^4*g^3*x^4 + 4*B^2*a*b^3*d^4*g^3*x^3 + 6*B^
2*a^2*b^2*d^4*g^3*x^2 + 4*B^2*a^3*b*d^4*g^3*x)*log((b*x + a)^n)^2 + 3*(B^2*
b^4*d^4*g^3*x^4 + 4*B^2*a*b^3*d^4*g^3*x^3 + 6*B^2*a^2*b^2*d^4*g^3*x^2 + 4*B
^2*a^3*b*d^4*g^3*x)*log((d*x + c)^n)^2 + (6*B^2*b^4*d^4*g^3*x^4 + 6*B^2*a^4
*d^4*g^3*n*log(b*x + a) + 2*(a*b^3*d^4*g^3*(n + 12) - b^4*c*d^3*g^3*n)*B^2*
x^3 + 3*(3*a^2*b^2*d^4*g^3*(n + 4) + b^4*c^2*d^2*g^3*n - 4*a*b^3*c*d^3*g^3*
n)*B^2*x^2 + 6*(a^3*b*d^4*g^3*(3*n + 4) - b^4*c^3*d*g^3*n + 4*a*b^3*c^2*d^2
*g^3*n - 6*a^2*b^2*c*d^3*g^3*n)*B^2*x + 6*(b^4*c^4*g^3*n - 4*a*b^3*c^3*d*g^
3*n + 6*a^2*b^2*c^2*d^2*g^3*n - 4*a^3*b*c*d^3*g^3*n)*B^2*log(d*x + c))*log(
(b*x + a)^n) - (6*B^2*b^4*d^4*g^3*x^4 + 6*B^2*a^4*d^4*g^3*n*log(b*x + a) +
2*(a*b^3*d^4*g^3*(n + 12) - b^4*c*d^3*g^3*n)*B^2*x^3 + 3*(3*a^2*b^2*d^4*g^3
*(n + 4) + b^4*c^2*d^2*g^3*n - 4*a*b^3*c*d^3*g^3*n)*B^2*x^2 + 6*(a^3*b*d^4*
g^3*(3*n + 4) - b^4*c^3*d*g^3*n + 4*a*b^3*c^2*d^2*g^3*n - 6*a^2*b^2*c*d^3*g
^3*n)*B^2*x + 6*(b^4*c^4*g^3*n - 4*a*b^3*c^3*d*g^3*n + 6*a^2*b^2*c^2*d^2*g^
3*n - 4*a^3*b*c*d^3*g^3*n)*B^2*log(d*x + c) + 6*(B^2*b^4*d^4*g^3*x^4 + 4*B^
2*a*b^3*d^4*g^3*x^3 + 6*B^2*a^2*b^2*d^4*g^3*x^2 + 4*B^2*a^3*b*d^4*g^3*x)*lo
g((b*x + a)^n))*log((d*x + c)^n))/(b*d^4)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="fr
icas")
```

```
[Out] integral(A^2*b^3*g^3*x^3 + 3*A^2*a*b^2*g^3*x^2 + 3*A^2*a^2*b*g^3*x + A^2*a^
3*g^3 + (B^2*b^3*g^3*x^3 + 3*B^2*a*b^2*g^3*x^2 + 3*B^2*a^2*b*g^3*x + B^2*a^
3*g^3)*log(((b*x + a)/(d*x + c))^n*e)^2 + 2*(A*B*b^3*g^3*x^3 + 3*A*B*a*b^2*
g^3*x^2 + 3*A*B*a^2*b*g^3*x + A*B*a^3*g^3)*log(((b*x + a)/(d*x + c))^n*e),
x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$g^{3}\left(\int A^{2}a^{3}\,dx + \int A^{2}b^{3}x^{3}\,dx + \int B^{2}a^{3}\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^{n}\right)^{2}dx + \int 2ABa^{3}\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^{n}\right)dx + \int 3A^{2}ab^{2}x^{2}\,dx + \int 3A^{2}a^{2}bx\,dx + \int B^{2}b^{3}x^{3}\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^{n}\right)^{2}dx + \int 2ABb^{3}x^{3}\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^{n}\right)dx + \int 3B^{2}ab^{2}x^{2}\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^{n}\right)^{2}dx + \int 3B^{2}a^{2}bx\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^{n}\right)^{2}dx + \int 6ABab^{2}x^{2}\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^{n}\right)dx + \int 6ABa^{2}bx\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^{n}\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)**3*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
```

```
[Out] g**3*(Integral(A**2*a**3, x) + Integral(A**2*b**3*x**3, x) + Integral(B**2*
a**3*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)**2, x) + Integral(2*A*B*a**3*l
og(e*(a/(c + d*x) + b*x/(c + d*x))**n), x) + Integral(3*A**2*a*b**2*x**2, x
) + Integral(3*A**2*a**2*b*x, x) + Integral(B**2*b**3*x**3*log(e*(a/(c + d*
x) + b*x/(c + d*x))**n)**2, x) + Integral(2*A*B*b**3*x**3*log(e*(a/(c + d*x
) + b*x/(c + d*x))**n), x) + Integral(3*B**2*a*b**2*x**2*log(e*(a/(c + d*x)
 + b*x/(c + d*x))**n)**2, x) + Integral(3*B**2*a**2*b*x*log(e*(a/(c + d*x)
+ b*x/(c + d*x))**n)**2, x) + Integral(6*A*B*a*b**2*x**2*log(e*(a/(c + d*x)
 + b*x/(c + d*x))**n), x) + Integral(6*A*B*a**2*b*x*log(e*(a/(c + d*x) + b*
x/(c + d*x))**n), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="gi
ac")
```

```
[Out] integrate((b*g*x + a*g)^3*(B*log(((b*x + a)/(d*x + c))^n*e) + A)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a\,g+b\,g\,x)^3\,\left(A+B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*g + b*g*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2,x)
```

```
[Out] int((a*g + b*g*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2, x)
```

3.12 $\int (ag+bgx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 \, dx$

Optimal. Leaf size=274

$$-\frac{B(bc-ad)g^2n(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3bd}+\frac{g^2(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3b}+\frac{B(bc-ad)^2g^2n}{}$$

```
[Out] -1/3*B*(-a*d+b*c)*g^2*n*(b*x+a)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b/d+1/3*g
^2*(b*x+a)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/b+1/3*B*(-a*d+b*c)^2*g^2*n*(
b*x+a)*(2*A+B*n+2*B*ln(e*((b*x+a)/(d*x+c))^n))/b/d^2+1/3*B*(-a*d+b*c)^3*g^2
*n*(2*A+3*B*n+2*B*ln(e*((b*x+a)/(d*x+c))^n))*ln((-a*d+b*c)/b/(d*x+c))/b/d^3
+2/3*B^2*(-a*d+b*c)^3*g^2*n^2*polylog(2,d*(b*x+a)/b/(d*x+c))/b/d^3
```

Rubi [A]
time = 0.22, antiderivative size = 274, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2549, 2381, 2384, 2354, 2438}

$$\frac{2B^2g^2n^2(bc-ad)^3\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{3bd^3}+\frac{Bg^2n(bc-ad)^3\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(2B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+2A+3Bn\right)}{3bd^3}+\frac{Bg^2n(a+bx)(bc-ad)^2\left(2B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+2A+Bn\right)}{3bd^2}-\frac{Bg^2n(a+bx)^2(bc-ad)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3bd}+\frac{g^2(a+bx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(a*g + b*g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

```
[Out] -1/3*(B*(b*c - a*d)*g^2*n*(a + b*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]
))/(b*d) + (g^2*(a + b*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(3*b)
 + (B*(b*c - a*d)^2*g^2*n*(a + b*x)*(2*A + B*n + 2*B*Log[e*((a + b*x)/(c +
d*x))^n]))/(3*b*d^2) + (B*(b*c - a*d)^3*g^2*n*(2*A + 3*B*n + 2*B*Log[e*((a
+ b*x)/(c + d*x))^n])*Log[(b*c - a*d)/(b*(c + d*x))])/(3*b*d^3) + (2*B^2*(b
*c - a*d)^3*g^2*n^2*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/(3*b*d^3)
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2381

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + Dist[b*n*(p/(d*(q + 1))), Int[(f*x)^
m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, d
, e, f, m, n, q}, x] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 2384

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Dist[f/(e*(q + 1)), Int[(f*x)^(m - 1)*(d + e*x)^(q + 1
)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x
] && ILtQ[q, -1] && GtQ[m, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2549

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(m +
1)*(g/b)^m, Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x], x,
(a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ
[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[b*f - a*g, 0] && (GtQ[p, 0] || LtQ
[m, -1])
```

Rubi steps

$$\int (ag+bgx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx = \frac{g^2(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3b} - \frac{(2Bn)\int \frac{(bc-ad)}{}}{}$$

$$= \frac{g^2(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3b} - \frac{(2B(bc-ad)g^2}{}$$

$$= \frac{g^2(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3b} - \frac{(2B(bc-ad)g^2}{}$$

$$= \frac{g^2(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3b} - \frac{(2B(bc-ad)g^2}{}$$

$$= \frac{2AB(bc-ad)^2g^2nx}{3d^2} - \frac{B(bc-ad)g^2n(a+bx)^2\left(A+B\right.}{3bd}$$

$$= \frac{2AB(bc-ad)^2g^2nx}{3d^2} + \frac{2B^2(bc-ad)^2g^2n(a+bx)\log\left(\right.}{3bd^2}$$

$$= \frac{2AB(bc-ad)^2g^2nx}{3d^2} + \frac{2B^2(bc-ad)^2g^2n(a+bx)\log\left(\right.}{3bd^2}$$

$$= \frac{2AB(bc-ad)^2g^2nx}{3d^2} + \frac{B^2(bc-ad)^2g^2n^2x}{3d^2} + \frac{2B^2(bc-}{}$$

$$= \frac{2AB(bc-ad)^2g^2nx}{3d^2} + \frac{B^2(bc-ad)^2g^2n^2x}{3d^2} + \frac{2B^2(bc-}{}$$

$$= \frac{2AB(bc-ad)^2g^2nx}{3d^2} + \frac{B^2(bc-ad)^2g^2n^2x}{3d^2} + \frac{2B^2(bc-}{}$$

Mathematica [A]
time = 0.16, size = 303, normalized size = 1.11

$$\frac{g^2\left((a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2+\frac{B(bc-ad)n\left(2Abd(bc-ad)x+2Bd(bc-ad)(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-d^2(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)-2B(bc-ad)^2n\log(c+dx)-2(bc-ad)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log(c+dx)+B(bc-ad)n(bdx+(-bc+ad)\log(c+dx))+B(bc-ad)^2n\left(\left(2\log\left(\frac{d(a+bx)}{-bc+ad}\right)-\log(c+dx)\right)\log(c+dx)+2\text{Li}_2\left(\frac{b(c+dx)}{bc-ad}\right)\right)\right)}{d^3}\right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*g + b*g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

```
[Out] (g^2*((a + b*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 + (B*(b*c - a*d)
*n*(2*A*b*d*(b*c - a*d)*x + 2*B*d*(b*c - a*d)*(a + b*x)*Log[e*((a + b*x)/(c
 + d*x))^n] - d^2*(a + b*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - 2*B*
(b*c - a*d)^2*n*Log[c + d*x] - 2*(b*c - a*d)^2*(A + B*Log[e*((a + b*x)/(c +
```

```
 d*x))^n])*Log[c + d*x] + B*(b*c - a*d)*n*(b*d*x + (-(b*c) + a*d)*Log[c + d
*x]) + B*(b*c - a*d)^2*n*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*
x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/d^3))/(3*b)
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int (bgx + ag)^2 \left(A + B \ln \left(e \left(\frac{bx + a}{dx + c}\right)^n\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*g*x+a*g)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

```
[Out] int((b*g*x+a*g)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1412 vs. 2(267) = 534.
time = 0.82, size = 1412, normalized size = 5.15

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="ma
xima")
```

```
[Out] 2/3*A*B*b^2*g^2*x^3*log((b*x/(d*x + c) + a/(d*x + c))^n*e) + 1/3*A^2*b^2*g^
2*x^3 + 2*A*B*a*b*g^2*x^2*log((b*x/(d*x + c) + a/(d*x + c))^n*e) + A^2*a*b*
g^2*x^2 + 1/3*A*B*b^2*g^2*n*(2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^
3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2)) - 2*A*B*
a*b*g^2*n*(a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d
)) + 2*A*B*a^2*g^2*n*(a*log(b*x + a)/b - c*log(d*x + c)/d) + 2*A*B*a^2*g^2*
x*log((b*x/(d*x + c) + a/(d*x + c))^n*e) + A^2*a^2*g^2*x - 1/3*((3*n^2 + 2*
n)*b^2*c^3*g^2 - (7*n^2 + 6*n)*a*b*c^2*d*g^2 + 2*(2*n^2 + 3*n)*a^2*c*d^2*g^
2)*B^2*log(d*x + c)/d^3 - 2/3*(b^3*c^3*g^2*n^2 - 3*a*b^2*c^2*d*g^2*n^2 + 3*
a^2*b*c*d^2*g^2*n^2 - a^3*d^3*g^2*n^2)*(log(b*x + a)*log((b*d*x + a*d)/(b*c
 - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B^2/(b*d^3) - 1/3*(B^2*a^
3*d^3*g^2*n^2*log(b*x + a)^2 - B^2*b^3*d^3*g^2*x^3 - (a*b^2*d^3*g^2*(n + 3)
 - b^3*c*d^2*g^2*n)*B^2*x^2 - 2*(b^3*c^3*g^2*n^2 - 3*a*b^2*c^2*d*g^2*n^2 +
3*a^2*b*c*d^2*g^2*n^2)*B^2*log(b*x + a)*log(d*x + c) + (b^3*c^3*g^2*n^2 - 3
*a*b^2*c^2*d*g^2*n^2 + 3*a^2*b*c*d^2*g^2*n^2)*B^2*log(d*x + c)^2 - ((n^2 +
2*n)*b^3*c^2*d*g^2 - 2*(n^2 + 3*n)*a*b^2*c*d^2*g^2 + (n^2 + 4*n + 3)*a^2*b*
d^3*g^2)*B^2*x - (2*a*b^2*c^2*d*g^2*n^2 - 5*a^2*b*c*d^2*g^2*n^2 + (3*n^2 +
2*n)*a^3*d^3*g^2)*B^2*log(b*x + a) - (B^2*b^3*d^3*g^2*x^3 + 3*B^2*a*b^2*d^3
*g^2*x^2 + 3*B^2*a^2*b*d^3*g^2*x)*log((b*x + a)^n)^2 - (B^2*b^3*d^3*g^2*x^3
 + 3*B^2*a*b^2*d^3*g^2*x^2 + 3*B^2*a^2*b*d^3*g^2*x)*log((d*x + c)^n)^2 - (2
*B^2*b^3*d^3*g^2*x^3 + 2*B^2*a^3*d^3*g^2*n*log(b*x + a) + (a*b^2*d^3*g^2*(n
```

```
 + 6) - b^3*c*d^2*g^2*n)*B^2*x^2 + 2*(a^2*b*d^3*g^2*(2*n + 3) + b^3*c^2*d*g
^2*n - 3*a*b^2*c*d^2*g^2*n)*B^2*x - 2*(b^3*c^3*g^2*n - 3*a*b^2*c^2*d*g^2*n
+ 3*a^2*b*c*d^2*g^2*n)*B^2*log(d*x + c))*log((b*x + a)^n) + (2*B^2*b^3*d^3*
g^2*x^3 + 2*B^2*a^3*d^3*g^2*n*log(b*x + a) + (a*b^2*d^3*g^2*(n + 6) - b^3*c
*d^2*g^2*n)*B^2*x^2 + 2*(a^2*b*d^3*g^2*(2*n + 3) + b^3*c^2*d*g^2*n - 3*a*b^
2*c*d^2*g^2*n)*B^2*x - 2*(b^3*c^3*g^2*n - 3*a*b^2*c^2*d*g^2*n + 3*a^2*b*c*d
^2*g^2*n)*B^2*log(d*x + c) + 2*(B^2*b^3*d^3*g^2*x^3 + 3*B^2*a*b^2*d^3*g^2*x
^2 + 3*B^2*a^2*b*d^3*g^2*x)*log((b*x + a)^n))*log((d*x + c)^n))/(b*d^3)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="fr
icas")
```

```
[Out] integral(A^2*b^2*g^2*x^2 + 2*A^2*a*b*g^2*x + A^2*a^2*g^2 + (B^2*b^2*g^2*x^2
 + 2*B^2*a*b*g^2*x + B^2*a^2*g^2)*log(((b*x + a)/(d*x + c))^n*e)^2 + 2*(A*B
*b^2*g^2*x^2 + 2*A*B*a*b*g^2*x + A*B*a^2*g^2)*log(((b*x + a)/(d*x + c))^n*e
), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$g^{2}\left(\int A^{2}a^{2}\,dx+\int A^{2}b^{2}x^{2}\,dx+\int B^{2}a^{2}\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^{n}\right)^{2}dx+\int 2ABa^{2}\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^{n}\right)dx+\int 2A^{2}abx\,dx+\int B^{2}b^{2}x^{2}\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^{n}\right)^{2}dx+\int 2ABb^{2}x^{2}\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^{n}\right)dx+\int 2B^{2}abx\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^{n}\right)^{2}dx+\int 4ABabx\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^{n}\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)**2*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
```

```
[Out] g**2*(Integral(A**2*a**2, x) + Integral(A**2*b**2*x**2, x) + Integral(B**2*
a**2*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)**2, x) + Integral(2*A*B*a**2*l
og(e*(a/(c + d*x) + b*x/(c + d*x))**n), x) + Integral(2*A**2*a*b*x, x) + In
tegral(B**2*b**2*x**2*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)**2, x) + Inte
gral(2*A*B*b**2*x**2*log(e*(a/(c + d*x) + b*x/(c + d*x))**n), x) + Integral
(2*B**2*a*b*x*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)**2, x) + Integral(4*A
*B*a*b*x*log(e*(a/(c + d*x) + b*x/(c + d*x))**n), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="gi
ac")
```

```
[Out] integrate((b*g*x + a*g)^2*(B*log(((b*x + a)/(d*x + c))^n*e) + A)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a\,g+b\,g\,x)^2\,\left(A+B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*g + b*g*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2,x)
```

```
[Out] int((a*g + b*g*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2, x)
```

3.13 $\int (ag+bgx) \left(A+B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 \, dx$

Optimal. Leaf size=196

$$-\frac{B(bc-ad)gn(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{bd}+\frac{g(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2b}-\frac{B(bc-ad)^2gn(A -}{}$$

```
[Out] -B*(-a*d+b*c)*g*n*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b/d+1/2*g*(b*x+a)
^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/b-B*(-a*d+b*c)^2*g*n*(A+B*n+B*ln(e*((b
*x+a)/(d*x+c))^n))*ln((-a*d+b*c)/b/(d*x+c))/b/d^2-B^2*(-a*d+b*c)^2*g*n^2*po
lylog(2,d*(b*x+a)/b/(d*x+c))/b/d^2
```

Rubi [A]
time = 0.13, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {2549, 2381, 2384, 2354, 2438}

$$-\frac{B^2gn^2(bc-ad)^2\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{bd^2}-\frac{Bgn(bc-ad)^2\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A+Bn\right)}{bd^2}-\frac{Bgn(a+bx)(bc-ad)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{bd}+\frac{g(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(a*g + b*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

```
[Out] -((B*(b*c - a*d)*g*n*(a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(b*d
)) + (g*(a + b*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(2*b) - (B*(b
*c - a*d)^2*g*n*(A + B*n + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[(b*c - a*d
)/(b*(c + d*x))])/(b*d^2) - (B^2*(b*c - a*d)^2*g*n^2*PolyLog[2, (d*(a + b*x
))/(b*(c + d*x))])/(b*d^2)
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2381

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + Dist[b*n*(p/(d*(q + 1))), Int[(f*x)^
m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, d
, e, f, m, n, q}, x] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 2384

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Dist[f/(e*(q + 1)), Int[(f*x)^(m - 1)*(d + e*x)^(q + 1
)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x
] && ILtQ[q, -1] && GtQ[m, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2549

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(m +
1)*(g/b)^m, Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x], x,
(a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ
[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[b*f - a*g, 0] && (GtQ[p, 0] || LtQ
[m, -1])
```

Rubi steps

$$\begin{aligned}
\int (ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx &= \frac{g(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2b} - \frac{(Bn)\int \frac{(bc-ad)g^2(a}{}}{} \\
&= \frac{g(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2b} - \frac{(B(bc-ad)gn)\int}{} \\
&= \frac{g(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2b} - \frac{(B(bc-ad)gn)\int}{} \\
&= \frac{g(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2b} - \frac{(B(bc-ad)gn)\int}{} \\
&= -\frac{AB(bc-ad)gnx}{d} + \frac{g(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right.}{2b} \\
&= -\frac{AB(bc-ad)gnx}{d} - \frac{B^2(bc-ad)gn(a+bx)\log\left(e\left(\frac{a+b}{c+d}\right.\right.}{bd} \\
&= -\frac{AB(bc-ad)gnx}{d} - \frac{B^2(bc-ad)gn(a+bx)\log\left(e\left(\frac{a+b}{c+d}\right.\right.}{bd} \\
&= -\frac{AB(bc-ad)gnx}{d} - \frac{B^2(bc-ad)gn(a+bx)\log\left(e\left(\frac{a+b}{c+d}\right.\right.}{bd} \\
&= -\frac{AB(bc-ad)gnx}{d} - \frac{B^2(bc-ad)gn(a+bx)\log\left(e\left(\frac{a+b}{c+d}\right.\right.}{bd} \\
&= -\frac{AB(bc-ad)gnx}{d} - \frac{B^2(bc-ad)gn(a+bx)\log\left(e\left(\frac{a+b}{c+d}\right.\right.}{bd}
\end{aligned}$$

Mathematica [A]
time = 0.14, size = 215, normalized size = 1.10

$$\frac{g\left((a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2-\frac{B(bc-ad)n\left(2Abdx+2Bd(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-2B(bc-ad)n\log(c+dx)-2(bc-ad)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log(c+dx)+B(bc-ad)n\left(\left(2\log\left(\frac{d(a+bx)}{-bc+ad}\right)-\log(c+dx)\right)\log(c+dx)+2\text{Li}_2\left(\frac{b(c+dx)}{bc-ad}\right)\right)\right)}{d^2}\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*g + b*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

```
[Out] (g*((a + b*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 - (B*(b*c - a*d)*n
*(2*A*b*d*x + 2*B*d*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n] - 2*B*(b*c - a
*d)*n*Log[c + d*x] - 2*(b*c - a*d)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*L
og[c + d*x] + B*(b*c - a*d)*n*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c
```

```
 + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/d^2))/(2
*b)
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int (bgx + ag)\left(A + B\ln\left(e\left(\frac{bx + a}{dx + c}\right)^{n}\right)\right)^{2}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*g*x+a*g)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

```
[Out] int((b*g*x+a*g)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 786 vs. $2(196) = 392$.
time = 0.89, size = 786, normalized size = 4.01

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="maxi
ma")
```

```
[Out] A*B*b*g*x^2*log((b*x/(d*x + c) + a/(d*x + c))^n*e) + 1/2*A^2*b*g*x^2 - A*B*
b*g*n*(a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d)) +
 2*A*B*a*g*n*(a*log(b*x + a)/b - c*log(d*x + c)/d) + 2*A*B*a*g*x*log((b*x/(
d*x + c) + a/(d*x + c))^n*e) + A^2*a*g*x + ((n^2 + n)*b*c^2*g - (n^2 + 2*n)
*a*c*d*g)*B^2*log(d*x + c)/d^2 + (b^2*c^2*g*n^2 - 2*a*b*c*d*g*n^2 + a^2*d^2
*g*n^2)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x +
a*d)/(b*c - a*d)))*B^2/(b*d^2) - 1/2*(B^2*a^2*d^2*g*n^2*log(b*x + a)^2 - B^
2*b^2*d^2*g*x^2 + 2*(b^2*c^2*g*n^2 - 2*a*b*c*d*g*n^2)*B^2*log(b*x + a)*log(
d*x + c) - (b^2*c^2*g*n^2 - 2*a*b*c*d*g*n^2)*B^2*log(d*x + c)^2 - 2*(a*b*d^
2*g*(n + 1) - b^2*c*d*g*n)*B^2*x + 2*(a*b*c*d*g*n^2 - (n^2 + n)*a^2*d^2*g)*
B^2*log(b*x + a) - (B^2*b^2*d^2*g*x^2 + 2*B^2*a*b*d^2*g*x)*log((b*x + a)^n)
^2 - (B^2*b^2*d^2*g*x^2 + 2*B^2*a*b*d^2*g*x)*log((d*x + c)^n)^2 - 2*(B^2*b^
2*d^2*g*x^2 + B^2*a^2*d^2*g*n*log(b*x + a) + (a*b*d^2*g*(n + 2) - b^2*c*d*g
*n)*B^2*x + (b^2*c^2*g*n - 2*a*b*c*d*g*n)*B^2*log(d*x + c))*log((b*x + a)^n
) + 2*(B^2*b^2*d^2*g*x^2 + B^2*a^2*d^2*g*n*log(b*x + a) + (a*b*d^2*g*(n + 2
) - b^2*c*d*g*n)*B^2*x + (b^2*c^2*g*n - 2*a*b*c*d*g*n)*B^2*log(d*x + c) + (
B^2*b^2*d^2*g*x^2 + 2*B^2*a*b*d^2*g*x)*log((b*x + a)^n))*log((d*x + c)^n))/
(b*d^2)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="fric
as")
```

```
[Out] integral(A^2*b*g*x + A^2*a*g + (B^2*b*g*x + B^2*a*g)*log(((b*x + a)/(d*x +
c))^n*e)^2 + 2*(A*B*b*g*x + A*B*a*g)*log(((b*x + a)/(d*x + c))^n*e), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$g\left(\int A^{2}a\,dx + \int A^{2}bx\,dx + \int B^{2}a\log\left(e\left(\frac{a}{c+dx} + \frac{bx}{c+dx}\right)^{n}\right)^{2}dx + \int 2ABa\log\left(e\left(\frac{a}{c+dx} + \frac{bx}{c+dx}\right)^{n}\right)dx + \int B^{2}bx\log\left(e\left(\frac{a}{c+dx} + \frac{bx}{c+dx}\right)^{n}\right)^{2}dx + \int 2ABbx\log\left(e\left(\frac{a}{c+dx} + \frac{bx}{c+dx}\right)^{n}\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
```

```
[Out] g*(Integral(A**2*a, x) + Integral(A**2*b*x, x) + Integral(B**2*a*log(e*(a/(
c + d*x) + b*x/(c + d*x))**n)**2, x) + Integral(2*A*B*a*log(e*(a/(c + d*x)
+ b*x/(c + d*x))**n), x) + Integral(B**2*b*x*log(e*(a/(c + d*x) + b*x/(c +
d*x))**n)**2, x) + Integral(2*A*B*b*x*log(e*(a/(c + d*x) + b*x/(c + d*x))**
n), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="giac
")
```

```
[Out] integrate((b*g*x + a*g)*(B*log(((b*x + a)/(d*x + c))^n*e) + A)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a\,g+b\,g\,x)\,\left(A+B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right)\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*g + b*g*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2,x)
```

```
[Out] int((a*g + b*g*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2, x)
```

3.14 $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ag+bgx}\,dx$

Optimal. Leaf size=138

$$-\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{bg}+\frac{2Bn\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\text{Li}_2\left(\frac{b(c+dx)}{d(a+bx)}\right)}{bg}+\frac{2B^2n^2\text{Li}_3\left(\frac{b(c+dx)}{d(a+bx)}\right)}{bg}$$

```
[Out] -(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2*ln(1-b*(d*x+c)/d/(b*x+a))/b/g+2*B*n*(A+B
*ln(e*((b*x+a)/(d*x+c))^n))*polylog(2,b*(d*x+c)/d/(b*x+a))/b/g+2*B^2*n^2*po
lylog(3,b*(d*x+c)/d/(b*x+a))/b/g
```

Rubi [A]

time = 0.12, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {2549, 2379, 2421, 6724}

$$\frac{2Bn\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{bg}+\frac{2B^2n^2\text{PolyLog}\left(3,\frac{b(c+dx)}{d(a+bx)}\right)}{bg}-\frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{bg}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(a*g + b*g*x),x]
```

```
[Out] -(((A + B*Log[e*((a + b*x)/(c + d*x))^n])^2*Log[1 - (b*(c + d*x))/(d*(a + b
*x))])/(b*g)) + (2*B*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*PolyLog[2, (b
*(c + d*x))/(d*(a + b*x))])/(b*g) + (2*B^2*n^2*PolyLog[3, (b*(c + d*x))/(d*
(a + b*x))])/(b*g)
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2549

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(m +
```

```
1)*(g/b)^m, Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x], x,
(a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ
[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[b*f - a*g, 0] && (GtQ[p, 0] || LtQ
[m, -1])
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ag+bgx}\,dx = \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log(ag+bgx)}{bg} - \frac{(2Bn)\int \frac{(c+dx)\left(-\frac{d(a+bx)}{(c+dx)^2}+\frac{b}{c+dx}\right)\left(A+\cdots\right.}{a\cdots}}{bg}$$

$$= \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log(ag+bgx)}{bg} - \frac{(2Bn)\int \frac{(bc-ad)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\cdots\right.}{(a+bx)(c+dx)}}{bg}$$

$$= \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log(ag+bgx)}{bg} - \frac{(2B(bc-ad)n)\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\cdots\right.\right.\right.}{(a+bx)\cdots}}{bg}$$

$$= \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log(ag+bgx)}{bg} - \frac{(2B(bc-ad)n)\int\left(\frac{d\left(-A-B\log\left(e\left(\cdots\right.\right.\right.}{(bc-\cdots}\right.}{\cdots}$$

$$= \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log(ag+bgx)}{bg} - \frac{(2Bn)\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log(ag\cdots}{a+bx}}{g}$$

$$= \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log(ag+bgx)}{bg} - \frac{(2Bn)\int\left(\frac{A\log(ag+bgx)}{a+bx}+\frac{B\log\left(e\left(\cdots\right.\right.}{\cdots}\right.}{g}$$

$$= \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log(ag+bgx)}{bg} - \frac{(2ABn)\int\frac{\log(ag+bgx)}{a+bx}\,dx}{g} - \frac{(2B^2\cdots}{\cdots}$$

$$= \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log(ag+bgx)}{bg} + \frac{2ABn\log\left(\frac{b(c+dx)}{bc-ad}\right)\log(ag+bg\cdots}{bg}$$

$$= \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log(ag+bgx)}{bg} + \frac{2ABn\log\left(\frac{b(c+dx)}{bc-ad}\right)\log(ag+bg\cdots}{bg}$$

$$= -\frac{ABn\log^2(g(a+bx))}{bg} + \frac{2B^2n\log(g(a+bx))\log\left((a+bx)^n\right)\log(-c-dx\cdots}{bg}$$

$$= -\frac{ABn\log^2(g(a+bx))}{bg} + \frac{2B^2n\log(g(a+bx))\log\left((a+bx)^n\right)\log(-c-dx\cdots}{bg}$$

$$= -\frac{ABn\log^2(g(a+bx))}{bg} + \frac{B^2n^2\log^3(g(a+bx))}{3bg} - \frac{B^2n^2\log^2(g(a+bx))\log\cdots}{bg}$$

$$= -\frac{ABn\log^2(g(a+bx))}{bg} + \frac{B^2n^2\log^3(g(a+bx))}{3bg} - \frac{B^2n^2\log^2(g(a+bx))\log\cdots}{bg}$$

$$= -\frac{ABn\log^2(g(a+bx))}{bg} + \frac{B^2n^2\log^3(g(a+bx))}{3bg} - \frac{B^2n^2\log^2(g(a+bx))\log\cdots}{bg}$$

$$= -\frac{ABn\log^2(g(a+bx))}{bg} + \frac{B^2n^2\log^3(g(a+bx))}{3bg} - \frac{B^2n^2\log^2(g(a+bx))\log\cdots}{bg}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 537 vs. 2(138) = 276.
time = 0.20, size = 537, normalized size = 3.89

$$\frac{3\log(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-Bn\log\left(\frac{a+bx}{c+dx}\right)\right)^2+3Bn\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-Bn\log\left(\frac{a+bx}{c+dx}\right)\right)\left(\log^2\left(\frac{a}{b}+x\right)-2\log(a+bx)\left(\log\left(\frac{a}{b}+x\right)-\log\left(\frac{c}{d}+x\right)-\log\left(\frac{a+bx}{c+dx}\right)\right)-2\left(\log\left(\frac{c}{d}+x\right)\log\left(\frac{d(a+bx)}{-bc+ad}\right)+\text{Li}_2\left(\frac{b(c+dx)}{bc-ad}\right)\right)\right)+B^2n^2\left(\log^3\left(\frac{a}{b}+x\right)+3\log^2\left(\frac{c}{d}+x\right)\log\left(\frac{d(a+bx)}{-bc+ad}\right)+3\log(a+bx)\left(-\log\left(\frac{a}{b}+x\right)+\log\left(\frac{c}{d}+x\right)+\log\left(\frac{a+bx}{c+dx}\right)\right)^2+3\log^2\left(\frac{a}{b}+x\right)\left(-\log\left(\frac{c}{d}+x\right)+\log\left(\frac{b(c+dx)}{bc-ad}\right)\right)+6\log\left(\frac{a}{b}+x\right)\text{Li}_2\left(\frac{d(a+bx)}{-bc+ad}\right)+6\log\left(\frac{c}{d}+x\right)\text{Li}_2\left(\frac{b(c+dx)}{bc-ad}\right)-3\left(\log\left(\frac{a}{b}+x\right)-\log\left(\frac{c}{d}+x\right)-\log\left(\frac{a+bx}{c+dx}\right)\right)\left(\log^2\left(\frac{a}{b}+x\right)-2\left(\log\left(\frac{c}{d}+x\right)\log\left(\frac{d(a+bx)}{-bc+ad}\right)+\text{Li}_2\left(\frac{b(c+dx)}{bc-ad}\right)\right)\right)-6\text{Li}_3\left(\frac{d(a+bx)}{-bc+ad}\right)-6\text{Li}_3\left(\frac{b(c+dx)}{bc-ad}\right)\right)}{3bg}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(a*g + b*g*x),x]
```

```
[Out] (3*Log[a + b*x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n] - B*n*Log[(a + b*x)/(
c + d*x)])^2 + 3*B*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n] - B*n*Log[(a + b
*x)/(c + d*x)])*(Log[a/b + x]^2 - 2*Log[a + b*x]*(Log[a/b + x] - Log[c/d +
x] - Log[(a + b*x)/(c + d*x)]) - 2*(Log[c/d + x]*Log[(d*(a + b*x))/(-(b*c)
+ a*d)] + PolyLog[2, (b*(c + d*x))/(b*c - a*d)])) + B^2*n^2*(Log[a/b + x]^3
 + 3*Log[c/d + x]^2*Log[(d*(a + b*x))/(-(b*c) + a*d)] + 3*Log[a + b*x]*(-Lo
g[a/b + x] + Log[c/d + x] + Log[(a + b*x)/(c + d*x)])^2 + 3*Log[a/b + x]^2*
(-Log[c/d + x] + Log[(b*(c + d*x))/(b*c - a*d)]) + 6*Log[a/b + x]*PolyLog[2
, (d*(a + b*x))/(-(b*c) + a*d)] + 6*Log[c/d + x]*PolyLog[2, (b*(c + d*x))/(
b*c - a*d)] - 3*(Log[a/b + x] - Log[c/d + x] - Log[(a + b*x)/(c + d*x)])*(L
og[a/b + x]^2 - 2*(Log[c/d + x]*Log[(d*(a + b*x))/(-(b*c) + a*d)] + PolyLog
[2, (b*(c + d*x))/(b*c - a*d)])) - 6*PolyLog[3, (d*(a + b*x))/(-(b*c) + a*d
)] - 6*PolyLog[3, (b*(c + d*x))/(b*c - a*d)]))/(3*b*g)
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2}{bgx+ag}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g),x)
```

```
[Out] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g),x, algorithm="maxi
ma")
```

```
[Out] B^2*log(b*x + a)*log((d*x + c)^n)^2/(b*g) + A^2*log(b*g*x + a*g)/(b*g) - in
tegrate(-(2*A*B*b*c + B^2*b*c + (B^2*b*d*x + B^2*b*c)*log((b*x + a)^n)^2 +
```

```
(2*A*B*b*d + B^2*b*d)*x + 2*(A*B*b*c + B^2*b*c + (A*B*b*d + B^2*b*d)*x)*log
((b*x + a)^n) - 2*(A*B*b*c + B^2*b*c + (A*B*b*d + B^2*b*d)*x + (B^2*b*d*n*x
 + B^2*a*d*n)*log(b*x + a) + (B^2*b*d*x + B^2*b*c)*log((b*x + a)^n))*log((d
*x + c)^n))/(b^2*d*g*x^2 + a*b*c*g + (b^2*c*g + a*b*d*g)*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g),x, algorithm="fric
as")
```

```
[Out] integral((B^2*log(((b*x + a)/(d*x + c))^n*e)^2 + 2*A*B*log(((b*x + a)/(d*x
+ c))^n*e) + A^2)/(b*g*x + a*g), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A^2}{a+bx}\,dx + \int \frac{B^2 \log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2}{a+bx}\,dx + \int \frac{2AB\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}{a+bx}\,dx}{g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(b*g*x+a*g),x)
```

```
[Out] (Integral(A**2/(a + b*x), x) + Integral(B**2*log(e*(a/(c + d*x) + b*x/(c +
d*x))**n)**2/(a + b*x), x) + Integral(2*A*B*log(e*(a/(c + d*x) + b*x/(c + d
*x))**n)/(a + b*x), x))/g
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g),x, algorithm="giac
")
```

```
[Out] integrate((B*log(((b*x + a)/(d*x + c))^n*e) + A)^2/(b*g*x + a*g), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\right)}^2}{a\,g+b\,g\,x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/(a*g + b*g*x),x)
```

```
[Out] int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/(a*g + b*g*x), x)
```

3.15 $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2}\,dx$

Optimal. Leaf size=136

$$-\frac{2B^2n^2(c+dx)}{(bc-ad)g^2(a+bx)}-\frac{2Bn(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)g^2(a+bx)}-\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)g^2(a+bx)}$$

```
[Out] -2*B^2*n^2*(d*x+c)/(-a*d+b*c)/g^2/(b*x+a)-2*B*n*(d*x+c)*(A+B*ln(e*((b*x+a)/
(d*x+c))^n))/(-a*d+b*c)/g^2/(b*x+a)-(d*x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))
^2/(-a*d+b*c)/g^2/(b*x+a)
```

Rubi [A]
time = 0.07, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {2549, 2342, 2341}

$$-\frac{2Bn(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{g^2(a+bx)(bc-ad)}-\frac{(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{g^2(a+bx)(bc-ad)}-\frac{2B^2n^2(c+dx)}{g^2(a+bx)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(a*g + b*g*x)^2,x]
```

```
[Out] (-2*B^2*n^2*(c + d*x))/((b*c - a*d)*g^2*(a + b*x)) - (2*B*n*(c + d*x)*(A +
B*Log[e*((a + b*x)/(c + d*x))^n]))/((b*c - a*d)*g^2*(a + b*x)) - ((c + d*x)
*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/((b*c - a*d)*g^2*(a + b*x))
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2549

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(m +
1)*(g/b)^m, Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x], x,
(a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ
```

```
[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[b*f - a*g, 0] && (GtQ[p, 0] || LtQ
[m, -1])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2}\,dx &= -\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{bg^2(a+bx)}+\frac{(2Bn)\int\frac{(bc-ad)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{g(a+bx)^2(c+dx)}\,dx}{bg} \\
&= -\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{bg^2(a+bx)}+\frac{(2B(bc-ad)n)\int\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)^2(c+dx)}\,dx}{bg^2} \\
&= -\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{bg^2(a+bx)}+\frac{(2B(bc-ad)n)\int\left(\frac{b\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)(a+bx)^2}-\frac{bd(\cdots}{\cdots}\right.}{b\cdots} \\
&= -\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{bg^2(a+bx)}+\frac{(2Bn)\int\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)^2}\,dx}{g^2}-\frac{(2Bdn)\int\frac{A+\cdots}{\cdots}}{(bc\cdots} \\
&= -\frac{2Bn\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{bg^2(a+bx)}-\frac{2Bdn\log(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\cdots}{b(bc-ad)g^2} \\
&= -\frac{2Bn\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{bg^2(a+bx)}-\frac{2Bdn\log(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\cdots}{b(bc-ad)g^2} \\
&= -\frac{2Bn\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{bg^2(a+bx)}-\frac{2Bdn\log(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\cdots}{b(bc-ad)g^2} \\
&= -\frac{2B^2n^2}{bg^2(a+bx)}-\frac{2B^2dn^2\log(a+bx)}{b(bc-ad)g^2}-\frac{2Bn\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{bg^2(a+bx)}-\frac{2B\cdots}{\cdots} \\
&= -\frac{2B^2n^2}{bg^2(a+bx)}-\frac{2B^2dn^2\log(a+bx)}{b(bc-ad)g^2}+\frac{B^2dn^2\log^2(a+bx)}{b(bc-ad)g^2}-\frac{2Bn\left(A+B\cdots\right.}{bg^2(\cdots} \\
&= -\frac{2B^2n^2}{bg^2(a+bx)}-\frac{2B^2dn^2\log(a+bx)}{b(bc-ad)g^2}+\frac{B^2dn^2\log^2(a+bx)}{b(bc-ad)g^2}-\frac{2Bn\left(A+B\cdots\right.}{bg^2(\cdots}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 0.38, size = 330, normalized size = 2.43

$$
-\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2+\frac{Bn\left(2(bc-ad)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)+2d(a+bx)\log(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)-2d(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log(c+dx)+2Bn(bc-ad+d(a+bx)\log(a+bx)-d(a+bx)\log(c+dx))-Bdn(a+bx)\left(\log(a+bx)\left(\log(a+bx)-2\log\left(\frac{b(c+dx)}{bc-ad}\right)\right)-2\text{Li}_2\left(\frac{d(a+bx)}{-bc+ad}\right)\right)+Bdn(a+bx)\left(\left(2\log\left(\frac{d(a+bx)}{-bc+ad}\right)-\log(c+dx)\right)\log(c+dx)+2\text{Li}_2\left(\frac{b(c+dx)}{bc-ad}\right)\right)\right)}{bc-ad}}{bg^2(a+bx)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(a*g + b*g*x)^2,x]
```

```
[Out] -(((A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 + (B*n*(2*(b*c - a*d)*(A + B*Lo
g[e*((a + b*x)/(c + d*x))^n]) + 2*d*(a + b*x)*Log[a + b*x]*(A + B*Log[e*((a
 + b*x)/(c + d*x))^n]) - 2*d*(a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n
])*Log[c + d*x] + 2*B*n*(b*c - a*d + d*(a + b*x)*Log[a + b*x] - d*(a + b*x)
*Log[c + d*x]) - B*d*n*(a + b*x)*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c
+ d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) + B*d*n
*(a + b*x)*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*
x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(b*c - a*d))/(b*g^2*(a + b*
x)))
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)\right)^{2}}{\left(bgx+ag\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^2,x)
```

```
[Out] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^2,x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 433 vs. 2(138) = 276.
time = 0.31, size = 433, normalized size = 3.18

$$-2\,ABn\left(\frac{1}{b^{2}g^{2}x+abg^{2}}+\frac{d\log\left(bx+a\right)}{(b^{2}c-abd)g^{2}}-\frac{d\log\left(dx+c\right)}{(b^{2}c-abd)g^{2}}\right)-\left(2\,n\left(\frac{1}{b^{2}g^{2}x+abg^{2}}+\frac{d\log\left(bx+a\right)}{(b^{2}c-abd)g^{2}}-\frac{d\log\left(dx+c\right)}{(b^{2}c-abd)g^{2}}\right)\log\left(\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^{n}e\right)-\frac{\left((bdx+ad)\log\left(bx+a\right)^{2}+(bdx+ad)\log\left(dx+c\right)^{2}-2\,bc+2\,ad-2\,(bdx+ad)\log\left(bx+a\right)+2\,(bdx+ad-(bdx+ad)\log\left(bx+a\right))\log\left(dx+c\right)\right)n^{2}}{ab^{2}cg^{2}-a^{2}bdg^{2}+(b^{3}cg^{2}-ab^{2}dg^{2})x}\right)B^{2}-\frac{B^{2}\log\left(\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^{n}e\right)^{2}}{b^{2}g^{2}x+abg^{2}}-\frac{2\,AB\log\left(\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^{n}e\right)}{b^{2}g^{2}x+abg^{2}}-\frac{A^{2}}{b^{2}g^{2}x+abg^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^2,x, algorithm="ma
xima")
```

```
[Out] -2*A*B*n*(1/(b^2*g^2*x + a*b*g^2) + d*log(b*x + a)/((b^2*c - a*b*d)*g^2) -
d*log(d*x + c)/((b^2*c - a*b*d)*g^2)) - (2*n*(1/(b^2*g^2*x + a*b*g^2) + d*l
og(b*x + a)/((b^2*c - a*b*d)*g^2) - d*log(d*x + c)/((b^2*c - a*b*d)*g^2))*l
og((b*x/(d*x + c) + a/(d*x + c))^n*e) - ((b*d*x + a*d)*log(b*x + a)^2 + (b*
d*x + a*d)*log(d*x + c)^2 - 2*b*c + 2*a*d - 2*(b*d*x + a*d)*log(b*x + a) +
2*(b*d*x + a*d - (b*d*x + a*d)*log(b*x + a))*log(d*x + c))*n^2/(a*b^2*c*g^2
 - a^2*b*d*g^2 + (b^3*c*g^2 - a*b^2*d*g^2)*x))*B^2 - B^2*log((b*x/(d*x + c)
 + a/(d*x + c))^n*e)^2/(b^2*g^2*x + a*b*g^2) - 2*A*B*log((b*x/(d*x + c) + a
/(d*x + c))^n*e)/(b^2*g^2*x + a*b*g^2) - A^2/(b^2*g^2*x + a*b*g^2)
```

Fricas [A]
time = 0.41, size = 212, normalized size = 1.56

$$-\frac{(A^{2}+2\,AB+B^{2})bc-(A^{2}+2\,AB+B^{2})ad+2\,(B^{2}bc-B^{2}ad)n^{2}+(B^{2}bdn^{2}x+B^{2}bcn^{2})\log\left(\frac{bx+a}{dx+c}\right)^{2}+2\,((AB+B^{2})bc-(AB+B^{2})ad)n+2\,(B^{2}bcn^{2}+(AB+B^{2})bcn+(B^{2}bdn^{2}+(AB+B^{2})bdn)x)\log\left(\frac{bx+a}{dx+c}\right)}{(b^{3}c-ab^{2}d)g^{2}x+(ab^{2}c-a^{2}bd)g^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^2,x, algorithm="fr
icas")
```

```
[Out] -((A^2 + 2*A*B + B^2)*b*c - (A^2 + 2*A*B + B^2)*a*d + 2*(B^2*b*c - B^2*a*d)
*n^2 + (B^2*b*d*n^2*x + B^2*b*c*n^2)*log((b*x + a)/(d*x + c))^2 + 2*((A*B +
 B^2)*b*c - (A*B + B^2)*a*d)*n + 2*(B^2*b*c*n^2 + (A*B + B^2)*b*c*n + (B^2*
b*d*n^2 + (A*B + B^2)*b*d*n)*x)*log((b*x + a)/(d*x + c)))/((b^3*c - a*b^2*d
)*g^2*x + (a*b^2*c - a^2*b*d)*g^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A^2}{a^2+2abx+b^2x^2}\,dx + \int \frac{B^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2}{a^2+2abx+b^2x^2}\,dx + \int \frac{2AB\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}{a^2+2abx+b^2x^2}\,dx}{g^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(b*g*x+a*g)**2,x)
```

```
[Out] (Integral(A**2/(a**2 + 2*a*b*x + b**2*x**2), x) + Integral(B**2*log(e*(a/(c
 + d*x) + b*x/(c + d*x))**n)**2/(a**2 + 2*a*b*x + b**2*x**2), x) + Integral
(2*A*B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(a**2 + 2*a*b*x + b**2*x**2)
, x))/g**2
```

Giac [A]
time = 5.03, size = 163, normalized size = 1.20

$$-\left(\frac{(dx+c)B^2n^2\log\left(\frac{bx+a}{dx+c}\right)^2}{(bx+a)g^2} + \frac{2\,(B^2n^2+ABn+B^2n)(dx+c)\log\left(\frac{bx+a}{dx+c}\right)}{(bx+a)g^2} + \frac{(2\,B^2n^2+2\,ABn+2\,B^2n+A^2+2\,AB+B^2)(dx+c)}{(bx+a)g^2}\right)\left(\frac{bc}{(bc-ad)^2} - \frac{ad}{(bc-ad)^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^2,x, algorithm="gi
ac")
```

```
[Out] -((d*x + c)*B^2*n^2*log((b*x + a)/(d*x + c))^2/((b*x + a)*g^2) + 2*(B^2*n^2
 + A*B*n + B^2*n)*(d*x + c)*log((b*x + a)/(d*x + c))/((b*x + a)*g^2) + (2*B
^2*n^2 + 2*A*B*n + 2*B^2*n + A^2 + 2*A*B + B^2)*(d*x + c)/((b*x + a)*g^2))*
(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)
```

Mupad [B]
time = 5.59, size = 238, normalized size = 1.75

$$-\frac{A^2+2\,A\,B\,n+2\,B^2\,n^2}{x\,b^2\,g^2+a\,b\,g^2} - \ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)^2\,\left(\frac{B^2}{b\,\left(a\,g^2+b\,g^2\,x\right)} - \frac{B^2\,d}{b\,g^2\,\left(a\,d-b\,c\right)}\right) - \ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\,\left(\frac{2\,B^2\,n}{x\,b^2\,g^2+a\,b\,g^2} + \frac{2\,A\,B}{x\,b^2\,g^2+a\,b\,g^2}\right) - \frac{B\,d\,n\,\mathrm{atan}\left(\frac{\left(2\,b\,d\,x+\frac{c\,b^2\,g^2+a\,d\,b\,g^2}{b\,g^2}\right)\,1{}\mathrm{i}}{a\,d-b\,c}\right)\,\left(A+B\,n\right)\,4{}\mathrm{i}}{b\,g^2\,\left(a\,d-b\,c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/(a*g + b*g*x)^2,x)
```

```
[Out] - (A^2 + 2*B^2*n^2 + 2*A*B*n)/(b^2*g^2*x + a*b*g^2) - log(e*((a + b*x)/(c +
 d*x))^n)^2*(B^2/(b*(a*g^2 + b*g^2*x)) - (B^2*d)/(b*g^2*(a*d - b*c))) - log
(e*((a + b*x)/(c + d*x))^n)*((2*B^2*n)/(b^2*g^2*x + a*b*g^2) + (2*A*B)/(b^2
*g^2*x + a*b*g^2)) - (B*d*n*atan(((2*b*d*x + (b^2*c*g^2 + a*b*d*g^2)/(b*g^2
))*1i)/(a*d - b*c))*(A + B*n)*4i)/(b*g^2*(a*d - b*c))
```

3.16 $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3}\,dx$

Optimal. Leaf size=288

$$\frac{2B^2dn^2(c+dx)}{(bc-ad)^2g^3(a+bx)}-\frac{bB^2n^2(c+dx)^2}{4(bc-ad)^2g^3(a+bx)^2}+\frac{2Bdn(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^2g^3(a+bx)}-\frac{bBn(c+dx)^2\left(A+\right.}{2(bc-ad)^2g}$$

```
[Out] 2*B^2*d*n^2*(d*x+c)/(-a*d+b*c)^2/g^3/(b*x+a)-1/4*b*B^2*n^2*(d*x+c)^2/(-a*d+
b*c)^2/g^3/(b*x+a)^2+2*B*d*n*(d*x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+
b*c)^2/g^3/(b*x+a)-1/2*b*B*n*(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*
d+b*c)^2/g^3/(b*x+a)^2+d*(d*x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*
c)^2/g^3/(b*x+a)-1/2*b*(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*
c)^2/g^3/(b*x+a)^2
```

Rubi [A]

time = 0.15, antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {2549, 2395, 2342, 2341}

$$-\frac{bBn(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2g^3(a+bx)^2(bc-ad)^2}+\frac{2Bdn(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{g^3(a+bx)(bc-ad)^2}-\frac{b(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2g^3(a+bx)^2(bc-ad)^2}+\frac{d(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{g^3(a+bx)(bc-ad)^2}-\frac{bB^2n^2(c+dx)^2}{4g^3(a+bx)^2(bc-ad)^2}+\frac{2B^2dn^2(c+dx)}{g^3(a+bx)(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(a*g + b*g*x)^3,x]
```

```
[Out] (2*B^2*d*n^2*(c + d*x))/((b*c - a*d)^2*g^3*(a + b*x)) - (b*B^2*n^2*(c + d*x
)^2)/(4*(b*c - a*d)^2*g^3*(a + b*x)^2) + (2*B*d*n*(c + d*x)*(A + B*Log[e*((
a + b*x)/(c + d*x))^n]))/((b*c - a*d)^2*g^3*(a + b*x)) - (b*B*n*(c + d*x)^2
*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(2*(b*c - a*d)^2*g^3*(a + b*x)^2)
+ (d*(c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/((b*c - a*d)^2*g^3
*(a + b*x)) - (b*(c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(2*(
b*c - a*d)^2*g^3*(a + b*x)^2)
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2549

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(m +
1)*(g/b)^m, Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x], x,
(a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ
[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[b*f - a*g, 0] && (GtQ[p, 0] || LtQ
[m, -1])
```

Rubi steps

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3}\,dx = -\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2bg^3(a+bx)^2}+\frac{(Bn)\int\frac{(bc-ad)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{g^2(a+bx)^3(c+dx)}\,dx}{bg}$$

$$= -\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2bg^3(a+bx)^2}+\frac{(B(bc-ad)n)\int\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)^3(c+dx)}\,dx}{bg^3}$$

$$= -\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2bg^3(a+bx)^2}+\frac{(B(bc-ad)n)\int\left(\frac{b\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)(a+bx)^3}-\frac{bd(A}{}\right.}{}$$

$$= -\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2bg^3(a+bx)^2}+\frac{(Bn)\int\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)^3}\,dx}{g^3}+\frac{(Bd^2n)\int\frac{A+B}{}}{(bc-}$$

$$= -\frac{Bn\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2bg^3(a+bx)^2}+\frac{Bdn\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b(bc-ad)g^3(a+bx)}+\frac{Bd^2n\log(a}{}$$

$$= -\frac{Bn\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2bg^3(a+bx)^2}+\frac{Bdn\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b(bc-ad)g^3(a+bx)}+\frac{Bd^2n\log(a}{}$$

$$= -\frac{Bn\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2bg^3(a+bx)^2}+\frac{Bdn\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b(bc-ad)g^3(a+bx)}+\frac{Bd^2n\log(a}{}$$

$$= -\frac{B^2n^2}{4bg^3(a+bx)^2}+\frac{3B^2dn^2}{2b(bc-ad)g^3(a+bx)}+\frac{3B^2d^2n^2\log(a+bx)}{2b(bc-ad)^2g^3}-\frac{Bn(A}{}$$

$$= -\frac{B^2n^2}{4bg^3(a+bx)^2}+\frac{3B^2dn^2}{2b(bc-ad)g^3(a+bx)}+\frac{3B^2d^2n^2\log(a+bx)}{2b(bc-ad)^2g^3}-\frac{B^2d^2n}{2b(b}$$

$$= -\frac{B^2n^2}{4bg^3(a+bx)^2}+\frac{3B^2dn^2}{2b(bc-ad)g^3(a+bx)}+\frac{3B^2d^2n^2\log(a+bx)}{2b(bc-ad)^2g^3}-\frac{B^2d^2n}{2b(b}$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 0.34, size = 463, normalized size = 1.61

$$-\frac{2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2+\frac{Bn\left(2(bc-ad)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)+4d(-bc+ad)(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)-4d^2(a+bx)^2\log(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)+4d^2(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log(c+dx)-4Bdn(a+bx)(bc-ad+d(a+bx)\log(a+bx)-d(a+bx)\log(c+dx))+Bn((bc-ad)^2+2d(-bc+ad)(a+bx)-2d^2(a+bx)^2\log(a+bx)+2d^2(a+bx)^2\log(c+dx))+2Bd^2n(a+bx)^2\left(\log(a+bx)\left(\log(a+bx)-2\log\left(\frac{b(c+dx)}{bc-ad}\right)\right)-2\text{Li}_2\left(\frac{d(a+bx)}{-bc+ad}\right)\right)-2Bd^2n(a+bx)^2\left(\left(2\log\left(\frac{d(a+bx)}{-bc+ad}\right)-\log(c+dx)\right)\log(c+dx)+2\text{Li}_2\left(\frac{b(c+dx)}{bc-ad}\right)\right)\right)}{(bc-ad)^2}}{4bg^3(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(a*g + b*g*x)^3,x]
```

```
[Out] -1/4*(2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 + (B*n*(2*(b*c - a*d)^2*(A
 + B*Log[e*((a + b*x)/(c + d*x))^n]) + 4*d*(-(b*c) + a*d)*(a + b*x)*(A + B*
```

```
Log[e*((a + b*x)/(c + d*x))^n]) - 4*d^2*(a + b*x)^2*Log[a + b*x]*(A + B*Log
[e*((a + b*x)/(c + d*x))^n]) + 4*d^2*(a + b*x)^2*(A + B*Log[e*((a + b*x)/(c
 + d*x))^n])*Log[c + d*x] - 4*B*d*n*(a + b*x)*(b*c - a*d + d*(a + b*x)*Log[
a + b*x] - d*(a + b*x)*Log[c + d*x]) + B*n*((b*c - a*d)^2 + 2*d*(-(b*c) + a
*d)*(a + b*x) - 2*d^2*(a + b*x)^2*Log[a + b*x] + 2*d^2*(a + b*x)^2*Log[c +
d*x]) + 2*B*d^2*n*(a + b*x)^2*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d
*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) - 2*B*d^2*
n*(a + b*x)^2*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c +
 d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(b*c - a*d)^2)/(b*g^3*(a
 + b*x)^2)
```

Maple [**F**]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{\left(A + B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)\right)^{2}}{\left(bgx + ag\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^3,x)
```

```
[Out] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^3,x)
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 864 vs. 2(286) = 572.
time = 0.34, size = 864, normalized size = 3.00

$$\frac{1}{2} ABn\left(\frac{2\,bdx - bc + 3\,ad}{(b^4c - ab^3d)g^3x^2 + 2\,(ab^3c - a^2b^2d)g^3x + (a^2b^2c - a^3bd)g^3} + \frac{2\,d^2\log(bx+a)}{(b^3c^2 - 2\,ab^2cd + a^2bd^2)g^3} - \frac{2\,d^2\log(dx+c)}{(b^3c^2 - 2\,ab^2cd + a^2bd^2)g^3}\right) + \frac{1}{4}\left(2\,n\left(\frac{2\,bdx - bc + 3\,ad}{(b^4c - ab^3d)g^3x^2 + 2\,(ab^3c - a^2b^2d)g^3x + (a^2b^2c - a^3bd)g^3} + \frac{2\,d^2\log(bx+a)}{(b^3c^2 - 2\,ab^2cd + a^2bd^2)g^3} - \frac{2\,d^2\log(dx+c)}{(b^3c^2 - 2\,ab^2cd + a^2bd^2)g^3}\right)\log\left(\left(\frac{bx}{dx+c} + \frac{a}{dx+c}\right)^{n}e\right) - \frac{\left(b^2c^2 - 8\,abcd + 7\,a^2d^2 + 2\,(b^2d^2x^2 + 2\,abd^2x + a^2d^2)\log(bx+a)^2 + 2\,(b^2d^2x^2 + 2\,abd^2x + a^2d^2)\log(dx+c)^2 - 6\,(b^2cd - abd^2)x - 6\,(b^2d^2x^2 + 2\,abd^2x + a^2d^2)\log(bx+a) + 2\,(3\,b^2d^2x^2 + 6\,abd^2x + 3\,a^2d^2 - 2\,(b^2d^2x^2 + 2\,abd^2x + a^2d^2)\log(bx+a))\log(dx+c)\right)n^2}{a^2b^3c^2g^3 - 2\,a^3b^2cdg^3 + a^4bd^2g^3 + (b^5c^2g^3 - 2\,ab^4cdg^3 + a^2b^3d^2g^3)x^2 + 2\,(ab^4c^2g^3 - 2\,a^2b^3cdg^3 + a^3b^2d^2g^3)x}\right)B^2 - \frac{B^2\log\left(\left(\frac{bx}{dx+c} + \frac{a}{dx+c}\right)^{n}e\right)^2}{2\,(b^3g^3x^2 + 2\,ab^2g^3x + a^2bg^3)} - \frac{AB\log\left(\left(\frac{bx}{dx+c} + \frac{a}{dx+c}\right)^{n}e\right)}{b^3g^3x^2 + 2\,ab^2g^3x + a^2bg^3} - \frac{A^2}{2\,(b^3g^3x^2 + 2\,ab^2g^3x + a^2bg^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^3,x, algorithm="ma
xima")
```

```
[Out] 1/2*A*B*n*((2*b*d*x - b*c + 3*a*d)/((b^4*c - a*b^3*d)*g^3*x^2 + 2*(a*b^3*c
- a^2*b^2*d)*g^3*x + (a^2*b^2*c - a^3*b*d)*g^3) + 2*d^2*log(b*x + a)/((b^3*
c^2 - 2*a*b^2*c*d + a^2*b*d^2)*g^3) - 2*d^2*log(d*x + c)/((b^3*c^2 - 2*a*b^
2*c*d + a^2*b*d^2)*g^3)) + 1/4*(2*n*((2*b*d*x - b*c + 3*a*d)/((b^4*c - a*b^
3*d)*g^3*x^2 + 2*(a*b^3*c - a^2*b^2*d)*g^3*x + (a^2*b^2*c - a^3*b*d)*g^3) +
 2*d^2*log(b*x + a)/((b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*g^3) - 2*d^2*log(d
*x + c)/((b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*g^3))*log((b*x/(d*x + c) + a/(
d*x + c))^n*e) - (b^2*c^2 - 8*a*b*c*d + 7*a^2*d^2 + 2*(b^2*d^2*x^2 + 2*a*b*
d^2*x + a^2*d^2)*log(b*x + a)^2 + 2*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*l
og(d*x + c)^2 - 6*(b^2*c*d - a*b*d^2)*x - 6*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^
2*d^2)*log(b*x + a) + 2*(3*b^2*d^2*x^2 + 6*a*b*d^2*x + 3*a^2*d^2 - 2*(b^2*d
^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*log(b*x + a))*log(d*x + c))*n^2/(a^2*b^3*c^
2*g^3 - 2*a^3*b^2*c*d*g^3 + a^4*b*d^2*g^3 + (b^5*c^2*g^3 - 2*a*b^4*c*d*g^3
```

```
+ a^2*b^3*d^2*g^3)*x^2 + 2*(a*b^4*c^2*g^3 - 2*a^2*b^3*c*d*g^3 + a^3*b^2*d^2
*g^3)*x))*B^2 - 1/2*B^2*log((b*x/(d*x + c) + a/(d*x + c))^n*e)^2/(b^3*g^3*x
^2 + 2*a*b^2*g^3*x + a^2*b*g^3) - A*B*log((b*x/(d*x + c) + a/(d*x + c))^n*e
)/(b^3*g^3*x^2 + 2*a*b^2*g^3*x + a^2*b*g^3) - 1/2*A^2/(b^3*g^3*x^2 + 2*a*b^
2*g^3*x + a^2*b*g^3)
```

Fricas [A]
time = 0.44, size = 528, normalized size = 1.83

$$-\frac{2\,(A^2+2\,AB+B^2)b^2c^2-4\,(A^2+2\,AB+B^2)abcd+2\,(A^2+2\,AB+B^2)a^2d^2+(B^2b^2c^2-8\,B^2abcd+7\,B^2a^2d^2)n^2-2\,(B^2b^2d^2n^2x^2+2\,B^2abd^2n^2x-(B^2b^2c^2-2\,B^2abcd)n^2)\log\left(\frac{bx+a}{dx+c}\right)^2+2\,((AB+B^2)b^2c^2-4\,(AB+B^2)abcd+3\,(AB+B^2)a^2d^2)n-2\,(3\,(B^2b^2cd-B^2abd^2)n^2+2\,((AB+B^2)b^2cd-(AB+B^2)abd^2)n)x+2\,((B^2b^2c^2-4\,B^2abcd)n^2-(3\,B^2b^2d^2n^2+2\,(AB+B^2)b^2d^2n)x^2+2\,((AB+B^2)b^2c^2-2\,(AB+B^2)abcd)n-2\,(2\,(AB+B^2)abd^2n+(B^2b^2cd+2\,B^2abd^2)n^2)x)\log\left(\frac{bx+a}{dx+c}\right)}{4\,((b^5c^2-2\,ab^4cd+a^2b^3d^2)g^3x^2+2\,(ab^4c^2-2\,a^2b^3cd+a^3b^2d^2)g^3x+(a^2b^3c^2-2\,a^3b^2cd+a^4bd^2)g^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^3,x, algorithm="fr
icas")
```

```
[Out] -1/4*(2*(A^2 + 2*A*B + B^2)*b^2*c^2 - 4*(A^2 + 2*A*B + B^2)*a*b*c*d + 2*(A^
2 + 2*A*B + B^2)*a^2*d^2 + (B^2*b^2*c^2 - 8*B^2*a*b*c*d + 7*B^2*a^2*d^2)*n^
2 - 2*(B^2*b^2*d^2*n^2*x^2 + 2*B^2*a*b*d^2*n^2*x - (B^2*b^2*c^2 - 2*B^2*a*b
*c*d)*n^2)*log((b*x + a)/(d*x + c))^2 + 2*((A*B + B^2)*b^2*c^2 - 4*(A*B + B
^2)*a*b*c*d + 3*(A*B + B^2)*a^2*d^2)*n - 2*(3*(B^2*b^2*c*d - B^2*a*b*d^2)*n
^2 + 2*((A*B + B^2)*b^2*c*d - (A*B + B^2)*a*b*d^2)*n)*x + 2*((B^2*b^2*c^2 -
 4*B^2*a*b*c*d)*n^2 - (3*B^2*b^2*d^2*n^2 + 2*(A*B + B^2)*b^2*d^2*n)*x^2 + 2
*((A*B + B^2)*b^2*c^2 - 2*(A*B + B^2)*a*b*c*d)*n - 2*(2*(A*B + B^2)*a*b*d^2
*n + (B^2*b^2*c*d + 2*B^2*a*b*d^2)*n^2)*x)*log((b*x + a)/(d*x + c)))/((b^5*
c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*g^3*x^2 + 2*(a*b^4*c^2 - 2*a^2*b^3*c*d + a
^3*b^2*d^2)*g^3*x + (a^2*b^3*c^2 - 2*a^3*b^2*c*d + a^4*b*d^2)*g^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A^2}{a^3+3a^2bx+3ab^2x^2+b^3x^3}\,dx+\int \frac{B^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2}{a^3+3a^2bx+3ab^2x^2+b^3x^3}\,dx+\int \frac{2AB\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}{a^3+3a^2bx+3ab^2x^2+b^3x^3}\,dx}{g^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(b*g*x+a*g)**3,x)
```

```
[Out] (Integral(A**2/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3), x) + Integr
al(B**2*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)**2/(a**3 + 3*a**2*b*x + 3*a
*b**2*x**2 + b**3*x**3), x) + Integral(2*A*B*log(e*(a/(c + d*x) + b*x/(c +
d*x))**n)/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3), x))/g**3
```

Giac [A]
time = 7.78, size = 458, normalized size = 1.59

$$-\frac{1}{4}\left(\frac{2\left(B^2bn^2-\frac{2\,(bx+a)B^2dn^2}{dx+c}\right)\log\left(\frac{bx+a}{dx+c}\right)^2}{\frac{(bx+a)^2bcg^3}{(dx+c)^2}-\frac{(bx+a)^2adg^3}{(dx+c)^2}}+\frac{2\left(B^2bn^2-\frac{4\,(bx+a)B^2dn^2}{dx+c}+2\,ABbn+2\,B^2bn-\frac{4\,(bx+a)ABdn}{dx+c}-\frac{4\,(bx+a)B^2dn}{dx+c}\right)\log\left(\frac{bx+a}{dx+c}\right)}{\frac{(bx+a)^2bcg^3}{(dx+c)^2}-\frac{(bx+a)^2adg^3}{(dx+c)^2}}+\frac{B^2bn^2-\frac{8\,(bx+a)B^2dn^2}{dx+c}+2\,ABbn+2\,B^2bn-\frac{8\,(bx+a)ABdn}{dx+c}-\frac{8\,(bx+a)B^2dn}{dx+c}+2\,A^2b+4\,ABb+2\,B^2b-\frac{4\,(bx+a)A^2d}{dx+c}-\frac{8\,(bx+a)ABd}{dx+c}-\frac{4\,(bx+a)B^2d}{dx+c}}{\frac{(bx+a)^2bcg^3}{(dx+c)^2}-\frac{(bx+a)^2adg^3}{(dx+c)^2}}\right)\left(\frac{bc}{(bc-ad)^2}-\frac{ad}{(bc-ad)^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^3,x, algorithm="gi
ac")
```

```
[Out] -1/4*(2*(B^2*b*n^2 - 2*(b*x + a)*B^2*d*n^2/(d*x + c))*log((b*x + a)/(d*x +
c))^2/((b*x + a)^2*b*c*g^3/(d*x + c)^2 - (b*x + a)^2*a*d*g^3/(d*x + c)^2) +
 2*(B^2*b*n^2 - 4*(b*x + a)*B^2*d*n^2/(d*x + c) + 2*A*B*b*n + 2*B^2*b*n - 4
*(b*x + a)*A*B*d*n/(d*x + c) - 4*(b*x + a)*B^2*d*n/(d*x + c))*log((b*x + a)
/(d*x + c))/((b*x + a)^2*b*c*g^3/(d*x + c)^2 - (b*x + a)^2*a*d*g^3/(d*x + c
)^2) + (B^2*b*n^2 - 8*(b*x + a)*B^2*d*n^2/(d*x + c) + 2*A*B*b*n + 2*B^2*b*n
 - 8*(b*x + a)*A*B*d*n/(d*x + c) - 8*(b*x + a)*B^2*d*n/(d*x + c) + 2*A^2*b
+ 4*A*B*b + 2*B^2*b - 4*(b*x + a)*A^2*d/(d*x + c) - 8*(b*x + a)*A*B*d/(d*x
+ c) - 4*(b*x + a)*B^2*d/(d*x + c))/((b*x + a)^2*b*c*g^3/(d*x + c)^2 - (b*x
 + a)^2*a*d*g^3/(d*x + c)^2))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)
```

Mupad [B]
time = 6.17, size = 506, normalized size = 1.76

$$-\ln\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^2\left(\frac{B^2}{2b\left(a^2g^3+2abg^3x+b^2g^3x^2\right)}-\frac{B^2d^2}{2bg^3\left(a^2d^2-2abcd+b^2c^2\right)}\right)-\frac{\frac{2A^2ad-2A^2bc+7B^2adn^2-B^2bcn^2+6ABadn-2ABbcn}{2(ad-bc)}+\frac{dx\left(3bB^2n^2+2AbBn\right)}{ad-bc}}{2a^2bg^3+4ab^2g^3x+2b^3g^3x^2}-\ln\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\left(\frac{AB}{a^2bg^3+2ab^2g^3x+b^3g^3x^2}+\frac{B^2d^2\left(\frac{bg^3n(ad-bc)(2ad-bc)}{2d^2}+\frac{b^2g^3nx(ad-bc)}{d}+\frac{abg^3n(ad-bc)}{2d}\right)}{bg^3\left(a^2d^2-2abcd+b^2c^2\right)\left(a^2bg^3+2ab^2g^3x+b^3g^3x^2\right)}\right)-\frac{Bd^2n\,\mathrm{atan}\left(\frac{\left(2bdx-\frac{2b^3c^2g^3-2a^2bd^2g^3}{2bg^3(ad-bc)}\right)1\mathrm{i}}{ad-bc}\right)(2A+3Bn)\,1\mathrm{i}}{bg^3(ad-bc)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/(a*g + b*g*x)^3,x)
```

```
[Out] - log(e*((a + b*x)/(c + d*x))^n)^2*(B^2/(2*b*(a^2*g^3 + b^2*g^3*x^2 + 2*a*b
*g^3*x)) - (B^2*d^2)/(2*b*g^3*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))) - ((2*A^2*a
*d - 2*A^2*b*c + 7*B^2*a*d*n^2 - B^2*b*c*n^2 + 6*A*B*a*d*n - 2*A*B*b*c*n)/(
2*(a*d - b*c)) + (d*x*(3*B^2*b*n^2 + 2*A*B*b*n))/(a*d - b*c))/(2*a^2*b*g^3
+ 2*b^3*g^3*x^2 + 4*a*b^2*g^3*x) - log(e*((a + b*x)/(c + d*x))^n)*((A*B)/(a
^2*b*g^3 + b^3*g^3*x^2 + 2*a*b^2*g^3*x) + (B^2*d^2*((b*g^3*n*(a*d - b*c)*(2
*a*d - b*c))/(2*d^2) + (b^2*g^3*n*x*(a*d - b*c))/d + (a*b*g^3*n*(a*d - b*c)
)/(2*d)))/(b*g^3*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)*(a^2*b*g^3 + b^3*g^3*x^2 +
 2*a*b^2*g^3*x))) - (B*d^2*n*atan(((2*b*d*x - (2*b^3*c^2*g^3 - 2*a^2*b*d^2*
g^3)/(2*b*g^3*(a*d - b*c)))*1i)/(a*d - b*c))*(2*A + 3*B*n)*1i)/(b*g^3*(a*d
- b*c)^2)
```

3.17 $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4}\,dx$

Optimal. Leaf size=448

$$-\frac{2B^2d^2n^2(c+dx)}{(bc-ad)^3g^4(a+bx)}+\frac{bB^2dn^2(c+dx)^2}{2(bc-ad)^3g^4(a+bx)^2}-\frac{2b^2B^2n^2(c+dx)^3}{27(bc-ad)^3g^4(a+bx)^3}-\frac{2Bd^2n(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^3g^4(a+bx)}$$

```
[Out] -2*B^2*d^2*n^2*(d*x+c)/(-a*d+b*c)^3/g^4/(b*x+a)+1/2*b*B^2*d*n^2*(d*x+c)^2/(
-a*d+b*c)^3/g^4/(b*x+a)^2-2/27*b^2*B^2*n^2*(d*x+c)^3/(-a*d+b*c)^3/g^4/(b*x+
a)^3-2*B*d^2*n*(d*x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^3/g^4/(b*
x+a)+b*B*d*n*(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^3/g^4/(b*
x+a)^2-2/9*b^2*B*n*(d*x+c)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^3/g
^4/(b*x+a)^3-d^2*(d*x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*c)^3/g^4
/(b*x+a)+b*d*(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*c)^3/g^4/(
b*x+a)^2-1/3*b^2*(d*x+c)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*c)^3/g
^4/(b*x+a)^3
```

Rubi [A]
time = 0.23, antiderivative size = 448, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {2549, 2395, 2342, 2341}

$$-\frac{b^2(c+dx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3g^4(a+bx)^3(bc-ad)^3}-\frac{2b^2Bn(c+dx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{9g^4(a+bx)^3(bc-ad)^3}-\frac{d^2(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{g^4(a+bx)(bc-ad)^3}-\frac{2Bd^2n(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{g^4(a+bx)(bc-ad)^3}+\frac{bd(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{g^4(a+bx)^2(bc-ad)^3}+\frac{bBdn(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{g^4(a+bx)^2(bc-ad)^3}-\frac{2b^2B^2n^2(c+dx)^3}{27g^4(a+bx)^3(bc-ad)^3}-\frac{2B^2d^2n^2(c+dx)}{g^4(a+bx)(bc-ad)^3}+\frac{bB^2dn^2(c+dx)^2}{2g^4(a+bx)^2(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(a*g + b*g*x)^4,x]
```

```
[Out] (-2*B^2*d^2*n^2*(c + d*x))/((b*c - a*d)^3*g^4*(a + b*x)) + (b*B^2*d*n^2*(c
+ d*x)^2)/(2*(b*c - a*d)^3*g^4*(a + b*x)^2) - (2*b^2*B^2*n^2*(c + d*x)^3)/(
27*(b*c - a*d)^3*g^4*(a + b*x)^3) - (2*B*d^2*n*(c + d*x)*(A + B*Log[e*((a +
 b*x)/(c + d*x))^n]))/((b*c - a*d)^3*g^4*(a + b*x)) + (b*B*d*n*(c + d*x)^2*
(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/((b*c - a*d)^3*g^4*(a + b*x)^2) - (
2*b^2*B*n*(c + d*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(9*(b*c - a*d
)^3*g^4*(a + b*x)^3) - (d^2*(c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]
)^2)/((b*c - a*d)^3*g^4*(a + b*x)) + (b*d*(c + d*x)^2*(A + B*Log[e*((a + b*
x)/(c + d*x))^n])^2)/((b*c - a*d)^3*g^4*(a + b*x)^2) - (b^2*(c + d*x)^3*(A
+ B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(3*(b*c - a*d)^3*g^4*(a + b*x)^3)
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2549

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(m +
1)*(g/b)^m, Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x], x,
(a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ
[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[b*f - a*g, 0] && (GtQ[p, 0] || LtQ
[m, -1])
```

Rubi steps

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4}\,dx = -\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3bg^4(a+bx)^3} + \frac{(2Bn)\int \frac{(bc-ad)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{g^3(a+bx)^4(c+dx)}\,dx}{3bg}$$

$$= -\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3bg^4(a+bx)^3} + \frac{(2B(bc-ad)n)\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)^4(c+dx)}\,dx}{3bg^4}$$

$$= -\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3bg^4(a+bx)^3} + \frac{(2B(bc-ad)n)\int \left(\frac{b\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)(a+bx)^4} - \frac{bd(\cdots}{\cdots}\right.}{\cdots}$$

$$= -\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3bg^4(a+bx)^3} + \frac{(2Bn)\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)^4}\,dx}{3g^4} - \frac{(2Bd^3n)\int \frac{A\cdots}{\cdots}}{3(bc\cdots}$$

$$= -\frac{2Bn\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{9bg^4(a+bx)^3} + \frac{Bdn\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3b(bc-ad)g^4(a+bx)^2} - \frac{2Bd^2n(A\cdots}{3b(bc\cdots}$$

$$= -\frac{2Bn\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{9bg^4(a+bx)^3} + \frac{Bdn\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3b(bc-ad)g^4(a+bx)^2} - \frac{2Bd^2n(A\cdots}{3b(bc\cdots}$$

$$= -\frac{2Bn\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{9bg^4(a+bx)^3} + \frac{Bdn\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3b(bc-ad)g^4(a+bx)^2} - \frac{2Bd^2n(A\cdots}{3b(bc\cdots}$$

$$= -\frac{2B^2n^2}{27bg^4(a+bx)^3} + \frac{5B^2dn^2}{18b(bc-ad)g^4(a+bx)^2} - \frac{11B^2d^2n^2}{9b(bc-ad)^2g^4(a+bx)} - \cdots$$

$$= -\frac{2B^2n^2}{27bg^4(a+bx)^3} + \frac{5B^2dn^2}{18b(bc-ad)g^4(a+bx)^2} - \frac{11B^2d^2n^2}{9b(bc-ad)^2g^4(a+bx)} - \cdots$$

$$= -\frac{2B^2n^2}{27bg^4(a+bx)^3} + \frac{5B^2dn^2}{18b(bc-ad)g^4(a+bx)^2} - \frac{11B^2d^2n^2}{9b(bc-ad)^2g^4(a+bx)} - \cdots$$

Mathematica [**C**] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 0.47, size = 609, normalized size = 1.36

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(a*g + b*g*x)^4,x]
```

```
[Out] -1/54*(18*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 + (B*n*(12*(b*c - a*d)^3
*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - 18*d*(b*c - a*d)^2*(a + b*x)*(A +
```

```
 B*Log[e*((a + b*x)/(c + d*x))^n]) + 36*d^2*(b*c - a*d)*(a + b*x)^2*(A + B*
Log[e*((a + b*x)/(c + d*x))^n]) + 36*d^3*(a + b*x)^3*Log[a + b*x]*(A + B*Lo
g[e*((a + b*x)/(c + d*x))^n]) - 36*d^3*(a + b*x)^3*(A + B*Log[e*((a + b*x)/
(c + d*x))^n])*Log[c + d*x] + 36*B*d^2*n*(a + b*x)^2*(b*c - a*d + d*(a + b*
x)*Log[a + b*x] - d*(a + b*x)*Log[c + d*x]) - 9*B*d*n*(a + b*x)*((b*c - a*d
)^2 + 2*d*(-(b*c) + a*d)*(a + b*x) - 2*d^2*(a + b*x)^2*Log[a + b*x] + 2*d^2
*(a + b*x)^2*Log[c + d*x]) + 2*B*n*(2*(b*c - a*d)^3 - 3*d*(b*c - a*d)^2*(a
+ b*x) + 6*d^2*(b*c - a*d)*(a + b*x)^2 + 6*d^3*(a + b*x)^3*Log[a + b*x] - 6
*d^3*(a + b*x)^3*Log[c + d*x]) - 18*B*d^3*n*(a + b*x)^3*(Log[a + b*x]*(Log[
a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(
-(b*c) + a*d)]) + 18*B*d^3*n*(a + b*x)^3*((2*Log[(d*(a + b*x))/(-(b*c) + a*
d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])
))/(b*c - a*d)^3)/(b*g^4*(a + b*x)^3)
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)\right)^{2}}{\left(bgx+ag\right)^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^4,x)
```

```
[Out] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^4,x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1435 vs. 2(446) = 892.
time = 0.48, size = 1435, normalized size = 3.20

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^4,x, algorithm="ma
xima")
```

```
[Out] -1/9*A*B*n*((6*b^2*d^2*x^2 + 2*b^2*c^2 - 7*a*b*c*d + 11*a^2*d^2 - 3*(b^2*c*
d - 5*a*b*d^2)*x)/((b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2)*g^4*x^3 + 3*(a*b^5
*c^2 - 2*a^2*b^4*c*d + a^3*b^3*d^2)*g^4*x^2 + 3*(a^2*b^4*c^2 - 2*a^3*b^3*c*
d + a^4*b^2*d^2)*g^4*x + (a^3*b^3*c^2 - 2*a^4*b^2*c*d + a^5*b*d^2)*g^4) + 6
*d^3*log(b*x + a)/((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*
g^4) - 6*d^3*log(d*x + c)/((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3
*b*d^3)*g^4)) - 1/54*(6*n*((6*b^2*d^2*x^2 + 2*b^2*c^2 - 7*a*b*c*d + 11*a^2*
d^2 - 3*(b^2*c*d - 5*a*b*d^2)*x)/((b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2)*g^4
*x^3 + 3*(a*b^5*c^2 - 2*a^2*b^4*c*d + a^3*b^3*d^2)*g^4*x^2 + 3*(a^2*b^4*c^2
 - 2*a^3*b^3*c*d + a^4*b^2*d^2)*g^4*x + (a^3*b^3*c^2 - 2*a^4*b^2*c*d + a^5*
```

```
b*d^2)*g^4) + 6*d^3*log(b*x + a)/((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^
2 - a^3*b*d^3)*g^4) - 6*d^3*log(d*x + c)/((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*
b^2*c*d^2 - a^3*b*d^3)*g^4))*log((b*x/(d*x + c) + a/(d*x + c))^n*e) + (4*b^
3*c^3 - 27*a*b^2*c^2*d + 108*a^2*b*c*d^2 - 85*a^3*d^3 + 66*(b^3*c*d^2 - a*b
^2*d^3)*x^2 - 18*(b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*
log(b*x + a)^2 - 18*(b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^
3)*log(d*x + c)^2 - 3*(5*b^3*c^2*d - 54*a*b^2*c*d^2 + 49*a^2*b*d^3)*x + 66*
(b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(b*x + a) - 6*
(11*b^3*d^3*x^3 + 33*a*b^2*d^3*x^2 + 33*a^2*b*d^3*x + 11*a^3*d^3 - 6*(b^3*d
^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(b*x + a))*log(d*x +
 c))*n^2/(a^3*b^4*c^3*g^4 - 3*a^4*b^3*c^2*d*g^4 + 3*a^5*b^2*c*d^2*g^4 - a^6
*b*d^3*g^4 + (b^7*c^3*g^4 - 3*a*b^6*c^2*d*g^4 + 3*a^2*b^5*c*d^2*g^4 - a^3*b
^4*d^3*g^4)*x^3 + 3*(a*b^6*c^3*g^4 - 3*a^2*b^5*c^2*d*g^4 + 3*a^3*b^4*c*d^2*
g^4 - a^4*b^3*d^3*g^4)*x^2 + 3*(a^2*b^5*c^3*g^4 - 3*a^3*b^4*c^2*d*g^4 + 3*a
^4*b^3*c*d^2*g^4 - a^5*b^2*d^3*g^4)*x))*B^2 - 1/3*B^2*log((b*x/(d*x + c) +
a/(d*x + c))^n*e)^2/(b^4*g^4*x^3 + 3*a*b^3*g^4*x^2 + 3*a^2*b^2*g^4*x + a^3*
b*g^4) - 2/3*A*B*log((b*x/(d*x + c) + a/(d*x + c))^n*e)/(b^4*g^4*x^3 + 3*a*
b^3*g^4*x^2 + 3*a^2*b^2*g^4*x + a^3*b*g^4) - 1/3*A^2/(b^4*g^4*x^3 + 3*a*b^3
*g^4*x^2 + 3*a^2*b^2*g^4*x + a^3*b*g^4)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 946 vs. $2(446) = 892$.
time = 0.41, size = 946, normalized size = 2.11

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^4,x, algorithm="fr
icas")
```

```
[Out] -1/54*(18*(A^2 + 2*A*B + B^2)*b^3*c^3 - 54*(A^2 + 2*A*B + B^2)*a*b^2*c^2*d
+ 54*(A^2 + 2*A*B + B^2)*a^2*b*c*d^2 - 18*(A^2 + 2*A*B + B^2)*a^3*d^3 + (4*
B^2*b^3*c^3 - 27*B^2*a*b^2*c^2*d + 108*B^2*a^2*b*c*d^2 - 85*B^2*a^3*d^3)*n^
2 + 6*(11*(B^2*b^3*c*d^2 - B^2*a*b^2*d^3)*n^2 + 6*((A*B + B^2)*b^3*c*d^2 -
(A*B + B^2)*a*b^2*d^3)*n)*x^2 + 18*(B^2*b^3*d^3*n^2*x^3 + 3*B^2*a*b^2*d^3*n
^2*x^2 + 3*B^2*a^2*b*d^3*n^2*x + (B^2*b^3*c^3 - 3*B^2*a*b^2*c^2*d + 3*B^2*a
^2*b*c*d^2)*n^2)*log((b*x + a)/(d*x + c))^2 + 6*(2*(A*B + B^2)*b^3*c^3 - 9*
(A*B + B^2)*a*b^2*c^2*d + 18*(A*B + B^2)*a^2*b*c*d^2 - 11*(A*B + B^2)*a^3*d
^3)*n - 3*((5*B^2*b^3*c^2*d - 54*B^2*a*b^2*c*d^2 + 49*B^2*a^2*b*d^3)*n^2 +
6*((A*B + B^2)*b^3*c^2*d - 6*(A*B + B^2)*a*b^2*c*d^2 + 5*(A*B + B^2)*a^2*b*
d^3)*n)*x + 6*((11*B^2*b^3*d^3*n^2 + 6*(A*B + B^2)*b^3*d^3*n)*x^3 + (2*B^2*
b^3*c^3 - 9*B^2*a*b^2*c^2*d + 18*B^2*a^2*b*c*d^2)*n^2 + 3*(6*(A*B + B^2)*a*
b^2*d^3*n + (2*B^2*b^3*c*d^2 + 9*B^2*a*b^2*d^3)*n^2)*x^2 + 6*((A*B + B^2)*b
^3*c^3 - 3*(A*B + B^2)*a*b^2*c^2*d + 3*(A*B + B^2)*a^2*b*c*d^2)*n + 3*(6*(A
*B + B^2)*a^2*b*d^3*n - (B^2*b^3*c^2*d - 6*B^2*a*b^2*c*d^2 - 6*B^2*a^2*b*d^
```

```
3)*n^2)*x)*log((b*x + a)/(d*x + c)))/((b^7*c^3 - 3*a*b^6*c^2*d + 3*a^2*b^5*
c*d^2 - a^3*b^4*d^3)*g^4*x^3 + 3*(a*b^6*c^3 - 3*a^2*b^5*c^2*d + 3*a^3*b^4*c
*d^2 - a^4*b^3*d^3)*g^4*x^2 + 3*(a^2*b^5*c^3 - 3*a^3*b^4*c^2*d + 3*a^4*b^3*
c*d^2 - a^5*b^2*d^3)*g^4*x + (a^3*b^4*c^3 - 3*a^4*b^3*c^2*d + 3*a^5*b^2*c*d
^2 - a^6*b*d^3)*g^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A^2}{a^4+4a^3bx+6a^2b^2x^2+4ab^3x^3+b^4x^4}\,dx + \int \frac{B^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2}{a^4+4a^3bx+6a^2b^2x^2+4ab^3x^3+b^4x^4}\,dx + \int \frac{2AB\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}{a^4+4a^3bx+6a^2b^2x^2+4ab^3x^3+b^4x^4}\,dx}{g^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(b*g*x+a*g)**4,x)
```

```
[Out] (Integral(A**2/(a**4 + 4*a**3*b*x + 6*a**2*b**2*x**2 + 4*a*b**3*x**3 + b**4
*x**4), x) + Integral(B**2*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)**2/(a**4
 + 4*a**3*b*x + 6*a**2*b**2*x**2 + 4*a*b**3*x**3 + b**4*x**4), x) + Integra
l(2*A*B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(a**4 + 4*a**3*b*x + 6*a**2
*b**2*x**2 + 4*a*b**3*x**3 + b**4*x**4), x))/g**4
```

Giac [A]
time = 5.63, size = 810, normalized size = 1.81

$$-\frac{1}{54}\left(\frac{18\left(B^2b^2n^2-\frac{3(bx+a)B^2bdn^2}{dx+c}+\frac{3(bx+a)^2B^2d^2n^2}{(dx+c)^2}\right)\log\left(\frac{bx+a}{dx+c}\right)^2}{\frac{(bx+a)^3b^2c^2g^4}{(dx+c)^3}-\frac{2(bx+a)^3abcdg^4}{(dx+c)^3}+\frac{(bx+a)^3a^2d^2g^4}{(dx+c)^3}}+\frac{6\left(2B^2b^2n^2-\frac{9(bx+a)B^2bdn^2}{dx+c}+\frac{18(bx+a)^2B^2d^2n^2}{(dx+c)^2}+6ABb^2n+6B^2b^2n-\frac{18(bx+a)ABbdn}{dx+c}-\frac{18(bx+a)B^2bdn}{dx+c}+\frac{18(bx+a)^2ABd^2n}{(dx+c)^2}+\frac{18(bx+a)^2B^2d^2n}{(dx+c)^2}\right)\log\left(\frac{bx+a}{dx+c}\right)}{\frac{(bx+a)^3b^2c^2g^4}{(dx+c)^3}-\frac{2(bx+a)^3abcdg^4}{(dx+c)^3}+\frac{(bx+a)^3a^2d^2g^4}{(dx+c)^3}}+\frac{4B^2b^2n^2-\frac{27(bx+a)B^2bdn^2}{dx+c}+\frac{108(bx+a)^2B^2d^2n^2}{(dx+c)^2}+12ABb^2n+12B^2b^2n-\frac{54(bx+a)ABbdn}{dx+c}-\frac{54(bx+a)B^2bdn}{dx+c}+\frac{108(bx+a)^2ABd^2n}{(dx+c)^2}+\frac{108(bx+a)^2B^2d^2n}{(dx+c)^2}+18A^2b^2+36ABb^2+18B^2b^2-\frac{54(bx+a)A^2bd}{dx+c}-\frac{108(bx+a)ABbd}{dx+c}-\frac{54(bx+a)B^2bd}{dx+c}+\frac{54(bx+a)^2A^2d^2}{(dx+c)^2}+\frac{108(bx+a)^2ABd^2}{(dx+c)^2}+\frac{54(bx+a)^2B^2d^2}{(dx+c)^2}}{\frac{(bx+a)^3b^2c^2g^4}{(dx+c)^3}-\frac{2(bx+a)^3abcdg^4}{(dx+c)^3}+\frac{(bx+a)^3a^2d^2g^4}{(dx+c)^3}}\right)\left(\frac{bc}{(bc-ad)^2}-\frac{ad}{(bc-ad)^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^4,x, algorithm="gi
ac")
```

```
[Out] -1/54*(18*(B^2*b^2*n^2 - 3*(b*x + a)*B^2*b*d*n^2/(d*x + c) + 3*(b*x + a)^2*
B^2*d^2*n^2/(d*x + c)^2)*log((b*x + a)/(d*x + c))^2/((b*x + a)^3*b^2*c^2*g^
4/(d*x + c)^3 - 2*(b*x + a)^3*a*b*c*d*g^4/(d*x + c)^3 + (b*x + a)^3*a^2*d^2
*g^4/(d*x + c)^3) + 6*(2*B^2*b^2*n^2 - 9*(b*x + a)*B^2*b*d*n^2/(d*x + c) +
18*(b*x + a)^2*B^2*d^2*n^2/(d*x + c)^2 + 6*A*B*b^2*n + 6*B^2*b^2*n - 18*(b*
x + a)*A*B*b*d*n/(d*x + c) - 18*(b*x + a)*B^2*b*d*n/(d*x + c) + 18*(b*x + a
)^2*A*B*d^2*n/(d*x + c)^2 + 18*(b*x + a)^2*B^2*d^2*n/(d*x + c)^2)*log((b*x
+ a)/(d*x + c))/((b*x + a)^3*b^2*c^2*g^4/(d*x + c)^3 - 2*(b*x + a)^3*a*b*c*
d*g^4/(d*x + c)^3 + (b*x + a)^3*a^2*d^2*g^4/(d*x + c)^3) + (4*B^2*b^2*n^2 -
 27*(b*x + a)*B^2*b*d*n^2/(d*x + c) + 108*(b*x + a)^2*B^2*d^2*n^2/(d*x + c)
^2 + 12*A*B*b^2*n + 12*B^2*b^2*n - 54*(b*x + a)*A*B*b*d*n/(d*x + c) - 54*(b
*x + a)*B^2*b*d*n/(d*x + c) + 108*(b*x + a)^2*A*B*d^2*n/(d*x + c)^2 + 108*(
b*x + a)^2*B^2*d^2*n/(d*x + c)^2 + 18*A^2*b^2 + 36*A*B*b^2 + 18*B^2*b^2 - 5
4*(b*x + a)*A^2*b*d/(d*x + c) - 108*(b*x + a)*A*B*b*d/(d*x + c) - 54*(b*x +
 a)*B^2*b*d/(d*x + c) + 54*(b*x + a)^2*A^2*d^2/(d*x + c)^2 + 108*(b*x + a)^
```

```
2*A*B*d^2/(d*x + c)^2 + 54*(b*x + a)^2*B^2*d^2/(d*x + c)^2)/((b*x + a)^3*b^
2*c^2*g^4/(d*x + c)^3 - 2*(b*x + a)^3*a*b*c*d*g^4/(d*x + c)^3 + (b*x + a)^3
*a^2*d^2*g^4/(d*x + c)^3))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)
```

Mupad [B]
time = 7.69, size = 1038, normalized size = 2.32

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/(a*g + b*g*x)^4,x)
```

```
[Out] ((18*A^2*a^2*d^2 + 18*A^2*b^2*c^2 + 85*B^2*a^2*d^2*n^2 + 4*B^2*b^2*c^2*n^2
- 36*A^2*a*b*c*d + 66*A*B*a^2*d^2*n + 12*A*B*b^2*c^2*n - 23*B^2*a*b*c*d*n^2
 - 42*A*B*a*b*c*d*n)/(6*(a*d - b*c)) + (x*(49*B^2*a*b*d^2*n^2 - 5*B^2*b^2*c
*d*n^2 + 30*A*B*a*b*d^2*n - 6*A*B*b^2*c*d*n))/(2*(a*d - b*c)) + (d*x^2*(11*
B^2*b^2*d*n^2 + 6*A*B*b^2*d*n))/(a*d - b*c))/(x*(27*a^2*b^3*c*g^4 - 27*a^3*
b^2*d*g^4) - x^2*(27*a^2*b^3*d*g^4 - 27*a*b^4*c*g^4) + x^3*(9*b^5*c*g^4 - 9
*a*b^4*d*g^4) + 9*a^3*b^2*c*g^4 - 9*a^4*b*d*g^4) - log(e*((a + b*x)/(c + d*
x))^n)*((2*A*B)/(3*a^3*b*g^4 + 3*b^4*g^4*x^3 + 9*a^2*b^2*g^4*x + 9*a*b^3*g^
4*x^2) + (2*B^2*d^3*(x*(b*((b*g^4*n*(a*d - b*c)*(3*a*d - b*c))/(2*d^2) + (a
*b*g^4*n*(a*d - b*c))/d) + (2*a*b^2*g^4*n*(a*d - b*c))/d + (b^2*g^4*n*(a*d
- b*c)*(3*a*d - b*c))/d^2) + a*((b*g^4*n*(a*d - b*c)*(3*a*d - b*c))/(2*d^2)
 + (a*b*g^4*n*(a*d - b*c))/d) + (b*g^4*n*(a*d - b*c)*(3*a^2*d^2 + b^2*c^2 -
 3*a*b*c*d))/d^3 + (3*b^3*g^4*n*x^2*(a*d - b*c))/d))/(3*b*g^4*(a^3*d^3 - b^
3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)*(3*a^3*b*g^4 + 3*b^4*g^4*x^3 + 9*a^2
*b^2*g^4*x + 9*a*b^3*g^4*x^2))) - log(e*((a + b*x)/(c + d*x))^n)^2*(B^2/(3*
b*(a^3*g^4 + b^3*g^4*x^3 + 3*a*b^2*g^4*x^2 + 3*a^2*b*g^4*x)) - (B^2*d^3)/(3
*b*g^4*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))) - (B*d^3*n*ata
n((B*d^3*n*(6*A + 11*B*n)*((b^4*c^3*g^4 + a^3*b*d^3*g^4 - a*b^3*c^2*d*g^4 -
 a^2*b^2*c*d^2*g^4)/(b^3*c^2*g^4 + a^2*b*d^2*g^4 - 2*a*b^2*c*d*g^4) + 2*b*d
*x)*(b^3*c^2*g^4 + a^2*b*d^2*g^4 - 2*a*b^2*c*d*g^4)*1i)/(b*g^4*(11*B^2*d^3*
n^2 + 6*A*B*d^3*n)*(a*d - b*c)^3))*(6*A + 11*B*n)*2i)/(9*b*g^4*(a*d - b*c)^
3)
```

3.18 $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^5}\,dx$

Optimal. Leaf size=615

$$\frac{2B^2d^3n^2(c+dx)}{(bc-ad)^4g^5(a+bx)}-\frac{3bB^2d^2n^2(c+dx)^2}{4(bc-ad)^4g^5(a+bx)^2}+\frac{2b^2B^2dn^2(c+dx)^3}{9(bc-ad)^4g^5(a+bx)^3}-\frac{b^3B^2n^2(c+dx)^4}{32(bc-ad)^4g^5(a+bx)^4}+\frac{2Bd^3n(}{}$$

```
[Out] 2*B^2*d^3*n^2*(d*x+c)/(-a*d+b*c)^4/g^5/(b*x+a)-3/4*b*B^2*d^2*n^2*(d*x+c)^2/
(-a*d+b*c)^4/g^5/(b*x+a)^2+2/9*b^2*B^2*d*n^2*(d*x+c)^3/(-a*d+b*c)^4/g^5/(b*
x+a)^3-1/32*b^3*B^2*n^2*(d*x+c)^4/(-a*d+b*c)^4/g^5/(b*x+a)^4+2*B*d^3*n*(d*x
+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^4/g^5/(b*x+a)-3/2*b*B*d^2*n*
(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^4/g^5/(b*x+a)^2+2/3*b^
2*B*d*n*(d*x+c)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^4/g^5/(b*x+a)^
3-1/8*b^3*B*n*(d*x+c)^4*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^4/g^5/(b
*x+a)^4+d^3*(d*x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*c)^4/g^5/(b*x
+a)-3/2*b*d^2*(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*c)^4/g^5/
(b*x+a)^2+b^2*d*(d*x+c)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*c)^4/g^
5/(b*x+a)^3-1/4*b^3*(d*x+c)^4*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*c)^
4/g^5/(b*x+a)^4
```

Rubi [A]
time = 0.30, antiderivative size = 615, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {2549, 2395, 2342, 2341}

$$-\frac{b^3(c+dx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{4g^5(a+bx)^4(bc-ad)^4}-\frac{b^3Bn(c+dx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{8g^5(a+bx)^4(bc-ad)^4}+\frac{b^2d(c+dx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{g^5(a+bx)^3(bc-ad)^4}+\frac{2b^2Bdn(c+dx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3g^5(a+bx)^3(bc-ad)^4}+\frac{d^3(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{g^5(a+bx)(bc-ad)^4}+\frac{2Bd^3n(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{g^5(a+bx)(bc-ad)^4}-\frac{3bd^2(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2g^5(a+bx)^2(bc-ad)^4}-\frac{3bBd^2n(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2g^5(a+bx)^2(bc-ad)^4}-\frac{b^3B^2n^2(c+dx)^4}{32g^5(a+bx)^4(bc-ad)^4}+\frac{2b^2B^2dn^2(c+dx)^3}{9g^5(a+bx)^3(bc-ad)^4}+\frac{2B^2d^3n^2(c+dx)}{g^5(a+bx)(bc-ad)^4}-\frac{3bB^2d^2n^2(c+dx)^2}{4g^5(a+bx)^2(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(a*g + b*g*x)^5,x]
```

```
[Out] (2*B^2*d^3*n^2*(c + d*x))/((b*c - a*d)^4*g^5*(a + b*x)) - (3*b*B^2*d^2*n^2*
(c + d*x)^2)/(4*(b*c - a*d)^4*g^5*(a + b*x)^2) + (2*b^2*B^2*d*n^2*(c + d*x)
^3)/(9*(b*c - a*d)^4*g^5*(a + b*x)^3) - (b^3*B^2*n^2*(c + d*x)^4)/(32*(b*c
- a*d)^4*g^5*(a + b*x)^4) + (2*B*d^3*n*(c + d*x)*(A + B*Log[e*((a + b*x)/(c
 + d*x))^n]))/((b*c - a*d)^4*g^5*(a + b*x)) - (3*b*B*d^2*n*(c + d*x)^2*(A +
 B*Log[e*((a + b*x)/(c + d*x))^n]))/(2*(b*c - a*d)^4*g^5*(a + b*x)^2) + (2*
b^2*B*d*n*(c + d*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(3*(b*c - a*d
)^4*g^5*(a + b*x)^3) - (b^3*B*n*(c + d*x)^4*(A + B*Log[e*((a + b*x)/(c + d*
x))^n]))/(8*(b*c - a*d)^4*g^5*(a + b*x)^4) + (d^3*(c + d*x)*(A + B*Log[e*((
a + b*x)/(c + d*x))^n])^2)/((b*c - a*d)^4*g^5*(a + b*x)) - (3*b*d^2*(c + d*
x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(2*(b*c - a*d)^4*g^5*(a + b*
x)^2) + (b^2*d*(c + d*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/((b*c
- a*d)^4*g^5*(a + b*x)^3) - (b^3*(c + d*x)^4*(A + B*Log[e*((a + b*x)/(c + d
*x))^n])^2)/(4*(b*c - a*d)^4*g^5*(a + b*x)^4)
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2549

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(m +
1)*(g/b)^m, Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x], x,
(a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ
[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[b*f - a*g, 0] && (GtQ[p, 0] || LtQ
[m, -1])
```

Rubi steps

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^5}\,dx = -\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4bg^5(a+bx)^4} + \frac{(Bn)\int \frac{(bc-ad)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{g^4(a+bx)^5(c+dx)}\,dx}{2bg}$$

$$= -\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4bg^5(a+bx)^4} + \frac{(B(bc-ad)n)\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)^5(c+dx)}\,dx}{2bg^5}$$

$$= -\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4bg^5(a+bx)^4} + \frac{(B(bc-ad)n)\int \left(\frac{b\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)(a+bx)^5} - \frac{bd}{\ldots}\right.}{\ldots}$$

$$= -\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4bg^5(a+bx)^4} + \frac{(Bn)\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)^5}\,dx}{2g^5} + \frac{(Bd^4n)\int \frac{A+\ldots}{\ldots}}{2(bc\ldots}$$

$$= -\frac{Bn\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{8bg^5(a+bx)^4} + \frac{Bdn\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{6b(bc-ad)g^5(a+bx)^3} - \frac{Bd^2n(A\ldots}{4b(bc-\ldots}$$

$$= -\frac{Bn\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{8bg^5(a+bx)^4} + \frac{Bdn\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{6b(bc-ad)g^5(a+bx)^3} - \frac{Bd^2n(A\ldots}{4b(bc-\ldots}$$

$$= -\frac{Bn\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{8bg^5(a+bx)^4} + \frac{Bdn\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{6b(bc-ad)g^5(a+bx)^3} - \frac{Bd^2n(A\ldots}{4b(bc-\ldots}$$

$$= -\frac{B^2n^2}{32bg^5(a+bx)^4} + \frac{7B^2dn^2}{72b(bc-ad)g^5(a+bx)^3} - \frac{13B^2d^2n^2}{48b(bc-ad)^2g^5(a+bx)^2}\ldots$$

$$= -\frac{B^2n^2}{32bg^5(a+bx)^4} + \frac{7B^2dn^2}{72b(bc-ad)g^5(a+bx)^3} - \frac{13B^2d^2n^2}{48b(bc-ad)^2g^5(a+bx)^2}\ldots$$

$$= -\frac{B^2n^2}{32bg^5(a+bx)^4} + \frac{7B^2dn^2}{72b(bc-ad)g^5(a+bx)^3} - \frac{13B^2d^2n^2}{48b(bc-ad)^2g^5(a+bx)^2}\ldots$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 0.68, size = 776, normalized size = 1.26

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(a*g + b*g*x)^5,x]
```

```
[Out] -1/288*(72*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 + (B*n*(36*(b*c - a*d)^
4*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + 48*d*(-(b*c) + a*d)^3*(a + b*x)*
```

```
(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + 72*d^2*(b*c - a*d)^2*(a + b*x)^2*(
A + B*Log[e*((a + b*x)/(c + d*x))^n]) + 144*d^3*(-(b*c) + a*d)*(a + b*x)^3*
(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - 144*d^4*(a + b*x)^4*Log[a + b*x]*(
A + B*Log[e*((a + b*x)/(c + d*x))^n]) + 144*d^4*(a + b*x)^4*(A + B*Log[e*((
a + b*x)/(c + d*x))^n])*Log[c + d*x] - 144*B*d^3*n*(a + b*x)^3*(b*c - a*d +
 d*(a + b*x)*Log[a + b*x] - d*(a + b*x)*Log[c + d*x]) + 36*B*d^2*n*(a + b*x
)^2*((b*c - a*d)^2 + 2*d*(-(b*c) + a*d)*(a + b*x) - 2*d^2*(a + b*x)^2*Log[a
 + b*x] + 2*d^2*(a + b*x)^2*Log[c + d*x]) - 8*B*d*n*(a + b*x)*(2*(b*c - a*d
)^3 - 3*d*(b*c - a*d)^2*(a + b*x) + 6*d^2*(b*c - a*d)*(a + b*x)^2 + 6*d^3*(
a + b*x)^3*Log[a + b*x] - 6*d^3*(a + b*x)^3*Log[c + d*x]) + 3*B*n*(3*(b*c -
 a*d)^4 + 4*d*(-(b*c) + a*d)^3*(a + b*x) + 6*d^2*(b*c - a*d)^2*(a + b*x)^2
+ 12*d^3*(-(b*c) + a*d)*(a + b*x)^3 - 12*d^4*(a + b*x)^4*Log[a + b*x] + 12*
d^4*(a + b*x)^4*Log[c + d*x]) + 72*B*d^4*n*(a + b*x)^4*(Log[a + b*x]*(Log[a
 + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-
(b*c) + a*d)]) - 72*B*d^4*n*(a + b*x)^4*((2*Log[(d*(a + b*x))/(-(b*c) + a*d
)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]))
)/(b*c - a*d)^4)/(b*g^5*(a + b*x)^4)
```

Maple [**F**]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)\right)^{2}}{\left(bgx+ag\right)^{5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^5,x)
```

```
[Out] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^5,x)
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2139 vs. $2(607) = 1214$.
time = 0.51, size = 2139, normalized size = 3.48

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^5,x, algorithm="ma
xima")
```

```
[Out] 1/24*A*B*n*((12*b^3*d^3*x^3 - 3*b^3*c^3 + 13*a*b^2*c^2*d - 23*a^2*b*c*d^2 +
 25*a^3*d^3 - 6*(b^3*c*d^2 - 7*a*b^2*d^3)*x^2 + 4*(b^3*c^2*d - 5*a*b^2*c*d^
2 + 13*a^2*b*d^3)*x)/((b^8*c^3 - 3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 - a^3*b^5*
d^3)*g^5*x^4 + 4*(a*b^7*c^3 - 3*a^2*b^6*c^2*d + 3*a^3*b^5*c*d^2 - a^4*b^4*d
^3)*g^5*x^3 + 6*(a^2*b^6*c^3 - 3*a^3*b^5*c^2*d + 3*a^4*b^4*c*d^2 - a^5*b^3*
d^3)*g^5*x^2 + 4*(a^3*b^5*c^3 - 3*a^4*b^4*c^2*d + 3*a^5*b^3*c*d^2 - a^6*b^2
```

```
*d^3)*g^5*x + (a^4*b^4*c^3 - 3*a^5*b^3*c^2*d + 3*a^6*b^2*c*d^2 - a^7*b*d^3)
*g^5) + 12*d^4*log(b*x + a)/((b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 -
 4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5) - 12*d^4*log(d*x + c)/((b^5*c^4 - 4*a*b^
4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5)) + 1/288*(1
2*n*((12*b^3*d^3*x^3 - 3*b^3*c^3 + 13*a*b^2*c^2*d - 23*a^2*b*c*d^2 + 25*a^3
*d^3 - 6*(b^3*c*d^2 - 7*a*b^2*d^3)*x^2 + 4*(b^3*c^2*d - 5*a*b^2*c*d^2 + 13*
a^2*b*d^3)*x)/((b^8*c^3 - 3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 - a^3*b^5*d^3)*g^
5*x^4 + 4*(a*b^7*c^3 - 3*a^2*b^6*c^2*d + 3*a^3*b^5*c*d^2 - a^4*b^4*d^3)*g^5
*x^3 + 6*(a^2*b^6*c^3 - 3*a^3*b^5*c^2*d + 3*a^4*b^4*c*d^2 - a^5*b^3*d^3)*g^
5*x^2 + 4*(a^3*b^5*c^3 - 3*a^4*b^4*c^2*d + 3*a^5*b^3*c*d^2 - a^6*b^2*d^3)*g
^5*x + (a^4*b^4*c^3 - 3*a^5*b^3*c^2*d + 3*a^6*b^2*c*d^2 - a^7*b*d^3)*g^5) +
 12*d^4*log(b*x + a)/((b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*
b^2*c*d^3 + a^4*b*d^4)*g^5) - 12*d^4*log(d*x + c)/((b^5*c^4 - 4*a*b^4*c^3*d
 + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5))*log((b*x/(d*x + c
) + a/(d*x + c))^n*e) - (9*b^4*c^4 - 64*a*b^3*c^3*d + 216*a^2*b^2*c^2*d^2 -
 576*a^3*b*c*d^3 + 415*a^4*d^4 - 300*(b^4*c*d^3 - a*b^3*d^4)*x^3 + 6*(13*b^
4*c^2*d^2 - 176*a*b^3*c*d^3 + 163*a^2*b^2*d^4)*x^2 + 72*(b^4*d^4*x^4 + 4*a*
b^3*d^4*x^3 + 6*a^2*b^2*d^4*x^2 + 4*a^3*b*d^4*x + a^4*d^4)*log(b*x + a)^2 +
 72*(b^4*d^4*x^4 + 4*a*b^3*d^4*x^3 + 6*a^2*b^2*d^4*x^2 + 4*a^3*b*d^4*x + a^
4*d^4)*log(d*x + c)^2 - 4*(7*b^4*c^3*d - 60*a*b^3*c^2*d^2 + 324*a^2*b^2*c*d
^3 - 271*a^3*b*d^4)*x - 300*(b^4*d^4*x^4 + 4*a*b^3*d^4*x^3 + 6*a^2*b^2*d^4*
x^2 + 4*a^3*b*d^4*x + a^4*d^4)*log(b*x + a) + 12*(25*b^4*d^4*x^4 + 100*a*b^
3*d^4*x^3 + 150*a^2*b^2*d^4*x^2 + 100*a^3*b*d^4*x + 25*a^4*d^4 - 12*(b^4*d^
4*x^4 + 4*a*b^3*d^4*x^3 + 6*a^2*b^2*d^4*x^2 + 4*a^3*b*d^4*x + a^4*d^4)*log(
b*x + a))*log(d*x + c))*n^2/(a^4*b^5*c^4*g^5 - 4*a^5*b^4*c^3*d*g^5 + 6*a^6*
b^3*c^2*d^2*g^5 - 4*a^7*b^2*c*d^3*g^5 + a^8*b*d^4*g^5 + (b^9*c^4*g^5 - 4*a*
b^8*c^3*d*g^5 + 6*a^2*b^7*c^2*d^2*g^5 - 4*a^3*b^6*c*d^3*g^5 + a^4*b^5*d^4*g
^5)*x^4 + 4*(a*b^8*c^4*g^5 - 4*a^2*b^7*c^3*d*g^5 + 6*a^3*b^6*c^2*d^2*g^5 -
4*a^4*b^5*c*d^3*g^5 + a^5*b^4*d^4*g^5)*x^3 + 6*(a^2*b^7*c^4*g^5 - 4*a^3*b^6
*c^3*d*g^5 + 6*a^4*b^5*c^2*d^2*g^5 - 4*a^5*b^4*c*d^3*g^5 + a^6*b^3*d^4*g^5)
*x^2 + 4*(a^3*b^6*c^4*g^5 - 4*a^4*b^5*c^3*d*g^5 + 6*a^5*b^4*c^2*d^2*g^5 - 4
*a^6*b^3*c*d^3*g^5 + a^7*b^2*d^4*g^5)*x))*B^2 - 1/4*B^2*log((b*x/(d*x + c)
+ a/(d*x + c))^n*e)^2/(b^5*g^5*x^4 + 4*a*b^4*g^5*x^3 + 6*a^2*b^3*g^5*x^2 +
4*a^3*b^2*g^5*x + a^4*b*g^5) - 1/2*A*B*log((b*x/(d*x + c) + a/(d*x + c))^n*
e)/(b^5*g^5*x^4 + 4*a*b^4*g^5*x^3 + 6*a^2*b^3*g^5*x^2 + 4*a^3*b^2*g^5*x + a
^4*b*g^5) - 1/4*A^2/(b^5*g^5*x^4 + 4*a*b^4*g^5*x^3 + 6*a^2*b^3*g^5*x^2 + 4*
a^3*b^2*g^5*x + a^4*b*g^5)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1438 vs. $2(607) = 1214$.
time = 0.46, size = 1438, normalized size = 2.34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^5,x, algorithm="fr
icas")
```

```
[Out] -1/288*(72*(A^2 + 2*A*B + B^2)*b^4*c^4 - 288*(A^2 + 2*A*B + B^2)*a*b^3*c^3*
d + 432*(A^2 + 2*A*B + B^2)*a^2*b^2*c^2*d^2 - 288*(A^2 + 2*A*B + B^2)*a^3*b
*c*d^3 + 72*(A^2 + 2*A*B + B^2)*a^4*d^4 - 12*(25*(B^2*b^4*c*d^3 - B^2*a*b^3
*d^4)*n^2 + 12*((A*B + B^2)*b^4*c*d^3 - (A*B + B^2)*a*b^3*d^4)*n)*x^3 + (9*
B^2*b^4*c^4 - 64*B^2*a*b^3*c^3*d + 216*B^2*a^2*b^2*c^2*d^2 - 576*B^2*a^3*b*
c*d^3 + 415*B^2*a^4*d^4)*n^2 + 6*((13*B^2*b^4*c^2*d^2 - 176*B^2*a*b^3*c*d^3
 + 163*B^2*a^2*b^2*d^4)*n^2 + 12*((A*B + B^2)*b^4*c^2*d^2 - 8*(A*B + B^2)*a
*b^3*c*d^3 + 7*(A*B + B^2)*a^2*b^2*d^4)*n)*x^2 - 72*(B^2*b^4*d^4*n^2*x^4 +
4*B^2*a*b^3*d^4*n^2*x^3 + 6*B^2*a^2*b^2*d^4*n^2*x^2 + 4*B^2*a^3*b*d^4*n^2*x
 - (B^2*b^4*c^4 - 4*B^2*a*b^3*c^3*d + 6*B^2*a^2*b^2*c^2*d^2 - 4*B^2*a^3*b*c
*d^3)*n^2)*log((b*x + a)/(d*x + c))^2 + 12*(3*(A*B + B^2)*b^4*c^4 - 16*(A*B
 + B^2)*a*b^3*c^3*d + 36*(A*B + B^2)*a^2*b^2*c^2*d^2 - 48*(A*B + B^2)*a^3*b
*c*d^3 + 25*(A*B + B^2)*a^4*d^4)*n - 4*((7*B^2*b^4*c^3*d - 60*B^2*a*b^3*c^2
*d^2 + 324*B^2*a^2*b^2*c*d^3 - 271*B^2*a^3*b*d^4)*n^2 + 12*((A*B + B^2)*b^4
*c^3*d - 6*(A*B + B^2)*a*b^3*c^2*d^2 + 18*(A*B + B^2)*a^2*b^2*c*d^3 - 13*(A
*B + B^2)*a^3*b*d^4)*n)*x - 12*((25*B^2*b^4*d^4*n^2 + 12*(A*B + B^2)*b^4*d^
4*n)*x^4 + 4*(12*(A*B + B^2)*a*b^3*d^4*n + (3*B^2*b^4*c*d^3 + 22*B^2*a*b^3*
d^4)*n^2)*x^3 - (3*B^2*b^4*c^4 - 16*B^2*a*b^3*c^3*d + 36*B^2*a^2*b^2*c^2*d^
2 - 48*B^2*a^3*b*c*d^3)*n^2 + 6*(12*(A*B + B^2)*a^2*b^2*d^4*n - (B^2*b^4*c^
2*d^2 - 8*B^2*a*b^3*c*d^3 - 18*B^2*a^2*b^2*d^4)*n^2)*x^2 - 12*((A*B + B^2)*
b^4*c^4 - 4*(A*B + B^2)*a*b^3*c^3*d + 6*(A*B + B^2)*a^2*b^2*c^2*d^2 - 4*(A*
B + B^2)*a^3*b*c*d^3)*n + 4*(12*(A*B + B^2)*a^3*b*d^4*n + (B^2*b^4*c^3*d -
6*B^2*a*b^3*c^2*d^2 + 18*B^2*a^2*b^2*c*d^3 + 12*B^2*a^3*b*d^4)*n^2)*x)*log(
(b*x + a)/(d*x + c)))/((b^9*c^4 - 4*a*b^8*c^3*d + 6*a^2*b^7*c^2*d^2 - 4*a^3
*b^6*c*d^3 + a^4*b^5*d^4)*g^5*x^4 + 4*(a*b^8*c^4 - 4*a^2*b^7*c^3*d + 6*a^3*
b^6*c^2*d^2 - 4*a^4*b^5*c*d^3 + a^5*b^4*d^4)*g^5*x^3 + 6*(a^2*b^7*c^4 - 4*a
^3*b^6*c^3*d + 6*a^4*b^5*c^2*d^2 - 4*a^5*b^4*c*d^3 + a^6*b^3*d^4)*g^5*x^2 +
 4*(a^3*b^6*c^4 - 4*a^4*b^5*c^3*d + 6*a^5*b^4*c^2*d^2 - 4*a^6*b^3*c*d^3 + a
^7*b^2*d^4)*g^5*x + (a^4*b^5*c^4 - 4*a^5*b^4*c^3*d + 6*a^6*b^3*c^2*d^2 - 4*
a^7*b^2*c*d^3 + a^8*b*d^4)*g^5)
```

<u>Sympy</u> [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(b*g*x+a*g)**5,x)
```

```
[Out] Timed out
```

<u>Giac</u> [**A**]
time = 4.87, size = 1166, normalized size = 1.90

$$-\frac{1}{288}\left(\frac{72\left(B^{2}b^{3}n^{2}-\frac{4(bx+a)B^{2}b^{2}dn^{2}}{dx+c}+\frac{6(bx+a)^{2}B^{2}bd^{2}n^{2}}{(dx+c)^{2}}-\frac{4(bx+a)^{3}B^{2}d^{3}n^{2}}{(dx+c)^{3}}\right)\log\left(\frac{bx+a}{dx+c}\right)^{2}}{\frac{(bx+a)^{4}b^{3}c^{3}g^{5}}{(dx+c)^{4}}-\frac{3(bx+a)^{4}ab^{2}c^{2}dg^{5}}{(dx+c)^{4}}+\frac{3(bx+a)^{4}a^{2}bcd^{2}g^{5}}{(dx+c)^{4}}-\frac{(bx+a)^{4}a^{3}d^{3}g^{5}}{(dx+c)^{4}}}+\frac{12\left(3B^{2}b^{3}n^{2}-\frac{16(bx+a)B^{2}b^{2}dn^{2}}{dx+c}+\frac{36(bx+a)^{2}B^{2}bd^{2}n^{2}}{(dx+c)^{2}}-\frac{48(bx+a)^{3}B^{2}d^{3}n^{2}}{(dx+c)^{3}}+12ABb^{3}n+12B^{2}b^{3}n-\frac{48(bx+a)ABb^{2}dn}{dx+c}-\frac{48(bx+a)B^{2}b^{2}dn}{dx+c}+\frac{72(bx+a)^{2}ABbd^{2}n}{(dx+c)^{2}}+\frac{72(bx+a)^{2}B^{2}bd^{2}n}{(dx+c)^{2}}-\frac{48(bx+a)^{3}ABd^{3}n}{(dx+c)^{3}}-\frac{48(bx+a)^{3}B^{2}d^{3}n}{(dx+c)^{3}}\right)\log\left(\frac{bx+a}{dx+c}\right)}{\frac{(bx+a)^{4}b^{3}c^{3}g^{5}}{(dx+c)^{4}}-\frac{3(bx+a)^{4}ab^{2}c^{2}dg^{5}}{(dx+c)^{4}}+\frac{3(bx+a)^{4}a^{2}bcd^{2}g^{5}}{(dx+c)^{4}}-\frac{(bx+a)^{4}a^{3}d^{3}g^{5}}{(dx+c)^{4}}}+\frac{9B^{2}b^{3}n^{2}-\frac{64(bx+a)B^{2}b^{2}dn^{2}}{dx+c}+\frac{216(bx+a)^{2}B^{2}bd^{2}n^{2}}{(dx+c)^{2}}-\frac{576(bx+a)^{3}B^{2}d^{3}n^{2}}{(dx+c)^{3}}+36ABb^{3}n+36B^{2}b^{3}n-\frac{192(bx+a)ABb^{2}dn}{dx+c}-\frac{192(bx+a)B^{2}b^{2}dn}{dx+c}+\frac{432(bx+a)^{2}ABbd^{2}n}{(dx+c)^{2}}+\frac{432(bx+a)^{2}B^{2}bd^{2}n}{(dx+c)^{2}}-\frac{576(bx+a)^{3}ABd^{3}n}{(dx+c)^{3}}-\frac{576(bx+a)^{3}B^{2}d^{3}n}{(dx+c)^{3}}+72A^{2}b^{3}+144ABb^{3}+72B^{2}b^{3}-\frac{288(bx+a)A^{2}b^{2}d}{dx+c}-\frac{576(bx+a)ABb^{2}d}{dx+c}-\frac{288(bx+a)B^{2}b^{2}d}{dx+c}+\frac{432(bx+a)^{2}A^{2}bd^{2}}{(dx+c)^{2}}+\frac{864(bx+a)^{2}ABbd^{2}}{(dx+c)^{2}}+\frac{432(bx+a)^{2}B^{2}bd^{2}}{(dx+c)^{2}}-\frac{288(bx+a)^{3}A^{2}d^{3}}{(dx+c)^{3}}-\frac{576(bx+a)^{3}ABd^{3}}{(dx+c)^{3}}-\frac{288(bx+a)^{3}B^{2}d^{3}}{(dx+c)^{3}}}{\frac{(bx+a)^{4}b^{3}c^{3}g^{5}}{(dx+c)^{4}}-\frac{3(bx+a)^{4}ab^{2}c^{2}dg^{5}}{(dx+c)^{4}}+\frac{3(bx+a)^{4}a^{2}bcd^{2}g^{5}}{(dx+c)^{4}}-\frac{(bx+a)^{4}a^{3}d^{3}g^{5}}{(dx+c)^{4}}}\right)\left(\frac{bc}{(bc-ad)^{2}}-\frac{ad}{(bc-ad)^{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^5,x, algorithm="gi
ac")
```

```
[Out] -1/288*(72*(B^2*b^3*n^2 - 4*(b*x + a)*B^2*b^2*d*n^2/(d*x + c) + 6*(b*x + a)
^2*B^2*b*d^2*n^2/(d*x + c)^2 - 4*(b*x + a)^3*B^2*d^3*n^2/(d*x + c)^3)*log((
b*x + a)/(d*x + c))^2/((b*x + a)^4*b^3*c^3*g^5/(d*x + c)^4 - 3*(b*x + a)^4*
a*b^2*c^2*d*g^5/(d*x + c)^4 + 3*(b*x + a)^4*a^2*b*c*d^2*g^5/(d*x + c)^4 - (
b*x + a)^4*a^3*d^3*g^5/(d*x + c)^4) + 12*(3*B^2*b^3*n^2 - 16*(b*x + a)*B^2*
b^2*d*n^2/(d*x + c) + 36*(b*x + a)^2*B^2*b*d^2*n^2/(d*x + c)^2 - 48*(b*x +
a)^3*B^2*d^3*n^2/(d*x + c)^3 + 12*A*B*b^3*n + 12*B^2*b^3*n - 48*(b*x + a)*A
*B*b^2*d*n/(d*x + c) - 48*(b*x + a)*B^2*b^2*d*n/(d*x + c) + 72*(b*x + a)^2*
A*B*b*d^2*n/(d*x + c)^2 + 72*(b*x + a)^2*B^2*b*d^2*n/(d*x + c)^2 - 48*(b*x
+ a)^3*A*B*d^3*n/(d*x + c)^3 - 48*(b*x + a)^3*B^2*d^3*n/(d*x + c)^3)*log((b
*x + a)/(d*x + c))/((b*x + a)^4*b^3*c^3*g^5/(d*x + c)^4 - 3*(b*x + a)^4*a*b
^2*c^2*d*g^5/(d*x + c)^4 + 3*(b*x + a)^4*a^2*b*c*d^2*g^5/(d*x + c)^4 - (b*x
 + a)^4*a^3*d^3*g^5/(d*x + c)^4) + (9*B^2*b^3*n^2 - 64*(b*x + a)*B^2*b^2*d*
n^2/(d*x + c) + 216*(b*x + a)^2*B^2*b*d^2*n^2/(d*x + c)^2 - 576*(b*x + a)^3
*B^2*d^3*n^2/(d*x + c)^3 + 36*A*B*b^3*n + 36*B^2*b^3*n - 192*(b*x + a)*A*B*
b^2*d*n/(d*x + c) - 192*(b*x + a)*B^2*b^2*d*n/(d*x + c) + 432*(b*x + a)^2*A
*B*b*d^2*n/(d*x + c)^2 + 432*(b*x + a)^2*B^2*b*d^2*n/(d*x + c)^2 - 576*(b*x
 + a)^3*A*B*d^3*n/(d*x + c)^3 - 576*(b*x + a)^3*B^2*d^3*n/(d*x + c)^3 + 72*
A^2*b^3 + 144*A*B*b^3 + 72*B^2*b^3 - 288*(b*x + a)*A^2*b^2*d/(d*x + c) - 57
6*(b*x + a)*A*B*b^2*d/(d*x + c) - 288*(b*x + a)*B^2*b^2*d/(d*x + c) + 432*(
b*x + a)^2*A^2*b*d^2/(d*x + c)^2 + 864*(b*x + a)^2*A*B*b*d^2/(d*x + c)^2 +
432*(b*x + a)^2*B^2*b*d^2/(d*x + c)^2 - 288*(b*x + a)^3*A^2*d^3/(d*x + c)^3
 - 576*(b*x + a)^3*A*B*d^3/(d*x + c)^3 - 288*(b*x + a)^3*B^2*d^3/(d*x + c)^
3)/((b*x + a)^4*b^3*c^3*g^5/(d*x + c)^4 - 3*(b*x + a)^4*a*b^2*c^2*d*g^5/(d*
x + c)^4 + 3*(b*x + a)^4*a^2*b*c*d^2*g^5/(d*x + c)^4 - (b*x + a)^4*a^3*d^3*
g^5/(d*x + c)^4))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)
```

Mupad [B]
time = 9.22, size = 1769, normalized size = 2.88

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/(a*g + b*g*x)^5,x)
```

```
[Out] (B*d^4*n*atan((B*d^4*n*(12*A + 25*B*n)*(24*b^5*c^4*g^5 - 24*a^4*b*d^4*g^5 -
 48*a*b^4*c^3*d*g^5 + 48*a^3*b^2*c*d^3*g^5)*1i)/(24*b*g^5*(25*B^2*d^4*n^2 +
 12*A*B*d^4*n)*(a*d - b*c)^4) + (B*d^5*n*x*(12*A + 25*B*n)*(b^4*c^3*g^5 - a
^3*b*d^3*g^5 - 3*a*b^3*c^2*d*g^5 + 3*a^2*b^2*c*d^2*g^5)*2i)/(g^5*(25*B^2*d^
4*n^2 + 12*A*B*d^4*n)*(a*d - b*c)^4))*(12*A + 25*B*n)*1i)/(12*b*g^5*(a*d -
b*c)^4) - ((72*A^2*a^3*d^3 - 72*A^2*b^3*c^3 + 415*B^2*a^3*d^3*n^2 - 9*B^2*b
```

^3*c^3*n^2 + 216*A^2*a*b^2*c^2*d - 216*A^2*a^2*b*c*d^2 + 300*A*B*a^3*d^3*n - 36*A*B*b^3*c^3*n + 55*B^2*a*b^2*c^2*d*n^2 - 161*B^2*a^2*b*c*d^2*n^2 + 156 *A*B*a*b^2*c^2*d*n - 276*A*B*a^2*b*c*d^2*n)/(12*(a*d - b*c)) + (x^2*(163*B^ 2*a*b^2*d^3*n^2 - 13*B^2*b^3*c*d^2*n^2 + 84*A*B*a*b^2*d^3*n - 12*A*B*b^3*c* d^2*n))/(2*(a*d - b*c)) + (x*(271*B^2*a^2*b*d^3*n^2 + 7*B^2*b^3*c^2*d*n^2 - 53*B^2*a*b^2*c*d^2*n^2 + 156*A*B*a^2*b*d^3*n + 12*A*B*b^3*c^2*d*n - 60*A*B *a*b^2*c*d^2*n))/(3*(a*d - b*c)) + (d*x^3*(25*B^2*b^3*d^2*n^2 + 12*A*B*b^3* d^2*n))/(a*d - b*c))/(x*(96*a^3*b^4*c^2*g^5 + 96*a^5*b^2*d^2*g^5 - 192*a^4* b^3*c*d*g^5) + x^3*(96*a*b^6*c^2*g^5 + 96*a^3*b^4*d^2*g^5 - 192*a^2*b^5*c*d *g^5) + x^4*(24*b^7*c^2*g^5 + 24*a^2*b^5*d^2*g^5 - 48*a*b^6*c*d*g^5) + x^2* (144*a^2*b^5*c^2*g^5 + 144*a^4*b^3*d^2*g^5 - 288*a^3*b^4*c*d*g^5) + 24*a^6* b*d^2*g^5 + 24*a^4*b^3*c^2*g^5 - 48*a^5*b^2*c*d*g^5) - log(e*((a + b*x)/(c + d*x))^n)^2*(B^2/(4*b*(a^4*g^5 + b^4*g^5*x^4 + 4*a*b^3*g^5*x^3 + 6*a^2*b^2 *g^5*x^2 + 4*a^3*b*g^5*x)) - (B^2*d^4)/(4*b*g^5*(a^4*d^4 + b^4*c^4 + 6*a^2* b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))) - log(e*((a + b*x)/(c + d*x))^n)*((A*B)/(2*a^4*b*g^5 + 2*b^5*g^5*x^4 + 8*a^3*b^2*g^5*x + 8*a*b^4*g^5*x^ 3 + 12*a^2*b^3*g^5*x^2) + (B^2*d^4*(x*(b*(a*((b*g^5*n*(a*d - b*c)*(4*a*d - b*c))/(6*d^2) + (a*b*g^5*n*(a*d - b*c))/(2*d)) + (b*g^5*n*(a*d - b*c)*(6*a^ 2*d^2 + b^2*c^2 - 4*a*b*c*d))/(6*d^3)) + a*(b*((b*g^5*n*(a*d - b*c)*(4*a*d - b*c))/(6*d^2) + (a*b*g^5*n*(a*d - b*c))/(2*d)) + (a*b^2*g^5*n*(a*d - b*c))/d + (b^2*g^5*n*(a*d - b*c)*(4*a*d - b*c))/(3*d^2)) + (b^2*g^5*n*(a*d - b* c)*(6*a^2*d^2 + b^2*c^2 - 4*a*b*c*d))/(2*d^3)) + a*(a*((b*g^5*n*(a*d - b*c) *(4*a*d - b*c))/(6*d^2) + (a*b*g^5*n*(a*d - b*c))/(2*d)) + (b*g^5*n*(a*d - b*c)*(6*a^2*d^2 + b^2*c^2 - 4*a*b*c*d))/(6*d^3)) + x^2*(b*(b*((b*g^5*n*(a*d - b*c)*(4*a*d - b*c))/(6*d^2) + (a*b*g^5*n*(a*d - b*c))/(2*d)) + (a*b^2*g^ 5*n*(a*d - b*c))/d + (b^2*g^5*n*(a*d - b*c)*(4*a*d - b*c))/(3*d^2)) + (3*a* b^3*g^5*n*(a*d - b*c))/(2*d) + (b^3*g^5*n*(a*d - b*c)*(4*a*d - b*c))/(2*d^2)) + (2*b^4*g^5*n*x^3*(a*d - b*c))/d + (b*g^5*n*(a*d - b*c)*(4*a^3*d^3 - b^ 3*c^3 + 4*a*b^2*c^2*d - 6*a^2*b*c*d^2))/(2*d^4)))/(2*b*g^5*(2*a^4*b*g^5 + 2 *b^5*g^5*x^4 + 8*a^3*b^2*g^5*x + 8*a*b^4*g^5*x^3 + 12*a^2*b^3*g^5*x^2)*(a^4 *d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)))

3.19 $\int \frac{(ag+bgx)^2}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$

Optimal. Leaf size=38

$$\text{Int}\left(\frac{(ag+bgx)^2}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)},x\right)$$

```
[Out] Unintegrable((b*g*x+a*g)^2/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(ag+bgx)^2}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a*g + b*g*x)^2/(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
[Out] Defer[Int][(a*g + b*g*x)^2/(A + B*Log[e*((a + b*x)/(c + d*x))^n]), x]
```

Rubi steps

$$\int \frac{(ag+bgx)^2}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \int \left(\frac{a^2g^2}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)} + \frac{2abg^2x}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)} + \frac{b^2g^2x^2}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right.}$$

$$= \left(a^2g^2\right)\int \frac{1}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx + \left(2abg^2\right)\int \frac{x}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx +$$

Mathematica [A]
time = 0.47, size = 0, normalized size = 0.00

$$\int \frac{(ag+bgx)^2}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a*g + b*g*x)^2/(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
[Out] Integrate[(a*g + b*g*x)^2/(A + B*Log[e*((a + b*x)/(c + d*x))^n]), x]
```

Maple [A]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{(bgx+ag)^2}{A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*g*x+a*g)^2/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

```
[Out] int((b*g*x+a*g)^2/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="maxi
ma")
```

```
[Out] integrate((b*g*x + a*g)^2/(B*log(((b*x + a)/(d*x + c))^n*e) + A), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fric
as")
```

```
[Out] integral((b^2*g^2*x^2 + 2*a*b*g^2*x + a^2*g^2)/(B*log(((b*x + a)/(d*x + c))
^n*e) + A), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$g^2\left(\int \frac{a^2}{A+B\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}\,dx+\int \frac{b^2x^2}{A+B\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}\,dx+\int \frac{2abx}{A+B\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)**2/(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

```
[Out] g**2*(Integral(a**2/(A + B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)), x) + I
ntegral(b**2*x**2/(A + B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)), x) + Int
egral(2*a*b*x/(A + B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)), x))
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="giac
")
[Out] integrate((b*g*x + a*g)^2/(B*log(((b*x + a)/(d*x + c))^n*e) + A), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{(a\,g+b\,g\,x)^2}{A+B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*g + b*g*x)^2/(A + B*log(e*((a + b*x)/(c + d*x))^n)),x)
[Out] int((a*g + b*g*x)^2/(A + B*log(e*((a + b*x)/(c + d*x))^n)), x)
```

3.20 $\int \frac{ag+bgx}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$

Optimal. Leaf size=36

$$\operatorname{Int}\left(\frac{ag+bgx}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)},x\right)$$

```
[Out] Unintegrable((b*g*x+a*g)/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{ag+bgx}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a*g + b*g*x)/(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
[Out] Defer[Int][(a*g + b*g*x)/(A + B*Log[e*((a + b*x)/(c + d*x))^n]), x]
```

Rubi steps

$$\begin{aligned}\int \frac{ag+bgx}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx &= \int \left(\frac{ag}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}+\frac{bgx}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\right)dx\\ &= (ag)\int \frac{1}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx+(bg)\int \frac{x}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx\end{aligned}$$

Mathematica [A]
time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{ag+bgx}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a*g + b*g*x)/(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
[Out] Integrate[(a*g + b*g*x)/(A + B*Log[e*((a + b*x)/(c + d*x))^n]), x]
```

Maple [A]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{bgx+ag}{A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*g*x+a*g)/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

```
[Out] int((b*g*x+a*g)/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="maxima
")
```

```
[Out] integrate((b*g*x + a*g)/(B*log(((b*x + a)/(d*x + c))^n*e) + A), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fricas
")
```

```
[Out] integral((b*g*x + a*g)/(B*log(((b*x + a)/(d*x + c))^n*e) + A), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$g\left(\int \frac{a}{A + B\log\left(e\left(\frac{a}{c+dx} + \frac{bx}{c+dx}\right)^n\right)}\,dx + \int \frac{bx}{A + B\log\left(e\left(\frac{a}{c+dx} + \frac{bx}{c+dx}\right)^n\right)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)/(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

```
[Out] g*(Integral(a/(A + B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)), x) + Integra
l(b*x/(A + B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)), x))
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="giac")
```

```
[Out] integrate((b*g*x + a*g)/(B*log(((b*x + a)/(d*x + c))^n*e) + A), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{a\,g+b\,g\,x}{A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*g + b*g*x)/(A + B*log(e*((a + b*x)/(c + d*x))^n)),x)
```

```
[Out] int((a*g + b*g*x)/(A + B*log(e*((a + b*x)/(c + d*x))^n)), x)
```

3.21 $\int \frac{1}{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx$

Optimal. Leaf size=38

$$\text{Int}\left(\frac{1}{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)},x\right)$$

```
[Out] Unintegrable(1/(b*g*x+a*g)/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((a*g + b*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])),x]
[Out] Defer[Int][1/((a*g + b*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])), x]
```

Rubi steps

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx = \int \frac{1}{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx$$

Mathematica [A]
time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((a*g + b*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])),x]
[Out] Integrate[1/((a*g + b*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])), x]
```

Maple [A]
time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{1}{(bgx+ag)\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*g*x+a*g)/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

```
[Out] int(1/(b*g*x+a*g)/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="maxi
ma")
```

```
[Out] integrate(1/((b*g*x + a*g)*(B*log(((b*x + a)/(d*x + c))^n*e) + A)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fric
as")
```

```
[Out] integral(1/(A*b*g*x + A*a*g + (B*b*g*x + B*a*g)*log(((b*x + a)/(d*x + c))^n
*e)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{Aa+Abx+Ba\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^{n}\right)+Bbx\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^{n}\right)}\,dx}{g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)/(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

```
[Out] Integral(1/(A*a + A*b*x + B*a*log(e*(a/(c + d*x) + b*x/(c + d*x))**n) + B*b
*x*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)), x)/g
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="giac
")
```

```
[Out] integrate(1/((b*g*x + a*g)*(B*log(((b*x + a)/(d*x + c))^n*e) + A)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{(a\,g+b\,g\,x)\,\left(A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*g + b*g*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))),x)
```

```
[Out] int(1/((a*g + b*g*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))), x)
```

3.22 $\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx$

Optimal. Leaf size=94

$$\frac{e^{\frac{A}{Bn}}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{1}{n}}(c+dx)\text{Ei}\left(-\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{Bn}\right)}{B(bc-ad)g^2n(a+bx)}$$

[Out] exp(A/B/n)*(e*((b*x+a)/(d*x+c))^n)^(1/n)*(d*x+c)*Ei((-A-B*ln(e*((b*x+a)/(d*x+c))^n))/B/n)/B/(-a*d+b*c)/g^2/n/(b*x+a)

Rubi [A]

time = 0.08, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {2549, 2347, 2209}

$$\frac{e^{\frac{A}{Bn}}(c+dx)\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{1}{n}}\text{Ei}\left(-\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{Bn}\right)}{Bg^2n(a+bx)(bc-ad)}$$

Antiderivative was successfully verified.

[In] Int[1/((a*g + b*g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])),x]

[Out] (E^(A/(B*n))*(e*((a + b*x)/(c + d*x))^n)^n^(-1)*(c + d*x)*ExpIntegralEi[-((A + B*Log[e*((a + b*x)/(c + d*x))^n])/(B*n))])/(B*(b*c - a*d)*g^2*n*(a + b*x))

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2549

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(m +
1)*(g/b)^m, Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x], x,
(a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ
```

```
[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[b*f - a*g, 0] && (GtQ[p, 0] || LtQ
[m, -1])
```

Rubi steps

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx=\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx$$

Mathematica [A]
time = 0.09, size = 94, normalized size = 1.00

$$\frac{e^{\frac{A}{Bn}}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{1}{n}}(c+dx)\text{Ei}\left(-\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{Bn}\right)}{B(bc-ad)g^2n(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a*g + b*g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])),x]
```

```
[Out] (E^(A/(B*n))*(e*((a + b*x)/(c + d*x))^n)^n^(-1)*(c + d*x)*ExpIntegralEi[-((
A + B*Log[e*((a + b*x)/(c + d*x))^n])/(B*n))])/(B*(b*c - a*d)*g^2*n*(a + b*
x))
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{1}{(bgx+ag)^2\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*g*x+a*g)^2/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

```
[Out] int(1/(b*g*x+a*g)^2/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="ma
xima")
```

[Out] integrate(1/((b*g*x + a*g)^2*(B*log(((b*x + a)/(d*x + c))^n*e) + A)), x)

Fricas [A]
time = 0.34, size = 56, normalized size = 0.60

$$\frac{e^{\left(\frac{A+B}{Bn}\right)}\,\mathrm{log\_integral}\left(\frac{(dx+c)e^{\left(-\frac{A+B}{Bn}\right)}}{bx+a}\right)}{(Bbc-Bad)g^2n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*g*x+a*g)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fricas")

[Out] e^((A + B)/(B*n))*log_integral((d*x + c)*e^(-(A + B)/(B*n))/(b*x + a))/((B*b*c - B*a*d)*g^2*n)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{Aa^2+2Aabx+Ab^2x^2+Ba^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)+2Babx\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)+Bb^2x^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}\,dx}{g^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*g*x+a*g)**2/(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)

[Out] Integral(1/(A*a**2 + 2*A*a*b*x + A*b**2*x**2 + B*a**2*log(e*(a/(c + d*x) + b*x/(c + d*x))**n) + 2*B*a*b*x*log(e*(a/(c + d*x) + b*x/(c + d*x))**n) + B*b**2*x**2*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)), x)/g**2

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*g*x+a*g)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="giac")

[Out] integrate(1/((b*g*x + a*g)^2*(B*log(((b*x + a)/(d*x + c))^n*e) + A)), x)

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a\,g+b\,g\,x\right)}^2\,\left(A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a*g + b*g*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))),x)

[Out] int(1/((a*g + b*g*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))), x)

3.23 $\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx$

Optimal. Leaf size=197

$$\frac{be^{\frac{2A}{Bn}}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{2/n}(c+dx)^2\text{Ei}\left(-\frac{2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{Bn}\right)}{B(bc-ad)^2g^3n(a+bx)^2}-\frac{de^{\frac{A}{Bn}}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{1}{n}}(c+dx)\text{Ei}\left(-\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{Bn}\right)}{B(bc-ad)^2g^3n(a+bx)}$$

```
[Out] b*exp(2*A/B/n)*(e*((b*x+a)/(d*x+c))^n)^(2/n)*(d*x+c)^2*Ei(-2*(A+B*ln(e*((b*
x+a)/(d*x+c))^n))/B/n)/B/(-a*d+b*c)^2/g^3/n/(b*x+a)^2-d*exp(A/B/n)*(e*((b*x
+a)/(d*x+c))^n)^(1/n)*(d*x+c)*Ei((-A-B*ln(e*((b*x+a)/(d*x+c))^n))/B/n)/B/(-
a*d+b*c)^2/g^3/n/(b*x+a)
```

Rubi [A]
time = 0.19, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {2549, 2395, 2347, 2209}

$$\frac{be^{\frac{2A}{Bn}}(c+dx)^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{2/n}\text{Ei}\left(-\frac{2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{Bn}\right)}{Bg^3n(a+bx)^2(bc-ad)^2}-\frac{de^{\frac{A}{Bn}}(c+dx)\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{1}{n}}\text{Ei}\left(-\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{Bn}\right)}{Bg^3n(a+bx)(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a*g + b*g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])),x]
```

```
[Out] (b*E^((2*A)/(B*n))*(e*((a + b*x)/(c + d*x))^n)^(2/n)*(c + d*x)^2*ExpIntegra
lEi[(-2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(B*n)])/(B*(b*c - a*d)^2*g^
3*n*(a + b*x)^2) - (d*E^(A/(B*n))*(e*((a + b*x)/(c + d*x))^n)^n^(-1)*(c + d
*x)*ExpIntegralEi[-((A + B*Log[e*((a + b*x)/(c + d*x))^n])/(B*n))])/(B*(b*c
 - a*d)^2*g^3*n*(a + b*x))
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
```

```
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2549

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(m +
1)*(g/b)^m, Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x], x,
(a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ
[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[b*f - a*g, 0] && (GtQ[p, 0] || LtQ
[m, -1])
```

Rubi steps

$$\int \frac{1}{(ag+bgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx = \int \frac{1}{(ag+bgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx$$

Mathematica [A]
time = 0.18, size = 172, normalized size = 0.87

$$\frac{e^{\frac{A}{Bn}}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{1}{n}}(c+dx)\left(be^{\frac{A}{Bn}}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{1}{n}}(c+dx)\text{Ei}\left(-\frac{2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{Bn}\right)-d(a+bx)\text{Ei}\left(-\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{Bn}\right)\right)}{B(bc-ad)^2g^3n(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a*g + b*g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])),x]
```

```
[Out] (E^(A/(B*n))*(e*((a + b*x)/(c + d*x))^n)^n^(-1)*(c + d*x)*(b*E^(A/(B*n))*(e
*((a + b*x)/(c + d*x))^n)^n^(-1)*(c + d*x)*ExpIntegralEi[(-2*(A + B*Log[e*(
(a + b*x)/(c + d*x))^n]))/(B*n)] - d*(a + b*x)*ExpIntegralEi[-((A + B*Log[e
*((a + b*x)/(c + d*x))^n])/(B*n))]))/(B*(b*c - a*d)^2*g^3*n*(a + b*x)^2)
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{1}{(bgx+ag)^3 \left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*g*x+a*g)^3/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

```
[Out] int(1/(b*g*x+a*g)^3/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)^3/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="ma
xima")
```

```
[Out] integrate(1/((b*g*x + a*g)^3*(B*log(((b*x + a)/(d*x + c))^n*e) + A)), x)
```

Fricas [**A**]
time = 0.40, size = 137, normalized size = 0.70

$$-\frac{de^{\left(\frac{A+B}{Bn}\right)}\operatorname{log\_integral}\left(\frac{(dx+c)e^{\left(-\frac{A+B}{Bn}\right)}}{bx+a}\right)-be^{\left(\frac{2(A+B)}{Bn}\right)}\operatorname{log\_integral}\left(\frac{(d^2x^2+2\,cdx+c^2)e^{\left(-\frac{2(A+B)}{Bn}\right)}}{b^2x^2+2\,abx+a^2}\right)}{(Bb^2c^2-2\,Babcd+Ba^2d^2)g^3n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)^3/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fr
icas")
```

```
[Out] -(d*e^((A + B)/(B*n))*log_integral((d*x + c)*e^(-(A + B)/(B*n))/(b*x + a))
- b*e^(2*(A + B)/(B*n))*log_integral((d^2*x^2 + 2*c*d*x + c^2)*e^(-2*(A + B
)/(B*n))/(b^2*x^2 + 2*a*b*x + a^2)))/((B*b^2*c^2 - 2*B*a*b*c*d + B*a^2*d^2)
*g^3*n)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)**3/(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)^3/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="gi
ac")
```

```
[Out] integrate(1/((b*g*x + a*g)^3*(B*log(((b*x + a)/(d*x + c))^n*e) + A)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a\,g+b\,g\,x\right)}^3\,\left(A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*g + b*g*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))),x)
[Out] int(1/((a*g + b*g*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))), x)
```

3.24 $\int \frac{(ag+bgx)^2}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$

Optimal. Leaf size=38

$$\text{Int}\left(\frac{(ag+bgx)^2}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2},x\right)$$

```
[Out] Unintegrable((b*g*x+a*g)^2/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(ag+bgx)^2}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a*g + b*g*x)^2/(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
[Out] Defer[Int][(a*g + b*g*x)^2/(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2, x]
```

Rubi steps

$$\int \frac{(ag+bgx)^2}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = \int \left(\frac{a^2g^2}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}+\frac{2abg^2x}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}+\frac{b^2}{\left(A+B\log\right.}\right.$$
$$=\left(a^2g^2\right)\int \frac{1}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx+\left(2abg^2\right)\int \frac{x}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)\right.\right.}$$

Mathematica [A]
time = 0.63, size = 0, normalized size = 0.00

$$\int \frac{(ag+bgx)^2}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a*g + b*g*x)^2/(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
[Out] Integrate[(a*g + b*g*x)^2/(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2, x]
```

Maple [A]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{(bgx+ag)^2}{\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*g*x+a*g)^2/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

```
[Out] int((b*g*x+a*g)^2/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="ma
xima")
```

```
[Out] -(b^3*d*g^2*x^4 + a^3*c*g^2 + (b^3*c*g^2 + 3*a*b^2*d*g^2)*x^3 + 3*(a*b^2*c*
g^2 + a^2*b*d*g^2)*x^2 + (3*a^2*b*c*g^2 + a^3*d*g^2)*x)/((b*c*n - a*d*n)*B^
2*log((b*x + a)^n) - (b*c*n - a*d*n)*B^2*log((d*x + c)^n) + (b*c*n - a*d*n)
*A*B + (b*c*n - a*d*n)*B^2) + integrate((4*b^3*d*g^2*x^3 + 3*a^2*b*c*g^2 +
a^3*d*g^2 + 3*(b^3*c*g^2 + 3*a*b^2*d*g^2)*x^2 + 6*(a*b^2*c*g^2 + a^2*b*d*g^
2)*x)/((b*c*n - a*d*n)*B^2*log((b*x + a)^n) - (b*c*n - a*d*n)*B^2*log((d*x
+ c)^n) + (b*c*n - a*d*n)*A*B + (b*c*n - a*d*n)*B^2), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="fr
icas")
```

```
[Out] integral((b^2*g^2*x^2 + 2*a*b*g^2*x + a^2*g^2)/(B^2*log(((b*x + a)/(d*x + c
))^n*e)^2 + 2*A*B*log(((b*x + a)/(d*x + c))^n*e) + A^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$g^2\left(\int \frac{a^2}{A^2+2AB\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)+B^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2}\,dx+\int \frac{b^2x^2}{A^2+2AB\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)+B^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2}\,dx+\int \frac{2abx}{A^2+2AB\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)+B^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)**2/(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
```

```
[Out] g**2*(Integral(a**2/(A**2 + 2*A*B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n) +
 B**2*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)**2), x) + Integral(b**2*x**2/
(A**2 + 2*A*B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n) + B**2*log(e*(a/(c +
d*x) + b*x/(c + d*x))**n)**2), x) + Integral(2*a*b*x/(A**2 + 2*A*B*log(e*(a
/(c + d*x) + b*x/(c + d*x))**n) + B**2*log(e*(a/(c + d*x) + b*x/(c + d*x))*
*n)**2), x))
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="gi
ac")
```

```
[Out] integrate((b*g*x + a*g)^2/(B*log(((b*x + a)/(d*x + c))^n*e) + A)^2, x)
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{{\left(a\,g+b\,g\,x\right)}^2}{{\left(A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*g + b*g*x)^2/(A + B*log(e*((a + b*x)/(c + d*x))^n))^2,x)
```

```
[Out] int((a*g + b*g*x)^2/(A + B*log(e*((a + b*x)/(c + d*x))^n))^2, x)
```

3.25 $\int \frac{ag+bgx}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$

Optimal. Leaf size=36

$$\text{Int}\left(\frac{ag+bgx}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2},x\right)$$

```
[Out] Unintegrable((b*g*x+a*g)/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{ag+bgx}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a*g + b*g*x)/(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
[Out] Defer[Int][(a*g + b*g*x)/(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2, x]
```

Rubi steps

$$\begin{aligned}\int \frac{ag+bgx}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx &= \int \left(\frac{ag}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}+\frac{bgx}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\right)dx\\ &= (ag)\int \frac{1}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx+(bg)\int \frac{x}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx\end{aligned}$$

Mathematica [A]
time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{ag+bgx}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a*g + b*g*x)/(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
[Out] Integrate[(a*g + b*g*x)/(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2, x]
```

Maple [A]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{bgx+ag}{\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*g*x+a*g)/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

```
[Out] int((b*g*x+a*g)/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="maxi
ma")
```

```
[Out] -(b^2*d*g*x^3 + a^2*c*g + (b^2*c*g + 2*a*b*d*g)*x^2 + (2*a*b*c*g + a^2*d*g)
*x)/((b*c*n - a*d*n)*B^2*log((b*x + a)^n) - (b*c*n - a*d*n)*B^2*log((d*x +
c)^n) + (b*c*n - a*d*n)*A*B + (b*c*n - a*d*n)*B^2) + integrate((3*b^2*d*g*x
^2 + 2*a*b*c*g + a^2*d*g + 2*(b^2*c*g + 2*a*b*d*g)*x)/((b*c*n - a*d*n)*B^2*
log((b*x + a)^n) - (b*c*n - a*d*n)*B^2*log((d*x + c)^n) + (b*c*n - a*d*n)*A
*B + (b*c*n - a*d*n)*B^2), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="fric
as")
```

```
[Out] integral((b*g*x + a*g)/(B^2*log(((b*x + a)/(d*x + c))^n*e)^2 + 2*A*B*log(((
b*x + a)/(d*x + c))^n*e) + A^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$g\left(\int \frac{a}{A^{2} + 2AB\log\left(e\left(\frac{a}{c+dx} + \frac{bx}{c+dx}\right)^{n}\right) + B^{2}\log\left(e\left(\frac{a}{c+dx} + \frac{bx}{c+dx}\right)^{n}\right)^{2}}\, dx + \int \frac{bx}{A^{2} + 2AB\log\left(e\left(\frac{a}{c+dx} + \frac{bx}{c+dx}\right)^{n}\right) + B^{2}\log\left(e\left(\frac{a}{c+dx} + \frac{bx}{c+dx}\right)^{n}\right)^{2}}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)/(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
```

```
[Out] g*(Integral(a/(A**2 + 2*A*B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n) + B**2*
log(e*(a/(c + d*x) + b*x/(c + d*x))**n)**2), x) + Integral(b*x/(A**2 + 2*A*
B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n) + B**2*log(e*(a/(c + d*x) + b*x/(
c + d*x))**n)**2), x))
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="giac
")
```

```
[Out] integrate((b*g*x + a*g)/(B*log(((b*x + a)/(d*x + c))^n*e) + A)^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{a\,g+b\,g\,x}{{\left(A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*g + b*g*x)/(A + B*log(e*((a + b*x)/(c + d*x))^n))^2,x)
```

```
[Out] int((a*g + b*g*x)/(A + B*log(e*((a + b*x)/(c + d*x))^n))^2, x)
```

3.26 $\int \frac{1}{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$

Optimal. Leaf size=38

$$\text{Int}\left(\frac{1}{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2},x\right)$$

```
[Out] Unintegrable(1/(b*g*x+a*g)/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((a*g + b*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2),x]
[Out] Defer[Int][1/((a*g + b*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2), x]
```

Rubi steps

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = \int \frac{1}{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$

Mathematica [A]
time = 0.37, size = 0, normalized size = 0.00

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((a*g + b*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2),x]
[Out] Integrate[1/((a*g + b*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2), x]
```

Maple [A]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{1}{(bgx+ag)\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*g*x+a*g)/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

```
[Out] int(1/(b*g*x+a*g)/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="ma
xima")
```

```
[Out] d*integrate(1/((b*c*g*n - a*d*g*n)*B^2*log((b*x + a)^n) - (b*c*g*n - a*d*g*
n)*B^2*log((d*x + c)^n) + (b*c*g*n - a*d*g*n)*A*B + (b*c*g*n - a*d*g*n)*B^2
), x) - (d*x + c)/((b*c*g*n - a*d*g*n)*B^2*log((b*x + a)^n) - (b*c*g*n - a*
d*g*n)*B^2*log((d*x + c)^n) + (b*c*g*n - a*d*g*n)*A*B + (b*c*g*n - a*d*g*n)
*B^2)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="fr
icas")
```

```
[Out] integral(1/(A^2*b*g*x + A^2*a*g + (B^2*b*g*x + B^2*a*g)*log(((b*x + a)/(d*x
 + c))^n*e)^2 + 2*(A*B*b*g*x + A*B*a*g)*log(((b*x + a)/(d*x + c))^n*e)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{A^{2}a+A^{2}bx+2ABa\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^{n}\right)+2ABbx\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^{n}\right)+B^{2}a\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^{n}\right)^{2}+B^{2}bx\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^{n}\right)^{2}}\,dx}{g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)/(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
```

```
[Out] Integral(1/(A**2*a + A**2*b*x + 2*A*B*a*log(e*(a/(c + d*x) + b*x/(c + d*x))
**n) + 2*A*B*b*x*log(e*(a/(c + d*x) + b*x/(c + d*x))**n) + B**2*a*log(e*(a/
(c + d*x) + b*x/(c + d*x))**n)**2 + B**2*b*x*log(e*(a/(c + d*x) + b*x/(c +
d*x))**n)**2), x)/g
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="gi
ac")
```

```
[Out] integrate(1/((b*g*x + a*g)*(B*log(((b*x + a)/(d*x + c))^n*e) + A)^2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{(a\,g+b\,g\,x)\,\left(A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*g + b*g*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2),x)
```

```
[Out] int(1/((a*g + b*g*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2), x)
```

3.27 $\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$

Optimal. Leaf size=153

$$-\frac{e^{\frac{A}{Bn}}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{1}{n}}(c+dx)\text{Ei}\left(-\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{Bn}\right)}{B^2(bc-ad)g^2n^2(a+bx)}-\frac{c+dx}{B(bc-ad)g^2n(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}$$

```
[Out] -exp(A/B/n)*(e*((b*x+a)/(d*x+c))^n)^(1/n)*(d*x+c)*Ei((-A-B*ln(e*((b*x+a)/(d
*x+c))^n))/B/n)/B^2/(-a*d+b*c)/g^2/n^2/(b*x+a)+(-d*x-c)/B/(-a*d+b*c)/g^2/n/
(b*x+a)/(A+B*ln(e*((b*x+a)/(d*x+c))^n))
```

Rubi [A]
time = 0.10, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {2549, 2343, 2347, 2209}

$$-\frac{e^{\frac{A}{Bn}}(c+dx)\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{1}{n}}\text{Ei}\left(-\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{Bn}\right)}{B^2g^2n^2(a+bx)(bc-ad)}-\frac{c+dx}{Bg^2n(a+bx)(bc-ad)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a*g + b*g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2),x]
```

```
[Out] -((E^(A/(B*n))*(e*((a + b*x)/(c + d*x))^n)^n^(-1)*(c + d*x)*ExpIntegralEi[-
((A + B*Log[e*((a + b*x)/(c + d*x))^n])/(B*n))])/(B^2*(b*c - a*d)*g^2*n^2*(
a + b*x))) - (c + d*x)/(B*(b*c - a*d)*g^2*n*(a + b*x)*(A + B*Log[e*((a + b*
x)/(c + d*x))^n]))
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2343

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^(p + 1)/(b*d*n*(p + 1))), x] -
Dist[(m + 1)/(b*n*(p + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p + 1), x], x]
/; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
```

```
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2549

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(m +
1)*(g/b)^m, Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x], x,
(a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ
[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[b*f - a*g, 0] && (GtQ[p, 0] || LtQ
[m, -1])
```

Rubi steps

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$

Mathematica [A]
time = 0.13, size = 146, normalized size = 0.95

$$-\frac{(c+dx)\left(Bn+e^{\frac{A}{Bn}}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{1}{n}}\operatorname{Ei}\left(-\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{Bn}\right)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\right)}{B^2(bc-ad)g^2n^2(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a*g + b*g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2),x]
```

```
[Out] -(((c + d*x)*(B*n + E^(A/(B*n))*(e*((a + b*x)/(c + d*x))^n)^n^(-1)*ExpInteg
ralEi[-((A + B*Log[e*((a + b*x)/(c + d*x))^n])/(B*n))]*(A + B*Log[e*((a + b
*x)/(c + d*x))^n])))/(B^2*(b*c - a*d)*g^2*n^2*(a + b*x)*(A + B*Log[e*((a +
b*x)/(c + d*x))^n])))
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{1}{(bgx+ag)^2\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*g*x+a*g)^2/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

```
[Out] int(1/(b*g*x+a*g)^2/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="
maxima")
```

```
[Out] -(d*x + c)/((a*b*c*g^2*n - a^2*d*g^2*n)*A*B + (a*b*c*g^2*n - a^2*d*g^2*n)*B
^2 + ((b^2*c*g^2*n - a*b*d*g^2*n)*A*B + (b^2*c*g^2*n - a*b*d*g^2*n)*B^2)*x
+ ((b^2*c*g^2*n - a*b*d*g^2*n)*B^2*x + (a*b*c*g^2*n - a^2*d*g^2*n)*B^2)*log
((b*x + a)^n) - ((b^2*c*g^2*n - a*b*d*g^2*n)*B^2*x + (a*b*c*g^2*n - a^2*d*g
^2*n)*B^2)*log((d*x + c)^n)) + integrate(-1/(A*B*a^2*g^2*n + B^2*a^2*g^2*n
+ (A*B*b^2*g^2*n + B^2*b^2*g^2*n)*x^2 + 2*(A*B*a*b*g^2*n + B^2*a*b*g^2*n)*x
 + (B^2*b^2*g^2*n*x^2 + 2*B^2*a*b*g^2*n*x + B^2*a^2*g^2*n)*log((b*x + a)^n)
 - (B^2*b^2*g^2*n*x^2 + 2*B^2*a*b*g^2*n*x + B^2*a^2*g^2*n)*log((d*x + c)^n)
), x)
```

Fricas [A]
time = 0.34, size = 241, normalized size = 1.58

$$-\frac{Bdnx + Bcn + \left((Bbnx + Ban)e^{\left(\frac{A+B}{Bn}\right)}\log\left(\frac{bx+a}{dx+c}\right) + ((A+B)bx + (A+B)a)e^{\left(\frac{A+B}{Bn}\right)}\right)\text{log\_integral}\left(\frac{(dx+c)e^{\left(-\frac{A+B}{Bn}\right)}}{bx+a}\right)}{((AB^2+B^3)b^2c - (AB^2+B^3)abd)g^2n^2x + ((AB^2+B^3)abc - (AB^2+B^3)a^2d)g^2n^2 + ((B^3b^2c - B^3abd)g^2n^3x + (B^3abc - B^3a^2d)g^2n^3)\log\left(\frac{bx+a}{dx+c}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="
fricas")
```

```
[Out] -(B*d*n*x + B*c*n + ((B*b*n*x + B*a*n)*e^((A + B)/(B*n))*log((b*x + a)/(d*x
 + c)) + ((A + B)*b*x + (A + B)*a)*e^((A + B)/(B*n)))*log_integral((d*x + c
)*e^(-(A + B)/(B*n))/(b*x + a)))/((((A*B^2 + B^3)*b^2*c - (A*B^2 + B^3)*a*b*
d)*g^2*n^2*x + ((A*B^2 + B^3)*a*b*c - (A*B^2 + B^3)*a^2*d)*g^2*n^2 + ((B^3*
b^2*c - B^3*a*b*d)*g^2*n^3*x + (B^3*a*b*c - B^3*a^2*d)*g^2*n^3)*log((b*x +
a)/(d*x + c)))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)**2/(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="
giac")
[Out] integrate(1/((b*g*x + a*g)^2*(B*log(((b*x + a)/(d*x + c))^n*e) + A)^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a\,g+b\,g\,x)^2\,\left(A+B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*g + b*g*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2),x)
[Out] int(1/((a*g + b*g*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2), x)
```

3.28 $\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$

Optimal. Leaf size=314

$$-\frac{2be^{\frac{2A}{Bn}}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{2/n}(c+dx)^2\text{Ei}\left(-\frac{2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{Bn}\right)}{B^2(bc-ad)^2g^3n^2(a+bx)^2}+\frac{de^{\frac{A}{Bn}}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{1}{n}}(c+dx)\text{Ei}\left(-\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{Bn}\right)}{B^2(bc-ad)^2g^3n^2(a+bx)}$$

```
[Out] -2*b*exp(2*A/B/n)*(e*((b*x+a)/(d*x+c))^n)^(2/n)*(d*x+c)^2*Ei(-2*(A+B*ln(e*(
(b*x+a)/(d*x+c))^n))/B/n)/B^2/(-a*d+b*c)^2/g^3/n^2/(b*x+a)^2+d*exp(A/B/n)*(
e*((b*x+a)/(d*x+c))^n)^(1/n)*(d*x+c)*Ei((-A-B*ln(e*((b*x+a)/(d*x+c))^n))/B/
n)/B^2/(-a*d+b*c)^2/g^3/n^2/(b*x+a)+d*(d*x+c)/B/(-a*d+b*c)^2/g^3/n/(b*x+a)/
(A+B*ln(e*((b*x+a)/(d*x+c))^n))-b*(d*x+c)^2/B/(-a*d+b*c)^2/g^3/n/(b*x+a)^2/
(A+B*ln(e*((b*x+a)/(d*x+c))^n))
```

Rubi [A]
time = 0.24, antiderivative size = 314, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2549, 2395, 2343, 2347, 2209}

$$-\frac{2be^{\frac{2A}{Bn}}(c+dx)^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{2/n}\text{Ei}\left(-\frac{2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{Bn}\right)}{B^2g^3n^2(a+bx)^2(bc-ad)^2}+\frac{de^{\frac{A}{Bn}}(c+dx)\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{1}{n}}\text{Ei}\left(-\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{Bn}\right)}{B^2g^3n^2(a+bx)(bc-ad)^2}-\frac{b(c+dx)^2}{Bg^3n(a+bx)^2(bc-ad)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}+\frac{d(c+dx)}{Bg^3n(a+bx)(bc-ad)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a*g + b*g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2),x]
```

```
[Out] (-2*b*E^((2*A)/(B*n))*(e*((a + b*x)/(c + d*x))^n)^(2/n)*(c + d*x)^2*ExpInte
gralEi[(-2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(B*n)])/(B^2*(b*c - a*d)
^2*g^3*n^2*(a + b*x)^2) + (d*E^(A/(B*n))*(e*((a + b*x)/(c + d*x))^n)^n^(-1)
*(c + d*x)*ExpIntegralEi[-((A + B*Log[e*((a + b*x)/(c + d*x))^n])/(B*n))])/
(B^2*(b*c - a*d)^2*g^3*n^2*(a + b*x)) + (d*(c + d*x))/(B*(b*c - a*d)^2*g^3*
n*(a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])) - (b*(c + d*x)^2)/(B*(b
*c - a*d)^2*g^3*n*(a + b*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2343

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^(p + 1)/(b*d*n*(p + 1))), x] -
Dist[(m + 1)/(b*n*(p + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p + 1), x], x]
```

```
/; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2549

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(m +
1)*(g/b)^m, Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x], x,
(a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ
[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[b*f - a*g, 0] && (GtQ[p, 0] || LtQ
[m, -1])
```

Rubi steps

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$

Mathematica [A]
time = 0.38, size = 254, normalized size = 0.81

$$\frac{(c+dx)\left(B(-bc+ad)n-2be^{\frac{2A}{Bn}}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{2/n}(c+dx)\text{Ei}\left(-\frac{2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{Bn}\right)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)+de^{\frac{A}{Bn}}(a+bx)\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{1}{n}}\text{Ei}\left(-\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{Bn}\right)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\right)}{B^2(bc-ad)^2g^3n^2(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a*g + b*g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2),x]
```

```
[Out] ((c + d*x)*(B*(-(b*c) + a*d)*n - 2*b*E^((2*A)/(B*n))*(e*((a + b*x)/(c + d*x
))^n)^(2/n)*(c + d*x)*ExpIntegralEi[(-2*(A + B*Log[e*((a + b*x)/(c + d*x))^
n]))/(B*n)]*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + d*E^(A/(B*n))*(a + b*x
)*(e*((a + b*x)/(c + d*x))^n)^n^(-1)*ExpIntegralEi[-((A + B*Log[e*((a + b*x
```

```
)/(c + d*x))^n])/(B*n))]*(A + B*Log[e*((a + b*x)/(c + d*x))^n])))/(B^2*(b*c
 - a*d)^2*g^3*n^2*(a + b*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))
```

Maple [**F**]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{1}{(bgx+ag)^3 \left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*g*x+a*g)^3/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

```
[Out] int(1/(b*g*x+a*g)^3/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)^3/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="
maxima")
```

```
[Out] -(d*x + c)/((a^2*b*c*g^3*n - a^3*d*g^3*n)*A*B + (a^2*b*c*g^3*n - a^3*d*g^3*
n)*B^2 + ((b^3*c*g^3*n - a*b^2*d*g^3*n)*A*B + (b^3*c*g^3*n - a*b^2*d*g^3*n)
*B^2)*x^2 + 2*((a*b^2*c*g^3*n - a^2*b*d*g^3*n)*A*B + (a*b^2*c*g^3*n - a^2*b
*d*g^3*n)*B^2)*x + ((b^3*c*g^3*n - a*b^2*d*g^3*n)*B^2*x^2 + 2*(a*b^2*c*g^3*
n - a^2*b*d*g^3*n)*B^2*x + (a^2*b*c*g^3*n - a^3*d*g^3*n)*B^2)*log((b*x + a)
^n) - ((b^3*c*g^3*n - a*b^2*d*g^3*n)*B^2*x^2 + 2*(a*b^2*c*g^3*n - a^2*b*d*g
^3*n)*B^2*x + (a^2*b*c*g^3*n - a^3*d*g^3*n)*B^2)*log((d*x + c)^n)) - integr
ate((b*d*x + 2*b*c - a*d)/(((b^4*c*g^3*n - a*b^3*d*g^3*n)*A*B + (b^4*c*g^3*
n - a*b^3*d*g^3*n)*B^2)*x^3 + (a^3*b*c*g^3*n - a^4*d*g^3*n)*A*B + (a^3*b*c*
g^3*n - a^4*d*g^3*n)*B^2 + 3*((a*b^3*c*g^3*n - a^2*b^2*d*g^3*n)*A*B + (a*b^
3*c*g^3*n - a^2*b^2*d*g^3*n)*B^2)*x^2 + 3*((a^2*b^2*c*g^3*n - a^3*b*d*g^3*n
)*A*B + (a^2*b^2*c*g^3*n - a^3*b*d*g^3*n)*B^2)*x + ((b^4*c*g^3*n - a*b^3*d*
g^3*n)*B^2*x^3 + 3*(a*b^3*c*g^3*n - a^2*b^2*d*g^3*n)*B^2*x^2 + 3*(a^2*b^2*c
*g^3*n - a^3*b*d*g^3*n)*B^2*x + (a^3*b*c*g^3*n - a^4*d*g^3*n)*B^2)*log((b*x
 + a)^n) - ((b^4*c*g^3*n - a*b^3*d*g^3*n)*B^2*x^3 + 3*(a*b^3*c*g^3*n - a^2*
b^2*d*g^3*n)*B^2*x^2 + 3*(a^2*b^2*c*g^3*n - a^3*b*d*g^3*n)*B^2*x + (a^3*b*c
*g^3*n - a^4*d*g^3*n)*B^2)*log((d*x + c)^n)), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 639 vs. 2(318) = 636.
time = 0.37, size = 639, normalized size = 2.04

$$-\frac{(Bbcd - Bad^2)nx + (Bbc^2 - Bacd)n - \left((Bb^2dnx^2 + 2\,Babdnx + Ba^2dn)e^{\left(\frac{A+B}{Bn}\right)}\log\left(\frac{bx+a}{dx+c}\right) + ((A+B)b^2dx^2 + 2\,(A+B)abdx + (A+B)a^2d)e^{\left(\frac{A+B}{Bn}\right)}\right)\log\_integral\left(\frac{(dx+c)e^{\left(-\frac{A+B}{Bn}\right)}}{bx+a}\right) + 2\left((Bb^3nx^2 + 2\,Bab^2nx + Ba^2bn)e^{\left(\frac{2\,(A+B)}{Bn}\right)}\log\left(\frac{bx+a}{dx+c}\right) + ((A+B)b^3x^2 + 2\,(A+B)ab^2x + (A+B)a^2b)e^{\left(\frac{2\,(A+B)}{Bn}\right)}\right)\log\_integral\left(\frac{(d^2x^2+2\,cdx+c^2)e^{\left(-\frac{2\,(A+B)}{Bn}\right)}}{b^2x^2+2\,abx+a^2}\right)}{((AB^2+B^3)b^4c^2 - 2\,(AB^2+B^3)ab^3cd + (AB^2+B^3)a^2b^2d^2)g^3n^2x^2 + 2\,((AB^2+B^3)ab^3c^2 - 2\,(AB^2+B^3)a^2b^2cd + (AB^2+B^3)a^3bd^2)g^3n^2x + ((AB^2+B^3)a^2b^2c^2 - 2\,(AB^2+B^3)a^3bcd + (AB^2+B^3)a^4d^2)g^3n^2 + ((B^3b^4c^2 - 2\,B^3ab^3cd + B^3a^2b^2d^2)g^3n^3x^2 + 2\,(B^3ab^3c^2 - 2\,B^3a^2b^2cd + B^3a^3bd^2)g^3n^3x + (B^3a^2b^2c^2 - 2\,B^3a^3bcd + B^3a^4d^2)g^3n^3)\log\left(\frac{bx+a}{dx+c}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)^3/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="
fricas")
```

```
[Out] -((B*b*c*d - B*a*d^2)*n*x + (B*b*c^2 - B*a*c*d)*n - ((B*b^2*d*n*x^2 + 2*B*a
*b*d*n*x + B*a^2*d*n)*e^((A + B)/(B*n))*log((b*x + a)/(d*x + c)) + ((A + B)
*b^2*d*x^2 + 2*(A + B)*a*b*d*x + (A + B)*a^2*d)*e^((A + B)/(B*n)))*log_inte
gral((d*x + c)*e^(-(A + B)/(B*n))/(b*x + a)) + 2*((B*b^3*n*x^2 + 2*B*a*b^2*
n*x + B*a^2*b*n)*e^(2*(A + B)/(B*n))*log((b*x + a)/(d*x + c)) + ((A + B)*b^
3*x^2 + 2*(A + B)*a*b^2*x + (A + B)*a^2*b)*e^(2*(A + B)/(B*n)))*log_integra
l((d^2*x^2 + 2*c*d*x + c^2)*e^(-2*(A + B)/(B*n))/(b^2*x^2 + 2*a*b*x + a^2))
)/(((A*B^2 + B^3)*b^4*c^2 - 2*(A*B^2 + B^3)*a*b^3*c*d + (A*B^2 + B^3)*a^2*b
^2*d^2)*g^3*n^2*x^2 + 2*((A*B^2 + B^3)*a*b^3*c^2 - 2*(A*B^2 + B^3)*a^2*b^2*
c*d + (A*B^2 + B^3)*a^3*b*d^2)*g^3*n^2*x + ((A*B^2 + B^3)*a^2*b^2*c^2 - 2*(
A*B^2 + B^3)*a^3*b*c*d + (A*B^2 + B^3)*a^4*d^2)*g^3*n^2 + ((B^3*b^4*c^2 - 2
*B^3*a*b^3*c*d + B^3*a^2*b^2*d^2)*g^3*n^3*x^2 + 2*(B^3*a*b^3*c^2 - 2*B^3*a^
2*b^2*c*d + B^3*a^3*b*d^2)*g^3*n^3*x + (B^3*a^2*b^2*c^2 - 2*B^3*a^3*b*c*d +
 B^3*a^4*d^2)*g^3*n^3)*log((b*x + a)/(d*x + c)))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)**3/(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)^3/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="
giac")
```

```
[Out] integrate(1/((b*g*x + a*g)^3*(B*log(((b*x + a)/(d*x + c))^n*e) + A)^2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(a\,g+b\,g\,x\right)}^3\,{\left(A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*g + b*g*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2),x)
```

```
[Out] int(1/((a*g + b*g*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2), x)
```

3.29 $\int (cg+dgx)^4 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) dx$

Optimal. Leaf size=188

$$-\frac{B(bc-ad)^4g^4nx}{5b^4}-\frac{B(bc-ad)^3g^4n(c+dx)^2}{10b^3d}-\frac{B(bc-ad)^2g^4n(c+dx)^3}{15b^2d}-\frac{B(bc-ad)g^4n(c+dx)^4}{20bd}-\frac{B(bc-$$

```
[Out] -1/5*B*(-a*d+b*c)^4*g^4*n*x/b^4-1/10*B*(-a*d+b*c)^3*g^4*n*(d*x+c)^2/b^3/d-1
/15*B*(-a*d+b*c)^2*g^4*n*(d*x+c)^3/b^2/d-1/20*B*(-a*d+b*c)*g^4*n*(d*x+c)^4/
b/d-1/5*B*(-a*d+b*c)^5*g^4*n*ln(b*x+a)/b^5/d+1/5*g^4*(d*x+c)^5*(A+B*ln(e*((
b*x+a)/(d*x+c))^n))/d
```

Rubi [A]
time = 0.08, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {2547, 21, 45}

$$\frac{g^4(c+dx)^5\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{5d}-\frac{Bg^4n(bc-ad)^5\log(a+bx)}{5b^5d}-\frac{Bg^4nx(bc-ad)^4}{5b^4}-\frac{Bg^4n(c+dx)^2(bc-ad)^3}{10b^3d}-\frac{Bg^4n(c+dx)^3(bc-ad)^2}{15b^2d}-\frac{Bg^4n(c+dx)^4(bc-ad)}{20bd}$$

Antiderivative was successfully verified.

```
[In] Int[(c*g + d*g*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

```
[Out] -1/5*(B*(b*c - a*d)^4*g^4*n*x)/b^4 - (B*(b*c - a*d)^3*g^4*n*(c + d*x)^2)/(1
0*b^3*d) - (B*(b*c - a*d)^2*g^4*n*(c + d*x)^3)/(15*b^2*d) - (B*(b*c - a*d)*
g^4*n*(c + d*x)^4)/(20*b*d) - (B*(b*c - a*d)^5*g^4*n*Log[a + b*x])/(5*b^5*d
) + (g^4*(c + d*x)^5*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(5*d)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2547

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*((A +
 B*Log[e*((a + b*x)/(c + d*x))^n])/(g*(m + 1))), x] - Dist[B*n*((b*c - a*d)
```

```
/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /; FreeQ
[{a, b, c, d, e, f, g, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, -2]
```

Rubi steps

$$\int (cg+dgx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)dx = \frac{g^4(c+dx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{5d} - \frac{(Bn)\int \frac{(bc-ad)g^5(c}{a+bx}}{5dg}$$

$$= \frac{g^4(c+dx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{5d} - \frac{(B(bc-ad)g^4n)}{5d}$$

$$= \frac{g^4(c+dx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{5d} - \frac{(B(bc-ad)g^4n)}{}$$

$$= -\frac{B(bc-ad)^4g^4nx}{5b^4} - \frac{B(bc-ad)^3g^4n(c+dx)^2}{10b^3d} - \frac{B(bc}{}$$

Mathematica [A]
time = 0.07, size = 146, normalized size = 0.78

$$\frac{g^4\left(-\frac{B(bc-ad)n\left(12bd(bc-ad)^3x+6b^2(bc-ad)^2(c+dx)^2+4b^3(bc-ad)(c+dx)^3+3b^4(c+dx)^4+12(bc-ad)^4\log(a+bx)\right)}{12b^5}+(c+dx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\right)}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*g + d*g*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

```
[Out] (g^4*(-1/12*(B*(b*c - a*d)*n*(12*b*d*(b*c - a*d)^3*x + 6*b^2*(b*c - a*d)^2*
(c + d*x)^2 + 4*b^3*(b*c - a*d)*(c + d*x)^3 + 3*b^4*(c + d*x)^4 + 12*(b*c -
 a*d)^4*Log[a + b*x]))/b^5 + (c + d*x)^5*(A + B*Log[e*((a + b*x)/(c + d*x))
^n])))/(5*d)
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int (dgx+cg)^4\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*g*x+c*g)^4*(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

```
[Out] int((d*g*x+c*g)^4*(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 681 vs. $2(177) = 354$.
time = 0.31, size = 681, normalized size = 3.62

$$\frac{1}{5}Bd^4g^4x^5\log\left(\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n e\right)+\frac{1}{5}Ad^4g^4x^5+Bcd^3g^4x^4\log\left(\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n e\right)+Acd^3g^4x^4+2Bc^2d^2g^4x^3\log\left(\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n e\right)+2Ac^2d^2g^4x^3+2Bc^3dg^4x^2\log\left(\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n e\right)+2Ac^3dg^4x^2+\frac{1}{60}Bd^4g^4n\left(\frac{12a^5\log(bx+a)}{b^5}-\frac{12c^5\log(dx+c)}{d^5}-\frac{3(b^4cd^3-ab^3d^4)x^4-4(b^4c^2d^2-a^2b^2d^4)x^3+6(b^4c^3d-a^3bd^4)x^2-12(b^4c^4-a^4d^4)x}{b^4d^4}\right)-\frac{1}{6}Bcd^3g^4n\left(\frac{6a^4\log(bx+a)}{b^4}-\frac{6c^4\log(dx+c)}{d^4}+\frac{2(b^3cd^2-ab^2d^3)x^3-3(b^3c^2d-a^2bd^3)x^2+6(b^3c^3-a^3d^3)x}{b^3d^3}\right)+Bc^2d^2g^4n\left(\frac{2a^3\log(bx+a)}{b^3}-\frac{2c^3\log(dx+c)}{d^3}-\frac{(b^2cd-abd^2)x^2-2(b^2c^2-a^2d^2)x}{b^2d^2}\right)-2Bc^3dg^4n\left(\frac{a^2\log(bx+a)}{b^2}-\frac{c^2\log(dx+c)}{d^2}+\frac{(bc-ad)x}{bd}\right)+Bc^4g^4n\left(\frac{a\log(bx+a)}{b}-\frac{c\log(dx+c)}{d}\right)+Bc^4g^4x\log\left(\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n e\right)+Ac^4g^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*g*x+c*g)^4*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="maxi
ma")
```

```
[Out] 1/5*B*d^4*g^4*x^5*log((b*x/(d*x + c) + a/(d*x + c))^n*e) + 1/5*A*d^4*g^4*x^
5 + B*c*d^3*g^4*x^4*log((b*x/(d*x + c) + a/(d*x + c))^n*e) + A*c*d^3*g^4*x^
4 + 2*B*c^2*d^2*g^4*x^3*log((b*x/(d*x + c) + a/(d*x + c))^n*e) + 2*A*c^2*d^
2*g^4*x^3 + 2*B*c^3*d*g^4*x^2*log((b*x/(d*x + c) + a/(d*x + c))^n*e) + 2*A*
c^3*d*g^4*x^2 + 1/60*B*d^4*g^4*n*(12*a^5*log(b*x + a)/b^5 - 12*c^5*log(d*x
+ c)/d^5 - (3*(b^4*c*d^3 - a*b^3*d^4)*x^4 - 4*(b^4*c^2*d^2 - a^2*b^2*d^4)*x
^3 + 6*(b^4*c^3*d - a^3*b*d^4)*x^2 - 12*(b^4*c^4 - a^4*d^4)*x)/(b^4*d^4)) -
 1/6*B*c*d^3*g^4*n*(6*a^4*log(b*x + a)/b^4 - 6*c^4*log(d*x + c)/d^4 + (2*(b
^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^
3*d^3)*x)/(b^3*d^3)) + B*c^2*d^2*g^4*n*(2*a^3*log(b*x + a)/b^3 - 2*c^3*log(
d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2
)) - 2*B*c^3*d*g^4*n*(a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c -
a*d)*x/(b*d)) + B*c^4*g^4*n*(a*log(b*x + a)/b - c*log(d*x + c)/d) + B*c^4*g
^4*x*log((b*x/(d*x + c) + a/(d*x + c))^n*e) + A*c^4*g^4*x
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 497 vs. $2(177) = 354$.
time = 0.43, size = 497, normalized size = 2.64

$$\frac{12(A+B)b^5d^5g^4x^5-12Bb^5c^5g^4n\log(dx+c)+12(5Bab^4c^4d-10Ba^2b^3c^3d^2+10Ba^3b^2c^2d^3-5Ba^4bcd^4+Ba^5d^5)g^4n\log(bx+a)+3(20(A+B)b^5cd^4g^4-(Bb^5cd^4-Bab^4d^5)g^4n)x^4+4(30(A+B)b^5c^2d^3g^4-(4Bb^5c^2d^3-5Bab^4cd^4+Ba^2b^3d^5)g^4n)x^3+6(20(A+B)b^5c^3d^2g^4-(6Bb^5c^3d^2-10Bab^4c^2d^3+5Ba^2b^3cd^4-Ba^3b^2d^5)g^4n)x^2+12(5(A+B)b^5c^4dg^4-(4Bb^5c^4d-10Bab^4c^3d^2+10Ba^2b^3c^2d^3-5Ba^3b^2cd^4+Ba^4bd^5)g^4n)x+12(Bb^5d^5g^4nx^5+5Bb^5cd^4g^4nx^4+10Bb^5c^2d^3g^4nx^3+10Bb^5c^3d^2g^4nx^2+5Bb^5c^4dg^4nx)\log\left(\frac{bx+a}{dx+c}\right)}{60b^5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*g*x+c*g)^4*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fric
as")
```

```
[Out] 1/60*(12*(A + B)*b^5*d^5*g^4*x^5 - 12*B*b^5*c^5*g^4*n*log(d*x + c) + 12*(5*
B*a*b^4*c^4*d - 10*B*a^2*b^3*c^3*d^2 + 10*B*a^3*b^2*c^2*d^3 - 5*B*a^4*b*c*d
^4 + B*a^5*d^5)*g^4*n*log(b*x + a) + 3*(20*(A + B)*b^5*c*d^4*g^4 - (B*b^5*c
*d^4 - B*a*b^4*d^5)*g^4*n)*x^4 + 4*(30*(A + B)*b^5*c^2*d^3*g^4 - (4*B*b^5*c
^2*d^3 - 5*B*a*b^4*c*d^4 + B*a^2*b^3*d^5)*g^4*n)*x^3 + 6*(20*(A + B)*b^5*c^
3*d^2*g^4 - (6*B*b^5*c^3*d^2 - 10*B*a*b^4*c^2*d^3 + 5*B*a^2*b^3*c*d^4 - B*a
^3*b^2*d^5)*g^4*n)*x^2 + 12*(5*(A + B)*b^5*c^4*d*g^4 - (4*B*b^5*c^4*d - 10*
B*a*b^4*c^3*d^2 + 10*B*a^2*b^3*c^2*d^3 - 5*B*a^3*b^2*c*d^4 + B*a^4*b*d^5)*g
^4*n)*x + 12*(B*b^5*d^5*g^4*n*x^5 + 5*B*b^5*c*d^4*g^4*n*x^4 + 10*B*b^5*c^2*
d^3*g^4*n*x^3 + 10*B*b^5*c^3*d^2*g^4*n*x^2 + 5*B*b^5*c^4*d*g^4*n*x)*log((b*
x + a)/(d*x + c)))/(b^5*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*g*x+c*g)**4*(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1862 vs. 2(177) = 354.
time = 4.87, size = 1862, normalized size = 9.90

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*g*x+c*g)^4*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="giac
")
```

```
[Out] 1/60*(12*(B*b^6*c^6*g^4*n - 6*B*a*b^5*c^5*d*g^4*n + 15*B*a^2*b^4*c^4*d^2*g^
4*n - 20*B*a^3*b^3*c^3*d^3*g^4*n + 15*B*a^4*b^2*c^2*d^4*g^4*n - 6*B*a^5*b*c
*d^5*g^4*n + B*a^6*d^6*g^4*n)*log((b*x + a)/(d*x + c))/(b^5*d - 5*(b*x + a)
*b^4*d^2/(d*x + c) + 10*(b*x + a)^2*b^3*d^3/(d*x + c)^2 - 10*(b*x + a)^3*b^
2*d^4/(d*x + c)^3 + 5*(b*x + a)^4*b*d^5/(d*x + c)^4 - (b*x + a)^5*d^6/(d*x
+ c)^5) - (25*B*b^10*c^6*g^4*n - 150*B*a*b^9*c^5*d*g^4*n - 77*(b*x + a)*B*b
^9*c^6*d*g^4*n/(d*x + c) + 375*B*a^2*b^8*c^4*d^2*g^4*n + 462*(b*x + a)*B*a*
b^8*c^5*d^2*g^4*n/(d*x + c) + 94*(b*x + a)^2*B*b^8*c^6*d^2*g^4*n/(d*x + c)^
2 - 500*B*a^3*b^7*c^3*d^3*g^4*n - 1155*(b*x + a)*B*a^2*b^7*c^4*d^3*g^4*n/(d
*x + c) - 564*(b*x + a)^2*B*a*b^7*c^5*d^3*g^4*n/(d*x + c)^2 - 54*(b*x + a)^
3*B*b^7*c^6*d^3*g^4*n/(d*x + c)^3 + 375*B*a^4*b^6*c^2*d^4*g^4*n + 1540*(b*x
 + a)*B*a^3*b^6*c^3*d^4*g^4*n/(d*x + c) + 1410*(b*x + a)^2*B*a^2*b^6*c^4*d^
4*g^4*n/(d*x + c)^2 + 324*(b*x + a)^3*B*a*b^6*c^5*d^4*g^4*n/(d*x + c)^3 + 1
2*(b*x + a)^4*B*b^6*c^6*d^4*g^4*n/(d*x + c)^4 - 150*B*a^5*b^5*c*d^5*g^4*n -
 1155*(b*x + a)*B*a^4*b^5*c^2*d^5*g^4*n/(d*x + c) - 1880*(b*x + a)^2*B*a^3*
b^5*c^3*d^5*g^4*n/(d*x + c)^2 - 810*(b*x + a)^3*B*a^2*b^5*c^4*d^5*g^4*n/(d*
x + c)^3 - 72*(b*x + a)^4*B*a*b^5*c^5*d^5*g^4*n/(d*x + c)^4 + 25*B*a^6*b^4*
d^6*g^4*n + 462*(b*x + a)*B*a^5*b^4*c*d^6*g^4*n/(d*x + c) + 1410*(b*x + a)^
2*B*a^4*b^4*c^2*d^6*g^4*n/(d*x + c)^2 + 1080*(b*x + a)^3*B*a^3*b^4*c^3*d^6*
g^4*n/(d*x + c)^3 + 180*(b*x + a)^4*B*a^2*b^4*c^4*d^6*g^4*n/(d*x + c)^4 - 7
7*(b*x + a)*B*a^6*b^3*d^7*g^4*n/(d*x + c) - 564*(b*x + a)^2*B*a^5*b^3*c*d^7
*g^4*n/(d*x + c)^2 - 810*(b*x + a)^3*B*a^4*b^3*c^2*d^7*g^4*n/(d*x + c)^3 -
240*(b*x + a)^4*B*a^3*b^3*c^3*d^7*g^4*n/(d*x + c)^4 + 94*(b*x + a)^2*B*a^6*
b^2*d^8*g^4*n/(d*x + c)^2 + 324*(b*x + a)^3*B*a^5*b^2*c*d^8*g^4*n/(d*x + c)
^3 + 180*(b*x + a)^4*B*a^4*b^2*c^2*d^8*g^4*n/(d*x + c)^4 - 54*(b*x + a)^3*B
```

```
*a^6*b*d^9*g^4*n/(d*x + c)^3 - 72*(b*x + a)^4*B*a^5*b*c*d^9*g^4*n/(d*x + c)
^4 + 12*(b*x + a)^4*B*a^6*d^10*g^4*n/(d*x + c)^4 - 12*A*b^10*c^6*g^4 - 12*B
*b^10*c^6*g^4 + 72*A*a*b^9*c^5*d*g^4 + 72*B*a*b^9*c^5*d*g^4 - 180*A*a^2*b^8
*c^4*d^2*g^4 - 180*B*a^2*b^8*c^4*d^2*g^4 + 240*A*a^3*b^7*c^3*d^3*g^4 + 240*
B*a^3*b^7*c^3*d^3*g^4 - 180*A*a^4*b^6*c^2*d^4*g^4 - 180*B*a^4*b^6*c^2*d^4*g
^4 + 72*A*a^5*b^5*c*d^5*g^4 + 72*B*a^5*b^5*c*d^5*g^4 - 12*A*a^6*b^4*d^6*g^4
 - 12*B*a^6*b^4*d^6*g^4)/(b^9*d - 5*(b*x + a)*b^8*d^2/(d*x + c) + 10*(b*x +
 a)^2*b^7*d^3/(d*x + c)^2 - 10*(b*x + a)^3*b^6*d^4/(d*x + c)^3 + 5*(b*x + a
)^4*b^5*d^5/(d*x + c)^4 - (b*x + a)^5*b^4*d^6/(d*x + c)^5) + 12*(B*b^6*c^6*
g^4*n - 6*B*a*b^5*c^5*d*g^4*n + 15*B*a^2*b^4*c^4*d^2*g^4*n - 20*B*a^3*b^3*c
^3*d^3*g^4*n + 15*B*a^4*b^2*c^2*d^4*g^4*n - 6*B*a^5*b*c*d^5*g^4*n + B*a^6*d
^6*g^4*n)*log(b - (b*x + a)*d/(d*x + c))/(b^5*d) - 12*(B*b^6*c^6*g^4*n - 6*
B*a*b^5*c^5*d*g^4*n + 15*B*a^2*b^4*c^4*d^2*g^4*n - 20*B*a^3*b^3*c^3*d^3*g^4
*n + 15*B*a^4*b^2*c^2*d^4*g^4*n - 6*B*a^5*b*c*d^5*g^4*n + B*a^6*d^6*g^4*n)*
log((b*x + a)/(d*x + c))/(b^5*d))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)
```

Mupad [B]
time = 4.48, size = 1045, normalized size = 5.56

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*g + d*g*x)^4*(A + B*log(e*((a + b*x)/(c + d*x))^n)),x)
```

```
[Out] x^2*(((5*a*d + 5*b*c)*((((d^3*g^4*(5*A*a*d + 25*A*b*c + B*a*d*n - B*b*c*n))
/(5*b) - (A*d^3*g^4*(5*a*d + 5*b*c))/(5*b))*(5*a*d + 5*b*c))/(5*b*d) - (c*d
^2*g^4*(5*A*a*d + 10*A*b*c + B*a*d*n - B*b*c*n))/b + (A*a*c*d^3*g^4)/b))/(1
0*b*d) - (a*c*((d^3*g^4*(5*A*a*d + 25*A*b*c + B*a*d*n - B*b*c*n))/(5*b) - (
A*d^3*g^4*(5*a*d + 5*b*c))/(5*b)))/(2*b*d) + (c^2*d*g^4*(5*A*a*d + 5*A*b*c
+ B*a*d*n - B*b*c*n))/b) - x^3*((((d^3*g^4*(5*A*a*d + 25*A*b*c + B*a*d*n -
B*b*c*n))/(5*b) - (A*d^3*g^4*(5*a*d + 5*b*c))/(5*b))*(5*a*d + 5*b*c))/(15*b
*d) - (c*d^2*g^4*(5*A*a*d + 10*A*b*c + B*a*d*n - B*b*c*n))/(3*b) + (A*a*c*d
^3*g^4)/(3*b)) + x^4*((d^3*g^4*(5*A*a*d + 25*A*b*c + B*a*d*n - B*b*c*n))/(2
0*b) - (A*d^3*g^4*(5*a*d + 5*b*c))/(20*b)) + log(e*((a + b*x)/(c + d*x))^n)
*((B*d^4*g^4*x^5)/5 + B*c^4*g^4*x + 2*B*c^3*d*g^4*x^2 + B*c*d^3*g^4*x^4 + 2
*B*c^2*d^2*g^4*x^3) + x*((c^3*g^4*(10*A*a*d + 5*A*b*c + 2*B*a*d*n - 2*B*b*c
*n))/b - ((5*a*d + 5*b*c)*(((5*a*d + 5*b*c)*((((d^3*g^4*(5*A*a*d + 25*A*b*c
 + B*a*d*n - B*b*c*n))/(5*b) - (A*d^3*g^4*(5*a*d + 5*b*c))/(5*b))*(5*a*d +
5*b*c))/(5*b*d) - (c*d^2*g^4*(5*A*a*d + 10*A*b*c + B*a*d*n - B*b*c*n))/b +
(A*a*c*d^3*g^4)/b))/(5*b*d) - (a*c*((d^3*g^4*(5*A*a*d + 25*A*b*c + B*a*d*n
- B*b*c*n))/(5*b) - (A*d^3*g^4*(5*a*d + 5*b*c))/(5*b)))/(b*d) + (2*c^2*d*g^
4*(5*A*a*d + 5*A*b*c + B*a*d*n - B*b*c*n))/b))/(5*b*d) + (a*c*((((d^3*g^4*(
5*A*a*d + 25*A*b*c + B*a*d*n - B*b*c*n))/(5*b) - (A*d^3*g^4*(5*a*d + 5*b*c)
)/(5*b))*(5*a*d + 5*b*c))/(5*b*d) - (c*d^2*g^4*(5*A*a*d + 10*A*b*c + B*a*d*
n - B*b*c*n))/b + (A*a*c*d^3*g^4)/b))/(b*d)) + (log(a + b*x)*((B*a^5*d^4*g^
```

```
4*n)/5 + B*a*b^4*c^4*g^4*n - B*a^4*b*c*d^3*g^4*n - 2*B*a^2*b^3*c^3*d*g^4*n
+ 2*B*a^3*b^2*c^2*d^2*g^4*n))/b^5 + (A*d^4*g^4*x^5)/5 - (B*c^5*g^4*n*log(c
+ d*x))/(5*d)
```

3.30 $\int (cg+dgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) dx$

Optimal. Leaf size=156

$$-\frac{B(bc-ad)^3g^3nx}{4b^3}-\frac{B(bc-ad)^2g^3n(c+dx)^2}{8b^2d}-\frac{B(bc-ad)g^3n(c+dx)^3}{12bd}-\frac{B(bc-ad)^4g^3n\log(a+bx)}{4b^4d}+\frac{g^3(c$$

```
[Out] -1/4*B*(-a*d+b*c)^3*g^3*n*x/b^3-1/8*B*(-a*d+b*c)^2*g^3*n*(d*x+c)^2/b^2/d-1/
12*B*(-a*d+b*c)*g^3*n*(d*x+c)^3/b/d-1/4*B*(-a*d+b*c)^4*g^3*n*ln(b*x+a)/b^4/
d+1/4*g^3*(d*x+c)^4*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/d
```

Rubi [A]
time = 0.07, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {2547, 21, 45}

$$\frac{g^3(c+dx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{4d}-\frac{Bg^3n(bc-ad)^4\log(a+bx)}{4b^4d}-\frac{Bg^3nx(bc-ad)^3}{4b^3}-\frac{Bg^3n(c+dx)^2(bc-ad)^2}{8b^2d}-\frac{Bg^3n(c+dx)^3(bc-ad)}{12bd}$$

Antiderivative was successfully verified.

```
[In] Int[(c*g + d*g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

```
[Out] -1/4*(B*(b*c - a*d)^3*g^3*n*x)/b^3 - (B*(b*c - a*d)^2*g^3*n*(c + d*x)^2)/(8
*b^2*d) - (B*(b*c - a*d)*g^3*n*(c + d*x)^3)/(12*b*d) - (B*(b*c - a*d)^4*g^3
*n*Log[a + b*x])/(4*b^4*d) + (g^3*(c + d*x)^4*(A + B*Log[e*((a + b*x)/(c +
d*x))^n]))/(4*d)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2547

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*((A +
 B*Log[e*((a + b*x)/(c + d*x))^n])/(g*(m + 1))), x] - Dist[B*n*((b*c - a*d)
/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /; FreeQ
```

```
[{a, b, c, d, e, f, g, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, -2]
```

Rubi steps

$$\int (cg+dgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) dx = \frac{g^3(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4d} - \frac{(Bn)\int \frac{(bc-ad)g^4(c}{a+bx}}{4dg}$$

$$= \frac{g^3(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4d} - \frac{(B(bc-ad)g^3n)}{4d}$$

$$= \frac{g^3(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4d} - \frac{(B(bc-ad)g^3n)}{}$$

$$= -\frac{B(bc-ad)^3g^3nx}{4b^3} - \frac{B(bc-ad)^2g^3n(c+dx)^2}{8b^2d} - \frac{B(bc}{}$$

Mathematica [A]
time = 0.07, size = 124, normalized size = 0.79

$$\frac{g^3\left(-\frac{B(bc-ad)n\left(6bd(bc-ad)^2x+3b^2(bc-ad)(c+dx)^2+2b^3(c+dx)^3+6(bc-ad)^3\log(a+bx)\right)}{6b^4}+(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*g + d*g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
[Out] (g^3*(-1/6*(B*(b*c - a*d)*n*(6*b*d*(b*c - a*d)^2*x + 3*b^2*(b*c - a*d)*(c +
 d*x)^2 + 2*b^3*(c + d*x)^3 + 6*(b*c - a*d)^3*Log[a + b*x]))/b^4 + (c + d*x
)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n])))/(4*d)
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int (dgx+cg)^3\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*g*x+c*g)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
[Out] int((d*g*x+c*g)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 483 vs. 2(147) = 294.
time = 0.36, size = 483, normalized size = 3.10

$$\frac{1}{4}Bd^3g^3x^4\log\left(\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n e\right)+\frac{1}{4}Ad^3g^3x^4+Bcd^2g^3x^3\log\left(\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n e\right)+Acd^2g^3x^3+\frac{3}{2}Bc^2dg^3x^2\log\left(\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n e\right)+\frac{3}{2}Ac^2dg^3x^2-\frac{1}{24}Bd^3g^3n\left(\frac{6a^4\log(bx+a)}{b^4}-\frac{6c^4\log(dx+c)}{d^4}+\frac{2(b^3cd^2-ab^2d^3)x^3-3(b^3c^2d-a^2bd^3)x^2+6(b^3c^3-a^3d^3)x}{b^3d^3}\right)+\frac{1}{2}Bcd^2g^3n\left(\frac{2a^3\log(bx+a)}{b^3}-\frac{2c^3\log(dx+c)}{d^3}-\frac{(b^2cd-abd^2)x^2-2(b^2c^2-a^2d^2)x}{b^2d^2}\right)-\frac{3}{2}Bc^2dg^3n\left(\frac{a^2\log(bx+a)}{b^2}-\frac{c^2\log(dx+c)}{d^2}+\frac{(bc-ad)x}{bd}\right)+Bc^3g^3n\left(\frac{a\log(bx+a)}{b}-\frac{c\log(dx+c)}{d}\right)+Bc^3g^3x\log\left(\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n e\right)+Ac^3g^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*g*x+c*g)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="maxi
ma")
```

```
[Out] 1/4*B*d^3*g^3*x^4*log((b*x/(d*x + c) + a/(d*x + c))^n*e) + 1/4*A*d^3*g^3*x^
4 + B*c*d^2*g^3*x^3*log((b*x/(d*x + c) + a/(d*x + c))^n*e) + A*c*d^2*g^3*x^
3 + 3/2*B*c^2*d*g^3*x^2*log((b*x/(d*x + c) + a/(d*x + c))^n*e) + 3/2*A*c^2*
d*g^3*x^2 - 1/24*B*d^3*g^3*n*(6*a^4*log(b*x + a)/b^4 - 6*c^4*log(d*x + c)/d
^4 + (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^
3*c^3 - a^3*d^3)*x)/(b^3*d^3)) + 1/2*B*c*d^2*g^3*n*(2*a^3*log(b*x + a)/b^3
- 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)
*x)/(b^2*d^2)) - 3/2*B*c^2*d*g^3*n*(a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)
/d^2 + (b*c - a*d)*x/(b*d)) + B*c^3*g^3*n*(a*log(b*x + a)/b - c*log(d*x + c
)/d) + B*c^3*g^3*x*log((b*x/(d*x + c) + a/(d*x + c))^n*e) + A*c^3*g^3*x
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 370 vs. $2(147) = 294$.
time = 0.39, size = 370, normalized size = 2.37

$$\frac{6\,(A+B)b^4d^4g^3x^4 - 6\,Bb^4c^4g^3n\log\left(dx+c\right) + 6\,(4\,Bab^3c^3d - 6\,Ba^2b^2c^2d^2 + 4\,Ba^3bcd^3 - Ba^4d^4)g^3n\log\left(bx+a\right) + 2\,(12\,(A+B)b^4cd^3g^3 - (Bb^4cd^3 - Bab^3d^4)g^3n)x^3 + 3\,(12\,(A+B)b^4c^2d^2g^3 - (3\,Bb^4c^2d^2 - 4\,Bab^3cd^3 + Ba^2b^2d^4)g^3n)x^2 + 6\,(4\,(A+B)b^4c^3dg^3 - (3\,Bb^4c^3d - 6\,Bab^3c^2d^2 + 4\,Ba^2b^2cd^3 - Ba^3bd^4)g^3n)x + 6\,(Bb^4d^4g^3nx^4 + 4\,Bb^4cd^3g^3nx^3 + 6\,Bb^4c^2d^2g^3nx^2 + 4\,Bb^4c^3dg^3nx)\log\left(\frac{bx+a}{dx+c}\right)}{24\,b^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*g*x+c*g)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fric
as")
```

```
[Out] 1/24*(6*(A + B)*b^4*d^4*g^3*x^4 - 6*B*b^4*c^4*g^3*n*log(d*x + c) + 6*(4*B*a
*b^3*c^3*d - 6*B*a^2*b^2*c^2*d^2 + 4*B*a^3*b*c*d^3 - B*a^4*d^4)*g^3*n*log(b
*x + a) + 2*(12*(A + B)*b^4*c*d^3*g^3 - (B*b^4*c*d^3 - B*a*b^3*d^4)*g^3*n)*
x^3 + 3*(12*(A + B)*b^4*c^2*d^2*g^3 - (3*B*b^4*c^2*d^2 - 4*B*a*b^3*c*d^3 +
B*a^2*b^2*d^4)*g^3*n)*x^2 + 6*(4*(A + B)*b^4*c^3*d*g^3 - (3*B*b^4*c^3*d - 6
*B*a*b^3*c^2*d^2 + 4*B*a^2*b^2*c*d^3 - B*a^3*b*d^4)*g^3*n)*x + 6*(B*b^4*d^4
*g^3*n*x^4 + 4*B*b^4*c*d^3*g^3*n*x^3 + 6*B*b^4*c^2*d^2*g^3*n*x^2 + 4*B*b^4*
c^3*d*g^3*n*x)*log((b*x + a)/(d*x + c)))/(b^4*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*g*x+c*g)**3*(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1390 vs. $2(147) = 294$.

time = 7.03, size = 1390, normalized size = 8.91

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*g*x+c*g)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="giac
")
```

```
[Out] 1/24*(6*(B*b^5*c^5*g^3*n - 5*B*a*b^4*c^4*d*g^3*n + 10*B*a^2*b^3*c^3*d^2*g^3
*n - 10*B*a^3*b^2*c^2*d^3*g^3*n + 5*B*a^4*b*c*d^4*g^3*n - B*a^5*d^5*g^3*n)*
log((b*x + a)/(d*x + c))/(b^4*d - 4*(b*x + a)*b^3*d^2/(d*x + c) + 6*(b*x +
a)^2*b^2*d^3/(d*x + c)^2 - 4*(b*x + a)^3*b*d^4/(d*x + c)^3 + (b*x + a)^4*d^
5/(d*x + c)^4) - (11*B*b^8*c^5*g^3*n - 55*B*a*b^7*c^4*d*g^3*n - 26*(b*x + a
)*B*b^7*c^5*d*g^3*n/(d*x + c) + 110*B*a^2*b^6*c^3*d^2*g^3*n + 130*(b*x + a)
*B*a*b^6*c^4*d^2*g^3*n/(d*x + c) + 21*(b*x + a)^2*B*b^6*c^5*d^2*g^3*n/(d*x
+ c)^2 - 110*B*a^3*b^5*c^2*d^3*g^3*n - 260*(b*x + a)*B*a^2*b^5*c^3*d^3*g^3*
n/(d*x + c) - 105*(b*x + a)^2*B*a*b^5*c^4*d^3*g^3*n/(d*x + c)^2 - 6*(b*x +
a)^3*B*b^5*c^5*d^3*g^3*n/(d*x + c)^3 + 55*B*a^4*b^4*c*d^4*g^3*n + 260*(b*x
+ a)*B*a^3*b^4*c^2*d^4*g^3*n/(d*x + c) + 210*(b*x + a)^2*B*a^2*b^4*c^3*d^4*
g^3*n/(d*x + c)^2 + 30*(b*x + a)^3*B*a*b^4*c^4*d^4*g^3*n/(d*x + c)^3 - 11*B
*a^5*b^3*d^5*g^3*n - 130*(b*x + a)*B*a^4*b^3*c*d^5*g^3*n/(d*x + c) - 210*(b
*x + a)^2*B*a^3*b^3*c^2*d^5*g^3*n/(d*x + c)^2 - 60*(b*x + a)^3*B*a^2*b^3*c^
3*d^5*g^3*n/(d*x + c)^3 + 26*(b*x + a)*B*a^5*b^2*d^6*g^3*n/(d*x + c) + 105*
(b*x + a)^2*B*a^4*b^2*c*d^6*g^3*n/(d*x + c)^2 + 60*(b*x + a)^3*B*a^3*b^2*c^
2*d^6*g^3*n/(d*x + c)^3 - 21*(b*x + a)^2*B*a^5*b*d^7*g^3*n/(d*x + c)^2 - 30
*(b*x + a)^3*B*a^4*b*c*d^7*g^3*n/(d*x + c)^3 + 6*(b*x + a)^3*B*a^5*d^8*g^3*
n/(d*x + c)^3 - 6*A*b^8*c^5*g^3 - 6*B*b^8*c^5*g^3 + 30*A*a*b^7*c^4*d*g^3 +
30*B*a*b^7*c^4*d*g^3 - 60*A*a^2*b^6*c^3*d^2*g^3 - 60*B*a^2*b^6*c^3*d^2*g^3
+ 60*A*a^3*b^5*c^2*d^3*g^3 + 60*B*a^3*b^5*c^2*d^3*g^3 - 30*A*a^4*b^4*c*d^4*
g^3 - 30*B*a^4*b^4*c*d^4*g^3 + 6*A*a^5*b^3*d^5*g^3 + 6*B*a^5*b^3*d^5*g^3)/(
b^7*d - 4*(b*x + a)*b^6*d^2/(d*x + c) + 6*(b*x + a)^2*b^5*d^3/(d*x + c)^2 -
 4*(b*x + a)^3*b^4*d^4/(d*x + c)^3 + (b*x + a)^4*b^3*d^5/(d*x + c)^4) + 6*(
B*b^5*c^5*g^3*n - 5*B*a*b^4*c^4*d*g^3*n + 10*B*a^2*b^3*c^3*d^2*g^3*n - 10*B
*a^3*b^2*c^2*d^3*g^3*n + 5*B*a^4*b*c*d^4*g^3*n - B*a^5*d^5*g^3*n)*log(-b +
(b*x + a)*d/(d*x + c))/(b^4*d) - 6*(B*b^5*c^5*g^3*n - 5*B*a*b^4*c^4*d*g^3*n
 + 10*B*a^2*b^3*c^3*d^2*g^3*n - 10*B*a^3*b^2*c^2*d^3*g^3*n + 5*B*a^4*b*c*d^
4*g^3*n - B*a^5*d^5*g^3*n)*log((b*x + a)/(d*x + c))/(b^4*d))*(b*c/(b*c - a*
d)^2 - a*d/(b*c - a*d)^2)
```

Mupad [B]
time = 4.37, size = 588, normalized size = 3.77

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*g + d*g*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n)),x)
```

```
[Out] x^3*((d^2*g^3*(4*A*a*d + 16*A*b*c + B*a*d*n - B*b*c*n))/(12*b) - (A*d^2*g^3
*(4*a*d + 4*b*c))/(12*b)) - x^2*((((d^2*g^3*(4*A*a*d + 16*A*b*c + B*a*d*n -
 B*b*c*n))/(4*b) - (A*d^2*g^3*(4*a*d + 4*b*c))/(4*b))*(4*a*d + 4*b*c))/(8*b
*d) - (c*d*g^3*(4*A*a*d + 6*A*b*c + B*a*d*n - B*b*c*n))/(2*b) + (A*a*c*d^2*
g^3)/(2*b)) + log(e*((a + b*x)/(c + d*x))^n)*((B*d^3*g^3*x^4)/4 + B*c^3*g^3
*x + (3*B*c^2*d*g^3*x^2)/2 + B*c*d^2*g^3*x^3) + x*(((4*a*d + 4*b*c)*((((d^2
*g^3*(4*A*a*d + 16*A*b*c + B*a*d*n - B*b*c*n))/(4*b) - (A*d^2*g^3*(4*a*d +
4*b*c))/(4*b))*(4*a*d + 4*b*c))/(4*b*d) - (c*d*g^3*(4*A*a*d + 6*A*b*c + B*a
*d*n - B*b*c*n))/b + (A*a*c*d^2*g^3)/b))/(4*b*d) + (c^2*g^3*(12*A*a*d + 8*A
*b*c + 3*B*a*d*n - 3*B*b*c*n))/(2*b) - (a*c*((d^2*g^3*(4*A*a*d + 16*A*b*c +
 B*a*d*n - B*b*c*n))/(4*b) - (A*d^2*g^3*(4*a*d + 4*b*c))/(4*b)))/(b*d)) - (
log(a + b*x)*(B*a^4*d^3*g^3*n - 4*B*a*b^3*c^3*g^3*n - 4*B*a^3*b*c*d^2*g^3*n
 + 6*B*a^2*b^2*c^2*d*g^3*n))/(4*b^4) + (A*d^3*g^3*x^4)/4 - (B*c^4*g^3*n*log
(c + d*x))/(4*d)
```

3.31 $\int(cg+dgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$

Optimal. Leaf size=124

$$-\frac{B(bc-ad)^2g^2nx}{3b^2}-\frac{B(bc-ad)g^2n(c+dx)^2}{6bd}-\frac{B(bc-ad)^3g^2n\log(a+bx)}{3b^3d}+\frac{g^2(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3d}$$

```
[Out] -1/3*B*(-a*d+b*c)^2*g^2*n*x/b^2-1/6*B*(-a*d+b*c)*g^2*n*(d*x+c)^2/b/d-1/3*B*
(-a*d+b*c)^3*g^2*n*ln(b*x+a)/b^3/d+1/3*g^2*(d*x+c)^3*(A+B*ln(e*((b*x+a)/(d*
x+c))^n))/d
```

Rubi [A]
time = 0.05, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {2547, 21, 45}

$$\frac{g^2(c+dx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3d}-\frac{Bg^2n(bc-ad)^3\log(a+bx)}{3b^3d}-\frac{Bg^2nx(bc-ad)^2}{3b^2}-\frac{Bg^2n(c+dx)^2(bc-ad)}{6bd}$$

Antiderivative was successfully verified.

```
[In] Int[(c*g + d*g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

```
[Out] -1/3*(B*(b*c - a*d)^2*g^2*n*x)/b^2 - (B*(b*c - a*d)*g^2*n*(c + d*x)^2)/(6*b
*d) - (B*(b*c - a*d)^3*g^2*n*Log[a + b*x])/(3*b^3*d) + (g^2*(c + d*x)^3*(A
+ B*Log[e*((a + b*x)/(c + d*x))^n]))/(3*d)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2547

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*((A +
 B*Log[e*((a + b*x)/(c + d*x))^n])/(g*(m + 1))), x] - Dist[B*n*((b*c - a*d)
/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /; FreeQ
[{a, b, c, d, e, f, g, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
```

```
& NeQ[m, -2]
```

Rubi steps

$$\begin{aligned}\int (cg+dgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)dx &= \frac{g^2(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3d}-\frac{(Bn)\int\frac{(bc-ad)g^3(c+}{a+bx}}{3dg}\\ &= \frac{g^2(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3d}-\frac{(B(bc-ad)g^2n)\int}{3d}\\ &= \frac{g^2(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3d}-\frac{(B(bc-ad)g^2n)\int}{}\\ &= -\frac{B(bc-ad)^2g^2nx}{3b^2}-\frac{B(bc-ad)g^2n(c+dx)^2}{6bd}-\frac{B(bc-}{}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 101, normalized size = 0.81

$$\frac{g^2\left(-\frac{B(bc-ad)n\left(2bd(bc-ad)x+b^2(c+dx)^2+2(bc-ad)^2\log(a+bx)\right)}{2b^3}+(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*g + d*g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

```
[Out] (g^2*(-1/2*(B*(b*c - a*d)*n*(2*b*d*(b*c - a*d)*x + b^2*(c + d*x)^2 + 2*(b*c
 - a*d)^2*Log[a + b*x]))/b^3 + (c + d*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x
))^n])))/(3*d)
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int (dgx+cg)^2\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*g*x+c*g)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

```
[Out] int((d*g*x+c*g)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 312 vs. 2(117) = 234.
time = 0.31, size = 312, normalized size = 2.52

$$\frac{1}{3}Bd^2g^2x^3\log\left(\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n e\right)+\frac{1}{3}Ad^2g^2x^3+Bcdg^2x^2\log\left(\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n e\right)+Acdg^2x^2+\frac{1}{6}Bd^2g^2n\left(\frac{2a^3\log(bx+a)}{b^3}-\frac{2c^3\log(dx+c)}{d^3}-\frac{(b^2cd-abd^2)x^2-2(b^2c^2-a^2d^2)x}{b^2d^2}\right)-Bcdg^2n\left(\frac{a^2\log(bx+a)}{b^2}-\frac{c^2\log(dx+c)}{d^2}+\frac{(bc-ad)x}{bd}\right)+Bc^2g^2n\left(\frac{a\log(bx+a)}{b}-\frac{c\log(dx+c)}{d}\right)+Bc^2g^2x\log\left(\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n e\right)+Ac^2g^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*g*x+c*g)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="maxi
ma")
```

```
[Out] 1/3*B*d^2*g^2*x^3*log((b*x/(d*x + c) + a/(d*x + c))^n*e) + 1/3*A*d^2*g^2*x^
3 + B*c*d*g^2*x^2*log((b*x/(d*x + c) + a/(d*x + c))^n*e) + A*c*d*g^2*x^2 +
1/6*B*d^2*g^2*n*(2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*
d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2)) - B*c*d*g^2*n*(a^2*l
og(b*x + a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d)) + B*c^2*g^2*n
*(a*log(b*x + a)/b - c*log(d*x + c)/d) + B*c^2*g^2*x*log((b*x/(d*x + c) + a
/(d*x + c))^n*e) + A*c^2*g^2*x
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 254 vs. $2(117) = 234$.
time = 0.37, size = 254, normalized size = 2.05

$$\frac{2(A+B)b^3d^3g^2x^3 - 2Bb^3c^3g^2n\log(dx+c) + 2(3Bab^2c^2d - 3Ba^2bcd^2 + Ba^3d^3)g^2n\log(bx+a) + (6(A+B)b^3cd^2g^2 - (Bb^3cd^2 - Bab^2d^3)g^2n)x^2 + 2(3(A+B)b^3c^2dg^2 - (2Bb^3c^2d - 3Bab^2cd^2 + Ba^2bd^3)g^2n)x + 2(Bb^3d^3g^2nx^3 + 3Bb^3cd^2g^2nx^2 + 3Bb^3c^2dg^2nx)\log\left(\frac{bx+a}{dx+c}\right)}{6b^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*g*x+c*g)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fric
as")
```

```
[Out] 1/6*(2*(A + B)*b^3*d^3*g^2*x^3 - 2*B*b^3*c^3*g^2*n*log(d*x + c) + 2*(3*B*a*
b^2*c^2*d - 3*B*a^2*b*c*d^2 + B*a^3*d^3)*g^2*n*log(b*x + a) + (6*(A + B)*b^
3*c*d^2*g^2 - (B*b^3*c*d^2 - B*a*b^2*d^3)*g^2*n)*x^2 + 2*(3*(A + B)*b^3*c^2
*d*g^2 - (2*B*b^3*c^2*d - 3*B*a*b^2*c*d^2 + B*a^2*b*d^3)*g^2*n)*x + 2*(B*b^
3*d^3*g^2*n*x^3 + 3*B*b^3*c*d^2*g^2*n*x^2 + 3*B*b^3*c^2*d*g^2*n*x)*log((b*x
 + a)/(d*x + c)))/(b^3*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*g*x+c*g)**2*(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 980 vs. $2(117) = 234$.
time = 4.03, size = 980, normalized size = 7.90

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*g*x+c*g)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="giac
")
```

```
[Out] 1/6*(2*(B*b^4*c^4*g^2*n - 4*B*a*b^3*c^3*d*g^2*n + 6*B*a^2*b^2*c^2*d^2*g^2*n
 - 4*B*a^3*b*c*d^3*g^2*n + B*a^4*d^4*g^2*n)*log((b*x + a)/(d*x + c))/(b^3*d
 - 3*(b*x + a)*b^2*d^2/(d*x + c) + 3*(b*x + a)^2*b*d^3/(d*x + c)^2 - (b*x +
 a)^3*d^4/(d*x + c)^3) - (3*B*b^6*c^4*g^2*n - 12*B*a*b^5*c^3*d*g^2*n - 5*(b
*x + a)*B*b^5*c^4*d*g^2*n/(d*x + c) + 18*B*a^2*b^4*c^2*d^2*g^2*n + 20*(b*x
+ a)*B*a*b^4*c^3*d^2*g^2*n/(d*x + c) + 2*(b*x + a)^2*B*b^4*c^4*d^2*g^2*n/(d
*x + c)^2 - 12*B*a^3*b^3*c*d^3*g^2*n - 30*(b*x + a)*B*a^2*b^3*c^2*d^3*g^2*n
/(d*x + c) - 8*(b*x + a)^2*B*a*b^3*c^3*d^3*g^2*n/(d*x + c)^2 + 3*B*a^4*b^2*
d^4*g^2*n + 20*(b*x + a)*B*a^3*b^2*c*d^4*g^2*n/(d*x + c) + 12*(b*x + a)^2*B
*a^2*b^2*c^2*d^4*g^2*n/(d*x + c)^2 - 5*(b*x + a)*B*a^4*b*d^5*g^2*n/(d*x + c
) - 8*(b*x + a)^2*B*a^3*b*c*d^5*g^2*n/(d*x + c)^2 + 2*(b*x + a)^2*B*a^4*d^6
*g^2*n/(d*x + c)^2 - 2*A*b^6*c^4*g^2 - 2*B*b^6*c^4*g^2 + 8*A*a*b^5*c^3*d*g^
2 + 8*B*a*b^5*c^3*d*g^2 - 12*A*a^2*b^4*c^2*d^2*g^2 - 12*B*a^2*b^4*c^2*d^2*g
^2 + 8*A*a^3*b^3*c*d^3*g^2 + 8*B*a^3*b^3*c*d^3*g^2 - 2*A*a^4*b^2*d^4*g^2 -
2*B*a^4*b^2*d^4*g^2)/(b^5*d - 3*(b*x + a)*b^4*d^2/(d*x + c) + 3*(b*x + a)^2
*b^3*d^3/(d*x + c)^2 - (b*x + a)^3*b^2*d^4/(d*x + c)^3) + 2*(B*b^4*c^4*g^2*
n - 4*B*a*b^3*c^3*d*g^2*n + 6*B*a^2*b^2*c^2*d^2*g^2*n - 4*B*a^3*b*c*d^3*g^2
*n + B*a^4*d^4*g^2*n)*log(b - (b*x + a)*d/(d*x + c))/(b^3*d) - 2*(B*b^4*c^4
*g^2*n - 4*B*a*b^3*c^3*d*g^2*n + 6*B*a^2*b^2*c^2*d^2*g^2*n - 4*B*a^3*b*c*d^
3*g^2*n + B*a^4*d^4*g^2*n)*log((b*x + a)/(d*x + c))/(b^3*d))*(b*c/(b*c - a*
d)^2 - a*d/(b*c - a*d)^2)
```

Mupad [B]
time = 4.31, size = 303, normalized size = 2.44

$$\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\,\left(B\,c^2\,g^2\,x+B\,c\,d\,g^2\,x^2+\frac{B\,d^2\,g^2\,x^3}{3}\right)-x\,\left(\frac{\left(3\,a\,d+3\,b\,c\right)\,\left(\frac{d\,g^2\,\left(3\,A\,a\,d+9\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n\right)}{3\,b}-\frac{A\,d\,g^2\,\left(3\,a\,d+3\,b\,c\right)}{3\,b}\right)}{3\,b\,d}-\frac{c\,g^2\,\left(3\,A\,a\,d+3\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n\right)}{b}+\frac{A\,a\,c\,d\,g^2}{b}\right)+x^2\,\left(\frac{d\,g^2\,\left(3\,A\,a\,d+9\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n\right)}{6\,b}-\frac{A\,d\,g^2\,\left(3\,a\,d+3\,b\,c\right)}{6\,b}\right)+\frac{\ln\left(a+b\,x\right)\,\left(B\,n\,a^3\,d^2\,g^2-3\,B\,n\,a^2\,b\,c\,d\,g^2+3\,B\,n\,a\,b^2\,c^2\,g^2\right)}{3\,b^3}+\frac{A\,d^2\,g^2\,x^3}{3}-\frac{B\,c^3\,g^2\,n\,\ln\left(c+d\,x\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*g + d*g*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n)),x)
```

```
[Out] log(e*((a + b*x)/(c + d*x))^n)*((B*d^2*g^2*x^3)/3 + B*c^2*g^2*x + B*c*d*g^2
*x^2) - x*(((3*a*d + 3*b*c)*((d*g^2*(3*A*a*d + 9*A*b*c + B*a*d*n - B*b*c*n)
)/(3*b) - (A*d*g^2*(3*a*d + 3*b*c))/(3*b)))/(3*b*d) - (c*g^2*(3*A*a*d + 3*A
*b*c + B*a*d*n - B*b*c*n))/b + (A*a*c*d*g^2)/b) + x^2*((d*g^2*(3*A*a*d + 9*
A*b*c + B*a*d*n - B*b*c*n))/(6*b) - (A*d*g^2*(3*a*d + 3*b*c))/(6*b)) + (log
(a + b*x)*(B*a^3*d^2*g^2*n + 3*B*a*b^2*c^2*g^2*n - 3*B*a^2*b*c*d*g^2*n))/(3
*b^3) + (A*d^2*g^2*x^3)/3 - (B*c^3*g^2*n*log(c + d*x))/(3*d)
```

3.32 $\int (cg+dgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$

Optimal. Leaf size=86

$$-\frac{B(bc-ad)gnx}{2b}-\frac{B(bc-ad)^2gn\log(a+bx)}{2b^2d}+\frac{g(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2d}$$

```
[Out] -1/2*B*(-a*d+b*c)*g*n*x/b-1/2*B*(-a*d+b*c)^2*g*n*ln(b*x+a)/b^2/d+1/2*g*(d*x
+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/d
```

Rubi [A]
time = 0.04, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {2547, 21, 45}

$$\frac{g(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2d}-\frac{Bgn(bc-ad)^2\log(a+bx)}{2b^2d}-\frac{Bgnx(bc-ad)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c*g + d*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

```
[Out] -1/2*(B*(b*c - a*d)*g*n*x)/b - (B*(b*c - a*d)^2*g*n*Log[a + b*x])/(2*b^2*d)
 + (g*(c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(2*d)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2547

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*((A +
 B*Log[e*((a + b*x)/(c + d*x))^n])/(g*(m + 1))), x] - Dist[B*n*((b*c - a*d)
/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /; FreeQ
[{a, b, c, d, e, f, g, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, -2]
```

Rubi steps

$$\int (cg+dgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx = \frac{g(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2d} - \frac{(Bn)\int \frac{(bc-ad)g^2(c+dx)}{a+bx}}{2dg}$$

$$= \frac{g(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2d} - \frac{(B(bc-ad)gn)\int \frac{c+}{a+}}{2d}$$

$$= \frac{g(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2d} - \frac{(B(bc-ad)gn)\int \left(\right.}{2d}$$

$$= -\frac{B(bc-ad)gnx}{2b} - \frac{B(bc-ad)^2gn\log(a+bx)}{2b^2d} + \frac{g(c+d}{}$$

Mathematica [A]
time = 0.03, size = 74, normalized size = 0.86

$$\frac{g\left(-\frac{B(bc-ad)n(bdx+(bc-ad)\log(a+bx))}{b^2}+(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*g + d*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
[Out] (g*(-((B*(b*c - a*d)*n*(b*d*x + (b*c - a*d)*Log[a + b*x]))/b^2) + (c + d*x)
^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])))/(2*d)
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int (dgx+cg)\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*g*x+c*g)*(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
[Out] int((d*g*x+c*g)*(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

Maxima [A]
time = 0.29, size = 158, normalized size = 1.84

$$\frac{1}{2}Bdgx^2\log\left(\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n e\right)+\frac{1}{2}Adgx^2-\frac{1}{2}Bdgn\left(\frac{a^2\log(bx+a)}{b^2}-\frac{c^2\log(dx+c)}{d^2}+\frac{(bc-ad)x}{bd}\right)+Bcgn\left(\frac{a\log(bx+a)}{b}-\frac{c\log(dx+c)}{d}\right)+Bcgx\log\left(\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n e\right)+Acgx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*g*x+c*g)*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="maxima
")
```

```
[Out] 1/2*B*d*g*x^2*log((b*x/(d*x + c) + a/(d*x + c))^n*e) + 1/2*A*d*g*x^2 - 1/2*
B*d*g*n*(a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d))
 + B*c*g*n*(a*log(b*x + a)/b - c*log(d*x + c)/d) + B*c*g*x*log((b*x/(d*x +
c) + a/(d*x + c))^n*e) + A*c*g*x
```

Fricas [A]
time = 0.42, size = 140, normalized size = 1.63

$$\frac{(A+B)b^2d^2gx^2 - Bb^2c^2gn\log(dx+c) + (2\,Babcd - Ba^2d^2)gn\log(bx+a) + (2\,(A+B)b^2cdg - (Bb^2cd - Babd^2)gn)x + (Bb^2d^2gnx^2 + 2\,Bb^2cdgnx)\log\left(\frac{bx+a}{dx+c}\right)}{2\,b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*g*x+c*g)*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fricas
")
```

```
[Out] 1/2*((A + B)*b^2*d^2*g*x^2 - B*b^2*c^2*g*n*log(d*x + c) + (2*B*a*b*c*d - B*
a^2*d^2)*g*n*log(b*x + a) + (2*(A + B)*b^2*c*d*g - (B*b^2*c*d - B*a*b*d^2)*
g*n)*x + (B*b^2*d^2*g*n*x^2 + 2*B*b^2*c*d*g*n*x)*log((b*x + a)/(d*x + c)))/
(b^2*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 382 vs. $2(73) = 146$.
time = 163.30, size = 382, normalized size = 4.44

$$\begin{cases} cgx\left(A + B\log\left(e\left(\frac{a}{c}\right)^n\right)\right) & \text{for}\: b = 0 \wedge d = 0 \\ cg\left(Ax + \frac{Ba\log\left(e\left(\frac{a}{c} + \frac{bx}{c}\right)^n\right)}{b} - Bnx + Bx\log\left(e\left(\frac{a}{c} + \frac{bx}{c}\right)^n\right)\right) & \text{for}\: d = 0 \\ Acgx + \frac{Adgx^2}{2} + \frac{Bc^2g\log\left(e\left(\frac{a}{c+dx}\right)^n\right)}{2d} + \frac{Bcgnx}{2} + Bcgx\log\left(e\left(\frac{a}{c+dx}\right)^n\right) + \frac{Bdgnx^2}{4} + \frac{Bdgx^2\log\left(e\left(\frac{a}{c+dx}\right)^n\right)}{2} & \text{for}\: b = 0 \\ Acgx + \frac{Adgx^2}{2} - \frac{Ba^2dgn\log\left(\frac{c}{d}+x\right)}{2b^2} - \frac{Ba^2dg\log\left(e\left(\frac{a}{c+dx} + \frac{bx}{c+dx}\right)^n\right)}{2b^2} + \frac{Bacgn\log\left(\frac{c}{d}+x\right)}{b} + \frac{Bacg\log\left(e\left(\frac{a}{c+dx} + \frac{bx}{c+dx}\right)^n\right)}{b} + \frac{Badgnx}{2b} - \frac{Bc^2gn\log\left(\frac{c}{d}+x\right)}{2d} - \frac{Bcgnx}{2} + Bcgx\log\left(e\left(\frac{a}{c+dx} + \frac{bx}{c+dx}\right)^n\right) + \frac{Bdgx^2\log\left(e\left(\frac{a}{c+dx} + \frac{bx}{c+dx}\right)^n\right)}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*g*x+c*g)*(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

```
[Out] Piecewise((c*g*x*(A + B*log(e*(a/c)**n)), Eq(b, 0) & Eq(d, 0)), (c*g*(A*x +
 B*a*log(e*(a/c + b*x/c)**n)/b - B*n*x + B*x*log(e*(a/c + b*x/c)**n)), Eq(d
, 0)), (A*c*g*x + A*d*g*x**2/2 + B*c**2*g*log(e*(a/(c + d*x))**n)/(2*d) + B
*c*g*n*x/2 + B*c*g*x*log(e*(a/(c + d*x))**n) + B*d*g*n*x**2/4 + B*d*g*x**2*
log(e*(a/(c + d*x))**n)/2, Eq(b, 0)), (A*c*g*x + A*d*g*x**2/2 - B*a**2*d*g*
n*log(c/d + x)/(2*b**2) - B*a**2*d*g*log(e*(a/(c + d*x) + b*x/(c + d*x))**n
)/(2*b**2) + B*a*c*g*n*log(c/d + x)/b + B*a*c*g*log(e*(a/(c + d*x) + b*x/(c
 + d*x))**n)/b + B*a*d*g*n*x/(2*b) - B*c**2*g*n*log(c/d + x)/(2*d) - B*c*g*
n*x/2 + B*c*g*x*log(e*(a/(c + d*x) + b*x/(c + d*x))**n) + B*d*g*x**2*log(e*
(a/(c + d*x) + b*x/(c + d*x))**n)/2, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 572 vs. $2(81) = 162$.
time = 5.61, size = 572, normalized size = 6.65

$$\frac{1}{2}\left(\frac{(Bb^3c^3gn - 3\,Bab^2c^2dgn + 3\,Ba^2bcd^2gn - Ba^3d^3gn)\log\left(\frac{bx+a}{dx+c}\right)}{b^2d - \frac{2\,(bx+a)bd^2}{dx+c} + \frac{(bx+a)^2d^3}{(dx+c)^2}} - \frac{Bb^4c^3gn - 3\,Bab^3c^2dgn - \frac{(bx+a)Bb^3c^3dgn}{dx+c} + 3\,Ba^2b^2cd^2gn + \frac{3\,(bx+a)Bab^2c^2d^2gn}{dx+c} - Ba^3bd^3gn - \frac{3\,(bx+a)Ba^2bcd^3gn}{dx+c} + \frac{(bx+a)Ba^3d^4gn}{dx+c} - Ab^4c^3g - Bb^4c^3g + 3\,Aab^3c^2dg + 3\,Bab^3c^2dg - 3\,Aa^2b^2cd^2g - 3\,Ba^2b^2cd^2g + Aa^3bd^3g + Ba^3bd^3g}{b^3d - \frac{2\,(bx+a)b^2d^2}{dx+c} + \frac{(bx+a)^2bd^3}{(dx+c)^2}} + \frac{(Bb^3c^3gn - 3\,Bab^2c^2dgn + 3\,Ba^2bcd^2gn - Ba^3d^3gn)\log\left(-b + \frac{(bx+a)d}{dx+c}\right)}{b^2d} - \frac{(Bb^3c^3gn - 3\,Bab^2c^2dgn + 3\,Ba^2bcd^2gn - Ba^3d^3gn)\log\left(\frac{bx+a}{dx+c}\right)}{b^2d}\right)\left(\frac{bc}{(bc-ad)^2} - \frac{ad}{(bc-ad)^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*g*x+c*g)*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="giac")
```

```
[Out] 1/2*((B*b^3*c^3*g*n - 3*B*a*b^2*c^2*d*g*n + 3*B*a^2*b*c*d^2*g*n - B*a^3*d^3
*g*n)*log((b*x + a)/(d*x + c))/(b^2*d - 2*(b*x + a)*b*d^2/(d*x + c) + (b*x
+ a)^2*d^3/(d*x + c)^2) - (B*b^4*c^3*g*n - 3*B*a*b^3*c^2*d*g*n - (b*x + a)*
B*b^3*c^3*d*g*n/(d*x + c) + 3*B*a^2*b^2*c*d^2*g*n + 3*(b*x + a)*B*a*b^2*c^2
*d^2*g*n/(d*x + c) - B*a^3*b*d^3*g*n - 3*(b*x + a)*B*a^2*b*c*d^3*g*n/(d*x +
 c) + (b*x + a)*B*a^3*d^4*g*n/(d*x + c) - A*b^4*c^3*g - B*b^4*c^3*g + 3*A*a
*b^3*c^2*d*g + 3*B*a*b^3*c^2*d*g - 3*A*a^2*b^2*c*d^2*g - 3*B*a^2*b^2*c*d^2*
g + A*a^3*b*d^3*g + B*a^3*b*d^3*g)/(b^3*d - 2*(b*x + a)*b^2*d^2/(d*x + c) +
 (b*x + a)^2*b*d^3/(d*x + c)^2) + (B*b^3*c^3*g*n - 3*B*a*b^2*c^2*d*g*n + 3*
B*a^2*b*c*d^2*g*n - B*a^3*d^3*g*n)*log(-b + (b*x + a)*d/(d*x + c))/(b^2*d)
- (B*b^3*c^3*g*n - 3*B*a*b^2*c^2*d*g*n + 3*B*a^2*b*c*d^2*g*n - B*a^3*d^3*g*
n)*log((b*x + a)/(d*x + c))/(b^2*d))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2
)
```

Mupad [B]
time = 4.10, size = 134, normalized size = 1.56

$$x\left(\frac{g\,(2\,A\,a\,d+4\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{2\,b}-\frac{A\,g\,(2\,a\,d+2\,b\,c)}{2\,b}\right)+\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\,\left(\frac{B\,d\,g\,x^2}{2}+B\,c\,g\,x\right)-\frac{\ln\left(a+b\,x\right)\,\left(B\,a^2\,d\,g\,n-2\,B\,a\,b\,c\,g\,n\right)}{2\,b^2}+\frac{A\,d\,g\,x^2}{2}-\frac{B\,c^2\,g\,n\,\ln\left(c+d\,x\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*g + d*g*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n)),x)
```

```
[Out] x*((g*(2*A*a*d + 4*A*b*c + B*a*d*n - B*b*c*n))/(2*b) - (A*g*(2*a*d + 2*b*c)
)/(2*b)) + log(e*((a + b*x)/(c + d*x))^n)*((B*d*g*x^2)/2 + B*c*g*x) - (log(
a + b*x)*(B*a^2*d*g*n - 2*B*a*b*c*g*n))/(2*b^2) + (A*d*g*x^2)/2 - (B*c^2*g*
n*log(c + d*x))/(2*d)
```

3.33 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{cg+dgx}\,dx$

Optimal. Leaf size=80

$$-\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{dg}-\frac{Bn\mathrm{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{dg}$$

```
[Out] -(A+B*ln(e*((b*x+a)/(d*x+c))^n))*ln((-a*d+b*c)/b/(d*x+c))/d/g-B*n*polylog(2
,d*(b*x+a)/b/(d*x+c))/d/g
```

Rubi [A]
time = 0.15, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {2543, 2458, 2378, 2370, 2352}

$$-\frac{Bn\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{dg}-\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{dg}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(c*g + d*g*x),x]
```

```
[Out] -(((A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[(b*c - a*d)/(b*(c + d*x))])/(
d*g)) - (B*n*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/(d*g)
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2370

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)/(x_))^(q_.)*(
x_)^(m_.), x_Symbol] :> Int[(e + d*x)^q*(a + b*Log[c*x^n])^p, x] /; FreeQ[{
a, b, c, d, e, m, n, p}, x] && EqQ[m, q] && IntegerQ[q]
```

Rule 2378

```
Int[((a_.) + Log[(c_.)*(x_)^(n_)]*(b_.))/((x_)*((d_) + (e_.)*(x_)^(r_.))),
x_Symbol] :> Dist[1/n, Subst[Int[(a + b*Log[c*x])/(x*(d + e*x^(r/n))), x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IntegerQ[r/n]
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
```

```
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2543

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))/((f_.) + (g_.)*(x_)), x_Symbol] :> Simp[(-Log[(b*c - a*d)/(b*(c + d*x
))])*((A + B*Log[e*((a + b*x)/(c + d*x))^n])/g), x] + Dist[B*n*((b*c - a*d)
/g), Int[Log[(b*c - a*d)/(b*(c + d*x))]/((a + b*x)*(c + d*x)), x], x] /; Fr
eeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[d*f - c*
g, 0]
```

Rubi steps

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{cg+dgx}\,dx = \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log(cg+dgx)}{dg} - \frac{(Bn)\int \frac{(c+dx)\left(-\frac{d(a+bx)}{(c+dx)^2}+\frac{b}{c+dx}\right)\log(cg+dgx)}{a+bx} \cdots}{dg}$$

$$= \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log(cg+dgx)}{dg} - \frac{(Bn)\int\left(\frac{b\log(cg+dgx)}{a+bx}-\frac{d\log(cg+dgx)}{c+dx}\right) \cdots}{dg}$$

$$= \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log(cg+dgx)}{dg} + \frac{(Bn)\int\frac{\log(cg+dgx)}{c+dx}\,dx}{g} - \frac{(bBn)\int \cdots}{dg}$$

$$= -\frac{Bn\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(cg+dgx)}{dg} + \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log(cg+dgx)}{dg} + \cdots$$

$$= -\frac{Bn\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(cg+dgx)}{dg} + \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log(cg+dgx)}{dg} + \cdots$$

$$= \frac{Bn\log^2(g(c+dx))}{2dg} - \frac{Bn\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(cg+dgx)}{dg} + \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx} \cdots\right)\right)\right)}{d \cdots}$$

Mathematica [A]
time = 0.03, size = 101, normalized size = 1.26

$$\frac{\log(g(c+dx))\left(2A-2Bn\log\left(\frac{d(a+bx)}{-bc+ad}\right)+2B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+Bn\log(g(c+dx))\right)-2Bn\,\mathrm{Li}_2\left(\frac{b(c+dx)}{bc-ad}\right)}{2dg}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(c*g + d*g*x),x]
```

```
[Out] (Log[g*(c + d*x)]*(2*A - 2*B*n*Log[(d*(a + b*x))/(-(b*c) + a*d)] + 2*B*Log[
e*((a + b*x)/(c + d*x))^n] + B*n*Log[g*(c + d*x)]) - 2*B*n*PolyLog[2, (b*(c
 + d*x))/(b*c - a*d)])/(2*d*g)
```

Maple [F]
time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{A + B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)}{dgx + cg}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(d*g*x+c*g),x)
```

```
[Out] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(d*g*x+c*g),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*g*x+c*g),x, algorithm="maxima
")
```

```
[Out] -1/2*B*((2*n*log(b*x + a)*log(d*x + c) - n*log(d*x + c)^2 - 2*log(d*x + c)*
log((b*x + a)^n) + 2*log(d*x + c)*log((d*x + c)^n))/(d*g) - 2*integrate((n*
log(b*x + a) + 1)/(d*g*x + c*g), x)) + A*log(d*g*x + c*g)/(d*g)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*g*x+c*g),x, algorithm="fricas
")
```

```
[Out] integral((B*log(((b*x + a)/(d*x + c))^n*e) + A)/(d*g*x + c*g), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A}{c+dx}\, dx + \int \frac{B\log\left(e\left(\frac{a}{c+dx} + \frac{bx}{c+dx}\right)^{n}\right)}{c+dx}\, dx}{g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))/(d*g*x+c*g),x)
```

```
[Out] (Integral(A/(c + d*x), x) + Integral(B*log(e*(a/(c + d*x) + b*x/(c + d*x))*
*n)/(c + d*x), x))/g
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 558 vs. 2(80) = 160.
time = 58.77, size = 558, normalized size = 6.98

$$\frac{1}{2}\left(\frac{\left(Bb^3c^3n - 3\,Bab^2c^2dn + 3\,Ba^2bcd^2n - Ba^3d^3n\right)\log\left(\frac{bx+a}{dx+c}\right)}{b^2dg - \frac{2\,(bx+a)bd^2g}{dx+c} + \frac{(bx+a)^2d^3g}{(dx+c)^2}} - \frac{Bb^4c^3n - 3\,Bab^3c^2dn - \frac{(bx+a)Bb^3c^3dn}{dx+c} + 3\,Ba^2b^2cd^2n + \frac{3\,(bx+a)Bab^2c^2d^2n}{dx+c} - Ba^3bd^3n - \frac{3\,(bx+a)Ba^2bcd^3n}{dx+c} + \frac{(bx+a)Ba^3d^4n}{dx+c} - Ab^4c^3 - Bb^4c^3 + 3\,Aab^3c^2d + 3\,Bab^3c^2d - 3\,Aa^2b^2cd^2 - 3\,Ba^2b^2cd^2 + Aa^3bd^3 + Ba^3bd^3}{b^3dg - \frac{2\,(bx+a)b^2d^2g}{dx+c} + \frac{(bx+a)^2bd^3g}{(dx+c)^2}} + \frac{\left(Bb^3c^3n - 3\,Bab^2c^2dn + 3\,Ba^2bcd^2n - Ba^3d^3n\right)\log\left(-b + \frac{(bx+a)d}{dx+c}\right)}{b^2dg} - \frac{\left(Bb^3c^3n - 3\,Bab^2c^2dn + 3\,Ba^2bcd^2n - Ba^3d^3n\right)\log\left(\frac{bx+a}{dx+c}\right)}{b^2dg}\right)\left(\frac{bc}{(bc-ad)^2} - \frac{ad}{(bc-ad)^2}\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*g*x+c*g),x, algorithm="giac")
```

```
[Out] 1/2*((B*b^3*c^3*n - 3*B*a*b^2*c^2*d*n + 3*B*a^2*b*c*d^2*n - B*a^3*d^3*n)*lo
g((b*x + a)/(d*x + c))/(b^2*d*g - 2*(b*x + a)*b*d^2*g/(d*x + c) + (b*x + a)
^2*d^3*g/(d*x + c)^2) - (B*b^4*c^3*n - 3*B*a*b^3*c^2*d*n - (b*x + a)*B*b^3*
c^3*d*n/(d*x + c) + 3*B*a^2*b^2*c*d^2*n + 3*(b*x + a)*B*a*b^2*c^2*d^2*n/(d*
x + c) - B*a^3*b*d^3*n - 3*(b*x + a)*B*a^2*b*c*d^3*n/(d*x + c) + (b*x + a)*
B*a^3*d^4*n/(d*x + c) - A*b^4*c^3 - B*b^4*c^3 + 3*A*a*b^3*c^2*d + 3*B*a*b^3
*c^2*d - 3*A*a^2*b^2*c*d^2 - 3*B*a^2*b^2*c*d^2 + A*a^3*b*d^3 + B*a^3*b*d^3)
/(b^3*d*g - 2*(b*x + a)*b^2*d^2*g/(d*x + c) + (b*x + a)^2*b*d^3*g/(d*x + c)
^2) + (B*b^3*c^3*n - 3*B*a*b^2*c^2*d*n + 3*B*a^2*b*c*d^2*n - B*a^3*d^3*n)*l
og(-b + (b*x + a)*d/(d*x + c))/(b^2*d*g) - (B*b^3*c^3*n - 3*B*a*b^2*c^2*d*n
 + 3*B*a^2*b*c*d^2*n - B*a^3*d^3*n)*log((b*x + a)/(d*x + c))/(b^2*d*g))*(b*
c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)^2
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)}{c\,g+d\,g\,x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log(e*((a + b*x)/(c + d*x))^n))/(c*g + d*g*x),x)
```

```
[Out] int((A + B*log(e*((a + b*x)/(c + d*x))^n))/(c*g + d*g*x), x)
```

3.34 $\int \frac{A+B \log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(cg+dgx)^2} \, dx$

Optimal. Leaf size=102

$$\frac{A(a+bx)}{(bc-ad)g^2(c+dx)}-\frac{Bn(a+bx)}{(bc-ad)g^2(c+dx)}+\frac{B(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(bc-ad)g^2(c+dx)}$$

```
[Out] A*(b*x+a)/(-a*d+b*c)/g^2/(d*x+c)-B*n*(b*x+a)/(-a*d+b*c)/g^2/(d*x+c)+B*(b*x+
a)*ln(e*((b*x+a)/(d*x+c))^n)/(-a*d+b*c)/g^2/(d*x+c)
```

Rubi [A]
time = 0.03, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.061$, Rules used = {2551, 2332}

$$\frac{A(a+bx)}{g^2(c+dx)(bc-ad)}+\frac{B(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{g^2(c+dx)(bc-ad)}-\frac{Bn(a+bx)}{g^2(c+dx)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(c*g + d*g*x)^2,x]
```

```
[Out] (A*(a + b*x))/((b*c - a*d)*g^2*(c + d*x)) - (B*n*(a + b*x))/((b*c - a*d)*g^
2*(c + d*x)) + (B*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n])/((b*c - a*d)*g^
2*(c + d*x))
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2551

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(m +
1)*(g/d)^m, Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a + b
*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ[b*c -
 a*d, 0] && IntegersQ[m, p] && EqQ[d*f - c*g, 0] && (GtQ[p, 0] || LtQ[m, -1
])
```

Rubi steps

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(cg+dgx)^2}\,dx = -\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{dg^2(c+dx)} + \frac{(Bn)\int\frac{bc-ad}{g(a+bx)(c+dx)^2}\,dx}{dg}$$

$$= -\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{dg^2(c+dx)} + \frac{(B(bc-ad)n)\int\frac{1}{(a+bx)(c+dx)^2}\,dx}{dg^2}$$

$$= -\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{dg^2(c+dx)} + \frac{(B(bc-ad)n)\int\left(\frac{b^2}{(bc-ad)^2(a+bx)} - \frac{d}{(bc-ad)(c+dx)^2} - \frac{}{(b}\right.}{dg^2}$$

$$= \frac{Bn}{dg^2(c+dx)} + \frac{bBn\log(a+bx)}{d(bc-ad)g^2} - \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{dg^2(c+dx)} - \frac{bBn\log(c+dx)}{d(bc-ad)g^2}$$

Mathematica [A]
time = 0.04, size = 114, normalized size = 1.12

$$-\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{dg(cg+dgx)} + \frac{B(bc-ad)n\left(\frac{1}{(bc-ad)(c+dx)} + \frac{b\log(a+bx)}{(bc-ad)^2} - \frac{b\log(c+dx)}{(bc-ad)^2}\right)}{dg^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(c*g + d*g*x)^2,x]
```

```
[Out] -((A + B*Log[e*((a + b*x)/(c + d*x))^n])/(d*g*(c*g + d*g*x))) + (B*(b*c - a
*d)*n*(1/((b*c - a*d)*(c + d*x)) + (b*Log[a + b*x])/(b*c - a*d)^2 - (b*Log[
c + d*x])/(b*c - a*d)^2))/(d*g^2)
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}{\left(dgx+cg\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(d*g*x+c*g)^2,x)
```

```
[Out] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(d*g*x+c*g)^2,x)
```

Maxima [A]
time = 0.30, size = 137, normalized size = 1.34

$$Bn\left(\frac{1}{d^2g^2x+cdg^2} + \frac{b\log\left(bx+a\right)}{(bcd-ad^2)g^2} - \frac{b\log\left(dx+c\right)}{(bcd-ad^2)g^2}\right) - \frac{B\log\left(\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n e\right)}{d^2g^2x+cdg^2} - \frac{A}{d^2g^2x+cdg^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*g*x+c*g)^2,x, algorithm="maxi
ma")
```

```
[Out] B*n*(1/(d^2*g^2*x + c*d*g^2) + b*log(b*x + a)/((b*c*d - a*d^2)*g^2) - b*log
(d*x + c)/((b*c*d - a*d^2)*g^2)) - B*log((b*x/(d*x + c) + a/(d*x + c))^n*e)
/(d^2*g^2*x + c*d*g^2) - A/(d^2*g^2*x + c*d*g^2)
```

Fricas [A]
time = 0.42, size = 96, normalized size = 0.94

$$-\frac{(A+B)bc-(A+B)ad-(Bbc-Bad)n-(Bbdnx+Badn)\log\left(\frac{bx+a}{dx+c}\right)}{(bcd^2-ad^3)g^2x+(bc^2d-acd^2)g^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*g*x+c*g)^2,x, algorithm="fric
as")
```

```
[Out] -((A + B)*b*c - (A + B)*a*d - (B*b*c - B*a*d)*n - (B*b*d*n*x + B*a*d*n)*log
((b*x + a)/(d*x + c)))/((b*c*d^2 - a*d^3)*g^2*x + (b*c^2*d - a*c*d^2)*g^2)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))/(d*g*x+c*g)**2,x)
```

```
[Out] Exception raised: NotImplementedError >> no valid subset found
```

Giac [A]
time = 2.69, size = 89, normalized size = 0.87

$$\left(\frac{(bx+a)Bn\log\left(\frac{bx+a}{dx+c}\right)}{(dx+c)g^2}-\frac{(Bn-A-B)(bx+a)}{(dx+c)g^2}\right)\left(\frac{bc}{(bc-ad)^2}-\frac{ad}{(bc-ad)^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*g*x+c*g)^2,x, algorithm="giac
")
```

```
[Out] ((b*x + a)*B*n*log((b*x + a)/(d*x + c))/((d*x + c)*g^2) - (B*n - A - B)*(b*
x + a)/((d*x + c)*g^2))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)
```

Mupad [B]
time = 4.02, size = 113, normalized size = 1.11

$$-\frac{A-B\,n}{x\,d^2\,g^2+c\,d\,g^2}-\frac{B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)}{d\,\left(c\,g^2+d\,g^2\,x\right)}+\frac{B\,b\,n\,\mathrm{atan}\left(\frac{b\,c\,2{}\mathrm{i}+b\,d\,x\,2{}\mathrm{i}}{a\,d-b\,c}+1{}\mathrm{i}\right)\,2{}\mathrm{i}}{d\,g^2\,\left(a\,d-b\,c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log(e*((a + b*x)/(c + d*x))^n))/(c*g + d*g*x)^2,x)
```

```
[Out] (B*b*n*atan((b*c*2i + b*d*x*2i)/(a*d - b*c) + 1i)*2i)/(d*g^2*(a*d - b*c)) -
 (B*log(e*((a + b*x)/(c + d*x))^n))/(d*(c*g^2 + d*g^2*x)) - (A - B*n)/(d^2*
g^2*x + c*d*g^2)
```

3.35 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(cg+dgx)^3}\,dx$

Optimal. Leaf size=151

$$\frac{Bn}{4dg^3(c+dx)^2}+\frac{bBn}{2d(bc-ad)g^3(c+dx)}+\frac{b^2Bn\log(a+bx)}{2d(bc-ad)^2g^3}-\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{2dg^3(c+dx)^2}-\frac{b^2Bn\log(c+dx)}{2d(bc-ad)^2g^3}$$

```
[Out] 1/4*B*n/d/g^3/(d*x+c)^2+1/2*b*B*n/d/(-a*d+b*c)/g^3/(d*x+c)+1/2*b^2*B*n*ln(b
*x+a)/d/(-a*d+b*c)^2/g^3+1/2*(-A-B*ln(e*((b*x+a)/(d*x+c))^n))/d/g^3/(d*x+c)
^2-1/2*b^2*B*n*ln(d*x+c)/d/(-a*d+b*c)^2/g^3
```

Rubi [A]
time = 0.08, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {2547, 21, 46}

$$-\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{2dg^3(c+dx)^2}+\frac{b^2Bn\log(a+bx)}{2dg^3(bc-ad)^2}-\frac{b^2Bn\log(c+dx)}{2dg^3(bc-ad)^2}+\frac{bBn}{2dg^3(c+dx)(bc-ad)}+\frac{Bn}{4dg^3(c+dx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(c*g + d*g*x)^3,x]
```

```
[Out] (B*n)/(4*d*g^3*(c + d*x)^2) + (b*B*n)/(2*d*(b*c - a*d)*g^3*(c + d*x)) + (b^
2*B*n*Log[a + b*x])/(2*d*(b*c - a*d)^2*g^3) - (A + B*Log[e*((a + b*x)/(c +
d*x))^n])/(2*d*g^3*(c + d*x)^2) - (b^2*B*n*Log[c + d*x])/(2*d*(b*c - a*d)^2
*g^3)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2547

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*((A +
```

```
 B*Log[e*((a + b*x)/(c + d*x))^n])/(g*(m + 1))), x] - Dist[B*n*((b*c - a*d)
/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /; FreeQ
[{a, b, c, d, e, f, g, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(cg+dgx)^3}\,dx &= -\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{2dg^3(c+dx)^2}+\frac{(Bn)\int \frac{bc-ad}{g^2(a+bx)(c+dx)^3}\,dx}{2dg}\\ &= -\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{2dg^3(c+dx)^2}+\frac{(B(bc-ad)n)\int \frac{1}{(a+bx)(c+dx)^3}\,dx}{2dg^3}\\ &= -\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{2dg^3(c+dx)^2}+\frac{(B(bc-ad)n)\int \left(\frac{b^3}{(bc-ad)^3(a+bx)}-\frac{d}{(bc-ad)(c+dx)^3}-\frac{(b}{}\right.}{2dg^3}\\ &= \frac{Bn}{4dg^3(c+dx)^2}+\frac{bBn}{2d(bc-ad)g^3(c+dx)}+\frac{b^2Bn\log(a+bx)}{2d(bc-ad)^2g^3}-\frac{A+B\log\left(e\left(\frac{}{}\right.\right.}{2dg^3(c+d}\end{aligned}$$

Mathematica [**A**]
time = 0.10, size = 115, normalized size = 0.76

$$\frac{-2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)+\frac{Bn\left((bc-ad)(3bc-ad+2bdx)+2b^2(c+dx)^2\log(a+bx)-2b^2(c+dx)^2\log(c+dx)\right)}{(bc-ad)^2}}{4dg^3(c+dx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(c*g + d*g*x)^3,x]
```

```
[Out] (-2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + (B*n*((b*c - a*d)*(3*b*c - a*d
 + 2*b*d*x) + 2*b^2*(c + d*x)^2*Log[a + b*x] - 2*b^2*(c + d*x)^2*Log[c + d*
x]))/(b*c - a*d)^2)/(4*d*g^3*(c + d*x)^2)
```

Maple [**F**]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}{\left(dgx+cg\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(d*g*x+c*g)^3,x)
```

```
[Out] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(d*g*x+c*g)^3,x)
```

Maxima [A]
time = 0.29, size = 260, normalized size = 1.72

$$\frac{1}{4}Bn\left(\frac{2\,bdx+3\,bc-ad}{(bcd^3-ad^4)g^3x^2+2\,(bc^2d^2-acd^3)g^3x+(bc^3d-ac^2d^2)g^3}+\frac{2\,b^2\log\left(bx+a\right)}{(b^2c^2d-2\,abcd^2+a^2d^3)g^3}-\frac{2\,b^2\log\left(dx+c\right)}{(b^2c^2d-2\,abcd^2+a^2d^3)g^3}\right)-\frac{B\log\left(\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^ne\right)}{2\,(d^3g^3x^2+2\,cd^2g^3x+c^2dg^3)}-\frac{A}{2\,(d^3g^3x^2+2\,cd^2g^3x+c^2dg^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*g*x+c*g)^3,x, algorithm="maxi
ma")
```

```
[Out] 1/4*B*n*((2*b*d*x + 3*b*c - a*d)/((b*c*d^3 - a*d^4)*g^3*x^2 + 2*(b*c^2*d^2
- a*c*d^3)*g^3*x + (b*c^3*d - a*c^2*d^2)*g^3) + 2*b^2*log(b*x + a)/((b^2*c^
2*d - 2*a*b*c*d^2 + a^2*d^3)*g^3) - 2*b^2*log(d*x + c)/((b^2*c^2*d - 2*a*b*
c*d^2 + a^2*d^3)*g^3)) - 1/2*B*log((b*x/(d*x + c) + a/(d*x + c))^n*e)/(d^3*
g^3*x^2 + 2*c*d^2*g^3*x + c^2*d*g^3) - 1/2*A/(d^3*g^3*x^2 + 2*c*d^2*g^3*x +
 c^2*d*g^3)
```

Fricas [A]
time = 0.36, size = 244, normalized size = 1.62

$$-\frac{2\,(A+B)b^2c^2-4\,(A+B)abcd+2\,(A+B)a^2d^2-2\,(Bb^2cd-Babd^2)nx-(3\,Bb^2c^2-4\,Babcd+Ba^2d^2)n-2\,(Bb^2d^2nx^2+2\,Bb^2cdnx+(2\,Babcd-Ba^2d^2)n)\log\left(\frac{bx+a}{dx+c}\right)}{4\,((b^2c^2d^3-2\,abcd^4+a^2d^5)g^3x^2+2\,(b^2c^3d^2-2\,abc^2d^3+a^2cd^4)g^3x+(b^2c^4d-2\,abc^3d^2+a^2c^2d^3)g^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*g*x+c*g)^3,x, algorithm="fric
as")
```

```
[Out] -1/4*(2*(A + B)*b^2*c^2 - 4*(A + B)*a*b*c*d + 2*(A + B)*a^2*d^2 - 2*(B*b^2*
c*d - B*a*b*d^2)*n*x - (3*B*b^2*c^2 - 4*B*a*b*c*d + B*a^2*d^2)*n - 2*(B*b^2
*d^2*n*x^2 + 2*B*b^2*c*d*n*x + (2*B*a*b*c*d - B*a^2*d^2)*n)*log((b*x + a)/(
d*x + c)))/((b^2*c^2*d^3 - 2*a*b*c*d^4 + a^2*d^5)*g^3*x^2 + 2*(b^2*c^3*d^2
- 2*a*b*c^2*d^3 + a^2*c*d^4)*g^3*x + (b^2*c^4*d - 2*a*b*c^3*d^2 + a^2*c^2*d
^3)*g^3)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))/(d*g*x+c*g)**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 3.92, size = 203, normalized size = 1.34

$$\frac{1}{4}\left(2\left(\frac{2\,(bx+a)Bbn}{(bcg^3-adg^3)(dx+c)}-\frac{(bx+a)^2Bdn}{(bcg^3-adg^3)(dx+c)^2}\right)\log\left(\frac{bx+a}{dx+c}\right)+\frac{(Bdn-2\,Ad-2\,Bd)(bx+a)^2}{(bcg^3-adg^3)(dx+c)^2}-\frac{4\,(Bbn-Ab-Bb)(bx+a)}{(bcg^3-adg^3)(dx+c)}\right)\left(\frac{bc}{(bc-ad)^2}-\frac{ad}{(bc-ad)^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*g*x+c*g)^3,x, algorithm="giac
")
```

```
[Out] 1/4*(2*(2*(b*x + a)*B*b*n/((b*c*g^3 - a*d*g^3)*(d*x + c)) - (b*x + a)^2*B*d
*n/((b*c*g^3 - a*d*g^3)*(d*x + c)^2))*log((b*x + a)/(d*x + c)) + (B*d*n - 2
*A*d - 2*B*d)*(b*x + a)^2/((b*c*g^3 - a*d*g^3)*(d*x + c)^2) - 4*(B*b*n - A*
b - B*b)*(b*x + a)/((b*c*g^3 - a*d*g^3)*(d*x + c)))*(b*c/(b*c - a*d)^2 - a*
d/(b*c - a*d)^2)
```

Mupad [B]
time = 4.55, size = 221, normalized size = 1.46

$$\frac{B\,b^2\,n\,\mathrm{atanh}\left(\frac{2\,a^2\,d^3\,g^3-2\,b^2\,c^2\,d\,g^3}{2\,d\,g^3\,{\left(a\,d-b\,c\right)}^2}+\frac{2\,b\,d\,x}{a\,d-b\,c}\right)}{d\,g^3\,{\left(a\,d-b\,c\right)}^2}-\frac{B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)}{2\,d\,\left(c^2\,g^3+2\,c\,d\,g^3\,x+d^2\,g^3\,x^2\right)}-\frac{\frac{2\,A\,a\,d-2\,A\,b\,c-B\,a\,d\,n+3\,B\,b\,c\,n}{2\,\left(a\,d-b\,c\right)}+\frac{B\,b\,d\,n\,x}{a\,d-b\,c}}{2\,c^2\,d\,g^3+4\,c\,d^2\,g^3\,x+2\,d^3\,g^3\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log(e*((a + b*x)/(c + d*x))^n))/(c*g + d*g*x)^3,x)
```

```
[Out] (B*b^2*n*atanh((2*a^2*d^3*g^3 - 2*b^2*c^2*d*g^3)/(2*d*g^3*(a*d - b*c)^2) +
(2*b*d*x)/(a*d - b*c)))/(d*g^3*(a*d - b*c)^2) - (B*log(e*((a + b*x)/(c + d*
x))^n))/(2*d*(c^2*g^3 + d^2*g^3*x^2 + 2*c*d*g^3*x)) - ((2*A*a*d - 2*A*b*c -
 B*a*d*n + 3*B*b*c*n)/(2*(a*d - b*c)) + (B*b*d*n*x)/(a*d - b*c))/(2*c^2*d*g
^3 + 2*d^3*g^3*x^2 + 4*c*d^2*g^3*x)
```

3.36 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(cg+dgx)^4}\,dx$

Optimal. Leaf size=183

$$\frac{Bn}{9dg^4(c+dx)^3}+\frac{bBn}{6d(bc-ad)g^4(c+dx)^2}+\frac{b^2Bn}{3d(bc-ad)^2g^4(c+dx)}+\frac{b^3Bn\log(a+bx)}{3d(bc-ad)^3g^4}-\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{3dg^4(c+dx)^3}$$

```
[Out] 1/9*B*n/d/g^4/(d*x+c)^3+1/6*b*B*n/d/(-a*d+b*c)/g^4/(d*x+c)^2+1/3*b^2*B*n/d/
(-a*d+b*c)^2/g^4/(d*x+c)+1/3*b^3*B*n*ln(b*x+a)/d/(-a*d+b*c)^3/g^4+1/3*(-A-B
*ln(e*((b*x+a)/(d*x+c))^n))/d/g^4/(d*x+c)^3-1/3*b^3*B*n*ln(d*x+c)/d/(-a*d+b
*c)^3/g^4
```

Rubi [A]
time = 0.10, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {2547, 21, 46}

$$-\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{3dg^4(c+dx)^3}+\frac{b^3Bn\log(a+bx)}{3dg^4(bc-ad)^3}-\frac{b^3Bn\log(c+dx)}{3dg^4(bc-ad)^3}+\frac{b^2Bn}{3dg^4(c+dx)(bc-ad)^2}+\frac{bBn}{6dg^4(c+dx)^2(bc-ad)}+\frac{Bn}{9dg^4(c+dx)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(c*g + d*g*x)^4,x]
```

```
[Out] (B*n)/(9*d*g^4*(c + d*x)^3) + (b*B*n)/(6*d*(b*c - a*d)*g^4*(c + d*x)^2) + (
b^2*B*n)/(3*d*(b*c - a*d)^2*g^4*(c + d*x)) + (b^3*B*n*Log[a + b*x])/(3*d*(b
*c - a*d)^3*g^4) - (A + B*Log[e*((a + b*x)/(c + d*x))^n])/(3*d*g^4*(c + d*x
)^3) - (b^3*B*n*Log[c + d*x])/(3*d*(b*c - a*d)^3*g^4)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2547

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*((A +
```

```
 B*Log[e*((a + b*x)/(c + d*x))^n])/(g*(m + 1))), x] - Dist[B*n*((b*c - a*d)
/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /; FreeQ
[{a, b, c, d, e, f, g, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, -2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(cg+dgx)^4}\,dx &= -\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{3dg^4(c+dx)^3}+\frac{(Bn)\int \frac{bc-ad}{g^3(a+bx)(c+dx)^4}\,dx}{3dg} \\
&= -\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{3dg^4(c+dx)^3}+\frac{(B(bc-ad)n)\int \frac{1}{(a+bx)(c+dx)^4}\,dx}{3dg^4} \\
&= -\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{3dg^4(c+dx)^3}+\frac{(B(bc-ad)n)\int \left(\frac{b^4}{(bc-ad)^4(a+bx)}-\frac{d}{(bc-ad)(c+dx)^4}-\frac{(b\cdots}{\cdots}\right)}{3dg\cdots} \\
&= \frac{Bn}{9dg^4(c+dx)^3}+\frac{bBn}{6d(bc-ad)g^4(c+dx)^2}+\frac{b^2Bn}{3d(bc-ad)^2g^4(c+dx)}+\frac{b^3Bn\,\mathrm{lo}\cdots}{3d(bc\cdots}
\end{aligned}
$$

Mathematica [A]
time = 0.12, size = 146, normalized size = 0.80

$$
\frac{-6\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)+\frac{Bn\left((bc-ad)\left(2a^2d^2-abd(7c+3dx)+b^2\left(11c^2+15cdx+6d^2x^2\right)\right)+6b^3(c+dx)^3\log(a+bx)-6b^3(c+dx)^3\log(c+dx)\right)}{(bc-ad)^3}}{18dg^4(c+dx)^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(c*g + d*g*x)^4,x]
```

```
[Out] (-6*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + (B*n*((b*c - a*d)*(2*a^2*d^2 -
 a*b*d*(7*c + 3*d*x) + b^2*(11*c^2 + 15*c*d*x + 6*d^2*x^2)) + 6*b^3*(c + d*
x)^3*Log[a + b*x] - 6*b^3*(c + d*x)^3*Log[c + d*x]))/(b*c - a*d)^3)/(18*d*g
^4*(c + d*x)^3)
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$
\int \frac{A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}{(dgx+cg)^4}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(d*g*x+c*g)^4,x)
```

```
[Out] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(d*g*x+c*g)^4,x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 434 vs. $2(172) = 344$.
time = 0.31, size = 434, normalized size = 2.37

$$\frac{1}{18}\,Bn\left(\frac{6\,b^2d^2x^2 + 11\,b^2c^2 - 7\,abcd + 2\,a^2d^2 + 3\,(5\,b^2cd - abd^2)x}{(b^2c^2d^4 - 2\,abcd^5 + a^2d^6)g^4x^3 + 3\,(b^2c^3d^3 - 2\,abc^2d^4 + a^2cd^5)g^4x^2 + 3\,(b^2c^4d^2 - 2\,abc^3d^3 + a^2c^2d^4)g^4x + (b^2c^5d - 2\,abc^4d^2 + a^2c^3d^3)g^4} + \frac{6\,b^3\log(bx + a)}{(b^3c^3d - 3\,ab^2c^2d^2 + 3\,a^2bcd^3 - a^3d^4)g^4} - \frac{6\,b^3\log(dx + c)}{(b^3c^3d - 3\,ab^2c^2d^2 + 3\,a^2bcd^3 - a^3d^4)g^4}\right) - \frac{B\log\left(\left(\frac{bx}{dx+c} + \frac{a}{dx+c}\right)^n e\right)}{3\,(d^4g^4x^3 + 3\,cd^3g^4x^2 + 3\,c^2d^2g^4x + c^3dg^4)} - \frac{A}{3\,(d^4g^4x^3 + 3\,cd^3g^4x^2 + 3\,c^2d^2g^4x + c^3dg^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*g*x+c*g)^4,x, algorithm="maxi
ma")
```

```
[Out] 1/18*B*n*((6*b^2*d^2*x^2 + 11*b^2*c^2 - 7*a*b*c*d + 2*a^2*d^2 + 3*(5*b^2*c*
d - a*b*d^2)*x)/((b^2*c^2*d^4 - 2*a*b*c*d^5 + a^2*d^6)*g^4*x^3 + 3*(b^2*c^3
*d^3 - 2*a*b*c^2*d^4 + a^2*c*d^5)*g^4*x^2 + 3*(b^2*c^4*d^2 - 2*a*b*c^3*d^3
+ a^2*c^2*d^4)*g^4*x + (b^2*c^5*d - 2*a*b*c^4*d^2 + a^2*c^3*d^3)*g^4) + 6*b
^3*log(b*x + a)/((b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*g^
4) - 6*b^3*log(d*x + c)/((b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3
*d^4)*g^4)) - 1/3*B*log((b*x/(d*x + c) + a/(d*x + c))^n*e)/(d^4*g^4*x^3 + 3
*c*d^3*g^4*x^2 + 3*c^2*d^2*g^4*x + c^3*d*g^4) - 1/3*A/(d^4*g^4*x^3 + 3*c*d^
3*g^4*x^2 + 3*c^2*d^2*g^4*x + c^3*d*g^4)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 447 vs. $2(172) = 344$.
time = 0.35, size = 447, normalized size = 2.44

$$-\frac{6\,(A + B)b^3c^3 - 18\,(A + B)ab^2c^2d + 18\,(A + B)a^2bcd^2 - 6\,(A + B)a^3d^3 - 6\,(Bb^3cd^2 - Bab^2d^3)nx^2 - 3\,(5\,Bb^3c^2d - 6\,Bab^2cd^2 + Ba^2bd^3)nx - (11\,Bb^3c^3 - 18\,Bab^2c^2d + 9\,Ba^2bcd^2 - 2\,Ba^3d^3)n - 6\,(Bb^3d^3nx^3 + 3\,Bb^3cd^2nx^2 + 3\,Bb^3c^2dnx + (3\,Bab^2c^2d - 3\,Ba^2bcd^2 + Ba^3d^3)n)\log\left(\frac{bx+a}{dx+c}\right)}{18\,((b^3c^3d^4 - 3\,ab^2c^2d^5 + 3\,a^2bcd^6 - a^3d^7)g^4x^3 + 3\,(b^3c^4d^3 - 3\,ab^2c^3d^4 + 3\,a^2bc^2d^5 - a^3cd^6)g^4x^2 + 3\,(b^3c^5d^2 - 3\,ab^2c^4d^3 + 3\,a^2bc^3d^4 - a^3c^2d^5)g^4x + (b^3c^6d - 3\,ab^2c^5d^2 + 3\,a^2bc^4d^3 - a^3c^3d^4)g^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*g*x+c*g)^4,x, algorithm="fric
as")
```

```
[Out] -1/18*(6*(A + B)*b^3*c^3 - 18*(A + B)*a*b^2*c^2*d + 18*(A + B)*a^2*b*c*d^2
- 6*(A + B)*a^3*d^3 - 6*(B*b^3*c*d^2 - B*a*b^2*d^3)*n*x^2 - 3*(5*B*b^3*c^2*
d - 6*B*a*b^2*c*d^2 + B*a^2*b*d^3)*n*x - (11*B*b^3*c^3 - 18*B*a*b^2*c^2*d +
 9*B*a^2*b*c*d^2 - 2*B*a^3*d^3)*n - 6*(B*b^3*d^3*n*x^3 + 3*B*b^3*c*d^2*n*x^
2 + 3*B*b^3*c^2*d*n*x + (3*B*a*b^2*c^2*d - 3*B*a^2*b*c*d^2 + B*a^3*d^3)*n)*
log((b*x + a)/(d*x + c)))/((b^3*c^3*d^4 - 3*a*b^2*c^2*d^5 + 3*a^2*b*c*d^6 -
 a^3*d^7)*g^4*x^3 + 3*(b^3*c^4*d^3 - 3*a*b^2*c^3*d^4 + 3*a^2*b*c^2*d^5 - a^
3*c*d^6)*g^4*x^2 + 3*(b^3*c^5*d^2 - 3*a*b^2*c^4*d^3 + 3*a^2*b*c^3*d^4 - a^3
*c^2*d^5)*g^4*x + (b^3*c^6*d - 3*a*b^2*c^5*d^2 + 3*a^2*b*c^4*d^3 - a^3*c^3*
d^4)*g^4)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))/(d*g*x+c*g)**4,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 399 vs. 2(172) = 344.
time = 3.91, size = 399, normalized size = 2.18

$$\frac{1}{18}\left(6\left(\frac{3\,(bx+a)Bb^2n}{(b^2c^2g^4-2\,abcdg^4+a^2d^2g^4)(dx+c)}-\frac{3\,(bx+a)^2Bbdn}{(b^2c^2g^4-2\,abcdg^4+a^2d^2g^4)(dx+c)^2}+\frac{(bx+a)^3Bd^2n}{(b^2c^2g^4-2\,abcdg^4+a^2d^2g^4)(dx+c)^3}\right)\log\left(\frac{bx+a}{dx+c}\right)-\frac{2\,(Bd^2n-3\,Ad^2-3\,Bd^2)(bx+a)^3}{(b^2c^2g^4-2\,abcdg^4+a^2d^2g^4)(dx+c)^3}+\frac{9\,(Bbdn-2\,Abd-2\,Bbd)(bx+a)^2}{(b^2c^2g^4-2\,abcdg^4+a^2d^2g^4)(dx+c)^2}-\frac{18\,(Bb^2n-Ab^2-Bb^2)(bx+a)}{(b^2c^2g^4-2\,abcdg^4+a^2d^2g^4)(dx+c)}\right)\left(\frac{bc}{(bc-ad)^2}-\frac{ad}{(bc-ad)^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*g*x+c*g)^4,x, algorithm="giac
")
```

```
[Out] 1/18*(6*(3*(b*x + a)*B*b^2*n/((b^2*c^2*g^4 - 2*a*b*c*d*g^4 + a^2*d^2*g^4)*(
d*x + c)) - 3*(b*x + a)^2*B*b*d*n/((b^2*c^2*g^4 - 2*a*b*c*d*g^4 + a^2*d^2*g
^4)*(d*x + c)^2) + (b*x + a)^3*B*d^2*n/((b^2*c^2*g^4 - 2*a*b*c*d*g^4 + a^2*
d^2*g^4)*(d*x + c)^3))*log((b*x + a)/(d*x + c)) - 2*(B*d^2*n - 3*A*d^2 - 3*
B*d^2)*(b*x + a)^3/((b^2*c^2*g^4 - 2*a*b*c*d*g^4 + a^2*d^2*g^4)*(d*x + c)^3
) + 9*(B*b*d*n - 2*A*b*d - 2*B*b*d)*(b*x + a)^2/((b^2*c^2*g^4 - 2*a*b*c*d*g
^4 + a^2*d^2*g^4)*(d*x + c)^2) - 18*(B*b^2*n - A*b^2 - B*b^2)*(b*x + a)/((b
^2*c^2*g^4 - 2*a*b*c*d*g^4 + a^2*d^2*g^4)*(d*x + c)))*(b*c/(b*c - a*d)^2 -
a*d/(b*c - a*d)^2)
```

Mupad [B]
time = 4.72, size = 349, normalized size = 1.91

$$\frac{B\,a^2\,d\,n}{9\,g^4\,(a\,d-b\,c)^2\,(c+d\,x)^3}-\frac{A\,a^2\,d}{3\,g^4\,(a\,d-b\,c)^2\,(c+d\,x)^3}-\frac{A\,b^2\,c^2}{3\,d\,g^4\,(a\,d-b\,c)^2\,(c+d\,x)^3}-\frac{B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)}{3\,d\,g^4\,(c+d\,x)^3}+\frac{2\,A\,a\,b\,c}{3\,g^4\,(a\,d-b\,c)^2\,(c+d\,x)^3}+\frac{B\,b^2\,d\,n\,x^2}{3\,g^4\,(a\,d-b\,c)^2\,(c+d\,x)^3}-\frac{7\,B\,a\,b\,c\,n}{18\,g^4\,(a\,d-b\,c)^2\,(c+d\,x)^3}+\frac{11\,B\,b^2\,c^2\,n}{18\,d\,g^4\,(a\,d-b\,c)^2\,(c+d\,x)^3}+\frac{5\,B\,b^2\,c\,n\,x}{6\,g^4\,(a\,d-b\,c)^2\,(c+d\,x)^3}-\frac{B\,a\,b\,d\,n\,x}{6\,g^4\,(a\,d-b\,c)^2\,(c+d\,x)^3}+\frac{B\,b^3\,n\,\mathrm{atan}\left(\frac{a\,d\,1{}\mathrm{i}+b\,c\,1{}\mathrm{i}+b\,d\,x\,2{}\mathrm{i}}{a\,d-b\,c}\right)\,2{}\mathrm{i}}{3\,d\,g^4\,(a\,d-b\,c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log(e*((a + b*x)/(c + d*x))^n))/(c*g + d*g*x)^4,x)
```

```
[Out] (B*a^2*d*n)/(9*g^4*(a*d - b*c)^2*(c + d*x)^3) - (A*a^2*d)/(3*g^4*(a*d - b*c
)^2*(c + d*x)^3) - (A*b^2*c^2)/(3*d*g^4*(a*d - b*c)^2*(c + d*x)^3) - (B*log
(e*((a + b*x)/(c + d*x))^n))/(3*d*g^4*(c + d*x)^3) + (B*b^3*n*atan((a*d*1i
+ b*c*1i + b*d*x*2i)/(a*d - b*c))*2i)/(3*d*g^4*(a*d - b*c)^3) + (2*A*a*b*c)
/(3*g^4*(a*d - b*c)^2*(c + d*x)^3) + (B*b^2*d*n*x^2)/(3*g^4*(a*d - b*c)^2*(
c + d*x)^3) - (7*B*a*b*c*n)/(18*g^4*(a*d - b*c)^2*(c + d*x)^3) + (11*B*b^2*
c^2*n)/(18*d*g^4*(a*d - b*c)^2*(c + d*x)^3) + (5*B*b^2*c*n*x)/(6*g^4*(a*d -
 b*c)^2*(c + d*x)^3) - (B*a*b*d*n*x)/(6*g^4*(a*d - b*c)^2*(c + d*x)^3)
```

3.37 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(cg+dgx)^5}\,dx$

Optimal. Leaf size=215

$$\frac{Bn}{16dg^5(c+dx)^4}+\frac{bBn}{12d(bc-ad)g^5(c+dx)^3}+\frac{b^2Bn}{8d(bc-ad)^2g^5(c+dx)^2}+\frac{b^3Bn}{4d(bc-ad)^3g^5(c+dx)}+\frac{b^4Bn\log(a}{4d(bc-ac}$$

```
[Out] 1/16*B*n/d/g^5/(d*x+c)^4+1/12*b*B*n/d/(-a*d+b*c)/g^5/(d*x+c)^3+1/8*b^2*B*n/
d/(-a*d+b*c)^2/g^5/(d*x+c)^2+1/4*b^3*B*n/d/(-a*d+b*c)^3/g^5/(d*x+c)+1/4*b^4
*B*n*ln(b*x+a)/d/(-a*d+b*c)^4/g^5+1/4*(-A-B*ln(e*((b*x+a)/(d*x+c))^n))/d/g^
5/(d*x+c)^4-1/4*b^4*B*n*ln(d*x+c)/d/(-a*d+b*c)^4/g^5
```

Rubi [A]
time = 0.13, antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {2547, 21, 46}

$$-\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{4dg^5(c+dx)^4}+\frac{b^4Bn\log(a+bx)}{4dg^5(bc-ad)^4}-\frac{b^4Bn\log(c+dx)}{4dg^5(bc-ad)^4}+\frac{b^3Bn}{4dg^5(c+dx)(bc-ad)^3}+\frac{b^2Bn}{8dg^5(c+dx)^2(bc-ad)^2}+\frac{bBn}{12dg^5(c+dx)^3(bc-ad)}+\frac{Bn}{16dg^5(c+dx)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(c*g + d*g*x)^5,x]
```

```
[Out] (B*n)/(16*d*g^5*(c + d*x)^4) + (b*B*n)/(12*d*(b*c - a*d)*g^5*(c + d*x)^3) +
 (b^2*B*n)/(8*d*(b*c - a*d)^2*g^5*(c + d*x)^2) + (b^3*B*n)/(4*d*(b*c - a*d)
^3*g^5*(c + d*x)) + (b^4*B*n*Log[a + b*x])/(4*d*(b*c - a*d)^4*g^5) - (A + B
*Log[e*((a + b*x)/(c + d*x))^n])/(4*d*g^5*(c + d*x)^4) - (b^4*B*n*Log[c + d
*x])/(4*d*(b*c - a*d)^4*g^5)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2547

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*((A +
```

```
B*Log[e*((a + b*x)/(c + d*x))^n])/(g*(m + 1))), x] - Dist[B*n*((b*c - a*d)
/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /; FreeQ
[{a, b, c, d, e, f, g, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, -2]
```

Rubi steps

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(cg+dgx)^5}\,dx = -\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{4dg^5(c+dx)^4}+\frac{(Bn)\int\frac{bc-ad}{g^4(a+bx)(c+dx)^5}\,dx}{4dg}$$
$$= -\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{4dg^5(c+dx)^4}+\frac{(B(bc-ad)n)\int\frac{1}{(a+bx)(c+dx)^5}\,dx}{4dg^5}$$
$$= -\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{4dg^5(c+dx)^4}+\frac{(B(bc-ad)n)\int\left(\frac{b^5}{(bc-ad)^5(a+bx)}-\frac{d}{(bc-ad)(c+dx)^5}-\frac{(b}{}\right.}{}$$
$$= \frac{Bn}{16dg^5(c+dx)^4}+\frac{bBn}{12d(bc-ad)g^5(c+dx)^3}+\frac{b^2Bn}{8d(bc-ad)^2g^5(c+dx)^2}+\frac{}{4d(b}$$

Mathematica [A]
time = 0.16, size = 162, normalized size = 0.75

$$\frac{-\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(c+dx)^4}+\frac{Bn\left(\frac{3(bc-ad)^4}{(c+dx)^4}+\frac{4b(bc-ad)^3}{(c+dx)^3}+\frac{6b^2(bc-ad)^2}{(c+dx)^2}+\frac{12b^3(bc-ad)}{c+dx}+12b^4\log(a+bx)-12b^4\log(c+dx)\right)}{12(bc-ad)^4}}{4dg^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(c*g + d*g*x)^5,x]
```

```
[Out] (-((A + B*Log[e*((a + b*x)/(c + d*x))^n])/(c + d*x)^4) + (B*n*((3*(b*c - a*
d)^4)/(c + d*x)^4 + (4*b*(b*c - a*d)^3)/(c + d*x)^3 + (6*b^2*(b*c - a*d)^2)
/(c + d*x)^2 + (12*b^3*(b*c - a*d))/(c + d*x) + 12*b^4*Log[a + b*x] - 12*b^
4*Log[c + d*x]))/(12*(b*c - a*d)^4))/(4*d*g^5)
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}{(dgx+cg)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(d*g*x+c*g)^5,x)
```

```
[Out] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(d*g*x+c*g)^5,x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 653 vs. $2(202) = 404$.
time = 0.32, size = 653, normalized size = 3.04

$$\frac{1}{48}\,Bn\left(\frac{12\,b^3d^3x^3 + 25\,b^3c^3 - 23\,ab^2c^2d + 13\,a^2bcd^2 - 3\,a^3d^3 + 6\,(7\,b^3cd^2 - ab^2d^3)x^2 + 4\,(13\,b^3c^2d - 5\,ab^2cd^2 + a^2bd^3)x}{(b^3c^3d^5 - 3\,ab^2c^2d^6 + 3\,a^2bcd^7 - a^3d^8)g^5x^4 + 4\,(b^3c^4d^4 - 3\,ab^2c^3d^5 + 3\,a^2bc^2d^6 - a^3cd^7)g^5x^3 + 6\,(b^3c^5d^3 - 3\,ab^2c^4d^4 + 3\,a^2bc^3d^5 - a^3c^2d^6)g^5x^2 + 4\,(b^3c^6d^2 - 3\,ab^2c^5d^3 + 3\,a^2bc^4d^4 - a^3c^3d^5)g^5x + (b^3c^7d - 3\,ab^2c^6d^2 + 3\,a^2bc^5d^3 - a^3c^4d^4)g^5} + \frac{12\,b^4\log(bx+a)}{(b^4c^4d - 4\,ab^3c^3d^2 + 6\,a^2b^2c^2d^3 - 4\,a^3bcd^4 + a^4d^5)g^5} - \frac{12\,b^4\log(dx+c)}{(b^4c^4d - 4\,ab^3c^3d^2 + 6\,a^2b^2c^2d^3 - 4\,a^3bcd^4 + a^4d^5)g^5}\right) - \frac{B\log\left(\left(\frac{bx}{dx+c} + \frac{a}{dx+c}\right)^n e\right)}{4\,(d^5g^5x^4 + 4\,cd^4g^5x^3 + 6\,c^2d^3g^5x^2 + 4\,c^3d^2g^5x + c^4dg^5)} - \frac{A}{4\,(d^5g^5x^4 + 4\,cd^4g^5x^3 + 6\,c^2d^3g^5x^2 + 4\,c^3d^2g^5x + c^4dg^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*g*x+c*g)^5,x, algorithm="maxi
ma")
```

```
[Out] 1/48*B*n*((12*b^3*d^3*x^3 + 25*b^3*c^3 - 23*a*b^2*c^2*d + 13*a^2*b*c*d^2 -
3*a^3*d^3 + 6*(7*b^3*c*d^2 - a*b^2*d^3)*x^2 + 4*(13*b^3*c^2*d - 5*a*b^2*c*d
^2 + a^2*b*d^3)*x)/((b^3*c^3*d^5 - 3*a*b^2*c^2*d^6 + 3*a^2*b*c*d^7 - a^3*d^
8)*g^5*x^4 + 4*(b^3*c^4*d^4 - 3*a*b^2*c^3*d^5 + 3*a^2*b*c^2*d^6 - a^3*c*d^7
)*g^5*x^3 + 6*(b^3*c^5*d^3 - 3*a*b^2*c^4*d^4 + 3*a^2*b*c^3*d^5 - a^3*c^2*d^
6)*g^5*x^2 + 4*(b^3*c^6*d^2 - 3*a*b^2*c^5*d^3 + 3*a^2*b*c^4*d^4 - a^3*c^3*d
^5)*g^5*x + (b^3*c^7*d - 3*a*b^2*c^6*d^2 + 3*a^2*b*c^5*d^3 - a^3*c^4*d^4)*g
^5) + 12*b^4*log(b*x + a)/((b^4*c^4*d - 4*a*b^3*c^3*d^2 + 6*a^2*b^2*c^2*d^3
 - 4*a^3*b*c*d^4 + a^4*d^5)*g^5) - 12*b^4*log(d*x + c)/((b^4*c^4*d - 4*a*b^
3*c^3*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4 + a^4*d^5)*g^5)) - 1/4*B*log(
(b*x/(d*x + c) + a/(d*x + c))^n*e)/(d^5*g^5*x^4 + 4*c*d^4*g^5*x^3 + 6*c^2*d
^3*g^5*x^2 + 4*c^3*d^2*g^5*x + c^4*d*g^5) - 1/4*A/(d^5*g^5*x^4 + 4*c*d^4*g^
5*x^3 + 6*c^2*d^3*g^5*x^2 + 4*c^3*d^2*g^5*x + c^4*d*g^5)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 687 vs. $2(202) = 404$.
time = 0.38, size = 687, normalized size = 3.20

$$-\frac{12\,(A+B)b^4c^4 - 48\,(A+B)ab^3c^3d + 72\,(A+B)a^2b^2c^2d^2 - 48\,(A+B)a^3bcd^3 + 12\,(A+B)a^4d^4 - 12\,(Bb^4cd^3 - Bab^3d^4)nx^3 - 6\,(7\,Bb^4c^2d^2 - 8\,Bab^3cd^3 + Ba^2b^2d^4)nx^2 - 4\,(13\,Bb^4c^3d - 18\,Bab^3c^2d^2 + 6\,Ba^2b^2cd^3 - Ba^3bd^4)nx - (25\,Bb^4c^4 - 48\,Bab^3c^3d + 36\,Ba^2b^2c^2d^2 - 16\,Ba^3bcd^3 + 3\,Ba^4d^4)n - 12\,(Bb^4d^4nx^4 + 4\,Bb^4cd^3nx^3 + 6\,Bb^4c^2d^2nx^2 + 4\,Bb^4c^3dnx + (4\,Bab^3c^3d - 6\,Ba^2b^2c^2d^2 + 4\,Ba^3bcd^3 - Ba^4d^4)n)\log\left(\frac{bx+a}{dx+c}\right)}{48\,((b^4c^4d^5 - 4\,ab^3c^3d^6 + 6\,a^2b^2c^2d^7 - 4\,a^3bcd^8 + a^4d^9)g^5x^4 + 4\,(b^4c^5d^4 - 4\,ab^3c^4d^5 + 6\,a^2b^2c^3d^6 - 4\,a^3bc^2d^7 + a^4cd^8)g^5x^3 + 6\,(b^4c^6d^3 - 4\,ab^3c^5d^4 + 6\,a^2b^2c^4d^5 - 4\,a^3bc^3d^6 + a^4c^2d^7)g^5x^2 + 4\,(b^4c^7d^2 - 4\,ab^3c^6d^3 + 6\,a^2b^2c^5d^4 - 4\,a^3bc^4d^5 + a^4c^3d^6)g^5x + (b^4c^8d - 4\,ab^3c^7d^2 + 6\,a^2b^2c^6d^3 - 4\,a^3bc^5d^4 + a^4c^4d^5)g^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*g*x+c*g)^5,x, algorithm="fric
as")
```

```
[Out] -1/48*(12*(A + B)*b^4*c^4 - 48*(A + B)*a*b^3*c^3*d + 72*(A + B)*a^2*b^2*c^2
*d^2 - 48*(A + B)*a^3*b*c*d^3 + 12*(A + B)*a^4*d^4 - 12*(B*b^4*c*d^3 - B*a*
b^3*d^4)*n*x^3 - 6*(7*B*b^4*c^2*d^2 - 8*B*a*b^3*c*d^3 + B*a^2*b^2*d^4)*n*x^
2 - 4*(13*B*b^4*c^3*d - 18*B*a*b^3*c^2*d^2 + 6*B*a^2*b^2*c*d^3 - B*a^3*b*d^
4)*n*x - (25*B*b^4*c^4 - 48*B*a*b^3*c^3*d + 36*B*a^2*b^2*c^2*d^2 - 16*B*a^3
*b*c*d^3 + 3*B*a^4*d^4)*n - 12*(B*b^4*d^4*n*x^4 + 4*B*b^4*c*d^3*n*x^3 + 6*B
*b^4*c^2*d^2*n*x^2 + 4*B*b^4*c^3*d*n*x + (4*B*a*b^3*c^3*d - 6*B*a^2*b^2*c^2
*d^2 + 4*B*a^3*b*c*d^3 - B*a^4*d^4)*n)*log((b*x + a)/(d*x + c)))/((b^4*c^4*
d^5 - 4*a*b^3*c^3*d^6 + 6*a^2*b^2*c^2*d^7 - 4*a^3*b*c*d^8 + a^4*d^9)*g^5*x^
4 + 4*(b^4*c^5*d^4 - 4*a*b^3*c^4*d^5 + 6*a^2*b^2*c^3*d^6 - 4*a^3*b*c^2*d^7
```

```
+ a^4*c*d^8)*g^5*x^3 + 6*(b^4*c^6*d^3 - 4*a*b^3*c^5*d^4 + 6*a^2*b^2*c^4*d^5
 - 4*a^3*b*c^3*d^6 + a^4*c^2*d^7)*g^5*x^2 + 4*(b^4*c^7*d^2 - 4*a*b^3*c^6*d^
3 + 6*a^2*b^2*c^5*d^4 - 4*a^3*b*c^4*d^5 + a^4*c^3*d^6)*g^5*x + (b^4*c^8*d -
 4*a*b^3*c^7*d^2 + 6*a^2*b^2*c^6*d^3 - 4*a^3*b*c^5*d^4 + a^4*c^4*d^5)*g^5)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))/(d*g*x+c*g)**5,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 676 vs. 2(202) = 404.
time = 4.26, size = 676, normalized size = 3.14

$$\frac{1}{48}\left(12\left(\frac{4(bx+a)Bb^3n}{(b^3c^3g^5-3ab^2c^2dg^5+3a^2bcd^2g^5-a^3d^3g^5)(dx+c)}-\frac{6(bx+a)^2Bb^2dn}{(b^3c^3g^5-3ab^2c^2dg^5+3a^2bcd^2g^5-a^3d^3g^5)(dx+c)^2}+\frac{4(bx+a)^3Bbd^2n}{(b^3c^3g^5-3ab^2c^2dg^5+3a^2bcd^2g^5-a^3d^3g^5)(dx+c)^3}-\frac{(bx+a)^4Bd^3n}{(b^3c^3g^5-3ab^2c^2dg^5+3a^2bcd^2g^5-a^3d^3g^5)(dx+c)^4}\right)\log\left(\frac{bx+a}{dx+c}\right)+\frac{3(Bd^3n-4Ad^3-4Bd^3)(bx+a)^4}{(b^3c^3g^5-3ab^2c^2dg^5+3a^2bcd^2g^5-a^3d^3g^5)(dx+c)^4}-\frac{16(Bbd^2n-3Abd^2-3Bbd^2)(bx+a)^3}{(b^3c^3g^5-3ab^2c^2dg^5+3a^2bcd^2g^5-a^3d^3g^5)(dx+c)^3}+\frac{36(Bb^2dn-2Ab^2d-2Bb^2d)(bx+a)^2}{(b^3c^3g^5-3ab^2c^2dg^5+3a^2bcd^2g^5-a^3d^3g^5)(dx+c)^2}-\frac{48(Bb^3n-Ab^3-Bb^3)(bx+a)}{(b^3c^3g^5-3ab^2c^2dg^5+3a^2bcd^2g^5-a^3d^3g^5)(dx+c)}\right)\left(\frac{bc}{(bc-ad)^2}-\frac{ad}{(bc-ad)^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*g*x+c*g)^5,x, algorithm="giac
")
```

```
[Out] 1/48*(12*(4*(b*x + a)*B*b^3*n/((b^3*c^3*g^5 - 3*a*b^2*c^2*d*g^5 + 3*a^2*b*c
*d^2*g^5 - a^3*d^3*g^5)*(d*x + c)) - 6*(b*x + a)^2*B*b^2*d*n/((b^3*c^3*g^5
- 3*a*b^2*c^2*d*g^5 + 3*a^2*b*c*d^2*g^5 - a^3*d^3*g^5)*(d*x + c)^2) + 4*(b*
x + a)^3*B*b*d^2*n/((b^3*c^3*g^5 - 3*a*b^2*c^2*d*g^5 + 3*a^2*b*c*d^2*g^5 -
a^3*d^3*g^5)*(d*x + c)^3) - (b*x + a)^4*B*d^3*n/((b^3*c^3*g^5 - 3*a*b^2*c^2
*d*g^5 + 3*a^2*b*c*d^2*g^5 - a^3*d^3*g^5)*(d*x + c)^4))*log((b*x + a)/(d*x
+ c)) + 3*(B*d^3*n - 4*A*d^3 - 4*B*d^3)*(b*x + a)^4/((b^3*c^3*g^5 - 3*a*b^2
*c^2*d*g^5 + 3*a^2*b*c*d^2*g^5 - a^3*d^3*g^5)*(d*x + c)^4) - 16*(B*b*d^2*n
- 3*A*b*d^2 - 3*B*b*d^2)*(b*x + a)^3/((b^3*c^3*g^5 - 3*a*b^2*c^2*d*g^5 + 3*
a^2*b*c*d^2*g^5 - a^3*d^3*g^5)*(d*x + c)^3) + 36*(B*b^2*d*n - 2*A*b^2*d - 2
*B*b^2*d)*(b*x + a)^2/((b^3*c^3*g^5 - 3*a*b^2*c^2*d*g^5 + 3*a^2*b*c*d^2*g^5
 - a^3*d^3*g^5)*(d*x + c)^2) - 48*(B*b^3*n - A*b^3 - B*b^3)*(b*x + a)/((b^3
*c^3*g^5 - 3*a*b^2*c^2*d*g^5 + 3*a^2*b*c*d^2*g^5 - a^3*d^3*g^5)*(d*x + c)))
*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)
```

Mupad [**B**]
time = 4.99, size = 603, normalized size = 2.80

$$\frac{B\,b^4\,n\,\mathrm{atanh}\left(\frac{4\,a^4\,d^5\,g^5-8\,a^3\,b\,c\,d^4\,g^5+8\,a\,b^3\,c^3\,d^2\,g^5-4\,b^4\,c^4\,d\,g^5}{4\,d\,g^5\,{\left(a\,d-b\,c\right)}^4}+\frac{2\,b\,d\,x\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}{{\left(a\,d-b\,c\right)}^4}\right)}{2\,d\,g^5\,{\left(a\,d-b\,c\right)}^4}-\frac{B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)}{4\,d\,\left(c^4\,g^5+4\,c^3\,d\,g^5\,x+6\,c^2\,d^2\,g^5\,x^2+4\,c\,d^3\,g^5\,x^3+d^4\,g^5\,x^4\right)}-\frac{\frac{12\,A\,a^3\,d^3-12\,A\,b^3\,c^3-3\,B\,a^3\,d^3\,n+25\,B\,b^3\,c^3\,n+36\,A\,a\,b^2\,c^2\,d-36\,A\,a^2\,b\,c\,d^2-23\,B\,a\,b^2\,c^2\,d\,n+13\,B\,a^2\,b\,c\,d^2\,n}{12\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}+\frac{b\,x\,\left(B\,n\,a^2\,d^3-5\,B\,n\,a\,b\,c\,d^2+13\,B\,n\,b^2\,c^2\,d\right)}{3\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}-\frac{b^2\,x^2\,\left(B\,a\,d^3\,n-7\,B\,b\,c\,d^2\,n\right)}{2\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}+\frac{B\,b^3\,d^3\,n\,x^3}{a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3}}{4\,c^4\,d\,g^5+16\,c^3\,d^2\,g^5\,x+24\,c^2\,d^3\,g^5\,x^2+16\,c\,d^4\,g^5\,x^3+4\,d^5\,g^5\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log(e*((a + b*x)/(c + d*x))^n))/(c*g + d*g*x)^5,x)
```

```
[Out] (B*b^4*n*atanh((4*a^4*d^5*g^5 - 4*b^4*c^4*d*g^5 - 8*a^3*b*c*d^4*g^5 + 8*a*b
^3*c^3*d^2*g^5)/(4*d*g^5*(a*d - b*c)^4) + (2*b*d*x*(a^3*d^3 - b^3*c^3 + 3*a
*b^2*c^2*d - 3*a^2*b*c*d^2))/(a*d - b*c)^4))/(2*d*g^5*(a*d - b*c)^4) - (B*l
og(e*((a + b*x)/(c + d*x))^n))/(4*d*(c^4*g^5 + d^4*g^5*x^4 + 4*c*d^3*g^5*x^
3 + 6*c^2*d^2*g^5*x^2 + 4*c^3*d*g^5*x)) - ((12*A*a^3*d^3 - 12*A*b^3*c^3 - 3
*B*a^3*d^3*n + 25*B*b^3*c^3*n + 36*A*a*b^2*c^2*d - 36*A*a^2*b*c*d^2 - 23*B*
a*b^2*c^2*d*n + 13*B*a^2*b*c*d^2*n)/(12*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d
- 3*a^2*b*c*d^2)) + (b*x*(B*a^2*d^3*n + 13*B*b^2*c^2*d*n - 5*B*a*b*c*d^2*n)
)/(3*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) - (b^2*x^2*(B*a*d
^3*n - 7*B*b*c*d^2*n))/(2*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^
2)) + (B*b^3*d^3*n*x^3)/(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)
)/(4*c^4*d*g^5 + 4*d^5*g^5*x^4 + 16*c^3*d^2*g^5*x + 16*c*d^4*g^5*x^3 + 24*c
^2*d^3*g^5*x^2)
```

3.38 $\int (cg+dgx)^4 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 \, dx$

Optimal. Leaf size=544

$$\frac{13B^2(bc-ad)^4g^4n^2x}{30b^4}+\frac{7B^2(bc-ad)^3g^4n^2(c+dx)^2}{60b^3d}+\frac{B^2(bc-ad)^2g^4n^2(c+dx)^3}{30b^2d}-\frac{2B(bc-ad)^4g^4n(a+bx)}{5b}$$

```
[Out] 13/30*B^2*(-a*d+b*c)^4*g^4*n^2*x/b^4+7/60*B^2*(-a*d+b*c)^3*g^4*n^2*(d*x+c)^
2/b^3/d+1/30*B^2*(-a*d+b*c)^2*g^4*n^2*(d*x+c)^3/b^2/d-2/5*B*(-a*d+b*c)^4*g^
4*n*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b^5-1/5*B*(-a*d+b*c)^3*g^4*n*(d
*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b^3/d-2/15*B*(-a*d+b*c)^2*g^4*n*(d*
x+c)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b^2/d-1/10*B*(-a*d+b*c)*g^4*n*(d*x+c
)^4*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b/d+1/5*g^4*(d*x+c)^5*(A+B*ln(e*((b*x+a
)/(d*x+c))^n))^2/d+13/30*B^2*(-a*d+b*c)^5*g^4*n^2*ln((b*x+a)/(d*x+c))/b^5/d
+5/6*B^2*(-a*d+b*c)^5*g^4*n^2*ln(d*x+c)/b^5/d+2/5*B*(-a*d+b*c)^5*g^4*n*(A+B
*ln(e*((b*x+a)/(d*x+c))^n))*ln(1-b*(d*x+c)/d/(b*x+a))/b^5/d-2/5*B^2*(-a*d+b
*c)^5*g^4*n^2*polylog(2,b*(d*x+c)/d/(b*x+a))/b^5/d
```

Rubi [A]

time = 0.48, antiderivative size = 544, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$, Rules used = {2551, 2356, 2389, 2379, 2438, 2351, 31, 46}

$$-\frac{2B^2g^4n^2(bc-ad)^5\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{5b^5d}+\frac{2Bg^4n(bc-ad)^5\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{5b^5d}-\frac{2Bg^4n(a+bx)(bc-ad)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{5b^5}-\frac{Bg^4n(c+dx)^2(bc-ad)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{5b^3d}-\frac{2Bg^4n(c+dx)^3(bc-ad)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{15b^2d}-\frac{Bg^4n(c+dx)^4(bc-ad)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{10bd}+\frac{g^4(c+dx)^5\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{5d}+\frac{13B^2g^4n^2(bc-ad)^5\log\left(\frac{a+bx}{c+dx}\right)}{30b^5d}+\frac{5B^2g^4n^2(bc-ad)^5\log(c+dx)}{6b^5d}+\frac{13B^2g^4n^2x(bc-ad)^4}{30b^4}+\frac{7B^2g^4n^2(c+dx)^2(bc-ad)^3}{60b^3d}+\frac{B^2g^4n^2(c+dx)^3(bc-ad)^2}{30b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*g + d*g*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

```
[Out] (13*B^2*(b*c - a*d)^4*g^4*n^2*x)/(30*b^4) + (7*B^2*(b*c - a*d)^3*g^4*n^2*(c
 + d*x)^2)/(60*b^3*d) + (B^2*(b*c - a*d)^2*g^4*n^2*(c + d*x)^3)/(30*b^2*d)
- (2*B*(b*c - a*d)^4*g^4*n*(a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])
)/(5*b^5) - (B*(b*c - a*d)^3*g^4*n*(c + d*x)^2*(A + B*Log[e*((a + b*x)/(c +
 d*x))^n]))/(5*b^3*d) - (2*B*(b*c - a*d)^2*g^4*n*(c + d*x)^3*(A + B*Log[e*(
(a + b*x)/(c + d*x))^n]))/(15*b^2*d) - (B*(b*c - a*d)*g^4*n*(c + d*x)^4*(A
+ B*Log[e*((a + b*x)/(c + d*x))^n]))/(10*b*d) + (g^4*(c + d*x)^5*(A + B*Log
[e*((a + b*x)/(c + d*x))^n])^2)/(5*d) + (13*B^2*(b*c - a*d)^5*g^4*n^2*Log[(
a + b*x)/(c + d*x)])/(30*b^5*d) + (5*B^2*(b*c - a*d)^5*g^4*n^2*Log[c + d*x]
)/(6*b^5*d) + (2*B*(b*c - a*d)^5*g^4*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n
])*Log[1 - (b*(c + d*x))/(d*(a + b*x))])/(5*b^5*d) - (2*B^2*(b*c - a*d)^5*g
^4*n^2*PolyLog[2, (b*(c + d*x))/(d*(a + b*x))])/(5*b^5*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2551

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(m +
1)*(g/d)^m, Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a + b
```

```
*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ[b*c -
 a*d, 0] && IntegersQ[m, p] && EqQ[d*f - c*g, 0] && (GtQ[p, 0] || LtQ[m, -1
])
```

Rubi steps

$$\int (cg+dgx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx = \frac{g^4(c+dx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{5d} - \frac{(2Bn)\int \frac{(bc-ad)g^5}{\ldots}}{\ldots}$$

$$= \frac{g^4(c+dx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{5d} - \frac{(2B(bc-ad)g^4n\ldots}{\ldots}$$

$$= \frac{g^4(c+dx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{5d} - \frac{(2B(bc-ad)g^4n\ldots}{\ldots}$$

$$= \frac{g^4(c+dx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{5d} - \frac{(2B(bc-ad)g^4n\ldots}{\ldots}$$

$$= -\frac{2AB(bc-ad)^4g^4nx}{5b^4} - \frac{B(bc-ad)^3g^4n(c+dx)^2\left(A+\ldots\right.}{5b^3d}$$

$$= -\frac{2AB(bc-ad)^4g^4nx}{5b^4} - \frac{2B^2(bc-ad)^4g^4n(a+bx)\log\left(\ldots\right.}{5b^5}$$

$$= -\frac{2AB(bc-ad)^4g^4nx}{5b^4} - \frac{2B^2(bc-ad)^4g^4n(a+bx)\log\left(\ldots\right.}{5b^5}$$

$$= -\frac{2AB(bc-ad)^4g^4nx}{5b^4} + \frac{13B^2(bc-ad)^4g^4n^2x}{30b^4} + \frac{7B^2(b\ldots}{\ldots}$$

$$= -\frac{2AB(bc-ad)^4g^4nx}{5b^4} + \frac{13B^2(bc-ad)^4g^4n^2x}{30b^4} + \frac{7B^2(b\ldots}{\ldots}$$

$$= -\frac{2AB(bc-ad)^4g^4nx}{5b^4} + \frac{13B^2(bc-ad)^4g^4n^2x}{30b^4} + \frac{7B^2(b\ldots}{\ldots}$$

Mathematica [A]
time = 0.34, size = 533, normalized size = 0.98

$$\frac{g^4\left((c+dx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2-\frac{B(bc-ad)n\left(24Abd(bc-ad)^3x-12B(bc-ad)^3n(bdx+(bc-ad)\log(a+bx))-4B(bc-ad)^2n\left(2bd(bc-ad)x+b^2(c+dx)^2+2(bc-ad)^2\log(a+bx)\right)-B(bc-ad)n\left(6bd(bc-ad)^2x+3b^2(bc-ad)(c+dx)^2+2b^3(c+dx)^3+6(bc-ad)^3\log(a+bx)\right)+24Bd(bc-ad)^3(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+12b^2(bc-ad)^2(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)+8b^3(bc-ad)(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)+6b^4(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)+24(bc-ad)^4\log(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)-24B(bc-ad)^4n\log(c+dx)-12B(bc-ad)^4n\left(\log(a+bx)\left(\log(a+bx)-2\log\left(\frac{b(c+dx)}{bc-ad}\right)\right)-2\mathrm{Li}_2\left(\frac{d(a+bx)}{-bc+ad}\right)\right)\right)}{12b^5}\right)}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*g + d*g*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

```
[Out] (g^4*((c + d*x)^5*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 - (B*(b*c - a*d)
*n*(24*A*b*d*(b*c - a*d)^3*x - 12*B*(b*c - a*d)^3*n*(b*d*x + (b*c - a*d)*Lo
g[a + b*x]) - 4*B*(b*c - a*d)^2*n*(2*b*d*(b*c - a*d)*x + b^2*(c + d*x)^2 +
2*(b*c - a*d)^2*Log[a + b*x]) - B*(b*c - a*d)*n*(6*b*d*(b*c - a*d)^2*x + 3*
b^2*(b*c - a*d)*(c + d*x)^2 + 2*b^3*(c + d*x)^3 + 6*(b*c - a*d)^3*Log[a + b
*x]) + 24*B*d*(b*c - a*d)^3*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n] + 12*b
^2*(b*c - a*d)^2*(c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + 8*b^3
*(b*c - a*d)*(c + d*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + 6*b^4*(c
+ d*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + 24*(b*c - a*d)^4*Log[a +
b*x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - 24*B*(b*c - a*d)^4*n*Log[c +
d*x] - 12*B*(b*c - a*d)^4*n*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x
))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)])))/(12*b^5)))
/(5*d)
```

Maple [F]
time = 0.09, size = 0, normalized size = 0.00

$$\int (dgx+cg)^4 \left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*g*x+c*g)^4*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

```
[Out] int((d*g*x+c*g)^4*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 2693 vs. 2(525) = 1050.
time = 0.82, size = 2693, normalized size = 4.95

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*g*x+c*g)^4*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="ma
xima")
```

```
[Out] 2/5*A*B*d^4*g^4*x^5*log((b*x/(d*x + c) + a/(d*x + c))^n*e) + 1/5*A^2*d^4*g^
4*x^5 + 2*A*B*c*d^3*g^4*x^4*log((b*x/(d*x + c) + a/(d*x + c))^n*e) + A^2*c*
d^3*g^4*x^4 + 4*A*B*c^2*d^2*g^4*x^3*log((b*x/(d*x + c) + a/(d*x + c))^n*e)
+ 2*A^2*c^2*d^2*g^4*x^3 + 4*A*B*c^3*d*g^4*x^2*log((b*x/(d*x + c) + a/(d*x +
 c))^n*e) + 2*A^2*c^3*d*g^4*x^2 + 1/30*A*B*d^4*g^4*n*(12*a^5*log(b*x + a)/b
^5 - 12*c^5*log(d*x + c)/d^5 - (3*(b^4*c*d^3 - a*b^3*d^4)*x^4 - 4*(b^4*c^2*
d^2 - a^2*b^2*d^4)*x^3 + 6*(b^4*c^3*d - a^3*b*d^4)*x^2 - 12*(b^4*c^4 - a^4*
d^4)*x)/(b^4*d^4)) - 1/3*A*B*c*d^3*g^4*n*(6*a^4*log(b*x + a)/b^4 - 6*c^4*lo
g(d*x + c)/d^4 + (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)
*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3)) + 2*A*B*c^2*d^2*g^4*n*(2*a^3*log
(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*
```

c^2 - a^2*d^2)*x)/(b^2*d^2)) - 4*A*B*c^3*d*g^4*n*(a^2*log(b*x + a)/b^2 - c^
2*log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d)) + 2*A*B*c^4*g^4*n*(a*log(b*x + a)
/b - c*log(d*x + c)/d) + 2*A*B*c^4*g^4*x*log((b*x/(d*x + c) + a/(d*x + c))^
n*e) + A^2*c^4*g^4*x - 1/30*(77*a*b^3*c^4*d*g^4*n^2 - 94*a^2*b^2*c^3*d^2*g^
4*n^2 + 54*a^3*b*c^2*d^3*g^4*n^2 - 12*a^4*c*d^4*g^4*n^2 - (25*n^2 - 12*n)*b
^4*c^5*g^4)*B^2*log(d*x + c)/(b^4*d) - 2/5*(b^5*c^5*g^4*n^2 - 5*a*b^4*c^4*d
*g^4*n^2 + 10*a^2*b^3*c^3*d^2*g^4*n^2 - 10*a^3*b^2*c^2*d^3*g^4*n^2 + 5*a^4*
b*c*d^4*g^4*n^2 - a^5*d^5*g^4*n^2)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a
*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B^2/(b^5*d) + 1/60*(12*B^2*b^
5*d^5*g^4*x^5 + 24*B^2*b^5*c^5*g^4*n^2*log(b*x + a)*log(d*x + c) - 12*B^2*b
^5*c^5*g^4*n^2*log(d*x + c)^2 - 6*(b^5*c*d^4*g^4*(n - 10) - a*b^4*d^5*g^4*n
)*B^2*x^4 + 2*((n^2 - 16*n + 60)*b^5*c^2*d^3*g^4 - 2*(n^2 - 10*n)*a*b^4*c*d
^4*g^4 + (n^2 - 4*n)*a^2*b^3*d^5*g^4)*B^2*x^3 + ((13*n^2 - 72*n + 120)*b^5*
c^3*d^2*g^4 - 3*(11*n^2 - 40*n)*a*b^4*c^2*d^3*g^4 + 3*(9*n^2 - 20*n)*a^2*b^
3*c*d^4*g^4 - (7*n^2 - 12*n)*a^3*b^2*d^5*g^4)*B^2*x^2 - 12*(5*a*b^4*c^4*d*g
^4*n^2 - 10*a^2*b^3*c^3*d^2*g^4*n^2 + 10*a^3*b^2*c^2*d^3*g^4*n^2 - 5*a^4*b*
c*d^4*g^4*n^2 + a^5*d^5*g^4*n^2)*B^2*log(b*x + a)^2 + 2*((23*n^2 - 48*n + 3
0)*b^5*c^4*d*g^4 - (79*n^2 - 120*n)*a*b^4*c^3*d^2*g^4 + 6*(17*n^2 - 20*n)*a
^2*b^3*c^2*d^3*g^4 - (59*n^2 - 60*n)*a^3*b^2*c*d^4*g^4 + (13*n^2 - 12*n)*a^
4*b*d^5*g^4)*B^2*x - 2*(12*(4*n^2 - 5*n)*a*b^4*c^4*d*g^4 - 12*(13*n^2 - 10*
n)*a^2*b^3*c^3*d^2*g^4 + 4*(49*n^2 - 30*n)*a^3*b^2*c^2*d^3*g^4 - (113*n^2 -
 60*n)*a^4*b*c*d^4*g^4 + (25*n^2 - 12*n)*a^5*d^5*g^4)*B^2*log(b*x + a) + 12
*(B^2*b^5*d^5*g^4*x^5 + 5*B^2*b^5*c*d^4*g^4*x^4 + 10*B^2*b^5*c^2*d^3*g^4*x^
3 + 10*B^2*b^5*c^3*d^2*g^4*x^2 + 5*B^2*b^5*c^4*d*g^4*x)*log((b*x + a)^n)^2
+ 12*(B^2*b^5*d^5*g^4*x^5 + 5*B^2*b^5*c*d^4*g^4*x^4 + 10*B^2*b^5*c^2*d^3*g^
4*x^3 + 10*B^2*b^5*c^3*d^2*g^4*x^2 + 5*B^2*b^5*c^4*d*g^4*x)*log((d*x + c)^n
)^2 + 2*(12*B^2*b^5*d^5*g^4*x^5 - 12*B^2*b^5*c^5*g^4*n*log(d*x + c) - 3*(b^
5*c*d^4*g^4*(n - 20) - a*b^4*d^5*g^4*n)*B^2*x^4 - 4*(2*b^5*c^2*d^3*g^4*(2*n
 - 15) - 5*a*b^4*c*d^4*g^4*n + a^2*b^3*d^5*g^4*n)*B^2*x^3 - 6*(2*b^5*c^3*d^
2*g^4*(3*n - 10) - 10*a*b^4*c^2*d^3*g^4*n + 5*a^2*b^3*c*d^4*g^4*n - a^3*b^2
*d^5*g^4*n)*B^2*x^2 - 12*(b^5*c^4*d*g^4*(4*n - 5) - 10*a*b^4*c^3*d^2*g^4*n
+ 10*a^2*b^3*c^2*d^3*g^4*n - 5*a^3*b^2*c*d^4*g^4*n + a^4*b*d^5*g^4*n)*B^2*x
 + 12*(5*a*b^4*c^4*d*g^4*n - 10*a^2*b^3*c^3*d^2*g^4*n + 10*a^3*b^2*c^2*d^3*
g^4*n - 5*a^4*b*c*d^4*g^4*n + a^5*d^5*g^4*n)*B^2*log(b*x + a))*log((b*x + a
)^n) - 2*(12*B^2*b^5*d^5*g^4*x^5 - 12*B^2*b^5*c^5*g^4*n*log(d*x + c) - 3*(b
^5*c*d^4*g^4*(n - 20) - a*b^4*d^5*g^4*n)*B^2*x^4 - 4*(2*b^5*c^2*d^3*g^4*(2*
n - 15) - 5*a*b^4*c*d^4*g^4*n + a^2*b^3*d^5*g^4*n)*B^2*x^3 - 6*(2*b^5*c^3*d
^2*g^4*(3*n - 10) - 10*a*b^4*c^2*d^3*g^4*n + 5*a^2*b^3*c*d^4*g^4*n - a^3*b^
2*d^5*g^4*n)*B^2*x^2 - 12*(b^5*c^4*d*g^4*(4*n - 5) - 10*a*b^4*c^3*d^2*g^4*n
 + 10*a^2*b^3*c^2*d^3*g^4*n - 5*a^3*b^2*c*d^4*g^4*n + a^4*b*d^5*g^4*n)*B^2*
x + 12*(5*a*b^4*c^4*d*g^4*n - 10*a^2*b^3*c^3*d^2*g^4*n + 10*a^3*b^2*c^2*d^3
*g^4*n - 5*a^4*b*c*d^4*g^4*n + a^5*d^5*g^4*n)*B^2*log(b*x + a) + 12*(B^2*b^
5*d^5*g^4*x^5 + 5*B^2*b^5*c*d^4*g^4*x^4 + 10*B^2*b^5*c^2*d^3*g^4*x^3 + 10*B
^2*b^5*c^3*d^2*g^4*x^2 + 5*B^2*b^5*c^4*d*g^4*x)*log((b*x + a)^n))*log((d*x
+ c)^n))/(b^5*d)

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*g*x+c*g)^4*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="fr
icas")
```

```
[Out] integral(A^2*d^4*g^4*x^4 + 4*A^2*c*d^3*g^4*x^3 + 6*A^2*c^2*d^2*g^4*x^2 + 4*
A^2*c^3*d*g^4*x + A^2*c^4*g^4 + (B^2*d^4*g^4*x^4 + 4*B^2*c*d^3*g^4*x^3 + 6*
B^2*c^2*d^2*g^4*x^2 + 4*B^2*c^3*d*g^4*x + B^2*c^4*g^4)*log(((b*x + a)/(d*x
+ c))^n*e)^2 + 2*(A*B*d^4*g^4*x^4 + 4*A*B*c*d^3*g^4*x^3 + 6*A*B*c^2*d^2*g^4
*x^2 + 4*A*B*c^3*d*g^4*x + A*B*c^4*g^4)*log(((b*x + a)/(d*x + c))^n*e), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*g*x+c*g)**4*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*g*x+c*g)^4*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="gi
ac")
```

```
[Out] integrate((d*g*x + c*g)^4*(B*log(((b*x + a)/(d*x + c))^n*e) + A)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (c\,g+d\,g\,x)^4\,{\left(A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\right)}^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*g + d*g*x)^4*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2,x)
```

```
[Out] int((c*g + d*g*x)^4*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2, x)
```

3.39 $\int (cg+dgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$

Optimal. Leaf size=454

$$\frac{5B^2(bc-ad)^3g^3n^2x}{12b^3}+\frac{B^2(bc-ad)^2g^3n^2(c+dx)^2}{12b^2d}-\frac{B(bc-ad)^3g^3n(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2b^4}-\frac{B(bc-$$

```
[Out] 5/12*B^2*(-a*d+b*c)^3*g^3*n^2*x/b^3+1/12*B^2*(-a*d+b*c)^2*g^3*n^2*(d*x+c)^2
/b^2/d-1/2*B*(-a*d+b*c)^3*g^3*n*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b^4
-1/4*B*(-a*d+b*c)^2*g^3*n*(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b^2/d-1
/6*B*(-a*d+b*c)*g^3*n*(d*x+c)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b/d+1/4*g^3
*(d*x+c)^4*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/d+5/12*B^2*(-a*d+b*c)^4*g^3*n^
2*ln((b*x+a)/(d*x+c))/b^4/d+11/12*B^2*(-a*d+b*c)^4*g^3*n^2*ln(d*x+c)/b^4/d+
1/2*B*(-a*d+b*c)^4*g^3*n*(A+B*ln(e*((b*x+a)/(d*x+c))^n))*ln(1-b*(d*x+c)/d/(
b*x+a))/b^4/d-1/2*B^2*(-a*d+b*c)^4*g^3*n^2*polylog(2,b*(d*x+c)/d/(b*x+a))/b
^4/d
```

Rubi [A]
time = 0.34, antiderivative size = 454, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$, Rules used = {2551, 2356, 2389, 2379, 2438, 2351, 31, 46}

$$-\frac{B^2g^3n^2(bc-ad)^4\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{2b^4d}+\frac{Bg^3n(bc-ad)^4\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2b^4d}-\frac{Bg^3n(a+bx)(bc-ad)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2b^4}-\frac{Bg^3n(c+dx)^2(bc-ad)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{4b^2d}-\frac{Bg^3n(c+dx)^3(bc-ad)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{6bd}+\frac{g^3(c+dx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{4d}+\frac{5B^2g^3n^2(bc-ad)^4\log\left(\frac{a+bx}{c+dx}\right)}{12b^4d}+\frac{11B^2g^3n^2(bc-ad)^4\log(c+dx)}{12b^4d}+\frac{5B^2g^3n^2x(bc-ad)^3}{12b^3}+\frac{B^2g^3n^2(c+dx)^2(bc-ad)^2}{12b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*g + d*g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

```
[Out] (5*B^2*(b*c - a*d)^3*g^3*n^2*x)/(12*b^3) + (B^2*(b*c - a*d)^2*g^3*n^2*(c +
d*x)^2)/(12*b^2*d) - (B*(b*c - a*d)^3*g^3*n*(a + b*x)*(A + B*Log[e*((a + b*
x)/(c + d*x))^n]))/(2*b^4) - (B*(b*c - a*d)^2*g^3*n*(c + d*x)^2*(A + B*Log[
e*((a + b*x)/(c + d*x))^n]))/(4*b^2*d) - (B*(b*c - a*d)*g^3*n*(c + d*x)^3*(
A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(6*b*d) + (g^3*(c + d*x)^4*(A + B*Lo
g[e*((a + b*x)/(c + d*x))^n])^2)/(4*d) + (5*B^2*(b*c - a*d)^4*g^3*n^2*Log[(
a + b*x)/(c + d*x)])/(12*b^4*d) + (11*B^2*(b*c - a*d)^4*g^3*n^2*Log[c + d*x
])/(12*b^4*d) + (B*(b*c - a*d)^4*g^3*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n
])*Log[1 - (b*(c + d*x))/(d*(a + b*x))])/(2*b^4*d) - (B^2*(b*c - a*d)^4*g^3
*n^2*PolyLog[2, (b*(c + d*x))/(d*(a + b*x))])/(2*b^4*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2551

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(m +
1)*(g/d)^m, Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a + b
*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ[b*c -
```

```
 a*d, 0] && IntegersQ[m, p] && EqQ[d*f - c*g, 0] && (GtQ[p, 0] || LtQ[m, -1
])
```

Rubi steps

$$\int (cg+dgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx = \frac{g^3(c+dx)^4 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4d} - \frac{(Bn)\int \frac{(bc-ad)g^4(c}{}}{}$$

$$= \frac{g^3(c+dx)^4 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4d} - \frac{(B(bc-ad)g^3n)}{}$$

$$= \frac{g^3(c+dx)^4 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4d} - \frac{(B(bc-ad)g^3n)}{}$$

$$= \frac{g^3(c+dx)^4 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4d} - \frac{(B(bc-ad)g^3n)}{}$$

$$= -\frac{AB(bc-ad)^3g^3nx}{2b^3} - \frac{B(bc-ad)^2g^3n(c+dx)^2\left(A+B\right.}{4b^2d}$$

$$= -\frac{AB(bc-ad)^3g^3nx}{2b^3} - \frac{B^2(bc-ad)^3g^3n(a+bx)\log\left(e\left(\right.\right.}{2b^4}$$

$$= -\frac{AB(bc-ad)^3g^3nx}{2b^3} - \frac{B^2(bc-ad)^3g^3n(a+bx)\log\left(e\left(\right.\right.}{2b^4}$$

$$= -\frac{AB(bc-ad)^3g^3nx}{2b^3} + \frac{5B^2(bc-ad)^3g^3n^2x}{12b^3} + \frac{B^2(bc-}{}$$

$$= -\frac{AB(bc-ad)^3g^3nx}{2b^3} + \frac{5B^2(bc-ad)^3g^3n^2x}{12b^3} + \frac{B^2(bc-}{}$$

$$= -\frac{AB(bc-ad)^3g^3nx}{2b^3} + \frac{5B^2(bc-ad)^3g^3n^2x}{12b^3} + \frac{B^2(bc-}{}$$

Mathematica [A]

time = 0.23, size = 409, normalized size = 0.90

$$\frac{g^3\left((c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2-\frac{B(bc-ad)n\left(6Abd(bc-ad)^2x-3B(bc-ad)^2n(bdx+(bc-ad)\log(a+bx))-B(bc-ad)n\left(2bd(bc-ad)x+b^2(c+dx)^2+2(bc-ad)^2\log(a+bx)\right)+6Bd(bc-ad)^2(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+3b^2(bc-ad)(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)+2b^3(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)+6(bc-ad)^3\log(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)-6B(bc-ad)^3n\log(c+dx)-3B(bc-ad)^3n\left(\log(a+bx)\left(\log(a+bx)-2\log\left(\frac{b(c+dx)}{bc-ad}\right)\right)-2\text{Li}_2\left(\frac{d(a+bx)}{-bc+ad}\right)\right)\right)}{3b^4}\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*g + d*g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

```
[Out] (g^3*((c + d*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 - (B*(b*c - a*d)
*n*(6*A*b*d*(b*c - a*d)^2*x - 3*B*(b*c - a*d)^2*n*(b*d*x + (b*c - a*d)*Log[
a + b*x]) - B*(b*c - a*d)*n*(2*b*d*(b*c - a*d)*x + b^2*(c + d*x)^2 + 2*(b*c
 - a*d)^2*Log[a + b*x]) + 6*B*d*(b*c - a*d)^2*(a + b*x)*Log[e*((a + b*x)/(c
 + d*x))^n] + 3*b^2*(b*c - a*d)*(c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*
x))^n]) + 2*b^3*(c + d*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + 6*(b*c
 - a*d)^3*Log[a + b*x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - 6*B*(b*c -
a*d)^3*n*Log[c + d*x] - 3*B*(b*c - a*d)^3*n*(Log[a + b*x]*(Log[a + b*x] - 2
*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d
)])))/(3*b^4)))/(4*d)
```

Maple [F]
time = 0.09, size = 0, normalized size = 0.00

$$\int (dgx + cg)^3 \left(A + B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*g*x+c*g)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

```
[Out] int((d*g*x+c*g)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1991 vs. 2(438) = 876.
time = 0.81, size = 1991, normalized size = 4.39

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*g*x+c*g)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="ma
xima")
```

```
[Out] 1/2*A*B*d^3*g^3*x^4*log((b*x/(d*x + c) + a/(d*x + c))^n*e) + 1/4*A^2*d^3*g^
3*x^4 + 2*A*B*c*d^2*g^3*x^3*log((b*x/(d*x + c) + a/(d*x + c))^n*e) + A^2*c*
d^2*g^3*x^3 + 3*A*B*c^2*d*g^3*x^2*log((b*x/(d*x + c) + a/(d*x + c))^n*e) +
3/2*A^2*c^2*d*g^3*x^2 - 1/12*A*B*d^3*g^3*n*(6*a^4*log(b*x + a)/b^4 - 6*c^4*
log(d*x + c)/d^4 + (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^
3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3)) + A*B*c*d^2*g^3*n*(2*a^3*log(b
*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^
2 - a^2*d^2)*x)/(b^2*d^2)) - 3*A*B*c^2*d*g^3*n*(a^2*log(b*x + a)/b^2 - c^2*
log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d)) + 2*A*B*c^3*g^3*n*(a*log(b*x + a)/b
 - c*log(d*x + c)/d) + 2*A*B*c^3*g^3*x*log((b*x/(d*x + c) + a/(d*x + c))^n*
e) + A^2*c^3*g^3*x - 1/12*(26*a*b^2*c^3*d*g^3*n^2 - 21*a^2*b*c^2*d^2*g^3*n^
2 + 6*a^3*c*d^3*g^3*n^2 - (11*n^2 - 6*n)*b^3*c^4*g^3)*B^2*log(d*x + c)/(b^3
*d) - 1/2*(b^4*c^4*g^3*n^2 - 4*a*b^3*c^3*d*g^3*n^2 + 6*a^2*b^2*c^2*d^2*g^3*
n^2 - 4*a^3*b*c*d^3*g^3*n^2 + a^4*d^4*g^3*n^2)*(log(b*x + a)*log((b*d*x + a
```

```
*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B^2/(b^4*d) + 1/1
2*(3*B^2*b^4*d^4*g^3*x^4 + 6*B^2*b^4*c^4*g^3*n^2*log(b*x + a)*log(d*x + c)
- 3*B^2*b^4*c^4*g^3*n^2*log(d*x + c)^2 - 2*(b^4*c*d^3*g^3*(n - 6) - a*b^3*d
^4*g^3*n)*B^2*x^3 + ((n^2 - 9*n + 18)*b^4*c^2*d^2*g^3 - 2*(n^2 - 6*n)*a*b^3
*c*d^3*g^3 + (n^2 - 3*n)*a^2*b^2*d^4*g^3)*B^2*x^2 - 3*(4*a*b^3*c^3*d*g^3*n^
2 - 6*a^2*b^2*c^2*d^2*g^3*n^2 + 4*a^3*b*c*d^3*g^3*n^2 - a^4*d^4*g^3*n^2)*B^
2*log(b*x + a)^2 + ((7*n^2 - 18*n + 12)*b^4*c^3*d*g^3 - (19*n^2 - 36*n)*a*b
^3*c^2*d^2*g^3 + (17*n^2 - 24*n)*a^2*b^2*c*d^3*g^3 - (5*n^2 - 6*n)*a^3*b*d^
4*g^3)*B^2*x - (6*(3*n^2 - 4*n)*a*b^3*c^3*d*g^3 - 9*(5*n^2 - 4*n)*a^2*b^2*c
^2*d^2*g^3 + 2*(19*n^2 - 12*n)*a^3*b*c*d^3*g^3 - (11*n^2 - 6*n)*a^4*d^4*g^3
)*B^2*log(b*x + a) + 3*(B^2*b^4*d^4*g^3*x^4 + 4*B^2*b^4*c*d^3*g^3*x^3 + 6*B
^2*b^4*c^2*d^2*g^3*x^2 + 4*B^2*b^4*c^3*d*g^3*x)*log((b*x + a)^n)^2 + 3*(B^2
*b^4*d^4*g^3*x^4 + 4*B^2*b^4*c*d^3*g^3*x^3 + 6*B^2*b^4*c^2*d^2*g^3*x^2 + 4*
B^2*b^4*c^3*d*g^3*x)*log((d*x + c)^n)^2 + (6*B^2*b^4*d^4*g^3*x^4 - 6*B^2*b^
4*c^4*g^3*n*log(d*x + c) - 2*(b^4*c*d^3*g^3*(n - 12) - a*b^3*d^4*g^3*n)*B^2
*x^3 - 3*(3*b^4*c^2*d^2*g^3*(n - 4) - 4*a*b^3*c*d^3*g^3*n + a^2*b^2*d^4*g^3
*n)*B^2*x^2 - 6*(b^4*c^3*d*g^3*(3*n - 4) - 6*a*b^3*c^2*d^2*g^3*n + 4*a^2*b^
2*c*d^3*g^3*n - a^3*b*d^4*g^3*n)*B^2*x + 6*(4*a*b^3*c^3*d*g^3*n - 6*a^2*b^2
*c^2*d^2*g^3*n + 4*a^3*b*c*d^3*g^3*n - a^4*d^4*g^3*n)*B^2*log(b*x + a))*log
((b*x + a)^n) - (6*B^2*b^4*d^4*g^3*x^4 - 6*B^2*b^4*c^4*g^3*n*log(d*x + c) -
 2*(b^4*c*d^3*g^3*(n - 12) - a*b^3*d^4*g^3*n)*B^2*x^3 - 3*(3*b^4*c^2*d^2*g^
3*(n - 4) - 4*a*b^3*c*d^3*g^3*n + a^2*b^2*d^4*g^3*n)*B^2*x^2 - 6*(b^4*c^3*d
*g^3*(3*n - 4) - 6*a*b^3*c^2*d^2*g^3*n + 4*a^2*b^2*c*d^3*g^3*n - a^3*b*d^4*
g^3*n)*B^2*x + 6*(4*a*b^3*c^3*d*g^3*n - 6*a^2*b^2*c^2*d^2*g^3*n + 4*a^3*b*c
*d^3*g^3*n - a^4*d^4*g^3*n)*B^2*log(b*x + a) + 6*(B^2*b^4*d^4*g^3*x^4 + 4*B
^2*b^4*c*d^3*g^3*x^3 + 6*B^2*b^4*c^2*d^2*g^3*x^2 + 4*B^2*b^4*c^3*d*g^3*x)*l
og((b*x + a)^n))*log((d*x + c)^n))/(b^4*d)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*g*x+c*g)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="fr
icas")
```

```
[Out] integral(A^2*d^3*g^3*x^3 + 3*A^2*c*d^2*g^3*x^2 + 3*A^2*c^2*d*g^3*x + A^2*c^
3*g^3 + (B^2*d^3*g^3*x^3 + 3*B^2*c*d^2*g^3*x^2 + 3*B^2*c^2*d*g^3*x + B^2*c^
3*g^3)*log(((b*x + a)/(d*x + c))^n*e)^2 + 2*(A*B*d^3*g^3*x^3 + 3*A*B*c*d^2*
g^3*x^2 + 3*A*B*c^2*d*g^3*x + A*B*c^3*g^3)*log(((b*x + a)/(d*x + c))^n*e),
x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$g^{3}\left(\int A^{2}c^{3}\,dx + \int A^{2}d^{3}x^{3}\,dx + \int B^{2}c^{3}\log{\left(e\left(\frac{a}{c+dx} + \frac{bx}{c+dx}\right)^{n}\right)}^{2}\,dx + \int 2ABc^{3}\log{\left(e\left(\frac{a}{c+dx} + \frac{bx}{c+dx}\right)^{n}\right)}\,dx + \int 3A^{2}cd^{2}x^{2}\,dx + \int 3A^{2}c^{2}dx\,dx + \int B^{2}d^{3}x^{3}\log{\left(e\left(\frac{a}{c+dx} + \frac{bx}{c+dx}\right)^{n}\right)}^{2}\,dx + \int 2ABd^{3}x^{3}\log{\left(e\left(\frac{a}{c+dx} + \frac{bx}{c+dx}\right)^{n}\right)}\,dx + \int 3B^{2}cd^{2}x^{2}\log{\left(e\left(\frac{a}{c+dx} + \frac{bx}{c+dx}\right)^{n}\right)}^{2}\,dx + \int 3B^{2}c^{2}dx\log{\left(e\left(\frac{a}{c+dx} + \frac{bx}{c+dx}\right)^{n}\right)}^{2}\,dx + \int 6ABcd^{2}x^{2}\log{\left(e\left(\frac{a}{c+dx} + \frac{bx}{c+dx}\right)^{n}\right)}\,dx + \int 6ABc^{2}dx\log{\left(e\left(\frac{a}{c+dx} + \frac{bx}{c+dx}\right)^{n}\right)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*g*x+c*g)**3*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
```

```
[Out] g**3*(Integral(A**2*c**3, x) + Integral(A**2*d**3*x**3, x) + Integral(B**2*
c**3*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)**2, x) + Integral(2*A*B*c**3*l
og(e*(a/(c + d*x) + b*x/(c + d*x))**n), x) + Integral(3*A**2*c*d**2*x**2, x
) + Integral(3*A**2*c**2*d*x, x) + Integral(B**2*d**3*x**3*log(e*(a/(c + d*
x) + b*x/(c + d*x))**n)**2, x) + Integral(2*A*B*d**3*x**3*log(e*(a/(c + d*x
) + b*x/(c + d*x))**n), x) + Integral(3*B**2*c*d**2*x**2*log(e*(a/(c + d*x)
 + b*x/(c + d*x))**n)**2, x) + Integral(3*B**2*c**2*d*x*log(e*(a/(c + d*x)
+ b*x/(c + d*x))**n)**2, x) + Integral(6*A*B*c*d**2*x**2*log(e*(a/(c + d*x)
 + b*x/(c + d*x))**n), x) + Integral(6*A*B*c**2*d*x*log(e*(a/(c + d*x) + b*
x/(c + d*x))**n), x))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*g*x+c*g)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="gi
ac")
```

```
[Out] integrate((d*g*x + c*g)^3*(B*log(((b*x + a)/(d*x + c))^n*e) + A)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int (c\,g+d\,g\,x)^3\,\left(A+B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*g + d*g*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2,x)
```

```
[Out] int((c*g + d*g*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2, x)
```

3.40 $\int (cg+dgx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$

Optimal. Leaf size=361

$$\frac{B^2(bc-ad)^2g^2n^2x}{3b^2}-\frac{2B(bc-ad)^2g^2n(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3b^3}-\frac{B(bc-ad)g^2n(c+dx)^2\left(A+B\log\right.}{3bd}$$

```
[Out] 1/3*B^2*(-a*d+b*c)^2*g^2*n^2*x/b^2-2/3*B*(-a*d+b*c)^2*g^2*n*(b*x+a)*(A+B*ln
(e*((b*x+a)/(d*x+c))^n))/b^3-1/3*B*(-a*d+b*c)*g^2*n*(d*x+c)^2*(A+B*ln(e*((b
*x+a)/(d*x+c))^n))/b/d+1/3*g^2*(d*x+c)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/
d+1/3*B^2*(-a*d+b*c)^3*g^2*n^2*ln((b*x+a)/(d*x+c))/b^3/d+B^2*(-a*d+b*c)^3*g
^2*n^2*ln(d*x+c)/b^3/d+2/3*B*(-a*d+b*c)^3*g^2*n*(A+B*ln(e*((b*x+a)/(d*x+c))
^n))*ln(1-b*(d*x+c)/d/(b*x+a))/b^3/d-2/3*B^2*(-a*d+b*c)^3*g^2*n^2*polylog(2
,b*(d*x+c)/d/(b*x+a))/b^3/d
```

Rubi [A]
time = 0.25, antiderivative size = 361, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$, Rules used = {2551, 2356, 2389, 2379, 2438, 2351, 31, 46}

$$-\frac{2B^2g^2n^2(bc-ad)^3\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{3b^3d}+\frac{2Bg^2n(bc-ad)^3\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3b^3d}-\frac{2Bg^2n(a+bx)(bc-ad)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3b^3}-\frac{Bg^2n(c+dx)^2(bc-ad)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3bd}+\frac{g^2(c+dx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3d}+\frac{B^2g^2n^2(bc-ad)^3\log\left(\frac{a+bx}{c+dx}\right)}{3b^3d}+\frac{B^2g^2n^2(bc-ad)^3\log(c+dx)}{b^3d}+\frac{B^2g^2n^2x(bc-ad)^2}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c*g + d*g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

```
[Out] (B^2*(b*c - a*d)^2*g^2*n^2*x)/(3*b^2) - (2*B*(b*c - a*d)^2*g^2*n*(a + b*x)*
(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(3*b^3) - (B*(b*c - a*d)*g^2*n*(c +
 d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(3*b*d) + (g^2*(c + d*x)^3*
(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(3*d) + (B^2*(b*c - a*d)^3*g^2*n^
2*Log[(a + b*x)/(c + d*x)])/(3*b^3*d) + (B^2*(b*c - a*d)^3*g^2*n^2*Log[c +
d*x])/(b^3*d) + (2*B*(b*c - a*d)^3*g^2*n*(A + B*Log[e*((a + b*x)/(c + d*x))
^n])*Log[1 - (b*(c + d*x))/(d*(a + b*x))])/(3*b^3*d) - (2*B^2*(b*c - a*d)^3
*g^2*n^2*PolyLog[2, (b*(c + d*x))/(d*(a + b*x))])/(3*b^3*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2551

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(m +
1)*(g/d)^m, Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a + b
*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ[b*c -
 a*d, 0] && IntegersQ[m, p] && EqQ[d*f - c*g, 0] && (GtQ[p, 0] || LtQ[m, -1
])
```

Rubi steps

$$\int (cg+dgx)^2 \left(A+B\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx = \frac{g^2(c+dx)^3 \left(A+B\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3d} - \frac{(2Bn)\int \frac{(bc-ad)g^3}{\cdots}}{\cdots}$$

$$= \frac{g^2(c+dx)^3 \left(A+B\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3d} - \frac{(2B(bc-ad)g^2n \cdots}{\cdots}$$

$$= \frac{g^2(c+dx)^3 \left(A+B\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3d} - \frac{(2B(bc-ad)g^2n \cdots}{\cdots}$$

$$= \frac{g^2(c+dx)^3 \left(A+B\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3d} - \frac{(2B(bc-ad)g^2n \cdots}{\cdots}$$

$$= -\frac{2AB(bc-ad)^2g^2nx}{3b^2} - \frac{B(bc-ad)g^2n(c+dx)^2 \left(A+ \cdots\right.}{3bd}$$

$$= -\frac{2AB(bc-ad)^2g^2nx}{3b^2} - \frac{2B^2(bc-ad)^2g^2n(a+bx)\log \left(\cdots\right.}{3b^3}$$

$$= -\frac{2AB(bc-ad)^2g^2nx}{3b^2} - \frac{2B^2(bc-ad)^2g^2n(a+bx)\log \left(\cdots\right.}{3b^3}$$

$$= -\frac{2AB(bc-ad)^2g^2nx}{3b^2} + \frac{B^2(bc-ad)^2g^2n^2x}{3b^2} + \frac{B^2(bc- \cdots}{\cdots}$$

$$= -\frac{2AB(bc-ad)^2g^2nx}{3b^2} + \frac{B^2(bc-ad)^2g^2n^2x}{3b^2} + \frac{B^2(bc- \cdots}{\cdots}$$

$$= -\frac{2AB(bc-ad)^2g^2nx}{3b^2} + \frac{B^2(bc-ad)^2g^2n^2x}{3b^2} + \frac{B^2(bc- \cdots}{\cdots}$$

Mathematica [A]
time = 0.16, size = 303, normalized size = 0.84

$$\frac{g^2\left((c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2-\frac{B(bc-ad)n\left(2Abd(bc-ad)x-B(bc-ad)n(bdx+(bc-ad)\log(a+bx))+2Bd(bc-ad)(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+b^2(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)+2(bc-ad)^2\log(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)-2B(bc-ad)^2n\log(c+dx)-B(bc-ad)^2n\left(\log(a+bx)\left(\log(a+bx)-2\log\left(\frac{b(c+dx)}{bc-ad}\right)\right)-2\text{Li}_2\left(\frac{d(a+bx)}{-bc+ad}\right)\right)\right)}{b^3}\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*g + d*g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

```
[Out] (g^2*((c + d*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 - (B*(b*c - a*d)
*n*(2*A*b*d*(b*c - a*d)*x - B*(b*c - a*d)*n*(b*d*x + (b*c - a*d)*Log[a + b*
x]) + 2*B*d*(b*c - a*d)*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n] + b^2*(c +
 d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + 2*(b*c - a*d)^2*Log[a + b*
```

```
x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - 2*B*(b*c - a*d)^2*n*Log[c + d*x
] - B*(b*c - a*d)^2*n*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*
c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)])))/b^3))/(3*d)
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int (dgx+cg)^2 \left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*g*x+c*g)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

```
[Out] int((d*g*x+c*g)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1382 vs. 2(350) = 700.
time = 0.75, size = 1382, normalized size = 3.83

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*g*x+c*g)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="ma
xima")
```

```
[Out] 2/3*A*B*d^2*g^2*x^3*log((b*x/(d*x + c) + a/(d*x + c))^n*e) + 1/3*A^2*d^2*g^
2*x^3 + 2*A*B*c*d*g^2*x^2*log((b*x/(d*x + c) + a/(d*x + c))^n*e) + A^2*c*d*
g^2*x^2 + 1/3*A*B*d^2*g^2*n*(2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^
3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2)) - 2*A*B*
c*d*g^2*n*(a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d
)) + 2*A*B*c^2*g^2*n*(a*log(b*x + a)/b - c*log(d*x + c)/d) + 2*A*B*c^2*g^2*
x*log((b*x/(d*x + c) + a/(d*x + c))^n*e) + A^2*c^2*g^2*x - 1/3*(5*a*b*c^2*d
*g^2*n^2 - 2*a^2*c*d^2*g^2*n^2 - (3*n^2 - 2*n)*b^2*c^3*g^2)*B^2*log(d*x + c
)/(b^2*d) - 2/3*(b^3*c^3*g^2*n^2 - 3*a*b^2*c^2*d*g^2*n^2 + 3*a^2*b*c*d^2*g^
2*n^2 - a^3*d^3*g^2*n^2)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) +
 dilog(-(b*d*x + a*d)/(b*c - a*d)))*B^2/(b^3*d) + 1/3*(2*B^2*b^3*c^3*g^2*n^
2*log(b*x + a)*log(d*x + c) - B^2*b^3*c^3*g^2*n^2*log(d*x + c)^2 + B^2*b^3*
d^3*g^2*x^3 - (b^3*c*d^2*g^2*(n - 3) - a*b^2*d^3*g^2*n)*B^2*x^2 - (3*a*b^2*
c^2*d*g^2*n^2 - 3*a^2*b*c*d^2*g^2*n^2 + a^3*d^3*g^2*n^2)*B^2*log(b*x + a)^2
 + ((n^2 - 4*n + 3)*b^3*c^2*d*g^2 - 2*(n^2 - 3*n)*a*b^2*c*d^2*g^2 + (n^2 -
2*n)*a^2*b*d^3*g^2)*B^2*x - (2*(2*n^2 - 3*n)*a*b^2*c^2*d*g^2 - (7*n^2 - 6*n
)*a^2*b*c*d^2*g^2 + (3*n^2 - 2*n)*a^3*d^3*g^2)*B^2*log(b*x + a) + (B^2*b^3*
d^3*g^2*x^3 + 3*B^2*b^3*c*d^2*g^2*x^2 + 3*B^2*b^3*c^2*d*g^2*x)*log((b*x + a
)^n)^2 + (B^2*b^3*d^3*g^2*x^3 + 3*B^2*b^3*c*d^2*g^2*x^2 + 3*B^2*b^3*c^2*d*g
^2*x)*log((d*x + c)^n)^2 + (2*B^2*b^3*d^3*g^2*x^3 - 2*B^2*b^3*c^3*g^2*n*log
(d*x + c) - (b^3*c*d^2*g^2*(n - 6) - a*b^2*d^3*g^2*n)*B^2*x^2 - 2*(b^3*c^2*
```

```
d*g^2*(2*n - 3) - 3*a*b^2*c*d^2*g^2*n + a^2*b*d^3*g^2*n)*B^2*x + 2*(3*a*b^2
*c^2*d*g^2*n - 3*a^2*b*c*d^2*g^2*n + a^3*d^3*g^2*n)*B^2*log(b*x + a))*log((
b*x + a)^n) - (2*B^2*b^3*d^3*g^2*x^3 - 2*B^2*b^3*c^3*g^2*n*log(d*x + c) - (
b^3*c*d^2*g^2*(n - 6) - a*b^2*d^3*g^2*n)*B^2*x^2 - 2*(b^3*c^2*d*g^2*(2*n -
3) - 3*a*b^2*c*d^2*g^2*n + a^2*b*d^3*g^2*n)*B^2*x + 2*(3*a*b^2*c^2*d*g^2*n
- 3*a^2*b*c*d^2*g^2*n + a^3*d^3*g^2*n)*B^2*log(b*x + a) + 2*(B^2*b^3*d^3*g^
2*x^3 + 3*B^2*b^3*c*d^2*g^2*x^2 + 3*B^2*b^3*c^2*d*g^2*x)*log((b*x + a)^n))*
log((d*x + c)^n))/(b^3*d)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*g*x+c*g)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="fr
icas")
```

```
[Out] integral(A^2*d^2*g^2*x^2 + 2*A^2*c*d*g^2*x + A^2*c^2*g^2 + (B^2*d^2*g^2*x^2
 + 2*B^2*c*d*g^2*x + B^2*c^2*g^2)*log(((b*x + a)/(d*x + c))^n*e)^2 + 2*(A*B
*d^2*g^2*x^2 + 2*A*B*c*d*g^2*x + A*B*c^2*g^2)*log(((b*x + a)/(d*x + c))^n*e
), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$g^{2}\left(\int A^{2}c^{2}\,dx+\int A^{2}d^{2}x^{2}\,dx+\int B^{2}c^{2}\log{\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^{n}\right)}^{2}\,dx+\int 2ABc^{2}\log{\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^{n}\right)}\,dx+\int 2A^{2}cdx\,dx+\int B^{2}d^{2}x^{2}\log{\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^{n}\right)}^{2}\,dx+\int 2ABd^{2}x^{2}\log{\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^{n}\right)}\,dx+\int 2B^{2}cdx\log{\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^{n}\right)}^{2}\,dx+\int 4ABcdx\log{\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^{n}\right)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*g*x+c*g)**2*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
```

```
[Out] g**2*(Integral(A**2*c**2, x) + Integral(A**2*d**2*x**2, x) + Integral(B**2*
c**2*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)**2, x) + Integral(2*A*B*c**2*l
og(e*(a/(c + d*x) + b*x/(c + d*x))**n), x) + Integral(2*A**2*c*d*x, x) + In
tegral(B**2*d**2*x**2*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)**2, x) + Inte
gral(2*A*B*d**2*x**2*log(e*(a/(c + d*x) + b*x/(c + d*x))**n), x) + Integral
(2*B**2*c*d*x*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)**2, x) + Integral(4*A
*B*c*d*x*log(e*(a/(c + d*x) + b*x/(c + d*x))**n), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*g*x+c*g)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="gi
ac")
```

```
[Out] integrate((d*g*x + c*g)^2*(B*log(((b*x + a)/(d*x + c))^n*e) + A)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (c\,g+d\,g\,x)^2\,\left(A+B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*g + d*g*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2,x)
```

```
[Out] int((c*g + d*g*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2, x)
```

3.41 $\int (cg+dgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$

Optimal. Leaf size=220

$$-\frac{B(bc-ad)gn(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b^2}+\frac{g(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2d}+\frac{B^2(bc-ad)^2gn^2\log}{b^2d}$$

```
[Out] -B*(-a*d+b*c)*g*n*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b^2+1/2*g*(d*x+c)
^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/d+B^2*(-a*d+b*c)^2*g*n^2*ln(d*x+c)/b^2
/d+B*(-a*d+b*c)^2*g*n*(A+B*ln(e*((b*x+a)/(d*x+c))^n))*ln(1-b*(d*x+c)/d/(b*x
+a))/b^2/d-B^2*(-a*d+b*c)^2*g*n^2*polylog(2,b*(d*x+c)/d/(b*x+a))/b^2/d
```

Rubi [A]
time = 0.16, antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$,
Rules used = {2551, 2356, 2389, 2379, 2438, 2351, 31}

$$-\frac{B^2gn^2(bc-ad)^2\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b^2d}+\frac{Bgn(bc-ad)^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b^2d}-\frac{Bgn(a+bx)(bc-ad)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b^2}+\frac{g(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2d}+\frac{B^2gn^2(bc-ad)^2\log(c+dx)}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*g + d*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

```
[Out] -((B*(b*c - a*d)*g*n*(a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/b^2)
 + (g*(c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(2*d) + (B^2*(b
*c - a*d)^2*g*n^2*Log[c + d*x])/(b^2*d) + (B*(b*c - a*d)^2*g*n*(A + B*Log[e
*((a + b*x)/(c + d*x))^n])*Log[1 - (b*(c + d*x))/(d*(a + b*x))])/(b^2*d) -
(B^2*(b*c - a*d)^2*g*n^2*PolyLog[2, (b*(c + d*x))/(d*(a + b*x))])/(b^2*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
```

```
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2551

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(m +
1)*(g/d)^m, Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a + b
*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ[b*c -
 a*d, 0] && IntegersQ[m, p] && EqQ[d*f - c*g, 0] && (GtQ[p, 0] || LtQ[m, -1
])
```

Rubi steps

$$\int (cg+dgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx = \frac{g(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2d} - \frac{(Bn)\int \frac{(bc-ad)g^2(c+}{}}{}$$

$$= \frac{g(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2d} - \frac{(B(bc-ad)gn)\int}{}$$

$$= \frac{g(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2d} - \frac{(B(bc-ad)gn)\int}{}$$

$$= \frac{g(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2d} - \frac{(B(bc-ad)gn)\int}{}$$

$$= -\frac{AB(bc-ad)gnx}{b} - \frac{B(bc-ad)^2gn\log(a+bx)\left(A+B\right.}{b^2d}$$

$$= -\frac{AB(bc-ad)gnx}{b} - \frac{B^2(bc-ad)gn(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right.\right.}{b^2}$$

$$= -\frac{AB(bc-ad)gnx}{b} - \frac{B^2(bc-ad)gn(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right.\right.}{b^2}$$

$$= -\frac{AB(bc-ad)gnx}{b} - \frac{B^2(bc-ad)gn(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right.\right.}{b^2}$$

$$= -\frac{AB(bc-ad)gnx}{b} + \frac{B^2(bc-ad)^2gn^2\log^2(a+bx)}{2b^2d} - \frac{B^2}{}$$

$$= -\frac{AB(bc-ad)gnx}{b} + \frac{B^2(bc-ad)^2gn^2\log^2(a+bx)}{2b^2d} - \frac{B^2}{}$$

Mathematica [A]
time = 0.14, size = 216, normalized size = 0.98

$$\frac{g\left((c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2+\frac{B(bc-ad)n\left(B(bc-ad)n\log^2(a+bx)-2\left(Abdx+Bd(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+B(-bc+ad)n\log(c+dx)\right)-2(bc-ad)\log(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+Bn\log\left(\frac{b(c+dx)}{bc-ad}\right)\right)+2B(-bc+ad)n\,\mathrm{Li}_2\left(\frac{d(a+bx)}{-bc+ad}\right)\right)}{b^2}\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*g + d*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

```
[Out] (g*((c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 + (B*(b*c - a*d)*n
*(B*(b*c - a*d)*n*Log[a + b*x]^2 - 2*(A*b*d*x + B*d*(a + b*x)*Log[e*((a + b
*x)/(c + d*x))^n] + B*(-(b*c) + a*d)*n*Log[c + d*x]) - 2*(b*c - a*d)*Log[a
+ b*x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n] + B*n*Log[(b*(c + d*x))/(b*c -
```

```
 a*d)]) + 2*B*(-(b*c) + a*d)*n*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]))/b
^2))/(2*d)
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int (dgx + cg) \left(A + B \ln \left(e \left(\frac{bx + a}{dx + c}\right)^{n}\right)\right)^{2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*g*x+c*g)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

```
[Out] int((d*g*x+c*g)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 788 vs. $2(220) = 440$.
time = 0.73, size = 788, normalized size = 3.58

$$ABdgx^2 \log\left(\left(\frac{bx}{dx+c} + \frac{a}{dx+c}\right)^n e\right) + \frac{1}{2}A^2dgx^2 - ABdgn\left(\frac{a^2\log(bx+a)}{b^2} - \frac{c^2\log(dx+c)}{d^2} + \frac{(bc-ad)x}{bd}\right) + 2ABcgn\left(\frac{a\log(bx+a)}{b} - \frac{c\log(dx+c)}{d}\right) + 2ABcgx\log\left(\left(\frac{bx}{dx+c} + \frac{a}{dx+c}\right)^n e\right) + A^2cgx - \ldots$$ [illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*g*x+c*g)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="maxi
ma")
```

```
[Out] A*B*d*g*x^2*log((b*x/(d*x + c) + a/(d*x + c))^n*e) + 1/2*A^2*d*g*x^2 - A*B*
d*g*n*(a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d)) +
 2*A*B*c*g*n*(a*log(b*x + a)/b - c*log(d*x + c)/d) + 2*A*B*c*g*x*log((b*x/(
d*x + c) + a/(d*x + c))^n*e) + A^2*c*g*x - (a*c*d*g*n^2 - (n^2 - n)*b*c^2*g
)*B^2*log(d*x + c)/(b*d) - (b^2*c^2*g*n^2 - 2*a*b*c*d*g*n^2 + a^2*d^2*g*n^2
)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(
b*c - a*d)))*B^2/(b^2*d) + 1/2*(2*B^2*b^2*c^2*g*n^2*log(b*x + a)*log(d*x +
c) - B^2*b^2*c^2*g*n^2*log(d*x + c)^2 + B^2*b^2*d^2*g*x^2 - (2*a*b*c*d*g*n^
2 - a^2*d^2*g*n^2)*B^2*log(b*x + a)^2 - 2*(b^2*c*d*g*(n - 1) - a*b*d^2*g*n)
*B^2*x - 2*((n^2 - 2*n)*a*b*c*d*g - (n^2 - n)*a^2*d^2*g)*B^2*log(b*x + a) +
 (B^2*b^2*d^2*g*x^2 + 2*B^2*b^2*c*d*g*x)*log((b*x + a)^n)^2 + (B^2*b^2*d^2*
g*x^2 + 2*B^2*b^2*c*d*g*x)*log((d*x + c)^n)^2 + 2*(B^2*b^2*d^2*g*x^2 - B^2*
b^2*c^2*g*n*log(d*x + c) - (b^2*c*d*g*(n - 2) - a*b*d^2*g*n)*B^2*x + (2*a*b
*c*d*g*n - a^2*d^2*g*n)*B^2*log(b*x + a))*log((b*x + a)^n) - 2*(B^2*b^2*d^2
*g*x^2 - B^2*b^2*c^2*g*n*log(d*x + c) - (b^2*c*d*g*(n - 2) - a*b*d^2*g*n)*B
^2*x + (2*a*b*c*d*g*n - a^2*d^2*g*n)*B^2*log(b*x + a) + (B^2*b^2*d^2*g*x^2
+ 2*B^2*b^2*c*d*g*x)*log((b*x + a)^n))*log((d*x + c)^n))/(b^2*d)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*g*x+c*g)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="fric
as")
```

```
[Out] integral(A^2*d*g*x + A^2*c*g + (B^2*d*g*x + B^2*c*g)*log(((b*x + a)/(d*x +
c))^n*e)^2 + 2*(A*B*d*g*x + A*B*c*g)*log(((b*x + a)/(d*x + c))^n*e), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$g\left(\int A^2c\,dx+\int A^2dx\,dx+\int B^2c\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2dx+\int 2ABc\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)dx+\int B^2dx\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2dx+\int 2ABdx\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*g*x+c*g)*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
```

```
[Out] g*(Integral(A**2*c, x) + Integral(A**2*d*x, x) + Integral(B**2*c*log(e*(a/(
c + d*x) + b*x/(c + d*x))**n)**2, x) + Integral(2*A*B*c*log(e*(a/(c + d*x)
+ b*x/(c + d*x))**n), x) + Integral(B**2*d*x*log(e*(a/(c + d*x) + b*x/(c +
d*x))**n)**2, x) + Integral(2*A*B*d*x*log(e*(a/(c + d*x) + b*x/(c + d*x))**
n), x))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*g*x+c*g)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="giac
")
```

```
[Out] integrate((d*g*x + c*g)*(B*log(((b*x + a)/(d*x + c))^n*e) + A)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int (c\,g+d\,g\,x)\,\left(A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*g + d*g*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2,x)
```

```
[Out] int((c*g + d*g*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2, x)
```

3.42 $\int \frac{\left(A+B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{cg+dgx} \, dx$

Optimal. Leaf size=137

$$-\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log\left(\frac{bc-ad}{b(c+dx)}\right)}{dg}-\frac{2Bn\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\text{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{dg}+\frac{2B^2n^2\text{Li}_3\left(\frac{d(a+bx)}{b(c+dx)}\right)}{dg}$$

```
[Out] -(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2*ln((-a*d+b*c)/b/(d*x+c))/d/g-2*B*n*(A+B*
ln(e*((b*x+a)/(d*x+c))^n))*polylog(2,d*(b*x+a)/b/(d*x+c))/d/g+2*B^2*n^2*pol
ylog(3,d*(b*x+a)/b/(d*x+c))/d/g
```

Rubi [A]
time = 0.08, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {2551, 2354, 2421, 6724}

$$-\frac{2Bn\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{dg}+\frac{2B^2n^2\text{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)}{dg}-\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{dg}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(c*g + d*g*x),x]
```

```
[Out] -(((A + B*Log[e*((a + b*x)/(c + d*x))^n])^2*Log[(b*c - a*d)/(b*(c + d*x))])
/(d*g)) - (2*B*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*PolyLog[2, (d*(a +
b*x))/(b*(c + d*x))])/(d*g) + (2*B^2*n^2*PolyLog[3, (d*(a + b*x))/(b*(c + d
*x))])/(d*g)
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2551

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(m +
```

```
1)*(g/d)^m, Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a + b
*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ[b*c -
 a*d, 0] && IntegersQ[m, p] && EqQ[d*f - c*g, 0] && (GtQ[p, 0] || LtQ[m, -1
])
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{cg+dgx}\,dx = \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log(cg+dgx)}{dg} - \frac{(2Bn)\int \frac{(c+dx)\left(-\frac{d(a+bx)}{(c+dx)^2}+\frac{b}{c+dx}\right)\left(A\right.}{}}{dg}$$

$$= \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log(cg+dgx)}{dg} - \frac{(2Bn)\int \frac{(bc-ad)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right.\right.}{(a+bx)(c+dx)}}{dg}$$

$$= \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log(cg+dgx)}{dg} - \frac{(2B(bc-ad)n)\int \frac{\left(A+B\log\left(e\left(\frac{a+}{c+}\right.\right.\right.}{(a+b}}{dg}$$

$$= \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log(cg+dgx)}{dg} - \frac{(2B(bc-ad)n)\int \left(\frac{d\left(-A-B\log\left(\right.\right.}{(b}\right.}{}$$

$$= \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log(cg+dgx)}{dg} - \frac{(2Bn)\int \frac{\left(-A-B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log}{c+dx}}{g}$$

$$= \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log(cg+dgx)}{dg} - \frac{(2Bn)\int \left(\frac{A\log(cg+dgx)}{-c-dx}+\frac{B\log\left(e\right.}{}\right.}{g}$$

$$= \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log(cg+dgx)}{dg} - \frac{(2ABn)\int \frac{\log(cg+dgx)}{-c-dx}\,dx}{g} - (2B$$

$$= -\frac{2ABn\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(cg+dgx)}{dg} + \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log(cg}{dg}$$

$$= -\frac{2ABn\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(cg+dgx)}{dg} + \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log(cg}{dg}$$

$$= -\frac{B^2\log^2\left((a+bx)^n\right)\log(g(c+dx))}{dg} + \frac{ABn\log^2(g(c+dx))}{dg} - \frac{2B^2n\log(}{}$$

$$= \frac{B^2\log^2\left((a+bx)^n\right)\log\left(\frac{b(c+dx)}{bc-ad}\right)}{dg} - \frac{B^2\log^2\left((a+bx)^n\right)\log(g(c+dx))}{dg} +$$

$$= \frac{B^2\log^2\left((a+bx)^n\right)\log\left(\frac{b(c+dx)}{bc-ad}\right)}{dg} - \frac{B^2\log^2\left((a+bx)^n\right)\log(g(c+dx))}{dg} +$$

$$= \frac{B^2\log^2\left((a+bx)^n\right)\log\left(\frac{b(c+dx)}{bc-ad}\right)}{dg} - \frac{B^2\log^2\left((a+bx)^n\right)\log(g(c+dx))}{dg} +$$

$$= \frac{B^2\log^2\left((a+bx)^n\right)\log\left(\frac{b(c+dx)}{bc-ad}\right)}{dg} - \frac{B^2\log^2\left((a+bx)^n\right)\log(g(c+dx))}{dg} +$$

$$= \frac{B^2\log^2\left((a+bx)^n\right)\log\left(\frac{b(c+dx)}{bc-ad}\right)}{} - \frac{B^2\log^2\left((a+bx)^n\right)\log(g(c+dx))}{} +$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 537 vs. $2(137) = 274$.
time = 0.19, size = 537, normalized size = 3.92

$$\frac{3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-Bn\log\left(\frac{a+bx}{c+dx}\right)\right)^2\log(c+dx)-3Bn\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-Bn\log\left(\frac{a+bx}{c+dx}\right)\right)\left(\log^2\left(\frac{c}{d}+x\right)+2\left(\log\left(\frac{a}{b}+x\right)-\log\left(\frac{c}{d}+x\right)-\log\left(\frac{a+bx}{c+dx}\right)\right)\log(c+dx)-2\left(\log\left(\frac{a}{b}+x\right)\log\left(\frac{b(c+dx)}{bc-ad}\right)+\mathrm{Li}_2\left(\frac{d(a+bx)}{-bc+ad}\right)\right)\right)+B^2n^2\left(\log^3\left(\frac{c}{d}+x\right)+3\log^2\left(\frac{c}{d}+x\right)\left(-\log\left(\frac{a}{b}+x\right)+\log\left(\frac{d(a+bx)}{-bc+ad}\right)\right)+3\left(-\log\left(\frac{a}{b}+x\right)+\log\left(\frac{c}{d}+x\right)+\log\left(\frac{a+bx}{c+dx}\right)\right)^2\log(c+dx)+3\log^2\left(\frac{a}{b}+x\right)\log\left(\frac{b(c+dx)}{bc-ad}\right)+6\log\left(\frac{a}{b}+x\right)\mathrm{Li}_2\left(\frac{d(a+bx)}{-bc+ad}\right)+3\left(\log\left(\frac{a}{b}+x\right)-\log\left(\frac{c}{d}+x\right)-\log\left(\frac{a+bx}{c+dx}\right)\right)\left(\log^2\left(\frac{c}{d}+x\right)-2\left(\log\left(\frac{a}{b}+x\right)\log\left(\frac{b(c+dx)}{bc-ad}\right)+\mathrm{Li}_2\left(\frac{d(a+bx)}{-bc+ad}\right)\right)\right)+6\log\left(\frac{c}{d}+x\right)\mathrm{Li}_2\left(\frac{b(c+dx)}{bc-ad}\right)-6\mathrm{Li}_3\left(\frac{d(a+bx)}{-bc+ad}\right)-6\mathrm{Li}_3\left(\frac{b(c+dx)}{bc-ad}\right)\right)}{3dg}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(c*g + d*g*x),x]
```

```
[Out] (3*(A + B*Log[e*((a + b*x)/(c + d*x))^n] - B*n*Log[(a + b*x)/(c + d*x)])^2*
Log[c + d*x] - 3*B*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n] - B*n*Log[(a + b
*x)/(c + d*x)])*(Log[c/d + x]^2 + 2*(Log[a/b + x] - Log[c/d + x] - Log[(a +
 b*x)/(c + d*x)])*Log[c + d*x] - 2*(Log[a/b + x]*Log[(b*(c + d*x))/(b*c - a
*d)] + PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)])) + B^2*n^2*(Log[c/d + x]^3
 + 3*Log[c/d + x]^2*(-Log[a/b + x] + Log[(d*(a + b*x))/(-(b*c) + a*d)]) + 3
*(-Log[a/b + x] + Log[c/d + x] + Log[(a + b*x)/(c + d*x)])^2*Log[c + d*x] +
 3*Log[a/b + x]^2*Log[(b*(c + d*x))/(b*c - a*d)] + 6*Log[a/b + x]*PolyLog[2
, (d*(a + b*x))/(-(b*c) + a*d)] + 3*(Log[a/b + x] - Log[c/d + x] - Log[(a +
 b*x)/(c + d*x)])*(Log[c/d + x]^2 - 2*(Log[a/b + x]*Log[(b*(c + d*x))/(b*c
- a*d)] + PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)])) + 6*Log[c/d + x]*PolyL
og[2, (b*(c + d*x))/(b*c - a*d)] - 6*PolyLog[3, (d*(a + b*x))/(-(b*c) + a*d
)] - 6*PolyLog[3, (b*(c + d*x))/(b*c - a*d)]))/(3*d*g)
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2}{dgx+cg}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(d*g*x+c*g),x)
```

```
[Out] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(d*g*x+c*g),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*g*x+c*g),x, algorithm="maxi
ma")
```

```
[Out] B^2*log(d*x + c)*log((d*x + c)^n)^2/(d*g) + A^2*log(d*g*x + c*g)/(d*g) - in
tegrate(-(B^2*log((b*x + a)^n)^2 + 2*A*B + B^2 + 2*(A*B + B^2)*log((b*x + a
```

```
)^n) - 2*(B^2*n*log(d*x + c) + B^2*log((b*x + a)^n) + A*B + B^2)*log((d*x +
 c)^n))/(d*g*x + c*g), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*g*x+c*g),x, algorithm="fric
as")
```

```
[Out] integral((B^2*log(((b*x + a)/(d*x + c))^n*e)^2 + 2*A*B*log(((b*x + a)/(d*x
+ c))^n*e) + A^2)/(d*g*x + c*g), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A^2}{c+dx}\,dx + \int \frac{B^2 \log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2}{c+dx}\,dx + \int \frac{2AB\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}{c+dx}\,dx}{g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(d*g*x+c*g),x)
```

```
[Out] (Integral(A**2/(c + d*x), x) + Integral(B**2*log(e*(a/(c + d*x) + b*x/(c +
d*x))**n)**2/(c + d*x), x) + Integral(2*A*B*log(e*(a/(c + d*x) + b*x/(c + d
*x))**n)/(c + d*x), x))/g
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*g*x+c*g),x, algorithm="giac
")
```

```
[Out] integrate((B*log(((b*x + a)/(d*x + c))^n*e) + A)^2/(d*g*x + c*g), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\right)^2}{c\,g+d\,g\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/(c*g + d*g*x),x)
```

```
[Out] int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/(c*g + d*g*x), x)
```

3.43 $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(cg+dgx)^2}\,dx$

Optimal. Leaf size=163

$$-\frac{2ABn(a+bx)}{(bc-ad)g^2(c+dx)}+\frac{2B^2n^2(a+bx)}{(bc-ad)g^2(c+dx)}-\frac{2B^2n(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(bc-ad)g^2(c+dx)}+\frac{(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)g^2(c+dx)}$$

```
[Out] -2*A*B*n*(b*x+a)/(-a*d+b*c)/g^2/(d*x+c)+2*B^2*n^2*(b*x+a)/(-a*d+b*c)/g^2/(d
*x+c)-2*B^2*n*(b*x+a)*ln(e*((b*x+a)/(d*x+c))^n)/(-a*d+b*c)/g^2/(d*x+c)+(b*x
+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*c)/g^2/(d*x+c)
```

Rubi [A]
time = 0.05, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {2551, 2333, 2332}

$$\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{g^2(c+dx)(bc-ad)}-\frac{2ABn(a+bx)}{g^2(c+dx)(bc-ad)}-\frac{2B^2n(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{g^2(c+dx)(bc-ad)}+\frac{2B^2n^2(a+bx)}{g^2(c+dx)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(c*g + d*g*x)^2,x]
```

```
[Out] (-2*A*B*n*(a + b*x))/((b*c - a*d)*g^2*(c + d*x)) + (2*B^2*n^2*(a + b*x))/((
b*c - a*d)*g^2*(c + d*x)) - (2*B^2*n*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^
n])/((b*c - a*d)*g^2*(c + d*x)) + ((a + b*x)*(A + B*Log[e*((a + b*x)/(c + d
*x))^n])^2)/((b*c - a*d)*g^2*(c + d*x))
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2551

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(m +
1)*(g/d)^m, Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a + b
*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ[b*c -
 a*d, 0] && IntegersQ[m, p] && EqQ[d*f - c*g, 0] && (GtQ[p, 0] || LtQ[m, -1
```

```
])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(cg+dgx)^2}\,dx &= -\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{dg^2(c+dx)}+\frac{(2Bn)\int\frac{(bc-ad)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{g(a+bx)(c+dx)^2}\,dx}{dg}\\
&= -\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{dg^2(c+dx)}+\frac{(2B(bc-ad)n)\int\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)(c+dx)^2}\,dx}{dg^2}\\
&= -\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{dg^2(c+dx)}+\frac{(2B(bc-ad)n)\int\left(\frac{b^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^2(a+bx)}-\right.}{}\\
&= -\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{dg^2(c+dx)}-\frac{(2Bn)\int\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(c+dx)^2}\,dx}{g^2}-\frac{(2bBn)\int\frac{A}{}}{(b}\\
&= \frac{2Bn\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{dg^2(c+dx)}+\frac{2bBn\log(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{d(bc-ad)g^2}\\
&= \frac{2Bn\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{dg^2(c+dx)}+\frac{2bBn\log(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{d(bc-ad)g^2}\\
&= \frac{2Bn\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{dg^2(c+dx)}+\frac{2bBn\log(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{d(bc-ad)g^2}\\
&= -\frac{2B^2n^2}{dg^2(c+dx)}-\frac{2bB^2n^2\log(a+bx)}{d(bc-ad)g^2}+\frac{2Bn\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{dg^2(c+dx)}+\frac{2b}{}\\
&= -\frac{2B^2n^2}{dg^2(c+dx)}-\frac{2bB^2n^2\log(a+bx)}{d(bc-ad)g^2}-\frac{bB^2n^2\log^2(a+bx)}{d(bc-ad)g^2}+\frac{2Bn(A+}{dg^2}\\
&= -\frac{2B^2n^2}{dg^2(c+dx)}-\frac{2bB^2n^2\log(a+bx)}{d(bc-ad)g^2}-\frac{bB^2n^2\log^2(a+bx)}{d(bc-ad)g^2}+\frac{2Bn(A+}{dg^2}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 0.29, size = 331, normalized size = 2.03

$$
\frac{-\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2+\frac{Bn\left(2(bc-ad)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)+2b(c+dx)\log(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)-2b(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log(c+dx)-2Bn(bc-ad+b(c+dx)\log(a+bx)-b(c+dx)\log(c+dx))-bBn(c+dx)\left(\log(a+bx)\left(\log(a+bx)-2\log\left(\frac{b(c+dx)}{bc-ad}\right)\right)-2\text{Li}_2\left(\frac{d(a+bx)}{-bc+ad}\right)\right)+bBn(c+dx)\left(\left(2\log\left(\frac{d(a+bx)}{-bc+ad}\right)-\log(c+dx)\right)\log(c+dx)+2\text{Li}_2\left(\frac{b(c+dx)}{bc-ad}\right)\right)\right)}{bc-ad}}{dg^2(c+dx)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(c*g + d*g*x)^2,x]
```

```
[Out] (-(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 + (B*n*(2*(b*c - a*d)*(A + B*Log
[e*((a + b*x)/(c + d*x))^n]) + 2*b*(c + d*x)*Log[a + b*x]*(A + B*Log[e*((a
+ b*x)/(c + d*x))^n]) - 2*b*(c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]
)*Log[c + d*x] - 2*B*n*(b*c - a*d + b*(c + d*x)*Log[a + b*x] - b*(c + d*x)*
Log[c + d*x]) - b*B*n*(c + d*x)*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c +
 d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) + b*B*n*
(c + d*x)*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x
] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(b*c - a*d))/(d*g^2*(c + d*x
))
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)\right)^{2}}{\left(dgx+cg\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(d*g*x+c*g)^2,x)
```

```
[Out] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(d*g*x+c*g)^2,x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 431 vs. $2(165) = 330$.
time = 0.30, size = 431, normalized size = 2.64

$$2\,ABn\left(\frac{1}{d^2g^2x+cdg^2}+\frac{b\log\left(bx+a\right)}{(bcd-ad^2)g^2}-\frac{b\log\left(dx+c\right)}{(bcd-ad^2)g^2}\right)+\left(2\,n\left(\frac{1}{d^2g^2x+cdg^2}+\frac{b\log\left(bx+a\right)}{(bcd-ad^2)g^2}-\frac{b\log\left(dx+c\right)}{(bcd-ad^2)g^2}\right)\log\left(\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^{n}e\right)-\frac{\left((bdx+bc)\log\left(bx+a\right)^2+(bdx+bc)\log\left(dx+c\right)^2+2\,bc-2\,ad+2\,(bdx+bc)\log\left(bx+a\right)-2\,(bdx+bc+(bdx+bc)\log\left(bx+a\right))\log\left(dx+c\right)\right)n^2}{bc^2dg^2-acd^2g^2+(bcd^2g^2-ad^3g^2)x}\right)B^2-\frac{B^2\log\left(\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^{n}e\right)^2}{d^2g^2x+cdg^2}-\frac{2\,AB\log\left(\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^{n}e\right)}{d^2g^2x+cdg^2}-\frac{A^2}{d^2g^2x+cdg^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*g*x+c*g)^2,x, algorithm="ma
xima")
```

```
[Out] 2*A*B*n*(1/(d^2*g^2*x + c*d*g^2) + b*log(b*x + a)/((b*c*d - a*d^2)*g^2) - b
*log(d*x + c)/((b*c*d - a*d^2)*g^2)) + (2*n*(1/(d^2*g^2*x + c*d*g^2) + b*lo
g(b*x + a)/((b*c*d - a*d^2)*g^2) - b*log(d*x + c)/((b*c*d - a*d^2)*g^2))*lo
g((b*x/(d*x + c) + a/(d*x + c))^n*e) - ((b*d*x + b*c)*log(b*x + a)^2 + (b*d
*x + b*c)*log(d*x + c)^2 + 2*b*c - 2*a*d + 2*(b*d*x + b*c)*log(b*x + a) - 2
*(b*d*x + b*c + (b*d*x + b*c)*log(b*x + a))*log(d*x + c))*n^2/(b*c^2*d*g^2
- a*c*d^2*g^2 + (b*c*d^2*g^2 - a*d^3*g^2)*x))*B^2 - B^2*log((b*x/(d*x + c)
+ a/(d*x + c))^n*e)^2/(d^2*g^2*x + c*d*g^2) - 2*A*B*log((b*x/(d*x + c) + a/
(d*x + c))^n*e)/(d^2*g^2*x + c*d*g^2) - A^2/(d^2*g^2*x + c*d*g^2)
```

Fricas [A]
time = 0.35, size = 215, normalized size = 1.32

$$-\frac{(A^2+2\,AB+B^2)bc-(A^2+2\,AB+B^2)ad+2\,(B^2bc-B^2ad)n^2-(B^2bdn^2x+B^2adn^2)\log\left(\frac{bx+a}{dx+c}\right)^2-2\,((AB+B^2)bc-(AB+B^2)ad)n+2\,(B^2adn^2-(AB+B^2)adn+(B^2bdn^2-(AB+B^2)bdn)x)\log\left(\frac{bx+a}{dx+c}\right)}{(bcd^2-ad^3)g^2x+(bc^2d-acd^2)g^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*g*x+c*g)^2,x, algorithm="fr
icas")
```

```
[Out] -((A^2 + 2*A*B + B^2)*b*c - (A^2 + 2*A*B + B^2)*a*d + 2*(B^2*b*c - B^2*a*d)
*n^2 - (B^2*b*d*n^2*x + B^2*a*d*n^2)*log((b*x + a)/(d*x + c))^2 - 2*((A*B +
 B^2)*b*c - (A*B + B^2)*a*d)*n + 2*(B^2*a*d*n^2 - (A*B + B^2)*a*d*n + (B^2*
b*d*n^2 - (A*B + B^2)*b*d*n)*x)*log((b*x + a)/(d*x + c)))/((b*c*d^2 - a*d^3
)*g^2*x + (b*c^2*d - a*c*d^2)*g^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A^2}{c^2+2cdx+d^2x^2}\,dx + \int \frac{B^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2}{c^2+2cdx+d^2x^2}\,dx + \int \frac{2AB\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}{c^2+2cdx+d^2x^2}\,dx}{g^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(d*g*x+c*g)**2,x)
```

```
[Out] (Integral(A**2/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(B**2*log(e*(a/(c
 + d*x) + b*x/(c + d*x))**n)**2/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral
(2*A*B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(c**2 + 2*c*d*x + d**2*x**2)
, x))/g**2
```

Giac [A]
time = 4.58, size = 164, normalized size = 1.01

$$\left(\frac{(bx+a)B^2n^2\log\left(\frac{bx+a}{dx+c}\right)^2}{(dx+c)g^2} - \frac{2\,(B^2n^2-ABn-B^2n)(bx+a)\log\left(\frac{bx+a}{dx+c}\right)}{(dx+c)g^2} + \frac{(2\,B^2n^2-2\,ABn-2\,B^2n+A^2+2\,AB+B^2)(bx+a)}{(dx+c)g^2}\right)\left(\frac{bc}{(bc-ad)^2} - \frac{ad}{(bc-ad)^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*g*x+c*g)^2,x, algorithm="gi
ac")
```

```
[Out] ((b*x + a)*B^2*n^2*log((b*x + a)/(d*x + c))^2/((d*x + c)*g^2) - 2*(B^2*n^2
- A*B*n - B^2*n)*(b*x + a)*log((b*x + a)/(d*x + c))/((d*x + c)*g^2) + (2*B^
2*n^2 - 2*A*B*n - 2*B^2*n + A^2 + 2*A*B + B^2)*(b*x + a)/((d*x + c)*g^2))*(
b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)
```

Mupad [B]
time = 5.65, size = 237, normalized size = 1.45

$$\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\left(\frac{2\,B^2\,n}{x\,d^2\,g^2+c\,d\,g^2}-\frac{2\,A\,B}{x\,d^2\,g^2+c\,d\,g^2}\right)-\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)^2\left(\frac{B^2}{d\,\left(c\,g^2+d\,g^2\,x\right)}+\frac{B^2\,b}{d\,g^2\,\left(a\,d-b\,c\right)}\right)-\frac{A^2-2\,A\,B\,n+2\,B^2\,n^2}{x\,d^2\,g^2+c\,d\,g^2}+\frac{B\,b\,n\,\mathrm{atan}\left(\frac{\left(2\,b\,d\,x+\frac{a\,d^2\,g^2+b\,c\,d\,g^2}{d\,g^2}\right)\,1{}\mathrm{i}}{a\,d-b\,c}\right)\,\left(A-B\,n\right)\,4{}\mathrm{i}}{d\,g^2\,\left(a\,d-b\,c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/(c*g + d*g*x)^2,x)
```

```
[Out] log(e*((a + b*x)/(c + d*x))^n)*((2*B^2*n)/(d^2*g^2*x + c*d*g^2) - (2*A*B)/(
d^2*g^2*x + c*d*g^2)) - log(e*((a + b*x)/(c + d*x))^n)^2*(B^2/(d*(c*g^2 + d
*g^2*x)) + (B^2*b)/(d*g^2*(a*d - b*c))) - (A^2 + 2*B^2*n^2 - 2*A*B*n)/(d^2*
g^2*x + c*d*g^2) + (B*b*n*atan(((2*b*d*x + (a*d^2*g^2 + b*c*d*g^2)/(d*g^2))
*1i)/(a*d - b*c))*(A - B*n)*4i)/(d*g^2*(a*d - b*c))
```

3.44 $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(cg+dgx)^3}\,dx$

Optimal. Leaf size=317

$$-\frac{B^2dn^2(a+bx)^2}{4(bc-ad)^2g^3(c+dx)^2}-\frac{2AbBn(a+bx)}{(bc-ad)^2g^3(c+dx)}+\frac{2bB^2n^2(a+bx)}{(bc-ad)^2g^3(c+dx)}-\frac{2bB^2n(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(bc-ad)^2g^3(c+dx)}+\frac{B}{}$$

```
[Out] -1/4*B^2*d*n^2*(b*x+a)^2/(-a*d+b*c)^2/g^3/(d*x+c)^2-2*A*b*B*n*(b*x+a)/(-a*d
+b*c)^2/g^3/(d*x+c)+2*b*B^2*n^2*(b*x+a)/(-a*d+b*c)^2/g^3/(d*x+c)-2*b*B^2*n*
(b*x+a)*ln(e*((b*x+a)/(d*x+c))^n)/(-a*d+b*c)^2/g^3/(d*x+c)+1/2*B*d*n*(b*x+a
)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^2/g^3/(d*x+c)^2-1/2*d*(b*x+a
)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*c)^2/g^3/(d*x+c)^2+b*(b*x+a)*
(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*c)^2/g^3/(d*x+c)
```

Rubi [A]
time = 0.11, antiderivative size = 317, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {2551, 2367, 2333, 2332, 2342, 2341}

$$\frac{Bdn(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2g^3(c+dx)^2(bc-ad)^2}+\frac{b(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{g^3(c+dx)(bc-ad)^2}-\frac{d(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2g^3(c+dx)^2(bc-ad)^2}-\frac{2AbBn(a+bx)}{g^3(c+dx)(bc-ad)^2}-\frac{2bB^2n(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{g^3(c+dx)(bc-ad)^2}+\frac{2bB^2n^2(a+bx)}{g^3(c+dx)(bc-ad)^2}-\frac{B^2dn^2(a+bx)^2}{4g^3(c+dx)^2(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(c*g + d*g*x)^3,x]
```

```
[Out] -1/4*(B^2*d*n^2*(a + b*x)^2)/((b*c - a*d)^2*g^3*(c + d*x)^2) - (2*A*b*B*n*(
a + b*x))/((b*c - a*d)^2*g^3*(c + d*x)) + (2*b*B^2*n^2*(a + b*x))/((b*c - a
*d)^2*g^3*(c + d*x)) - (2*b*B^2*n*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n])
/((b*c - a*d)^2*g^3*(c + d*x)) + (B*d*n*(a + b*x)^2*(A + B*Log[e*((a + b*x)
/(c + d*x))^n]))/(2*(b*c - a*d)^2*g^3*(c + d*x)^2) - (d*(a + b*x)^2*(A + B*
Log[e*((a + b*x)/(c + d*x))^n])^2)/(2*(b*c - a*d)^2*g^3*(c + d*x)^2) + (b*(
a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/((b*c - a*d)^2*g^3*(c +
d*x))
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2367

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^(r_.))^(
q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*x^n])^p, (d + e*x
^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
&& IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0] && IntegerQ[r]))
```

Rule 2551

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(m +
1)*(g/d)^m, Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a + b
*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ[b*c -
 a*d, 0] && IntegersQ[m, p] && EqQ[d*f - c*g, 0] && (GtQ[p, 0] || LtQ[m, -1
])
```

Rubi steps

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(cg+dgx)^3}\,dx = -\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2dg^3(c+dx)^2}+\frac{(Bn)\int \frac{(bc-ad)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{g^2(a+bx)(c+dx)^3}\,dx}{dg}$$

$$= -\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2dg^3(c+dx)^2}+\frac{(B(bc-ad)n)\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)(c+dx)^3}\,dx}{dg^3}$$

$$= -\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2dg^3(c+dx)^2}+\frac{(B(bc-ad)n)\int \left(\frac{b^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^3(a+bx)} - \frac{d}{\ldots}\right.}{\ldots}$$

$$= -\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2dg^3(c+dx)^2}-\frac{(Bn)\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(c+dx)^3}\,dx}{g^3}-\frac{(b^2Bn)\int \frac{A+\ldots}{\ldots}}{(bc\ldots}$$

$$= \frac{Bn\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2dg^3(c+dx)^2}+\frac{bBn\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{d(bc-ad)g^3(c+dx)}+\frac{b^2Bn\log(a\ldots}{\ldots}$$

$$= \frac{Bn\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2dg^3(c+dx)^2}+\frac{bBn\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{d(bc-ad)g^3(c+dx)}+\frac{b^2Bn\log(a\ldots}{\ldots}$$

$$= \frac{Bn\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2dg^3(c+dx)^2}+\frac{bBn\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{d(bc-ad)g^3(c+dx)}+\frac{b^2Bn\log(a\ldots}{\ldots}$$

$$= -\frac{B^2n^2}{4dg^3(c+dx)^2}-\frac{3bB^2n^2}{2d(bc-ad)g^3(c+dx)}-\frac{3b^2B^2n^2\log(a+bx)}{2d(bc-ad)^2g^3}+\frac{Bn(\ldots}{\ldots}$$

$$= -\frac{B^2n^2}{4dg^3(c+dx)^2}-\frac{3bB^2n^2}{2d(bc-ad)g^3(c+dx)}-\frac{3b^2B^2n^2\log(a+bx)}{2d(bc-ad)^2g^3}-\frac{b^2B^2\ldots}{2d\ldots}$$

$$= -\frac{B^2n^2}{4dg^3(c+dx)^2}-\frac{3bB^2n^2}{2d(bc-ad)g^3(c+dx)}-\frac{3b^2B^2n^2\log(a+bx)}{2d(bc-ad)^2g^3}-\frac{b^2B^2\ldots}{2d\ldots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 0.31, size = 464, normalized size = 1.46

$$\frac{-2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2+\frac{Bn\left(2(bc-ad)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)+4b(bc-ad)(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)+4b^2(c+dx)^2\log(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)-4b^2(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log(c+dx)-4bBn(c+dx)(bc-ad+b(c+dx)\log(a+bx)-b(c+dx)\log(c+dx))-Bn\left((bc-ad)^2+2b(bc-ad)(c+dx)+2b^2(c+dx)^2\log(a+bx)-2b^2(c+dx)^2\log(c+dx)\right)-2b^2Bn(c+dx)^2\left(\log(a+bx)\left(\log(a+bx)-2\log\left(\frac{b(c+dx)}{bc-ad}\right)\right)-2\text{Li}_2\left(\frac{d(a+bx)}{-bc+ad}\right)\right)+2b^2Bn(c+dx)^2\left(\left(2\log\left(\frac{d(a+bx)}{-bc+ad}\right)-\log(c+dx)\right)\log(c+dx)+2\text{Li}_2\left(\frac{b(c+dx)}{bc-ad}\right)\right)\right)}{(bc-ad)^2}}{4dg^3(c+dx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(c*g + d*g*x)^3,x]
```

```
[Out] (-2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 + (B*n*(2*(b*c - a*d)^2*(A + B
*Log[e*((a + b*x)/(c + d*x))^n]) + 4*b*(b*c - a*d)*(c + d*x)*(A + B*Log[e*(
```

(a + b*x)/(c + d*x))^n]) + 4*b^2*(c + d*x)^2*Log[a + b*x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - 4*b^2*(c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[c + d*x] - 4*b*B*n*(c + d*x)*(b*c - a*d + b*(c + d*x)*Log[a + b*x] - b*(c + d*x)*Log[c + d*x]) - B*n*((b*c - a*d)^2 + 2*b*(b*c - a*d)*(c + d*x) + 2*b^2*(c + d*x)^2*Log[a + b*x] - 2*b^2*(c + d*x)^2*Log[c + d*x]) - 2*b^2*B*n*(c + d*x)^2*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) + 2*b^2*B*n*(c + d*x)^2*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(b*c - a*d)^2)/(4*d*g^3*(c + d*x)^2)

Maple [**F**]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)\right)^{2}}{\left(dgx+cg\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(d*g*x+c*g)^3,x)

[Out] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(d*g*x+c*g)^3,x)

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 864 vs. 2(315) = 630.
time = 0.33, size = 864, normalized size = 2.73

$$\frac{1}{2}ABn\left(\frac{2bdx+3bc-ad}{(bcd^3-ad^4)g^3x^2+2(bc^2d^2-acd^3)g^3x+(bc^3d-ac^2d^2)g^3}+\frac{2b^2\log(bx+a)}{(b^2c^2d-2abcd^2+a^2d^3)g^3}-\frac{2b^2\log(dx+c)}{(b^2c^2d-2abcd^2+a^2d^3)g^3}\right)+\frac{1}{4}\left(2n\left(\frac{2bdx+3bc-ad}{(bcd^3-ad^4)g^3x^2+2(bc^2d^2-acd^3)g^3x+(bc^3d-ac^2d^2)g^3}+\frac{2b^2\log(bx+a)}{(b^2c^2d-2abcd^2+a^2d^3)g^3}-\frac{2b^2\log(dx+c)}{(b^2c^2d-2abcd^2+a^2d^3)g^3}\right)\log\left(\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n e\right)-\frac{\left(7b^2c^2-8abcd+a^2d^2+2(b^2d^2x^2+2b^2cdx+b^2c^2)\log(bx+a)^2+2(b^2d^2x^2+2b^2cdx+b^2c^2)\log(dx+c)^2+6(b^2cd-abd^2)x+6(b^2d^2x^2+2b^2cdx+b^2c^2)\log(bx+a)-2(3b^2d^2x^2+6b^2cdx+3b^2c^2+2(b^2d^2x^2+2b^2cdx+b^2c^2)\log(bx+a))\log(dx+c)\right)n^2}{b^2c^4dg^3-2abc^3d^2g^3+a^2c^2d^3g^3+(b^2c^2d^3g^3-2abcd^4g^3+a^2d^5g^3)x^2+2(b^2c^3d^2g^3-2abc^2d^3g^3+a^2cd^4g^3)x}\right)B^2-\frac{B^2\log\left(\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n e\right)^2}{2(d^3g^3x^2+2cd^2g^3x+c^2dg^3)}-\frac{AB\log\left(\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n e\right)}{d^3g^3x^2+2cd^2g^3x+c^2dg^3}-\frac{A^2}{2(d^3g^3x^2+2cd^2g^3x+c^2dg^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*g*x+c*g)^3,x, algorithm="maxima")

[Out] 1/2*A*B*n*((2*b*d*x + 3*b*c - a*d)/((b*c*d^3 - a*d^4)*g^3*x^2 + 2*(b*c^2*d^2 - a*c*d^3)*g^3*x + (b*c^3*d - a*c^2*d^2)*g^3) + 2*b^2*log(b*x + a)/((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*g^3) - 2*b^2*log(d*x + c)/((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*g^3)) + 1/4*(2*n*((2*b*d*x + 3*b*c - a*d)/((b*c*d^3 - a*d^4)*g^3*x^2 + 2*(b*c^2*d^2 - a*c*d^3)*g^3*x + (b*c^3*d - a*c^2*d^2)*g^3) + 2*b^2*log(b*x + a)/((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*g^3) - 2*b^2*log(d*x + c)/((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*g^3))*log((b*x/(d*x + c) + a/(d*x + c))^n*e) - (7*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(b*x + a)^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(d*x + c)^2 + 6*(b^2*c*d - a*b*d^2)*x + 6*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(b*x + a) - 2*(3*b^2*d^2*x^2 + 6*b^2*c*d*x + 3*b^2*c^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(b*x + a))*log(d*x + c))*n^2/(b^2*c^4*d*g^3 - 2*a*b*c^3*d^2*g^3 + a^2*c^2*d^3*g^3 + (b^2*c^2*d^3*g^3 - 2*a*b*c*d^4*

```
g^3 + a^2*d^5*g^3)*x^2 + 2*(b^2*c^3*d^2*g^3 - 2*a*b*c^2*d^3*g^3 + a^2*c*d^4
*g^3)*x))*B^2 - 1/2*B^2*log((b*x/(d*x + c) + a/(d*x + c))^n*e)^2/(d^3*g^3*x
^2 + 2*c*d^2*g^3*x + c^2*d*g^3) - A*B*log((b*x/(d*x + c) + a/(d*x + c))^n*e
)/(d^3*g^3*x^2 + 2*c*d^2*g^3*x + c^2*d*g^3) - 1/2*A^2/(d^3*g^3*x^2 + 2*c*d^
2*g^3*x + c^2*d*g^3)
```

Fricas [A]
time = 0.34, size = 530, normalized size = 1.67

$$-\frac{2\,(A^2+2\,AB+B^2)b^2c^2-4\,(A^2+2\,AB+B^2)abcd+2\,(A^2+2\,AB+B^2)a^2d^2+(7\,B^2b^2c^2-8\,B^2abcd+B^2a^2d^2)n^2-2\,(B^2b^2d^2n^2x^2+2\,B^2b^2cdn^2x+(2\,B^2abcd-B^2a^2d^2)n^2)\log\left(\frac{bx+a}{dx+c}\right)^2-2\,(3\,(AB+B^2)b^2c^2-4\,(AB+B^2)abcd+(AB+B^2)a^2d^2)n+2\,(3\,(B^2b^2cd-B^2abd^2)n^2-2\,((AB+B^2)b^2cd-(AB+B^2)abd^2)n)x+2\,((4\,B^2abcd-B^2a^2d^2)n^2+(3\,B^2b^2d^2n^2-2\,(AB+B^2)b^2d^2n)x^2-2\,(2\,(AB+B^2)abcd-(AB+B^2)a^2d^2)n-2\,(2\,(AB+B^2)b^2cdn-(2\,B^2b^2cd+B^2abd^2)n^2)x)\log\left(\frac{bx+a}{dx+c}\right)}{4\,((b^2c^2d^3-2\,abcd^4+a^2d^5)g^3x^2+2\,(b^2c^3d^2-2\,abc^2d^3+a^2cd^4)g^3x+(b^2c^4d-2\,abc^3d^2+a^2c^2d^3)g^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*g*x+c*g)^3,x, algorithm="fr
icas")
```

```
[Out] -1/4*(2*(A^2 + 2*A*B + B^2)*b^2*c^2 - 4*(A^2 + 2*A*B + B^2)*a*b*c*d + 2*(A^
2 + 2*A*B + B^2)*a^2*d^2 + (7*B^2*b^2*c^2 - 8*B^2*a*b*c*d + B^2*a^2*d^2)*n^
2 - 2*(B^2*b^2*d^2*n^2*x^2 + 2*B^2*b^2*c*d*n^2*x + (2*B^2*a*b*c*d - B^2*a^2
*d^2)*n^2)*log((b*x + a)/(d*x + c))^2 - 2*(3*(A*B + B^2)*b^2*c^2 - 4*(A*B +
 B^2)*a*b*c*d + (A*B + B^2)*a^2*d^2)*n + 2*(3*(B^2*b^2*c*d - B^2*a*b*d^2)*n
^2 - 2*((A*B + B^2)*b^2*c*d - (A*B + B^2)*a*b*d^2)*n)*x + 2*((4*B^2*a*b*c*d
 - B^2*a^2*d^2)*n^2 + (3*B^2*b^2*d^2*n^2 - 2*(A*B + B^2)*b^2*d^2*n)*x^2 - 2
*(2*(A*B + B^2)*a*b*c*d - (A*B + B^2)*a^2*d^2)*n - 2*(2*(A*B + B^2)*b^2*c*d
*n - (2*B^2*b^2*c*d + B^2*a*b*d^2)*n^2)*x)*log((b*x + a)/(d*x + c)))/((b^2*
c^2*d^3 - 2*a*b*c*d^4 + a^2*d^5)*g^3*x^2 + 2*(b^2*c^3*d^2 - 2*a*b*c^2*d^3 +
 a^2*c*d^4)*g^3*x + (b^2*c^4*d - 2*a*b*c^3*d^2 + a^2*c^2*d^3)*g^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A^2}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx+\int \frac{B^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx+\int \frac{2AB\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx}{g^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(d*g*x+c*g)**3,x)
```

```
[Out] (Integral(A**2/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x) + Integr
al(B**2*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)**2/(c**3 + 3*c**2*d*x + 3*c
*d**2*x**2 + d**3*x**3), x) + Integral(2*A*B*log(e*(a/(c + d*x) + b*x/(c +
d*x))**n)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x))/g**3
```

Giac [A]
time = 5.50, size = 387, normalized size = 1.22

$$\frac{1}{4}\left(2\left(\frac{2\,(bx+a)B^2bn^2}{(bcg^3-adg^3)(dx+c)}-\frac{(bx+a)^2B^2dn^2}{(bcg^3-adg^3)(dx+c)^2}\right)\log\left(\frac{bx+a}{dx+c}\right)^2+2\left(\frac{(B^2dn^2-2\,ABdn-2\,B^2dn)(bx+a)^2}{(bcg^3-adg^3)(dx+c)^2}-\frac{4\,(B^2bn^2-ABbn-B^2bn)(bx+a)}{(bcg^3-adg^3)(dx+c)}\right)\log\left(\frac{bx+a}{dx+c}\right)-\frac{(B^2dn^2-2\,ABdn-2\,B^2dn+2\,A^2d+4\,ABd+2\,B^2d)(bx+a)^2}{(bcg^3-adg^3)(dx+c)^2}+\frac{4\,(2\,B^2bn^2-2\,ABbn-2\,B^2bn+A^2b+2\,ABb+B^2b)(bx+a)}{(bcg^3-adg^3)(dx+c)}\right)\left(\frac{bc}{(bc-ad)^2}-\frac{ad}{(bc-ad)^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*g*x+c*g)^3,x, algorithm="gi
ac")
```

```
[Out] 1/4*(2*(2*(b*x + a)*B^2*b*n^2/((b*c*g^3 - a*d*g^3)*(d*x + c)) - (b*x + a)^2
*B^2*d*n^2/((b*c*g^3 - a*d*g^3)*(d*x + c)^2))*log((b*x + a)/(d*x + c))^2 +
2*((B^2*d*n^2 - 2*A*B*d*n - 2*B^2*d*n)*(b*x + a)^2/((b*c*g^3 - a*d*g^3)*(d*
x + c)^2) - 4*(B^2*b*n^2 - A*B*b*n - B^2*b*n)*(b*x + a)/((b*c*g^3 - a*d*g^3
)*(d*x + c)))*log((b*x + a)/(d*x + c)) - (B^2*d*n^2 - 2*A*B*d*n - 2*B^2*d*n
 + 2*A^2*d + 4*A*B*d + 2*B^2*d)*(b*x + a)^2/((b*c*g^3 - a*d*g^3)*(d*x + c)^
2) + 4*(2*B^2*b*n^2 - 2*A*B*b*n - 2*B^2*b*n + A^2*b + 2*A*B*b + B^2*b)*(b*x
 + a)/((b*c*g^3 - a*d*g^3)*(d*x + c)))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)
^2)
```

Mupad [B]
time = 5.47, size = 505, normalized size = 1.59

$$-\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)^2\left(\frac{B^2}{2\,d\,\left(c^2\,g^3+2\,c\,d\,g^3\,x+d^2\,g^3\,x^2\right)}-\frac{B^2\,b^2}{2\,d\,g^3\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}\right)-\frac{\frac{2\,A^2\,a\,d-2\,A^2\,b\,c+B^2\,a\,d\,n^2-7\,B^2\,b\,c\,n^2-2\,A\,B\,a\,d\,n+6\,A\,B\,b\,c\,n}{2\,\left(a\,d-b\,c\right)}-\frac{b\,x\,\left(3\,B^2\,d\,n^2-2\,A\,B\,d\,n\right)}{a\,d-b\,c}}{2\,c^2\,d\,g^3+4\,c\,d^2\,g^3\,x+2\,d^3\,g^3\,x^2}-\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\left(\frac{A\,B}{c^2\,d\,g^3+2\,c\,d^2\,g^3\,x+d^3\,g^3\,x^2}+\frac{B^2\,b^2\,\left(\frac{d^2\,g^3\,n\,x\,\left(a\,d-b\,c\right)}{b}-\frac{d\,g^3\,n\,\left(a\,d-b\,c\right)\,\left(a\,d-2\,b\,c\right)}{2\,b^2}+\frac{c\,d\,g^3\,n\,\left(a\,d-b\,c\right)}{2\,b}\right)}{d\,g^3\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)\,\left(c^2\,d\,g^3+2\,c\,d^2\,g^3\,x+d^3\,g^3\,x^2\right)}\right)-\frac{B\,b^2\,n\,\mathrm{atan}\left(\frac{\left(2\,b\,d\,x+\frac{2\,a^2\,d^3\,g^3-2\,b^2\,c^2\,d\,g^3}{2\,d\,g^3\,\left(a\,d-b\,c\right)}\right)\,1{}\mathrm{i}}{a\,d-b\,c}\right)\,\left(2\,A-3\,B\,n\right)\,1{}\mathrm{i}}{d\,g^3\,{\left(a\,d-b\,c\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/(c*g + d*g*x)^3,x)
```

```
[Out] - log(e*((a + b*x)/(c + d*x))^n)^2*(B^2/(2*d*(c^2*g^3 + d^2*g^3*x^2 + 2*c*d
*g^3*x)) - (B^2*b^2)/(2*d*g^3*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))) - ((2*A^2*a
*d - 2*A^2*b*c + B^2*a*d*n^2 - 7*B^2*b*c*n^2 - 2*A*B*a*d*n + 6*A*B*b*c*n)/(
2*(a*d - b*c)) - (b*x*(3*B^2*d*n^2 - 2*A*B*d*n))/(a*d - b*c))/(2*c^2*d*g^3
+ 2*d^3*g^3*x^2 + 4*c*d^2*g^3*x) - log(e*((a + b*x)/(c + d*x))^n)*((A*B)/(c
^2*d*g^3 + d^3*g^3*x^2 + 2*c*d^2*g^3*x) + (B^2*b^2*((d^2*g^3*n*x*(a*d - b*c
))/b - (d*g^3*n*(a*d - b*c)*(a*d - 2*b*c))/(2*b^2) + (c*d*g^3*n*(a*d - b*c)
)/(2*b)))/(d*g^3*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)*(c^2*d*g^3 + d^3*g^3*x^2 +
 2*c*d^2*g^3*x))) - (B*b^2*n*atan(((2*b*d*x + (2*a^2*d^3*g^3 - 2*b^2*c^2*d*
g^3)/(2*d*g^3*(a*d - b*c)))*1i)/(a*d - b*c))*(2*A - 3*B*n)*1i)/(d*g^3*(a*d
- b*c)^2)
```

3.45 $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(cg+dgx)^4}\,dx$

Optimal. Leaf size=429

$$\frac{2B^2d^2n^2(a+bx)^3}{27(bc-ad)^3g^4(c+dx)^3}-\frac{bB^2dn^2(a+bx)^2}{2(bc-ad)^3g^4(c+dx)^2}+\frac{2b^2B^2n^2(a+bx)}{(bc-ad)^3g^4(c+dx)}-\frac{2Bd^2n(a+bx)^3\left(A+B\log\left(e\left(\frac{a+}{c+}\right.\right.\right.}{9(bc-ad)^3g^4(c+dx)^3}$$

```
[Out] 2/27*B^2*d^2*n^2*(b*x+a)^3/(-a*d+b*c)^3/g^4/(d*x+c)^3-1/2*b*B^2*d*n^2*(b*x+
a)^2/(-a*d+b*c)^3/g^4/(d*x+c)^2+2*b^2*B^2*n^2*(b*x+a)/(-a*d+b*c)^3/g^4/(d*x
+c)-2/9*B*d^2*n*(b*x+a)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^3/g^4/
(d*x+c)^3+b*B*d*n*(b*x+a)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^3/g^
4/(d*x+c)^2-2*b^2*B*n*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^3/
g^4/(d*x+c)-1/3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/d/g^4/(d*x+c)^3+2/3*b^3*B
*n*(A+B*ln(e*((b*x+a)/(d*x+c))^n))*ln((b*x+a)/(d*x+c))/d/(-a*d+b*c)^3/g^4-1
/3*b^3*B^2*n^2*ln((b*x+a)/(d*x+c))^2/d/(-a*d+b*c)^3/g^4
```

Rubi [A]
time = 0.18, antiderivative size = 429, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2551, 2356, 45, 2372, 2338}

$$\frac{2b^3Bn\log\left(\frac{a+bx}{c+dx}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3dg^4(bc-ad)^3}-\frac{2b^2Bn(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{g^4(c+dx)(bc-ad)^3}-\frac{2Bd^2n(a+bx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{9g^4(c+dx)^3(bc-ad)^3}+\frac{bBdn(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{g^4(c+dx)^2(bc-ad)^3}-\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3dg^4(c+dx)^3}-\frac{b^3B^2n^2\log^2\left(\frac{a+bx}{c+dx}\right)}{3dg^4(bc-ad)^3}+\frac{2b^2B^2n^2(a+bx)}{g^4(c+dx)(bc-ad)^3}+\frac{2B^2d^2n^2(a+bx)^3}{27g^4(c+dx)^3(bc-ad)^3}-\frac{bB^2dn^2(a+bx)^2}{2g^4(c+dx)^2(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(c*g + d*g*x)^4,x]
```

```
[Out] (2*B^2*d^2*n^2*(a + b*x)^3)/(27*(b*c - a*d)^3*g^4*(c + d*x)^3) - (b*B^2*d*n
^2*(a + b*x)^2)/(2*(b*c - a*d)^3*g^4*(c + d*x)^2) + (2*b^2*B^2*n^2*(a + b*x
))/((b*c - a*d)^3*g^4*(c + d*x)) - (2*B*d^2*n*(a + b*x)^3*(A + B*Log[e*((a
+ b*x)/(c + d*x))^n]))/(9*(b*c - a*d)^3*g^4*(c + d*x)^3) + (b*B*d*n*(a + b*
x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/((b*c - a*d)^3*g^4*(c + d*x)^2
) - (2*b^2*B*n*(a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/((b*c - a*
d)^3*g^4*(c + d*x)) - (A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(3*d*g^4*(c
+ d*x)^3) + (2*b^3*B*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[(a + b*x)
/(c + d*x)])/(3*d*(b*c - a*d)^3*g^4) - (b^3*B^2*n^2*Log[(a + b*x)/(c + d*x)
]^2)/(3*d*(b*c - a*d)^3*g^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rule 2551

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(m +
1)*(g/d)^m, Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a + b
*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ[b*c -
 a*d, 0] && IntegersQ[m, p] && EqQ[d*f - c*g, 0] && (GtQ[p, 0] || LtQ[m, -1
])
```

Rubi steps

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(cg+dgx)^4}\,dx = -\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3dg^4(c+dx)^3} + \frac{(2Bn)\int \frac{(bc-ad)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{g^3(a+bx)(c+dx)^4}\,dx}{3dg}$$

$$= -\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3dg^4(c+dx)^3} + \frac{(2B(bc-ad)n)\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)(c+dx)^4}\,dx}{3dg^4}$$

$$= -\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3dg^4(c+dx)^3} + \frac{(2B(bc-ad)n)\int \left(\frac{b^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^4(a+bx)} - \right.}{}$$

$$= -\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3dg^4(c+dx)^3} - \frac{(2Bn)\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(c+dx)^4}\,dx}{3g^4} - \frac{(2b^3Bn)\int}{3(b}$$

$$= \frac{2Bn\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{9dg^4(c+dx)^3} + \frac{bBn\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3d(bc-ad)g^4(c+dx)^2} + \frac{2b^2Bn(A}{3d(bc}$$

$$= \frac{2Bn\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{9dg^4(c+dx)^3} + \frac{bBn\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3d(bc-ad)g^4(c+dx)^2} + \frac{2b^2Bn(A}{3d(bc}$$

$$= \frac{2Bn\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{9dg^4(c+dx)^3} + \frac{bBn\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3d(bc-ad)g^4(c+dx)^2} + \frac{2b^2Bn(A}{3d(bc}$$

$$= -\frac{2B^2n^2}{27dg^4(c+dx)^3} - \frac{5bB^2n^2}{18d(bc-ad)g^4(c+dx)^2} - \frac{11b^2B^2n^2}{9d(bc-ad)^2g^4(c+dx)} -$$

$$= -\frac{2B^2n^2}{27dg^4(c+dx)^3} - \frac{5bB^2n^2}{18d(bc-ad)g^4(c+dx)^2} - \frac{11b^2B^2n^2}{9d(bc-ad)^2g^4(c+dx)} -$$

$$= -\frac{2B^2n^2}{27dg^4(c+dx)^3} - \frac{5bB^2n^2}{18d(bc-ad)g^4(c+dx)^2} - \frac{11b^2B^2n^2}{9d(bc-ad)^2g^4(c+dx)} -$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 0.45, size = 609, normalized size = 1.42

$$\frac{-18\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 + \text{[illegible]}}{54dg^4(c+dx)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(c*g + d*g*x)^4,x]
[Out] (-18*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 + (B*n*(12*(b*c - a*d)^3*(A +
 B*Log[e*((a + b*x)/(c + d*x))^n]) + 18*b*(b*c - a*d)^2*(c + d*x)*(A + B*Lo
```

```
g[e*((a + b*x)/(c + d*x))^n]) + 36*b^2*(b*c - a*d)*(c + d*x)^2*(A + B*Log[e
*((a + b*x)/(c + d*x))^n]) + 36*b^3*(c + d*x)^3*Log[a + b*x]*(A + B*Log[e*(
(a + b*x)/(c + d*x))^n]) - 36*b^3*(c + d*x)^3*(A + B*Log[e*((a + b*x)/(c +
d*x))^n])*Log[c + d*x] - 36*b^2*B*n*(c + d*x)^2*(b*c - a*d + b*(c + d*x)*Lo
g[a + b*x] - b*(c + d*x)*Log[c + d*x]) - 9*b*B*n*(c + d*x)*((b*c - a*d)^2 +
 2*b*(b*c - a*d)*(c + d*x) + 2*b^2*(c + d*x)^2*Log[a + b*x] - 2*b^2*(c + d*
x)^2*Log[c + d*x]) - 2*B*n*(2*(b*c - a*d)^3 + 3*b*(b*c - a*d)^2*(c + d*x) +
 6*b^2*(b*c - a*d)*(c + d*x)^2 + 6*b^3*(c + d*x)^3*Log[a + b*x] - 6*b^3*(c
+ d*x)^3*Log[c + d*x]) - 18*b^3*B*n*(c + d*x)^3*(Log[a + b*x]*(Log[a + b*x]
 - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) +
 a*d)]) + 18*b^3*B*n*(c + d*x)^3*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Lo
g[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(b*c
- a*d)^3)/(54*d*g^4*(c + d*x)^3)
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)\right)^{2}}{\left(dgx+cg\right)^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(d*g*x+c*g)^4,x)
```

```
[Out] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(d*g*x+c*g)^4,x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1438 vs. $2(422) = 844$.
time = 0.41, size = 1438, normalized size = 3.35

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*g*x+c*g)^4,x, algorithm="ma
xima")
```

```
[Out] 1/9*A*B*n*((6*b^2*d^2*x^2 + 11*b^2*c^2 - 7*a*b*c*d + 2*a^2*d^2 + 3*(5*b^2*c
*d - a*b*d^2)*x)/((b^2*c^2*d^4 - 2*a*b*c*d^5 + a^2*d^6)*g^4*x^3 + 3*(b^2*c^
3*d^3 - 2*a*b*c^2*d^4 + a^2*c*d^5)*g^4*x^2 + 3*(b^2*c^4*d^2 - 2*a*b*c^3*d^3
 + a^2*c^2*d^4)*g^4*x + (b^2*c^5*d - 2*a*b*c^4*d^2 + a^2*c^3*d^3)*g^4) + 6*
b^3*log(b*x + a)/((b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*g
^4) - 6*b^3*log(d*x + c)/((b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^
3*d^4)*g^4)) + 1/54*(6*n*((6*b^2*d^2*x^2 + 11*b^2*c^2 - 7*a*b*c*d + 2*a^2*d
^2 + 3*(5*b^2*c*d - a*b*d^2)*x)/((b^2*c^2*d^4 - 2*a*b*c*d^5 + a^2*d^6)*g^4*
x^3 + 3*(b^2*c^3*d^3 - 2*a*b*c^2*d^4 + a^2*c*d^5)*g^4*x^2 + 3*(b^2*c^4*d^2
- 2*a*b*c^3*d^3 + a^2*c^2*d^4)*g^4*x + (b^2*c^5*d - 2*a*b*c^4*d^2 + a^2*c^3
```

```
*d^3)*g^4) + 6*b^3*log(b*x + a)/((b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d
^3 - a^3*d^4)*g^4) - 6*b^3*log(d*x + c)/((b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a
^2*b*c*d^3 - a^3*d^4)*g^4))*log((b*x/(d*x + c) + a/(d*x + c))^n*e) - (85*b^
3*c^3 - 108*a*b^2*c^2*d + 27*a^2*b*c*d^2 - 4*a^3*d^3 + 66*(b^3*c*d^2 - a*b^
2*d^3)*x^2 + 18*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*l
og(b*x + a)^2 + 18*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3
)*log(d*x + c)^2 + 3*(49*b^3*c^2*d - 54*a*b^2*c*d^2 + 5*a^2*b*d^3)*x + 66*(
b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*log(b*x + a) - 6*(
11*b^3*d^3*x^3 + 33*b^3*c*d^2*x^2 + 33*b^3*c^2*d*x + 11*b^3*c^3 + 6*(b^3*d^
3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*log(b*x + a))*log(d*x +
c))*n^2/(b^3*c^6*d*g^4 - 3*a*b^2*c^5*d^2*g^4 + 3*a^2*b*c^4*d^3*g^4 - a^3*c^
3*d^4*g^4 + (b^3*c^3*d^4*g^4 - 3*a*b^2*c^2*d^5*g^4 + 3*a^2*b*c*d^6*g^4 - a^
3*d^7*g^4)*x^3 + 3*(b^3*c^4*d^3*g^4 - 3*a*b^2*c^3*d^4*g^4 + 3*a^2*b*c^2*d^5
*g^4 - a^3*c*d^6*g^4)*x^2 + 3*(b^3*c^5*d^2*g^4 - 3*a*b^2*c^4*d^3*g^4 + 3*a^
2*b*c^3*d^4*g^4 - a^3*c^2*d^5*g^4)*x))*B^2 - 1/3*B^2*log((b*x/(d*x + c) + a
/(d*x + c))^n*e)^2/(d^4*g^4*x^3 + 3*c*d^3*g^4*x^2 + 3*c^2*d^2*g^4*x + c^3*d
*g^4) - 2/3*A*B*log((b*x/(d*x + c) + a/(d*x + c))^n*e)/(d^4*g^4*x^3 + 3*c*d
^3*g^4*x^2 + 3*c^2*d^2*g^4*x + c^3*d*g^4) - 1/3*A^2/(d^4*g^4*x^3 + 3*c*d^3*
g^4*x^2 + 3*c^2*d^2*g^4*x + c^3*d*g^4)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 948 vs. $2(422) = 844$.
time = 0.35, size = 948, normalized size = 2.21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*g*x+c*g)^4,x, algorithm="fr
icas")
```

```
[Out] -1/54*(18*(A^2 + 2*A*B + B^2)*b^3*c^3 - 54*(A^2 + 2*A*B + B^2)*a*b^2*c^2*d
+ 54*(A^2 + 2*A*B + B^2)*a^2*b*c*d^2 - 18*(A^2 + 2*A*B + B^2)*a^3*d^3 + (85
*B^2*b^3*c^3 - 108*B^2*a*b^2*c^2*d + 27*B^2*a^2*b*c*d^2 - 4*B^2*a^3*d^3)*n^
2 + 6*(11*(B^2*b^3*c*d^2 - B^2*a*b^2*d^3)*n^2 - 6*((A*B + B^2)*b^3*c*d^2 -
(A*B + B^2)*a*b^2*d^3)*n)*x^2 - 18*(B^2*b^3*d^3*n^2*x^3 + 3*B^2*b^3*c*d^2*n
^2*x^2 + 3*B^2*b^3*c^2*d*n^2*x + (3*B^2*a*b^2*c^2*d - 3*B^2*a^2*b*c*d^2 + B
^2*a^3*d^3)*n^2)*log((b*x + a)/(d*x + c))^2 - 6*(11*(A*B + B^2)*b^3*c^3 - 1
8*(A*B + B^2)*a*b^2*c^2*d + 9*(A*B + B^2)*a^2*b*c*d^2 - 2*(A*B + B^2)*a^3*d
^3)*n + 3*((49*B^2*b^3*c^2*d - 54*B^2*a*b^2*c*d^2 + 5*B^2*a^2*b*d^3)*n^2 -
6*(5*(A*B + B^2)*b^3*c^2*d - 6*(A*B + B^2)*a*b^2*c*d^2 + (A*B + B^2)*a^2*b*
d^3)*n)*x + 6*((11*B^2*b^3*d^3*n^2 - 6*(A*B + B^2)*b^3*d^3*n)*x^3 + (18*B^2
*a*b^2*c^2*d - 9*B^2*a^2*b*c*d^2 + 2*B^2*a^3*d^3)*n^2 - 3*(6*(A*B + B^2)*b^
3*c*d^2*n - (9*B^2*b^3*c*d^2 + 2*B^2*a*b^2*d^3)*n^2)*x^2 - 6*(3*(A*B + B^2)
*a*b^2*c^2*d - 3*(A*B + B^2)*a^2*b*c*d^2 + (A*B + B^2)*a^3*d^3)*n - 3*(6*(A
*B + B^2)*b^3*c^2*d*n - (6*B^2*b^3*c^2*d + 6*B^2*a*b^2*c*d^2 - B^2*a^2*b*d^
```

```
3)*n^2)*x)*log((b*x + a)/(d*x + c)))/((b^3*c^3*d^4 - 3*a*b^2*c^2*d^5 + 3*a^
2*b*c*d^6 - a^3*d^7)*g^4*x^3 + 3*(b^3*c^4*d^3 - 3*a*b^2*c^3*d^4 + 3*a^2*b*c
^2*d^5 - a^3*c*d^6)*g^4*x^2 + 3*(b^3*c^5*d^2 - 3*a*b^2*c^4*d^3 + 3*a^2*b*c^
3*d^4 - a^3*c^2*d^5)*g^4*x + (b^3*c^6*d - 3*a*b^2*c^5*d^2 + 3*a^2*b*c^4*d^3
 - a^3*c^3*d^4)*g^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A^2}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx + \int \frac{B^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx + \int \frac{2AB\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx}{g^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(d*g*x+c*g)**4,x)
```

```
[Out] (Integral(A**2/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**4
*x**4), x) + Integral(B**2*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)**2/(c**4
 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**4*x**4), x) + Integra
l(2*A*B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(c**4 + 4*c**3*d*x + 6*c**2
*d**2*x**2 + 4*c*d**3*x**3 + d**4*x**4), x))/g**4
```

Giac [A]
time = 3.46, size = 746, normalized size = 1.74

$$\frac{1}{54}\left(18\left(\frac{3(bx+a)B^2b^2n^2}{(b^2c^2g^4-2abcdg^4+a^2d^2g^4)(dx+c)}-\frac{3(bx+a)^2B^2bdn^2}{(b^2c^2g^4-2abcdg^4+a^2d^2g^4)(dx+c)^2}+\frac{(bx+a)^3B^2d^2n^2}{(b^2c^2g^4-2abcdg^4+a^2d^2g^4)(dx+c)^3}\right)\log\left(\frac{bx+a}{dx+c}\right)^2-6\left(\frac{2(B^2d^2n^2-3ABd^2n-3B^2d^2n)(bx+a)^3}{(b^2c^2g^4-2abcdg^4+a^2d^2g^4)(dx+c)^3}-\frac{9(B^2bdn^2-2ABbdn-2B^2bdn)(bx+a)^2}{(b^2c^2g^4-2abcdg^4+a^2d^2g^4)(dx+c)^2}+\frac{18(B^2b^2n^2-ABb^2n-B^2b^2n)(bx+a)}{(b^2c^2g^4-2abcdg^4+a^2d^2g^4)(dx+c)}\right)\log\left(\frac{bx+a}{dx+c}\right)+\frac{2(2B^2d^2n^2-6ABd^2n-6B^2d^2n+9A^2d^2+18ABd^2+9B^2d^2)(bx+a)^3}{(b^2c^2g^4-2abcdg^4+a^2d^2g^4)(dx+c)^3}-\frac{27(B^2bdn^2-2ABbdn-2B^2bdn+2A^2bd+4ABbd+2B^2bd)(bx+a)^2}{(b^2c^2g^4-2abcdg^4+a^2d^2g^4)(dx+c)^2}+\frac{54(2B^2b^2n^2-2ABb^2n-2B^2b^2n+A^2b^2+2ABb^2+B^2b^2)(bx+a)}{(b^2c^2g^4-2abcdg^4+a^2d^2g^4)(dx+c)}\right)\left(\frac{bc}{(bc-ad)^2}-\frac{ad}{(bc-ad)^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*g*x+c*g)^4,x, algorithm="gi
ac")
```

```
[Out] 1/54*(18*(3*(b*x + a)*B^2*b^2*n^2/((b^2*c^2*g^4 - 2*a*b*c*d*g^4 + a^2*d^2*g
^4)*(d*x + c)) - 3*(b*x + a)^2*B^2*b*d*n^2/((b^2*c^2*g^4 - 2*a*b*c*d*g^4 +
a^2*d^2*g^4)*(d*x + c)^2) + (b*x + a)^3*B^2*d^2*n^2/((b^2*c^2*g^4 - 2*a*b*c
*d*g^4 + a^2*d^2*g^4)*(d*x + c)^3))*log((b*x + a)/(d*x + c))^2 - 6*(2*(B^2*
d^2*n^2 - 3*A*B*d^2*n - 3*B^2*d^2*n)*(b*x + a)^3/((b^2*c^2*g^4 - 2*a*b*c*d*
g^4 + a^2*d^2*g^4)*(d*x + c)^3) - 9*(B^2*b*d*n^2 - 2*A*B*b*d*n - 2*B^2*b*d*
n)*(b*x + a)^2/((b^2*c^2*g^4 - 2*a*b*c*d*g^4 + a^2*d^2*g^4)*(d*x + c)^2) +
18*(B^2*b^2*n^2 - A*B*b^2*n - B^2*b^2*n)*(b*x + a)/((b^2*c^2*g^4 - 2*a*b*c*
d*g^4 + a^2*d^2*g^4)*(d*x + c)))*log((b*x + a)/(d*x + c)) + 2*(2*B^2*d^2*n^
2 - 6*A*B*d^2*n - 6*B^2*d^2*n + 9*A^2*d^2 + 18*A*B*d^2 + 9*B^2*d^2)*(b*x +
a)^3/((b^2*c^2*g^4 - 2*a*b*c*d*g^4 + a^2*d^2*g^4)*(d*x + c)^3) - 27*(B^2*b*
d*n^2 - 2*A*B*b*d*n - 2*B^2*b*d*n + 2*A^2*b*d + 4*A*B*b*d + 2*B^2*b*d)*(b*x
 + a)^2/((b^2*c^2*g^4 - 2*a*b*c*d*g^4 + a^2*d^2*g^4)*(d*x + c)^2) + 54*(2*B
^2*b^2*n^2 - 2*A*B*b^2*n - 2*B^2*b^2*n + A^2*b^2 + 2*A*B*b^2 + B^2*b^2)*(b*
x + a)/((b^2*c^2*g^4 - 2*a*b*c*d*g^4 + a^2*d^2*g^4)*(d*x + c)))*(b*c/(b*c -
 a*d)^2 - a*d/(b*c - a*d)^2)
```

Mupad [B]

time = 7.16, size = 1040, normalized size = 2.42

$$-\ln\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^2\left(\frac{B^2}{3d\left(c^3g^4+3c^2dg^4x+3cd^2g^4x^2+d^3g^4x^3\right)}+\frac{B^2b^3}{3dg^4\left(a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3\right)}\right)-\frac{\frac{18A^2a^2d^2-36A^2abcd+18A^2b^2c^2-12ABa^2d^2n+42ABabcdn-66ABb^2c^2n+4B^2a^2d^2n^2-23B^2abcdn^2+85B^2b^2c^2n^2}{6(ad-bc)}-\frac{x\left(-49cB^2b^2dn^2+5aB^2bd^2n^2+30AcBb^2dn-6AaBbd^2n\right)}{2(ad-bc)}+\frac{bx^2\left(11B^2bd^2n^2-6ABbd^2n\right)}{ad-bc}}{x\left(27ac^2d^3g^4-27bc^3d^2g^4\right)-x^2\left(27bc^2d^3g^4-27acd^4g^4\right)+x^3\left(9ad^5g^4-9bcd^4g^4\right)+9ac^3d^2g^4-9bc^4dg^4}-\ln\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\left(\frac{2AB}{3c^3dg^4+9c^2d^2g^4x+9cd^3g^4x^2+3d^4g^4x^3}+\frac{2B^2b^3\left(x\left(d\left(\frac{dg^4n(ad-bc)(ad-3bc)}{2b^2}-\frac{cdg^4n(ad-bc)}{b}\right)-\frac{2cd^2g^4n(ad-bc)}{b}+\frac{d^2g^4n(ad-bc)(ad-3bc)}{b^2}\right)+c\left(\frac{dg^4n(ad-bc)(ad-3bc)}{2b^2}-\frac{cdg^4n(ad-bc)}{b}\right)-\frac{dg^4n(ad-bc)\left(a^2d^2-3abcd+3b^2c^2\right)}{b^3}-\frac{3d^3g^4nx^2(ad-bc)}{b}\right)}{3dg^4\left(a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3\right)\left(3c^3dg^4+9c^2d^2g^4x+9cd^3g^4x^2+3d^4g^4x^3\right)}\right)-\frac{Bb^3n\,\mathrm{atan}\left(\frac{Bb^3n(6A-11Bn)\left(\frac{a^3d^4g^4-a^2bcd^3g^4-ab^2c^2d^2g^4+b^3c^3dg^4}{a^2d^3g^4-2abcd^2g^4+b^2c^2dg^4}+2bdx\right)\left(a^2d^3g^4-2abcd^2g^4+b^2c^2dg^4\right)1\mathrm{i}}{dg^4\left(11B^2b^3n^2-6ABb^3n\right)(ad-bc)^3}\right)(6A-11Bn)\,2\mathrm{i}}{9dg^4(ad-bc)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/(c*g + d*g*x)^4,x)
```

```
[Out] - log(e*((a + b*x)/(c + d*x))^n)^2*(B^2/(3*d*(c^3*g^4 + d^3*g^4*x^3 + 3*c*d
^2*g^4*x^2 + 3*c^2*d*g^4*x)) + (B^2*b^3)/(3*d*g^4*(a^3*d^3 - b^3*c^3 + 3*a*
b^2*c^2*d - 3*a^2*b*c*d^2))) - ((18*A^2*a^2*d^2 + 18*A^2*b^2*c^2 + 4*B^2*a^
2*d^2*n^2 + 85*B^2*b^2*c^2*n^2 - 36*A^2*a*b*c*d - 12*A*B*a^2*d^2*n - 66*A*B
*b^2*c^2*n - 23*B^2*a*b*c*d*n^2 + 42*A*B*a*b*c*d*n)/(6*(a*d - b*c)) - (x*(5
*B^2*a*b*d^2*n^2 - 49*B^2*b^2*c*d*n^2 - 6*A*B*a*b*d^2*n + 30*A*B*b^2*c*d*n)
)/(2*(a*d - b*c)) + (b*x^2*(11*B^2*b*d^2*n^2 - 6*A*B*b*d^2*n))/(a*d - b*c))
/(x*(27*a*c^2*d^3*g^4 - 27*b*c^3*d^2*g^4) - x^2*(27*b*c^2*d^3*g^4 - 27*a*c*
d^4*g^4) + x^3*(9*a*d^5*g^4 - 9*b*c*d^4*g^4) + 9*a*c^3*d^2*g^4 - 9*b*c^4*d*
g^4) - log(e*((a + b*x)/(c + d*x))^n)*((2*A*B)/(3*c^3*d*g^4 + 3*d^4*g^4*x^3
 + 9*c^2*d^2*g^4*x + 9*c*d^3*g^4*x^2) + (2*B^2*b^3*(x*(d*((d*g^4*n*(a*d - b
*c)*(a*d - 3*b*c))/(2*b^2) - (c*d*g^4*n*(a*d - b*c))/b) - (2*c*d^2*g^4*n*(a
*d - b*c))/b + (d^2*g^4*n*(a*d - b*c)*(a*d - 3*b*c))/b^2) + c*((d*g^4*n*(a*
d - b*c)*(a*d - 3*b*c))/(2*b^2) - (c*d*g^4*n*(a*d - b*c))/b) - (d*g^4*n*(a*
d - b*c)*(a^2*d^2 + 3*b^2*c^2 - 3*a*b*c*d))/b^3 - (3*d^3*g^4*n*x^2*(a*d - b
*c))/b))/(3*d*g^4*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)*(3*c^
3*d*g^4 + 3*d^4*g^4*x^3 + 9*c^2*d^2*g^4*x + 9*c*d^3*g^4*x^2))) - (B*b^3*n*a
tan((B*b^3*n*(6*A - 11*B*n)*((a^3*d^4*g^4 + b^3*c^3*d*g^4 - a^2*b*c*d^3*g^4
 - a*b^2*c^2*d^2*g^4)/(a^2*d^3*g^4 + b^2*c^2*d*g^4 - 2*a*b*c*d^2*g^4) + 2*b
*d*x)*(a^2*d^3*g^4 + b^2*c^2*d*g^4 - 2*a*b*c*d^2*g^4)*1i)/(d*g^4*(11*B^2*b^
3*n^2 - 6*A*B*b^3*n)*(a*d - b*c)^3))*(6*A - 11*B*n)*2i)/(9*d*g^4*(a*d - b*c
)^3)
```

3.46 $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(cg+dgx)^5}\,dx$

Optimal. Leaf size=536

$$-\frac{B^2d^3n^2(a+bx)^4}{32(bc-ad)^4g^5(c+dx)^4}+\frac{2bB^2d^2n^2(a+bx)^3}{9(bc-ad)^4g^5(c+dx)^3}-\frac{3b^2B^2dn^2(a+bx)^2}{4(bc-ad)^4g^5(c+dx)^2}+\frac{2b^3B^2n^2(a+bx)}{(bc-ad)^4g^5(c+dx)}+\frac{Bd^3n(a$$

```
[Out] -1/32*B^2*d^3*n^2*(b*x+a)^4/(-a*d+b*c)^4/g^5/(d*x+c)^4+2/9*b*B^2*d^2*n^2*(b
*x+a)^3/(-a*d+b*c)^4/g^5/(d*x+c)^3-3/4*b^2*B^2*d*n^2*(b*x+a)^2/(-a*d+b*c)^4
/g^5/(d*x+c)^2+2*b^3*B^2*n^2*(b*x+a)/(-a*d+b*c)^4/g^5/(d*x+c)+1/8*B*d^3*n*(
b*x+a)^4*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^4/g^5/(d*x+c)^4-2/3*b*B
*d^2*n*(b*x+a)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^4/g^5/(d*x+c)^3
+3/2*b^2*B*d*n*(b*x+a)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^4/g^5/(
d*x+c)^2-2*b^3*B*n*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^4/g^5
/(d*x+c)-1/4*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/d/g^5/(d*x+c)^4+1/2*b^4*B*n*
(A+B*ln(e*((b*x+a)/(d*x+c))^n))*ln((b*x+a)/(d*x+c))/d/(-a*d+b*c)^4/g^5-1/4*
b^4*B^2*n^2*ln((b*x+a)/(d*x+c))^2/d/(-a*d+b*c)^4/g^5
```

Rubi [A]
time = 0.20, antiderivative size = 536, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2551, 2356, 45, 2372, 2338}

$$\frac{b^4Bn\log\left(\frac{a+bx}{c+dx}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2dg^5(bc-ad)^4}-\frac{2b^3Bn(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{g^5(c+dx)(bc-ad)^4}+\frac{3b^2Bdn(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2g^5(c+dx)^2(bc-ad)^4}+\frac{Bd^3n(a+bx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{8g^5(c+dx)^4(bc-ad)^4}-\frac{2bBd^2n(a+bx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3g^5(c+dx)^3(bc-ad)^4}-\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{4dg^5(c+dx)^4}-\frac{b^4B^2n^2\log^2\left(\frac{a+bx}{c+dx}\right)}{4dg^5(bc-ad)^4}+\frac{2b^3B^2n^2(a+bx)}{g^5(c+dx)(bc-ad)^4}-\frac{3b^2B^2dn^2(a+bx)^2}{4g^5(c+dx)^2(bc-ad)^4}-\frac{B^2d^3n^2(a+bx)^4}{32g^5(c+dx)^4(bc-ad)^4}+\frac{2bB^2d^2n^2(a+bx)^3}{9g^5(c+dx)^3(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(c*g + d*g*x)^5,x]
```

```
[Out] -1/32*(B^2*d^3*n^2*(a + b*x)^4)/((b*c - a*d)^4*g^5*(c + d*x)^4) + (2*b*B^2*
d^2*n^2*(a + b*x)^3)/(9*(b*c - a*d)^4*g^5*(c + d*x)^3) - (3*b^2*B^2*d*n^2*(
a + b*x)^2)/(4*(b*c - a*d)^4*g^5*(c + d*x)^2) + (2*b^3*B^2*n^2*(a + b*x))/(
(b*c - a*d)^4*g^5*(c + d*x)) + (B*d^3*n*(a + b*x)^4*(A + B*Log[e*((a + b*x)
/(c + d*x))^n]))/(8*(b*c - a*d)^4*g^5*(c + d*x)^4) - (2*b*B*d^2*n*(a + b*x)
^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(3*(b*c - a*d)^4*g^5*(c + d*x)^3
) + (3*b^2*B*d*n*(a + b*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(2*(b*
c - a*d)^4*g^5*(c + d*x)^2) - (2*b^3*B*n*(a + b*x)*(A + B*Log[e*((a + b*x)/
(c + d*x))^n]))/((b*c - a*d)^4*g^5*(c + d*x)) - (A + B*Log[e*((a + b*x)/(c
+ d*x))^n])^2/(4*d*g^5*(c + d*x)^4) + (b^4*B*n*(A + B*Log[e*((a + b*x)/(c +
 d*x))^n])*Log[(a + b*x)/(c + d*x)])/(2*d*(b*c - a*d)^4*g^5) - (b^4*B^2*n^2
*Log[(a + b*x)/(c + d*x)]^2)/(4*d*(b*c - a*d)^4*g^5)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rule 2551

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(m +
1)*(g/d)^m, Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a + b
*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ[b*c -
 a*d, 0] && IntegersQ[m, p] && EqQ[d*f - c*g, 0] && (GtQ[p, 0] || LtQ[m, -1
])
```

Rubi steps

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(cg+dgx)^5}\,dx = -\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4dg^5(c+dx)^4} + \frac{(Bn)\int \frac{(bc-ad)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{g^4(a+bx)(c+dx)^5}\,dx}{2dg}$$

$$= -\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4dg^5(c+dx)^4} + \frac{(B(bc-ad)n)\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)(c+dx)^5}\,dx}{2dg^5}$$

$$= -\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4dg^5(c+dx)^4} + \frac{(B(bc-ad)n)\int \left(\frac{b^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^5(a+bx)} - \frac{d(\ldots}{\ldots}\right.}{\ldots}$$

$$= -\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4dg^5(c+dx)^4} - \frac{(Bn)\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(c+dx)^5}\,dx}{2g^5} - \frac{(b^4Bn)\int \frac{A+B\ldots}{\ldots}}{2(bc-\ldots}$$

$$= \frac{Bn\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{8dg^5(c+dx)^4} + \frac{bBn\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{6d(bc-ad)g^5(c+dx)^3} + \frac{b^2Bn(A+B\ldots}{4d(bc-ad\ldots}$$

$$= \frac{Bn\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{8dg^5(c+dx)^4} + \frac{bBn\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{6d(bc-ad)g^5(c+dx)^3} + \frac{b^2Bn(A+B\ldots}{4d(bc-ad\ldots}$$

$$= \frac{Bn\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{8dg^5(c+dx)^4} + \frac{bBn\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{6d(bc-ad)g^5(c+dx)^3} + \frac{b^2Bn(A+B\ldots}{4d(bc-ad\ldots}$$

$$= -\frac{B^2n^2}{32dg^5(c+dx)^4} - \frac{7bB^2n^2}{72d(bc-ad)g^5(c+dx)^3} - \frac{13b^2B^2n^2}{48d(bc-ad)^2g^5(c+dx)^2} \ldots$$

$$= -\frac{B^2n^2}{32dg^5(c+dx)^4} - \frac{7bB^2n^2}{72d(bc-ad)g^5(c+dx)^3} - \frac{13b^2B^2n^2}{48d(bc-ad)^2g^5(c+dx)^2} \ldots$$

$$= -\frac{B^2n^2}{32dg^5(c+dx)^4} - \frac{7bB^2n^2}{72d(bc-ad)g^5(c+dx)^3} - \frac{13b^2B^2n^2}{48d(bc-ad)^2g^5(c+dx)^2} \ldots$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 0.62, size = 776, normalized size = 1.45

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(c*g + d*g*x)^5,x]
```

```
[Out] (-72*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 + (B*n*(36*(b*c - a*d)^4*(A +
 B*Log[e*((a + b*x)/(c + d*x))^n]) + 48*b*(b*c - a*d)^3*(c + d*x)*(A + B*Lo
```

```
g[e*((a + b*x)/(c + d*x))^n]) + 72*b^2*(b*c - a*d)^2*(c + d*x)^2*(A + B*Log
[e*((a + b*x)/(c + d*x))^n]) + 144*b^3*(b*c - a*d)*(c + d*x)^3*(A + B*Log[e
*((a + b*x)/(c + d*x))^n]) + 144*b^4*(c + d*x)^4*Log[a + b*x]*(A + B*Log[e*
((a + b*x)/(c + d*x))^n]) - 144*b^4*(c + d*x)^4*(A + B*Log[e*((a + b*x)/(c
+ d*x))^n])*Log[c + d*x] - 144*b^3*B*n*(c + d*x)^3*(b*c - a*d + b*(c + d*x)
*Log[a + b*x] - b*(c + d*x)*Log[c + d*x]) - 36*b^2*B*n*(c + d*x)^2*((b*c -
a*d)^2 + 2*b*(b*c - a*d)*(c + d*x) + 2*b^2*(c + d*x)^2*Log[a + b*x] - 2*b^2
*(c + d*x)^2*Log[c + d*x]) - 8*b*B*n*(c + d*x)*(2*(b*c - a*d)^3 + 3*b*(b*c
- a*d)^2*(c + d*x) + 6*b^2*(b*c - a*d)*(c + d*x)^2 + 6*b^3*(c + d*x)^3*Log[
a + b*x] - 6*b^3*(c + d*x)^3*Log[c + d*x]) - 3*B*n*(3*(b*c - a*d)^4 + 4*b*(
b*c - a*d)^3*(c + d*x) + 6*b^2*(b*c - a*d)^2*(c + d*x)^2 + 12*b^3*(b*c - a*
d)*(c + d*x)^3 + 12*b^4*(c + d*x)^4*Log[a + b*x] - 12*b^4*(c + d*x)^4*Log[c
 + d*x]) - 72*b^4*B*n*(c + d*x)^4*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c
 + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) + 72*b
^4*B*n*(c + d*x)^4*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Lo
g[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(b*c - a*d)^4)/(288
*d*g^5*(c + d*x)^4)
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)\right)^{2}}{\left(dgx+cg\right)^{5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(d*g*x+c*g)^5,x)
```

```
[Out] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(d*g*x+c*g)^5,x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 2141 vs. 2(524) = 1048.
time = 0.48, size = 2141, normalized size = 3.99

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*g*x+c*g)^5,x, algorithm="ma
xima")
```

```
[Out] 1/24*A*B*n*((12*b^3*d^3*x^3 + 25*b^3*c^3 - 23*a*b^2*c^2*d + 13*a^2*b*c*d^2
- 3*a^3*d^3 + 6*(7*b^3*c*d^2 - a*b^2*d^3)*x^2 + 4*(13*b^3*c^2*d - 5*a*b^2*c
*d^2 + a^2*b*d^3)*x)/((b^3*c^3*d^5 - 3*a*b^2*c^2*d^6 + 3*a^2*b*c*d^7 - a^3*
d^8)*g^5*x^4 + 4*(b^3*c^4*d^4 - 3*a*b^2*c^3*d^5 + 3*a^2*b*c^2*d^6 - a^3*c*d
^7)*g^5*x^3 + 6*(b^3*c^5*d^3 - 3*a*b^2*c^4*d^4 + 3*a^2*b*c^3*d^5 - a^3*c^2*
d^6)*g^5*x^2 + 4*(b^3*c^6*d^2 - 3*a*b^2*c^5*d^3 + 3*a^2*b*c^4*d^4 - a^3*c^3
```

```
*d^5)*g^5*x + (b^3*c^7*d - 3*a*b^2*c^6*d^2 + 3*a^2*b*c^5*d^3 - a^3*c^4*d^4)
*g^5) + 12*b^4*log(b*x + a)/((b^4*c^4*d - 4*a*b^3*c^3*d^2 + 6*a^2*b^2*c^2*d
^3 - 4*a^3*b*c*d^4 + a^4*d^5)*g^5) - 12*b^4*log(d*x + c)/((b^4*c^4*d - 4*a*
b^3*c^3*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4 + a^4*d^5)*g^5)) + 1/288*(1
2*n*((12*b^3*d^3*x^3 + 25*b^3*c^3 - 23*a*b^2*c^2*d + 13*a^2*b*c*d^2 - 3*a^3
*d^3 + 6*(7*b^3*c*d^2 - a*b^2*d^3)*x^2 + 4*(13*b^3*c^2*d - 5*a*b^2*c*d^2 +
a^2*b*d^3)*x)/((b^3*c^3*d^5 - 3*a*b^2*c^2*d^6 + 3*a^2*b*c*d^7 - a^3*d^8)*g^
5*x^4 + 4*(b^3*c^4*d^4 - 3*a*b^2*c^3*d^5 + 3*a^2*b*c^2*d^6 - a^3*c*d^7)*g^5
*x^3 + 6*(b^3*c^5*d^3 - 3*a*b^2*c^4*d^4 + 3*a^2*b*c^3*d^5 - a^3*c^2*d^6)*g^
5*x^2 + 4*(b^3*c^6*d^2 - 3*a*b^2*c^5*d^3 + 3*a^2*b*c^4*d^4 - a^3*c^3*d^5)*g
^5*x + (b^3*c^7*d - 3*a*b^2*c^6*d^2 + 3*a^2*b*c^5*d^3 - a^3*c^4*d^4)*g^5) +
 12*b^4*log(b*x + a)/((b^4*c^4*d - 4*a*b^3*c^3*d^2 + 6*a^2*b^2*c^2*d^3 - 4*
a^3*b*c*d^4 + a^4*d^5)*g^5) - 12*b^4*log(d*x + c)/((b^4*c^4*d - 4*a*b^3*c^3
*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4 + a^4*d^5)*g^5))*log((b*x/(d*x + c
) + a/(d*x + c))^n*e) - (415*b^4*c^4 - 576*a*b^3*c^3*d + 216*a^2*b^2*c^2*d^
2 - 64*a^3*b*c*d^3 + 9*a^4*d^4 + 300*(b^4*c*d^3 - a*b^3*d^4)*x^3 + 6*(163*b
^4*c^2*d^2 - 176*a*b^3*c*d^3 + 13*a^2*b^2*d^4)*x^2 + 72*(b^4*d^4*x^4 + 4*b^
4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + b^4*c^4)*log(b*x + a)^2 +
 72*(b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + b^
4*c^4)*log(d*x + c)^2 + 4*(271*b^4*c^3*d - 324*a*b^3*c^2*d^2 + 60*a^2*b^2*c
*d^3 - 7*a^3*b*d^4)*x + 300*(b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*
x^2 + 4*b^4*c^3*d*x + b^4*c^4)*log(b*x + a) - 12*(25*b^4*d^4*x^4 + 100*b^4*
c*d^3*x^3 + 150*b^4*c^2*d^2*x^2 + 100*b^4*c^3*d*x + 25*b^4*c^4 + 12*(b^4*d^
4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + b^4*c^4)*log(
b*x + a))*log(d*x + c))*n^2/(b^4*c^8*d*g^5 - 4*a*b^3*c^7*d^2*g^5 + 6*a^2*b^
2*c^6*d^3*g^5 - 4*a^3*b*c^5*d^4*g^5 + a^4*c^4*d^5*g^5 + (b^4*c^4*d^5*g^5 -
4*a*b^3*c^3*d^6*g^5 + 6*a^2*b^2*c^2*d^7*g^5 - 4*a^3*b*c*d^8*g^5 + a^4*d^9*g
^5)*x^4 + 4*(b^4*c^5*d^4*g^5 - 4*a*b^3*c^4*d^5*g^5 + 6*a^2*b^2*c^3*d^6*g^5
- 4*a^3*b*c^2*d^7*g^5 + a^4*c*d^8*g^5)*x^3 + 6*(b^4*c^6*d^3*g^5 - 4*a*b^3*c
^5*d^4*g^5 + 6*a^2*b^2*c^4*d^5*g^5 - 4*a^3*b*c^3*d^6*g^5 + a^4*c^2*d^7*g^5)
*x^2 + 4*(b^4*c^7*d^2*g^5 - 4*a*b^3*c^6*d^3*g^5 + 6*a^2*b^2*c^5*d^4*g^5 - 4
*a^3*b*c^4*d^5*g^5 + a^4*c^3*d^6*g^5)*x))*B^2 - 1/4*B^2*log((b*x/(d*x + c)
+ a/(d*x + c))^n*e)^2/(d^5*g^5*x^4 + 4*c*d^4*g^5*x^3 + 6*c^2*d^3*g^5*x^2 +
4*c^3*d^2*g^5*x + c^4*d*g^5) - 1/2*A*B*log((b*x/(d*x + c) + a/(d*x + c))^n*
e)/(d^5*g^5*x^4 + 4*c*d^4*g^5*x^3 + 6*c^2*d^3*g^5*x^2 + 4*c^3*d^2*g^5*x + c
^4*d*g^5) - 1/4*A^2/(d^5*g^5*x^4 + 4*c*d^4*g^5*x^3 + 6*c^2*d^3*g^5*x^2 + 4*
c^3*d^2*g^5*x + c^4*d*g^5)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1442 vs. $2(524) = 1048$.
time = 0.41, size = 1442, normalized size = 2.69

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*g*x+c*g)^5,x, algorithm="fr
icas")
```

```
[Out] -1/288*(72*(A^2 + 2*A*B + B^2)*b^4*c^4 - 288*(A^2 + 2*A*B + B^2)*a*b^3*c^3*
d + 432*(A^2 + 2*A*B + B^2)*a^2*b^2*c^2*d^2 - 288*(A^2 + 2*A*B + B^2)*a^3*b
*c*d^3 + 72*(A^2 + 2*A*B + B^2)*a^4*d^4 + 12*(25*(B^2*b^4*c*d^3 - B^2*a*b^3
*d^4)*n^2 - 12*((A*B + B^2)*b^4*c*d^3 - (A*B + B^2)*a*b^3*d^4)*n)*x^3 + (41
5*B^2*b^4*c^4 - 576*B^2*a*b^3*c^3*d + 216*B^2*a^2*b^2*c^2*d^2 - 64*B^2*a^3*
b*c*d^3 + 9*B^2*a^4*d^4)*n^2 + 6*((163*B^2*b^4*c^2*d^2 - 176*B^2*a*b^3*c*d^
3 + 13*B^2*a^2*b^2*d^4)*n^2 - 12*(7*(A*B + B^2)*b^4*c^2*d^2 - 8*(A*B + B^2)
*a*b^3*c*d^3 + (A*B + B^2)*a^2*b^2*d^4)*n)*x^2 - 72*(B^2*b^4*d^4*n^2*x^4 +
4*B^2*b^4*c*d^3*n^2*x^3 + 6*B^2*b^4*c^2*d^2*n^2*x^2 + 4*B^2*b^4*c^3*d*n^2*x
 + (4*B^2*a*b^3*c^3*d - 6*B^2*a^2*b^2*c^2*d^2 + 4*B^2*a^3*b*c*d^3 - B^2*a^4
*d^4)*n^2)*log((b*x + a)/(d*x + c))^2 - 12*(25*(A*B + B^2)*b^4*c^4 - 48*(A*
B + B^2)*a*b^3*c^3*d + 36*(A*B + B^2)*a^2*b^2*c^2*d^2 - 16*(A*B + B^2)*a^3*
b*c*d^3 + 3*(A*B + B^2)*a^4*d^4)*n + 4*((271*B^2*b^4*c^3*d - 324*B^2*a*b^3*
c^2*d^2 + 60*B^2*a^2*b^2*c*d^3 - 7*B^2*a^3*b*d^4)*n^2 - 12*(13*(A*B + B^2)*
b^4*c^3*d - 18*(A*B + B^2)*a*b^3*c^2*d^2 + 6*(A*B + B^2)*a^2*b^2*c*d^3 - (A
*B + B^2)*a^3*b*d^4)*n)*x + 12*((25*B^2*b^4*d^4*n^2 - 12*(A*B + B^2)*b^4*d^
4*n)*x^4 - 4*(12*(A*B + B^2)*b^4*c*d^3*n - (22*B^2*b^4*c*d^3 + 3*B^2*a*b^3*
d^4)*n^2)*x^3 + (48*B^2*a*b^3*c^3*d - 36*B^2*a^2*b^2*c^2*d^2 + 16*B^2*a^3*b
*c*d^3 - 3*B^2*a^4*d^4)*n^2 - 6*(12*(A*B + B^2)*b^4*c^2*d^2*n - (18*B^2*b^4
*c^2*d^2 + 8*B^2*a*b^3*c*d^3 - B^2*a^2*b^2*d^4)*n^2)*x^2 - 12*(4*(A*B + B^2
)*a*b^3*c^3*d - 6*(A*B + B^2)*a^2*b^2*c^2*d^2 + 4*(A*B + B^2)*a^3*b*c*d^3 -
 (A*B + B^2)*a^4*d^4)*n - 4*(12*(A*B + B^2)*b^4*c^3*d*n - (12*B^2*b^4*c^3*d
 + 18*B^2*a*b^3*c^2*d^2 - 6*B^2*a^2*b^2*c*d^3 + B^2*a^3*b*d^4)*n^2)*x)*log(
(b*x + a)/(d*x + c)))/((b^4*c^4*d^5 - 4*a*b^3*c^3*d^6 + 6*a^2*b^2*c^2*d^7 -
 4*a^3*b*c*d^8 + a^4*d^9)*g^5*x^4 + 4*(b^4*c^5*d^4 - 4*a*b^3*c^4*d^5 + 6*a^
2*b^2*c^3*d^6 - 4*a^3*b*c^2*d^7 + a^4*c*d^8)*g^5*x^3 + 6*(b^4*c^6*d^3 - 4*a
*b^3*c^5*d^4 + 6*a^2*b^2*c^4*d^5 - 4*a^3*b*c^3*d^6 + a^4*c^2*d^7)*g^5*x^2 +
 4*(b^4*c^7*d^2 - 4*a*b^3*c^6*d^3 + 6*a^2*b^2*c^5*d^4 - 4*a^3*b*c^4*d^5 + a
^4*c^3*d^6)*g^5*x + (b^4*c^8*d - 4*a*b^3*c^7*d^2 + 6*a^2*b^2*c^6*d^3 - 4*a^
3*b*c^5*d^4 + a^4*c^4*d^5)*g^5)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(d*g*x+c*g)**5,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1225 vs. $2(524) = 1048$.

time = 6.63, size = 1225, normalized size = 2.29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*g*x+c*g)^5,x, algorithm="gi
ac")
```

```
[Out] 1/288*(72*(4*(b*x + a)*B^2*b^3*n^2/((b^3*c^3*g^5 - 3*a*b^2*c^2*d*g^5 + 3*a^
2*b*c*d^2*g^5 - a^3*d^3*g^5)*(d*x + c)) - 6*(b*x + a)^2*B^2*b^2*d*n^2/((b^3
*c^3*g^5 - 3*a*b^2*c^2*d*g^5 + 3*a^2*b*c*d^2*g^5 - a^3*d^3*g^5)*(d*x + c)^2
) + 4*(b*x + a)^3*B^2*b*d^2*n^2/((b^3*c^3*g^5 - 3*a*b^2*c^2*d*g^5 + 3*a^2*b
*c*d^2*g^5 - a^3*d^3*g^5)*(d*x + c)^3) - (b*x + a)^4*B^2*d^3*n^2/((b^3*c^3*
g^5 - 3*a*b^2*c^2*d*g^5 + 3*a^2*b*c*d^2*g^5 - a^3*d^3*g^5)*(d*x + c)^4))*lo
g((b*x + a)/(d*x + c))^2 + 12*(3*(B^2*d^3*n^2 - 4*A*B*d^3*n - 4*B^2*d^3*n)*
(b*x + a)^4/((b^3*c^3*g^5 - 3*a*b^2*c^2*d*g^5 + 3*a^2*b*c*d^2*g^5 - a^3*d^3
*g^5)*(d*x + c)^4) - 16*(B^2*b*d^2*n^2 - 3*A*B*b*d^2*n - 3*B^2*b*d^2*n)*(b*
x + a)^3/((b^3*c^3*g^5 - 3*a*b^2*c^2*d*g^5 + 3*a^2*b*c*d^2*g^5 - a^3*d^3*g^
5)*(d*x + c)^3) + 36*(B^2*b^2*d*n^2 - 2*A*B*b^2*d*n - 2*B^2*b^2*d*n)*(b*x +
 a)^2/((b^3*c^3*g^5 - 3*a*b^2*c^2*d*g^5 + 3*a^2*b*c*d^2*g^5 - a^3*d^3*g^5)*
(d*x + c)^2) - 48*(B^2*b^3*n^2 - A*B*b^3*n - B^2*b^3*n)*(b*x + a)/((b^3*c^3
*g^5 - 3*a*b^2*c^2*d*g^5 + 3*a^2*b*c*d^2*g^5 - a^3*d^3*g^5)*(d*x + c)))*log
((b*x + a)/(d*x + c)) - 9*(B^2*d^3*n^2 - 4*A*B*d^3*n - 4*B^2*d^3*n + 8*A^2*
d^3 + 16*A*B*d^3 + 8*B^2*d^3)*(b*x + a)^4/((b^3*c^3*g^5 - 3*a*b^2*c^2*d*g^5
 + 3*a^2*b*c*d^2*g^5 - a^3*d^3*g^5)*(d*x + c)^4) + 32*(2*B^2*b*d^2*n^2 - 6*
A*B*b*d^2*n - 6*B^2*b*d^2*n + 9*A^2*b*d^2 + 18*A*B*b*d^2 + 9*B^2*b*d^2)*(b*
x + a)^3/((b^3*c^3*g^5 - 3*a*b^2*c^2*d*g^5 + 3*a^2*b*c*d^2*g^5 - a^3*d^3*g^
5)*(d*x + c)^3) - 216*(B^2*b^2*d*n^2 - 2*A*B*b^2*d*n - 2*B^2*b^2*d*n + 2*A^
2*b^2*d + 4*A*B*b^2*d + 2*B^2*b^2*d)*(b*x + a)^2/((b^3*c^3*g^5 - 3*a*b^2*c^
2*d*g^5 + 3*a^2*b*c*d^2*g^5 - a^3*d^3*g^5)*(d*x + c)^2) + 288*(2*B^2*b^3*n^
2 - 2*A*B*b^3*n - 2*B^2*b^3*n + A^2*b^3 + 2*A*B*b^3 + B^2*b^3)*(b*x + a)/((
b^3*c^3*g^5 - 3*a*b^2*c^2*d*g^5 + 3*a^2*b*c*d^2*g^5 - a^3*d^3*g^5)*(d*x + c
)))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)
```

Mupad [B]
time = 9.08, size = 1765, normalized size = 3.29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/(c*g + d*g*x)^5,x)
```

```
[Out] (B*b^4*n*atan((B*b^4*n*(12*A - 25*B*n)*(24*a^4*d^5*g^5 - 24*b^4*c^4*d*g^5 -
 48*a^3*b*c*d^4*g^5 + 48*a*b^3*c^3*d^2*g^5)*1i)/(24*d*g^5*(25*B^2*b^4*n^2 -
 12*A*B*b^4*n)*(a*d - b*c)^4) + (B*b^5*n*x*(12*A - 25*B*n)*(a^3*d^4*g^5 - b
```

```
^3*c^3*d*g^5 - 3*a^2*b*c*d^3*g^5 + 3*a*b^2*c^2*d^2*g^5)*2i)/(g^5*(25*B^2*b^
4*n^2 - 12*A*B*b^4*n)*(a*d - b*c)^4))*(12*A - 25*B*n)*1i)/(12*d*g^5*(a*d -
b*c)^4) - ((72*A^2*a^3*d^3 - 72*A^2*b^3*c^3 + 9*B^2*a^3*d^3*n^2 - 415*B^2*b
^3*c^3*n^2 + 216*A^2*a*b^2*c^2*d - 216*A^2*a^2*b*c*d^2 - 36*A*B*a^3*d^3*n +
 300*A*B*b^3*c^3*n + 161*B^2*a*b^2*c^2*d*n^2 - 55*B^2*a^2*b*c*d^2*n^2 - 276
*A*B*a*b^2*c^2*d*n + 156*A*B*a^2*b*c*d^2*n)/(12*(a*d - b*c)) + (x^2*(13*B^2
*a*b^2*d^3*n^2 - 163*B^2*b^3*c*d^2*n^2 - 12*A*B*a*b^2*d^3*n + 84*A*B*b^3*c*
d^2*n))/(2*(a*d - b*c)) - (x*(7*B^2*a^2*b*d^3*n^2 + 271*B^2*b^3*c^2*d*n^2 -
 53*B^2*a*b^2*c*d^2*n^2 - 12*A*B*a^2*b*d^3*n - 156*A*B*b^3*c^2*d*n + 60*A*B
*a*b^2*c*d^2*n))/(3*(a*d - b*c)) - (b*x^3*(25*B^2*b^2*d^3*n^2 - 12*A*B*b^2*
d^3*n))/(a*d - b*c))/(x*(96*a^2*c^3*d^4*g^5 + 96*b^2*c^5*d^2*g^5 - 192*a*b*
c^4*d^3*g^5) + x^3*(96*a^2*c*d^6*g^5 + 96*b^2*c^3*d^4*g^5 - 192*a*b*c^2*d^5
*g^5) + x^4*(24*a^2*d^7*g^5 + 24*b^2*c^2*d^5*g^5 - 48*a*b*c*d^6*g^5) + x^2*
(144*a^2*c^2*d^5*g^5 + 144*b^2*c^4*d^3*g^5 - 288*a*b*c^3*d^4*g^5) + 24*b^2*
c^6*d*g^5 + 24*a^2*c^4*d^3*g^5 - 48*a*b*c^5*d^2*g^5) - log(e*((a + b*x)/(c
+ d*x))^n)^2*(B^2/(4*d*(c^4*g^5 + d^4*g^5*x^4 + 4*c*d^3*g^5*x^3 + 6*c^2*d^2
*g^5*x^2 + 4*c^3*d*g^5*x)) - (B^2*b^4)/(4*d*g^5*(a^4*d^4 + b^4*c^4 + 6*a^2*
b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))) - log(e*((a + b*x)/(c + d*x)
)^n)*((A*B)/(2*c^4*d*g^5 + 2*d^5*g^5*x^4 + 8*c^3*d^2*g^5*x + 8*c*d^4*g^5*x^
3 + 12*c^2*d^3*g^5*x^2) - (B^2*b^4*(x*(d*(c*((d*g^5*n*(a*d - b*c)*(a*d - 4*
b*c))/(6*b^2) - (c*d*g^5*n*(a*d - b*c))/(2*b)) - (d*g^5*n*(a*d - b*c)*(a^2*
d^2 + 6*b^2*c^2 - 4*a*b*c*d))/(6*b^3)) + c*(d*((d*g^5*n*(a*d - b*c)*(a*d -
4*b*c))/(6*b^2) - (c*d*g^5*n*(a*d - b*c))/(2*b)) - (c*d^2*g^5*n*(a*d - b*c)
)/b + (d^2*g^5*n*(a*d - b*c)*(a*d - 4*b*c))/(3*b^2)) - (d^2*g^5*n*(a*d - b*
c)*(a^2*d^2 + 6*b^2*c^2 - 4*a*b*c*d))/(2*b^3)) + c*(c*((d*g^5*n*(a*d - b*c)
*(a*d - 4*b*c))/(6*b^2) - (c*d*g^5*n*(a*d - b*c))/(2*b)) - (d*g^5*n*(a*d -
b*c)*(a^2*d^2 + 6*b^2*c^2 - 4*a*b*c*d))/(6*b^3)) + x^2*(d*(d*((d*g^5*n*(a*d
 - b*c)*(a*d - 4*b*c))/(6*b^2) - (c*d*g^5*n*(a*d - b*c))/(2*b)) - (c*d^2*g^
5*n*(a*d - b*c))/b + (d^2*g^5*n*(a*d - b*c)*(a*d - 4*b*c))/(3*b^2)) - (3*c*
d^3*g^5*n*(a*d - b*c))/(2*b) + (d^3*g^5*n*(a*d - b*c)*(a*d - 4*b*c))/(2*b^2
)) - (2*d^4*g^5*n*x^3*(a*d - b*c))/b + (d*g^5*n*(a*d - b*c)*(a^3*d^3 - 4*b^
3*c^3 + 6*a*b^2*c^2*d - 4*a^2*b*c*d^2))/(2*b^4)))/(2*d*g^5*(2*c^4*d*g^5 + 2
*d^5*g^5*x^4 + 8*c^3*d^2*g^5*x + 8*c*d^4*g^5*x^3 + 12*c^2*d^3*g^5*x^2)*(a^4
*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)))
```

3.47 $\int \frac{(cg+dgx)^2}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$

Optimal. Leaf size=38

$$\operatorname{Int}\left(\frac{(cg+dgx)^2}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)},x\right)$$

```
[Out] Unintegrable((d*g*x+c*g)^2/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(cg+dgx)^2}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(c*g + d*g*x)^2/(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
[Out] Defer[Int][(c*g + d*g*x)^2/(A + B*Log[e*((a + b*x)/(c + d*x))^n]), x]
```

Rubi steps

$$\begin{aligned}\int \frac{(cg+dgx)^2}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx &= \int \left(\frac{c^2g^2}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}+\frac{2cdg^2x}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}+\frac{d^2g^2x^2}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\right.\\ &= \left(c^2g^2\right)\int \frac{1}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx+\left(2cdg^2\right)\int \frac{x}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx+\left(\right.\end{aligned}$$

Mathematica [A]
time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{(cg+dgx)^2}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c*g + d*g*x)^2/(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
[Out] Integrate[(c*g + d*g*x)^2/(A + B*Log[e*((a + b*x)/(c + d*x))^n]), x]
```

Maple [A]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{(dgx+cg)^2}{A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*g*x+c*g)^2/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

```
[Out] int((d*g*x+c*g)^2/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*g*x+c*g)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="maxi
ma")
```

```
[Out] integrate((d*g*x + c*g)^2/(B*log(((b*x + a)/(d*x + c))^n*e) + A), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*g*x+c*g)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fric
as")
```

```
[Out] integral((d^2*g^2*x^2 + 2*c*d*g^2*x + c^2*g^2)/(B*log(((b*x + a)/(d*x + c))
^n*e) + A), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$g^2\left(\int \frac{c^2}{A+B\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}\,dx+\int \frac{d^2x^2}{A+B\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}\,dx+\int \frac{2cdx}{A+B\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*g*x+c*g)**2/(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

```
[Out] g**2*(Integral(c**2/(A + B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)), x) + I
ntegral(d**2*x**2/(A + B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)), x) + Int
egral(2*c*d*x/(A + B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)), x))
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*g*x+c*g)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="giac
")
```

```
[Out] integrate((d*g*x + c*g)^2/(B*log(((b*x + a)/(d*x + c))^n*e) + A), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{(c\,g+d\,g\,x)^2}{A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*g + d*g*x)^2/(A + B*log(e*((a + b*x)/(c + d*x))^n)),x)
```

```
[Out] int((c*g + d*g*x)^2/(A + B*log(e*((a + b*x)/(c + d*x))^n)), x)
```

3.48 $\int \frac{cg+dgx}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$

Optimal. Leaf size=36

$$\text{Int}\left(\frac{cg+dgx}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)},x\right)$$

```
[Out] Unintegrable((d*g*x+c*g)/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{cg+dgx}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(c*g + d*g*x)/(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
[Out] Defer[Int][(c*g + d*g*x)/(A + B*Log[e*((a + b*x)/(c + d*x))^n]), x]
```

Rubi steps

$$\begin{aligned}\int \frac{cg+dgx}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx &= \int \left(\frac{cg}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}+\frac{dgx}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\right)dx\\ &= (cg)\int \frac{1}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx+(dg)\int \frac{x}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx\end{aligned}$$

Mathematica [A]
time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{cg+dgx}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c*g + d*g*x)/(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
[Out] Integrate[(c*g + d*g*x)/(A + B*Log[e*((a + b*x)/(c + d*x))^n]), x]
```

Maple [A]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{dgx+cg}{A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*g*x+c*g)/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

```
[Out] int((d*g*x+c*g)/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*g*x+c*g)/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="maxima
")
```

```
[Out] integrate((d*g*x + c*g)/(B*log(((b*x + a)/(d*x + c))^n*e) + A), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*g*x+c*g)/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fricas
")
```

```
[Out] integral((d*g*x + c*g)/(B*log(((b*x + a)/(d*x + c))^n*e) + A), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$g\left(\int \frac{c}{A+B\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^{n}\right)}\,dx+\int \frac{dx}{A+B\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^{n}\right)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*g*x+c*g)/(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

```
[Out] g*(Integral(c/(A + B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)), x) + Integra
l(d*x/(A + B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)), x))
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*g*x+c*g)/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="giac")
[Out] integrate((d*g*x + c*g)/(B*log(((b*x + a)/(d*x + c))^n*e) + A), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{c\,g+d\,g\,x}{A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*g + d*g*x)/(A + B*log(e*((a + b*x)/(c + d*x))^n)),x)
[Out] int((c*g + d*g*x)/(A + B*log(e*((a + b*x)/(c + d*x))^n)), x)
```

3.49 $\int \frac{1}{(cg+dgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx$

Optimal. Leaf size=38

$$\text{Int}\left(\frac{1}{(cg+dgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)},x\right)$$

```
[Out] Unintegrable(1/(d*g*x+c*g)/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(cg+dgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((c*g + d*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])),x]
[Out] Defer[Int][1/((c*g + d*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])), x]
```

Rubi steps

$$\int \frac{1}{(cg+dgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx=\int \frac{1}{(cg+dgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx$$

Mathematica [A]
time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{1}{(cg+dgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((c*g + d*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])),x]
[Out] Integrate[1/((c*g + d*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])), x]
```

Maple [A]
time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{1}{(dgx+cg)\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*g*x+c*g)/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

```
[Out] int(1/(d*g*x+c*g)/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*g*x+c*g)/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="maxi
ma")
```

```
[Out] integrate(1/((d*g*x + c*g)*(B*log(((b*x + a)/(d*x + c))^n*e) + A)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*g*x+c*g)/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fric
as")
```

```
[Out] integral(1/(A*d*g*x + A*c*g + (B*d*g*x + B*c*g)*log(((b*x + a)/(d*x + c))^n
*e)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{Ac+Adx+Bc\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^{n}\right)+Bdx\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^{n}\right)}\,dx}{g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*g*x+c*g)/(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

```
[Out] Integral(1/(A*c + A*d*x + B*c*log(e*(a/(c + d*x) + b*x/(c + d*x))**n) + B*d
*x*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)), x)/g
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*g*x+c*g)/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="giac
")
```

```
[Out] integrate(1/((d*g*x + c*g)*(B*log(((b*x + a)/(d*x + c))^n*e) + A)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{(c\,g+d\,g\,x)\,\left(A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*g + d*g*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))),x)
```

```
[Out] int(1/((c*g + d*g*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))), x)
```

3.50 $\int \frac{1}{(cg+dgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx$

Optimal. Leaf size=96

$$\frac{e^{-\frac{A}{Bn}}(a+bx)\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-1/n}\operatorname{Ei}\left(\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{Bn}\right)}{B(bc-ad)g^2n(c+dx)}$$

```
[Out] (b*x+a)*Ei((A+B*ln(e*((b*x+a)/(d*x+c))^n))/B/n)/B/(-a*d+b*c)/exp(A/B/n)/g^2
/n/((e*((b*x+a)/(d*x+c))^n)^(1/n))/(d*x+c)
```

Rubi [A]
time = 0.07, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {2551, 2337, 2209}

$$\frac{(a+bx)e^{-\frac{A}{Bn}}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-1/n}\operatorname{Ei}\left(\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{Bn}\right)}{Bg^2n(c+dx)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*g + d*g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])),x]
```

```
[Out] ((a + b*x)*ExpIntegralEi[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(B*n)])/(B*
(b*c - a*d)*E^(A/(B*n))*g^2*n*(e*((a + b*x)/(c + d*x))^n)^n^(-1)*(c + d*x))
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2551

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(m +
1)*(g/d)^m, Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a + b
*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ[b*c -
 a*d, 0] && IntegersQ[m, p] && EqQ[d*f - c*g, 0] && (GtQ[p, 0] || LtQ[m, -1
```

```
])
```

Rubi steps

$$\int \frac{1}{(cg+dgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx=\int \frac{1}{(cg+dgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx$$

Mathematica [A]
time = 0.09, size = 96, normalized size = 1.00

$$\frac{e^{-\frac{A}{Bn}}(a+bx)\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-1/n}\text{Ei}\left(\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{Bn}\right)}{B(bc-ad)g^2n(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*g + d*g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])),x]
[Out] ((a + b*x)*ExpIntegralEi[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(B*n)])/(B*
(b*c - a*d)*E^(A/(B*n))*g^2*n*(e*((a + b*x)/(c + d*x))^n)^n^(-1)*(c + d*x))
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{1}{(dgx+cg)^2\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*g*x+c*g)^2/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
[Out] int(1/(d*g*x+c*g)^2/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*g*x+c*g)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="ma
xima")
[Out] integrate(1/((d*g*x + c*g)^2*(B*log(((b*x + a)/(d*x + c))^n*e) + A)), x)
```

Fricas [A]
time = 0.34, size = 56, normalized size = 0.58

$$\frac{e^{\left(-\frac{A+B}{Bn}\right)}\,\mathrm{log\_integral}\left(\frac{(bx+a)e^{\left(\frac{A+B}{Bn}\right)}}{dx+c}\right)}{(Bbc-Bad)g^2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*g*x+c*g)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fr
icas")
```

```
[Out] e^(-(A + B)/(B*n))*log_integral((b*x + a)*e^((A + B)/(B*n))/(d*x + c))/((B*
b*c - B*a*d)*g^2*n)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{Ac^2+2Acdx+Ad^2x^2+Bc^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)+2Bcdx\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)+Bd^2x^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}\,dx}{g^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*g*x+c*g)**2/(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

```
[Out] Integral(1/(A*c**2 + 2*A*c*d*x + A*d**2*x**2 + B*c**2*log(e*(a/(c + d*x) +
b*x/(c + d*x))**n) + 2*B*c*d*x*log(e*(a/(c + d*x) + b*x/(c + d*x))**n) + B*
d**2*x**2*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)), x)/g**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*g*x+c*g)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="gi
ac")
```

```
[Out] integrate(1/((d*g*x + c*g)^2*(B*log(((b*x + a)/(d*x + c))^n*e) + A)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(c\,g+d\,g\,x\right)}^2\,\left(A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*g + d*g*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))),x)
```

```
[Out] int(1/((c*g + d*g*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))), x)
```

3.51 $\int \frac{1}{(cg+dgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx$

Optimal. Leaf size=199

$$\frac{be^{-\frac{A}{Bn}}(a+bx)\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-1/n}\operatorname{Ei}\left(\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{Bn}\right)}{B(bc-ad)^2g^3n(c+dx)}-\frac{de^{-\frac{2A}{Bn}}(a+bx)^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-2/n}\operatorname{Ei}\left(\frac{2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{Bn}\right)}{B(bc-ad)^2g^3n(c+dx)^2}$$

```
[Out] b*(b*x+a)*Ei((A+B*ln(e*((b*x+a)/(d*x+c))^n))/B/n)/B/(-a*d+b*c)^2/exp(A/B/n)
/g^3/n/((e*((b*x+a)/(d*x+c))^n)^(1/n))/(d*x+c)-d*(b*x+a)^2*Ei(2*(A+B*ln(e*(
(b*x+a)/(d*x+c))^n))/B/n)/B/(-a*d+b*c)^2/exp(2*A/B/n)/g^3/n/((e*((b*x+a)/(d
*x+c))^n)^(2/n))/(d*x+c)^2
```

Rubi [A]

time = 0.15, antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2551, 2367, 2337, 2209, 2347}

$$\frac{b(a+bx)e^{-\frac{A}{Bn}}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-1/n}\operatorname{Ei}\left(\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{Bn}\right)}{Bg^3n(c+dx)(bc-ad)^2}-\frac{d(a+bx)^2e^{-\frac{2A}{Bn}}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-2/n}\operatorname{Ei}\left(\frac{2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{Bn}\right)}{Bg^3n(c+dx)^2(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*g + d*g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])),x]
```

```
[Out] (b*(a + b*x)*ExpIntegralEi[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(B*n)])/(
B*(b*c - a*d)^2*E^(A/(B*n))*g^3*n*(e*((a + b*x)/(c + d*x))^n)^n^(-1)*(c + d
*x)) - (d*(a + b*x)^2*ExpIntegralEi[(2*(A + B*Log[e*((a + b*x)/(c + d*x))^n
]))/(B*n)])/(B*(b*c - a*d)^2*E^((2*A)/(B*n))*g^3*n*(e*((a + b*x)/(c + d*x))
^n)^(2/n)*(c + d*x)^2)
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
```

```
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2367

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^(r_.))^(
q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*x^n])^p, (d + e*x
^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
&& IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0] && IntegerQ[r]))
```

Rule 2551

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(m +
1)*(g/d)^m, Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a + b
*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ[b*c -
 a*d, 0] && IntegersQ[m, p] && EqQ[d*f - c*g, 0] && (GtQ[p, 0] || LtQ[m, -1
])
```

Rubi steps

$$\int \frac{1}{(cg+dgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx=\int \frac{1}{(cg+dgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx$$

Mathematica [A]
time = 0.17, size = 174, normalized size = 0.87

$$\frac{e^{-\frac{2A}{Bn}}(a+bx)\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-2/n}\left(be^{\frac{A}{Bn}}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{1}{n}}(c+dx)\text{Ei}\left(\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{Bn}\right)-d(a+bx)\text{Ei}\left(\frac{2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{Bn}\right)\right)}{B(bc-ad)^2g^3n(c+dx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*g + d*g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])),x]
```

```
[Out] ((a + b*x)*(b*E^(A/(B*n))*(e*((a + b*x)/(c + d*x))^n)^n^(-1)*(c + d*x)*ExpI
ntegralEi[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(B*n)] - d*(a + b*x)*ExpIn
tegralEi[(2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(B*n)]))/(B*(b*c - a*d)
^2*E^((2*A)/(B*n))*g^3*n*(e*((a + b*x)/(c + d*x))^n)^(2/n)*(c + d*x)^2)
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{1}{(dgx+cg)^3\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*g*x+c*g)^3/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

```
[Out] int(1/(d*g*x+c*g)^3/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*g*x+c*g)^3/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="ma
xima")
```

```
[Out] integrate(1/((d*g*x + c*g)^3*(B*log(((b*x + a)/(d*x + c))^n*e) + A)), x)
```

Fricas [A]
time = 0.35, size = 135, normalized size = 0.68

$$\frac{\left(be^{\left(\frac{A+B}{Bn}\right)}\operatorname{log\_integral}\left(\frac{(bx+a)e^{\left(\frac{A+B}{Bn}\right)}}{dx+c}\right)-d\operatorname{log\_integral}\left(\frac{(b^2x^2+2\,abx+a^2)e^{\left(\frac{2\,(A+B)}{Bn}\right)}}{d^2x^2+2\,cdx+c^2}\right)\right)e^{\left(-\frac{2\,(A+B)}{Bn}\right)}}{(Bb^2c^2-2\,Babcd+Ba^2d^2)g^3n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*g*x+c*g)^3/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fr
icas")
```

```
[Out] (b*e^((A + B)/(B*n))*log_integral((b*x + a)*e^((A + B)/(B*n))/(d*x + c)) -
d*log_integral((b^2*x^2 + 2*a*b*x + a^2)*e^(2*(A + B)/(B*n))/(d^2*x^2 + 2*c
*d*x + c^2)))*e^(-2*(A + B)/(B*n))/((B*b^2*c^2 - 2*B*a*b*c*d + B*a^2*d^2)*g
^3*n)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*g*x+c*g)**3/(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*g*x+c*g)^3/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="gi
ac")
```

```
[Out] integrate(1/((d*g*x + c*g)^3*(B*log(((b*x + a)/(d*x + c))^n*e) + A)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(c\,g+d\,g\,x)^3\,\left(A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*g + d*g*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))),x)
```

```
[Out] int(1/((c*g + d*g*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))), x)
```

3.52 $\int \frac{(cg+dgx)^2}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$

Optimal. Leaf size=38

$$\mathrm{Int}\left(\frac{(cg+dgx)^2}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2},x\right)$$

```
[Out] Unintegrable((d*g*x+c*g)^2/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(cg+dgx)^2}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(c*g + d*g*x)^2/(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

```
[Out] Defer[Int][(c*g + d*g*x)^2/(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2, x]
```

Rubi steps

$$\int \frac{(cg+dgx)^2}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = \int \left(\frac{c^2g^2}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2} + \frac{2cdg^2x}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2} + \frac{d^2g}{\left(A+B\log\left(\right.\right.}\right.$$
$$= \left(c^2g^2\right)\int \frac{1}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx + \left(2cdg^2\right)\int \frac{x}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right.\right.}$$

Mathematica [A]
time = 0.63, size = 0, normalized size = 0.00

$$\int \frac{(cg+dgx)^2}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c*g + d*g*x)^2/(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

```
[Out] Integrate[(c*g + d*g*x)^2/(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2, x]
```

Maple [A]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{(dgx+cg)^2}{\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*g*x+c*g)^2/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

```
[Out] int((d*g*x+c*g)^2/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*g*x+c*g)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="ma
xima")
```

```
[Out] -(b*d^3*g^2*x^4 + a*c^3*g^2 + (3*b*c*d^2*g^2 + a*d^3*g^2)*x^3 + 3*(b*c^2*d*
g^2 + a*c*d^2*g^2)*x^2 + (b*c^3*g^2 + 3*a*c^2*d*g^2)*x)/((b*c*n - a*d*n)*B^
2*log((b*x + a)^n) - (b*c*n - a*d*n)*B^2*log((d*x + c)^n) + (b*c*n - a*d*n)
*A*B + (b*c*n - a*d*n)*B^2) + integrate((4*b*d^3*g^2*x^3 + b*c^3*g^2 + 3*a*
c^2*d*g^2 + 3*(3*b*c*d^2*g^2 + a*d^3*g^2)*x^2 + 6*(b*c^2*d*g^2 + a*c*d^2*g^
2)*x)/((b*c*n - a*d*n)*B^2*log((b*x + a)^n) - (b*c*n - a*d*n)*B^2*log((d*x
+ c)^n) + (b*c*n - a*d*n)*A*B + (b*c*n - a*d*n)*B^2), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*g*x+c*g)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="fr
icas")
```

```
[Out] integral((d^2*g^2*x^2 + 2*c*d*g^2*x + c^2*g^2)/(B^2*log(((b*x + a)/(d*x + c
))^n*e)^2 + 2*A*B*log(((b*x + a)/(d*x + c))^n*e) + A^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$g^{2}\left(\int \frac{c^{2}}{A^{2}+2AB\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^{n}\right)+B^{2}\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^{n}\right)^{2}}\,dx+\int \frac{d^{2}x^{2}}{A^{2}+2AB\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^{n}\right)+B^{2}\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^{n}\right)^{2}}\,dx+\int \frac{2cdx}{A^{2}+2AB\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^{n}\right)+B^{2}\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^{n}\right)^{2}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*g*x+c*g)**2/(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
```

```
[Out] g**2*(Integral(c**2/(A**2 + 2*A*B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n) +
 B**2*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)**2), x) + Integral(d**2*x**2/
(A**2 + 2*A*B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n) + B**2*log(e*(a/(c +
d*x) + b*x/(c + d*x))**n)**2), x) + Integral(2*c*d*x/(A**2 + 2*A*B*log(e*(a
/(c + d*x) + b*x/(c + d*x))**n) + B**2*log(e*(a/(c + d*x) + b*x/(c + d*x))*
*n)**2), x))
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*g*x+c*g)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="gi
ac")
```

```
[Out] integrate((d*g*x + c*g)^2/(B*log(((b*x + a)/(d*x + c))^n*e) + A)^2, x)
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{(c\,g+d\,g\,x)^2}{\left(A+B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*g + d*g*x)^2/(A + B*log(e*((a + b*x)/(c + d*x))^n))^2,x)
```

```
[Out] int((c*g + d*g*x)^2/(A + B*log(e*((a + b*x)/(c + d*x))^n))^2, x)
```

3.53 $\int \frac{cg+dgx}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$

Optimal. Leaf size=36

$$\text{Int}\left(\frac{cg+dgx}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2},x\right)$$

```
[Out] Unintegrable((d*g*x+c*g)/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{cg+dgx}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(c*g + d*g*x)/(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
[Out] Defer[Int][(c*g + d*g*x)/(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2, x]
```

Rubi steps

$$\begin{aligned}\int \frac{cg+dgx}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx &= \int \left(\frac{cg}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}+\frac{dgx}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\right)dx\\ &= (cg)\int \frac{1}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx+(dg)\int \frac{x}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,d\end{aligned}$$

Mathematica [A]
time = 0.63, size = 0, normalized size = 0.00

$$\int \frac{cg+dgx}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c*g + d*g*x)/(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
[Out] Integrate[(c*g + d*g*x)/(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2, x]
```

Maple [A]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{dgx + cg}{\left(A + B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*g*x+c*g)/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

```
[Out] int((d*g*x+c*g)/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*g*x+c*g)/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="maxi
ma")
```

```
[Out] -(b*d^2*g*x^3 + a*c^2*g + (2*b*c*d*g + a*d^2*g)*x^2 + (b*c^2*g + 2*a*c*d*g)
*x)/((b*c*n - a*d*n)*B^2*log((b*x + a)^n) - (b*c*n - a*d*n)*B^2*log((d*x +
c)^n) + (b*c*n - a*d*n)*A*B + (b*c*n - a*d*n)*B^2) + integrate((3*b*d^2*g*x
^2 + b*c^2*g + 2*a*c*d*g + 2*(2*b*c*d*g + a*d^2*g)*x)/((b*c*n - a*d*n)*B^2*
log((b*x + a)^n) - (b*c*n - a*d*n)*B^2*log((d*x + c)^n) + (b*c*n - a*d*n)*A
*B + (b*c*n - a*d*n)*B^2), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*g*x+c*g)/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="fric
as")
```

```
[Out] integral((d*g*x + c*g)/(B^2*log(((b*x + a)/(d*x + c))^n*e)^2 + 2*A*B*log(((
b*x + a)/(d*x + c))^n*e) + A^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$g\left(\int \frac{c}{A^{2} + 2AB\log\left(e\left(\frac{a}{c+dx} + \frac{bx}{c+dx}\right)^{n}\right) + B^{2}\log\left(e\left(\frac{a}{c+dx} + \frac{bx}{c+dx}\right)^{n}\right)^{2}}\, dx + \int \frac{dx}{A^{2} + 2AB\log\left(e\left(\frac{a}{c+dx} + \frac{bx}{c+dx}\right)^{n}\right) + B^{2}\log\left(e\left(\frac{a}{c+dx} + \frac{bx}{c+dx}\right)^{n}\right)^{2}}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*g*x+c*g)/(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
```

```
[Out] g*(Integral(c/(A**2 + 2*A*B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n) + B**2*
log(e*(a/(c + d*x) + b*x/(c + d*x))**n)**2), x) + Integral(d*x/(A**2 + 2*A*
B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n) + B**2*log(e*(a/(c + d*x) + b*x/(
c + d*x))**n)**2), x))
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*g*x+c*g)/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="giac
")
```

```
[Out] integrate((d*g*x + c*g)/(B*log(((b*x + a)/(d*x + c))^n*e) + A)^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{c\,g+d\,g\,x}{{\left(A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*g + d*g*x)/(A + B*log(e*((a + b*x)/(c + d*x))^n))^2,x)
```

```
[Out] int((c*g + d*g*x)/(A + B*log(e*((a + b*x)/(c + d*x))^n))^2, x)
```

3.54 $\int \frac{1}{(cg+dgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$

Optimal. Leaf size=38

$$\text{Int}\left(\frac{1}{(cg+dgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2},x\right)$$

```
[Out] Unintegrable(1/(d*g*x+c*g)/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(cg+dgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((c*g + d*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2),x]
[Out] Defer[Int][1/((c*g + d*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2), x]
```

Rubi steps

$$\int \frac{1}{(cg+dgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{1}{(cg+dgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$

Mathematica [A]
time = 0.38, size = 0, normalized size = 0.00

$$\int \frac{1}{(cg+dgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((c*g + d*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2),x]
[Out] Integrate[1/((c*g + d*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2), x]
```

Maple [A]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{1}{(dgx+cg)\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*g*x+c*g)/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

```
[Out] int(1/(d*g*x+c*g)/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*g*x+c*g)/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="ma
xima")
```

```
[Out] b*integrate(1/((b*c*g*n - a*d*g*n)*B^2*log((b*x + a)^n) - (b*c*g*n - a*d*g*
n)*B^2*log((d*x + c)^n) + (b*c*g*n - a*d*g*n)*A*B + (b*c*g*n - a*d*g*n)*B^2
), x) - (b*x + a)/((b*c*g*n - a*d*g*n)*B^2*log((b*x + a)^n) - (b*c*g*n - a*
d*g*n)*B^2*log((d*x + c)^n) + (b*c*g*n - a*d*g*n)*A*B + (b*c*g*n - a*d*g*n)
*B^2)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*g*x+c*g)/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="fr
icas")
```

```
[Out] integral(1/(A^2*d*g*x + A^2*c*g + (B^2*d*g*x + B^2*c*g)*log(((b*x + a)/(d*x
 + c))^n*e)^2 + 2*(A*B*d*g*x + A*B*c*g)*log(((b*x + a)/(d*x + c))^n*e)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{A^{2}c+A^{2}dx+2ABc\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^{n}\right)+2ABdx\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^{n}\right)+B^{2}c\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^{n}\right)^{2}+B^{2}dx\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^{n}\right)^{2}}\, dx}{g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*g*x+c*g)/(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
```

```
[Out] Integral(1/(A**2*c + A**2*d*x + 2*A*B*c*log(e*(a/(c + d*x) + b*x/(c + d*x))
**n) + 2*A*B*d*x*log(e*(a/(c + d*x) + b*x/(c + d*x))**n) + B**2*c*log(e*(a/
(c + d*x) + b*x/(c + d*x))**n)**2 + B**2*d*x*log(e*(a/(c + d*x) + b*x/(c +
d*x))**n)**2), x)/g
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*g*x+c*g)/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="gi
ac")
[Out] integrate(1/((d*g*x + c*g)*(B*log(((b*x + a)/(d*x + c))^n*e) + A)^2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{(c\,g+d\,g\,x)\,\left(A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*g + d*g*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2),x)
[Out] int(1/((c*g + d*g*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2), x)
```

3.55 $\int \frac{1}{(cg+dgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$

Optimal. Leaf size=154

$$\frac{e^{-\frac{A}{Bn}}(a+bx)\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-1/n}\operatorname{Ei}\left(\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{Bn}\right)}{B^2(bc-ad)g^2n^2(c+dx)}-\frac{a+bx}{B(bc-ad)g^2n(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}$$

```
[Out] (b*x+a)*Ei((A+B*ln(e*((b*x+a)/(d*x+c))^n))/B/n)/B^2/(-a*d+b*c)/exp(A/B/n)/g
^2/n^2/((e*((b*x+a)/(d*x+c))^n)^(1/n))/(d*x+c)+(-b*x-a)/B/(-a*d+b*c)/g^2/n/
(d*x+c)/(A+B*ln(e*((b*x+a)/(d*x+c))^n))
```

Rubi [A]

time = 0.08, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {2551, 2334, 2337, 2209}

$$\frac{(a+bx)e^{-\frac{A}{Bn}}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-1/n}\operatorname{Ei}\left(\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{Bn}\right)}{B^2g^2n^2(c+dx)(bc-ad)}-\frac{a+bx}{Bg^2n(c+dx)(bc-ad)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*g + d*g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2),x]
```

```
[Out] ((a + b*x)*ExpIntegralEi[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(B*n)])/(B^
2*(b*c - a*d)*E^(A/(B*n))*g^2*n^2*(e*((a + b*x)/(c + d*x))^n)^n^(-1)*(c + d
*x)) - (a + b*x)/(B*(b*c - a*d)*g^2*n*(c + d*x)*(A + B*Log[e*((a + b*x)/(c
+ d*x))^n]))
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2334

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x*((a + b
*Log[c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Dist[1/(b*n*(p + 1)), Int[(a + b*
Log[c*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, n}, x] && LtQ[p, -1] && Inte
gerQ[2*p]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
```

```
{a, b, c, n, p}, x]
```

Rule 2551

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(m +
1)*(g/d)^m, Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a + b
*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ[b*c -
 a*d, 0] && IntegersQ[m, p] && EqQ[d*f - c*g, 0] && (GtQ[p, 0] || LtQ[m, -1
])
```

Rubi steps

$$\int \frac{1}{(cg+dgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{1}{(cg+dgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$

Mathematica [A]
time = 0.12, size = 180, normalized size = 1.17

$$-\frac{e^{-\frac{A}{Bn}}(a+bx)\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-1/n}\left(Be^{\frac{A}{Bn}}n\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{1}{n}}-\text{Ei}\left(\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{Bn}\right)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\right)}{B^2(bc-ad)g^2n^2(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*g + d*g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2),x]
```

```
[Out] -(((a + b*x)*(B*E^(A/(B*n))*n*(e*((a + b*x)/(c + d*x))^n)^n^(-1) - ExpInteg
ralEi[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(B*n)]*(A + B*Log[e*((a + b*x)
/(c + d*x))^n])))/(B^2*(b*c - a*d)*E^(A/(B*n))*g^2*n^2*(e*((a + b*x)/(c + d
*x))^n)^n^(-1)*(c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])))
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{1}{(dgx+cg)^2\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*g*x+c*g)^2/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

```
[Out] int(1/(d*g*x+c*g)^2/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*g*x+c*g)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="
maxima")
```

```
[Out] -(b*x + a)/((b*c^2*g^2*n - a*c*d*g^2*n)*A*B + (b*c^2*g^2*n - a*c*d*g^2*n)*B
^2 + ((b*c*d*g^2*n - a*d^2*g^2*n)*A*B + (b*c*d*g^2*n - a*d^2*g^2*n)*B^2)*x
+ ((b*c*d*g^2*n - a*d^2*g^2*n)*B^2*x + (b*c^2*g^2*n - a*c*d*g^2*n)*B^2)*log
((b*x + a)^n) - ((b*c*d*g^2*n - a*d^2*g^2*n)*B^2*x + (b*c^2*g^2*n - a*c*d*g
^2*n)*B^2)*log((d*x + c)^n)) - integrate(-1/(A*B*c^2*g^2*n + B^2*c^2*g^2*n
+ (A*B*d^2*g^2*n + B^2*d^2*g^2*n)*x^2 + 2*(A*B*c*d*g^2*n + B^2*c*d*g^2*n)*x
 + (B^2*d^2*g^2*n*x^2 + 2*B^2*c*d*g^2*n*x + B^2*c^2*g^2*n)*log((b*x + a)^n)
 - (B^2*d^2*g^2*n*x^2 + 2*B^2*c*d*g^2*n*x + B^2*c^2*g^2*n)*log((d*x + c)^n)
), x)
```

Fricas [A]
time = 0.34, size = 254, normalized size = 1.65

$$-\frac{(Bbnx+Ban)e^{\left(\frac{A+B}{Bn}\right)}-((A+B)dx+(A+B)c+(Bdnx+Bcn)\log\left(\frac{bx+a}{dx+c}\right))\operatorname{log\_integral}\left(\frac{(bx+a)e^{\left(\frac{A+B}{Bn}\right)}}{dx+c}\right)}{((B^3bcd-B^3ad^2)g^2n^3x+(B^3bc^2-B^3acd)g^2n^3)e^{\left(\frac{A+B}{Bn}\right)}\log\left(\frac{bx+a}{dx+c}\right)+(((AB^2+B^3)bcd-(AB^2+B^3)ad^2)g^2n^2x+((AB^2+B^3)bc^2-(AB^2+B^3)acd)g^2n^2)e^{\left(\frac{A+B}{Bn}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*g*x+c*g)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="
fricas")
```

```
[Out] -((B*b*n*x + B*a*n)*e^((A + B)/(B*n)) - ((A + B)*d*x + (A + B)*c + (B*d*n*x
 + B*c*n)*log((b*x + a)/(d*x + c)))*log_integral((b*x + a)*e^((A + B)/(B*n)
)/(d*x + c)))/(((B^3*b*c*d - B^3*a*d^2)*g^2*n^3*x + (B^3*b*c^2 - B^3*a*c*d)
*g^2*n^3)*e^((A + B)/(B*n))*log((b*x + a)/(d*x + c)) + (((A*B^2 + B^3)*b*c*
d - (A*B^2 + B^3)*a*d^2)*g^2*n^2*x + ((A*B^2 + B^3)*b*c^2 - (A*B^2 + B^3)*a
*c*d)*g^2*n^2)*e^((A + B)/(B*n)))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*g*x+c*g)**2/(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 4.88, size = 140, normalized size = 0.91

$$-\left(\frac{bc}{(bc-ad)^2}-\frac{ad}{(bc-ad)^2}\right)\left(\frac{bx+a}{\left(B^2g^2n^2\log\left(\frac{bx+a}{dx+c}\right)+ABg^2n+B^2g^2n\right)(dx+c)}-\frac{\mathrm{Ei}\left(\frac{A}{Bn}+\frac{1}{n}+\log\left(\frac{bx+a}{dx+c}\right)\right)e^{\left(-\frac{A}{Bn}-\frac{1}{n}\right)}}{B^2g^2n^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*g*x+c*g)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="
giac")
```

```
[Out] -(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)*((b*x + a)/((B^2*g^2*n^2*log((b*x
+ a)/(d*x + c)) + A*B*g^2*n + B^2*g^2*n)*(d*x + c)) - Ei(A/(B*n) + 1/n + lo
g((b*x + a)/(d*x + c)))*e^(-A/(B*n) - 1/n)/(B^2*g^2*n^2))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(c\,g+d\,g\,x)^2\,\left(A+B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*g + d*g*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2),x)
```

```
[Out] int(1/((c*g + d*g*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2), x)
```

3.56 $\int \frac{1}{(cg+dgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$

Optimal. Leaf size=256

$$\frac{be^{-\frac{A}{Bn}}(a+bx)\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-1/n}\mathrm{Ei}\left(\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{Bn}\right)}{B^2(bc-ad)^2g^3n^2(c+dx)}-\frac{2de^{-\frac{2A}{Bn}}(a+bx)^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-2/n}\mathrm{Ei}\left(\frac{2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right.\right.\right.}{Bn}\right.}{B^2(bc-ad)^2g^3n^2(c+dx)^2}$$

```
[Out] b*(b*x+a)*Ei((A+B*ln(e*((b*x+a)/(d*x+c))^n))/B/n)/B^2/(-a*d+b*c)^2/exp(A/B/
n)/g^3/n^2/((e*((b*x+a)/(d*x+c))^n)^(1/n))/(d*x+c)-2*d*(b*x+a)^2*Ei(2*(A+B*
ln(e*((b*x+a)/(d*x+c))^n))/B/n)/B^2/(-a*d+b*c)^2/exp(2*A/B/n)/g^3/n^2/((e*(
(b*x+a)/(d*x+c))^n)^(2/n))/(d*x+c)^2+(-b*x-a)/B/(-a*d+b*c)/g^3/n/(d*x+c)^2/
(A+B*ln(e*((b*x+a)/(d*x+c))^n))
```

Rubi [A]
time = 0.19, antiderivative size = 256, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {2551, 2357, 2367, 2337, 2209, 2347}

$$-\frac{2d(a+bx)^2e^{-\frac{2A}{Bn}}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-2/n}\mathrm{Ei}\left(\frac{2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{Bn}\right)}{B^2g^3n^2(c+dx)^2(bc-ad)^2}+\frac{b(a+bx)e^{-\frac{A}{Bn}}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-1/n}\mathrm{Ei}\left(\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{Bn}\right)}{B^2g^3n^2(c+dx)(bc-ad)^2}-\frac{a+bx}{Bg^3n(c+dx)^2(bc-ad)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*g + d*g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2),x]
```

```
[Out] (b*(a + b*x)*ExpIntegralEi[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(B*n)])/(
B^2*(b*c - a*d)^2*E^(A/(B*n))*g^3*n^2*(e*((a + b*x)/(c + d*x))^n)^n^(-1)*(c
 + d*x)) - (2*d*(a + b*x)^2*ExpIntegralEi[(2*(A + B*Log[e*((a + b*x)/(c + d
*x))^n]))/(B*n)])/(B^2*(b*c - a*d)^2*E^((2*A)/(B*n))*g^3*n^2*(e*((a + b*x)/
(c + d*x))^n)^(2/n)*(c + d*x)^2) - (a + b*x)/(B*(b*c - a*d)*g^3*n*(c + d*x)
^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2357

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x
_Symbol] :> Simp[x*(d + e*x)^q*((a + b*Log[c*x^n])^(p + 1)/(b*n*(p + 1))),
x] + (-Dist[(q + 1)/(b*n*(p + 1)), Int[(d + e*x)^q*(a + b*Log[c*x^n])^(p +
1), x], x] + Dist[d*(q/(b*n*(p + 1))), Int[(d + e*x)^(q - 1)*(a + b*Log[c*x
^n])^(p + 1), x], x]) /; FreeQ[{a, b, c, d, e, n}, x] && LtQ[p, -1] && GtQ[
q, 0]
```

Rule 2367

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^(r_.))^(
q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*x^n])^p, (d + e*x
^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
&& IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0] && IntegerQ[r]))
```

Rule 2551

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(m +
1)*(g/d)^m, Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a + b
*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ[b*c -
 a*d, 0] && IntegersQ[m, p] && EqQ[d*f - c*g, 0] && (GtQ[p, 0] || LtQ[m, -1
])
```

Rubi steps

$$\int \frac{1}{(cg+dgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{1}{(cg+dgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$

Mathematica [A]
time = 0.33, size = 288, normalized size = 1.12

$$\frac{e^{-\frac{2A}{Bn}}(a+bx)\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-2/n}\left(-B(bc-ad)e^{\frac{2A}{Bn}}n\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{2/n}+be^{\frac{A}{Bn}}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{1}{n}}(c+dx)\text{Ei}\left(\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{Bn}\right)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)-2d(a+bx)\text{Ei}\left(\frac{2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{Bn}\right)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\right)}{B^2(bc-ad)^2g^3n^2(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*g + d*g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2),x]
```

```
[Out] ((a + b*x)*(-(B*(b*c - a*d)*E^((2*A)/(B*n))*n*(e*((a + b*x)/(c + d*x))^n)^(
2/n)) + b*E^(A/(B*n))*(e*((a + b*x)/(c + d*x))^n)^n^(-1)*(c + d*x)*ExpInteg
ralEi[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(B*n)]*(A + B*Log[e*((a + b*x)
/(c + d*x))^n]) - 2*d*(a + b*x)*ExpIntegralEi[(2*(A + B*Log[e*((a + b*x)/(c
 + d*x))^n]))/(B*n)]*(A + B*Log[e*((a + b*x)/(c + d*x))^n])))/(B^2*(b*c - a
*d)^2*E^((2*A)/(B*n))*g^3*n^2*(e*((a + b*x)/(c + d*x))^n)^(2/n)*(c + d*x)^2
*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(dgx+cg\right)^{3} \left(A+B\ln \left(e \left(\frac{bx+a}{dx+c}\right)^{n}\right)\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*g*x+c*g)^3/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

```
[Out] int(1/(d*g*x+c*g)^3/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*g*x+c*g)^3/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="
maxima")
```

```
[Out] -(b*x + a)/((b*c^3*g^3*n - a*c^2*d*g^3*n)*A*B + (b*c^3*g^3*n - a*c^2*d*g^3*
n)*B^2 + ((b*c*d^2*g^3*n - a*d^3*g^3*n)*A*B + (b*c*d^2*g^3*n - a*d^3*g^3*n)
*B^2)*x^2 + 2*((b*c^2*d*g^3*n - a*c*d^2*g^3*n)*A*B + (b*c^2*d*g^3*n - a*c*d
^2*g^3*n)*B^2)*x + ((b*c*d^2*g^3*n - a*d^3*g^3*n)*B^2*x^2 + 2*(b*c^2*d*g^3*
n - a*c*d^2*g^3*n)*B^2*x + (b*c^3*g^3*n - a*c^2*d*g^3*n)*B^2)*log((b*x + a)
^n) - ((b*c*d^2*g^3*n - a*d^3*g^3*n)*B^2*x^2 + 2*(b*c^2*d*g^3*n - a*c*d^2*g
^3*n)*B^2*x + (b*c^3*g^3*n - a*c^2*d*g^3*n)*B^2)*log((d*x + c)^n)) - integr
ate((b*d*x - b*c + 2*a*d)/(((b*c*d^3*g^3*n - a*d^4*g^3*n)*A*B + (b*c*d^3*g^
3*n - a*d^4*g^3*n)*B^2)*x^3 + (b*c^4*g^3*n - a*c^3*d*g^3*n)*A*B + (b*c^4*g^
3*n - a*c^3*d*g^3*n)*B^2 + 3*((b*c^2*d^2*g^3*n - a*c*d^3*g^3*n)*A*B + (b*c^
2*d^2*g^3*n - a*c*d^3*g^3*n)*B^2)*x^2 + 3*((b*c^3*d*g^3*n - a*c^2*d^2*g^3*n
)*A*B + (b*c^3*d*g^3*n - a*c^2*d^2*g^3*n)*B^2)*x + ((b*c*d^3*g^3*n - a*d^4*
g^3*n)*B^2*x^3 + 3*(b*c^2*d^2*g^3*n - a*c*d^3*g^3*n)*B^2*x^2 + 3*(b*c^3*d*g
^3*n - a*c^2*d^2*g^3*n)*B^2*x + (b*c^4*g^3*n - a*c^3*d*g^3*n)*B^2)*log((b*x
 + a)^n) - ((b*c*d^3*g^3*n - a*d^4*g^3*n)*B^2*x^3 + 3*(b*c^2*d^2*g^3*n - a*
c*d^3*g^3*n)*B^2*x^2 + 3*(b*c^3*d*g^3*n - a*c^2*d^2*g^3*n)*B^2*x + (b*c^4*g
^3*n - a*c^3*d*g^3*n)*B^2)*log((d*x + c)^n)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 652 vs. $2(261) = 522$.

time = 0.37, size = 652, normalized size = 2.55

$$-\frac{\left((Bb^2c - Babd)nx + (Babc - Ba^2d)n\right)e^{\left(\frac{2(A+B)}{Bn}\right)} + 2\left((A+B)d^3x^2 + 2(A+B)cd^2x + (A+B)c^2d + (Bd^3nx^2 + 2Bcd^2nx + Bc^2dn)\log\left(\frac{bx+a}{dx+c}\right)\right)\log\_integral\left(\frac{(b^2x^2+2abx+a^2)e^{\left(\frac{2(A+B)}{Bn}\right)}}{d^2x^2+2cdx+c^2}\right) - \left((Bbd^2nx^2 + 2Bbcdnx + Bbc^2n)e^{\left(\frac{A+B}{Bn}\right)}\log\left(\frac{bx+a}{dx+c}\right) + ((A+B)bd^2x^2 + 2(A+B)bcdx + (A+B)bc^2)e^{\left(\frac{A+B}{Bn}\right)}\right)\log\_integral\left(\frac{(bx+a)e^{\left(\frac{A+B}{Bn}\right)}}{dx+c}\right)}{\left((B^3b^2c^2d^2 - 2B^3abcd^3 + B^3a^2d^4)g^3n^3x^2 + 2(B^3b^2c^3d - 2B^3abc^2d^2 + B^3a^2cd^3)g^3n^3x + (B^3b^2c^4 - 2B^3abc^3d + B^3a^2c^2d^2)g^3n^3\right)e^{\left(\frac{2(A+B)}{Bn}\right)}\log\left(\frac{bx+a}{dx+c}\right) + \left(((AB^2+B^3)b^2c^2d^2 - 2(AB^2+B^3)abcd^3 + (AB^2+B^3)a^2d^4)g^3n^2x^2 + 2((AB^2+B^3)b^2c^3d - 2(AB^2+B^3)abc^2d^2 + (AB^2+B^3)a^2cd^3)g^3n^2x + ((AB^2+B^3)b^2c^4 - 2(AB^2+B^3)abc^3d + (AB^2+B^3)a^2c^2d^2)g^3n^2\right)e^{\left(\frac{2(A+B)}{Bn}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*g*x+c*g)^3/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="
fricas")
```

```
[Out] -(((B*b^2*c - B*a*b*d)*n*x + (B*a*b*c - B*a^2*d)*n)*e^(2*(A + B)/(B*n)) + 2
*((A + B)*d^3*x^2 + 2*(A + B)*c*d^2*x + (A + B)*c^2*d + (B*d^3*n*x^2 + 2*B*
c*d^2*n*x + B*c^2*d*n)*log((b*x + a)/(d*x + c)))*log_integral((b^2*x^2 + 2*
a*b*x + a^2)*e^(2*(A + B)/(B*n))/(d^2*x^2 + 2*c*d*x + c^2)) - ((B*b*d^2*n*x
^2 + 2*B*b*c*d*n*x + B*b*c^2*n)*e^((A + B)/(B*n))*log((b*x + a)/(d*x + c))
+ ((A + B)*b*d^2*x^2 + 2*(A + B)*b*c*d*x + (A + B)*b*c^2)*e^((A + B)/(B*n))
)*log_integral((b*x + a)*e^((A + B)/(B*n))/(d*x + c)))/(((B^3*b^2*c^2*d^2 -
 2*B^3*a*b*c*d^3 + B^3*a^2*d^4)*g^3*n^3*x^2 + 2*(B^3*b^2*c^3*d - 2*B^3*a*b*
c^2*d^2 + B^3*a^2*c*d^3)*g^3*n^3*x + (B^3*b^2*c^4 - 2*B^3*a*b*c^3*d + B^3*a
^2*c^2*d^2)*g^3*n^3)*e^(2*(A + B)/(B*n))*log((b*x + a)/(d*x + c)) + (((A*B^
2 + B^3)*b^2*c^2*d^2 - 2*(A*B^2 + B^3)*a*b*c*d^3 + (A*B^2 + B^3)*a^2*d^4)*g
^3*n^2*x^2 + 2*((A*B^2 + B^3)*b^2*c^3*d - 2*(A*B^2 + B^3)*a*b*c^2*d^2 + (A*
B^2 + B^3)*a^2*c*d^3)*g^3*n^2*x + ((A*B^2 + B^3)*b^2*c^4 - 2*(A*B^2 + B^3)*
a*b*c^3*d + (A*B^2 + B^3)*a^2*c^2*d^2)*g^3*n^2)*e^(2*(A + B)/(B*n)))
```

Sympy [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*g*x+c*g)**3/(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
```

```
[Out] Timed out
```

Giac [A]

time = 2.94, size = 312, normalized size = 1.22

$$\left(\frac{b\mathrm{Ei}\left(\frac{A}{Bn} + \frac{1}{n} + \log\left(\frac{bx+a}{dx+c}\right)\right)e^{\left(-\frac{A}{Bn} - \frac{1}{n}\right)}}{B^2bcg^3n^2 - B^2adg^3n^2} - \frac{2\,d\mathrm{Ei}\left(\frac{2A}{Bn} + \frac{2}{n} + 2\log\left(\frac{bx+a}{dx+c}\right)\right)e^{\left(-\frac{2A}{Bn} - \frac{2}{n}\right)}}{B^2bcg^3n^2 - B^2adg^3n^2} - \frac{\frac{(bx+a)b}{dx+c} - \frac{(bx+a)^2d}{(dx+c)^2}}{B^2bcg^3n^2\log\left(\frac{bx+a}{dx+c}\right) - B^2adg^3n^2\log\left(\frac{bx+a}{dx+c}\right) + ABbcg^3n + B^2bcg^3n - ABadg^3n - B^2adg^3n}\right)\left(\frac{bc}{(bc-ad)^2} - \frac{ad}{(bc-ad)^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*g*x+c*g)^3/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="
giac")
```

```
[Out] (b*Ei(A/(B*n) + 1/n + log((b*x + a)/(d*x + c)))*e^(-A/(B*n) - 1/n)/(B^2*b*c
*g^3*n^2 - B^2*a*d*g^3*n^2) - 2*d*Ei(2*A/(B*n) + 2/n + 2*log((b*x + a)/(d*x
```

```
 + c)))*e^(-2*A/(B*n) - 2/n)/(B^2*b*c*g^3*n^2 - B^2*a*d*g^3*n^2) - ((b*x +
a)*b/(d*x + c) - (b*x + a)^2*d/(d*x + c)^2)/(B^2*b*c*g^3*n^2*log((b*x + a)/
(d*x + c)) - B^2*a*d*g^3*n^2*log((b*x + a)/(d*x + c)) + A*B*b*c*g^3*n + B^2
*b*c*g^3*n - A*B*a*d*g^3*n - B^2*a*d*g^3*n))*(b*c/(b*c - a*d)^2 - a*d/(b*c
- a*d)^2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(c\,g+d\,g\,x\right)}^3\,{\left(A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*g + d*g*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2),x)
[Out] int(1/((c*g + d*g*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2), x)
```

3.57 $\int (f+gx)^4 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$

Optimal. Leaf size=364

$$\frac{B(bc-ad)g(a^3d^3g^3-a^2bd^2g^2(5df-cg)+ab^2dg(10d^2f^2-5cdfg+c^2g^2)-b^3(10d^3f^3-10cd^2f^2g+5c^2dfg^2}{5b^4d^4}$$

```
[Out] 1/5*B*(-a*d+b*c)*g*(a^3*d^3*g^3-a^2*b*d^2*g^2*(-c*g+5*d*f)+a*b^2*d*g*(c^2*g
^2-5*c*d*f*g+10*d^2*f^2)-b^3*(-c^3*g^3+5*c^2*d*f*g^2-10*c*d^2*f^2*g+10*d^3*
f^3))*n*x/b^4/d^4-1/10*B*(-a*d+b*c)*g^2*(a^2*d^2*g^2-a*b*d*g*(-c*g+5*d*f)+b
^2*(c^2*g^2-5*c*d*f*g+10*d^2*f^2))*n*x^2/b^3/d^3-1/15*B*(-a*d+b*c)*g^3*(-a*
d*g-b*c*g+5*b*d*f)*n*x^3/b^2/d^2-1/20*B*(-a*d+b*c)*g^4*n*x^4/b/d-1/5*B*(-a*
g+b*f)^5*n*ln(b*x+a)/b^5/g+1/5*(g*x+f)^5*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/g+
1/5*B*(-c*g+d*f)^5*n*ln(d*x+c)/d^5/g
```

Rubi [A]
time = 0.35, antiderivative size = 364, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2547, 84}

$$-\frac{Bg^2nx^2(bc-ad)\left(a^2d^2g^2-abdg(5df-cg)+b^2(c^2g^2-5cdfg+10d^2f^2)\right)}{10b^3d^3}+\frac{Bgnx(bc-ad)\left(a^3d^3g^3-a^2bd^2g^2(5df-cg)+ab^2dg(c^2g^2-5cdfg+10d^2f^2)-(b^3(-c^3g^3+5c^2dfg^2-10cd^2f^2g+10d^3f^3))\right)}{5b^4d^4}+\frac{(f+gx)^5\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{5g}-\frac{Bn(bf-ag)^5\log(a+bx)}{5b^5g}-\frac{Bg^3nx^3(bc-ad)(-adg-bcg+5bdf)}{15b^2d^2}-\frac{Bg^4nx^4(bc-ad)}{20bd}+\frac{Bn(df-cg)^5\log(c+dx)}{5d^5g}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

```
[Out] (B*(b*c - a*d)*g*(a^3*d^3*g^3 - a^2*b*d^2*g^2*(5*d*f - c*g) + a*b^2*d*g*(10
*d^2*f^2 - 5*c*d*f*g + c^2*g^2) - b^3*(10*d^3*f^3 - 10*c*d^2*f^2*g + 5*c^2*
d*f*g^2 - c^3*g^3))*n*x)/(5*b^4*d^4) - (B*(b*c - a*d)*g^2*(a^2*d^2*g^2 - a*
b*d*g*(5*d*f - c*g) + b^2*(10*d^2*f^2 - 5*c*d*f*g + c^2*g^2))*n*x^2)/(10*b^
3*d^3) - (B*(b*c - a*d)*g^3*(5*b*d*f - b*c*g - a*d*g)*n*x^3)/(15*b^2*d^2) -
 (B*(b*c - a*d)*g^4*n*x^4)/(20*b*d) - (B*(b*f - a*g)^5*n*Log[a + b*x])/(5*b
^5*g) + ((f + g*x)^5*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(5*g) + (B*(d*
f - c*g)^5*n*Log[c + d*x])/(5*d^5*g)
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 2547

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*((A +
 B*Log[e*((a + b*x)/(c + d*x))^n])/(g*(m + 1))), x] - Dist[B*n*((b*c - a*d)
/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /; FreeQ
```

```
[{a, b, c, d, e, f, g, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, -2]
```

Rubi steps

$$\int (f+gx)^4 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) dx = \frac{(f+gx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{5g} - \frac{(Bn)\int \frac{(bc-ad)(f+gx)^5}{(a+bx)(c+dx)}}{5g}$$

$$= \frac{(f+gx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{5g} - \frac{(B(bc-ad)n)\int \frac{(f}{(a+b}}{5g}$$

$$= \frac{(f+gx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{5g} - \frac{(B(bc-ad)n)\int \Big(\frac{g^2(}{}}{}$$

$$= \frac{B(bc-ad)g(a^3d^3g^3 - a^2bd^2g^2(5df-cg) + ab^2dg(10d^2f^2}{}$$

Mathematica [A]
time = 0.44, size = 285, normalized size = 0.78

$$\frac{\frac{B(-bc+ad)g^2nx\left(-12a^3d^3g^3+6a^2bd^2g^2(10df-2cg+dgx)-2ab^2dg\left(6c^2g^2-3cdg(10f+gx)+d^2\left(60f^2+15fgx+2g^2x^2\right)\right)+b^3\left(-12c^3g^3+6c^2dg^2(10f+gx)-2cd^2g\left(60f^2+15fgx+2g^2x^2\right)+d^3\left(120f^3+60f^2gx+20fg^2x^2+3g^3x^3\right)\right)\right)}{12b^4d^4} - \frac{B(bf-ag)^5n\log(a+bx)}{b^5} + (f+gx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) + \frac{B(df-cg)^5n\log(c+dx)}{d^5}}{5g}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

```
[Out] ((B*(-(b*c) + a*d)*g^2*n*x*(-12*a^3*d^3*g^3 + 6*a^2*b*d^2*g^2*(10*d*f - 2*c
*g + d*g*x) - 2*a*b^2*d*g*(6*c^2*g^2 - 3*c*d*g*(10*f + g*x) + d^2*(60*f^2 +
 15*f*g*x + 2*g^2*x^2)) + b^3*(-12*c^3*g^3 + 6*c^2*d*g^2*(10*f + g*x) - 2*c
*d^2*g*(60*f^2 + 15*f*g*x + 2*g^2*x^2) + d^3*(120*f^3 + 60*f^2*g*x + 20*f*g
^2*x^2 + 3*g^3*x^3))))/(12*b^4*d^4) - (B*(b*f - a*g)^5*n*Log[a + b*x])/b^5
+ (f + g*x)^5*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + (B*(d*f - c*g)^5*n*L
og[c + d*x])/d^5)/(5*g)
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int (gx+f)^4 \left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^4*(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

```
[Out] int((g*x+f)^4*(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

Maxima [A]
time = 0.30, size = 636, normalized size = 1.75

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^4*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="maxima")
```

```
[Out] 1/5*B*g^4*x^5*log((b*x/(d*x + c) + a/(d*x + c))^n*e) + 1/5*A*g^4*x^5 + B*f*
g^3*x^4*log((b*x/(d*x + c) + a/(d*x + c))^n*e) + A*f*g^3*x^4 + 2*B*f^2*g^2*
x^3*log((b*x/(d*x + c) + a/(d*x + c))^n*e) + 2*A*f^2*g^2*x^3 + 2*B*f^3*g*x^
2*log((b*x/(d*x + c) + a/(d*x + c))^n*e) + 2*A*f^3*g*x^2 + 1/60*B*g^4*n*(12
*a^5*log(b*x + a)/b^5 - 12*c^5*log(d*x + c)/d^5 - (3*(b^4*c*d^3 - a*b^3*d^4
)*x^4 - 4*(b^4*c^2*d^2 - a^2*b^2*d^4)*x^3 + 6*(b^4*c^3*d - a^3*b*d^4)*x^2 -
 12*(b^4*c^4 - a^4*d^4)*x)/(b^4*d^4)) - 1/6*B*f*g^3*n*(6*a^4*log(b*x + a)/b
^4 - 6*c^4*log(d*x + c)/d^4 + (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d
 - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3)) + B*f^2*g^2*n*(2*a^
3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*
(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2)) - 2*B*f^3*g*n*(a^2*log(b*x + a)/b^2 - c^2
*log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d)) + B*f^4*n*(a*log(b*x + a)/b - c*lo
g(d*x + c)/d) + B*f^4*x*log((b*x/(d*x + c) + a/(d*x + c))^n*e) + A*f^4*x
```

Fricas [A]
time = 0.78, size = 664, normalized size = 1.82

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^4*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fricas")
```

```
[Out] 1/60*(12*(A + B)*b^5*d^5*g^4*x^5 + 3*(20*(A + B)*b^5*d^5*f*g^3 - (B*b^5*c*d
^4 - B*a*b^4*d^5)*g^4*n)*x^4 + 4*(30*(A + B)*b^5*d^5*f^2*g^2 - (5*(B*b^5*c*
d^4 - B*a*b^4*d^5)*f*g^3 - (B*b^5*c^2*d^3 - B*a^2*b^3*d^5)*g^4)*n)*x^3 + 6*
(20*(A + B)*b^5*d^5*f^3*g - (10*(B*b^5*c*d^4 - B*a*b^4*d^5)*f^2*g^2 - 5*(B*
b^5*c^2*d^3 - B*a^2*b^3*d^5)*f*g^3 + (B*b^5*c^3*d^2 - B*a^3*b^2*d^5)*g^4)*n
)*x^2 + 12*(5*B*a*b^4*d^5*f^4 - 10*B*a^2*b^3*d^5*f^3*g + 10*B*a^3*b^2*d^5*f
^2*g^2 - 5*B*a^4*b*d^5*f*g^3 + B*a^5*d^5*g^4)*n*log(b*x + a) - 12*(5*B*b^5*
c*d^4*f^4 - 10*B*b^5*c^2*d^3*f^3*g + 10*B*b^5*c^3*d^2*f^2*g^2 - 5*B*b^5*c^4
*d*f*g^3 + B*b^5*c^5*g^4)*n*log(d*x + c) + 12*(5*(A + B)*b^5*d^5*f^4 - (10*
(B*b^5*c*d^4 - B*a*b^4*d^5)*f^3*g - 10*(B*b^5*c^2*d^3 - B*a^2*b^3*d^5)*f^2*
g^2 + 5*(B*b^5*c^3*d^2 - B*a^3*b^2*d^5)*f*g^3 - (B*b^5*c^4*d - B*a^4*b*d^5)
*g^4)*n)*x + 12*(B*b^5*d^5*g^4*n*x^5 + 5*B*b^5*d^5*f*g^3*n*x^4 + 10*B*b^5*d
^5*f^2*g^2*n*x^3 + 10*B*b^5*d^5*f^3*g*n*x^2 + 5*B*b^5*d^5*f^4*n*x)*log((b*x
 + a)/(d*x + c)))/(b^5*d^5)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**4*(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 11806 vs. 2(351) = 702.
time = 7.13, size = 11806, normalized size = 32.43

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^4*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="giac")
```

```
[Out] 1/60*(12*(5*B*b^6*c^2*d^4*f^4*n - 10*B*a*b^5*c*d^5*f^4*n - 20*(b*x + a)*B*b
^5*c^2*d^5*f^4*n/(d*x + c) + 5*B*a^2*b^4*d^6*f^4*n + 40*(b*x + a)*B*a*b^4*c
*d^6*f^4*n/(d*x + c) + 30*(b*x + a)^2*B*b^4*c^2*d^6*f^4*n/(d*x + c)^2 - 20*
(b*x + a)*B*a^2*b^3*d^7*f^4*n/(d*x + c) - 60*(b*x + a)^2*B*a*b^3*c*d^7*f^4*
n/(d*x + c)^2 - 20*(b*x + a)^3*B*b^3*c^2*d^7*f^4*n/(d*x + c)^3 + 30*(b*x +
a)^2*B*a^2*b^2*d^8*f^4*n/(d*x + c)^2 + 40*(b*x + a)^3*B*a*b^2*c*d^8*f^4*n/(
d*x + c)^3 + 5*(b*x + a)^4*B*b^2*c^2*d^8*f^4*n/(d*x + c)^4 - 20*(b*x + a)^3
*B*a^2*b*d^9*f^4*n/(d*x + c)^3 - 10*(b*x + a)^4*B*a*b*c*d^9*f^4*n/(d*x + c)
^4 + 5*(b*x + a)^4*B*a^2*d^10*f^4*n/(d*x + c)^4 - 10*B*b^6*c^3*d^3*f^3*g*n
+ 10*B*a*b^5*c^2*d^4*f^3*g*n + 50*(b*x + a)*B*b^5*c^3*d^4*f^3*g*n/(d*x + c)
 + 10*B*a^2*b^4*c*d^5*f^3*g*n - 70*(b*x + a)*B*a*b^4*c^2*d^5*f^3*g*n/(d*x +
 c) - 90*(b*x + a)^2*B*b^4*c^3*d^5*f^3*g*n/(d*x + c)^2 - 10*B*a^3*b^3*d^6*f
^3*g*n - 10*(b*x + a)*B*a^2*b^3*c*d^6*f^3*g*n/(d*x + c) + 150*(b*x + a)^2*B
*a*b^3*c^2*d^6*f^3*g*n/(d*x + c)^2 + 70*(b*x + a)^3*B*b^3*c^3*d^6*f^3*g*n/(
d*x + c)^3 + 30*(b*x + a)*B*a^3*b^2*d^7*f^3*g*n/(d*x + c) - 30*(b*x + a)^2*
B*a^2*b^2*c*d^7*f^3*g*n/(d*x + c)^2 - 130*(b*x + a)^3*B*a*b^2*c^2*d^7*f^3*g
*n/(d*x + c)^3 - 20*(b*x + a)^4*B*b^2*c^3*d^7*f^3*g*n/(d*x + c)^4 - 30*(b*x
 + a)^2*B*a^3*b*d^8*f^3*g*n/(d*x + c)^2 + 50*(b*x + a)^3*B*a^2*b*c*d^8*f^3*
g*n/(d*x + c)^3 + 40*(b*x + a)^4*B*a*b*c^2*d^8*f^3*g*n/(d*x + c)^4 + 10*(b*
x + a)^3*B*a^3*d^9*f^3*g*n/(d*x + c)^3 - 20*(b*x + a)^4*B*a^2*c*d^9*f^3*g*n
/(d*x + c)^4 + 10*B*b^6*c^4*d^2*f^2*g^2*n - 10*B*a*b^5*c^3*d^3*f^2*g^2*n -
50*(b*x + a)*B*b^5*c^4*d^3*f^2*g^2*n/(d*x + c) + 50*(b*x + a)*B*a*b^4*c^3*d
^4*f^2*g^2*n/(d*x + c) + 100*(b*x + a)^2*B*b^4*c^4*d^4*f^2*g^2*n/(d*x + c)^
2 - 10*B*a^3*b^3*c*d^5*f^2*g^2*n + 30*(b*x + a)*B*a^2*b^3*c^2*d^5*f^2*g^2*n
/(d*x + c) - 130*(b*x + a)^2*B*a*b^3*c^3*d^5*f^2*g^2*n/(d*x + c)^2 - 90*(b*
x + a)^3*B*b^3*c^4*d^5*f^2*g^2*n/(d*x + c)^3 + 10*B*a^4*b^2*d^6*f^2*g^2*n -
```

```
10*(b*x + a)*B*a^3*b^2*c*d^6*f^2*g^2*n/(d*x + c) - 30*(b*x + a)^2*B*a^2*b^
2*c^2*d^6*f^2*g^2*n/(d*x + c)^2 + 150*(b*x + a)^3*B*a*b^2*c^3*d^6*f^2*g^2*n
/(d*x + c)^3 + 30*(b*x + a)^4*B*b^2*c^4*d^6*f^2*g^2*n/(d*x + c)^4 - 20*(b*x
 + a)*B*a^4*b*d^7*f^2*g^2*n/(d*x + c) + 50*(b*x + a)^2*B*a^3*b*c*d^7*f^2*g^
2*n/(d*x + c)^2 - 30*(b*x + a)^3*B*a^2*b*c^2*d^7*f^2*g^2*n/(d*x + c)^3 - 60
*(b*x + a)^4*B*a*b*c^3*d^7*f^2*g^2*n/(d*x + c)^4 + 10*(b*x + a)^2*B*a^4*d^8
*f^2*g^2*n/(d*x + c)^2 - 30*(b*x + a)^3*B*a^3*c*d^8*f^2*g^2*n/(d*x + c)^3 +
 30*(b*x + a)^4*B*a^2*c^2*d^8*f^2*g^2*n/(d*x + c)^4 - 5*B*b^6*c^5*d*f*g^3*n
 + 5*B*a*b^5*c^4*d^2*f*g^3*n + 25*(b*x + a)*B*b^5*c^5*d^2*f*g^3*n/(d*x + c)
 - 25*(b*x + a)*B*a*b^4*c^4*d^3*f*g^3*n/(d*x + c) - 50*(b*x + a)^2*B*b^4*c^
5*d^3*f*g^3*n/(d*x + c)^2 + 50*(b*x + a)^2*B*a*b^3*c^4*d^4*f*g^3*n/(d*x + c
)^2 + 50*(b*x + a)^3*B*b^3*c^5*d^4*f*g^3*n/(d*x + c)^3 + 5*B*a^4*b^2*c*d^5*
f*g^3*n - 20*(b*x + a)*B*a^3*b^2*c^2*d^5*f*g^3*n/(d*x + c) + 30*(b*x + a)^2
*B*a^2*b^2*c^3*d^5*f*g^3*n/(d*x + c)^2 - 70*(b*x + a)^3*B*a*b^2*c^4*d^5*f*g
^3*n/(d*x + c)^3 - 20*(b*x + a)^4*B*b^2*c^5*d^5*f*g^3*n/(d*x + c)^4 - 5*B*a
^5*b*d^6*f*g^3*n + 15*(b*x + a)*B*a^4*b*c*d^6*f*g^3*n/(d*x + c) - 10*(b*x +
 a)^2*B*a^3*b*c^2*d^6*f*g^3*n/(d*x + c)^2 - 10*(b*x + a)^3*B*a^2*b*c^3*d^6*
f*g^3*n/(d*x + c)^3 + 40*(b*x + a)^4*B*a*b*c^4*d^6*f*g^3*n/(d*x + c)^4 + 5*
(b*x + a)*B*a^5*d^7*f*g^3*n/(d*x + c) - 20*(b*x + a)^2*B*a^4*c*d^7*f*g^3*n/
(d*x + c)^2 + 30*(b*x + a)^3*B*a^3*c^2*d^7*f*g^3*n/(d*x + c)^3 - 20*(b*x +
a)^4*B*a^2*c^3*d^7*f*g^3*n/(d*x + c)^4 + B*b^6*c^6*g^4*n - B*a*b^5*c^5*d*g^
4*n - 5*(b*x + a)*B*b^5*c^6*d*g^4*n/(d*x + c) + 5*(b*x + a)*B*a*b^4*c^5*d^2
*g^4*n/(d*x + c) + 10*(b*x + a)^2*B*b^4*c^6*d^2*g^4*n/(d*x + c)^2 - 10*(b*x
 + a)^2*B*a*b^3*c^5*d^3*g^4*n/(d*x + c)^2 - 10*(b*x + a)^3*B*b^3*c^6*d^3*g^
4*n/(d*x + c)^3 + 10*(b*x + a)^3*B*a*b^2*c^5*d^4*g^4*n/(d*x + c)^3 + 5*(b*x
 + a)^4*B*b^2*c^6*d^4*g^4*n/(d*x + c)^4 - B*a^5*b*c*d^5*g^4*n + 5*(b*x + a)
*B*a^4*b*c^2*d^5*g^4*n/(d*x + c) - 10*(b*x + a)^2*B*a^3*b*c^3*d^5*g^4*n/(d*
x + c)^2 + 10*(b*x + a)^3*B*a^2*b*c^4*d^5*g^4*n/(d*x + c)^3 - 10*(b*x + a)^
4*B*a*b*c^5*d^5*g^4*n/(d*x + c)^4 + B*a^6*d^6*g^4*n - 5*(b*x + a)*B*a^5*c*d
^6*g^4*n/(d*x + c) + 10*(b*x + a)^2*B*a^4*c^2*d^6*g^4*n/(d*x + c)^2 - 10*(b
*x + a)^3*B*a^3*c^3*d^6*g^4*n/(d*x + c)^3 + 5*(b*x + a)^4*B*a^2*c^4*d^6*g^4
*n/(d*x + c)^4)*log((b*x + a)/(d*x + c))/(b^5*d^5 - 5*(b*x + a)*b^4*d^6/(d*
x + c) + 10*(b*x + a)^2*b^3*d^7/(d*x + c)^2 - 10*(b*x + a)^3*b^2*d^8/(d*x +
 c)^3 + 5*(b*x + a)^4*b*d^9/(d*x + c)^4 - (b*x + a)^5*d^10/(d*x + c)^5) - (
120*B*b^10*c^3*d^3*f^3*g*n - 360*B*a*b^9*c^2*d^4*f^3*g*n - 480*(b*x + a)*B*
b^9*c^3*d^4*f^3*g*n/(d*x + c) + 360*B*a^2*b^8*c*d^5*f^3*g*n + 1440*(b*x + a
)*B*a*b^8*c^2*d^5*f^3*g*n/(d*x + c) + 720*(b*x + a)^2*B*b^8*c^3*d^5*f^3*g*n
/(d*x + c)^2 - 120*B*a^3*b^7*d^6*f^3*g*n - 1440*(b*x + a)*B*a^2*b^7*c*d^6*f
^3*g*n/(d*x + c) - 2160*(b*x + a)^2*B*a*b^7*c^2*d^6*f^3*g*n/(d*x + c)^2 - 4
80*(b*x + a)^3*B*b^7*c^3*d^6*f^3*g*n/(d*x + c)^3 + 480*(b*x + a)*B*a^3*b^6*
d^7*f^3*g*n/(d*x + c) + 2160*(b*x + a)^2*B*a^2*...
```

Mupad [B]

time = 4.68, size = 1433, normalized size = 3.94

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^4*(A + B*log(e*((a + b*x)/(c + d*x))^n)),x)
```

```
[Out] x^4*((5*A*a*d*g^4 + 5*A*b*c*g^4 + 20*A*b*d*f*g^3 + B*a*d*g^4*n - B*b*c*g^4*
n)/(20*b*d) - (A*g^4*(5*a*d + 5*b*c))/(20*b*d)) + x^2*((20*A*a*c*f*g^3 + 20
*A*b*d*f^3*g + 30*A*a*d*f^2*g^2 + 30*A*b*c*f^2*g^2 + 10*B*a*d*f^2*g^2*n - 1
0*B*b*c*f^2*g^2*n)/(10*b*d) + ((5*a*d + 5*b*c)*((((5*A*a*d*g^4 + 5*A*b*c*g^
4 + 20*A*b*d*f*g^3 + B*a*d*g^4*n - B*b*c*g^4*n)/(5*b*d) - (A*g^4*(5*a*d + 5
*b*c))/(5*b*d))*(5*a*d + 5*b*c))/(5*b*d) - (5*A*a*c*g^4 + 20*A*a*d*f*g^3 +
20*A*b*c*f*g^3 + 30*A*b*d*f^2*g^2 + 5*B*a*d*f*g^3*n - 5*B*b*c*f*g^3*n)/(5*b
*d) + (A*a*c*g^4)/(b*d)))/(10*b*d) - (a*c*((5*A*a*d*g^4 + 5*A*b*c*g^4 + 20*
A*b*d*f*g^3 + B*a*d*g^4*n - B*b*c*g^4*n)/(5*b*d) - (A*g^4*(5*a*d + 5*b*c))/
(5*b*d)))/(2*b*d)) - x^3*((((5*A*a*d*g^4 + 5*A*b*c*g^4 + 20*A*b*d*f*g^3 + B
*a*d*g^4*n - B*b*c*g^4*n)/(5*b*d) - (A*g^4*(5*a*d + 5*b*c))/(5*b*d))*(5*a*d
 + 5*b*c))/(15*b*d) - (5*A*a*c*g^4 + 20*A*a*d*f*g^3 + 20*A*b*c*f*g^3 + 30*A
*b*d*f^2*g^2 + 5*B*a*d*f*g^3*n - 5*B*b*c*f*g^3*n)/(15*b*d) + (A*a*c*g^4)/(3
*b*d)) + x*((5*A*b*d*f^4 + 20*A*a*d*f^3*g + 20*A*b*c*f^3*g + 30*A*a*c*f^2*g
^2 + 10*B*a*d*f^3*g*n - 10*B*b*c*f^3*g*n)/(5*b*d) - ((5*a*d + 5*b*c)*((20*A
*a*c*f*g^3 + 20*A*b*d*f^3*g + 30*A*a*d*f^2*g^2 + 30*A*b*c*f^2*g^2 + 10*B*a*
d*f^2*g^2*n - 10*B*b*c*f^2*g^2*n)/(5*b*d) + ((5*a*d + 5*b*c)*((((5*A*a*d*g^
4 + 5*A*b*c*g^4 + 20*A*b*d*f*g^3 + B*a*d*g^4*n - B*b*c*g^4*n)/(5*b*d) - (A*
g^4*(5*a*d + 5*b*c))/(5*b*d))*(5*a*d + 5*b*c))/(5*b*d) - (5*A*a*c*g^4 + 20*
A*a*d*f*g^3 + 20*A*b*c*f*g^3 + 30*A*b*d*f^2*g^2 + 5*B*a*d*f*g^3*n - 5*B*b*c
*f*g^3*n)/(5*b*d) + (A*a*c*g^4)/(b*d)))/(5*b*d) - (a*c*((5*A*a*d*g^4 + 5*A*
b*c*g^4 + 20*A*b*d*f*g^3 + B*a*d*g^4*n - B*b*c*g^4*n)/(5*b*d) - (A*g^4*(5*a
*d + 5*b*c))/(5*b*d)))/(b*d)))/(5*b*d) + (a*c*((((5*A*a*d*g^4 + 5*A*b*c*g^4
 + 20*A*b*d*f*g^3 + B*a*d*g^4*n - B*b*c*g^4*n)/(5*b*d) - (A*g^4*(5*a*d + 5*
b*c))/(5*b*d))*(5*a*d + 5*b*c))/(5*b*d) - (5*A*a*c*g^4 + 20*A*a*d*f*g^3 + 2
0*A*b*c*f*g^3 + 30*A*b*d*f^2*g^2 + 5*B*a*d*f*g^3*n - 5*B*b*c*f*g^3*n)/(5*b*
d) + (A*a*c*g^4)/(b*d)))/(b*d)) + log(e*((a + b*x)/(c + d*x))^n)*((B*g^4*x^
5)/5 + B*f^4*x + 2*B*f^2*g^2*x^3 + 2*B*f^3*g*x^2 + B*f*g^3*x^4) + (A*g^4*x^
5)/5 + (log(a + b*x)*((B*a^5*g^4*n)/5 + B*a*b^4*f^4*n + 2*B*a^3*b^2*f^2*g^2
*n - B*a^4*b*f*g^3*n - 2*B*a^2*b^3*f^3*g*n))/b^5 - (log(c + d*x)*(B*c^5*g^4
*n + 5*B*c*d^4*f^4*n + 10*B*c^3*d^2*f^2*g^2*n - 5*B*c^4*d*f*g^3*n - 10*B*c^
2*d^3*f^3*g*n))/(5*d^5)
```

3.58 $\int (f+gx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$

Optimal. Leaf size=235

$$-\frac{B(bc-ad)g(a^2d^2g^2-abdg(4df-cg)+b^2(6d^2f^2-4cdfg+c^2g^2))\,nx}{4b^3d^3}-\frac{B(bc-ad)g^2(4bdf-bcg-adg)n}{8b^2d^2}$$

```
[Out] -1/4*B*(-a*d+b*c)*g*(a^2*d^2*g^2-a*b*d*g*(-c*g+4*d*f)+b^2*(c^2*g^2-4*c*d*f*
g+6*d^2*f^2))*n*x/b^3/d^3-1/8*B*(-a*d+b*c)*g^2*(-a*d*g-b*c*g+4*b*d*f)*n*x^2
/b^2/d^2-1/12*B*(-a*d+b*c)*g^3*n*x^3/b/d-1/4*B*(-a*g+b*f)^4*n*ln(b*x+a)/b^4
/g+1/4*(g*x+f)^4*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/g+1/4*B*(-c*g+d*f)^4*n*ln(
d*x+c)/d^4/g
```

Rubi [A]
time = 0.21, antiderivative size = 235, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2547, 84}

$$-\frac{Bgnx(bc-ad)\left(a^2d^2g^2-abdg(4df-cg)+b^2(c^2g^2-4cdfg+6d^2f^2)\right)}{4b^3d^3}+\frac{(f+gx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{4g}-\frac{Bn(bf-ag)^4\log(a+bx)}{4b^4g}-\frac{Bg^2nx^2(bc-ad)(-adg-bcg+4bdf)}{8b^2d^2}-\frac{Bg^3nx^3(bc-ad)}{12bd}+\frac{Bn(df-cg)^4\log(c+dx)}{4d^4g}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

```
[Out] -1/4*(B*(b*c - a*d)*g*(a^2*d^2*g^2 - a*b*d*g*(4*d*f - c*g) + b^2*(6*d^2*f^2
 - 4*c*d*f*g + c^2*g^2))*n*x)/(b^3*d^3) - (B*(b*c - a*d)*g^2*(4*b*d*f - b*c
*g - a*d*g)*n*x^2)/(8*b^2*d^2) - (B*(b*c - a*d)*g^3*n*x^3)/(12*b*d) - (B*(b
*f - a*g)^4*n*Log[a + b*x])/(4*b^4*g) + ((f + g*x)^4*(A + B*Log[e*((a + b*x
)/(c + d*x))^n]))/(4*g) + (B*(d*f - c*g)^4*n*Log[c + d*x])/(4*d^4*g)
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 2547

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*((A +
 B*Log[e*((a + b*x)/(c + d*x))^n])/(g*(m + 1))), x] - Dist[B*n*((b*c - a*d)
/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /; FreeQ
[{a, b, c, d, e, f, g, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, -2]
```

Rubi steps

$$\int (f+gx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx = \frac{(f+gx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4g} - \frac{(Bn)\int\frac{(bc-ad)(f+gx)^4}{(a+bx)(c+dx)}}{4g}$$

$$= \frac{(f+gx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4g} - \frac{(B(bc-ad)n)\int\frac{(f}{(a+b}}{4g}$$

$$= \frac{(f+gx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4g} - \frac{(B(bc-ad)n)\int\left(\frac{g^2(}{}\right.}{}$$

$$= -\frac{B(bc-ad)g(a^2d^2g^2-abdg(4df-cg)+b^2(6d^2f^2-4cd}{4b^3d^3}$$

Mathematica [A]
time = 0.20, size = 219, normalized size = 0.93

$$\frac{(f+gx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)-\frac{Bn\left(6bd(bc-ad)g^2\left(a^2d^2g^2+abdg(-4df+cg)+b^2\left(6d^2f^2-4cdfg+c^2g^2\right)\right)x+3b^2d^2(bc-ad)g^3(4bdf-bcg-adg)x^2+2b^3d^3(bc-ad)g^4x^3+6d^4(bf-ag)^4\log(a+bx)-6b^4(df-cg)^4\log(c+dx)\right)}{6b^4d^4}}{4g}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

```
[Out] ((f + g*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - (B*n*(6*b*d*(b*c - a*
d)*g^2*(a^2*d^2*g^2 + a*b*d*g*(-4*d*f + c*g) + b^2*(6*d^2*f^2 - 4*c*d*f*g +
 c^2*g^2))*x + 3*b^2*d^2*(b*c - a*d)*g^3*(4*b*d*f - b*c*g - a*d*g)*x^2 + 2*
b^3*d^3*(b*c - a*d)*g^4*x^3 + 6*d^4*(b*f - a*g)^4*Log[a + b*x] - 6*b^4*(d*f
 - c*g)^4*Log[c + d*x]))/(6*b^4*d^4))/(4*g)
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int (gx+f)^3\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

```
[Out] int((g*x+f)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

Maxima [A]
time = 0.29, size = 447, normalized size = 1.90

$$\frac{1}{4}Bg^3x^4\log\left(\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^ne\right)+\frac{1}{4}Ag^3x^4+Bfg^2x^3\log\left(\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^ne\right)+Afg^2x^3+\frac{3}{2}Bf^2gx^2\log\left(\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^ne\right)+\frac{3}{2}Af^2gx^2-\frac{1}{24}Bg^3n\left(\frac{6a^4\log(bx+a)}{b^4}-\frac{6c^4\log(dx+c)}{d^4}+\frac{2(b^3cd^2-ab^2d^3)x^3-3(b^3c^2d-a^2bd^3)x^2+6(b^3c^3-a^3d^3)x}{b^3d^3}\right)+\frac{1}{2}Bfg^2n\left(\frac{2a^3\log(bx+a)}{b^3}-\frac{2c^3\log(dx+c)}{d^3}-\frac{(b^2cd-abd^2)x^2-2(b^2c^2-a^2d^2)x}{b^2d^2}\right)-\frac{3}{2}Bf^2gn\left(\frac{a^2\log(bx+a)}{b^2}-\frac{c^2\log(dx+c)}{d^2}+\frac{(bc-ad)x}{bd}\right)+Bf^3n\left(\frac{a\log(bx+a)}{b}-\frac{c\log(dx+c)}{d}\right)+Bf^3x\log\left(\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^ne\right)+Af^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="maxima")
```

```
[Out] 1/4*B*g^3*x^4*log((b*x/(d*x + c) + a/(d*x + c))^n*e) + 1/4*A*g^3*x^4 + B*f*
g^2*x^3*log((b*x/(d*x + c) + a/(d*x + c))^n*e) + A*f*g^2*x^3 + 3/2*B*f^2*g*
x^2*log((b*x/(d*x + c) + a/(d*x + c))^n*e) + 3/2*A*f^2*g*x^2 - 1/24*B*g^3*n
*(6*a^4*log(b*x + a)/b^4 - 6*c^4*log(d*x + c)/d^4 + (2*(b^3*c*d^2 - a*b^2*d
^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3
)) + 1/2*B*f*g^2*n*(2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2
*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2)) - 3/2*B*f^2*g*n*(
a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d)) + B*f^3*
n*(a*log(b*x + a)/b - c*log(d*x + c)/d) + B*f^3*x*log((b*x/(d*x + c) + a/(d
*x + c))^n*e) + A*f^3*x
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 465 vs. $2(224) = 448$.
time = 0.46, size = 465, normalized size = 1.98

$$\frac{6\,(A+B)b^4d^4g^3x^4+2\,(12\,(A+B)b^4d^4fg^2-(Bb^4cd^3-Bab^3d^4)g^3n)x^3+3\,(12\,(A+B)b^4d^4f^2g-(4\,(Bb^4cd^3-Bab^3d^4)fg^2-(Bb^4c^2d^2-Ba^2b^2d^4)g^3)n)x^2+6\,(4\,Bab^3d^4f^3-6\,Ba^2b^2d^4f^2g+4\,Ba^3bd^4fg^2-Ba^4d^4g^3)n\log(bx+a)-6\,(4\,Bb^4cd^3f^3-6\,Bb^4c^2d^2f^2g+4\,Bb^4c^3dfg^2-Bb^4c^4g^3)n\log(dx+c)+6\,(4\,(A+B)b^4d^4f^3-(6\,(Bb^4cd^3-Bab^3d^4)f^2g-4\,(Bb^4c^2d^2-Ba^2b^2d^4)fg^2+(Bb^4c^3d-Ba^3bd^4)g^3)n)x+6\,(Bb^4d^4g^3nx^4+4\,Bb^4d^4fg^2nx^3+6\,Bb^4d^4f^2gnx^2+4\,Bb^4d^4f^3nx)\log\left(\frac{bx+a}{dx+c}\right)}{24\,b^4d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fricas")
```

```
[Out] 1/24*(6*(A + B)*b^4*d^4*g^3*x^4 + 2*(12*(A + B)*b^4*d^4*f*g^2 - (B*b^4*c*d^
3 - B*a*b^3*d^4)*g^3*n)*x^3 + 3*(12*(A + B)*b^4*d^4*f^2*g - (4*(B*b^4*c*d^3
 - B*a*b^3*d^4)*f*g^2 - (B*b^4*c^2*d^2 - B*a^2*b^2*d^4)*g^3)*n)*x^2 + 6*(4*
B*a*b^3*d^4*f^3 - 6*B*a^2*b^2*d^4*f^2*g + 4*B*a^3*b*d^4*f*g^2 - B*a^4*d^4*g
^3)*n*log(b*x + a) - 6*(4*B*b^4*c*d^3*f^3 - 6*B*b^4*c^2*d^2*f^2*g + 4*B*b^4
*c^3*d*f*g^2 - B*b^4*c^4*g^3)*n*log(d*x + c) + 6*(4*(A + B)*b^4*d^4*f^3 - (
6*(B*b^4*c*d^3 - B*a*b^3*d^4)*f^2*g - 4*(B*b^4*c^2*d^2 - B*a^2*b^2*d^4)*f*g
^2 + (B*b^4*c^3*d - B*a^3*b*d^4)*g^3)*n)*x + 6*(B*b^4*d^4*g^3*n*x^4 + 4*B*b
^4*d^4*f*g^2*n*x^3 + 6*B*b^4*d^4*f^2*g*n*x^2 + 4*B*b^4*d^4*f^3*n*x)*log((b*
x + a)/(d*x + c)))/(b^4*d^4)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**3*(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 6660 vs. $2(224) = 448$.
time = 5.35, size = 6660, normalized size = 28.34

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="giac")
```

```
[Out] 1/24*(6*(4*B*b^5*c^2*d^3*f^3*n - 8*B*a*b^4*c*d^4*f^3*n - 12*(b*x + a)*B*b^4
*c^2*d^4*f^3*n/(d*x + c) + 4*B*a^2*b^3*d^5*f^3*n + 24*(b*x + a)*B*a*b^3*c*d
^5*f^3*n/(d*x + c) + 12*(b*x + a)^2*B*b^3*c^2*d^5*f^3*n/(d*x + c)^2 - 12*(b
*x + a)*B*a^2*b^2*d^6*f^3*n/(d*x + c) - 24*(b*x + a)^2*B*a*b^2*c*d^6*f^3*n/
(d*x + c)^2 - 4*(b*x + a)^3*B*b^2*c^2*d^6*f^3*n/(d*x + c)^3 + 12*(b*x + a)^
2*B*a^2*b*d^7*f^3*n/(d*x + c)^2 + 8*(b*x + a)^3*B*a*b*c*d^7*f^3*n/(d*x + c)
^3 - 4*(b*x + a)^3*B*a^2*d^8*f^3*n/(d*x + c)^3 - 6*B*b^5*c^3*d^2*f^2*g*n +
6*B*a*b^4*c^2*d^3*f^2*g*n + 24*(b*x + a)*B*b^4*c^3*d^3*f^2*g*n/(d*x + c) +
6*B*a^2*b^3*c*d^4*f^2*g*n - 36*(b*x + a)*B*a*b^3*c^2*d^4*f^2*g*n/(d*x + c)
- 30*(b*x + a)^2*B*b^3*c^3*d^4*f^2*g*n/(d*x + c)^2 - 6*B*a^3*b^2*d^5*f^2*g*
n + 54*(b*x + a)^2*B*a*b^2*c^2*d^5*f^2*g*n/(d*x + c)^2 + 12*(b*x + a)^3*B*b
^2*c^3*d^5*f^2*g*n/(d*x + c)^3 + 12*(b*x + a)*B*a^3*b*d^6*f^2*g*n/(d*x + c)
 - 18*(b*x + a)^2*B*a^2*b*c*d^6*f^2*g*n/(d*x + c)^2 - 24*(b*x + a)^3*B*a*b*
c^2*d^6*f^2*g*n/(d*x + c)^3 - 6*(b*x + a)^2*B*a^3*d^7*f^2*g*n/(d*x + c)^2 +
 12*(b*x + a)^3*B*a^2*c*d^7*f^2*g*n/(d*x + c)^3 + 4*B*b^5*c^4*d*f*g^2*n - 4
*B*a*b^4*c^3*d^2*f*g^2*n - 16*(b*x + a)*B*b^4*c^4*d^2*f*g^2*n/(d*x + c) + 1
6*(b*x + a)*B*a*b^3*c^3*d^3*f*g^2*n/(d*x + c) + 24*(b*x + a)^2*B*b^3*c^4*d^
3*f*g^2*n/(d*x + c)^2 - 4*B*a^3*b^2*c*d^4*f*g^2*n + 12*(b*x + a)*B*a^2*b^2*
c^2*d^4*f*g^2*n/(d*x + c) - 36*(b*x + a)^2*B*a*b^2*c^3*d^4*f*g^2*n/(d*x + c
)^2 - 12*(b*x + a)^3*B*b^2*c^4*d^4*f*g^2*n/(d*x + c)^3 + 4*B*a^4*b*d^5*f*g^
2*n - 8*(b*x + a)*B*a^3*b*c*d^5*f*g^2*n/(d*x + c) + 24*(b*x + a)^3*B*a*b*c^
3*d^5*f*g^2*n/(d*x + c)^3 - 4*(b*x + a)*B*a^4*d^6*f*g^2*n/(d*x + c) + 12*(b
*x + a)^2*B*a^3*c*d^6*f*g^2*n/(d*x + c)^2 - 12*(b*x + a)^3*B*a^2*c^2*d^6*f*
g^2*n/(d*x + c)^3 - B*b^5*c^5*g^3*n + B*a*b^4*c^4*d*g^3*n + 4*(b*x + a)*B*b
^4*c^5*d*g^3*n/(d*x + c) - 4*(b*x + a)*B*a*b^3*c^4*d^2*g^3*n/(d*x + c) - 6*
(b*x + a)^2*B*b^3*c^5*d^2*g^3*n/(d*x + c)^2 + 6*(b*x + a)^2*B*a*b^2*c^4*d^3
*g^3*n/(d*x + c)^2 + 4*(b*x + a)^3*B*b^2*c^5*d^3*g^3*n/(d*x + c)^3 + B*a^4*
b*c*d^4*g^3*n - 4*(b*x + a)*B*a^3*b*c^2*d^4*g^3*n/(d*x + c) + 6*(b*x + a)^2
*B*a^2*b*c^3*d^4*g^3*n/(d*x + c)^2 - 8*(b*x + a)^3*B*a*b*c^4*d^4*g^3*n/(d*x
 + c)^3 - B*a^5*d^5*g^3*n + 4*(b*x + a)*B*a^4*c*d^5*g^3*n/(d*x + c) - 6*(b*
x + a)^2*B*a^3*c^2*d^5*g^3*n/(d*x + c)^2 + 4*(b*x + a)^3*B*a^2*c^3*d^5*g^3*
n/(d*x + c)^3)*log((b*x + a)/(d*x + c))/(b^4*d^4 - 4*(b*x + a)*b^3*d^5/(d*x
 + c) + 6*(b*x + a)^2*b^2*d^6/(d*x + c)^2 - 4*(b*x + a)^3*b*d^7/(d*x + c)^3
 + (b*x + a)^4*d^8/(d*x + c)^4) - (36*B*b^8*c^3*d^2*f^2*g*n - 108*B*a*b^7*c
^2*d^3*f^2*g*n - 108*(b*x + a)*B*b^7*c^3*d^3*f^2*g*n/(d*x + c) + 108*B*a^2*
b^6*c*d^4*f^2*g*n + 324*(b*x + a)*B*a*b^6*c^2*d^4*f^2*g*n/(d*x + c) + 108*(
b*x + a)^2*B*b^6*c^3*d^4*f^2*g*n/(d*x + c)^2 - 36*B*a^3*b^5*d^5*f^2*g*n - 3
24*(b*x + a)*B*a^2*b^5*c*d^5*f^2*g*n/(d*x + c) - 324*(b*x + a)^2*B*a*b^5*c^
2*d^5*f^2*g*n/(d*x + c)^2 - 36*(b*x + a)^3*B*b^5*c^3*d^5*f^2*g*n/(d*x + c)^
3 + 108*(b*x + a)*B*a^3*b^4*d^6*f^2*g*n/(d*x + c) + 324*(b*x + a)^2*B*a^2*b
^4*c*d^6*f^2*g*n/(d*x + c)^2 + 108*(b*x + a)^3*B*a*b^4*c^2*d^6*f^2*g*n/(d*x
```

```
 + c)^3 - 108*(b*x + a)^2*B*a^3*b^3*d^7*f^2*g*n/(d*x + c)^2 - 108*(b*x + a)
^3*B*a^2*b^3*c*d^7*f^2*g*n/(d*x + c)^3 + 36*(b*x + a)^3*B*a^3*b^2*d^8*f^2*g
*n/(d*x + c)^3 - 36*B*b^8*c^4*d*f*g^2*n + 72*B*a*b^7*c^3*d^2*f*g^2*n + 120*
(b*x + a)*B*b^7*c^4*d^2*f*g^2*n/(d*x + c) - 264*(b*x + a)*B*a*b^6*c^3*d^3*f
*g^2*n/(d*x + c) - 132*(b*x + a)^2*B*b^6*c^4*d^3*f*g^2*n/(d*x + c)^2 - 72*B
*a^3*b^5*c*d^4*f*g^2*n + 72*(b*x + a)*B*a^2*b^5*c^2*d^4*f*g^2*n/(d*x + c) +
 312*(b*x + a)^2*B*a*b^5*c^3*d^4*f*g^2*n/(d*x + c)^2 + 48*(b*x + a)^3*B*b^5
*c^4*d^4*f*g^2*n/(d*x + c)^3 + 36*B*a^4*b^4*d^5*f*g^2*n + 168*(b*x + a)*B*a
^3*b^4*c*d^5*f*g^2*n/(d*x + c) - 144*(b*x + a)^2*B*a^2*b^4*c^2*d^5*f*g^2*n/
(d*x + c)^2 - 120*(b*x + a)^3*B*a*b^4*c^3*d^5*f*g^2*n/(d*x + c)^3 - 96*(b*x
 + a)*B*a^4*b^3*d^6*f*g^2*n/(d*x + c) - 120*(b*x + a)^2*B*a^3*b^3*c*d^6*f*g
^2*n/(d*x + c)^2 + 72*(b*x + a)^3*B*a^2*b^3*c^2*d^6*f*g^2*n/(d*x + c)^3 + 8
4*(b*x + a)^2*B*a^4*b^2*d^7*f*g^2*n/(d*x + c)^2 + 24*(b*x + a)^3*B*a^3*b^2*
c*d^7*f*g^2*n/(d*x + c)^3 - 24*(b*x + a)^3*B*a^4*b*d^8*f*g^2*n/(d*x + c)^3
+ 11*B*b^8*c^5*g^3*n - 19*B*a*b^7*c^4*d*g^3*n - 38*(b*x + a)*B*b^7*c^5*d*g^
3*n/(d*x + c) + 2*B*a^2*b^6*c^3*d^2*g^3*n + 70*(b*x + a)*B*a*b^6*c^4*d^2*g^
3*n/(d*x + c) + 45*(b*x + a)^2*B*b^6*c^5*d^2*g^3*n/(d*x + c)^2 - 2*B*a^3*b^
5*c^2*d^3*g^3*n - 8*(b*x + a)*B*a^2*b^5*c^3*d^3*g^3*n/(d*x + c) - 93*(b*x +
 a)^2*B*a*b^5*c^4*d^3*g^3*n/(d*x + c)^2 - 18*(b*x + a)^3*B*b^5*c^5*d^3*g^3*
n/(d*x + c)^3 + 19*B*a^4*b^4*c*d^4*g^3*n - 16*(b*x + a)*B*a^3*b^4*c^2*d^4*g
^3*n/(d*x + c) + 30*(b*x + a)^2*B*a^2*b^4*c^3*d^4*g^3*n/(d*x + c)^2 + 42*(b
*x + a)^3*B*a*b^4*c^4*d^4*g^3*n/(d*x + c)^3 - 11*B*a^5*b^3*d^5*g^3*n - 34*(
b*x + a)*B*a^4*b^3*c*d^5*g^3*n/(d*x + c) + 18*(b*x + a)^2*B*a^3*b^3*c^2*d^5
*g^3*n/(d*x + c)^2 - 24*(b*x + a)^3*B*a^2*b^3*c^3*d^5*g^3*n/(d*x + c)^3 + 2
6*(b*x + a)*B*a^5*b^2*d^6*g^3*n/(d*x + c) + 21*(b*x + a)^2*B*a^4*b^2*c*d^6*
g^3*n/(d*x + c)^2 - 21*(b*x + a)^2*B*a^5*b*d^7*...
```

Mupad [B]
time = 4.74, size = 766, normalized size = 3.26

$$x\left(\frac{4\,A\,b\,d\,f^3+12\,A\,a\,c\,f\,g^2+12\,A\,a\,d\,f^2\,g+12\,A\,b\,c\,f^2\,g+6\,B\,a\,d\,f^2\,g\,n-6\,B\,b\,c\,f^2\,g\,n}{4\,b\,d}+\text{[illegible]}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n)),x)
```

```
[Out] x*((4*A*b*d*f^3 + 12*A*a*c*f*g^2 + 12*A*a*d*f^2*g + 12*A*b*c*f^2*g + 6*B*a*
d*f^2*g*n - 6*B*b*c*f^2*g*n)/(4*b*d) + ((4*a*d + 4*b*c)*((((4*A*a*d*g^3 + 4
*A*b*c*g^3 + 12*A*b*d*f*g^2 + B*a*d*g^3*n - B*b*c*g^3*n)/(4*b*d) - (A*g^3*(
4*a*d + 4*b*c))/(4*b*d))*(4*a*d + 4*b*c))/(4*b*d) - (4*A*a*c*g^3 + 12*A*a*d
*f*g^2 + 12*A*b*c*f*g^2 + 12*A*b*d*f^2*g + 4*B*a*d*f*g^2*n - 4*B*b*c*f*g^2*
n)/(4*b*d) + (A*a*c*g^3)/(b*d)))/(4*b*d) - (a*c*((4*A*a*d*g^3 + 4*A*b*c*g^3
 + 12*A*b*d*f*g^2 + B*a*d*g^3*n - B*b*c*g^3*n)/(4*b*d) - (A*g^3*(4*a*d + 4*
b*c))/(4*b*d)))/(b*d)) - x^2*((((4*A*a*d*g^3 + 4*A*b*c*g^3 + 12*A*b*d*f*g^2
 + B*a*d*g^3*n - B*b*c*g^3*n)/(4*b*d) - (A*g^3*(4*a*d + 4*b*c))/(4*b*d))*(4
*a*d + 4*b*c))/(8*b*d) - (4*A*a*c*g^3 + 12*A*a*d*f*g^2 + 12*A*b*c*f*g^2 + 1
```

```
2*A*b*d*f^2*g + 4*B*a*d*f*g^2*n - 4*B*b*c*f*g^2*n)/(8*b*d) + (A*a*c*g^3)/(2
*b*d)) + x^3*((4*A*a*d*g^3 + 4*A*b*c*g^3 + 12*A*b*d*f*g^2 + B*a*d*g^3*n - B
*b*c*g^3*n)/(12*b*d) - (A*g^3*(4*a*d + 4*b*c))/(12*b*d)) + log(e*((a + b*x)
/(c + d*x))^n)*((B*g^3*x^4)/4 + B*f^3*x + (3*B*f^2*g*x^2)/2 + B*f*g^2*x^3)
+ (A*g^3*x^4)/4 - (log(a + b*x)*(B*a^4*g^3*n - 4*B*a*b^3*f^3*n - 4*B*a^3*b*
f*g^2*n + 6*B*a^2*b^2*f^2*g*n))/(4*b^4) + (log(c + d*x)*(B*c^4*g^3*n - 4*B*
c*d^3*f^3*n - 4*B*c^3*d*f*g^2*n + 6*B*c^2*d^2*f^2*g*n))/(4*d^4)
```

3.59 $\int (f+gx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) dx$

Optimal. Leaf size=157

$$-\frac{B(bc-ad)g(3bdf-bcg-adg)nx}{3b^2d^2}-\frac{B(bc-ad)g^2nx^2}{6bd}-\frac{B(bf-ag)^3n\log(a+bx)}{3b^3g}+\frac{(f+gx)^3\left(A+B\log\right.}{3g}$$

```
[Out] -1/3*B*(-a*d+b*c)*g*(-a*d*g-b*c*g+3*b*d*f)*n*x/b^2/d^2-1/6*B*(-a*d+b*c)*g^2
*n*x^2/b/d-1/3*B*(-a*g+b*f)^3*n*ln(b*x+a)/b^3/g+1/3*(g*x+f)^3*(A+B*ln(e*((b
*x+a)/(d*x+c))^n))/g+1/3*B*(-c*g+d*f)^3*n*ln(d*x+c)/d^3/g
```

Rubi [A]
time = 0.11, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2547, 84}

$$\frac{(f+gx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3g}-\frac{Bn(bf-ag)^3\log(a+bx)}{3b^3g}-\frac{Bgnx(bc-ad)(-adg-bcg+3bdf)}{3b^2d^2}-\frac{Bg^2nx^2(bc-ad)}{6bd}+\frac{Bn(df-cg)^3\log(c+dx)}{3d^3g}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

```
[Out] -1/3*(B*(b*c - a*d)*g*(3*b*d*f - b*c*g - a*d*g)*n*x)/(b^2*d^2) - (B*(b*c -
a*d)*g^2*n*x^2)/(6*b*d) - (B*(b*f - a*g)^3*n*Log[a + b*x])/(3*b^3*g) + ((f
+ g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(3*g) + (B*(d*f - c*g)^3*n
*Log[c + d*x])/(3*d^3*g)
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 2547

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*((A +
 B*Log[e*((a + b*x)/(c + d*x))^n])/(g*(m + 1))), x] - Dist[B*n*((b*c - a*d)
/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /; FreeQ
[{a, b, c, d, e, f, g, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, -2]
```

Rubi steps

$$\int (f+gx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx = \frac{(f+gx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3g} - \frac{(Bn)\int \frac{(bc-ad)(f+gx)^3}{(a+bx)(c+dx)}}{3g}$$

$$= \frac{(f+gx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3g} - \frac{(B(bc-ad)n)\int \frac{(f}{(a+b}}{3g}$$

$$= \frac{(f+gx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3g} - \frac{(B(bc-ad)n)\int \left(\frac{g^2(}{}\right.}{}$$

$$= -\frac{B(bc-ad)g(3bdf-bcg-adg)nx}{3b^2d^2} - \frac{B(bc-ad)g^2nx^2}{6bd}$$

Mathematica [A]
time = 0.10, size = 146, normalized size = 0.93

$$\frac{(f+gx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) - \frac{Bn\left(2bd(bc-ad)g^2(3bdf-bcg-adg)x+b^2d^2(bc-ad)g^3x^2+2d^3(bf-ag)^3\log(a+bx)-2b^3(df-cg)^3\log(c+dx)\right)}{2b^3d^3}}{3g}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

```
[Out] ((f + g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - (B*n*(2*b*d*(b*c - a*
d)*g^2*(3*b*d*f - b*c*g - a*d*g)*x + b^2*d^2*(b*c - a*d)*g^3*x^2 + 2*d^3*(b
*f - a*g)^3*Log[a + b*x] - 2*b^3*(d*f - c*g)^3*Log[c + d*x]))/(2*b^3*d^3))/
(3*g)
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int (gx+f)^2 \left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

```
[Out] int((g*x+f)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

Maxima [A]
time = 0.28, size = 285, normalized size = 1.82

$$\frac{1}{3}Bg^2x^3\log\left(\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n e\right)+\frac{1}{3}Ag^2x^3+Bfgx^2\log\left(\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n e\right)+Afgx^2+\frac{1}{6}Bg^2n\left(\frac{2a^3\log(bx+a)}{b^3}-\frac{2c^3\log(dx+c)}{d^3}-\frac{(b^2cd-abd^2)x^2-2(b^2c^2-a^2d^2)x}{b^2d^2}\right)-Bfgn\left(\frac{a^2\log(bx+a)}{b^2}-\frac{c^2\log(dx+c)}{d^2}+\frac{(bc-ad)x}{bd}\right)+Bf^2n\left(\frac{a\log(bx+a)}{b}-\frac{c\log(dx+c)}{d}\right)+Bf^2x\log\left(\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n e\right)+Af^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="maxima")
```

```
[Out] 1/3*B*g^2*x^3*log((b*x/(d*x + c) + a/(d*x + c))^n*e) + 1/3*A*g^2*x^3 + B*f*
g*x^2*log((b*x/(d*x + c) + a/(d*x + c))^n*e) + A*f*g*x^2 + 1/6*B*g^2*n*(2*a
^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2
*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2)) - B*f*g*n*(a^2*log(b*x + a)/b^2 - c^2*lo
g(d*x + c)/d^2 + (b*c - a*d)*x/(b*d)) + B*f^2*n*(a*log(b*x + a)/b - c*log(d
*x + c)/d) + B*f^2*x*log((b*x/(d*x + c) + a/(d*x + c))^n*e) + A*f^2*x
```

Fricas [**A**]
time = 0.44, size = 294, normalized size = 1.87

$$\frac{2(A+B)b^3d^3g^2x^3 + (6(A+B)b^3d^3fg - (Bb^3cd^2 - Bab^2d^3)g^2n)x^2 + 2(3Bab^2d^3f^2 - 3Ba^2bd^3fg + Ba^3d^3g^2)n\log(bx+a) - 2(3Bb^3cd^2f^2 - 3Bb^3c^2dfg + Bb^3c^3g^2)n\log(dx+c) + 2(3(A+B)b^3d^3f^2 - (3(Bb^3cd^2 - Bab^2d^3)fg - (Bb^3c^2d - Ba^2bd^3)g^2)n)x + 2(Bb^3d^3g^2nx^3 + 3Bb^3d^3fgnx^2 + 3Bb^3d^3f^2nx)\log\left(\frac{bx+a}{dx+c}\right)}{6b^3d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fricas")
```

```
[Out] 1/6*(2*(A + B)*b^3*d^3*g^2*x^3 + (6*(A + B)*b^3*d^3*f*g - (B*b^3*c*d^2 - B*
a*b^2*d^3)*g^2*n)*x^2 + 2*(3*B*a*b^2*d^3*f^2 - 3*B*a^2*b*d^3*f*g + B*a^3*d^
3*g^2)*n*log(b*x + a) - 2*(3*B*b^3*c*d^2*f^2 - 3*B*b^3*c^2*d*f*g + B*b^3*c^
3*g^2)*n*log(d*x + c) + 2*(3*(A + B)*b^3*d^3*f^2 - (3*(B*b^3*c*d^2 - B*a*b^
2*d^3)*f*g - (B*b^3*c^2*d - B*a^2*b*d^3)*g^2)*n)*x + 2*(B*b^3*d^3*g^2*n*x^3
 + 3*B*b^3*d^3*f*g*n*x^2 + 3*B*b^3*d^3*f^2*n*x)*log((b*x + a)/(d*x + c)))/(
b^3*d^3)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2*(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 3346 vs. 2(148) = 296.
time = 5.53, size = 3346, normalized size = 21.31

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="giac")
```

```
[Out] 1/6*(2*(3*B*b^4*c^2*d^2*f^2*n - 6*B*a*b^3*c*d^3*f^2*n - 6*(b*x + a)*B*b^3*c
^2*d^3*f^2*n/(d*x + c) + 3*B*a^2*b^2*d^4*f^2*n + 12*(b*x + a)*B*a*b^2*c*d^4
*f^2*n/(d*x + c) + 3*(b*x + a)^2*B*b^2*c^2*d^4*f^2*n/(d*x + c)^2 - 6*(b*x +
```

a)*B*a^2*b*d^5*f^2*n/(d*x + c) - 6*(b*x + a)^2*B*a*b*c*d^5*f^2*n/(d*x + c)
^2 + 3*(b*x + a)^2*B*a^2*d^6*f^2*n/(d*x + c)^2 - 3*B*b^4*c^3*d*f*g*n + 3*B*
a*b^3*c^2*d^2*f*g*n + 9*(b*x + a)*B*b^3*c^3*d^2*f*g*n/(d*x + c) + 3*B*a^2*b
^2*c*d^3*f*g*n - 15*(b*x + a)*B*a*b^2*c^2*d^3*f*g*n/(d*x + c) - 6*(b*x + a)
^2*B*b^2*c^3*d^3*f*g*n/(d*x + c)^2 - 3*B*a^3*b*d^4*f*g*n + 3*(b*x + a)*B*a^
2*b*c*d^4*f*g*n/(d*x + c) + 12*(b*x + a)^2*B*a*b*c^2*d^4*f*g*n/(d*x + c)^2
+ 3*(b*x + a)*B*a^3*d^5*f*g*n/(d*x + c) - 6*(b*x + a)^2*B*a^2*c*d^5*f*g*n/(
d*x + c)^2 + B*b^4*c^4*g^2*n - B*a*b^3*c^3*d*g^2*n - 3*(b*x + a)*B*b^3*c^4*
d*g^2*n/(d*x + c) + 3*(b*x + a)*B*a*b^2*c^3*d^2*g^2*n/(d*x + c) + 3*(b*x +
a)^2*B*b^2*c^4*d^2*g^2*n/(d*x + c)^2 - B*a^3*b*c*d^3*g^2*n + 3*(b*x + a)*B*
a^2*b*c^2*d^3*g^2*n/(d*x + c) - 6*(b*x + a)^2*B*a*b*c^3*d^3*g^2*n/(d*x + c)
^2 + B*a^4*d^4*g^2*n - 3*(b*x + a)*B*a^3*c*d^4*g^2*n/(d*x + c) + 3*(b*x + a
)^2*B*a^2*c^2*d^4*g^2*n/(d*x + c)^2)*log((b*x + a)/(d*x + c))/(b^3*d^3 - 3*
(b*x + a)*b^2*d^4/(d*x + c) + 3*(b*x + a)^2*b*d^5/(d*x + c)^2 - (b*x + a)^3
*d^6/(d*x + c)^3) - (6*B*b^6*c^3*d*f*g*n - 18*B*a*b^5*c^2*d^2*f*g*n - 12*(b
*x + a)*B*b^5*c^3*d^2*f*g*n/(d*x + c) + 18*B*a^2*b^4*c*d^3*f*g*n + 36*(b*x
+ a)*B*a*b^4*c^2*d^3*f*g*n/(d*x + c) + 6*(b*x + a)^2*B*b^4*c^3*d^3*f*g*n/(d
*x + c)^2 - 6*B*a^3*b^3*d^4*f*g*n - 36*(b*x + a)*B*a^2*b^3*c*d^4*f*g*n/(d*x
+ c) - 18*(b*x + a)^2*B*a*b^3*c^2*d^4*f*g*n/(d*x + c)^2 + 12*(b*x + a)*B*a
^3*b^2*d^5*f*g*n/(d*x + c) + 18*(b*x + a)^2*B*a^2*b^2*c*d^5*f*g*n/(d*x + c)
^2 - 6*(b*x + a)^2*B*a^3*b*d^6*f*g*n/(d*x + c)^2 - 3*B*b^6*c^4*g^2*n + 6*B*
a*b^5*c^3*d*g^2*n + 7*(b*x + a)*B*b^5*c^4*d*g^2*n/(d*x + c) - 16*(b*x + a)*
B*a*b^4*c^3*d^2*g^2*n/(d*x + c) - 4*(b*x + a)^2*B*b^4*c^4*d^2*g^2*n/(d*x +
c)^2 - 6*B*a^3*b^3*c*d^3*g^2*n + 6*(b*x + a)*B*a^2*b^3*c^2*d^3*g^2*n/(d*x +
c) + 10*(b*x + a)^2*B*a*b^3*c^3*d^3*g^2*n/(d*x + c)^2 + 3*B*a^4*b^2*d^4*g^
2*n + 8*(b*x + a)*B*a^3*b^2*c*d^4*g^2*n/(d*x + c) - 6*(b*x + a)^2*B*a^2*b^2
*c^2*d^4*g^2*n/(d*x + c)^2 - 5*(b*x + a)*B*a^4*b*d^5*g^2*n/(d*x + c) - 2*(b
*x + a)^2*B*a^3*b*c*d^5*g^2*n/(d*x + c)^2 + 2*(b*x + a)^2*B*a^4*d^6*g^2*n/(
d*x + c)^2 - 6*A*b^6*c^2*d^2*f^2 - 6*B*b^6*c^2*d^2*f^2 + 12*A*a*b^5*c*d^3*f
^2 + 12*B*a*b^5*c*d^3*f^2 + 12*(b*x + a)*A*b^5*c^2*d^3*f^2/(d*x + c) + 12*(
b*x + a)*B*b^5*c^2*d^3*f^2/(d*x + c) - 6*A*a^2*b^4*d^4*f^2 - 6*B*a^2*b^4*d^
4*f^2 - 24*(b*x + a)*A*a*b^4*c*d^4*f^2/(d*x + c) - 24*(b*x + a)*B*a*b^4*c*d
^4*f^2/(d*x + c) - 6*(b*x + a)^2*A*b^4*c^2*d^4*f^2/(d*x + c)^2 - 6*(b*x + a
)^2*B*b^4*c^2*d^4*f^2/(d*x + c)^2 + 12*(b*x + a)*A*a^2*b^3*d^5*f^2/(d*x + c
) + 12*(b*x + a)*B*a^2*b^3*d^5*f^2/(d*x + c) + 12*(b*x + a)^2*A*a*b^3*c*d^5
*f^2/(d*x + c)^2 + 12*(b*x + a)^2*B*a*b^3*c*d^5*f^2/(d*x + c)^2 - 6*(b*x +
a)^2*A*a^2*b^2*d^6*f^2/(d*x + c)^2 - 6*(b*x + a)^2*B*a^2*b^2*d^6*f^2/(d*x +
c)^2 + 6*A*b^6*c^3*d*f*g + 6*B*b^6*c^3*d*f*g - 6*A*a*b^5*c^2*d^2*f*g - 6*B
*a*b^5*c^2*d^2*f*g - 18*(b*x + a)*A*b^5*c^3*d^2*f*g/(d*x + c) - 18*(b*x + a
)*B*b^5*c^3*d^2*f*g/(d*x + c) - 6*A*a^2*b^4*c*d^3*f*g - 6*B*a^2*b^4*c*d^3*f
g + 30(b*x + a)*A*a*b^4*c^2*d^3*f*g/(d*x + c) + 30*(b*x + a)*B*a*b^4*c^2*
d^3*f*g/(d*x + c) + 12*(b*x + a)^2*A*b^4*c^3*d^3*f*g/(d*x + c)^2 + 12*(b*x
+ a)^2*B*b^4*c^3*d^3*f*g/(d*x + c)^2 + 6*A*a^3*b^3*d^4*f*g + 6*B*a^3*b^3*d^
4*f*g - 6*(b*x + a)*A*a^2*b^3*c*d^4*f*g/(d*x + c) - 6*(b*x + a)*B*a^2*b^3*c
*d^4*f*g/(d*x + c) - 24*(b*x + a)^2*A*a*b^3*c^2*d^4*f*g/(d*x + c)^2 - 24*(b

```
*x + a)^2*B*a*b^3*c^2*d^4*f*g/(d*x + c)^2 - 6*(b*x + a)*A*a^3*b^2*d^5*f*g/(
d*x + c) - 6*(b*x + a)*B*a^3*b^2*d^5*f*g/(d*x + c) + 12*(b*x + a)^2*A*a^2*b
^2*c*d^5*f*g/(d*x + c)^2 + 12*(b*x + a)^2*B*a^2*b^2*c*d^5*f*g/(d*x + c)^2 -
 2*A*b^6*c^4*g^2 - 2*B*b^6*c^4*g^2 + 2*A*a*b^5*c^3*d*g^2 + 2*B*a*b^5*c^3*d*
g^2 + 6*(b*x + a)*A*b^5*c^4*d*g^2/(d*x + c) + 6*(b*x + a)*B*b^5*c^4*d*g^2/(
d*x + c) - 6*(b*x + a)*A*a*b^4*c^3*d^2*g^2/(d*x + c) - 6*(b*x + a)*B*a*b^4*
c^3*d^2*g^2/(d*x + c) - 6*(b*x + a)^2*A*b^4*c^4*d^2*g^2/(d*x + c)^2 - 6*(b*
x + a)^2*B*b^4*c^4*d^2*g^2/(d*x + c)^2 + 2*A*a^3*b^3*c*d^3*g^2 + 2*B*a^3*b^
3*c*d^3*g^2 - 6*(b*x + a)*A*a^2*b^3*c^2*d^3*g^2/(d*x + c) - 6*(b*x + a)*B*a
^2*b^3*c^2*d^3*g^2/(d*x + c) + 12*(b*x + a)^2*A*a*b^3*c^3*d^3*g^2/(d*x + c)
^2 + 12*(b*x + a)^2*B*a*b^3*c^3*d^3*g^2/(d*x + c)^2 - 2*A*a^4*b^2*d^4*g^2 -
 2*B*a^4*b^2*d^4*g^2 + 6*(b*x + a)*A*a^3*b^2*c*d^4*g^2/(d*x + c) + 6*(b*x +
 a)*B*a^3*b^2*c*d^4*g^2/(d*x + c) - 6*(b*x + a)^2*A*a^2*b^2*c^2*d^4*g^2/(d*
x + c)^2 - 6*(b*x + a)^2*B*a^2*b^2*c^2*d^4*g^2/(d*x + c)^2)/(b^5*d^3 - 3*(b
*x + a)*b^4*d^4/(d*x + c) + 3*(b*x + a)^2*b^3*d^5/(d*x + c)^2 - (b*x + a)^3
*b^2*d^6/(d*x + c)^3) + 2*(3*B*b^4*c^2*d^2*f^2*n - 6*B*a*b^3*c*d^3*f^2*n +
3*B*a^2*b^2*d^4*f^2*n - 3*B*b^4*c^3*d*f*g*n + 3*B*a*b^3*c^2*d^2*f*g*n + 3*B
*a^2*b^2*c*d^3*f*g*n - 3*B*a^3*b*d^4*f*g*n + B*b^4*c^4*g^2*n - B*a*b^3*c^3*
d*g^2*n - B*a^3*b*c*d^3*g^2*n + B*a^4*d^4*g^2*n...
```

Mupad [B]
time = 4.18, size = 371, normalized size = 2.36

$$x^2\left(\frac{3\,A\,a\,d\,g^2+3\,A\,b\,c\,g^2+6\,A\,b\,d\,f\,g+B\,a\,d\,g^2\,n-B\,b\,c\,g^2\,n}{6\,b\,d}-\frac{A\,g^2\,\left(3\,a\,d+3\,b\,c\right)}{6\,b\,d}\right)-x\,\left(\frac{\left(3\,a\,d+3\,b\,c\right)\,\left(\frac{3\,A\,a\,d\,g^2+3\,A\,b\,c\,g^2+6\,A\,b\,d\,f\,g+B\,a\,d\,g^2\,n-B\,b\,c\,g^2\,n}{3\,b\,d}-\frac{A\,g^2\,\left(3\,a\,d+3\,b\,c\right)}{3\,b\,d}\right)}{3\,b\,d}-\frac{3\,A\,a\,c\,g^2+3\,A\,b\,d\,f^2+6\,A\,a\,d\,f\,g+6\,A\,b\,c\,f\,g+3\,B\,a\,d\,f\,g\,n-3\,B\,b\,c\,f\,g\,n}{3\,b\,d}+\frac{A\,a\,c\,g^2}{b\,d}\right)+\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\,\left(B\,f^2\,x+B\,f\,g\,x^2+\frac{B\,g^2\,x^3}{3}\right)+\frac{A\,g^2\,x^3}{3}+\frac{\ln\left(a+b\,x\right)\,\left(B\,n\,a^3\,g^2-3\,B\,n\,a^2\,b\,f\,g+3\,B\,n\,a\,b^2\,f^2\right)}{3\,b^3}-\frac{\ln\left(c+d\,x\right)\,\left(B\,n\,c^3\,g^2-3\,B\,n\,c^2\,d\,f\,g+3\,B\,n\,c\,d^2\,f^2\right)}{3\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n)),x)
```

```
[Out] x^2*((3*A*a*d*g^2 + 3*A*b*c*g^2 + 6*A*b*d*f*g + B*a*d*g^2*n - B*b*c*g^2*n)/
(6*b*d) - (A*g^2*(3*a*d + 3*b*c))/(6*b*d)) - x*(((3*a*d + 3*b*c)*((3*A*a*d*
g^2 + 3*A*b*c*g^2 + 6*A*b*d*f*g + B*a*d*g^2*n - B*b*c*g^2*n)/(3*b*d) - (A*g
^2*(3*a*d + 3*b*c))/(3*b*d)))/(3*b*d) - (3*A*a*c*g^2 + 3*A*b*d*f^2 + 6*A*a*
d*f*g + 6*A*b*c*f*g + 3*B*a*d*f*g*n - 3*B*b*c*f*g*n)/(3*b*d) + (A*a*c*g^2)/
(b*d)) + log(e*((a + b*x)/(c + d*x))^n)*((B*g^2*x^3)/3 + B*f^2*x + B*f*g*x^
2) + (A*g^2*x^3)/3 + (log(a + b*x)*(B*a^3*g^2*n + 3*B*a*b^2*f^2*n - 3*B*a^2
*b*f*g*n))/(3*b^3) - (log(c + d*x)*(B*c^3*g^2*n + 3*B*c*d^2*f^2*n - 3*B*c^2
*d*f*g*n))/(3*d^3)
```

3.60 $\int (f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$

Optimal. Leaf size=115

$$-\frac{B(bc-ad)gnx}{2bd}-\frac{B(bf-ag)^2n\log(a+bx)}{2b^2g}+\frac{(f+gx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2g}+\frac{B(df-cg)^2n\log(c+dx)}{2d^2g}$$

```
[Out] -1/2*B*(-a*d+b*c)*g*n*x/b/d-1/2*B*(-a*g+b*f)^2*n*ln(b*x+a)/b^2/g+1/2*(g*x+f
)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/g+1/2*B*(-c*g+d*f)^2*n*ln(d*x+c)/d^2/g
```

Rubi [A]
time = 0.07, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2547, 84}

$$\frac{(f+gx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2g}-\frac{Bn(bf-ag)^2\log(a+bx)}{2b^2g}-\frac{Bgnx(bc-ad)}{2bd}+\frac{Bn(df-cg)^2\log(c+dx)}{2d^2g}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

```
[Out] -1/2*(B*(b*c - a*d)*g*n*x)/(b*d) - (B*(b*f - a*g)^2*n*Log[a + b*x])/(2*b^2*
g) + ((f + g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(2*g) + (B*(d*f -
 c*g)^2*n*Log[c + d*x])/(2*d^2*g)
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 2547

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*((A +
 B*Log[e*((a + b*x)/(c + d*x))^n])/(g*(m + 1))), x] - Dist[B*n*((b*c - a*d)
/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /; FreeQ
[{a, b, c, d, e, f, g, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, -2]
```

Rubi steps

$$\int (f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)dx = \frac{(f+gx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2g} - \frac{(Bn)\int \frac{(bc-ad)(f+gx)^2}{(a+bx)(c+dx)}dx}{2g}$$
$$= \frac{(f+gx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2g} - \frac{(B(bc-ad)n)\int \frac{(f+g}{(a+bx)(}}{2g}$$
$$= \frac{(f+gx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2g} - \frac{(B(bc-ad)n)\int\left(\frac{g^2}{bd}+\right.}{}$$
$$= -\frac{B(bc-ad)gnx}{2bd} - \frac{B(bf-ag)^2n\log(a+bx)}{2b^2g} + \frac{(f+gx)^2\ (}{}$$

Mathematica [A]
time = 0.09, size = 120, normalized size = 1.04

$$\frac{-Bd^2(bf-ag)^2n\log(a+bx)+b\left(d(B(-bc+ad)g^2nx+Abd(f+gx)^2)+bBd^2(f+gx)^2\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+bB(df-cg)^2n\log(c+dx)\right)}{2b^2d^2g}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

```
[Out] (-(B*d^2*(b*f - a*g)^2*n*Log[a + b*x]) + b*(d*(B*(-(b*c) + a*d)*g^2*n*x + A
*b*d*(f + g*x)^2) + b*B*d^2*(f + g*x)^2*Log[e*((a + b*x)/(c + d*x))^n] + b*
B*(d*f - c*g)^2*n*Log[c + d*x]))/(2*b^2*d^2*g)
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int (gx+f)\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)*(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

```
[Out] int((g*x+f)*(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

Maxima [A]
time = 0.28, size = 152, normalized size = 1.32

$$\frac{1}{2}Bgx^2\log\left(\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n e\right)+\frac{1}{2}Agx^2-\frac{1}{2}Bgn\left(\frac{a^2\log(bx+a)}{b^2}-\frac{c^2\log(dx+c)}{d^2}+\frac{(bc-ad)x}{bd}\right)+Bfn\left(\frac{a\log(bx+a)}{b}-\frac{c\log(dx+c)}{d}\right)+Bfx\log\left(\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n e\right)+Afx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="maxima")
```

```
[Out] 1/2*B*g*x^2*log((b*x/(d*x + c) + a/(d*x + c))^n*e) + 1/2*A*g*x^2 - 1/2*B*g*
n*(a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d)) + B*f
*n*(a*log(b*x + a)/b - c*log(d*x + c)/d) + B*f*x*log((b*x/(d*x + c) + a/(d*
x + c))^n*e) + A*f*x
```

Fricas [A]
time = 0.36, size = 156, normalized size = 1.36

$$\frac{(A+B)b^2d^2gx^2 + (2\,Babd^2f - Ba^2d^2g)n\log(bx+a) - (2\,Bb^2cdf - Bb^2c^2g)n\log(dx+c) + (2\,(A+B)b^2d^2f - (Bb^2cd - Babd^2)gn)x + (Bb^2d^2gnx^2 + 2\,Bb^2d^2fnx)\log\left(\frac{bx+a}{dx+c}\right)}{2\,b^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fricas")
```

```
[Out] 1/2*((A + B)*b^2*d^2*g*x^2 + (2*B*a*b*d^2*f - B*a^2*d^2*g)*n*log(b*x + a) -
 (2*B*b^2*c*d*f - B*b^2*c^2*g)*n*log(d*x + c) + (2*(A + B)*b^2*d^2*f - (B*b
^2*c*d - B*a*b*d^2)*g*n)*x + (B*b^2*d^2*g*n*x^2 + 2*B*b^2*d^2*f*n*x)*log((b
*x + a)/(d*x + c)))/(b^2*d^2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 493 vs. $2(97) = 194$.
time = 168.92, size = 493, normalized size = 4.29

$$\begin{cases} \left(A + B\log\left(e\left(\frac{a}{c}\right)^n\right)\right)\left(fx + \frac{gx^2}{2}\right) & \text{for}\: b = 0 \wedge d = 0 \\ Afx + \frac{Agx^2}{2} - \frac{Ba^2g\log\left(e\left(\frac{a}{c}+\frac{bx}{c}\right)^n\right)}{2b^2} + \frac{Baf\log\left(e\left(\frac{a}{c}+\frac{bx}{c}\right)^n\right)}{b} + \frac{Bagnx}{2b} - Bfnx + Bfx\log\left(e\left(\frac{a}{c}+\frac{bx}{c}\right)^n\right) - \frac{Bgnx^2}{4} + \frac{Bgx^2\log\left(e\left(\frac{a}{c}+\frac{bx}{c}\right)^n\right)}{2} & \text{for}\: d = 0 \\ Afx + \frac{Agx^2}{2} - \frac{Bc^2g\log\left(e\left(\frac{a}{c+dx}\right)^n\right)}{2d^2} + \frac{Bcf\log\left(e\left(\frac{a}{c+dx}\right)^n\right)}{d} - \frac{Bcgnx}{2d} + Bfnx + Bfx\log\left(e\left(\frac{a}{c+dx}\right)^n\right) + \frac{Bgnx^2}{4} + \frac{Bgx^2\log\left(e\left(\frac{a}{c+dx}\right)^n\right)}{2} & \text{for}\: b = 0 \\ Afx + \frac{Agx^2}{2} - \frac{Ba^2gn\log\left(\frac{c}{d}+x\right)}{2b^2} - \frac{Ba^2g\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}{2b^2} + \frac{Bafn\log\left(\frac{c}{d}+x\right)}{b} + \frac{Baf\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}{b} + \frac{Bagnx}{2b} + \frac{Bc^2gn\log\left(\frac{c}{d}+x\right)}{2d^2} - \frac{Bcfn\log\left(\frac{c}{d}+x\right)}{d} - \frac{Bcgnx}{2d} + Bfx\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right) + \frac{Bgx^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

```
[Out] Piecewise(((A + B*log(e*(a/c)**n))*(f*x + g*x**2/2), Eq(b, 0) & Eq(d, 0)),
(A*f*x + A*g*x**2/2 - B*a**2*g*log(e*(a/c + b*x/c)**n)/(2*b**2) + B*a*f*log
(e*(a/c + b*x/c)**n)/b + B*a*g*n*x/(2*b) - B*f*n*x + B*f*x*log(e*(a/c + b*x
/c)**n) - B*g*n*x**2/4 + B*g*x**2*log(e*(a/c + b*x/c)**n)/2, Eq(d, 0)), (A*
f*x + A*g*x**2/2 - B*c**2*g*log(e*(a/(c + d*x))**n)/(2*d**2) + B*c*f*log(e*
(a/(c + d*x))**n)/d - B*c*g*n*x/(2*d) + B*f*n*x + B*f*x*log(e*(a/(c + d*x))
**n) + B*g*n*x**2/4 + B*g*x**2*log(e*(a/(c + d*x))**n)/2, Eq(b, 0)), (A*f*x
 + A*g*x**2/2 - B*a**2*g*n*log(c/d + x)/(2*b**2) - B*a**2*g*log(e*(a/(c + d
*x) + b*x/(c + d*x))**n)/(2*b**2) + B*a*f*n*log(c/d + x)/b + B*a*f*log(e*(a
/(c + d*x) + b*x/(c + d*x))**n)/b + B*a*g*n*x/(2*b) + B*c**2*g*n*log(c/d +
x)/(2*d**2) - B*c*f*n*log(c/d + x)/d - B*c*g*n*x/(2*d) + B*f*x*log(e*(a/(c
+ d*x) + b*x/(c + d*x))**n) + B*g*x**2*log(e*(a/(c + d*x) + b*x/(c + d*x))*
*n)/2, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1189 vs. $2(108) = 216$.
time = 4.31, size = 1189, normalized size = 10.34

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="giac")
```

```
[Out] 1/2*((2*B*b^3*c^2*d*f*n - 4*B*a*b^2*c*d^2*f*n - 2*(b*x + a)*B*b^2*c^2*d^2*f
*n/(d*x + c) + 2*B*a^2*b*d^3*f*n + 4*(b*x + a)*B*a*b*c*d^3*f*n/(d*x + c) -
2*(b*x + a)*B*a^2*d^4*f*n/(d*x + c) - B*b^3*c^3*g*n + B*a*b^2*c^2*d*g*n + 2
*(b*x + a)*B*b^2*c^3*d*g*n/(d*x + c) + B*a^2*b*c*d^2*g*n - 4*(b*x + a)*B*a*
b*c^2*d^2*g*n/(d*x + c) - B*a^3*d^3*g*n + 2*(b*x + a)*B*a^2*c*d^3*g*n/(d*x
+ c))*log((b*x + a)/(d*x + c))/(b^2*d^2 - 2*(b*x + a)*b*d^3/(d*x + c) + (b*
x + a)^2*d^4/(d*x + c)^2) - (B*b^4*c^3*g*n - 3*B*a*b^3*c^2*d*g*n - (b*x + a
)*B*b^3*c^3*d*g*n/(d*x + c) + 3*B*a^2*b^2*c*d^2*g*n + 3*(b*x + a)*B*a*b^2*c
^2*d^2*g*n/(d*x + c) - B*a^3*b*d^3*g*n - 3*(b*x + a)*B*a^2*b*c*d^3*g*n/(d*x
 + c) + (b*x + a)*B*a^3*d^4*g*n/(d*x + c) - 2*A*b^4*c^2*d*f - 2*B*b^4*c^2*d
*f + 4*A*a*b^3*c*d^2*f + 4*B*a*b^3*c*d^2*f + 2*(b*x + a)*A*b^3*c^2*d^2*f/(d
*x + c) + 2*(b*x + a)*B*b^3*c^2*d^2*f/(d*x + c) - 2*A*a^2*b^2*d^3*f - 2*B*a
^2*b^2*d^3*f - 4*(b*x + a)*A*a*b^2*c*d^3*f/(d*x + c) - 4*(b*x + a)*B*a*b^2*
c*d^3*f/(d*x + c) + 2*(b*x + a)*A*a^2*b*d^4*f/(d*x + c) + 2*(b*x + a)*B*a^2
*b*d^4*f/(d*x + c) + A*b^4*c^3*g + B*b^4*c^3*g - A*a*b^3*c^2*d*g - B*a*b^3*
c^2*d*g - 2*(b*x + a)*A*b^3*c^3*d*g/(d*x + c) - 2*(b*x + a)*B*b^3*c^3*d*g/(
d*x + c) - A*a^2*b^2*c*d^2*g - B*a^2*b^2*c*d^2*g + 4*(b*x + a)*A*a*b^2*c^2*
d^2*g/(d*x + c) + 4*(b*x + a)*B*a*b^2*c^2*d^2*g/(d*x + c) + A*a^3*b*d^3*g +
 B*a^3*b*d^3*g - 2*(b*x + a)*A*a^2*b*c*d^3*g/(d*x + c) - 2*(b*x + a)*B*a^2*
b*c*d^3*g/(d*x + c))/(b^3*d^2 - 2*(b*x + a)*b^2*d^3/(d*x + c) + (b*x + a)^2
*b*d^4/(d*x + c)^2) + (2*B*b^3*c^2*d*f*n - 4*B*a*b^2*c*d^2*f*n + 2*B*a^2*b*
d^3*f*n - B*b^3*c^3*g*n + B*a*b^2*c^2*d*g*n + B*a^2*b*c*d^2*g*n - B*a^3*d^3
*g*n)*log(-b + (b*x + a)*d/(d*x + c))/(b^2*d^2) - (2*B*b^3*c^2*d*f*n - 4*B*
a*b^2*c*d^2*f*n + 2*B*a^2*b*d^3*f*n - B*b^3*c^3*g*n + B*a*b^2*c^2*d*g*n + B
*a^2*b*c*d^2*g*n - B*a^3*d^3*g*n)*log((b*x + a)/(d*x + c))/(b^2*d^2))*(b*c/
(b*c - a*d)^2 - a*d/(b*c - a*d)^2)
```

Mupad [B]
time = 4.26, size = 153, normalized size = 1.33

$$x\left(\frac{2\,A\,a\,d\,g+2\,A\,b\,c\,g+2\,A\,b\,d\,f+B\,a\,d\,g\,n-B\,b\,c\,g\,n}{2\,b\,d}-\frac{A\,g\,\left(2\,a\,d+2\,b\,c\right)}{2\,b\,d}\right)+\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\,\left(\frac{B\,g\,x^2}{2}+B\,f\,x\right)-\frac{\ln\left(a+b\,x\right)\,\left(B\,a^2\,g\,n-2\,B\,a\,b\,f\,n\right)}{2\,b^2}+\frac{\ln\left(c+d\,x\right)\,\left(B\,c^2\,g\,n-2\,B\,c\,d\,f\,n\right)}{2\,d^2}+\frac{A\,g\,x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n)),x)
```

```
[Out] x*((2*A*a*d*g + 2*A*b*c*g + 2*A*b*d*f + B*a*d*g*n - B*b*c*g*n)/(2*b*d) - (A
*g*(2*a*d + 2*b*c))/(2*b*d)) + log(e*((a + b*x)/(c + d*x))^n)*(B*f*x + (B*g
*x^2)/2) - (log(a + b*x)*(B*a^2*g*n - 2*B*a*b*f*n))/(2*b^2) + (log(c + d*x)
*(B*c^2*g*n - 2*B*c*d*f*n))/(2*d^2) + (A*g*x^2)/2
```

3.61 $\int \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$

Optimal. Leaf size=56

$$Ax+\frac{B(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{b}-\frac{B(bc-ad)n\log(c+dx)}{bd}$$

```
[Out] A*x+B*(b*x+a)*ln(e*((b*x+a)/(d*x+c))^n)/b-B*(-a*d+b*c)*n*ln(d*x+c)/b/d
```

Rubi [A]
time = 0.02, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}=0.091$, Rules used = {2535, 31}

$$\frac{B(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{b}-\frac{Bn(bc-ad)\log(c+dx)}{bd}+Ax$$

Antiderivative was successfully verified.

```
[In] Int[A + B*Log[e*((a + b*x)/(c + d*x))^n],x]
```

```
[Out] A*x + (B*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n])/b - (B*(b*c - a*d)*n*Log
[c + d*x])/(b*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2535

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.), x_Symbol] :> Simp[(a + b*x)*((A + B*Log[e*((a + b*x)/(c + d*x)
)^n])^p/b), x] - Dist[B*n*p*((b*c - a*d)/b), Int[(A + B*Log[e*((a + b*x)/(c
 + d*x))^n])^(p - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, A, B, n}, x
] && NeQ[b*c - a*d, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)dx&=Ax+B\int\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)dx\\&=Ax+\frac{B(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{b}-\frac{(B(bc-ad)n)\int\frac{1}{c+dx}\,dx}{b}\\&=Ax+\frac{B(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{b}-\frac{B(bc-ad)n\log(c+dx)}{bd}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 56, normalized size = 1.00

$$Ax+\frac{B(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{b}-\frac{B(bc-ad)n\log(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[A + B*Log[e*((a + b*x)/(c + d*x))^n],x]
```

```
[Out] A*x + (B*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n])/b - (B*(b*c - a*d)*n*Log
[c + d*x])/(b*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 121 vs. 2(56) = 112.
time = 0.06, size = 122, normalized size = 2.18

method	result	size
default	$Ax+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)x-\frac{Bnc\ln(dx+c)a}{ad-cb}+\frac{Bn\,c^2\ln(dx+c)b}{(ad-cb)d}+\frac{Bn\,a^2\ln(bx+a)d}{(ad-cb)b}-\frac{Bna\ln(bx+a)c}{ad-cb}$	122

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(A+B*ln(e*((b*x+a)/(d*x+c))^n),x,method=_RETURNVERBOSE)
```

```
[Out] A*x+B*ln(e*((b*x+a)/(d*x+c))^n)*x-B*n*c/(a*d-b*c)*ln(d*x+c)*a+B*n*c^2/(a*d-
b*c)/d*ln(d*x+c)*b+B*n*a^2/(a*d-b*c)/b*ln(b*x+a)*d-B*n*a/(a*d-b*c)*ln(b*x+a
)*c
```

Maxima [A]
time = 0.28, size = 53, normalized size = 0.95

$$Bn\left(\frac{a\log(bx+a)}{b}-\frac{c\log(dx+c)}{d}\right)+Bx\log\left(\left(\frac{bx+a}{dx+c}\right)^ne\right)+Ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(A+B*log(e*((b*x+a)/(d*x+c))^n),x, algorithm="maxima")
```

```
[Out] B*n*(a*log(b*x + a)/b - c*log(d*x + c)/d) + B*x*log(((b*x + a)/(d*x + c))^n
*e) + A*x
```

Fricas [A]
time = 0.37, size = 58, normalized size = 1.04

$$\frac{Bbdnx\log\left(\frac{bx+a}{dx+c}\right)+Badn\log(bx+a)-Bbcn\log(dx+c)+(A+B)bdx}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(A+B*log(e*((b*x+a)/(d*x+c))^n),x, algorithm="fricas")
```

```
[Out] (B*b*d*n*x*log((b*x + a)/(d*x + c)) + B*a*d*n*log(b*x + a) - B*b*c*n*log(d*
x + c) + (A + B)*b*d*x)/(b*d)
```

Sympy [A]
time = 3.34, size = 150, normalized size = 2.68

$$Ax + B\left(\begin{cases} x\log\left(e\left(\frac{a}{c}\right)^{n}\right) & \text{for}\: b = 0 \wedge d = 0 \\ \frac{a\log\left(e\left(\frac{a}{c} + \frac{bx}{c}\right)^{n}\right)}{b} - nx + x\log\left(e\left(\frac{a}{c} + \frac{bx}{c}\right)^{n}\right) & \text{for}\: d = 0 \\ \frac{c\log\left(e\left(\frac{a}{c+dx}\right)^{n}\right)}{d} + nx + x\log\left(e\left(\frac{a}{c+dx}\right)^{n}\right) & \text{for}\: b = 0 \\ \frac{an\log\left(c+dx\right)}{b} + \frac{a\log\left(e\left(\frac{a}{c+dx} + \frac{bx}{c+dx}\right)^{n}\right)}{b} - \frac{cn\log\left(c+dx\right)}{d} + x\log\left(e\left(\frac{a}{c+dx} + \frac{bx}{c+dx}\right)^{n}\right) & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(A+B*ln(e*((b*x+a)/(d*x+c))**n),x)
```

```
[Out] A*x + B*Piecewise((x*log(e*(a/c)**n), Eq(b, 0) & Eq(d, 0)), (a*log(e*(a/c +
 b*x/c)**n)/b - n*x + x*log(e*(a/c + b*x/c)**n), Eq(d, 0)), (c*log(e*(a/(c
+ d*x))**n)/d + n*x + x*log(e*(a/(c + d*x))**n), Eq(b, 0)), (a*n*log(c + d*
x)/b + a*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/b - c*n*log(c + d*x)/d + x
*log(e*(a/(c + d*x) + b*x/(c + d*x))**n), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 237 vs. 2(57) = 114.
time = 5.52, size = 237, normalized size = 4.23

$$B\left(\frac{\left(b^{2}c^{2}n - 2\,abcdn + a^{2}d^{2}n\right)\log\left(\frac{bx+a}{dx+c}\right)}{bd - \frac{(bx+a)d^{2}}{dx+c}} + \frac{b^{2}c^{2} - 2\,abcd + a^{2}d^{2}}{bd - \frac{(bx+a)d^{2}}{dx+c}} + \frac{\left(b^{2}c^{2}n - 2\,abcdn + a^{2}d^{2}n\right)\log\left(b - \frac{(bx+a)d}{dx+c}\right)}{bd} - \frac{\left(b^{2}c^{2}n - 2\,abcdn + a^{2}d^{2}n\right)\log\left(\frac{bx+a}{dx+c}\right)}{bd}\right)\left(\frac{bc}{(bc-ad)^{2}} - \frac{ad}{(bc-ad)^{2}}\right) + Ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(A+B*log(e*((b*x+a)/(d*x+c))^n),x, algorithm="giac")
```

```
[Out] B*((b^2*c^2*n - 2*a*b*c*d*n + a^2*d^2*n)*log((b*x + a)/(d*x + c))/(b*d - (b
*x + a)*d^2/(d*x + c)) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)/(b*d - (b*x + a)*d
^2/(d*x + c)) + (b^2*c^2*n - 2*a*b*c*d*n + a^2*d^2*n)*log(b - (b*x + a)*d/(
d*x + c))/(b*d) - (b^2*c^2*n - 2*a*b*c*d*n + a^2*d^2*n)*log((b*x + a)/(d*x
+ c))/(b*d))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2) + A*x
```

Mupad [B]
time = 4.01, size = 52, normalized size = 0.93

$$A\,x+B\,x\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)+\frac{B\,a\,n\,\ln\left(a+b\,x\right)}{b}-\frac{B\,c\,n\,\ln\left(c+d\,x\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(A + B*log(e*((a + b*x)/(c + d*x))^n),x)
```

```
[Out] A*x + B*x*log(e*((a + b*x)/(c + d*x))^n) + (B*a*n*log(a + b*x))/b - (B*c*n*
log(c + d*x))/d
```

3.62 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx}\,dx$

Optimal. Leaf size=147

$$-\frac{Bn\log\left(-\frac{g(a+bx)}{bf-ag}\right)\log(f+gx)}{g}+\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log(f+gx)}{g}+\frac{Bn\log\left(-\frac{g(c+dx)}{df-cg}\right)\log(f+gx)}{g}-\frac{Bn}{}$$

```
[Out] -B*n*ln(-g*(b*x+a)/(-a*g+b*f))*ln(g*x+f)/g+(A+B*ln(e*((b*x+a)/(d*x+c))^n))*
ln(g*x+f)/g+B*n*ln(-g*(d*x+c)/(-c*g+d*f))*ln(g*x+f)/g-B*n*polylog(2,b*(g*x+
f)/(-a*g+b*f))/g+B*n*polylog(2,d*(g*x+f)/(-c*g+d*f))/g
```

Rubi [A]
time = 0.09, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2545, 2441, 2440, 2438}

$$-\frac{Bn\text{PolyLog}\left(2,\frac{b(f+gx)}{bf-ag}\right)}{g}+\frac{Bn\text{PolyLog}\left(2,\frac{d(f+gx)}{df-cg}\right)}{g}+\frac{\log(f+gx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{g}-\frac{Bn\log(f+gx)\log\left(-\frac{g(a+bx)}{bf-ag}\right)}{g}+\frac{Bn\log(f+gx)\log\left(-\frac{g(c+dx)}{df-cg}\right)}{g}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(f + g*x),x]
```

```
[Out] -((B*n*Log[-((g*(a + b*x))/(b*f - a*g))]*Log[f + g*x])/g) + ((A + B*Log[e*(
(a + b*x)/(c + d*x))^n])*Log[f + g*x])/g + (B*n*Log[-((g*(c + d*x))/(d*f -
c*g))]*Log[f + g*x])/g - (B*n*PolyLog[2, (b*(f + g*x))/(b*f - a*g)])/g + (B
*n*PolyLog[2, (d*(f + g*x))/(d*f - c*g)])/g
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2545

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))/((f_.) + (g_.)*(x_)), x_Symbol] :> Simp[Log[f + g*x]*((A + B*Log[e*((
a + b*x)/(c + d*x))^n])/g), x] + (-Dist[b*B*(n/g), Int[Log[f + g*x]/(a + b*
x), x], x] + Dist[B*d*(n/g), Int[Log[f + g*x]/(c + d*x), x], x]) /; FreeQ[{
a, b, c, d, e, f, g, A, B, n}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx}\,dx = \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log(f+gx)}{g} - \frac{(Bn)\int \frac{(c+dx)\left(-\frac{d(a+bx)}{(c+dx)^2}+\frac{b}{c+dx}\right)\log(f+gx)}{a+bx}\,d}{g}$$

$$= \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log(f+gx)}{g} - \frac{(Bn)\int \left(\frac{b\log(f+gx)}{a+bx}-\frac{d\log(f+gx)}{c+dx}\right)dx}{g}$$

$$= \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log(f+gx)}{g} - \frac{(bBn)\int \frac{\log(f+gx)}{a+bx}\,dx}{g} + \frac{(Bdn)\int \frac{\log(f+}{c+d}}{g}$$

$$= -\frac{Bn\log\left(-\frac{g(a+bx)}{bf-ag}\right)\log(f+gx)}{g} + \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log(f+gx)}{g} + \frac{B}{}$$

$$= -\frac{Bn\log\left(-\frac{g(a+bx)}{bf-ag}\right)\log(f+gx)}{g} + \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log(f+gx)}{g} + \frac{B}{}$$

$$= -\frac{Bn\log\left(-\frac{g(a+bx)}{bf-ag}\right)\log(f+gx)}{g} + \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log(f+gx)}{g} + \frac{B}{}$$

Mathematica [A]
time = 0.04, size = 122, normalized size = 0.83

$$\frac{\left(A-Bn\log\left(\frac{g(a+bx)}{-bf+ag}\right)+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+Bn\log\left(\frac{g(c+dx)}{-df+cg}\right)\right)\log(f+gx)-Bn\text{Li}_2\left(\frac{b(f+gx)}{bf-ag}\right)+Bn\text{Li}_2\left(\frac{d(f+gx)}{df-cg}\right)}{g}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(f + g*x),x]
[Out] ((A - B*n*Log[(g*(a + b*x))/(-(b*f) + a*g)] + B*Log[e*((a + b*x)/(c + d*x))
^n] + B*n*Log[(g*(c + d*x))/(-(d*f) + c*g)])*Log[f + g*x] - B*n*PolyLog[2,
(b*(f + g*x))/(b*f - a*g)] + B*n*PolyLog[2, (d*(f + g*x))/(d*f - c*g)])/g
```

Maple [F]
time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}{gx+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(g*x+f),x)
```

```
[Out] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(g*x+f),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(g*x+f),x, algorithm="maxima")
```

```
[Out] -B*integrate(-(log((b*x + a)^n) - log((d*x + c)^n) + 1)/(g*x + f), x) + A*l
og(g*x + f)/g
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(g*x+f),x, algorithm="fricas")
```

```
[Out] integral((B*log(((b*x + a)/(d*x + c))^n*e) + A)/(g*x + f), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B\log\left(e\left(\frac{a}{c+dx} + \frac{bx}{c+dx}\right)^{n}\right)}{f + gx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))/(g*x+f),x)
```

```
[Out] Integral((A + B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n))/(f + g*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(g*x+f),x, algorithm="giac")
```

```
[Out] integrate((B*log(((b*x + a)/(d*x + c))^n*e) + A)/(g*x + f), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A + B\ln\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f + gx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log(e*((a + b*x)/(c + d*x))^n))/(f + g*x),x)
[Out] int((A + B*log(e*((a + b*x)/(c + d*x))^n))/(f + g*x), x)
```

3.63 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(f+gx)^2}\,dx$

Optimal. Leaf size=91

$$\frac{(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bf-ag)(f+gx)}+\frac{B(bc-ad)n\log\left(\frac{f+gx}{c+dx}\right)}{(bf-ag)(df-cg)}$$

```
[Out] (b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*g+b*f)/(g*x+f)+B*(-a*d+b*c)*n*l
n((g*x+f)/(d*x+c))/(-a*g+b*f)/(-c*g+d*f)
```

Rubi [A]
time = 0.06, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2553, 2351, 31}

$$\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{(f+gx)(bf-ag)}+\frac{Bn(bc-ad)\log\left(\frac{f+gx}{c+dx}\right)}{(bf-ag)(df-cg)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(f + g*x)^2,x]
```

```
[Out] ((a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/((b*f - a*g)*(f + g*x))
+ (B*(b*c - a*d)*n*Log[(f + g*x)/(c + d*x)])/((b*f - a*g)*(d*f - c*g))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2553

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[b*c - a*d, Subst[
Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)),
 x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x
] && NeQ[b*c - a*d, 0] && IntegerQ[m] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(f+gx)^2}\,dx = -\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{g(f+gx)} + \frac{(Bn)\int \frac{bc-ad}{(a+bx)(c+dx)(f+gx)}\,dx}{g}$$
$$= -\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{g(f+gx)} + \frac{(B(bc-ad)n)\int \frac{1}{(a+bx)(c+dx)(f+gx)}\,dx}{g}$$
$$= -\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{g(f+gx)} + \frac{(B(bc-ad)n)\int \left(\frac{b^2}{(bc-ad)(bf-ag)(a+bx)} + \frac{d^2}{(bc-ad)(-df+}\right.}{g}$$
$$= \frac{bBn\log(a+bx)}{g(bf-ag)} - \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{g(f+gx)} - \frac{Bdn\log(c+dx)}{g(df-cg)} + \frac{B(bc-ad)n}{(bf-ag)}$$

Mathematica [A]
time = 0.10, size = 109, normalized size = 1.20

$$\frac{-\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx} + \frac{Bn(b(df-cg)\log(a+bx)+(-bdf+adg)\log(c+dx)+(bc-ad)g\log(f+gx))}{(bf-ag)(df-cg)}}{g}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(f + g*x)^2,x]
```

```
[Out] (-((A + B*Log[e*((a + b*x)/(c + d*x))^n])/(f + g*x)) + (B*n*(b*(d*f - c*g)*
Log[a + b*x] + (-(b*d*f) + a*d*g)*Log[c + d*x] + (b*c - a*d)*g*Log[f + g*x]
))/((b*f - a*g)*(d*f - c*g)))/g
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}{(gx+f)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(g*x+f)^2,x)
```

```
[Out] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(g*x+f)^2,x)
```

Maxima [A]
time = 0.30, size = 143, normalized size = 1.57

$$Bn\left(\frac{b\log(bx+a)}{bfg-ag^2} - \frac{d\log(dx+c)}{dfg-cg^2} + \frac{(bc-ad)\log(gx+f)}{bdf^2+acg^2-(bc+ad)fg}\right) - \frac{B\log\left(\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n e\right)}{g^2x+fg} - \frac{A}{g^2x+fg}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(g*x+f)^2,x, algorithm="maxima")
```

```
[Out] B*n*(b*log(b*x + a)/(b*f*g - a*g^2) - d*log(d*x + c)/(d*f*g - c*g^2) + (b*c
 - a*d)*log(g*x + f)/(b*d*f^2 + a*c*g^2 - (b*c + a*d)*f*g)) - B*log((b*x/(d
*x + c) + a/(d*x + c))^n*e)/(g^2*x + f*g) - A/(g^2*x + f*g)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 271 vs. $2(92) = 184$.
time = 3.40, size = 271, normalized size = 2.98

$$-\frac{(A+B)bdf^2+(A+B)acg^2-((A+B)bc+(A+B)ad)fg+(Bbdf^2+Bacg^2-(Bbc+Bad)fg)n\log\left(\frac{bx+a}{dx+c}\right)-((Bbdfg-Bbcg^2)nx+(Bbdf^2-Bbcfg)n)\log(bx+a)+((Bbdfg-Badg^2)nx+(Bbdf^2-Badfg)n)\log(dx+c)-((Bbc-Bad)g^2nx+(Bbc-Bad)fgn)\log(gx+f)}{bdf^3g+acfg^3-(bc+ad)f^2g^2+(bdf^2g^2+acg^4-(bc+ad)fg^3)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(g*x+f)^2,x, algorithm="fricas")
```

```
[Out] -((A + B)*b*d*f^2 + (A + B)*a*c*g^2 - ((A + B)*b*c + (A + B)*a*d)*f*g + (B*
b*d*f^2 + B*a*c*g^2 - (B*b*c + B*a*d)*f*g)*n*log((b*x + a)/(d*x + c)) - ((B
*b*d*f*g - B*b*c*g^2)*n*x + (B*b*d*f^2 - B*b*c*f*g)*n)*log(b*x + a) + ((B*b
*d*f*g - B*a*d*g^2)*n*x + (B*b*d*f^2 - B*a*d*f*g)*n)*log(d*x + c) - ((B*b*c
 - B*a*d)*g^2*n*x + (B*b*c - B*a*d)*f*g*n)*log(g*x + f))/(b*d*f^3*g + a*c*f
*g^3 - (b*c + a*d)*f^2*g^2 + (b*d*f^2*g^2 + a*c*g^4 - (b*c + a*d)*f*g^3)*x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))/(g*x+f)**2,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 455 vs. $2(92) = 184$.
time = 4.58, size = 455, normalized size = 5.00

$$\left(\frac{(Bb^2c^2n-2\,Babcdn+Ba^2d^2n)\log\left(-bf+\frac{(bx+a)df}{dx+c}+ag-\frac{(bx+a)cg}{dx+c}\right)}{bdf^2-bcfg-adfg+acg^2}+\frac{(Bb^2c^2n-2\,Babcdn+Ba^2d^2n)\log\left(\frac{bx+a}{dx+c}\right)}{bdf^2-\frac{(bx+a)d^2f^2}{dx+c}-bcfg-adfg+\frac{2\,(bx+a)cdfg}{dx+c}+acg^2-\frac{(bx+a)c^2g^2}{dx+c}}-\frac{(Bb^2c^2n-2\,Babcdn+Ba^2d^2n)\log\left(\frac{bx+a}{dx+c}\right)}{bdf^2-bcfg-adfg+acg^2}+\frac{Ab^2c^2+Bb^2c^2-2\,Aabcd-2\,Babcd+Aa^2d^2+Ba^2d^2}{bdf^2-\frac{(bx+a)d^2f^2}{dx+c}-bcfg-adfg+\frac{2\,(bx+a)cdfg}{dx+c}+acg^2-\frac{(bx+a)c^2g^2}{dx+c}}\right)\left(\frac{bc}{(bc-ad)^2}-\frac{ad}{(bc-ad)^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(g*x+f)^2,x, algorithm="giac")
```

```
[Out] ((B*b^2*c^2*n - 2*B*a*b*c*d*n + B*a^2*d^2*n)*log(-b*f + (b*x + a)*d*f/(d*x
+ c) + a*g - (b*x + a)*c*g/(d*x + c))/(b*d*f^2 - b*c*f*g - a*d*f*g + a*c*g^
2) + (B*b^2*c^2*n - 2*B*a*b*c*d*n + B*a^2*d^2*n)*log((b*x + a)/(d*x + c))/(
b*d*f^2 - (b*x + a)*d^2*f^2/(d*x + c) - b*c*f*g - a*d*f*g + 2*(b*x + a)*c*d
*f*g/(d*x + c) + a*c*g^2 - (b*x + a)*c^2*g^2/(d*x + c)) - (B*b^2*c^2*n - 2*
B*a*b*c*d*n + B*a^2*d^2*n)*log((b*x + a)/(d*x + c))/(b*d*f^2 - b*c*f*g - a*
```

```
d*f*g + a*c*g^2) + (A*b^2*c^2 + B*b^2*c^2 - 2*A*a*b*c*d - 2*B*a*b*c*d + A*a
^2*d^2 + B*a^2*d^2)/(b*d*f^2 - (b*x + a)*d^2*f^2/(d*x + c) - b*c*f*g - a*d*
f*g + 2*(b*x + a)*c*d*f*g/(d*x + c) + a*c*g^2 - (b*x + a)*c^2*g^2/(d*x + c)
))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)
```

Mupad [B]
time = 4.64, size = 140, normalized size = 1.54

$$\frac{B\,d\,n\,\ln\left(c+d\,x\right)}{c\,g^2-d\,f\,g}-\frac{\ln\left(f+g\,x\right)\,\left(B\,a\,d\,n-B\,b\,c\,n\right)}{a\,c\,g^2+b\,d\,f^2-a\,d\,f\,g-b\,c\,f\,g}-\frac{B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)}{g\,\left(f+g\,x\right)}-\frac{B\,b\,n\,\ln\left(a+b\,x\right)}{a\,g^2-b\,f\,g}-\frac{A}{x\,g^2+f\,g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log(e*((a + b*x)/(c + d*x))^n))/(f + g*x)^2,x)
```

```
[Out] (B*d*n*log(c + d*x))/(c*g^2 - d*f*g) - (log(f + g*x)*(B*a*d*n - B*b*c*n))/(
a*c*g^2 + b*d*f^2 - a*d*f*g - b*c*f*g) - (B*log(e*((a + b*x)/(c + d*x))^n))
/(g*(f + g*x)) - (B*b*n*log(a + b*x))/(a*g^2 - b*f*g) - A/(f*g + g^2*x)
```

3.64 $\int \frac{A+B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)}{(f+gx)^3} \, dx$

Optimal. Leaf size=190

$$-\frac{B(bc-ad)n}{2(bf-ag)(df-cg)(f+gx)}+\frac{b^2Bn\log(a+bx)}{2g(bf-ag)^2}-\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{2g(f+gx)^2}-\frac{Bd^2n\log(c+dx)}{2g(df-cg)^2}+\frac{B(bc-ad)(2}{2(}$$

```
[Out] -1/2*B*(-a*d+b*c)*n/(-a*g+b*f)/(-c*g+d*f)/(g*x+f)+1/2*b^2*B*n*ln(b*x+a)/g/(
-a*g+b*f)^2+1/2*(-A-B*ln(e*((b*x+a)/(d*x+c))^n))/g/(g*x+f)^2-1/2*B*d^2*n*ln
(d*x+c)/g/(-c*g+d*f)^2+1/2*B*(-a*d+b*c)*(-a*d*g-b*c*g+2*b*d*f)*n*ln(g*x+f)/
(-a*g+b*f)^2/(-c*g+d*f)^2
```

Rubi [A]
time = 0.15, antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2547, 84}

$$-\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{2g(f+gx)^2}+\frac{b^2Bn\log(a+bx)}{2g(bf-ag)^2}-\frac{Bn(bc-ad)}{2(f+gx)(bf-ag)(df-cg)}+\frac{Bn(bc-ad)\log(f+gx)(-adg-bcg+2bdf)}{2(bf-ag)^2(df-cg)^2}-\frac{Bd^2n\log(c+dx)}{2g(df-cg)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(f + g*x)^3,x]
```

```
[Out] -1/2*(B*(b*c - a*d)*n)/((b*f - a*g)*(d*f - c*g)*(f + g*x)) + (b^2*B*n*Log[a
 + b*x])/(2*g*(b*f - a*g)^2) - (A + B*Log[e*((a + b*x)/(c + d*x))^n])/(2*g*
(f + g*x)^2) - (B*d^2*n*Log[c + d*x])/(2*g*(d*f - c*g)^2) + (B*(b*c - a*d)*
(2*b*d*f - b*c*g - a*d*g)*n*Log[f + g*x])/(2*(b*f - a*g)^2*(d*f - c*g)^2)
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 2547

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*((A +
 B*Log[e*((a + b*x)/(c + d*x))^n])/(g*(m + 1))), x] - Dist[B*n*((b*c - a*d)
/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /; FreeQ
[{a, b, c, d, e, f, g, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, -2]
```

Rubi steps

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(f+gx)^3}\,dx = -\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{2g(f+gx)^2} + \frac{(Bn)\int\frac{bc-ad}{(a+bx)(c+dx)(f+gx)^2}\,dx}{2g}$$
$$= -\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{2g(f+gx)^2} + \frac{(B(bc-ad)n)\int\frac{1}{(a+bx)(c+dx)(f+gx)^2}\,dx}{2g}$$
$$= -\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{2g(f+gx)^2} + \frac{(B(bc-ad)n)\int\left(\frac{b^3}{(bc-ad)(bf-ag)^2(a+bx)} - \frac{d}{(bc-ad)(-df}\right.}{}$$
$$= -\frac{B(bc-ad)n}{2(bf-ag)(df-cg)(f+gx)} + \frac{b^2Bn\log(a+bx)}{2g(bf-ag)^2} - \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{2g(f+gx)^2}$$

Mathematica [A]
time = 0.37, size = 173, normalized size = 0.91

$$\frac{-\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(f+gx)^2} + B(bc-ad)n\left(\frac{b^2\log(a+bx)}{(bc-ad)(bf-ag)^2} + \frac{\frac{g(-df+cg)}{(bf-ag)(f+gx)} + \frac{d^2\log(c+dx)}{-bc+ad} - \frac{g(-2bdf+bcg+adg)\log(f+gx)}{(bf-ag)^2}}{(df-cg)^2}\right)}{2g}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(f + g*x)^3,x]
```

```
[Out] (-((A + B*Log[e*((a + b*x)/(c + d*x))^n])/(f + g*x)^2) + B*(b*c - a*d)*n*((
b^2*Log[a + b*x])/((b*c - a*d)*(b*f - a*g)^2) + ((g*(-(d*f) + c*g))/((b*f -
 a*g)*(f + g*x)) + (d^2*Log[c + d*x])/(-(b*c) + a*d) - (g*(-2*b*d*f + b*c*g
 + a*d*g)*Log[f + g*x])/(b*f - a*g)^2)/(d*f - c*g)^2))/(2*g)
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}{\left(gx+f\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(g*x+f)^3,x)
```

```
[Out] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(g*x+f)^3,x)
```

Maxima [A]
time = 0.31, size = 356, normalized size = 1.87

$$\frac{1}{2}\left(\frac{b^2\log(bx+a)}{b^2f^2g-2\,abfg^2+a^2g^3} - \frac{d^2\log(dx+c)}{d^2f^2g-2\,cdfg^2+c^2g^3} + \frac{(2\,(b^2cd-abd^2)f-(b^2c^2-a^2d^2)g)\log(gx+f)}{b^2d^2f^4+a^2c^2g^4-2\,(b^2cd+abd^2)f^3g+(b^2c^2+4\,abcd+a^2d^2)f^2g^2-2\,(abc^2+a^2cd)fg^3} - \frac{bc-ad}{bdf^3+acfg^2-(bc+ad)f^2g+(bdf^2g+acg^3-(bc+ad)fg^2)x}\right)Bn - \frac{B\log\left(\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^ne\right)}{2\,(g^3x^2+2\,fg^2x+f^2g)} - \frac{A}{2\,(g^3x^2+2\,fg^2x+f^2g)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(g*x+f)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(b^2*log(b*x + a)/(b^2*f^2*g - 2*a*b*f*g^2 + a^2*g^3) - d^2*log(d*x + c
)/(d^2*f^2*g - 2*c*d*f*g^2 + c^2*g^3) + (2*(b^2*c*d - a*b*d^2)*f - (b^2*c^2
 - a^2*d^2)*g)*log(g*x + f)/(b^2*d^2*f^4 + a^2*c^2*g^4 - 2*(b^2*c*d + a*b*d
^2)*f^3*g + (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*f^2*g^2 - 2*(a*b*c^2 + a^2*c*d)
*f*g^3) - (b*c - a*d)/(b*d*f^3 + a*c*f*g^2 - (b*c + a*d)*f^2*g + (b*d*f^2*g
 + a*c*g^3 - (b*c + a*d)*f*g^2)*x))*B*n - 1/2*B*log((b*x/(d*x + c) + a/(d*x
 + c))^n*e)/(g^3*x^2 + 2*f*g^2*x + f^2*g) - 1/2*A/(g^3*x^2 + 2*f*g^2*x + f^
2*g)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1094 vs. $2(181) = 362$.
time = 47.84, size = 1094, normalized size = 5.76

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(g*x+f)^3,x, algorithm="fricas")
```

```
[Out] -1/2*((A + B)*b^2*d^2*f^4 + (A + B)*a^2*c^2*g^4 - 2*((A + B)*b^2*c*d + (A +
 B)*a*b*d^2)*f^3*g + ((A + B)*b^2*c^2 + 4*(A + B)*a*b*c*d + (A + B)*a^2*d^2
)*f^2*g^2 - 2*((A + B)*a*b*c^2 + (A + B)*a^2*c*d)*f*g^3 + ((B*b^2*c*d - B*a
*b*d^2)*f^2*g^2 - (B*b^2*c^2 - B*a^2*d^2)*f*g^3 + (B*a*b*c^2 - B*a^2*c*d)*g
^4)*n*x + (B*b^2*d^2*f^4 + B*a^2*c^2*g^4 - 2*(B*b^2*c*d + B*a*b*d^2)*f^3*g
+ (B*b^2*c^2 + 4*B*a*b*c*d + B*a^2*d^2)*f^2*g^2 - 2*(B*a*b*c^2 + B*a^2*c*d)
*f*g^3)*n*log((b*x + a)/(d*x + c)) + ((B*b^2*c*d - B*a*b*d^2)*f^3*g - (B*b^
2*c^2 - B*a^2*d^2)*f^2*g^2 + (B*a*b*c^2 - B*a^2*c*d)*f*g^3)*n - ((B*b^2*d^2
*f^2*g^2 - 2*B*b^2*c*d*f*g^3 + B*b^2*c^2*g^4)*n*x^2 + 2*(B*b^2*d^2*f^3*g -
2*B*b^2*c*d*f^2*g^2 + B*b^2*c^2*f*g^3)*n*x + (B*b^2*d^2*f^4 - 2*B*b^2*c*d*f
^3*g + B*b^2*c^2*f^2*g^2)*n)*log(b*x + a) + ((B*b^2*d^2*f^2*g^2 - 2*B*a*b*d
^2*f*g^3 + B*a^2*d^2*g^4)*n*x^2 + 2*(B*b^2*d^2*f^3*g - 2*B*a*b*d^2*f^2*g^2
+ B*a^2*d^2*f*g^3)*n*x + (B*b^2*d^2*f^4 - 2*B*a*b*d^2*f^3*g + B*a^2*d^2*f^2
*g^2)*n)*log(d*x + c) - ((2*(B*b^2*c*d - B*a*b*d^2)*f*g^3 - (B*b^2*c^2 - B*
a^2*d^2)*g^4)*n*x^2 + 2*(2*(B*b^2*c*d - B*a*b*d^2)*f^2*g^2 - (B*b^2*c^2 - B
*a^2*d^2)*f*g^3)*n*x + (2*(B*b^2*c*d - B*a*b*d^2)*f^3*g - (B*b^2*c^2 - B*a^
2*d^2)*f^2*g^2)*n)*log(g*x + f))/(b^2*d^2*f^6*g + a^2*c^2*f^2*g^5 - 2*(b^2*
c*d + a*b*d^2)*f^5*g^2 + (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*f^4*g^3 - 2*(a*b*c
^2 + a^2*c*d)*f^3*g^4 + (b^2*d^2*f^4*g^3 + a^2*c^2*g^7 - 2*(b^2*c*d + a*b*d
^2)*f^3*g^4 + (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*f^2*g^5 - 2*(a*b*c^2 + a^2*c*
d)*f*g^6)*x^2 + 2*(b^2*d^2*f^5*g^2 + a^2*c^2*f*g^6 - 2*(b^2*c*d + a*b*d^2)*
f^4*g^3 + (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*f^3*g^4 - 2*(a*b*c^2 + a^2*c*d)*f
^2*g^5)*x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))/(g*x+f)**3,x)

[Out] Timed out

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 2952 vs. $2(181) = 362$.
time = 4.90, size = 2952, normalized size = 15.54

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(g*x+f)^3,x, algorithm="giac")

```
[Out] 1/2*((2*B*b^3*c^2*d*f*n - 4*B*a*b^2*c*d^2*f*n + 2*B*a^2*b*d^3*f*n - B*b^3*c
^3*g*n + B*a*b^2*c^2*d*g*n + B*a^2*b*c*d^2*g*n - B*a^3*d^3*g*n)*log(-b*f +
(b*x + a)*d*f/(d*x + c) + a*g - (b*x + a)*c*g/(d*x + c))/(b^2*d^2*f^4 - 2*b
^2*c*d*f^3*g - 2*a*b*d^2*f^3*g + b^2*c^2*f^2*g^2 + 4*a*b*c*d*f^2*g^2 + a^2*
d^2*f^2*g^2 - 2*a*b*c^2*f*g^3 - 2*a^2*c*d*f*g^3 + a^2*c^2*g^4) + (2*B*b^3*c
^2*d*f*n - 4*B*a*b^2*c*d^2*f*n - 2*(b*x + a)*B*b^2*c^2*d^2*f*n/(d*x + c) +
2*B*a^2*b*d^3*f*n + 4*(b*x + a)*B*a*b*c*d^3*f*n/(d*x + c) - 2*(b*x + a)*B*a
^2*d^4*f*n/(d*x + c) - B*b^3*c^3*g*n + B*a*b^2*c^2*d*g*n + 2*(b*x + a)*B*b^
2*c^3*d*g*n/(d*x + c) + B*a^2*b*c*d^2*g*n - 4*(b*x + a)*B*a*b*c^2*d^2*g*n/(
d*x + c) - B*a^3*d^3*g*n + 2*(b*x + a)*B*a^2*c*d^3*g*n/(d*x + c))*log((b*x
+ a)/(d*x + c))/(b^2*d^2*f^4 - 2*(b*x + a)*b*d^3*f^4/(d*x + c) + (b*x + a)^
2*d^4*f^4/(d*x + c)^2 - 2*b^2*c*d*f^3*g - 2*a*b*d^2*f^3*g + 6*(b*x + a)*b*c
*d^2*f^3*g/(d*x + c) + 2*(b*x + a)*a*d^3*f^3*g/(d*x + c) - 4*(b*x + a)^2*c*
d^3*f^3*g/(d*x + c)^2 + b^2*c^2*f^2*g^2 + 4*a*b*c*d*f^2*g^2 - 6*(b*x + a)*b
*c^2*d*f^2*g^2/(d*x + c) + a^2*d^2*f^2*g^2 - 6*(b*x + a)*a*c*d^2*f^2*g^2/(d
*x + c) + 6*(b*x + a)^2*c^2*d^2*f^2*g^2/(d*x + c)^2 - 2*a*b*c^2*f*g^3 + 2*(
b*x + a)*b*c^3*f*g^3/(d*x + c) - 2*a^2*c*d*f*g^3 + 6*(b*x + a)*a*c^2*d*f*g^
3/(d*x + c) - 4*(b*x + a)^2*c^3*d*f*g^3/(d*x + c)^2 + a^2*c^2*g^4 - 2*(b*x
+ a)*a*c^3*g^4/(d*x + c) + (b*x + a)^2*c^4*g^4/(d*x + c)^2) - (2*B*b^3*c^2*
d*f*n - 4*B*a*b^2*c*d^2*f*n + 2*B*a^2*b*d^3*f*n - B*b^3*c^3*g*n + B*a*b^2*c
^2*d*g*n + B*a^2*b*c*d^2*g*n - B*a^3*d^3*g*n)*log((b*x + a)/(d*x + c))/(b^2
*d^2*f^4 - 2*b^2*c*d*f^3*g - 2*a*b*d^2*f^3*g + b^2*c^2*f^2*g^2 + 4*a*b*c*d*
f^2*g^2 + a^2*d^2*f^2*g^2 - 2*a*b*c^2*f*g^3 - 2*a^2*c*d*f*g^3 + a^2*c^2*g^4
) + (B*b^4*c^3*f*g*n - 3*B*a*b^3*c^2*d*f*g*n - (b*x + a)*B*b^3*c^3*d*f*g*n/
(d*x + c) + 3*B*a^2*b^2*c*d^2*f*g*n + 3*(b*x + a)*B*a*b^2*c^2*d^2*f*g*n/(d*
x + c) - B*a^3*b*d^3*f*g*n - 3*(b*x + a)*B*a^2*b*c*d^3*f*g*n/(d*x + c) + (b
*x + a)*B*a^3*d^4*f*g*n/(d*x + c) - B*a*b^3*c^3*g^2*n + (b*x + a)*B*b^3*c^4
*g^2*n/(d*x + c) + 3*B*a^2*b^2*c^2*d*g^2*n - 3*(b*x + a)*B*a*b^2*c^3*d*g^2*
n/(d*x + c) - 3*B*a^3*b*c*d^2*g^2*n + 3*(b*x + a)*B*a^2*b*c^2*d^2*g^2*n/(d*
x + c) + B*a^4*d^3*g^2*n - (b*x + a)*B*a^3*c*d^3*g^2*n/(d*x + c) + 2*A*b^4*
c^2*d*f^2 + 2*B*b^4*c^2*d*f^2 - 4*A*a*b^3*c*d^2*f^2 - 4*B*a*b^3*c*d^2*f^2 -
```

```
2*(b*x + a)*A*b^3*c^2*d^2*f^2/(d*x + c) - 2*(b*x + a)*B*b^3*c^2*d^2*f^2/(d
*x + c) + 2*A*a^2*b^2*d^3*f^2 + 2*B*a^2*b^2*d^3*f^2 + 4*(b*x + a)*A*a*b^2*c
*d^3*f^2/(d*x + c) + 4*(b*x + a)*B*a*b^2*c*d^3*f^2/(d*x + c) - 2*(b*x + a)*
A*a^2*b*d^4*f^2/(d*x + c) - 2*(b*x + a)*B*a^2*b*d^4*f^2/(d*x + c) - A*b^4*c
^3*f*g - B*b^4*c^3*f*g - A*a*b^3*c^2*d*f*g - B*a*b^3*c^2*d*f*g + 2*(b*x + a
)*A*b^3*c^3*d*f*g/(d*x + c) + 2*(b*x + a)*B*b^3*c^3*d*f*g/(d*x + c) + 5*A*a
^2*b^2*c*d^2*f*g + 5*B*a^2*b^2*c*d^2*f*g - 2*(b*x + a)*A*a*b^2*c^2*d^2*f*g/
(d*x + c) - 2*(b*x + a)*B*a*b^2*c^2*d^2*f*g/(d*x + c) - 3*A*a^3*b*d^3*f*g -
 3*B*a^3*b*d^3*f*g - 2*(b*x + a)*A*a^2*b*c*d^3*f*g/(d*x + c) - 2*(b*x + a)*
B*a^2*b*c*d^3*f*g/(d*x + c) + 2*(b*x + a)*A*a^3*d^4*f*g/(d*x + c) + 2*(b*x
+ a)*B*a^3*d^4*f*g/(d*x + c) + A*a*b^3*c^3*g^2 + B*a*b^3*c^3*g^2 - A*a^2*b^
2*c^2*d*g^2 - B*a^2*b^2*c^2*d*g^2 - 2*(b*x + a)*A*a*b^2*c^3*d*g^2/(d*x + c)
 - 2*(b*x + a)*B*a*b^2*c^3*d*g^2/(d*x + c) - A*a^3*b*c*d^2*g^2 - B*a^3*b*c*
d^2*g^2 + 4*(b*x + a)*A*a^2*b*c^2*d^2*g^2/(d*x + c) + 4*(b*x + a)*B*a^2*b*c
^2*d^2*g^2/(d*x + c) + A*a^4*d^3*g^2 + B*a^4*d^3*g^2 - 2*(b*x + a)*A*a^3*c*
d^3*g^2/(d*x + c) - 2*(b*x + a)*B*a^3*c*d^3*g^2/(d*x + c))/(b^3*d^2*f^5 - 2
*(b*x + a)*b^2*d^3*f^5/(d*x + c) + (b*x + a)^2*b*d^4*f^5/(d*x + c)^2 - 2*b^
3*c*d*f^4*g - 3*a*b^2*d^2*f^4*g + 6*(b*x + a)*b^2*c*d^2*f^4*g/(d*x + c) + 4
*(b*x + a)*a*b*d^3*f^4*g/(d*x + c) - 4*(b*x + a)^2*b*c*d^3*f^4*g/(d*x + c)^
2 - (b*x + a)^2*a*d^4*f^4*g/(d*x + c)^2 + b^3*c^2*f^3*g^2 + 6*a*b^2*c*d*f^3
*g^2 - 6*(b*x + a)*b^2*c^2*d*f^3*g^2/(d*x + c) + 3*a^2*b*d^2*f^3*g^2 - 12*(
b*x + a)*a*b*c*d^2*f^3*g^2/(d*x + c) + 6*(b*x + a)^2*b*c^2*d^2*f^3*g^2/(d*x
 + c)^2 - 2*(b*x + a)*a^2*d^3*f^3*g^2/(d*x + c) + 4*(b*x + a)^2*a*c*d^3*f^3
*g^2/(d*x + c)^2 - 3*a*b^2*c^2*f^2*g^3 + 2*(b*x + a)*b^2*c^3*f^2*g^3/(d*x +
 c) - 6*a^2*b*c*d*f^2*g^3 + 12*(b*x + a)*a*b*c^2*d*f^2*g^3/(d*x + c) - 4*(b
*x + a)^2*b*c^3*d*f^2*g^3/(d*x + c)^2 - a^3*d^2*f^2*g^3 + 6*(b*x + a)*a^2*c
*d^2*f^2*g^3/(d*x + c) - 6*(b*x + a)^2*a*c^2*d^2*f^2*g^3/(d*x + c)^2 + 3*a^
2*b*c^2*f*g^4 - 4*(b*x + a)*a*b*c^3*f*g^4/(d*x + c) + (b*x + a)^2*b*c^4*f*g
^4/(d*x + c)^2 + 2*a^3*c*d*f*g^4 - 6*(b*x + a)*a^2*c^2*d*f*g^4/(d*x + c) +
4*(b*x + a)^2*a*c^3*d*f*g^4/(d*x + c)^2 - a^3*c^2*g^5 + 2*(b*x + a)*a^2*c^3
*g^5/(d*x + c) - (b*x + a)^2*a*c^4*g^5/(d*x + c)^2))*(b*c/(b*c - a*d)^2 - a
*d/(b*c - a*d)^2)
```

Mupad [B]
time = 6.20, size = 430, normalized size = 2.26

$$\frac{\ln(f+g\,x)\,\left(g\,\left(B\,a^2\,d^2\,n-B\,b^2\,c^2\,n\right)-2\,B\,a\,b\,d^2\,f\,n+2\,B\,b^2\,c\,d\,f\,n\right)}{2\,a^2\,c^2\,g^4-4\,a^2\,c\,d\,f\,g^3+2\,a^2\,d^2\,f^2\,g^2-4\,a\,b\,c^2\,f\,g^3+8\,a\,b\,c\,d\,f^2\,g^2-4\,a\,b\,d^2\,f^3\,g+2\,b^2\,c^2\,f^2\,g^2-4\,b^2\,c\,d\,f^3\,g+2\,b^2\,d^2\,f^4}-\frac{\frac{A\,a\,c\,g^2+A\,b\,d\,f^2-A\,a\,d\,f\,g-A\,b\,c\,f\,g-B\,a\,d\,f\,g\,n+B\,b\,c\,f\,g\,n}{a\,c\,g^2+b\,d\,f^2-a\,d\,f\,g-b\,c\,f\,g}-\frac{x\,\left(B\,a\,d\,g^2\,n-B\,b\,c\,g^2\,n\right)}{a\,c\,g^2+b\,d\,f^2-a\,d\,f\,g-b\,c\,f\,g}}{2\,f^2\,g+4\,f\,g^2\,x+2\,g^3\,x^2}-\frac{B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)}{2\,g\,\left(f^2+2\,f\,g\,x+g^2\,x^2\right)}+\frac{B\,b^2\,n\,\ln\left(a+b\,x\right)}{2\,a^2\,g^3-4\,a\,b\,f\,g^2+2\,b^2\,f^2\,g}-\frac{B\,d^2\,n\,\ln\left(c+d\,x\right)}{2\,c^2\,g^3-4\,c\,d\,f\,g^2+2\,d^2\,f^2\,g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log(e*((a + b*x)/(c + d*x))^n))/(f + g*x)^3,x)
```

```
[Out] (log(f + g*x)*(g*(B*a^2*d^2*n - B*b^2*c^2*n) - 2*B*a*b*d^2*f*n + 2*B*b^2*c*
d*f*n))/(2*a^2*c^2*g^4 + 2*b^2*d^2*f^4 + 2*a^2*d^2*f^2*g^2 + 2*b^2*c^2*f^2*
g^2 - 4*a*b*c^2*f*g^3 - 4*a*b*d^2*f^3*g - 4*a^2*c*d*f*g^3 - 4*b^2*c*d*f^3*g
 + 8*a*b*c*d*f^2*g^2) - ((A*a*c*g^2 + A*b*d*f^2 - A*a*d*f*g - A*b*c*f*g - B
```

```
*a*d*f*g*n + B*b*c*f*g*n)/(a*c*g^2 + b*d*f^2 - a*d*f*g - b*c*f*g) - (x*(B*a
*d*g^2*n - B*b*c*g^2*n))/(a*c*g^2 + b*d*f^2 - a*d*f*g - b*c*f*g))/(2*f^2*g
+ 2*g^3*x^2 + 4*f*g^2*x) - (B*log(e*((a + b*x)/(c + d*x))^n))/(2*g*(f^2 + g
^2*x^2 + 2*f*g*x)) + (B*b^2*n*log(a + b*x))/(2*a^2*g^3 + 2*b^2*f^2*g - 4*a*
b*f*g^2) - (B*d^2*n*log(c + d*x))/(2*c^2*g^3 + 2*d^2*f^2*g - 4*c*d*f*g^2)
```

3.65 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(f+gx)^4}\,dx$

Optimal. Leaf size=283

$$-\frac{B(bc-ad)n}{6(bf-ag)(df-cg)(f+gx)^2}-\frac{B(bc-ad)(2bdf-bcg-adg)n}{3(bf-ag)^2(df-cg)^2(f+gx)}+\frac{b^3Bn\log(a+bx)}{3g(bf-ag)^3}-\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{3g(f+gx)^3}$$

```
[Out] -1/6*B*(-a*d+b*c)*n/(-a*g+b*f)/(-c*g+d*f)/(g*x+f)^2-1/3*B*(-a*d+b*c)*(-a*d*
g-b*c*g+2*b*d*f)*n/(-a*g+b*f)^2/(-c*g+d*f)^2/(g*x+f)+1/3*b^3*B*n*ln(b*x+a)/
g/(-a*g+b*f)^3+1/3*(-A-B*ln(e*((b*x+a)/(d*x+c))^n))/g/(g*x+f)^3-1/3*B*d^3*n
*ln(d*x+c)/g/(-c*g+d*f)^3+1/3*B*(-a*d+b*c)*(a^2*d^2*g^2-a*b*d*g*(-c*g+3*d*f
)+b^2*(c^2*g^2-3*c*d*f*g+3*d^2*f^2))*n*ln(g*x+f)/(-a*g+b*f)^3/(-c*g+d*f)^3
```

Rubi [A]

time = 0.28, antiderivative size = 283, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2547, 84}

$$\frac{Bn(bc-ad)\log(f+gx)\left(a^2d^2g^2-abdg(3df-cg)+b^2(c^2g^2-3cdfg+3d^2f^2)\right)}{3(bf-ag)^3(df-cg)^3}-\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{3g(f+gx)^3}+\frac{b^3Bn\log(a+bx)}{3g(bf-ag)^3}-\frac{Bn(bc-ad)(-adg-bcg+2bdf)}{3(f+gx)(bf-ag)^2(df-cg)^2}-\frac{Bn(bc-ad)}{6(f+gx)^2(bf-ag)(df-cg)}-\frac{Bd^3n\log(c+dx)}{3g(df-cg)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(f + g*x)^4,x]
```

```
[Out] -1/6*(B*(b*c - a*d)*n)/((b*f - a*g)*(d*f - c*g)*(f + g*x)^2) - (B*(b*c - a*
d)*(2*b*d*f - b*c*g - a*d*g)*n)/(3*(b*f - a*g)^2*(d*f - c*g)^2*(f + g*x)) +
 (b^3*B*n*Log[a + b*x])/(3*g*(b*f - a*g)^3) - (A + B*Log[e*((a + b*x)/(c +
d*x))^n])/(3*g*(f + g*x)^3) - (B*d^3*n*Log[c + d*x])/(3*g*(d*f - c*g)^3) +
(B*(b*c - a*d)*(a^2*d^2*g^2 - a*b*d*g*(3*d*f - c*g) + b^2*(3*d^2*f^2 - 3*c*
d*f*g + c^2*g^2))*n*Log[f + g*x])/(3*(b*f - a*g)^3*(d*f - c*g)^3)
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 2547

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*((A +
 B*Log[e*((a + b*x)/(c + d*x))^n])/(g*(m + 1))), x] - Dist[B*n*((b*c - a*d)
/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /; FreeQ
[{a, b, c, d, e, f, g, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, -2]
```

Rubi steps

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(f+gx)^4}\,dx = -\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{3g(f+gx)^3} + \frac{(Bn)\int \frac{bc-ad}{(a+bx)(c+dx)(f+gx)^3}\,dx}{3g}$$

$$= -\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{3g(f+gx)^3} + \frac{(B(bc-ad)n)\int \frac{1}{(a+bx)(c+dx)(f+gx)^3}\,dx}{3g}$$

$$= -\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{3g(f+gx)^3} + \frac{(B(bc-ad)n)\int \left(\frac{b^4}{(bc-ad)(bf-ag)^3(a+bx)} + \frac{d\ldots}{(bc-ad)(-df\ldots}\right.}{\ldots}$$

$$= -\frac{B(bc-ad)n}{6(bf-ag)(df-cg)(f+gx)^2} - \frac{B(bc-ad)(2bdf-bcg-adg)n}{3(bf-ag)^2(df-cg)^2(f+gx)} + \frac{b^3Bn\ldots}{3g(b\ldots}$$

Mathematica [A]
time = 0.52, size = 264, normalized size = 0.93

$$\frac{-\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(f+gx)^3} + B(bc-ad)n\left(-\frac{g}{2(bf-ag)(df-cg)(f+gx)^2} + \frac{g(-2bdf+bcg+adg)}{(bf-ag)^2(df-cg)^2(f+gx)} + \frac{b^3\log(a+bx)}{(bc-ad)(bf-ag)^3} + \frac{d^3\log(c+dx)}{(bc-ad)(-df+cg)^3} + \frac{g\left(a^2d^2g^2+abdg(-3df+cg)+b^2\left(3d^2f^2-3cdfg+c^2g^2\right)\right)\log(f+gx)}{(bf-ag)^3(df-cg)^3}\right)}{3g}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(f + g*x)^4,x]
```

```
[Out] (-((A + B*Log[e*((a + b*x)/(c + d*x))^n])/(f + g*x)^3) + B*(b*c - a*d)*n*(-
1/2*g/((b*f - a*g)*(d*f - c*g)*(f + g*x)^2) + (g*(-2*b*d*f + b*c*g + a*d*g)
)/((b*f - a*g)^2*(d*f - c*g)^2*(f + g*x)) + (b^3*Log[a + b*x])/((b*c - a*d)
*(b*f - a*g)^3) + (d^3*Log[c + d*x])/((b*c - a*d)*(-(d*f) + c*g)^3) + (g*(a
^2*d^2*g^2 + a*b*d*g*(-3*d*f + c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^2)
)*Log[f + g*x])/((b*f - a*g)^3*(d*f - c*g)^3)))/(3*g)
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}{(gx+f)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(g*x+f)^4,x)
```

```
[Out] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(g*x+f)^4,x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 853 vs. 2(272) = 544.
time = 0.36, size = 853, normalized size = 3.01

$$\frac{1}{6}\left(\frac{2b^3\log(bx+a)}{b^3f^3g-3ab^2f^2g^2+3a^2bfg^3-a^3g^4} - \frac{2d^3\log(dx+c)}{d^3f^3g-3cd^2f^2g^2+3c^2dfg^3-c^3g^4} + \frac{2\left(3\left(b^3cd^2-ab^2d^3\right)f^2-3\left(b^3c^2d-a^2bd^3\right)fg+\left(b^3c^3-a^3d^3\right)g^2\right)\log(gx+f)}{b^3d^3f^6+a^3c^3g^6-3\left(b^3cd^2+ab^2d^3\right)f^5g+3\left(b^3c^2d+3ab^2cd^2+a^2bd^3\right)f^4g^2-\left(b^3c^3+9ab^2c^2d+9a^2bcd^2+a^3d^3\right)f^3g^3+3\left(ab^2c^3+3a^2bc^2d+a^3cd^2\right)f^2g^4-3\left(a^2bc^3+a^3c^2d\right)fg^5} - \frac{5\left(b^2cd-abd^2\right)f^2-3\left(b^2c^2-a^2d^2\right)fg+\left(abc^2-a^2cd\right)g^2+2\left(2\left(b^2cd-abd^2\right)fg-\left(b^2c^2-a^2d^2\right)g^2\right)x}{b^2d^2f^6+a^2c^2f^2g^4-2\left(b^2cd+abd^2\right)f^5g+\left(b^2c^2+4abcd+a^2d^2\right)f^4g^2-2\left(abc^2+a^2cd\right)f^3g^3+\left(b^2d^2f^4g^2+a^2c^2g^6-2\left(b^2cd+abd^2\right)f^3g^3+\left(b^2c^2+4abcd+a^2d^2\right)f^2g^4-2\left(abc^2+a^2cd\right)fg^5\right)x^2+2\left(b^2d^2f^5g+a^2c^2fg^5-2\left(b^2cd+abd^2\right)f^4g^2+\left(b^2c^2+4abcd+a^2d^2\right)f^3g^3-2\left(abc^2+a^2cd\right)f^2g^4\right)x}\right)Bn - \frac{B\log\left(\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^ne\right)}{3\left(g^4x^3+3fg^3x^2+3f^2g^2x+f^3g\right)} - \frac{A}{3\left(g^4x^3+3fg^3x^2+3f^2g^2x+f^3g\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(g*x+f)^4,x, algorithm="maxima")
```

```
[Out] 1/6*(2*b^3*log(b*x + a)/(b^3*f^3*g - 3*a*b^2*f^2*g^2 + 3*a^2*b*f*g^3 - a^3*
g^4) - 2*d^3*log(d*x + c)/(d^3*f^3*g - 3*c*d^2*f^2*g^2 + 3*c^2*d*f*g^3 - c^
3*g^4) + 2*(3*(b^3*c*d^2 - a*b^2*d^3)*f^2 - 3*(b^3*c^2*d - a^2*b*d^3)*f*g +
 (b^3*c^3 - a^3*d^3)*g^2)*log(g*x + f)/(b^3*d^3*f^6 + a^3*c^3*g^6 - 3*(b^3*
c*d^2 + a*b^2*d^3)*f^5*g + 3*(b^3*c^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3)*f^4*g^
2 - (b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*f^3*g^3 + 3*(a*b^2*
c^3 + 3*a^2*b*c^2*d + a^3*c*d^2)*f^2*g^4 - 3*(a^2*b*c^3 + a^3*c^2*d)*f*g^5)
 - (5*(b^2*c*d - a*b*d^2)*f^2 - 3*(b^2*c^2 - a^2*d^2)*f*g + (a*b*c^2 - a^2*
c*d)*g^2 + 2*(2*(b^2*c*d - a*b*d^2)*f*g - (b^2*c^2 - a^2*d^2)*g^2)*x)/(b^2*
d^2*f^6 + a^2*c^2*f^2*g^4 - 2*(b^2*c*d + a*b*d^2)*f^5*g + (b^2*c^2 + 4*a*b*
c*d + a^2*d^2)*f^4*g^2 - 2*(a*b*c^2 + a^2*c*d)*f^3*g^3 + (b^2*d^2*f^4*g^2 +
 a^2*c^2*g^6 - 2*(b^2*c*d + a*b*d^2)*f^3*g^3 + (b^2*c^2 + 4*a*b*c*d + a^2*d
^2)*f^2*g^4 - 2*(a*b*c^2 + a^2*c*d)*f*g^5)*x^2 + 2*(b^2*d^2*f^5*g + a^2*c^2
*f*g^5 - 2*(b^2*c*d + a*b*d^2)*f^4*g^2 + (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*f^
3*g^3 - 2*(a*b*c^2 + a^2*c*d)*f^2*g^4)*x))*B*n - 1/3*B*log((b*x/(d*x + c) +
 a/(d*x + c))^n*e)/(g^4*x^3 + 3*f*g^3*x^2 + 3*f^2*g^2*x + f^3*g) - 1/3*A/(g
^4*x^3 + 3*f*g^3*x^2 + 3*f^2*g^2*x + f^3*g)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(g*x+f)^4,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))/(g*x+f)**4,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 9570 vs. 2(272) = 544.
time = 3.23, size = 9570, normalized size = 33.82

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(g*x+f)^4,x, algorithm="giac")
```

```
[Out] 1/6*(2*(3*B*b^4*c^2*d^2*f^2*n - 6*B*a*b^3*c*d^3*f^2*n + 3*B*a^2*b^2*d^4*f^2
*n - 3*B*b^4*c^3*d*f*g*n + 3*B*a*b^3*c^2*d^2*f*g*n + 3*B*a^2*b^2*c*d^3*f*g*
n - 3*B*a^3*b*d^4*f*g*n + B*b^4*c^4*g^2*n - B*a*b^3*c^3*d*g^2*n - B*a^3*b*c
*d^3*g^2*n + B*a^4*d^4*g^2*n)*log(-b*f + (b*x + a)*d*f/(d*x + c) + a*g - (b
*x + a)*c*g/(d*x + c))/(b^3*d^3*f^6 - 3*b^3*c*d^2*f^5*g - 3*a*b^2*d^3*f^5*g
 + 3*b^3*c^2*d*f^4*g^2 + 9*a*b^2*c*d^2*f^4*g^2 + 3*a^2*b*d^3*f^4*g^2 - b^3*
c^3*f^3*g^3 - 9*a*b^2*c^2*d*f^3*g^3 - 9*a^2*b*c*d^2*f^3*g^3 - a^3*d^3*f^3*g
^3 + 3*a*b^2*c^3*f^2*g^4 + 9*a^2*b*c^2*d*f^2*g^4 + 3*a^3*c*d^2*f^2*g^4 - 3*
a^2*b*c^3*f*g^5 - 3*a^3*c^2*d*f*g^5 + a^3*c^3*g^6) + 2*(3*B*b^4*c^2*d^2*f^2
*n - 6*B*a*b^3*c*d^3*f^2*n - 6*(b*x + a)*B*b^3*c^2*d^3*f^2*n/(d*x + c) + 3*
B*a^2*b^2*d^4*f^2*n + 12*(b*x + a)*B*a*b^2*c*d^4*f^2*n/(d*x + c) + 3*(b*x +
 a)^2*B*b^2*c^2*d^4*f^2*n/(d*x + c)^2 - 6*(b*x + a)*B*a^2*b*d^5*f^2*n/(d*x
+ c) - 6*(b*x + a)^2*B*a*b*c*d^5*f^2*n/(d*x + c)^2 + 3*(b*x + a)^2*B*a^2*d^
6*f^2*n/(d*x + c)^2 - 3*B*b^4*c^3*d*f*g*n + 3*B*a*b^3*c^2*d^2*f*g*n + 9*(b*
x + a)*B*b^3*c^3*d^2*f*g*n/(d*x + c) + 3*B*a^2*b^2*c*d^3*f*g*n - 15*(b*x +
a)*B*a*b^2*c^2*d^3*f*g*n/(d*x + c) - 6*(b*x + a)^2*B*b^2*c^3*d^3*f*g*n/(d*x
 + c)^2 - 3*B*a^3*b*d^4*f*g*n + 3*(b*x + a)*B*a^2*b*c*d^4*f*g*n/(d*x + c) +
 12*(b*x + a)^2*B*a*b*c^2*d^4*f*g*n/(d*x + c)^2 + 3*(b*x + a)*B*a^3*d^5*f*g
*n/(d*x + c) - 6*(b*x + a)^2*B*a^2*c*d^5*f*g*n/(d*x + c)^2 + B*b^4*c^4*g^2*
n - B*a*b^3*c^3*d*g^2*n - 3*(b*x + a)*B*b^3*c^4*d*g^2*n/(d*x + c) + 3*(b*x
+ a)*B*a*b^2*c^3*d^2*g^2*n/(d*x + c) + 3*(b*x + a)^2*B*b^2*c^4*d^2*g^2*n/(d
*x + c)^2 - B*a^3*b*c*d^3*g^2*n + 3*(b*x + a)*B*a^2*b*c^2*d^3*g^2*n/(d*x +
c) - 6*(b*x + a)^2*B*a*b*c^3*d^3*g^2*n/(d*x + c)^2 + B*a^4*d^4*g^2*n - 3*(b
*x + a)*B*a^3*c*d^4*g^2*n/(d*x + c) + 3*(b*x + a)^2*B*a^2*c^2*d^4*g^2*n/(d*
x + c)^2)*log((b*x + a)/(d*x + c))/(b^3*d^3*f^6 - 3*(b*x + a)*b^2*d^4*f^6/(
d*x + c) + 3*(b*x + a)^2*b*d^5*f^6/(d*x + c)^2 - (b*x + a)^3*d^6*f^6/(d*x +
 c)^3 - 3*b^3*c*d^2*f^5*g - 3*a*b^2*d^3*f^5*g + 12*(b*x + a)*b^2*c*d^3*f^5*
g/(d*x + c) + 6*(b*x + a)*a*b*d^4*f^5*g/(d*x + c) - 15*(b*x + a)^2*b*c*d^4*
f^5*g/(d*x + c)^2 - 3*(b*x + a)^2*a*d^5*f^5*g/(d*x + c)^2 + 6*(b*x + a)^3*c
*d^5*f^5*g/(d*x + c)^3 + 3*b^3*c^2*d*f^4*g^2 + 9*a*b^2*c*d^2*f^4*g^2 - 18*(
b*x + a)*b^2*c^2*d^2*f^4*g^2/(d*x + c) + 3*a^2*b*d^3*f^4*g^2 - 24*(b*x + a)
*a*b*c*d^3*f^4*g^2/(d*x + c) + 30*(b*x + a)^2*b*c^2*d^3*f^4*g^2/(d*x + c)^2
 - 3*(b*x + a)*a^2*d^4*f^4*g^2/(d*x + c) + 15*(b*x + a)^2*a*c*d^4*f^4*g^2/(
d*x + c)^2 - 15*(b*x + a)^3*c^2*d^4*f^4*g^2/(d*x + c)^3 - b^3*c^3*f^3*g^3 -
 9*a*b^2*c^2*d*f^3*g^3 + 12*(b*x + a)*b^2*c^3*d*f^3*g^3/(d*x + c) - 9*a^2*b
*c*d^2*f^3*g^3 + 36*(b*x + a)*a*b*c^2*d^2*f^3*g^3/(d*x + c) - 30*(b*x + a)^
2*b*c^3*d^2*f^3*g^3/(d*x + c)^2 - a^3*d^3*f^3*g^3 + 12*(b*x + a)*a^2*c*d^3*
f^3*g^3/(d*x + c) - 30*(b*x + a)^2*a*c^2*d^3*f^3*g^3/(d*x + c)^2 + 20*(b*x
+ a)^3*c^3*d^3*f^3*g^3/(d*x + c)^3 + 3*a*b^2*c^3*f^2*g^4 - 3*(b*x + a)*b^2*
c^4*f^2*g^4/(d*x + c) + 9*a^2*b*c^2*d*f^2*g^4 - 24*(b*x + a)*a*b*c^3*d*f^2*
g^4/(d*x + c) + 15*(b*x + a)^2*b*c^4*d*f^2*g^4/(d*x + c)^2 + 3*a^3*c*d^2*f^
```

```
2*g^4 - 18*(b*x + a)*a^2*c^2*d^2*f^2*g^4/(d*x + c) + 30*(b*x + a)^2*a*c^3*d
^2*f^2*g^4/(d*x + c)^2 - 15*(b*x + a)^3*c^4*d^2*f^2*g^4/(d*x + c)^3 - 3*a^2
*b*c^3*f*g^5 + 6*(b*x + a)*a*b*c^4*f*g^5/(d*x + c) - 3*(b*x + a)^2*b*c^5*f*
g^5/(d*x + c)^2 - 3*a^3*c^2*d*f*g^5 + 12*(b*x + a)*a^2*c^3*d*f*g^5/(d*x + c
) - 15*(b*x + a)^2*a*c^4*d*f*g^5/(d*x + c)^2 + 6*(b*x + a)^3*c^5*d*f*g^5/(d
*x + c)^3 + a^3*c^3*g^6 - 3*(b*x + a)*a^2*c^4*g^6/(d*x + c) + 3*(b*x + a)^2
*a*c^5*g^6/(d*x + c)^2 - (b*x + a)^3*c^6*g^6/(d*x + c)^3) - 2*(3*B*b^4*c^2*
d^2*f^2*n - 6*B*a*b^3*c*d^3*f^2*n + 3*B*a^2*b^2*d^4*f^2*n - 3*B*b^4*c^3*d*f
*g*n + 3*B*a*b^3*c^2*d^2*f*g*n + 3*B*a^2*b^2*c*d^3*f*g*n - 3*B*a^3*b*d^4*f*
g*n + B*b^4*c^4*g^2*n - B*a*b^3*c^3*d*g^2*n - B*a^3*b*c*d^3*g^2*n + B*a^4*d
^4*g^2*n)*log((b*x + a)/(d*x + c))/(b^3*d^3*f^6 - 3*b^3*c*d^2*f^5*g - 3*a*b
^2*d^3*f^5*g + 3*b^3*c^2*d*f^4*g^2 + 9*a*b^2*c*d^2*f^4*g^2 + 3*a^2*b*d^3*f^
4*g^2 - b^3*c^3*f^3*g^3 - 9*a*b^2*c^2*d*f^3*g^3 - 9*a^2*b*c*d^2*f^3*g^3 - a
^3*d^3*f^3*g^3 + 3*a*b^2*c^3*f^2*g^4 + 9*a^2*b*c^2*d*f^2*g^4 + 3*a^3*c*d^2*
f^2*g^4 - 3*a^2*b*c^3*f*g^5 - 3*a^3*c^2*d*f*g^5 + a^3*c^3*g^6) + (6*B*b^6*c
^3*d*f^3*g*n - 18*B*a*b^5*c^2*d^2*f^3*g*n - 12*(b*x + a)*B*b^5*c^3*d^2*f^3*
g*n/(d*x + c) + 18*B*a^2*b^4*c*d^3*f^3*g*n + 36*(b*x + a)*B*a*b^4*c^2*d^3*f
^3*g*n/(d*x + c) + 6*(b*x + a)^2*B*b^4*c^3*d^3*f^3*g*n/(d*x + c)^2 - 6*B*a^
3*b^3*d^4*f^3*g*n - 36*(b*x + a)*B*a^2*b^3*c*d^4*f^3*g*n/(d*x + c) - 18*(b*
x + a)^2*B*a*b^3*c^2*d^4*f^3*g*n/(d*x + c)^2 + 12*(b*x + a)*B*a^3*b^2*d^5*f
^3*g*n/(d*x + c) + 18*(b*x + a)^2*B*a^2*b^2*c*d^5*f^3*g*n/(d*x + c)^2 - 6*(
b*x + a)^2*B*a^3*b*d^6*f^3*g*n/(d*x + c)^2 - 3*B*b^6*c^4*f^2*g^2*n - 6*B*a*
b^5*c^3*d*f^2*g^2*n + 17*(b*x + a)*B*b^5*c^4*d*f^2*g^2*n/(d*x + c) + 36*B*a
^2*b^4*c^2*d^2*f^2*g^2*n - 32*(b*x + a)*B*a*b^4*c^3*d^2*f^2*g^2*n/(d*x + c)
 - 14*(b*x + a)^2*B*b^4*c^4*d^2*f^2*g^2*n/(d*x + c)^2 - 42*B*a^3*b^3*c*d^3*
f^2*g^2*n - 6*(b*x + a)*B*a^2*b^3*c^2*d^3*f^2*g...
```

Mupad [B]
time = 9.23, size = 1182, normalized size = 4.18

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log(e*((a + b*x)/(c + d*x))^n))/(f + g*x)^4,x)
```

```
[Out] (B*d^3*n*log(c + d*x))/(3*c^3*g^4 - 3*d^3*f^3*g + 9*c*d^2*f^2*g^2 - 9*c^2*d
*f*g^3) - (log(f + g*x)*(g^2*(B*a^3*d^3*n - B*b^3*c^3*n) - g*(3*B*a^2*b*d^3
*f*n - 3*B*b^3*c^2*d*f*n) + 3*B*a*b^2*d^3*f^2*n - 3*B*b^3*c*d^2*f^2*n))/(3*
a^3*c^3*g^6 + 3*b^3*d^3*f^6 - 3*a^3*d^3*f^3*g^3 - 3*b^3*c^3*f^3*g^3 - 9*a^2
*b*c^3*f*g^5 - 9*a*b^2*d^3*f^5*g - 9*a^3*c^2*d*f*g^5 - 9*b^3*c*d^2*f^5*g +
9*a*b^2*c^3*f^2*g^4 + 9*a^2*b*d^3*f^4*g^2 + 9*a^3*c*d^2*f^2*g^4 + 9*b^3*c^2
*d*f^4*g^2 + 27*a*b^2*c*d^2*f^4*g^2 - 27*a*b^2*c^2*d*f^3*g^3 - 27*a^2*b*c*d
^2*f^3*g^3 + 27*a^2*b*c^2*d*f^2*g^4) - (B*log(e*((a + b*x)/(c + d*x))^n))/(
3*g*(f^3 + g^3*x^3 + 3*f^2*g*x + 3*f*g^2*x^2)) - (B*b^3*n*log(a + b*x))/(3*
a^3*g^4 - 3*b^3*f^3*g + 9*a*b^2*f^2*g^2 - 9*a^2*b*f*g^3) - ((2*A*a^2*c^2*g^
```

```
4 + 2*A*b^2*d^2*f^4 + 2*A*a^2*d^2*f^2*g^2 + 2*A*b^2*c^2*f^2*g^2 + 3*B*a^2*d
^2*f^2*g^2*n - 3*B*b^2*c^2*f^2*g^2*n - 4*A*a*b*c^2*f*g^3 - 4*A*a*b*d^2*f^3*
g - 4*A*a^2*c*d*f*g^3 - 4*A*b^2*c*d*f^3*g + 8*A*a*b*c*d*f^2*g^2 + B*a*b*c^2
*f*g^3*n - 5*B*a*b*d^2*f^3*g*n - B*a^2*c*d*f*g^3*n + 5*B*b^2*c*d*f^3*g*n)/(
2*(a^2*c^2*g^4 + b^2*d^2*f^4 + a^2*d^2*f^2*g^2 + b^2*c^2*f^2*g^2 - 2*a*b*c^
2*f*g^3 - 2*a*b*d^2*f^3*g - 2*a^2*c*d*f*g^3 - 2*b^2*c*d*f^3*g + 4*a*b*c*d*f
^2*g^2)) + (x*(B*a*b*c^2*g^4*n - B*a^2*c*d*g^4*n + 5*B*a^2*d^2*f*g^3*n - 5*
B*b^2*c^2*f*g^3*n - 9*B*a*b*d^2*f^2*g^2*n + 9*B*b^2*c*d*f^2*g^2*n))/(2*(a^2
*c^2*g^4 + b^2*d^2*f^4 + a^2*d^2*f^2*g^2 + b^2*c^2*f^2*g^2 - 2*a*b*c^2*f*g^
3 - 2*a*b*d^2*f^3*g - 2*a^2*c*d*f*g^3 - 2*b^2*c*d*f^3*g + 4*a*b*c*d*f^2*g^2
)) + (x^2*(B*a^2*d^2*g^4*n - B*b^2*c^2*g^4*n - 2*B*a*b*d^2*f*g^3*n + 2*B*b^
2*c*d*f*g^3*n))/(a^2*c^2*g^4 + b^2*d^2*f^4 + a^2*d^2*f^2*g^2 + b^2*c^2*f^2*
g^2 - 2*a*b*c^2*f*g^3 - 2*a*b*d^2*f^3*g - 2*a^2*c*d*f*g^3 - 2*b^2*c*d*f^3*g
 + 4*a*b*c*d*f^2*g^2))/(3*f^3*g + 3*g^4*x^3 + 9*f^2*g^2*x + 9*f*g^3*x^2)
```

3.66 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(f+gx)^5}\,dx$

Optimal. Leaf size=388

$$-\frac{B(bc-ad)n}{12(bf-ag)(df-cg)(f+gx)^3}-\frac{B(bc-ad)(2bdf-bcg-adg)n}{8(bf-ag)^2(df-cg)^2(f+gx)^2}-\frac{B(bc-ad)\left(a^2d^2g^2-abdg(3df-cg)+\right.}{4(bf-ag)^3(df-cg)}$$

```
[Out] -1/12*B*(-a*d+b*c)*n/(-a*g+b*f)/(-c*g+d*f)/(g*x+f)^3-1/8*B*(-a*d+b*c)*(-a*d
*g-b*c*g+2*b*d*f)*n/(-a*g+b*f)^2/(-c*g+d*f)^2/(g*x+f)^2-1/4*B*(-a*d+b*c)*(a
^2*d^2*g^2-a*b*d*g*(-c*g+3*d*f)+b^2*(c^2*g^2-3*c*d*f*g+3*d^2*f^2))*n/(-a*g+
b*f)^3/(-c*g+d*f)^3/(g*x+f)+1/4*b^4*B*n*ln(b*x+a)/g/(-a*g+b*f)^4+1/4*(-A-B*
ln(e*((b*x+a)/(d*x+c))^n))/g/(g*x+f)^4-1/4*B*d^4*n*ln(d*x+c)/g/(-c*g+d*f)^4
-1/4*B*(-a*d+b*c)*(-a*d*g-b*c*g+2*b*d*f)*(2*a*b*d^2*f*g-a^2*d^2*g^2-b^2*(c^
2*g^2-2*c*d*f*g+2*d^2*f^2))*n*ln(g*x+f)/(-a*g+b*f)^4/(-c*g+d*f)^4
```

Rubi [A]
time = 0.44, antiderivative size = 388, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2547, 84}

$$-\frac{Bn(bc-ad)\left(a^2d^2g^2-abdg(3df-cg)+b^2(c^2g^2-3cdfg+3d^2f^2)\right)}{4(f+gx)(bf-ag)^3(df-cg)^3}-\frac{Bn(bc-ad)\log(f+gx)(-adg-bcg+2bdf)\left(-a^2d^2g^2+2abd^2fg-(b^2(c^2g^2-2cdfg+2d^2f^2))\right)}{4(bf-ag)^4(df-cg)^4}-\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{4g(f+gx)^4}+\frac{b^4Bn\log(a+bx)}{4g(bf-ag)^4}-\frac{Bn(bc-ad)(-adg-bcg+2bdf)}{8(f+gx)^2(bf-ag)^2(df-cg)^2}-\frac{Bn(bc-ad)}{12(f+gx)^3(bf-ag)(df-cg)}-\frac{Bd^4n\log(c+dx)}{4g(df-cg)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(f + g*x)^5,x]
```

```
[Out] -1/12*(B*(b*c - a*d)*n)/((b*f - a*g)*(d*f - c*g)*(f + g*x)^3) - (B*(b*c - a
*d)*(2*b*d*f - b*c*g - a*d*g)*n)/(8*(b*f - a*g)^2*(d*f - c*g)^2*(f + g*x)^2
) - (B*(b*c - a*d)*(a^2*d^2*g^2 - a*b*d*g*(3*d*f - c*g) + b^2*(3*d^2*f^2 -
3*c*d*f*g + c^2*g^2))*n)/(4*(b*f - a*g)^3*(d*f - c*g)^3*(f + g*x)) + (b^4*B
*n*Log[a + b*x])/(4*g*(b*f - a*g)^4) - (A + B*Log[e*((a + b*x)/(c + d*x))^n
])/(4*g*(f + g*x)^4) - (B*d^4*n*Log[c + d*x])/(4*g*(d*f - c*g)^4) - (B*(b*c
 - a*d)*(2*b*d*f - b*c*g - a*d*g)*(2*a*b*d^2*f*g - a^2*d^2*g^2 - b^2*(2*d^2
*f^2 - 2*c*d*f*g + c^2*g^2))*n*Log[f + g*x])/(4*(b*f - a*g)^4*(d*f - c*g)^4
)
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 2547

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*((A +
```

```
B*Log[e*((a + b*x)/(c + d*x))^n])/(g*(m + 1))), x] - Dist[B*n*((b*c - a*d)
/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /; FreeQ
[{a, b, c, d, e, f, g, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, -2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(f+gx)^5}\,dx &= -\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{4g(f+gx)^4}+\frac{(Bn)\int\frac{bc-ad}{(a+bx)(c+dx)(f+gx)^4}\,dx}{4g}\\
&= -\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{4g(f+gx)^4}+\frac{(B(bc-ad)n)\int\frac{1}{(a+bx)(c+dx)(f+gx)^4}\,dx}{4g}\\
&= -\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{4g(f+gx)^4}+\frac{(B(bc-ad)n)\int\left(\frac{b^5}{(bc-ad)(bf-ag)^4(a+bx)}-\frac{d}{(bc-ad)(-df}\right.}{}\\
&= -\frac{B(bc-ad)n}{12(bf-ag)(df-cg)(f+gx)^3}-\frac{B(bc-ad)(2bdf-bcg-adg)n}{8(bf-ag)^2(df-cg)^2(f+gx)^2}-\frac{B(bc}{}
\end{aligned}
$$

Mathematica [A]
time = 0.69, size = 359, normalized size = 0.93

$$
\frac{-\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(f+gx)^4}+B(bc-ad)n\left(-\frac{g}{3(bf-ag)(df-cg)(f+gx)^3}+\frac{g(-2bdf+bcg+adg)}{2(bf-ag)^2(df-cg)^2(f+gx)^2}-\frac{g\left(a^2d^2g^2+abdg(-3df+cg)+b^2\left(3d^2f^2-3cdfg+c^2g^2\right)\right)}{(bf-ag)^3(df-cg)^3(f+gx)}+\frac{b^4\log(a+bx)}{(bc-ad)(bf-ag)^4}-\frac{d^4\log(c+dx)}{(bc-ad)(df-cg)^4}-\frac{g(-2bdf+bcg+adg)\left(-2abd^2fg+a^2d^2g^2+b^2\left(2d^2f^2-2cdfg+c^2g^2\right)\right)\log(f+gx)}{(bf-ag)^4(df-cg)^4}\right)}{4g}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(f + g*x)^5,x]
```

```
[Out] (-((A + B*Log[e*((a + b*x)/(c + d*x))^n])/(f + g*x)^4) + B*(b*c - a*d)*n*(-
1/3*g/((b*f - a*g)*(d*f - c*g)*(f + g*x)^3) + (g*(-2*b*d*f + b*c*g + a*d*g)
)/(2*(b*f - a*g)^2*(d*f - c*g)^2*(f + g*x)^2) - (g*(a^2*d^2*g^2 + a*b*d*g*(
-3*d*f + c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^2)))/((b*f - a*g)^3*(d*f
 - c*g)^3*(f + g*x)) + (b^4*Log[a + b*x])/((b*c - a*d)*(b*f - a*g)^4) - (d^
4*Log[c + d*x])/((b*c - a*d)*(d*f - c*g)^4) - (g*(-2*b*d*f + b*c*g + a*d*g)
*(-2*a*b*d^2*f*g + a^2*d^2*g^2 + b^2*(2*d^2*f^2 - 2*c*d*f*g + c^2*g^2))*Log
[f + g*x])/((b*f - a*g)^4*(d*f - c*g)^4)))/(4*g)
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$
\int \frac{A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}{(gx+f)^5}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(g*x+f)^5,x)

[Out] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(g*x+f)^5,x)

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1762 vs. $2(375) = 750$.
time = 0.47, size = 1762, normalized size = 4.54

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(g*x+f)^5,x, algorithm="maxima")

[Out] 1/24*(6*b^4*log(b*x + a)/(b^4*f^4*g - 4*a*b^3*f^3*g^2 + 6*a^2*b^2*f^2*g^3 - 4*a^3*b*f*g^4 + a^4*g^5) - 6*d^4*log(d*x + c)/(d^4*f^4*g - 4*c*d^3*f^3*g^2 + 6*c^2*d^2*f^2*g^3 - 4*c^3*d*f*g^4 + c^4*g^5) + 6*(4*(b^4*c*d^3 - a*b^3*d^4)*f^3 - 6*(b^4*c^2*d^2 - a^2*b^2*d^4)*f^2*g + 4*(b^4*c^3*d - a^3*b*d^4)*f*g^2 - (b^4*c^4 - a^4*d^4)*g^3)*log(g*x + f)/(b^4*d^4*f^8 + a^4*c^4*g^8 - 4*(b^4*c*d^3 + a*b^3*d^4)*f^7*g + 2*(3*b^4*c^2*d^2 + 8*a*b^3*c*d^3 + 3*a^2*b^2*d^4)*f^6*g^2 - 4*(b^4*c^3*d + 6*a*b^3*c^2*d^2 + 6*a^2*b^2*c*d^3 + a^3*b*d^4)*f^5*g^3 + (b^4*c^4 + 16*a*b^3*c^3*d + 36*a^2*b^2*c^2*d^2 + 16*a^3*b*c*d^3 + a^4*d^4)*f^4*g^4 - 4*(a*b^3*c^4 + 6*a^2*b^2*c^3*d + 6*a^3*b*c^2*d^2 + a^4*c*d^3)*f^3*g^5 + 2*(3*a^2*b^2*c^4 + 8*a^3*b*c^3*d + 3*a^4*c^2*d^2)*f^2*g^6 - 4*(a^3*b*c^4 + a^4*c^3*d)*f*g^7) - (26*(b^3*c*d^2 - a*b^2*d^3)*f^4 - 31*(b^3*c^2*d - a^2*b*d^3)*f^3*g + (11*b^3*c^3 + 15*a*b^2*c^2*d - 15*a^2*b*c*d^2 - 11*a^3*d^3)*f^2*g^2 - 7*(a*b^2*c^3 - a^3*c*d^2)*f*g^3 + 2*(a^2*b*c^3 - a^3*c^2*d)*g^4 + 6*(3*(b^3*c*d^2 - a*b^2*d^3)*f^2*g^2 - 3*(b^3*c^2*d - a^2*b*d^3)*f*g^3 + (b^3*c^3 - a^3*d^3)*g^4)*x^2 + 3*(14*(b^3*c*d^2 - a*b^2*d^3)*f^3*g - 15*(b^3*c^2*d - a^2*b*d^3)*f^2*g^2 + (5*b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 - 5*a^3*d^3)*f*g^3 - (a*b^2*c^3 - a^3*c*d^2)*g^4)*x)/(b^3*d^3*f^9 + a^3*c^3*f^3*g^6 - 3*(b^3*c*d^2 + a*b^2*d^3)*f^8*g + 3*(b^3*c^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3)*f^7*g^2 - (b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*f^6*g^3 + 3*(a*b^2*c^3 + 3*a^2*b*c^2*d + a^3*c*d^2)*f^5*g^4 - 3*(a^2*b*c^3 + a^3*c^2*d)*f^4*g^5 + (b^3*d^3*f^6*g^3 + a^3*c^3*g^9 - 3*(b^3*c*d^2 + a*b^2*d^3)*f^5*g^4 + 3*(b^3*c^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3)*f^4*g^5 - (b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*f^3*g^6 + 3*(a*b^2*c^3 + 3*a^2*b*c^2*d + a^3*c*d^2)*f^2*g^7 - 3*(a^2*b*c^3 + a^3*c^2*d)*f*g^8)*x^3 + 3*(b^3*d^3*f^7*g^2 + a^3*c^3*f*g^8 - 3*(b^3*c*d^2 + a*b^2*d^3)*f^6*g^3 + 3*(b^3*c^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3)*f^5*g^4 - (b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*f^4*g^5 + 3*(a*b^2*c^3 + 3*a^2*b*c^2*d + a^3*c*d^2)*f^3*g^6 - 3*(a^2*b*c^3 + a^3*c^2*d)*f^2*g^7)*x^2 + 3*(b^3*d^3*f^8*g + a^3*c^3*f^2*g^7 - 3*(b^3*c*d^2 + a*b^2*d^3)*f^7*g^2 + 3*(b^3*c^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3)*f^6*g^3 - (b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*f^5*g^4 + 3*(a*b^2*c^3 + 3*a^2*b*c^2*d + a^3*c*d^2)*f^4*g^5 - 3*(a^2*b*c^3 + a^3*c^2*d)*f^3*g^6)*x))*B*n - 1/4*B*log((b*x/(d*x + c) + a/(d*x + c))^n*e)/(g^5*x^4 + 4*f*g^4*x^3 + 6*f^2*g^3*x^2 + 4*f^3*g^2*

```
x + f^4*g) - 1/4*A/(g^5*x^4 + 4*f*g^4*x^3 + 6*f^2*g^3*x^2 + 4*f^3*g^2*x + f
^4*g)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(g*x+f)^5,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))/(g*x+f)**5,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 21485 vs. $2(375) = 750$.
time = 6.20, size = 21485, normalized size = 55.37

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(g*x+f)^5,x, algorithm="giac")
```

```
[Out] 1/24*(6*(4*B*b^5*c^2*d^3*f^3*n - 8*B*a*b^4*c*d^4*f^3*n + 4*B*a^2*b^3*d^5*f^
3*n - 6*B*b^5*c^3*d^2*f^2*g*n + 6*B*a*b^4*c^2*d^3*f^2*g*n + 6*B*a^2*b^3*c*d
^4*f^2*g*n - 6*B*a^3*b^2*d^5*f^2*g*n + 4*B*b^5*c^4*d*f*g^2*n - 4*B*a*b^4*c^
3*d^2*f*g^2*n - 4*B*a^3*b^2*c*d^4*f*g^2*n + 4*B*a^4*b*d^5*f*g^2*n - B*b^5*c
^5*g^3*n + B*a*b^4*c^4*d*g^3*n + B*a^4*b*c*d^4*g^3*n - B*a^5*d^5*g^3*n)*log
(-b*f + (b*x + a)*d*f/(d*x + c) + a*g - (b*x + a)*c*g/(d*x + c))/(b^4*d^4*f
^8 - 4*b^4*c*d^3*f^7*g - 4*a*b^3*d^4*f^7*g + 6*b^4*c^2*d^2*f^6*g^2 + 16*a*b
^3*c*d^3*f^6*g^2 + 6*a^2*b^2*d^4*f^6*g^2 - 4*b^4*c^3*d*f^5*g^3 - 24*a*b^3*c
^2*d^2*f^5*g^3 - 24*a^2*b^2*c*d^3*f^5*g^3 - 4*a^3*b*d^4*f^5*g^3 + b^4*c^4*f
^4*g^4 + 16*a*b^3*c^3*d*f^4*g^4 + 36*a^2*b^2*c^2*d^2*f^4*g^4 + 16*a^3*b*c*d
^3*f^4*g^4 + a^4*d^4*f^4*g^4 - 4*a*b^3*c^4*f^3*g^5 - 24*a^2*b^2*c^3*d*f^3*g
^5 - 24*a^3*b*c^2*d^2*f^3*g^5 - 4*a^4*c*d^3*f^3*g^5 + 6*a^2*b^2*c^4*f^2*g^6
 + 16*a^3*b*c^3*d*f^2*g^6 + 6*a^4*c^2*d^2*f^2*g^6 - 4*a^3*b*c^4*f*g^7 - 4*a
^4*c^3*d*f*g^7 + a^4*c^4*g^8) + 6*(4*B*b^5*c^2*d^3*f^3*n - 8*B*a*b^4*c*d^4*
```

f^3*n - 12*(b*x + a)*B*b^4*c^2*d^4*f^3*n/(d*x + c) + 4*B*a^2*b^3*d^5*f^3*n
+ 24*(b*x + a)*B*a*b^3*c*d^5*f^3*n/(d*x + c) + 12*(b*x + a)^2*B*b^3*c^2*d^5
*f^3*n/(d*x + c)^2 - 12*(b*x + a)*B*a^2*b^2*d^6*f^3*n/(d*x + c) - 24*(b*x +
 a)^2*B*a*b^2*c*d^6*f^3*n/(d*x + c)^2 - 4*(b*x + a)^3*B*b^2*c^2*d^6*f^3*n/(
d*x + c)^3 + 12*(b*x + a)^2*B*a^2*b*d^7*f^3*n/(d*x + c)^2 + 8*(b*x + a)^3*B
*a*b*c*d^7*f^3*n/(d*x + c)^3 - 4*(b*x + a)^3*B*a^2*d^8*f^3*n/(d*x + c)^3 -
6*B*b^5*c^3*d^2*f^2*g*n + 6*B*a*b^4*c^2*d^3*f^2*g*n + 24*(b*x + a)*B*b^4*c^
3*d^3*f^2*g*n/(d*x + c) + 6*B*a^2*b^3*c*d^4*f^2*g*n - 36*(b*x + a)*B*a*b^3*
c^2*d^4*f^2*g*n/(d*x + c) - 30*(b*x + a)^2*B*b^3*c^3*d^4*f^2*g*n/(d*x + c)^
2 - 6*B*a^3*b^2*d^5*f^2*g*n + 54*(b*x + a)^2*B*a*b^2*c^2*d^5*f^2*g*n/(d*x +
 c)^2 + 12*(b*x + a)^3*B*b^2*c^3*d^5*f^2*g*n/(d*x + c)^3 + 12*(b*x + a)*B*a
^3*b*d^6*f^2*g*n/(d*x + c) - 18*(b*x + a)^2*B*a^2*b*c*d^6*f^2*g*n/(d*x + c)
^2 - 24*(b*x + a)^3*B*a*b*c^2*d^6*f^2*g*n/(d*x + c)^3 - 6*(b*x + a)^2*B*a^3
*d^7*f^2*g*n/(d*x + c)^2 + 12*(b*x + a)^3*B*a^2*c*d^7*f^2*g*n/(d*x + c)^3 +
 4*B*b^5*c^4*d*f*g^2*n - 4*B*a*b^4*c^3*d^2*f*g^2*n - 16*(b*x + a)*B*b^4*c^4
*d^2*f*g^2*n/(d*x + c) + 16*(b*x + a)*B*a*b^3*c^3*d^3*f*g^2*n/(d*x + c) + 2
4*(b*x + a)^2*B*b^3*c^4*d^3*f*g^2*n/(d*x + c)^2 - 4*B*a^3*b^2*c*d^4*f*g^2*n
 + 12*(b*x + a)*B*a^2*b^2*c^2*d^4*f*g^2*n/(d*x + c) - 36*(b*x + a)^2*B*a*b^
2*c^3*d^4*f*g^2*n/(d*x + c)^2 - 12*(b*x + a)^3*B*b^2*c^4*d^4*f*g^2*n/(d*x +
 c)^3 + 4*B*a^4*b*d^5*f*g^2*n - 8*(b*x + a)*B*a^3*b*c*d^5*f*g^2*n/(d*x + c)
 + 24*(b*x + a)^3*B*a*b*c^3*d^5*f*g^2*n/(d*x + c)^3 - 4*(b*x + a)*B*a^4*d^6
*f*g^2*n/(d*x + c) + 12*(b*x + a)^2*B*a^3*c*d^6*f*g^2*n/(d*x + c)^2 - 12*(b
*x + a)^3*B*a^2*c^2*d^6*f*g^2*n/(d*x + c)^3 - B*b^5*c^5*g^3*n + B*a*b^4*c^4
*d*g^3*n + 4*(b*x + a)*B*b^4*c^5*d*g^3*n/(d*x + c) - 4*(b*x + a)*B*a*b^3*c^
4*d^2*g^3*n/(d*x + c) - 6*(b*x + a)^2*B*b^3*c^5*d^2*g^3*n/(d*x + c)^2 + 6*(
b*x + a)^2*B*a*b^2*c^4*d^3*g^3*n/(d*x + c)^2 + 4*(b*x + a)^3*B*b^2*c^5*d^3*
g^3*n/(d*x + c)^3 + B*a^4*b*c*d^4*g^3*n - 4*(b*x + a)*B*a^3*b*c^2*d^4*g^3*n
/(d*x + c) + 6*(b*x + a)^2*B*a^2*b*c^3*d^4*g^3*n/(d*x + c)^2 - 8*(b*x + a)^
3*B*a*b*c^4*d^4*g^3*n/(d*x + c)^3 - B*a^5*d^5*g^3*n + 4*(b*x + a)*B*a^4*c*d
^5*g^3*n/(d*x + c) - 6*(b*x + a)^2*B*a^3*c^2*d^5*g^3*n/(d*x + c)^2 + 4*(b*x
 + a)^3*B*a^2*c^3*d^5*g^3*n/(d*x + c)^3)*log((b*x + a)/(d*x + c))/(b^4*d^4*
f^8 - 4*(b*x + a)*b^3*d^5*f^8/(d*x + c) + 6*(b*x + a)^2*b^2*d^6*f^8/(d*x +
c)^2 - 4*(b*x + a)^3*b*d^7*f^8/(d*x + c)^3 + (b*x + a)^4*d^8*f^8/(d*x + c)^
4 - 4*b^4*c*d^3*f^7*g - 4*a*b^3*d^4*f^7*g + 20*(b*x + a)*b^3*c*d^4*f^7*g/(d
x + c) + 12(b*x + a)*a*b^2*d^5*f^7*g/(d*x + c) - 36*(b*x + a)^2*b^2*c*d^5
*f^7*g/(d*x + c)^2 - 12*(b*x + a)^2*a*b*d^6*f^7*g/(d*x + c)^2 + 28*(b*x + a
)^3*b*c*d^6*f^7*g/(d*x + c)^3 + 4*(b*x + a)^3*a*d^7*f^7*g/(d*x + c)^3 - 8*(
b*x + a)^4*c*d^7*f^7*g/(d*x + c)^4 + 6*b^4*c^2*d^2*f^6*g^2 + 16*a*b^3*c*d^3
*f^6*g^2 - 40*(b*x + a)*b^3*c^2*d^3*f^6*g^2/(d*x + c) + 6*a^2*b^2*d^4*f^6*g
^2 - 60*(b*x + a)*a*b^2*c*d^4*f^6*g^2/(d*x + c) + 90*(b*x + a)^2*b^2*c^2*d^
4*f^6*g^2/(d*x + c)^2 - 12*(b*x + a)*a^2*b*d^5*f^6*g^2/(d*x + c) + 72*(b*x
+ a)^2*a*b*c*d^5*f^6*g^2/(d*x + c)^2 - 84*(b*x + a)^3*b*c^2*d^5*f^6*g^2/(d*
x + c)^3 + 6*(b*x + a)^2*a^2*d^6*f^6*g^2/(d*x + c)^2 - 28*(b*x + a)^3*a*c*d
^6*f^6*g^2/(d*x + c)^3 + 28*(b*x + a)^4*c^2*d^6*f^6*g^2/(d*x + c)^4 - 4*b^4
*c^3*d*f^5*g^3 - 24*a*b^3*c^2*d^2*f^5*g^3 + 40*(b*x + a)*b^3*c^3*d^2*f^5*g^

```
3/(d*x + c) - 24*a^2*b^2*c*d^3*f^5*g^3 + 120*(b*x + a)*a*b^2*c^2*d^3*f^5*g^
3/(d*x + c) - 120*(b*x + a)^2*b^2*c^3*d^3*f^5*g^3/(d*x + c)^2 - 4*a^3*b*d^4
*f^5*g^3 + 60*(b*x + a)*a^2*b*c*d^4*f^5*g^3/(d*x + c) - 180*(b*x + a)^2*a*b
*c^2*d^4*f^5*g^3/(d*x + c)^2 + 140*(b*x + a)^3*b*c^3*d^4*f^5*g^3/(d*x + c)^
3 + 4*(b*x + a)*a^3*d^5*f^5*g^3/(d*x + c) - 36*(b*x + a)^2*a^2*c*d^5*f^5*g^
3/(d*x + c)^2 + 84*(b*x + a)^3*a*c^2*d^5*f^5*g^3/(d*x + c)^3 - 56*(b*x + a)
^4*c^3*d^5*f^5*g^3/(d*x + c)^4 + b^4*c^4*f^4*g^4 + 16*a*b^3*c^3*d*f^4*g^4 -
 20*(b*x + a)*b^3*c^4*d*f^4*g^4/(d*x + c) + 36*...
```

Mupad [B]
time = 13.77, size = 2569, normalized size = 6.62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log(e*((a + b*x)/(c + d*x))^n))/(f + g*x)^5,x)
```

```
[Out] ((x^3*(B*a^3*d^3*g^6*n - B*b^3*c^3*g^6*n - 3*B*a^2*b*d^3*f*g^5*n + 3*B*b^3*
c^2*d*f*g^5*n + 3*B*a*b^2*d^3*f^2*g^4*n - 3*B*b^3*c*d^2*f^2*g^4*n))/(a^3*c^
3*g^6 + b^3*d^3*f^6 - a^3*d^3*f^3*g^3 - b^3*c^3*f^3*g^3 - 3*a^2*b*c^3*f*g^5
 - 3*a*b^2*d^3*f^5*g - 3*a^3*c^2*d*f*g^5 - 3*b^3*c*d^2*f^5*g + 3*a*b^2*c^3*
f^2*g^4 + 3*a^2*b*d^3*f^4*g^2 + 3*a^3*c*d^2*f^2*g^4 + 3*b^3*c^2*d*f^4*g^2 +
 9*a*b^2*c*d^2*f^4*g^2 - 9*a*b^2*c^2*d*f^3*g^3 - 9*a^2*b*c*d^2*f^3*g^3 + 9*
a^2*b*c^2*d*f^2*g^4) - (6*A*a^3*c^3*g^6 + 6*A*b^3*d^3*f^6 - 6*A*a^3*d^3*f^3
*g^3 - 6*A*b^3*c^3*f^3*g^3 + 18*A*a*b^2*c^3*f^2*g^4 + 18*A*a^2*b*d^3*f^4*g^
2 + 18*A*a^3*c*d^2*f^2*g^4 + 18*A*b^3*c^2*d*f^4*g^2 - 11*B*a^3*d^3*f^3*g^3*
n + 11*B*b^3*c^3*f^3*g^3*n - 18*A*a^2*b*c^3*f*g^5 - 18*A*a*b^2*d^3*f^5*g -
18*A*a^3*c^2*d*f*g^5 - 18*A*b^3*c*d^2*f^5*g + 2*B*a^2*b*c^3*f*g^5*n - 26*B*
a*b^2*d^3*f^5*g*n - 2*B*a^3*c^2*d*f*g^5*n + 26*B*b^3*c*d^2*f^5*g*n + 54*A*a
*b^2*c*d^2*f^4*g^2 - 54*A*a*b^2*c^2*d*f^3*g^3 - 54*A*a^2*b*c*d^2*f^3*g^3 +
54*A*a^2*b*c^2*d*f^2*g^4 - 7*B*a*b^2*c^3*f^2*g^4*n + 31*B*a^2*b*d^3*f^4*g^2
*n + 7*B*a^3*c*d^2*f^2*g^4*n - 31*B*b^3*c^2*d*f^4*g^2*n + 15*B*a*b^2*c^2*d*
f^3*g^3*n - 15*B*a^2*b*c*d^2*f^3*g^3*n)/(6*(a^3*c^3*g^6 + b^3*d^3*f^6 - a^3
*d^3*f^3*g^3 - b^3*c^3*f^3*g^3 - 3*a^2*b*c^3*f*g^5 - 3*a*b^2*d^3*f^5*g - 3*
a^3*c^2*d*f*g^5 - 3*b^3*c*d^2*f^5*g + 3*a*b^2*c^3*f^2*g^4 + 3*a^2*b*d^3*f^4
*g^2 + 3*a^3*c*d^2*f^2*g^4 + 3*b^3*c^2*d*f^4*g^2 + 9*a*b^2*c*d^2*f^4*g^2 -
9*a*b^2*c^2*d*f^3*g^3 - 9*a^2*b*c*d^2*f^3*g^3 + 9*a^2*b*c^2*d*f^2*g^4)) + (
x^2*(B*a*b^2*c^3*g^6*n - B*a^3*c*d^2*g^6*n + 7*B*a^3*d^3*f*g^5*n - 7*B*b^3*
c^3*f*g^5*n + 20*B*a*b^2*d^3*f^3*g^3*n - 21*B*a^2*b*d^3*f^2*g^4*n - 20*B*b^
3*c*d^2*f^3*g^3*n + 21*B*b^3*c^2*d*f^2*g^4*n - 3*B*a*b^2*c^2*d*f*g^5*n + 3*
B*a^2*b*c*d^2*f*g^5*n))/(2*(a^3*c^3*g^6 + b^3*d^3*f^6 - a^3*d^3*f^3*g^3 - b
^3*c^3*f^3*g^3 - 3*a^2*b*c^3*f*g^5 - 3*a*b^2*d^3*f^5*g - 3*a^3*c^2*d*f*g^5
- 3*b^3*c*d^2*f^5*g + 3*a*b^2*c^3*f^2*g^4 + 3*a^2*b*d^3*f^4*g^2 + 3*a^3*c*d
^2*f^2*g^4 + 3*b^3*c^2*d*f^4*g^2 + 9*a*b^2*c*d^2*f^4*g^2 - 9*a*b^2*c^2*d*f^
3*g^3 - 9*a^2*b*c*d^2*f^3*g^3 + 9*a^2*b*c^2*d*f^2*g^4)) + (x*(13*B*a^3*d^3*
```

```
f^2*g^4*n - 13*B*b^3*c^3*f^2*g^4*n - B*a^2*b*c^3*g^6*n + B*a^3*c^2*d*g^6*n
+ 5*B*a*b^2*c^3*f*g^5*n - 5*B*a^3*c*d^2*f*g^5*n + 34*B*a*b^2*d^3*f^4*g^2*n
- 38*B*a^2*b*d^3*f^3*g^3*n - 34*B*b^3*c*d^2*f^4*g^2*n + 38*B*b^3*c^2*d*f^3*
g^3*n - 12*B*a*b^2*c^2*d*f^2*g^4*n + 12*B*a^2*b*c*d^2*f^2*g^4*n))/(3*(a^3*c
^3*g^6 + b^3*d^3*f^6 - a^3*d^3*f^3*g^3 - b^3*c^3*f^3*g^3 - 3*a^2*b*c^3*f*g^
5 - 3*a*b^2*d^3*f^5*g - 3*a^3*c^2*d*f*g^5 - 3*b^3*c*d^2*f^5*g + 3*a*b^2*c^3
*f^2*g^4 + 3*a^2*b*d^3*f^4*g^2 + 3*a^3*c*d^2*f^2*g^4 + 3*b^3*c^2*d*f^4*g^2
+ 9*a*b^2*c*d^2*f^4*g^2 - 9*a*b^2*c^2*d*f^3*g^3 - 9*a^2*b*c*d^2*f^3*g^3 + 9
*a^2*b*c^2*d*f^2*g^4)))/(4*f^4*g + 4*g^5*x^4 + 16*f^3*g^2*x + 16*f*g^4*x^3
+ 24*f^2*g^3*x^2) + (log(f + g*x)*(g*(6*B*a^2*b^2*d^4*f^2*n - 6*B*b^4*c^2*d
^2*f^2*n) - g^2*(4*B*a^3*b*d^4*f*n - 4*B*b^4*c^3*d*f*n) + g^3*(B*a^4*d^4*n
- B*b^4*c^4*n) - 4*B*a*b^3*d^4*f^3*n + 4*B*b^4*c*d^3*f^3*n))/(4*a^4*c^4*g^8
 + 4*b^4*d^4*f^8 + 4*a^4*d^4*f^4*g^4 + 4*b^4*c^4*f^4*g^4 + 24*a^2*b^2*c^4*f
^2*g^6 + 24*a^2*b^2*d^4*f^6*g^2 + 24*a^4*c^2*d^2*f^2*g^6 + 24*b^4*c^2*d^2*f
^6*g^2 - 16*a^3*b*c^4*f*g^7 - 16*a*b^3*d^4*f^7*g - 16*a^4*c^3*d*f*g^7 - 16*
b^4*c*d^3*f^7*g - 16*a*b^3*c^4*f^3*g^5 - 16*a^3*b*d^4*f^5*g^3 - 16*a^4*c*d^
3*f^3*g^5 - 16*b^4*c^3*d*f^5*g^3 + 64*a*b^3*c*d^3*f^6*g^2 + 64*a*b^3*c^3*d*
f^4*g^4 + 64*a^3*b*c*d^3*f^4*g^4 + 64*a^3*b*c^3*d*f^2*g^6 - 96*a*b^3*c^2*d^
2*f^5*g^3 - 96*a^2*b^2*c*d^3*f^5*g^3 - 96*a^2*b^2*c^3*d*f^3*g^5 - 96*a^3*b*
c^2*d^2*f^3*g^5 + 144*a^2*b^2*c^2*d^2*f^4*g^4) - (B*log(e*((a + b*x)/(c + d
*x))^n))/(4*g*(f^4 + g^4*x^4 + 4*f^3*g*x + 4*f*g^3*x^3 + 6*f^2*g^2*x^2)) +
(B*b^4*n*log(a + b*x))/(4*a^4*g^5 + 4*b^4*f^4*g - 16*a*b^3*f^3*g^2 + 24*a^2
*b^2*f^2*g^3 - 16*a^3*b*f*g^4) - (B*d^4*n*log(c + d*x))/(4*c^4*g^5 + 4*d^4*
f^4*g - 16*c*d^3*f^3*g^2 + 24*c^2*d^2*f^2*g^3 - 16*c^3*d*f*g^4)
```

3.67 $\int (f+gx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$

Optimal. Leaf size=923

$$\frac{B^2(bc-ad)^3g^3n^2x}{6b^3d^3}+\frac{B^2(bc-ad)^2g^2(4bdf-3bcg-adg)n^2x}{4b^3d^3}+\frac{B^2(bc-ad)^2g^3n^2(c+dx)^2}{12b^2d^4}-\frac{B(bc-ad)g(a^2}{}$$

```
[Out] 1/6*B^2*(-a*d+b*c)^3*g^3*n^2*x/b^3/d^3+1/4*B^2*(-a*d+b*c)^2*g^2*(-a*d*g-3*b
*c*g+4*b*d*f)*n^2*x/b^3/d^3+1/12*B^2*(-a*d+b*c)^2*g^3*n^2*(d*x+c)^2/b^2/d^4
-1/2*B*(-a*d+b*c)*g*(a^2*d^2*g^2-2*a*b*d*g*(-c*g+2*d*f)+b^2*(3*c^2*g^2-8*c*
d*f*g+6*d^2*f^2))*n*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b^4/d^3-1/4*B*(
-a*d+b*c)*g^2*(-a*d*g-3*b*c*g+4*b*d*f)*n*(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+
c))^n))/b^2/d^4-1/6*B*(-a*d+b*c)*g^3*n*(d*x+c)^3*(A+B*ln(e*((b*x+a)/(d*x+c)
)^n))/b/d^4-1/4*(-a*g+b*f)^4*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/b^4/g+1/4*(g
*x+f)^4*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/g-1/2*B*(-a*d+b*c)*(-a*d*g-b*c*g+
2*b*d*f)*(2*a*b*d^2*f*g-a^2*d^2*g^2-b^2*(c^2*g^2-2*c*d*f*g+2*d^2*f^2))*n*(A
+B*ln(e*((b*x+a)/(d*x+c))^n))*ln((-a*d+b*c)/b/(d*x+c))/b^4/d^4+1/6*B^2*(-a*
d+b*c)^4*g^3*n^2*ln((b*x+a)/(d*x+c))/b^4/d^4+1/4*B^2*(-a*d+b*c)^3*g^2*(-a*d
*g-3*b*c*g+4*b*d*f)*n^2*ln((b*x+a)/(d*x+c))/b^4/d^4+1/6*B^2*(-a*d+b*c)^4*g^
3*n^2*ln(d*x+c)/b^4/d^4+1/4*B^2*(-a*d+b*c)^3*g^2*(-a*d*g-3*b*c*g+4*b*d*f)*n
^2*ln(d*x+c)/b^4/d^4+1/2*B^2*(-a*d+b*c)^2*g*(a^2*d^2*g^2-2*a*b*d*g*(-c*g+2*
d*f)+b^2*(3*c^2*g^2-8*c*d*f*g+6*d^2*f^2))*n^2*ln(d*x+c)/b^4/d^4-1/2*B^2*(-a
*d+b*c)*(-a*d*g-b*c*g+2*b*d*f)*(2*a*b*d^2*f*g-a^2*d^2*g^2-b^2*(c^2*g^2-2*c*
d*f*g+2*d^2*f^2))*n^2*polylog(2,d*(b*x+a)/b/(d*x+c))/b^4/d^4
```

Rubi [A]
time = 1.17, antiderivative size = 923, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {2553, 2398, 2404, 2338, 2356, 46, 2351, 31, 2354, 2438}

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

```
[Out] (B^2*(b*c - a*d)^3*g^3*n^2*x)/(6*b^3*d^3) + (B^2*(b*c - a*d)^2*g^2*(4*b*d*f
 - 3*b*c*g - a*d*g)*n^2*x)/(4*b^3*d^3) + (B^2*(b*c - a*d)^2*g^3*n^2*(c + d*
x)^2)/(12*b^2*d^4) - (B*(b*c - a*d)*g*(a^2*d^2*g^2 - 2*a*b*d*g*(2*d*f - c*g
) + b^2*(6*d^2*f^2 - 8*c*d*f*g + 3*c^2*g^2))*n*(a + b*x)*(A + B*Log[e*((a +
 b*x)/(c + d*x))^n]))/(2*b^4*d^3) - (B*(b*c - a*d)*g^2*(4*b*d*f - 3*b*c*g -
 a*d*g)*n*(c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(4*b^2*d^4) -
 (B*(b*c - a*d)*g^3*n*(c + d*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(
6*b*d^4) - ((b*f - a*g)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(4*b^4*
g) + ((f + g*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(4*g) - (B*(b*c
```

```
 - a*d)*(2*b*d*f - b*c*g - a*d*g)*(2*a*b*d^2*f*g - a^2*d^2*g^2 - b^2*(2*d^2
*f^2 - 2*c*d*f*g + c^2*g^2))*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[(
b*c - a*d)/(b*(c + d*x))])/(2*b^4*d^4) + (B^2*(b*c - a*d)^4*g^3*n^2*Log[(a
+ b*x)/(c + d*x)])/(6*b^4*d^4) + (B^2*(b*c - a*d)^3*g^2*(4*b*d*f - 3*b*c*g
- a*d*g)*n^2*Log[(a + b*x)/(c + d*x)])/(4*b^4*d^4) + (B^2*(b*c - a*d)^4*g^3
*n^2*Log[c + d*x])/(6*b^4*d^4) + (B^2*(b*c - a*d)^3*g^2*(4*b*d*f - 3*b*c*g
- a*d*g)*n^2*Log[c + d*x])/(4*b^4*d^4) + (B^2*(b*c - a*d)^2*g*(a^2*d^2*g^2
- 2*a*b*d*g*(2*d*f - c*g) + b^2*(6*d^2*f^2 - 8*c*d*f*g + 3*c^2*g^2))*n^2*Lo
g[c + d*x])/(2*b^4*d^4) - (B^2*(b*c - a*d)*(2*b*d*f - b*c*g - a*d*g)*(2*a*b
*d^2*f*g - a^2*d^2*g^2 - b^2*(2*d^2*f^2 - 2*c*d*f*g + c^2*g^2))*n^2*PolyLog
[2, (d*(a + b*x))/(b*(c + d*x))])/(2*b^4*d^4)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
```

```
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_)*((
f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(d + e*x)^(q +
 1)*((a + b*Log[c*x^n])^p/((q + 1)*(e*f - d*g))), x] - Dist[b*n*(p/((q + 1)
*(e*f - d*g))), Int[(f + g*x)^(m + 1)*(d + e*x)^(q + 1)*((a + b*Log[c*x^n])
^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, q}, x] && NeQ[e*f
- d*g, 0] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 2404

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2553

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[b*c - a*d, Subst[
Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)),
 x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x
] && NeQ[b*c - a*d, 0] && IntegerQ[m] && IGtQ[p, 0]
```

Rubi steps

$$\int (f+gx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx = \frac{(f+gx)^4 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4g} - \frac{(Bn)\int \frac{(bc-ad)(f+gx)^4}{(a+\cdots}}{2\cdots}$$

$$= \frac{(f+gx)^4 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4g} - \frac{(B(bc-ad)n)\int \frac{(f+\cdots}{\cdots}}{\cdots}$$

$$= \frac{(f+gx)^4 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4g} - \frac{(B(bc-ad)n)\int \left(\frac{g^2\cdots}{\cdots}\right.}{\cdots}$$

$$= \frac{(f+gx)^4 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4g} - \frac{(B(bc-ad)g^3n)\int x\cdots}{\cdots}$$

$$= -\frac{AB(bc-ad)g(a^2d^2g^2 - abdg(4df-cg) + b^2(6d^2f^2 - 4c\cdots}{2b^3d^3}$$

$$= -\frac{AB(bc-ad)g(a^2d^2g^2 - abdg(4df-cg) + b^2(6d^2f^2 - 4c\cdots}{2b^3d^3}$$

$$= -\frac{AB(bc-ad)g(a^2d^2g^2 - abdg(4df-cg) + b^2(6d^2f^2 - 4c\cdots}{2b^3d^3}$$

$$= -\frac{AB(bc-ad)g(a^2d^2g^2 - abdg(4df-cg) + b^2(6d^2f^2 - 4c\cdots}{2b^3d^3}$$

$$= -\frac{AB(bc-ad)g(a^2d^2g^2 - abdg(4df-cg) + b^2(6d^2f^2 - 4c\cdots}{2b^3d^3}$$

$$= -\frac{AB(bc-ad)g(a^2d^2g^2 - abdg(4df-cg) + b^2(6d^2f^2 - 4c\cdots}{2b^3d^3}$$

Mathematica [A]
time = 0.69, size = 757, normalized size = 0.82

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

```
[Out] ((f + g*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 - (B*n*(6*A*b*d*(b*c
- a*d)*g^2*(a^2*d^2*g^2 + a*b*d*g*(-4*d*f + c*g) + b^2*(6*d^2*f^2 - 4*c*d*f
*g + c^2*g^2))*x + 6*B*d*(b*c - a*d)*g^2*(a^2*d^2*g^2 + a*b*d*g*(-4*d*f + c
```

```
*g) + b^2*(6*d^2*f^2 - 4*c*d*f*g + c^2*g^2))*(a + b*x)*Log[e*((a + b*x)/(c
+ d*x))^n] + 3*b^2*d^2*(b*c - a*d)*g^3*(4*b*d*f - b*c*g - a*d*g)*x^2*(A + B
*Log[e*((a + b*x)/(c + d*x))^n]) + 2*b^3*d^3*(b*c - a*d)*g^4*x^3*(A + B*Log
[e*((a + b*x)/(c + d*x))^n]) + 6*d^4*(b*f - a*g)^4*Log[a + b*x]*(A + B*Log[
e*((a + b*x)/(c + d*x))^n]) - 6*B*(b*c - a*d)^2*g^2*(a^2*d^2*g^2 + a*b*d*g*
(-4*d*f + c*g) + b^2*(6*d^2*f^2 - 4*c*d*f*g + c^2*g^2))*n*Log[c + d*x] - 6*
b^4*(d*f - c*g)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[c + d*x] + B*(
b*c - a*d)*g^4*n*(b*d*(b*c - a*d)*x*(2*b*c + 2*a*d - b*d*x) + 2*a^3*d^3*Log
[a + b*x] - 2*b^3*c^3*Log[c + d*x]) - 3*B*(b*c - a*d)*g^3*(-4*b*d*f + b*c*g
 + a*d*g)*n*(-(a^2*d^2*Log[a + b*x]) + b*(d*(-(b*c) + a*d)*x + b*c^2*Log[c
+ d*x])) - 3*B*d^4*(b*f - a*g)^4*n*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(
c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) + 3*b
^4*B*(d*f - c*g)^4*n*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*
Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(3*b^4*d^4))/(4*g
)
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int (gx+f)^3 \left(A + B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

```
[Out] int((g*x+f)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 2534 vs. 2(898) = 1796.
time = 0.83, size = 2534, normalized size = 2.75

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="maxima
")
```

```
[Out] 1/2*A*B*g^3*x^4*log((b*x/(d*x + c) + a/(d*x + c))^n*e) + 1/4*A^2*g^3*x^4 +
2*A*B*f*g^2*x^3*log((b*x/(d*x + c) + a/(d*x + c))^n*e) + A^2*f*g^2*x^3 + 3*
A*B*f^2*g*x^2*log((b*x/(d*x + c) + a/(d*x + c))^n*e) + 3/2*A^2*f^2*g*x^2 -
1/12*A*B*g^3*n*(6*a^4*log(b*x + a)/b^4 - 6*c^4*log(d*x + c)/d^4 + (2*(b^3*c
*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^
3)*x)/(b^3*d^3)) + A*B*f*g^2*n*(2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)
/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2)) - 3*A
*B*f^2*g*n*(a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c - a*d)*x/(b*
d)) + 2*A*B*f^3*n*(a*log(b*x + a)/b - c*log(d*x + c)/d) + 2*A*B*f^3*x*log((
```

```
b*x/(d*x + c) + a/(d*x + c))^n*e) + A^2*f^3*x - 1/12*(6*a^3*c*d^3*g^3*n^2 -
 3*(8*c*d^3*f*g^2*n^2 - c^2*d^2*g^3*n^2)*a^2*b + 2*(18*c*d^3*f^2*g*n^2 - 6*
c^2*d^2*f*g^2*n^2 + c^3*d*g^3*n^2)*a*b^2 - (36*(n^2 + n)*c^2*d^2*f^2*g - 12
*(3*n^2 + 2*n)*c^3*d*f*g^2 + (11*n^2 + 6*n)*c^4*g^3 - 24*c*d^3*f^3*n)*b^3)*
B^2*log(d*x + c)/(b^3*d^4) + 1/2*(4*a*b^3*d^4*f^3*n^2 - 6*a^2*b^2*d^4*f^2*g
*n^2 + 4*a^3*b*d^4*f*g^2*n^2 - a^4*d^4*g^3*n^2 - (4*c*d^3*f^3*n^2 - 6*c^2*d
^2*f^2*g*n^2 + 4*c^3*d*f*g^2*n^2 - c^4*g^3*n^2)*b^4)*(log(b*x + a)*log((b*d
*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B^2/(b^4*d^
4) + 1/12*(3*B^2*b^4*d^4*g^3*x^4 + 6*(4*c*d^3*f^3*n^2 - 6*c^2*d^2*f^2*g*n^2
 + 4*c^3*d*f*g^2*n^2 - c^4*g^3*n^2)*B^2*b^4*log(b*x + a)*log(d*x + c) - 3*(
4*c*d^3*f^3*n^2 - 6*c^2*d^2*f^2*g*n^2 + 4*c^3*d*f*g^2*n^2 - c^4*g^3*n^2)*B^
2*b^4*log(d*x + c)^2 + 2*(a*b^3*d^4*g^3*n - (c*d^3*g^3*n - 6*d^4*f*g^2)*b^4
)*B^2*x^3 + ((n^2 - 3*n)*a^2*b^2*d^4*g^3 - 2*(c*d^3*g^3*n^2 - 6*d^4*f*g^2*n
)*a*b^3 + ((n^2 + 3*n)*c^2*d^2*g^3 - 12*c*d^3*f*g^2*n + 18*d^4*f^2*g)*b^4)*
B^2*x^2 - 3*(4*a*b^3*d^4*f^3*n^2 - 6*a^2*b^2*d^4*f^2*g*n^2 + 4*a^3*b*d^4*f*
g^2*n^2 - a^4*d^4*g^3*n^2)*B^2*log(b*x + a)^2 - ((5*n^2 - 6*n)*a^3*b*d^4*g^
3 - (5*c*d^3*g^3*n^2 + 12*(n^2 - 2*n)*d^4*f*g^2)*a^2*b^2 + (24*c*d^3*f*g^2*
n^2 - 5*c^2*d^2*g^3*n^2 - 36*d^4*f^2*g*n)*a*b^3 - (12*(n^2 + 2*n)*c^2*d^2*f
*g^2 - (5*n^2 + 6*n)*c^3*d*g^3 - 36*c*d^3*f^2*g*n + 12*d^4*f^3)*b^4)*B^2*x
+ ((11*n^2 - 6*n)*a^4*d^4*g^3 - 2*(c*d^3*g^3*n^2 + 6*(3*n^2 - 2*n)*d^4*f*g^
2)*a^3*b + 3*(4*c*d^3*f*g^2*n^2 - c^2*d^2*g^3*n^2 + 12*(n^2 - n)*d^4*f^2*g)
*a^2*b^2 - 6*(6*c*d^3*f^2*g*n^2 - 4*c^2*d^2*f*g^2*n^2 + c^3*d*g^3*n^2 - 4*d
^4*f^3*n)*a*b^3)*B^2*log(b*x + a) + 3*(B^2*b^4*d^4*g^3*x^4 + 4*B^2*b^4*d^4*
f*g^2*x^3 + 6*B^2*b^4*d^4*f^2*g*x^2 + 4*B^2*b^4*d^4*f^3*x)*log((b*x + a)^n)
^2 + 3*(B^2*b^4*d^4*g^3*x^4 + 4*B^2*b^4*d^4*f*g^2*x^3 + 6*B^2*b^4*d^4*f^2*g
*x^2 + 4*B^2*b^4*d^4*f^3*x)*log((d*x + c)^n)^2 + (6*B^2*b^4*d^4*g^3*x^4 - 6
*(4*c*d^3*f^3*n - 6*c^2*d^2*f^2*g*n + 4*c^3*d*f*g^2*n - c^4*g^3*n)*B^2*b^4*
log(d*x + c) + 2*(a*b^3*d^4*g^3*n - (c*d^3*g^3*n - 12*d^4*f*g^2)*b^4)*B^2*x
^3 + 3*(4*a*b^3*d^4*f*g^2*n - a^2*b^2*d^4*g^3*n - (4*c*d^3*f*g^2*n - c^2*d^
2*g^3*n - 12*d^4*f^2*g)*b^4)*B^2*x^2 + 6*(6*a*b^3*d^4*f^2*g*n - 4*a^2*b^2*d
^4*f*g^2*n + a^3*b*d^4*g^3*n - (6*c*d^3*f^2*g*n - 4*c^2*d^2*f*g^2*n + c^3*d
*g^3*n - 4*d^4*f^3)*b^4)*B^2*x + 6*(4*a*b^3*d^4*f^3*n - 6*a^2*b^2*d^4*f^2*g
*n + 4*a^3*b*d^4*f*g^2*n - a^4*d^4*g^3*n)*B^2*log(b*x + a))*log((b*x + a)^n
) - (6*B^2*b^4*d^4*g^3*x^4 - 6*(4*c*d^3*f^3*n - 6*c^2*d^2*f^2*g*n + 4*c^3*d
*f*g^2*n - c^4*g^3*n)*B^2*b^4*log(d*x + c) + 2*(a*b^3*d^4*g^3*n - (c*d^3*g^
3*n - 12*d^4*f*g^2)*b^4)*B^2*x^3 + 3*(4*a*b^3*d^4*f*g^2*n - a^2*b^2*d^4*g^3
*n - (4*c*d^3*f*g^2*n - c^2*d^2*g^3*n - 12*d^4*f^2*g)*b^4)*B^2*x^2 + 6*(6*a
*b^3*d^4*f^2*g*n - 4*a^2*b^2*d^4*f*g^2*n + a^3*b*d^4*g^3*n - (6*c*d^3*f^2*g
*n - 4*c^2*d^2*f*g^2*n + c^3*d*g^3*n - 4*d^4*f^3)*b^4)*B^2*x + 6*(4*a*b^3*d
^4*f^3*n - 6*a^2*b^2*d^4*f^2*g*n + 4*a^3*b*d^4*f*g^2*n - a^4*d^4*g^3*n)*B^2
*log(b*x + a) + 6*(B^2*b^4*d^4*g^3*x^4 + 4*B^2*b^4*d^4*f*g^2*x^3 + 6*B^2*b^
4*d^4*f^2*g*x^2 + 4*B^2*b^4*d^4*f^3*x)*log((b*x + a)^n))*log((d*x + c)^n))/
(b^4*d^4)
```

Fricas [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="fricas
")
```

```
[Out] integral(A^2*g^3*x^3 + 3*A^2*f*g^2*x^2 + 3*A^2*f^2*g*x + A^2*f^3 + (B^2*g^3
*x^3 + 3*B^2*f*g^2*x^2 + 3*B^2*f^2*g*x + B^2*f^3)*log(((b*x + a)/(d*x + c))
^n*e)^2 + 2*(A*B*g^3*x^3 + 3*A*B*f*g^2*x^2 + 3*A*B*f^2*g*x + A*B*f^3)*log((
(b*x + a)/(d*x + c))^n*e), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(A + B\log\left(e\left(\frac{a}{c+dx} + \frac{bx}{c+dx}\right)^n\right)\right)^2 (f+gx)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**3*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
```

```
[Out] Integral((A + B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n))**2*(f + g*x)**3, x
)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int (f+g\,x)^3\left(A+B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2,x)
```

```
[Out] int((f + g*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2, x)
```

3.68 $\int (f+gx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$

Optimal. Leaf size=565

$$\frac{B^2(bc-ad)^2g^2n^2x}{3b^2d^2}-\frac{2B(bc-ad)g(3bdf-2bcg-adg)n(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3b^3d^2}-\frac{B(bc-ad)g^2n(c$$

```
[Out] 1/3*B^2*(-a*d+b*c)^2*g^2*n^2*x/b^2/d^2-2/3*B*(-a*d+b*c)*g*(-a*d*g-2*b*c*g+3
*b*d*f)*n*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b^3/d^2-1/3*B*(-a*d+b*c)*
g^2*n*(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b/d^3-1/3*(-a*g+b*f)^3*(A+B
*ln(e*((b*x+a)/(d*x+c))^n))^2/b^3/g+1/3*(g*x+f)^3*(A+B*ln(e*((b*x+a)/(d*x+c
))^n))^2/g+2/3*B*(-a*d+b*c)*(a^2*d^2*g^2-a*b*d*g*(-c*g+3*d*f)+b^2*(c^2*g^2-
3*c*d*f*g+3*d^2*f^2))*n*(A+B*ln(e*((b*x+a)/(d*x+c))^n))*ln((-a*d+b*c)/b/(d*
x+c))/b^3/d^3+1/3*B^2*(-a*d+b*c)^3*g^2*n^2*ln((b*x+a)/(d*x+c))/b^3/d^3+1/3*
B^2*(-a*d+b*c)^3*g^2*n^2*ln(d*x+c)/b^3/d^3+2/3*B^2*(-a*d+b*c)^2*g*(-a*d*g-2
*b*c*g+3*b*d*f)*n^2*ln(d*x+c)/b^3/d^3+2/3*B^2*(-a*d+b*c)*(a^2*d^2*g^2-a*b*d
*g*(-c*g+3*d*f)+b^2*(c^2*g^2-3*c*d*f*g+3*d^2*f^2))*n^2*polylog(2,d*(b*x+a)/
b/(d*x+c))/b^3/d^3
```

Rubi [A]
time = 0.75, antiderivative size = 565, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {2553, 2398, 2404, 2338, 2356, 46, 2351, 31, 2354, 2438}

$$\frac{2B^2n^2(bc-ad)\left(a^2d^2g^2-abdg(3df-cg)+b^2(c^2g^2-3cdfg+3d^2f^2)\right)\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{3b^3d^3}+\frac{2Bn(bc-ad)\left(a^2d^2g^2-abdg(3df-cg)+b^2(c^2g^2-3cdfg+3d^2f^2)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3b^3d^3}-\frac{2Bgn(a+bx)(bc-ad)(-adg-2bcg+3bdf)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3b^3d^2}-\frac{(bf-ag)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3b^3g}-\frac{Bg^2n(c+dx)^2(bc-ad)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3bd^3}+\frac{(f+gx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3g}+\frac{2B^2gn^2(bc-ad)^2\log(c+dx)(-adg-2bcg+3bdf)}{3b^3d^3}+\frac{B^2g^2n^2(bc-ad)^3\log\left(\frac{a+bx}{c+dx}\right)}{3b^3d^3}+\frac{B^2g^2n^2(bc-ad)^3\log(c+dx)}{3b^3d^3}+\frac{B^2g^2n^2x(bc-ad)^2}{3b^2d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

```
[Out] (B^2*(b*c - a*d)^2*g^2*n^2*x)/(3*b^2*d^2) - (2*B*(b*c - a*d)*g*(3*b*d*f - 2
*b*c*g - a*d*g)*n*(a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(3*b^3*
d^2) - (B*(b*c - a*d)*g^2*n*(c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^
n]))/(3*b*d^3) - ((b*f - a*g)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(
3*b^3*g) + ((f + g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(3*g) + (
2*B*(b*c - a*d)*(a^2*d^2*g^2 - a*b*d*g*(3*d*f - c*g) + b^2*(3*d^2*f^2 - 3*c
*d*f*g + c^2*g^2))*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[(b*c - a*d)
/(b*(c + d*x))])/(3*b^3*d^3) + (B^2*(b*c - a*d)^3*g^2*n^2*Log[(a + b*x)/(c
+ d*x)])/(3*b^3*d^3) + (B^2*(b*c - a*d)^3*g^2*n^2*Log[c + d*x])/(3*b^3*d^3)
 + (2*B^2*(b*c - a*d)^2*g*(3*b*d*f - 2*b*c*g - a*d*g)*n^2*Log[c + d*x])/(3*
b^3*d^3) + (2*B^2*(b*c - a*d)*(a^2*d^2*g^2 - a*b*d*g*(3*d*f - c*g) + b^2*(3
*d^2*f^2 - 3*c*d*f*g + c^2*g^2))*n^2*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))
])/(3*b^3*d^3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_)*((
f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(d + e*x)^(q +
 1)*((a + b*Log[c*x^n])^p/((q + 1)*(e*f - d*g))), x] - Dist[b*n*(p/((q + 1)
*(e*f - d*g))), Int[(f + g*x)^(m + 1)*(d + e*x)^(q + 1)*((a + b*Log[c*x^n])
^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, q}, x] && NeQ[e*f
- d*g, 0] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 2404

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2553

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[b*c - a*d, Subst[
Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)),
 x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x
] && NeQ[b*c - a*d, 0] && IntegerQ[m] && IGtQ[p, 0]
```

Rubi steps

$$\int (f+gx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx = \frac{(f+gx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3g} - \frac{(2Bn)\int \frac{(bc-ad)(f+g}{}}{}$$

$$= \frac{(f+gx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3g} - \frac{(2B(bc-ad)n)\int}{}$$

$$= \frac{(f+gx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3g} - \frac{(2B(bc-ad)n)\int}{}$$

$$= \frac{(f+gx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3g} - \frac{(2B(bc-ad)g^2n)}{}$$

$$= -\frac{2AB(bc-ad)g(3bdf-bcg-adg)nx}{3b^2d^2} - \frac{B(bc-ad)g^2n}{}$$

$$= -\frac{2AB(bc-ad)g(3bdf-bcg-adg)nx}{3b^2d^2} - \frac{2B^2(bc-ad)g}{}$$

$$= -\frac{2AB(bc-ad)g(3bdf-bcg-adg)nx}{3b^2d^2} - \frac{2B^2(bc-ad)g}{}$$

$$= -\frac{2AB(bc-ad)g(3bdf-bcg-adg)nx}{3b^2d^2} + \frac{B^2(bc-ad)^2g}{3b^2d^2}$$

$$= -\frac{2AB(bc-ad)g(3bdf-bcg-adg)nx}{3b^2d^2} + \frac{B^2(bc-ad)^2g}{3b^2d^2}$$

$$= -\frac{2AB(bc-ad)g(3bdf-bcg-adg)nx}{3b^2d^2} + \frac{B^2(bc-ad)^2g}{3b^2d^2}$$

Mathematica [A]
time = 0.38, size = 506, normalized size = 0.90

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

```
[Out] ((f + g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 - (B*n*(2*A*b*d*(b*c
- a*d)*g^2*(3*b*d*f - b*c*g - a*d*g)*x + 2*B*d*(b*c - a*d)*g^2*(3*b*d*f - b
*c*g - a*d*g)*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n] + b^2*d^2*(b*c - a*d
```

```
)*g^3*x^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + 2*d^3*(b*f - a*g)^3*Log[
a + b*x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + 2*B*(b*c - a*d)^2*g^2*(-3
*b*d*f + b*c*g + a*d*g)*n*Log[c + d*x] - 2*b^3*(d*f - c*g)^3*(A + B*Log[e*(
(a + b*x)/(c + d*x))^n])*Log[c + d*x] - B*(b*c - a*d)*g^3*n*(a^2*d^2*Log[a
+ b*x] - b*(d*(-(b*c) + a*d)*x + b*c^2*Log[c + d*x])) - B*d^3*(b*f - a*g)^3
*n*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*Poly
Log[2, (d*(a + b*x))/(-(b*c) + a*d)]) + b^3*B*(d*f - c*g)^3*n*((2*Log[(d*(a
 + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c
+ d*x))/(b*c - a*d)])))/(b^3*d^3))/(3*g)
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int (gx+f)^2 \left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

```
[Out] int((g*x+f)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1591 vs. 2(549) = 1098.
time = 0.75, size = 1591, normalized size = 2.82

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="maxima
")
```

```
[Out] 2/3*A*B*g^2*x^3*log((b*x/(d*x + c) + a/(d*x + c))^n*e) + 1/3*A^2*g^2*x^3 +
2*A*B*f*g*x^2*log((b*x/(d*x + c) + a/(d*x + c))^n*e) + A^2*f*g*x^2 + 1/3*A*
B*g^2*n*(2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*
d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2)) - 2*A*B*f*g*n*(a^2*log(b*x +
 a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d)) + 2*A*B*f^2*n*(a*log(
b*x + a)/b - c*log(d*x + c)/d) + 2*A*B*f^2*x*log((b*x/(d*x + c) + a/(d*x +
c))^n*e) + A^2*f^2*x + 1/3*(2*a^2*c*d^2*g^2*n^2 - (6*c*d^2*f*g*n^2 - c^2*d*
g^2*n^2)*a*b + (6*(n^2 + n)*c^2*d*f*g - (3*n^2 + 2*n)*c^3*g^2 - 6*c*d^2*f^2
*n)*b^2)*B^2*log(d*x + c)/(b^2*d^3) + 2/3*(3*a*b^2*d^3*f^2*n^2 - 3*a^2*b*d^
3*f*g*n^2 + a^3*d^3*g^2*n^2 - (3*c*d^2*f^2*n^2 - 3*c^2*d*f*g*n^2 + c^3*g^2*
n^2)*b^3)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x
+ a*d)/(b*c - a*d)))*B^2/(b^3*d^3) + 1/3*(B^2*b^3*d^3*g^2*x^3 + 2*(3*c*d^2*
f^2*n^2 - 3*c^2*d*f*g*n^2 + c^3*g^2*n^2)*B^2*b^3*log(b*x + a)*log(d*x + c)
- (3*c*d^2*f^2*n^2 - 3*c^2*d*f*g*n^2 + c^3*g^2*n^2)*B^2*b^3*log(d*x + c)^2
+ (a*b^2*d^3*g^2*n - (c*d^2*g^2*n - 3*d^3*f*g)*b^3)*B^2*x^2 - (3*a*b^2*d^3*
```

```
f^2*n^2 - 3*a^2*b*d^3*f*g*n^2 + a^3*d^3*g^2*n^2)*B^2*log(b*x + a)^2 + ((n^2
 - 2*n)*a^2*b*d^3*g^2 - 2*(c*d^2*g^2*n^2 - 3*d^3*f*g*n)*a*b^2 + ((n^2 + 2*n
)*c^2*d*g^2 - 6*c*d^2*f*g*n + 3*d^3*f^2)*b^3)*B^2*x - ((3*n^2 - 2*n)*a^3*d^
3*g^2 - (c*d^2*g^2*n^2 + 6*(n^2 - n)*d^3*f*g)*a^2*b + 2*(3*c*d^2*f*g*n^2 -
c^2*d*g^2*n^2 - 3*d^3*f^2*n)*a*b^2)*B^2*log(b*x + a) + (B^2*b^3*d^3*g^2*x^3
 + 3*B^2*b^3*d^3*f*g*x^2 + 3*B^2*b^3*d^3*f^2*x)*log((b*x + a)^n)^2 + (B^2*b
^3*d^3*g^2*x^3 + 3*B^2*b^3*d^3*f*g*x^2 + 3*B^2*b^3*d^3*f^2*x)*log((d*x + c)
^n)^2 + (2*B^2*b^3*d^3*g^2*x^3 - 2*(3*c*d^2*f^2*n - 3*c^2*d*f*g*n + c^3*g^2
*n)*B^2*b^3*log(d*x + c) + (a*b^2*d^3*g^2*n - (c*d^2*g^2*n - 6*d^3*f*g)*b^3
)*B^2*x^2 + 2*(3*a*b^2*d^3*f*g*n - a^2*b*d^3*g^2*n - (3*c*d^2*f*g*n - c^2*d
*g^2*n - 3*d^3*f^2)*b^3)*B^2*x + 2*(3*a*b^2*d^3*f^2*n - 3*a^2*b*d^3*f*g*n +
 a^3*d^3*g^2*n)*B^2*log(b*x + a))*log((b*x + a)^n) - (2*B^2*b^3*d^3*g^2*x^3
 - 2*(3*c*d^2*f^2*n - 3*c^2*d*f*g*n + c^3*g^2*n)*B^2*b^3*log(d*x + c) + (a*
b^2*d^3*g^2*n - (c*d^2*g^2*n - 6*d^3*f*g)*b^3)*B^2*x^2 + 2*(3*a*b^2*d^3*f*g
*n - a^2*b*d^3*g^2*n - (3*c*d^2*f*g*n - c^2*d*g^2*n - 3*d^3*f^2)*b^3)*B^2*x
 + 2*(3*a*b^2*d^3*f^2*n - 3*a^2*b*d^3*f*g*n + a^3*d^3*g^2*n)*B^2*log(b*x +
a) + 2*(B^2*b^3*d^3*g^2*x^3 + 3*B^2*b^3*d^3*f*g*x^2 + 3*B^2*b^3*d^3*f^2*x)*
log((b*x + a)^n))*log((d*x + c)^n))/(b^3*d^3)
```

Fricas [**F**]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="fricas
")
```

```
[Out] integral(A^2*g^2*x^2 + 2*A^2*f*g*x + A^2*f^2 + (B^2*g^2*x^2 + 2*B^2*f*g*x +
 B^2*f^2)*log(((b*x + a)/(d*x + c))^n*e)^2 + 2*(A*B*g^2*x^2 + 2*A*B*f*g*x +
 A*B*f^2)*log(((b*x + a)/(d*x + c))^n*e), x)
```

Sympy [**F**]

time = 0.00, size = 0, normalized size = 0.00

$$\int \left(A + B\log\left(e\left(\frac{a}{c+dx} + \frac{bx}{c+dx}\right)^{n}\right)\right)^{2}(f+gx)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
```

```
[Out] Integral((A + B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n))**2*(f + g*x)**2, x
)
```

Giac [**F**]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="giac")
[Out] integrate((g*x + f)^2*(B*log(((b*x + a)/(d*x + c))^n*e) + A)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (f+g\,x)^2\,\left(A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2,x)
[Out] int((f + g*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2, x)
```

3.69 $\int (f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$

Optimal. Leaf size=290

$$-\frac{B(bc-ad)gn(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b^2d}-\frac{(bf-ag)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2b^2g}+\frac{(f+gx)^2\left(A+B\l\right.}{2g}$$

```
[Out] -B*(-a*d+b*c)*g*n*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b^2/d-1/2*(-a*g+b
*f)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/b^2/g+1/2*(g*x+f)^2*(A+B*ln(e*((b*x
+a)/(d*x+c))^n))^2/g+B*(-a*d+b*c)*(-a*d*g-b*c*g+2*b*d*f)*n*(A+B*ln(e*((b*x+
a)/(d*x+c))^n))*ln((-a*d+b*c)/b/(d*x+c))/b^2/d^2+B^2*(-a*d+b*c)^2*g*n^2*ln(
d*x+c)/b^2/d^2+B^2*(-a*d+b*c)*(-a*d*g-b*c*g+2*b*d*f)*n^2*polylog(2,d*(b*x+a
)/b/(d*x+c))/b^2/d^2
```

Rubi [A]
time = 0.38, antiderivative size = 290, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {2553, 2398, 2404, 2338, 2351, 31, 2354, 2438}

$$\frac{B^2n^2(bc-ad)(-adg-bcg+2bdf)\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{b^2d^2}+\frac{Bn(bc-ad)(-adg-bcg+2bdf)\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b^2d^2}-\frac{(bf-ag)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2b^2g}-\frac{Bgn(a+bx)(bc-ad)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b^2d}+\frac{(f+gx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2g}+\frac{B^2gn^2(bc-ad)^2\log(c+dx)}{b^2d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

```
[Out] -((B*(b*c - a*d)*g*n*(a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(b^2
*d)) - ((b*f - a*g)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(2*b^2*g) +
 ((f + g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(2*g) + (B*(b*c - a
*d)*(2*b*d*f - b*c*g - a*d*g)*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[
(b*c - a*d)/(b*(c + d*x))])/(b^2*d^2) + (B^2*(b*c - a*d)^2*g*n^2*Log[c + d*
x])/(b^2*d^2) + (B^2*(b*c - a*d)*(2*b*d*f - b*c*g - a*d*g)*n^2*PolyLog[2, (
d*(a + b*x))/(b*(c + d*x))])/(b^2*d^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
```

```
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_)*((
f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(d + e*x)^(q +
 1)*((a + b*Log[c*x^n])^p/((q + 1)*(e*f - d*g))), x] - Dist[b*n*(p/((q + 1)
*(e*f - d*g))), Int[(f + g*x)^(m + 1)*(d + e*x)^(q + 1)*((a + b*Log[c*x^n])
^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, q}, x] && NeQ[e*f
- d*g, 0] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 2404

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2553

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[b*c - a*d, Subst[
Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)),
 x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x
] && NeQ[b*c - a*d, 0] && IntegerQ[m] && IGtQ[p, 0]
```

Rubi steps

$$\int (f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx = \frac{(f+gx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2g} - \frac{(Bn)\int \frac{(bc-ad)(f+gx)^2 \cdots}{(a \cdots}}{\cdots}$$

$$= \frac{(f+gx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2g} - \frac{(B(bc-ad)n)\int \frac{(f+\cdots}{\cdots}}{\cdots}$$

$$= \frac{(f+gx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2g} - \frac{(B(bc-ad)n)\int \left(\frac{g \cdots}{\cdots}\right.}{\cdots}$$

$$= \frac{(f+gx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2g} - \frac{(B(bc-ad)gn)\int (\cdots}{\cdots}$$

$$= -\frac{AB(bc-ad)gnx}{bd} - \frac{B(bf-ag)^2 n\log(a+bx)\left(A+B\log \cdots\right.}{b^2 g}$$

$$= -\frac{AB(bc-ad)gnx}{bd} - \frac{B^2(bc-ad)gn(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)\cdots\right.}{b^2 d}$$

$$= -\frac{AB(bc-ad)gnx}{bd} - \frac{B^2(bc-ad)gn(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)\cdots\right.}{b^2 d}$$

$$= -\frac{AB(bc-ad)gnx}{bd} - \frac{B^2(bc-ad)gn(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)\cdots\right.}{b^2 d}$$

$$= -\frac{AB(bc-ad)gnx}{bd} + \frac{B^2(bf-ag)^2 n^2\log^2(a+bx)}{2b^2 g} - \frac{B^2(\cdots}{\cdots}$$

$$= -\frac{AB(bc-ad)gnx}{bd} + \frac{B^2(bf-ag)^2 n^2\log^2(a+bx)}{2b^2 g} - \frac{B^2(\cdots}{\cdots}$$

<u>Mathematica</u> [A]
time = 0.22, size = 362, normalized size = 1.25

$$\frac{(f+gx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2-\frac{Bn\left(2Abd(bc-ad)g^2x+2Bd(bc-ad)g^2(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+2d^2(bf-ag)^2\log(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)-2B(bc-ad)^2g^2n\log(c+dx)-2b^2(df-cg)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log(c+dx)-Bd^2(bf-ag)^2n\left(\log(a+bx)\left(\log(a+bx)-2\log\left(\frac{b(c+dx)}{bc-ad}\right)\right)-2\text{Li}_2\left(\frac{d(a+bx)}{-bc+ad}\right)\right)+b^2B(df-cg)^2n\left(\left(2\log\left(\frac{d(a+bx)}{-bc+ad}\right)-\log(c+dx)\right)\log(c+dx)+2\text{Li}_2\left(\frac{b(c+dx)}{bc-ad}\right)\right)\right)}{b^2d^2}}{2g}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

```
[Out] ((f + g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 - (B*n*(2*A*b*d*(b*c
- a*d)*g^2*x + 2*B*d*(b*c - a*d)*g^2*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^
n] + 2*d^2*(b*f - a*g)^2*Log[a + b*x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n]
```

```
) - 2*B*(b*c - a*d)^2*g^2*n*Log[c + d*x] - 2*b^2*(d*f - c*g)^2*(A + B*Log[e
*((a + b*x)/(c + d*x))^n])*Log[c + d*x] - B*d^2*(b*f - a*g)^2*n*(Log[a + b*
x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a +
 b*x))/(-(b*c) + a*d)]) + b^2*B*(d*f - c*g)^2*n*((2*Log[(d*(a + b*x))/(-(b*
c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c -
 a*d)])))/(b^2*d^2))/(2*g)
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int (gx+f)\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)\right)^{2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

```
[Out] int((g*x+f)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 868 vs. $2(289) = 578$.
time = 0.75, size = 868, normalized size = 2.99

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="maxima")
```

```
[Out] A*B*g*x^2*log((b*x/(d*x + c) + a/(d*x + c))^n*e) + 1/2*A^2*g*x^2 - A*B*g*n*
(a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d)) + 2*A*B
*f*n*(a*log(b*x + a)/b - c*log(d*x + c)/d) + 2*A*B*f*x*log((b*x/(d*x + c) +
 a/(d*x + c))^n*e) + A^2*f*x - (a*c*d*g*n^2 - ((n^2 + n)*c^2*g - 2*c*d*f*n)
*b)*B^2*log(d*x + c)/(b*d^2) + (2*a*b*d^2*f*n^2 - a^2*d^2*g*n^2 - (2*c*d*f*
n^2 - c^2*g*n^2)*b^2)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + di
log(-(b*d*x + a*d)/(b*c - a*d)))*B^2/(b^2*d^2) + 1/2*(B^2*b^2*d^2*g*x^2 + 2
*(2*c*d*f*n^2 - c^2*g*n^2)*B^2*b^2*log(b*x + a)*log(d*x + c) - (2*c*d*f*n^2
 - c^2*g*n^2)*B^2*b^2*log(d*x + c)^2 - (2*a*b*d^2*f*n^2 - a^2*d^2*g*n^2)*B^
2*log(b*x + a)^2 + 2*(a*b*d^2*g*n - (c*d*g*n - d^2*f)*b^2)*B^2*x + 2*((n^2
- n)*a^2*d^2*g - (c*d*g*n^2 - 2*d^2*f*n)*a*b)*B^2*log(b*x + a) + (B^2*b^2*d
^2*g*x^2 + 2*B^2*b^2*d^2*f*x)*log((b*x + a)^n)^2 + (B^2*b^2*d^2*g*x^2 + 2*B
^2*b^2*d^2*f*x)*log((d*x + c)^n)^2 + 2*(B^2*b^2*d^2*g*x^2 - (2*c*d*f*n - c^
2*g*n)*B^2*b^2*log(d*x + c) + (a*b*d^2*g*n - (c*d*g*n - 2*d^2*f)*b^2)*B^2*x
 + (2*a*b*d^2*f*n - a^2*d^2*g*n)*B^2*log(b*x + a))*log((b*x + a)^n) - 2*(B^
2*b^2*d^2*g*x^2 - (2*c*d*f*n - c^2*g*n)*B^2*b^2*log(d*x + c) + (a*b*d^2*g*n
 - (c*d*g*n - 2*d^2*f)*b^2)*B^2*x + (2*a*b*d^2*f*n - a^2*d^2*g*n)*B^2*log(b
*x + a) + (B^2*b^2*d^2*g*x^2 + 2*B^2*b^2*d^2*f*x)*log((b*x + a)^n))*log((d*
x + c)^n))/(b^2*d^2)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="fricas")
[Out] integral(A^2*g*x + A^2*f + (B^2*g*x + B^2*f)*log(((b*x + a)/(d*x + c))^n*e)
^2 + 2*(A*B*g*x + A*B*f)*log(((b*x + a)/(d*x + c))^n*e), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(A + B\log\left(e\left(\frac{a}{c+dx} + \frac{bx}{c+dx}\right)^{n}\right)\right)^{2} (f+gx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
[Out] Integral((A + B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n))**2*(f + g*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="giac")
[Out] integrate((g*x + f)*(B*log(((b*x + a)/(d*x + c))^n*e) + A)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (f+g\,x)\,\left(A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^{n}\right)\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2,x)
[Out] int((f + g*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2, x)
```

3.70 $\int \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$

Optimal. Leaf size=135

$$\frac{(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{b}+\frac{2B(bc-ad)n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{bd}+\frac{2B^2(bc-ad)n^2\mathrm{Li}_2\left(\right.}{bd}$$

```
[Out] (b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/b+2*B*(-a*d+b*c)*n*(A+B*ln(e*((b*
x+a)/(d*x+c))^n))*ln((-a*d+b*c)/b/(d*x+c))/b/d+2*B^2*(-a*d+b*c)*n^2*polylog
(2,d*(b*x+a)/b/(d*x+c))/b/d
```

Rubi [A]

time = 0.17, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2535, 2543, 2458, 2378, 2370, 2352}

$$\frac{2B^2n^2(bc-ad)\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{bd}+\frac{2Bn(bc-ad)\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{bd}+\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

```
[Out] ((a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/b + (2*B*(b*c - a*d)*n
*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[(b*c - a*d)/(b*(c + d*x))])/(b*
d) + (2*B^2*(b*c - a*d)*n^2*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/(b*d)
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2370

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)/(x_))^(q_.)*(
x_)^(m_.), x_Symbol] :> Int[(e + d*x)^q*(a + b*Log[c*x^n])^p, x] /; FreeQ[{
a, b, c, d, e, m, n, p}, x] && EqQ[m, q] && IntegerQ[q]
```

Rule 2378

```
Int[((a_.) + Log[(c_.)*(x_)^(n_)]*(b_.))/((x_)*((d_) + (e_.)*(x_)^(r_.))),
x_Symbol] :> Dist[1/n, Subst[Int[(a + b*Log[c*x])/(x*(d + e*x^(r/n))), x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IntegerQ[r/n]
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
```

```
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2535

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.), x_Symbol] :> Simp[(a + b*x)*((A + B*Log[e*((a + b*x)/(c + d*x)
)^n])^p/b), x] - Dist[B*n*p*((b*c - a*d)/b), Int[(A + B*Log[e*((a + b*x)/(c
 + d*x))^n])^(p - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, A, B, n}, x
] && NeQ[b*c - a*d, 0] && IGtQ[p, 0]
```

Rule 2543

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))/((f_.) + (g_.)*(x_)), x_Symbol] :> Simp[(-Log[(b*c - a*d)/(b*(c + d*x
))])*((A + B*Log[e*((a + b*x)/(c + d*x))^n])/g), x] + Dist[B*n*((b*c - a*d)
/g), Int[Log[(b*c - a*d)/(b*(c + d*x))]/((a + b*x)*(c + d*x)), x], x] /; Fr
eeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[d*f - c*
g, 0]
```

Rubi steps

$$\int \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx = x\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 - (2Bn)\int \frac{(bc-ad)x(A+B\log(}{(a+bx)(c+d}$$

$$= x\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 - (2B(bc-ad)n)\int \frac{x(A+B\log(}{(a+bx)(}$$

$$= x\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 - (2B(bc-ad)n)\int \left(-\frac{a(A+B\,l}{(bc-}\right.$$

$$= x\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 + (2aBn)\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{a+bx}$$

$$= \frac{2aBn\log(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b} + x\left(A+B\log\left(e\left(\frac{a+b}{c+d}\right.\right.\right.$$

$$= \frac{2aBn\log(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b} + x\left(A+B\log\left(e\left(\frac{a+b}{c+d}\right.\right.\right.$$

$$= \frac{2aBn\log(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b} + x\left(A+B\log\left(e\left(\frac{a+b}{c+d}\right.\right.\right.$$

$$= \frac{2aBn\log(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b} + x\left(A+B\log\left(e\left(\frac{a+b}{c+d}\right.\right.\right.$$

$$= -\frac{aB^2n^2\log^2(a+bx)}{b} + \frac{2aBn\log(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b} +$$

$$= -\frac{aB^2n^2\log^2(a+bx)}{b} + \frac{2aBn\log(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b} +$$

Mathematica [A]
time = 0.12, size = 226, normalized size = 1.67

$$x\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 + \frac{Bn\left(2ad\log(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) - 2bc\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log(c+dx) - aBdn\left(\log(a+bx)\left(\log(a+bx)-2\log\left(\frac{b(c+dx)}{bc-ad}\right)\right) - 2\text{Li}_2\left(\frac{d(a+bx)}{-bc+ad}\right)\right) + bBcn\left(\left(2\log\left(\frac{d(a+bx)}{-bc+ad}\right)-\log(c+dx)\right)\log(c+dx) + 2\text{Li}_2\left(\frac{b(c+dx)}{bc-ad}\right)\right)\right)}{bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

```
[Out] x*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 + (B*n*(2*a*d*Log[a + b*x]*(A +
B*Log[e*((a + b*x)/(c + d*x))^n]) - 2*b*c*(A + B*Log[e*((a + b*x)/(c + d*x)
)^n])*Log[c + d*x] - a*B*d*n*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*
x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) + b*B*c*n*(
(2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyL
og[2, (b*(c + d*x))/(b*c - a*d)])))/(b*d)
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)\right)^{2}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
[Out] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="maxima")
[Out] 2*A*B*n*(a*log(b*x + a)/b - c*log(d*x + c)/d) + 2*A*B*x*log(((b*x + a)/(d*x
 + c))^n*e) + A^2*x + B^2*((2*b*c*n^2*log(b*x + a)*log(d*x + c) - b*c*n^2*l
og(d*x + c)^2 + b*d*x*log((b*x + a)^n)^2 + b*d*x*log((d*x + c)^n)^2 + 2*(a*
d*n*log(b*x + a) - b*c*n*log(d*x + c) + b*d*x)*log((b*x + a)^n) - 2*(a*d*n*
log(b*x + a) - b*c*n*log(d*x + c) + b*d*x*log((b*x + a)^n) + b*d*x)*log((d*
x + c)^n))/(b*d) - integrate(-(b^2*d*x^2 + a*b*c + (a*b*d*(2*n + 1) - b^2*c
*(2*n - 1))*x - 2*(b^2*c*n^2*x + 2*a*b*c*n^2 - a^2*d*n^2)*log(b*x + a))/(b^
2*d*x^2 + a*b*c + (b^2*c + a*b*d)*x), x))
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="fricas")
[Out] integral(B^2*log(((b*x + a)/(d*x + c))^n*e)^2 + 2*A*B*log(((b*x + a)/(d*x +
 c))^n*e) + A^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^{n}\right)\right)^{2}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
```

```
[Out] Integral((A + B*log(e*((a + b*x)/(c + d*x))**n))**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="giac")
```

```
[Out] integrate((B*log(((b*x + a)/(d*x + c))^n*e) + A)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\right)}^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2,x)
```

```
[Out] int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2, x)
```

3.71 $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{f+gx}\,dx$

Optimal. Leaf size=297

$$-\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log\left(\frac{bc-ad}{b(c+dx)}\right)}{g}+\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log\left(1-\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{g}-\frac{2Bn(A+B\log$$

```
[Out] -(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2*ln((-a*d+b*c)/b/(d*x+c))/g+(A+B*ln(e*((b
*x+a)/(d*x+c))^n))^2*ln(1-(-c*g+d*f)*(b*x+a)/(-a*g+b*f)/(d*x+c))/g-2*B*n*(A
+B*ln(e*((b*x+a)/(d*x+c))^n))*polylog(2,d*(b*x+a)/b/(d*x+c))/g+2*B*n*(A+B*l
n(e*((b*x+a)/(d*x+c))^n))*polylog(2,(-c*g+d*f)*(b*x+a)/(-a*g+b*f)/(d*x+c))/
g+2*B^2*n^2*polylog(3,d*(b*x+a)/b/(d*x+c))/g-2*B^2*n^2*polylog(3,(-c*g+d*f)
*(b*x+a)/(-a*g+b*f)/(d*x+c))/g
```

Rubi [A]

time = 0.36, antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {2553, 2404, 2354, 2421, 6724}

$$\frac{2Bn\text{PolyLog}\left(2,\frac{(a+bx)(df-cg)}{(c+dx)(bf-ag)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{g}-\frac{2Bn\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{g}-\frac{2B^2n^2\text{PolyLog}\left(3,\frac{(a+bx)(df-cg)}{(c+dx)(bf-ag)}\right)}{g}+\frac{2B^2n^2\text{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)}{g}+\frac{\log\left(1-\frac{(a+bx)(df-cg)}{(c+dx)(bf-ag)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{g}-\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{g}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(f + g*x),x]
```

```
[Out] -(((A + B*Log[e*((a + b*x)/(c + d*x))^n])^2*Log[(b*c - a*d)/(b*(c + d*x))])
/g) + ((A + B*Log[e*((a + b*x)/(c + d*x))^n])^2*Log[1 - ((d*f - c*g)*(a + b
*x))/((b*f - a*g)*(c + d*x))])/g - (2*B*n*(A + B*Log[e*((a + b*x)/(c + d*x)
)^n])*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/g + (2*B*n*(A + B*Log[e*((a
+ b*x)/(c + d*x))^n])*PolyLog[2, ((d*f - c*g)*(a + b*x))/((b*f - a*g)*(c +
d*x))])/g + (2*B^2*n^2*PolyLog[3, (d*(a + b*x))/(b*(c + d*x))])/g - (2*B^2*
n^2*PolyLog[3, ((d*f - c*g)*(a + b*x))/((b*f - a*g)*(c + d*x))])/g
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2404

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2553

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[b*c - a*d, Subst[
Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)),
 x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x
] && NeQ[b*c - a*d, 0] && IntegerQ[m] && IGtQ[p, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{f+gx}\,dx = \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log(f+gx)}{g} - \frac{(2Bn)\int \frac{(c+dx)\left(-\frac{d(a+bx)}{(c+dx)^2}+\frac{b}{c+dx}\right)\left(A+\ldots\right.}{a+b\ldots}}{g}$$

$$= \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log(f+gx)}{g} - \frac{(2Bn)\int \frac{(bc-ad)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\ldots}{(a+bx)(c+dx)}}{g}$$

$$= \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log(f+gx)}{g} - \frac{(2B(bc-ad)n)\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right.\right.\right.\ldots}{(a+bx)(c\ldots}}{g}$$

$$= \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log(f+gx)}{g} - \frac{(2B(bc-ad)n)\int \left(\frac{b\left(A+B\log\left(e\left(\frac{a+\ldots}{c+\ldots}\right.\right.\right.}{(bc-a\ldots}\right.\ldots}{\ldots}$$

$$= \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log(f+gx)}{g} - \frac{(2bBn)\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log(f\ldots}{a+bx}}{g}$$

$$= \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log(f+gx)}{g} - \frac{(2bBn)\int \left(\frac{A\log(f+gx)}{a+bx}+\frac{B\log\left(e\left(\frac{a+\ldots}{c+\ldots}\right.\right.}{\ldots}\right.\ldots}{g}$$

$$= \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log(f+gx)}{g} - \frac{(2AbBn)\int \frac{\log(f+gx)}{a+bx}\,dx}{g} - \frac{(2bB^2n\ldots}{\ldots}$$

$$= -\frac{2ABn\log\left(-\frac{g(a+bx)}{bf-ag}\right)\log(f+gx)}{g} + \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log(f+g\ldots}{g}$$

$$= -\frac{2ABn\log\left(-\frac{g(a+bx)}{bf-ag}\right)\log(f+gx)}{g} + \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log(f+g\ldots}{g}$$

$$= -\frac{2ABn\log\left(-\frac{g(a+bx)}{bf-ag}\right)\log(f+gx)}{g} - \frac{B^2\log^2\left((a+bx)^n\right)\log(f+gx)}{g} + \ldots$$

$$= -\frac{2ABn\log\left(-\frac{g(a+bx)}{bf-ag}\right)\log(f+gx)}{g} - \frac{B^2\log^2\left((a+bx)^n\right)\log(f+gx)}{g} + \ldots$$

$$= -\frac{2ABn\log\left(-\frac{g(a+bx)}{bf-ag}\right)\log(f+gx)}{g} - \frac{B^2\log^2\left((a+bx)^n\right)\log(f+gx)}{g} + \ldots$$

$$= -\frac{2ABn\log\left(-\frac{g(a+bx)}{bf-ag}\right)\log(f+gx)}{g} - \frac{B^2\log^2\left((a+bx)^n\right)\log(f+gx)}{g} + \ldots$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 1441 vs. $2(297) = 594$.
time = 0.26, size = 1441, normalized size = 4.85

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(f + g*x),x]
```

```
[Out] (-(B^2*n^2*Log[(-(b*c) + a*d)/(d*(a + b*x))]*Log[((b*f - a*g)*(c + d*x))/((
d*f - c*g)*(a + b*x))]^2) + A^2*Log[f + g*x] - 2*A*B*n*Log[a/b + x]*Log[f +
 g*x] + B^2*n^2*Log[a/b + x]^2*Log[f + g*x] + 2*A*B*n*Log[c/d + x]*Log[f +
g*x] - 2*B^2*n^2*Log[a/b + x]*Log[c/d + x]*Log[f + g*x] + B^2*n^2*Log[c/d +
 x]^2*Log[f + g*x] + 2*A*B*Log[e*((a + b*x)/(c + d*x))^n]*Log[f + g*x] - 2*
B^2*n*Log[a/b + x]*Log[e*((a + b*x)/(c + d*x))^n]*Log[f + g*x] + 2*B^2*n*Lo
g[c/d + x]*Log[e*((a + b*x)/(c + d*x))^n]*Log[f + g*x] + B^2*Log[e*((a + b*
x)/(c + d*x))^n]^2*Log[f + g*x] + 2*A*B*n*Log[a/b + x]*Log[(b*(f + g*x))/(b
*f - a*g)] - B^2*n^2*Log[a/b + x]^2*Log[(b*(f + g*x))/(b*f - a*g)] + 2*B^2*
n*Log[a/b + x]*Log[e*((a + b*x)/(c + d*x))^n]*Log[(b*(f + g*x))/(b*f - a*g)
] + 2*B^2*n^2*Log[a/b + x]*Log[(g*(c + d*x))/(-(d*f) + c*g)]*Log[(b*(f + g*
x))/(b*f - a*g)] - B^2*n^2*Log[(g*(c + d*x))/(-(d*f) + c*g)]^2*Log[(b*(f +
g*x))/(b*f - a*g)] + 2*B^2*n^2*Log[(g*(c + d*x))/(-(d*f) + c*g)]*Log[((b*f
- a*g)*(c + d*x))/((d*f - c*g)*(a + b*x))]*Log[(b*(f + g*x))/(b*f - a*g)] -
 B^2*n^2*Log[((b*f - a*g)*(c + d*x))/((d*f - c*g)*(a + b*x))]^2*Log[(b*(f +
 g*x))/(b*f - a*g)] - 2*A*B*n*Log[c/d + x]*Log[(d*(f + g*x))/(d*f - c*g)] +
 2*B^2*n^2*Log[a/b + x]*Log[c/d + x]*Log[(d*(f + g*x))/(d*f - c*g)] - B^2*n
^2*Log[c/d + x]^2*Log[(d*(f + g*x))/(d*f - c*g)] - 2*B^2*n*Log[c/d + x]*Log
[e*((a + b*x)/(c + d*x))^n]*Log[(d*(f + g*x))/(d*f - c*g)] - 2*B^2*n^2*Log[
a/b + x]*Log[(g*(c + d*x))/(-(d*f) + c*g)]*Log[(d*(f + g*x))/(d*f - c*g)] +
 B^2*n^2*Log[(g*(c + d*x))/(-(d*f) + c*g)]^2*Log[(d*(f + g*x))/(d*f - c*g)]
 - 2*B^2*n^2*Log[(g*(c + d*x))/(-(d*f) + c*g)]*Log[((b*f - a*g)*(c + d*x))/
((d*f - c*g)*(a + b*x))]*Log[(d*(f + g*x))/(d*f - c*g)] + B^2*n^2*Log[((b*f
 - a*g)*(c + d*x))/((d*f - c*g)*(a + b*x))]^2*Log[((-(b*c) + a*d)*(f + g*x)
)/((d*f - c*g)*(a + b*x))] + 2*B*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n] +
B*n*Log[((b*f - a*g)*(c + d*x))/((d*f - c*g)*(a + b*x))])*PolyLog[2, (g*(a
+ b*x))/(-(b*f) + a*g)] - 2*B*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n] + B*n
*Log[((b*f - a*g)*(c + d*x))/((d*f - c*g)*(a + b*x))])*PolyLog[2, (g*(c + d
*x))/(-(d*f) + c*g)] - 2*B^2*n^2*Log[((b*f - a*g)*(c + d*x))/((d*f - c*g)*(
a + b*x))]*PolyLog[2, (b*(c + d*x))/(d*(a + b*x))] + 2*B^2*n^2*Log[((b*f -
a*g)*(c + d*x))/((d*f - c*g)*(a + b*x))]*PolyLog[2, ((b*f - a*g)*(c + d*x))
/((d*f - c*g)*(a + b*x))] + 2*B^2*n^2*PolyLog[3, (b*(c + d*x))/(d*(a + b*x)
)] - 2*B^2*n^2*PolyLog[3, ((b*f - a*g)*(c + d*x))/((d*f - c*g)*(a + b*x))])
/g
```

Maple [F]

time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)\right)^{2}}{gx+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(g*x+f),x)
```

```
[Out] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(g*x+f),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(g*x+f),x, algorithm="maxima")
```

```
[Out] A^2*log(g*x + f)/g + integrate((B^2*log((b*x + a)^n)^2 + B^2*log((d*x + c)^
n)^2 + 2*A*B + B^2 + 2*(A*B + B^2)*log((b*x + a)^n) - 2*(B^2*log((b*x + a)^
n) + A*B + B^2)*log((d*x + c)^n))/(g*x + f), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(g*x+f),x, algorithm="fricas")
```

```
[Out] integral((B^2*log(((b*x + a)/(d*x + c))^n*e)^2 + 2*A*B*log(((b*x + a)/(d*x
+ c))^n*e) + A^2)/(g*x + f), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(A+B\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^{n}\right)\right)^{2}}{f+gx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(g*x+f),x)
```

```
[Out] Integral((A + B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n))**2/(f + g*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(g*x+f),x, algorithm="giac")
[Out] integrate((B*log(((b*x + a)/(d*x + c))^n*e) + A)^2/(g*x + f), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\right)^2}{f+g\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/(f + g*x),x)
[Out] int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/(f + g*x), x)
```

3.72 $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(f+gx)^2}\,dx$

Optimal. Leaf size=206

$$\frac{(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bf-ag)(f+gx)}+\frac{2B(bc-ad)n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(1-\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{(bf-ag)(df-cg)}+\frac{2B^2(bc-\cdots}{(b\cdots}$$

```
[Out] (b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*g+b*f)/(g*x+f)+2*B*(-a*d+b*c)
*n*(A+B*ln(e*((b*x+a)/(d*x+c))^n))*ln(1-(-c*g+d*f)*(b*x+a)/(-a*g+b*f)/(d*x+
c))/(-a*g+b*f)/(-c*g+d*f)+2*B^2*(-a*d+b*c)*n^2*polylog(2,(-c*g+d*f)*(b*x+a)
/(-a*g+b*f)/(d*x+c))/(-a*g+b*f)/(-c*g+d*f)
```

Rubi [A]
time = 0.12, antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2553, 2355, 2354, 2438}

$$\frac{2B^2n^2(bc-ad)\text{PolyLog}\left(2,\frac{(a+bx)(df-cg)}{(c+dx)(bf-ag)}\right)}{(bf-ag)(df-cg)}+\frac{2Bn(bc-ad)\log\left(1-\frac{(a+bx)(df-cg)}{(c+dx)(bf-ag)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{(bf-ag)(df-cg)}+\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{(f+gx)(bf-ag)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(f + g*x)^2,x]
```

```
[Out] ((a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/((b*f - a*g)*(f + g*x)
) + (2*B*(b*c - a*d)*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[1 - ((d*f
 - c*g)*(a + b*x))/((b*f - a*g)*(c + d*x))])/((b*f - a*g)*(d*f - c*g)) + (2
*B^2*(b*c - a*d)*n^2*PolyLog[2, ((d*f - c*g)*(a + b*x))/((b*f - a*g)*(c + d
*x))])/((b*f - a*g)*(d*f - c*g))
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2355

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[x*((a + b*Log[c*x^n])^p/(d*(d + e*x))), x] - Dist[b*n*(p/d),
Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n
, p}, x] && GtQ[p, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2553

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[b*c - a*d, Subst[
Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)),
 x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x
] && NeQ[b*c - a*d, 0] && IntegerQ[m] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(f+gx)^2}\,dx = -\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{g(f+gx)}+\frac{(2Bn)\int\frac{(bc-ad)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)(c+dx)(f+gx)}\,dx}{g}$$

$$= -\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{g(f+gx)}+\frac{(2B(bc-ad)n)\int\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)(c+dx)(f+gx)}\,dx}{g}$$

$$= -\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{g(f+gx)}+\frac{(2B(bc-ad)n)\int\left(\frac{b^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)(bf-ag)(a+bx)}+\frac{d^2}{(}\right.}{}$$

$$= -\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{g(f+gx)}+\frac{(2b^2Bn)\int\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{a+bx}\,dx}{g(bf-ag)}-\frac{(2Bd^2n)\int}{g}$$

$$= \frac{2bBn\log(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{g(bf-ag)}-\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{g(f+gx)}-\frac{2A}{}$$

$$= \frac{2bBn\log(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{g(bf-ag)}-\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{g(f+gx)}-\frac{2A}{}$$

$$= \frac{2bBn\log(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{g(bf-ag)}-\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{g(f+gx)}-\frac{2A}{}$$

$$= \frac{2bBn\log(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{g(bf-ag)}-\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{g(f+gx)}+\frac{2A}{}$$

$$= -\frac{bB^2n^2\log^2(a+bx)}{g(bf-ag)}+\frac{2bBn\log(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{g(bf-ag)}-\frac{(A+}{}$$

$$= -\frac{bB^2n^2\log^2(a+bx)}{g(bf-ag)}+\frac{2bBn\log(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{g(bf-ag)}-\frac{(A+}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 418 vs. 2(206) = 412.
time = 0.34, size = 418, normalized size = 2.03

$$\frac{-\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{f+gx}+\frac{Bn\left(2b(df-cg)\log(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)-2d(bf-ag)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log(c+dx)+2(bc-ad)g\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log(f+gx)-bB(df-cg)n\left(\log(a+bx)\left(\log(a+bx)-2\log\left(\frac{b(c+dx)}{bc-ad}\right)\right)-2\text{Li}_2\left(\frac{d(a+bx)}{-bc+ad}\right)\right)+Bd(bf-ag)n\left(\left(2\log\left(\frac{d(a+bx)}{-bc+ad}\right)-\log(c+dx)\right)\log(c+dx)+2\text{Li}_2\left(\frac{b(c+dx)}{bc-ad}\right)\right)-2B(bc-ad)gn\left(\left(\log\left(\frac{g(a+bx)}{-bf+ag}\right)-\log\left(\frac{g(c+dx)}{-df+cg}\right)\right)\log(f+gx)+\text{Li}_2\left(\frac{b(f+gx)}{bf-ag}\right)-\text{Li}_2\left(\frac{d(f+gx)}{df-cg}\right)\right)\right)}{(bf-ag)(df-cg)}}{g}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(f + g*x)^2,x]
```

```
[Out] (-((A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(f + g*x)) + (B*n*(2*b*(d*f - c
*g)*Log[a + b*x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - 2*d*(b*f - a*g)*(
A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[c + d*x] + 2*(b*c - a*d)*g*(A + B
*Log[e*((a + b*x)/(c + d*x))^n])*Log[f + g*x] - b*B*(d*f - c*g)*n*(Log[a +
b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a
 + b*x))/(-(b*c) + a*d)]) + B*d*(b*f - a*g)*n*((2*Log[(d*(a + b*x))/(-(b*c)
 + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a
*d)]) - 2*B*(b*c - a*d)*g*n*((Log[(g*(a + b*x))/(-(b*f) + a*g)] - Log[(g*(c
 + d*x))/(-(d*f) + c*g)])*Log[f + g*x] + PolyLog[2, (b*(f + g*x))/(b*f - a*
g)] - PolyLog[2, (d*(f + g*x))/(d*f - c*g)])))/((b*f - a*g)*(d*f - c*g)))/g
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2}{(gx+f)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(g*x+f)^2,x)
```

```
[Out] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(g*x+f)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(g*x+f)^2,x, algorithm="maxima
")
```

```
[Out] 2*A*B*n*(b*log(b*x + a)/(b*f*g - a*g^2) - d*log(d*x + c)/(d*f*g - c*g^2) +
(b*c - a*d)*log(g*x + f)/(b*d*f^2 + a*c*g^2 - (b*c + a*d)*f*g)) - B^2*(log(
(d*x + c)^n)^2/(g^2*x + f*g) + integrate(-(d*g*x + (d*g*x + c*g)*log((b*x +
 a)^n)^2 + c*g + 2*(d*g*x + c*g)*log((b*x + a)^n) + 2*(d*g*(n - 1)*x + d*f*
n - c*g - (d*g*x + c*g)*log((b*x + a)^n))*log((d*x + c)^n))/(d*g^3*x^3 + c*
```

```
f^2*g + (2*d*f*g^2 + c*g^3)*x^2 + (d*f^2*g + 2*c*f*g^2)*x), x)) - 2*A*B*log
((b*x/(d*x + c) + a/(d*x + c))^n*e)/(g^2*x + f*g) - A^2/(g^2*x + f*g)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(g*x+f)^2,x, algorithm="fricas
")
```

```
[Out] integral((B^2*log(((b*x + a)/(d*x + c))^n*e)^2 + 2*A*B*log(((b*x + a)/(d*x
+ c))^n*e) + A^2)/(g^2*x^2 + 2*f*g*x + f^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(A + B \log \left(e\left(\frac{a}{c+dx} + \frac{bx}{c+dx}\right)^{n}\right)\right)^{2}}{(f + gx)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(g*x+f)**2,x)
```

```
[Out] Integral((A + B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n))**2/(f + g*x)**2, x
)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(g*x+f)^2,x, algorithm="giac")
```

```
[Out] integrate((B*log(((b*x + a)/(d*x + c))^n*e) + A)^2/(g*x + f)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\right)}^2}{{\left(f+g\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/(f + g*x)^2,x)
```

```
[Out] int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/(f + g*x)^2, x)
```

3.73 $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(f+gx)^3}\,dx$

Optimal. Leaf size=389

$$\frac{B(bc-ad)gn(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bf-ag)^2(df-cg)(f+gx)}+\frac{b^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2g(bf-ag)^2}-\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2g(f+gx)^2}+B^2$$

```
[Out] B*(-a*d+b*c)*g*n*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*g+b*f)^2/(-c*g
+d*f)/(g*x+f)+1/2*b^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/g/(-a*g+b*f)^2-1/2*
(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/g/(g*x+f)^2+B^2*(-a*d+b*c)^2*g*n^2*ln((g*
x+f)/(d*x+c))/(-a*g+b*f)^2/(-c*g+d*f)^2+B*(-a*d+b*c)*(-a*d*g-b*c*g+2*b*d*f)
*n*(A+B*ln(e*((b*x+a)/(d*x+c))^n))*ln(1-(-c*g+d*f)*(b*x+a)/(-a*g+b*f)/(d*x+
c))/(-a*g+b*f)^2/(-c*g+d*f)^2+B^2*(-a*d+b*c)*(-a*d*g-b*c*g+2*b*d*f)*n^2*pol
ylog(2,(-c*g+d*f)*(b*x+a)/(-a*g+b*f)/(d*x+c))/(-a*g+b*f)^2/(-c*g+d*f)^2
```

Rubi [A]
time = 0.55, antiderivative size = 389, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2553, 2398, 2404, 2338, 2351, 31, 2354, 2438}

$$\frac{B^2n^2(bc-ad)(-adg-bcg+2bdf)\text{PolyLog}\left(2,\frac{(a+bx)(df-cg)}{(c+dx)(bf-ag)}\right)}{(bf-ag)^2(df-cg)^2}+\frac{b^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2g(bf-ag)^2}+\frac{Bgn(a+bx)(bc-ad)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{(f+gx)(bf-ag)^2(df-cg)}+\frac{Bn(bc-ad)(-adg-bcg+2bdf)\log\left(1-\frac{(a+bx)(df-cg)}{(c+dx)(bf-ag)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{(bf-ag)^2(df-cg)^2}-\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2g(f+gx)^2}+\frac{B^2gn^2(bc-ad)^2\log\left(\frac{f+gx}{c+dx}\right)}{(bf-ag)^2(df-cg)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(f + g*x)^3,x]
```

```
[Out] (B*(b*c - a*d)*g*n*(a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/((b*f
 - a*g)^2*(d*f - c*g)*(f + g*x)) + (b^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n
])^2)/(2*g*(b*f - a*g)^2) - (A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(2*g*(
f + g*x)^2) + (B^2*(b*c - a*d)^2*g*n^2*Log[(f + g*x)/(c + d*x)])/((b*f - a*
g)^2*(d*f - c*g)^2) + (B*(b*c - a*d)*(2*b*d*f - b*c*g - a*d*g)*n*(A + B*Log
[e*((a + b*x)/(c + d*x))^n])*Log[1 - ((d*f - c*g)*(a + b*x))/((b*f - a*g)*(
c + d*x))])/((b*f - a*g)^2*(d*f - c*g)^2) + (B^2*(b*c - a*d)*(2*b*d*f - b*c
*g - a*d*g)*n^2*PolyLog[2, ((d*f - c*g)*(a + b*x))/((b*f - a*g)*(c + d*x))]
)/((b*f - a*g)^2*(d*f - c*g)^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_)*((
f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(d + e*x)^(q +
 1)*((a + b*Log[c*x^n])^p/((q + 1)*(e*f - d*g))), x] - Dist[b*n*(p/((q + 1)
*(e*f - d*g))), Int[(f + g*x)^(m + 1)*(d + e*x)^(q + 1)*((a + b*Log[c*x^n])
^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, q}, x] && NeQ[e*f
- d*g, 0] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 2404

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2553

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[b*c - a*d, Subst[
Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)),
 x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x
] && NeQ[b*c - a*d, 0] && IntegerQ[m] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(f+gx)^3}\,dx = -\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2g(f+gx)^2} + \frac{(Bn)\int \frac{(bc-ad)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)(c+dx)(f+gx)^2}\,dx}{g}$$

$$= -\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2g(f+gx)^2} + \frac{(B(bc-ad)n)\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)(c+dx)(f+gx)^2}\,dx}{g}$$

$$= -\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2g(f+gx)^2} + \frac{(B(bc-ad)n)\int \left(\frac{b^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)(bf-ag)^2(a+bx)} - \cdots\right.}{\cdots}$$

$$= -\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2g(f+gx)^2} + \frac{(b^3Bn)\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{a+bx}\,dx}{g(bf-ag)^2} - \frac{(Bd^3n)\int \cdots}{g\cdots}$$

$$= -\frac{B(bc-ad)n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bf-ag)(df-cg)(f+gx)} + \frac{b^2Bn\log(a+bx)\left(A+B\log\left(e\cdots\right.\right.}{g(bf-ag)^2}$$

$$= -\frac{B(bc-ad)n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bf-ag)(df-cg)(f+gx)} + \frac{b^2Bn\log(a+bx)\left(A+B\log\left(e\cdots\right.\right.}{g(bf-ag)^2}$$

$$= -\frac{B(bc-ad)n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bf-ag)(df-cg)(f+gx)} + \frac{b^2Bn\log(a+bx)\left(A+B\log\left(e\cdots\right.\right.}{g(bf-ag)^2}$$

$$= \frac{bB^2(bc-ad)n^2\log(a+bx)}{(bf-ag)^2(df-cg)} - \frac{B(bc-ad)n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bf-ag)(df-cg)(f+gx)} + \frac{b^2\cdots}{\cdots}$$

$$= \frac{bB^2(bc-ad)n^2\log(a+bx)}{(bf-ag)^2(df-cg)} - \frac{b^2B^2n^2\log^2(a+bx)}{2g(bf-ag)^2} - \frac{B(bc-ad)n\left(A+\cdots\right.}{(bf-ag)(df-\cdots}$$

$$= \frac{bB^2(bc-ad)n^2\log(a+bx)}{(bf-ag)^2(df-cg)} - \frac{b^2B^2n^2\log^2(a+bx)}{2g(bf-ag)^2} - \frac{B(bc-ad)n\left(A+\cdots\right.}{(bf-ag)(df-\cdots}$$

Mathematica [A]
time = 1.04, size = 615, normalized size = 1.58

$$-\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2+\frac{Bn(f+gx)\left(2(bc-ad)g(bf-ag)(df-cg)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)-2b^2(df-cg)^2(f+gx)\log(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)+2d^2(bf-ag)^2(f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log(c+dx)+2(bc-ad)g(-2bdf+bcg+adg)(f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log(f+gx)-2B(bc-ad)gn(f+gx)(b(df-cg)\log(a+bx)+(-bdf+adg)\log(c+dx)+(bc-ad)g\log(f+gx))+b^2B(df-cg)^2n(f+gx)\left(\log(a+bx)\left(\log(a+bx)-2\log\left(\frac{b(c+dx)}{bc-ad}\right)\right)-2\operatorname{Li}_2\left(\frac{d(a+bx)}{-bc+ad}\right)\right)-Bd^2(bf-ag)^2n(f+gx)\left(\left(2\log\left(\frac{d(a+bx)}{-bc+ad}\right)-\log(c+dx)\right)\log(c+dx)+2\operatorname{Li}_2\left(\frac{b(c+dx)}{bc-ad}\right)\right)-2B(bc-ad)g(-2bdf+bcg+adg)n(f+gx)\left(\left(\log\left(\frac{g(a+bx)}{-bf+ag}\right)-\log\left(\frac{g(c+dx)}{-df+cg}\right)\right)\log(f+gx)+\operatorname{Li}_2\left(\frac{b(f+gx)}{bf-ag}\right)-\operatorname{Li}_2\left(\frac{d(f+gx)}{df-cg}\right)\right)\right)}{(bf-ag)^2(df-cg)^2}}{2g(f+gx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(f + g*x)^3,x]
```

```
[Out] -1/2*((A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 + (B*n*(f + g*x)*(2*(b*c - a
*d)*g*(b*f - a*g)*(d*f - c*g)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - 2*b^
```

```
2*(d*f - c*g)^2*(f + g*x)*Log[a + b*x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n
]) + 2*d^2*(b*f - a*g)^2*(f + g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*L
og[c + d*x] + 2*(b*c - a*d)*g*(-2*b*d*f + b*c*g + a*d*g)*(f + g*x)*(A + B*L
og[e*((a + b*x)/(c + d*x))^n])*Log[f + g*x] - 2*B*(b*c - a*d)*g*n*(f + g*x)
*(b*(d*f - c*g)*Log[a + b*x] + (-(b*d*f) + a*d*g)*Log[c + d*x] + (b*c - a*d
)*g*Log[f + g*x]) + b^2*B*(d*f - c*g)^2*n*(f + g*x)*(Log[a + b*x]*(Log[a +
b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*
c) + a*d)]) - B*d^2*(b*f - a*g)^2*n*(f + g*x)*((2*Log[(d*(a + b*x))/(-(b*c)
 + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a
*d)]) - 2*B*(b*c - a*d)*g*(-2*b*d*f + b*c*g + a*d*g)*n*(f + g*x)*((Log[(g*(
a + b*x))/(-(b*f) + a*g)] - Log[(g*(c + d*x))/(-(d*f) + c*g)])*Log[f + g*x]
 + PolyLog[2, (b*(f + g*x))/(b*f - a*g)] - PolyLog[2, (d*(f + g*x))/(d*f -
c*g)])))/((b*f - a*g)^2*(d*f - c*g)^2))/(g*(f + g*x)^2)
```

Maple [F]

time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)\right)^{2}}{\left(gx+f\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(g*x+f)^3,x)
```

```
[Out] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(g*x+f)^3,x)
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(g*x+f)^3,x, algorithm="maxima
")
```

```
[Out] (b^2*log(b*x + a)/(b^2*f^2*g - 2*a*b*f*g^2 + a^2*g^3) - d^2*log(d*x + c)/(d
^2*f^2*g - 2*c*d*f*g^2 + c^2*g^3) + (2*(b^2*c*d - a*b*d^2)*f - (b^2*c^2 - a
^2*d^2)*g)*log(g*x + f)/(b^2*d^2*f^4 + a^2*c^2*g^4 - 2*(b^2*c*d + a*b*d^2)*
f^3*g + (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*f^2*g^2 - 2*(a*b*c^2 + a^2*c*d)*f*g
^3) - (b*c - a*d)/(b*d*f^3 + a*c*f*g^2 - (b*c + a*d)*f^2*g + (b*d*f^2*g + a
*c*g^3 - (b*c + a*d)*f*g^2)*x))*A*B*n - 1/2*B^2*(log((d*x + c)^n)^2/(g^3*x^
2 + 2*f*g^2*x + f^2*g) + 2*integrate(-(d*g*x + (d*g*x + c*g)*log((b*x + a)^
n)^2 + c*g + 2*(d*g*x + c*g)*log((b*x + a)^n) + (d*g*(n - 2)*x + d*f*n - 2*
c*g - 2*(d*g*x + c*g)*log((b*x + a)^n))*log((d*x + c)^n))/(d*g^4*x^4 + c*f^
3*g + (3*d*f*g^3 + c*g^4)*x^3 + 3*(d*f^2*g^2 + c*f*g^3)*x^2 + (d*f^3*g + 3*
c*f^2*g^2)*x), x)) - A*B*log((b*x/(d*x + c) + a/(d*x + c))^n*e)/(g^3*x^2 +
2*f*g^2*x + f^2*g) - 1/2*A^2/(g^3*x^2 + 2*f*g^2*x + f^2*g)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(g*x+f)^3,x, algorithm="fricas
")
```

```
[Out] integral((B^2*log(((b*x + a)/(d*x + c))^n*e)^2 + 2*A*B*log(((b*x + a)/(d*x
+ c))^n*e) + A^2)/(g^3*x^3 + 3*f*g^2*x^2 + 3*f^2*g*x + f^3), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(g*x+f)**3,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(g*x+f)^3,x, algorithm="giac")
```

```
[Out] integrate((B*log(((b*x + a)/(d*x + c))^n*e) + A)^2/(g*x + f)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\right)^2}{{\left(f+g\,x\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/(f + g*x)^3,x)
```

```
[Out] int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/(f + g*x)^3, x)
```

3.74 $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(f+gx)^4}\,dx$

Optimal. Leaf size=747

$$\frac{B^2(bc-ad)^2g^2n^2(c+dx)}{3(bf-ag)^2(df-cg)^3(f+gx)}-\frac{B(bc-ad)g^2n(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3(bf-ag)(df-cg)^3(f+gx)^2}+\frac{2B(bc-ad)g(3bdf-bcg}{3(bf-}$$

```
[Out] 1/3*B^2*(-a*d+b*c)^2*g^2*n^2*(d*x+c)/(-a*g+b*f)^2/(-c*g+d*f)^3/(g*x+f)-1/3*
B*(-a*d+b*c)*g^2*n*(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*g+b*f)/(-c
*g+d*f)^3/(g*x+f)^2+2/3*B*(-a*d+b*c)*g*(-2*a*d*g-b*c*g+3*b*d*f)*n*(b*x+a)*(
A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*g+b*f)^3/(-c*g+d*f)^2/(g*x+f)+1/3*b^3*(A
+B*ln(e*((b*x+a)/(d*x+c))^n))^2/g/(-a*g+b*f)^3-1/3*(A+B*ln(e*((b*x+a)/(d*x+
c))^n))^2/g/(g*x+f)^3+1/3*B^2*(-a*d+b*c)^3*g^2*n^2*ln((b*x+a)/(d*x+c))/(-a*
g+b*f)^3/(-c*g+d*f)^3-1/3*B^2*(-a*d+b*c)^3*g^2*n^2*ln((g*x+f)/(d*x+c))/(-a*
g+b*f)^3/(-c*g+d*f)^3+2/3*B^2*(-a*d+b*c)^2*g*(-2*a*d*g-b*c*g+3*b*d*f)*n^2*l
n((g*x+f)/(d*x+c))/(-a*g+b*f)^3/(-c*g+d*f)^3+2/3*B*(-a*d+b*c)*(a^2*d^2*g^2-
a*b*d*g*(-c*g+3*d*f)+b^2*(c^2*g^2-3*c*d*f*g+3*d^2*f^2))*n*(A+B*ln(e*((b*x+a
)/(d*x+c))^n))*ln(1-(-c*g+d*f)*(b*x+a)/(-a*g+b*f)/(d*x+c))/(-a*g+b*f)^3/(-c
*g+d*f)^3+2/3*B^2*(-a*d+b*c)*(a^2*d^2*g^2-a*b*d*g*(-c*g+3*d*f)+b^2*(c^2*g^2
-3*c*d*f*g+3*d^2*f^2))*n^2*polylog(2,(-c*g+d*f)*(b*x+a)/(-a*g+b*f)/(d*x+c))
/(-a*g+b*f)^3/(-c*g+d*f)^3
```

Rubi [A]
time = 1.14, antiderivative size = 747, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {2553, 2398, 2404, 2338, 2356, 46, 2351, 31, 2354, 2438}

$$\frac{2B^2n^2(bc-ad)\left(a^2d^2g^2-abdg(3df-cg)+b^2(c^2g^2-3cdfg+3d^2f^2)\right)\text{PolyLog}\left(2,\frac{(a+bx)(df-cg)}{(c+dx)(bf-ag)}\right)}{3(bf-ag)^3(df-cg)^3}+\frac{2Bn(bc-ad)\left(a^2d^2g^2-abdg(3df-cg)+b^2(c^2g^2-3cdfg+3d^2f^2)\right)\log\left(1-\frac{(a+bx)(df-cg)}{(c+dx)(bf-ag)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3(bf-ag)^3(df-cg)^3}+\frac{b^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3g(bf-ag)^3}-\frac{Bg^2n(c+dx)^2(bc-ad)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3(f+gx)^2(bf-ag)(df-cg)^3}-\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3g(f+gx)^3}+\frac{2Bgn(a+bx)(bc-ad)(-2adg-bcg+3bdf)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3(f+gx)(bf-ag)^3(df-cg)^2}+\frac{B^2g^2n^2(c+dx)(bc-ad)^2}{3(f+gx)(bf-ag)^2(df-cg)^3}+\frac{B^2g^2n^2(bc-ad)^3\log\left(\frac{a+bx}{c+dx}\right)}{3(bf-ag)^3(df-cg)^3}-\frac{B^2g^2n^2(bc-ad)^3\log\left(\frac{f+gx}{c+dx}\right)}{3(bf-ag)^3(df-cg)^3}+\frac{2B^2gn^2(bc-ad)^2(-2adg-bcg+3bdf)\log\left(\frac{f+gx}{c+dx}\right)}{3(bf-ag)^3(df-cg)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(f + g*x)^4,x]
```

```
[Out] (B^2*(b*c - a*d)^2*g^2*n^2*(c + d*x))/(3*(b*f - a*g)^2*(d*f - c*g)^3*(f + g
*x)) - (B*(b*c - a*d)*g^2*n*(c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^
n]))/(3*(b*f - a*g)*(d*f - c*g)^3*(f + g*x)^2) + (2*B*(b*c - a*d)*g*(3*b*d*
f - b*c*g - 2*a*d*g)*n*(a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(3
*(b*f - a*g)^3*(d*f - c*g)^2*(f + g*x)) + (b^3*(A + B*Log[e*((a + b*x)/(c +
 d*x))^n])^2)/(3*g*(b*f - a*g)^3) - (A + B*Log[e*((a + b*x)/(c + d*x))^n])^
2/(3*g*(f + g*x)^3) + (B^2*(b*c - a*d)^3*g^2*n^2*Log[(a + b*x)/(c + d*x)])/
(3*(b*f - a*g)^3*(d*f - c*g)^3) - (B^2*(b*c - a*d)^3*g^2*n^2*Log[(f + g*x)/
(c + d*x)])/(3*(b*f - a*g)^3*(d*f - c*g)^3) + (2*B^2*(b*c - a*d)^2*g*(3*b*d
*f - b*c*g - 2*a*d*g)*n^2*Log[(f + g*x)/(c + d*x)])/(3*(b*f - a*g)^3*(d*f -
 c*g)^3) + (2*B*(b*c - a*d)*(a^2*d^2*g^2 - a*b*d*g*(3*d*f - c*g) + b^2*(3*d
```

```
^2*f^2 - 3*c*d*f*g + c^2*g^2))*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log
[1 - ((d*f - c*g)*(a + b*x))/((b*f - a*g)*(c + d*x))])/(3*(b*f - a*g)^3*(d*
f - c*g)^3) + (2*B^2*(b*c - a*d)*(a^2*d^2*g^2 - a*b*d*g*(3*d*f - c*g) + b^2
*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^2))*n^2*PolyLog[2, ((d*f - c*g)*(a + b*x))/
((b*f - a*g)*(c + d*x))])/(3*(b*f - a*g)^3*(d*f - c*g)^3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_)*((
f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(d + e*x)^(q +
 1)*((a + b*Log[c*x^n])^p/((q + 1)*(e*f - d*g))), x] - Dist[b*n*(p/((q + 1)
*(e*f - d*g))), Int[(f + g*x)^(m + 1)*(d + e*x)^(q + 1)*((a + b*Log[c*x^n])
^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, q}, x] && NeQ[e*f
- d*g, 0] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 2404

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2553

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[b*c - a*d, Subst[
Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)),
 x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x
] && NeQ[b*c - a*d, 0] && IntegerQ[m] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(f+gx)^4}\,dx = -\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3g(f+gx)^3} + \frac{(2Bn)\int \frac{(bc-ad)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)(c+dx)(f+gx)^3}\,dx}{3g}$$

$$= -\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3g(f+gx)^3} + \frac{(2B(bc-ad)n)\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)(c+dx)(f+gx)^3}\,dx}{3g}$$

$$= -\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3g(f+gx)^3} + \frac{(2B(bc-ad)n)\int \left(\frac{b^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)(bf-ag)^3(a+bx)} + \cdots\right.}{\cdots}$$

$$= -\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3g(f+gx)^3} + \frac{(2b^4Bn)\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{a+bx}\,dx}{3g(bf-ag)^3} - \frac{(2Bd^4n)\cdots}{3\cdots}$$

$$= -\frac{B(bc-ad)n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3(bf-ag)(df-cg)(f+gx)^2} - \frac{2B(bc-ad)(2bdf-bcg-adg)\cdots}{3(bf-ag)^2(df-c\cdots}$$

$$= -\frac{B(bc-ad)n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3(bf-ag)(df-cg)(f+gx)^2} - \frac{2B(bc-ad)(2bdf-bcg-adg)\cdots}{3(bf-ag)^2(df-c\cdots}$$

$$= -\frac{B(bc-ad)n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3(bf-ag)(df-cg)(f+gx)^2} - \frac{2B(bc-ad)(2bdf-bcg-adg)\cdots}{3(bf-ag)^2(df-c\cdots}$$

$$= -\frac{B^2(bc-ad)^2gn^2}{3(bf-ag)^2(df-cg)^2(f+gx)} + \frac{b^2B^2(bc-ad)n^2\log(a+bx)}{3(bf-ag)^3(df-cg)} + \frac{2bB^2(\cdots}{\cdots}$$

$$= -\frac{B^2(bc-ad)^2gn^2}{3(bf-ag)^2(df-cg)^2(f+gx)} + \frac{b^2B^2(bc-ad)n^2\log(a+bx)}{3(bf-ag)^3(df-cg)} + \frac{2bB^2(\cdots}{\cdots}$$

$$= -\frac{B^2(bc-ad)^2gn^2}{3(bf-ag)^2(df-cg)^2(f+gx)} + \frac{b^2B^2(bc-ad)n^2\log(a+bx)}{3(bf-ag)^3(df-cg)} + \frac{2bB^2(\cdots}{\cdots}$$

Mathematica [A]
time = 2.11, size = 918, normalized size = 1.23

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(f + g*x)^4,x]
```

```
[Out] -1/3*((A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 + (B*n*(f + g*x)*((b*c - a*d
)*g*(b*f - a*g)^2*(d*f - c*g)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + 2*
```

```
(b*c - a*d)*g*(b*f - a*g)*(-(d*f) + c*g)*(-2*b*d*f + b*c*g + a*d*g)*(f + g*
x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - 2*b^3*(d*f - c*g)^3*(f + g*x)^2
*Log[a + b*x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + 2*d^3*(b*f - a*g)^3*
(f + g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[c + d*x] - 2*(b*c -
a*d)*g*(a^2*d^2*g^2 + a*b*d*g*(-3*d*f + c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g +
 c^2*g^2))*(f + g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[f + g*x]
- 2*B*(b*c - a*d)*g*(2*b*d*f - b*c*g - a*d*g)*n*(f + g*x)^2*(b*(d*f - c*g)*
Log[a + b*x] + (-(b*d*f) + a*d*g)*Log[c + d*x] + (b*c - a*d)*g*Log[f + g*x]
) + B*(b*c - a*d)*g*n*(f + g*x)*((b*c - a*d)*g*(b*f - a*g)*(d*f - c*g) - b^
2*(d*f - c*g)^2*(f + g*x)*Log[a + b*x] + d^2*(b*f - a*g)^2*(f + g*x)*Log[c
+ d*x] + (b*c - a*d)*g*(-2*b*d*f + b*c*g + a*d*g)*(f + g*x)*Log[f + g*x]) +
 b^3*B*(d*f - c*g)^3*n*(f + g*x)^2*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(
c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) - B*d
^3*(b*f - a*g)^3*n*(f + g*x)^2*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[
c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]) + 2*B*(b*
c - a*d)*g*(a^2*d^2*g^2 + a*b*d*g*(-3*d*f + c*g) + b^2*(3*d^2*f^2 - 3*c*d*f
*g + c^2*g^2))*n*(f + g*x)^2*((Log[(g*(a + b*x))/(-(b*f) + a*g)] - Log[(g*(
c + d*x))/(-(d*f) + c*g)])*Log[f + g*x] + PolyLog[2, (b*(f + g*x))/(b*f - a
*g)] - PolyLog[2, (d*(f + g*x))/(d*f - c*g)])))/((b*f - a*g)^3*(d*f - c*g)^
3))/(g*(f + g*x)^3)
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)\right)^{2}}{\left(gx+f\right)^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(g*x+f)^4,x)
```

```
[Out] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(g*x+f)^4,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(g*x+f)^4,x, algorithm="maxima
")
```

```
[Out] 1/3*(2*b^3*log(b*x + a)/(b^3*f^3*g - 3*a*b^2*f^2*g^2 + 3*a^2*b*f*g^3 - a^3*
g^4) - 2*d^3*log(d*x + c)/(d^3*f^3*g - 3*c*d^2*f^2*g^2 + 3*c^2*d*f*g^3 - c^
3*g^4) + 2*(3*(b^3*c*d^2 - a*b^2*d^3)*f^2 - 3*(b^3*c^2*d - a^2*b*d^3)*f*g +
 (b^3*c^3 - a^3*d^3)*g^2)*log(g*x + f)/(b^3*d^3*f^6 + a^3*c^3*g^6 - 3*(b^3*
```

```
c*d^2 + a*b^2*d^3)*f^5*g + 3*(b^3*c^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3)*f^4*g^
2 - (b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*f^3*g^3 + 3*(a*b^2*
c^3 + 3*a^2*b*c^2*d + a^3*c*d^2)*f^2*g^4 - 3*(a^2*b*c^3 + a^3*c^2*d)*f*g^5)
 - (5*(b^2*c*d - a*b*d^2)*f^2 - 3*(b^2*c^2 - a^2*d^2)*f*g + (a*b*c^2 - a^2*
c*d)*g^2 + 2*(2*(b^2*c*d - a*b*d^2)*f*g - (b^2*c^2 - a^2*d^2)*g^2)*x)/(b^2*
d^2*f^6 + a^2*c^2*f^2*g^4 - 2*(b^2*c*d + a*b*d^2)*f^5*g + (b^2*c^2 + 4*a*b*
c*d + a^2*d^2)*f^4*g^2 - 2*(a*b*c^2 + a^2*c*d)*f^3*g^3 + (b^2*d^2*f^4*g^2 +
 a^2*c^2*g^6 - 2*(b^2*c*d + a*b*d^2)*f^3*g^3 + (b^2*c^2 + 4*a*b*c*d + a^2*d
^2)*f^2*g^4 - 2*(a*b*c^2 + a^2*c*d)*f*g^5)*x^2 + 2*(b^2*d^2*f^5*g + a^2*c^2
*f*g^5 - 2*(b^2*c*d + a*b*d^2)*f^4*g^2 + (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*f^
3*g^3 - 2*(a*b*c^2 + a^2*c*d)*f^2*g^4)*x))*A*B*n - 1/3*B^2*(log((d*x + c)^n
)^2/(g^4*x^3 + 3*f*g^3*x^2 + 3*f^2*g^2*x + f^3*g) + 3*integrate(-1/3*(3*d*g
*x + 3*(d*g*x + c*g)*log((b*x + a)^n)^2 + 3*c*g + 6*(d*g*x + c*g)*log((b*x
+ a)^n) + 2*(d*g*(n - 3)*x + d*f*n - 3*c*g - 3*(d*g*x + c*g)*log((b*x + a)^
n))*log((d*x + c)^n))/(d*g^5*x^5 + c*f^4*g + (4*d*f*g^4 + c*g^5)*x^4 + 2*(3
*d*f^2*g^3 + 2*c*f*g^4)*x^3 + 2*(2*d*f^3*g^2 + 3*c*f^2*g^3)*x^2 + (d*f^4*g
+ 4*c*f^3*g^2)*x), x)) - 2/3*A*B*log((b*x/(d*x + c) + a/(d*x + c))^n*e)/(g^
4*x^3 + 3*f*g^3*x^2 + 3*f^2*g^2*x + f^3*g) - 1/3*A^2/(g^4*x^3 + 3*f*g^3*x^2
 + 3*f^2*g^2*x + f^3*g)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(g*x+f)^4,x, algorithm="fricas
")
```

```
[Out] integral((B^2*log(((b*x + a)/(d*x + c))^n*e)^2 + 2*A*B*log(((b*x + a)/(d*x
+ c))^n*e) + A^2)/(g^4*x^4 + 4*f*g^3*x^3 + 6*f^2*g^2*x^2 + 4*f^3*g*x + f^4)
, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(g*x+f)**4,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(g*x+f)^4,x, algorithm="giac")
[Out] integrate((B*log(((b*x + a)/(d*x + c))^n*e) + A)^2/(g*x + f)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\right)^2}{{\left(f+g\,x\right)}^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/(f + g*x)^4,x)
[Out] int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/(f + g*x)^4, x)
```

3.75 $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(f+gx)^5}\,dx$

Optimal. Leaf size=1208

$$-\frac{B^2(bc-ad)^2g^3n^2(c+dx)^2}{12(bf-ag)^2(df-cg)^4(f+gx)^2}-\frac{B^2(bc-ad)^3g^3n^2(c+dx)}{6(bf-ag)^3(df-cg)^4(f+gx)}+\frac{B^2(bc-ad)^2g^2(4bdf-bcg-3adg)n}{4(bf-ag)^3(df-cg)^4(f+gx}$$

```
[Out] -1/12*B^2*(-a*d+b*c)^2*g^3*n^2*(d*x+c)^2/(-a*g+b*f)^2/(-c*g+d*f)^4/(g*x+f)^
2-1/6*B^2*(-a*d+b*c)^3*g^3*n^2*(d*x+c)/(-a*g+b*f)^3/(-c*g+d*f)^4/(g*x+f)+1/
4*B^2*(-a*d+b*c)^2*g^2*(-3*a*d*g-b*c*g+4*b*d*f)*n^2*(d*x+c)/(-a*g+b*f)^3/(-
c*g+d*f)^4/(g*x+f)+1/6*B*(-a*d+b*c)*g^3*n*(d*x+c)^3*(A+B*ln(e*((b*x+a)/(d*x
+c))^n))/(-a*g+b*f)/(-c*g+d*f)^4/(g*x+f)^3-1/4*B*(-a*d+b*c)*g^2*(-3*a*d*g-b
*c*g+4*b*d*f)*n*(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*g+b*f)^2/(-c*
g+d*f)^4/(g*x+f)^2+1/2*B*(-a*d+b*c)*g*(3*a^2*d^2*g^2-2*a*b*d*g*(-c*g+4*d*f)
+b^2*(c^2*g^2-4*c*d*f*g+6*d^2*f^2))*n*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n
))/(-a*g+b*f)^4/(-c*g+d*f)^3/(g*x+f)+1/4*b^4*(A+B*ln(e*((b*x+a)/(d*x+c))^n)
)^2/g/(-a*g+b*f)^4-1/4*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/g/(g*x+f)^4-1/6*B^
2*(-a*d+b*c)^4*g^3*n^2*ln((b*x+a)/(d*x+c))/(-a*g+b*f)^4/(-c*g+d*f)^4+1/4*B^
2*(-a*d+b*c)^3*g^2*(-3*a*d*g-b*c*g+4*b*d*f)*n^2*ln((b*x+a)/(d*x+c))/(-a*g+b
*f)^4/(-c*g+d*f)^4+1/6*B^2*(-a*d+b*c)^4*g^3*n^2*ln((g*x+f)/(d*x+c))/(-a*g+b
*f)^4/(-c*g+d*f)^4-1/4*B^2*(-a*d+b*c)^3*g^2*(-3*a*d*g-b*c*g+4*b*d*f)*n^2*ln
((g*x+f)/(d*x+c))/(-a*g+b*f)^4/(-c*g+d*f)^4+1/2*B^2*(-a*d+b*c)^2*g*(3*a^2*d
^2*g^2-2*a*b*d*g*(-c*g+4*d*f)+b^2*(c^2*g^2-4*c*d*f*g+6*d^2*f^2))*n^2*ln((g*
x+f)/(d*x+c))/(-a*g+b*f)^4/(-c*g+d*f)^4-1/2*B*(-a*d+b*c)*(-a*d*g-b*c*g+2*b*
d*f)*(2*a*b*d^2*f*g-a^2*d^2*g^2-b^2*(c^2*g^2-2*c*d*f*g+2*d^2*f^2))*n*(A+B*l
n(e*((b*x+a)/(d*x+c))^n))*ln(1-(-c*g+d*f)*(b*x+a)/(-a*g+b*f)/(d*x+c))/(-a*g
+b*f)^4/(-c*g+d*f)^4-1/2*B^2*(-a*d+b*c)*(-a*d*g-b*c*g+2*b*d*f)*(2*a*b*d^2*f
*g-a^2*d^2*g^2-b^2*(c^2*g^2-2*c*d*f*g+2*d^2*f^2))*n^2*polylog(2,(-c*g+d*f)*
(b*x+a)/(-a*g+b*f)/(d*x+c))/(-a*g+b*f)^4/(-c*g+d*f)^4
```

Rubi [A]

time = 1.77, antiderivative size = 1208, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {2553, 2398, 2404, 2338, 2356, 46, 2351, 31, 2354, 2438}

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(f + g*x)^5,x]
```

```
[Out] -1/12*(B^2*(b*c - a*d)^2*g^3*n^2*(c + d*x)^2)/((b*f - a*g)^2*(d*f - c*g)^4*
(f + g*x)^2) - (B^2*(b*c - a*d)^3*g^3*n^2*(c + d*x))/(6*(b*f - a*g)^3*(d*f
- c*g)^4*(f + g*x)) + (B^2*(b*c - a*d)^2*g^2*(4*b*d*f - b*c*g - 3*a*d*g)*n^
```

```
2*(c + d*x))/(4*(b*f - a*g)^3*(d*f - c*g)^4*(f + g*x)) + (B*(b*c - a*d)*g^3
*n*(c + d*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(6*(b*f - a*g)*(d*f
- c*g)^4*(f + g*x)^3) - (B*(b*c - a*d)*g^2*(4*b*d*f - b*c*g - 3*a*d*g)*n*(c
 + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(4*(b*f - a*g)^2*(d*f - c
*g)^4*(f + g*x)^2) + (B*(b*c - a*d)*g*(3*a^2*d^2*g^2 - 2*a*b*d*g*(4*d*f - c
*g) + b^2*(6*d^2*f^2 - 4*c*d*f*g + c^2*g^2))*n*(a + b*x)*(A + B*Log[e*((a +
 b*x)/(c + d*x))^n]))/(2*(b*f - a*g)^4*(d*f - c*g)^3*(f + g*x)) + (b^4*(A +
 B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(4*g*(b*f - a*g)^4) - (A + B*Log[e*((
a + b*x)/(c + d*x))^n])^2/(4*g*(f + g*x)^4) - (B^2*(b*c - a*d)^4*g^3*n^2*Lo
g[(a + b*x)/(c + d*x)])/(6*(b*f - a*g)^4*(d*f - c*g)^4) + (B^2*(b*c - a*d)^
3*g^2*(4*b*d*f - b*c*g - 3*a*d*g)*n^2*Log[(a + b*x)/(c + d*x)])/(4*(b*f - a
*g)^4*(d*f - c*g)^4) + (B^2*(b*c - a*d)^4*g^3*n^2*Log[(f + g*x)/(c + d*x)])
/(6*(b*f - a*g)^4*(d*f - c*g)^4) - (B^2*(b*c - a*d)^3*g^2*(4*b*d*f - b*c*g
- 3*a*d*g)*n^2*Log[(f + g*x)/(c + d*x)])/(4*(b*f - a*g)^4*(d*f - c*g)^4) +
(B^2*(b*c - a*d)^2*g*(3*a^2*d^2*g^2 - 2*a*b*d*g*(4*d*f - c*g) + b^2*(6*d^2*
f^2 - 4*c*d*f*g + c^2*g^2))*n^2*Log[(f + g*x)/(c + d*x)])/(2*(b*f - a*g)^4*
(d*f - c*g)^4) - (B*(b*c - a*d)*(2*b*d*f - b*c*g - a*d*g)*(2*a*b*d^2*f*g -
a^2*d^2*g^2 - b^2*(2*d^2*f^2 - 2*c*d*f*g + c^2*g^2))*n*(A + B*Log[e*((a + b
*x)/(c + d*x))^n])*Log[1 - ((d*f - c*g)*(a + b*x))/((b*f - a*g)*(c + d*x))]
)/(2*(b*f - a*g)^4*(d*f - c*g)^4) - (B^2*(b*c - a*d)*(2*b*d*f - b*c*g - a*d
*g)*(2*a*b*d^2*f*g - a^2*d^2*g^2 - b^2*(2*d^2*f^2 - 2*c*d*f*g + c^2*g^2))*n
^2*PolyLog[2, ((d*f - c*g)*(a + b*x))/((b*f - a*g)*(c + d*x))])/(2*(b*f - a
*g)^4*(d*f - c*g)^4)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
```

```
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_)*((
f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(d + e*x)^(q +
 1)*((a + b*Log[c*x^n])^p/((q + 1)*(e*f - d*g))), x] - Dist[b*n*(p/((q + 1)
*(e*f - d*g))), Int[(f + g*x)^(m + 1)*(d + e*x)^(q + 1)*((a + b*Log[c*x^n])
^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, q}, x] && NeQ[e*f
- d*g, 0] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 2404

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2553

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[b*c - a*d, Subst[
Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)),
 x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x
] && NeQ[b*c - a*d, 0] && IntegerQ[m] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(f+gx)^5}\,dx = -\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4g(f+gx)^4}+\frac{(Bn)\int\frac{(bc-ad)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)(c+dx)(f+gx)^4}\,dx}{2g}$$

$$= -\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4g(f+gx)^4}+\frac{(B(bc-ad)n)\int\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)(c+dx)(f+gx)^4}\,dx}{2g}$$

$$= -\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4g(f+gx)^4}+\frac{(B(bc-ad)n)\int\left(\frac{b^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)(bf-ag)^4(a+bx)}-\frac{d^5(}{(bc}\right.}{}$$

$$= -\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4g(f+gx)^4}+\frac{(b^5Bn)\int\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{a+bx}\,dx}{2g(bf-ag)^4}-\frac{(Bd^5n)\int\frac{A+}{}}{2g(}$$

$$= -\frac{B(bc-ad)n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{6(bf-ag)(df-cg)(f+gx)^3}-\frac{B(bc-ad)(2bdf-bcg-adg)n(}{4(bf-ag)^2(df-cg)}$$

$$= -\frac{B(bc-ad)n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{6(bf-ag)(df-cg)(f+gx)^3}-\frac{B(bc-ad)(2bdf-bcg-adg)n(}{4(bf-ag)^2(df-cg)}$$

$$= -\frac{B(bc-ad)n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{6(bf-ag)(df-cg)(f+gx)^3}-\frac{B(bc-ad)(2bdf-bcg-adg)n(}{4(bf-ag)^2(df-cg)}$$

$$= -\frac{B^2(bc-ad)^2gn^2}{12(bf-ag)^2(df-cg)^2(f+gx)^2}-\frac{5B^2(bc-ad)^2g(2bdf-bcg-adg)n^2}{12(bf-ag)^3(df-cg)^3(f+gx)}$$

$$= -\frac{B^2(bc-ad)^2gn^2}{12(bf-ag)^2(df-cg)^2(f+gx)^2}-\frac{5B^2(bc-ad)^2g(2bdf-bcg-adg)n^2}{12(bf-ag)^3(df-cg)^3(f+gx)}$$

$$= -\frac{B^2(bc-ad)^2gn^2}{12(bf-ag)^2(df-cg)^2(f+gx)^2}-\frac{5B^2(bc-ad)^2g(2bdf-bcg-adg)n^2}{12(bf-ag)^3(df-cg)^3(f+gx)}$$

Mathematica [A]
time = 5.15, size = 1329, normalized size = 1.10

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(f + g*x)^5,x]
```

```
[Out] -1/12*(3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 + (B*n*(f + g*x)*(2*(b*c
- a*d)*g*(b*f - a*g)^3*(d*f - c*g)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])
```

```
 - 3*(b*c - a*d)*g*(b*f - a*g)^2*(d*f - c*g)^2*(-2*b*d*f + b*c*g + a*d*g)*(
f + g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + 6*(b*c - a*d)*g*(b*f - a*
g)*(d*f - c*g)*(a^2*d^2*g^2 + a*b*d*g*(-3*d*f + c*g) + b^2*(3*d^2*f^2 - 3*c
*d*f*g + c^2*g^2))*(f + g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - 6*b
^4*(d*f - c*g)^4*(f + g*x)^3*Log[a + b*x]*(A + B*Log[e*((a + b*x)/(c + d*x)
)^n]) + 6*d^4*(b*f - a*g)^4*(f + g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^
n])*Log[c + d*x] + 6*(b*c - a*d)*g*(-2*b*d*f + b*c*g + a*d*g)*(-2*a*b*d^2*f
*g + a^2*d^2*g^2 + b^2*(2*d^2*f^2 - 2*c*d*f*g + c^2*g^2))*(f + g*x)^3*(A +
B*Log[e*((a + b*x)/(c + d*x))^n])*Log[f + g*x] - 6*B*(b*c - a*d)*g*(a^2*d^2
*g^2 + a*b*d*g*(-3*d*f + c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^2))*n*(f
 + g*x)^3*(b*(d*f - c*g)*Log[a + b*x] + (-(b*d*f) + a*d*g)*Log[c + d*x] + (
b*c - a*d)*g*Log[f + g*x]) + 3*B*(b*c - a*d)*g*(2*b*d*f - b*c*g - a*d*g)*n*
(f + g*x)^2*((b*c - a*d)*g*(b*f - a*g)*(d*f - c*g) - b^2*(d*f - c*g)^2*(f +
 g*x)*Log[a + b*x] + d^2*(b*f - a*g)^2*(f + g*x)*Log[c + d*x] + (b*c - a*d)
*g*(-2*b*d*f + b*c*g + a*d*g)*(f + g*x)*Log[f + g*x]) + B*(b*c - a*d)*g*n*(
f + g*x)*((b*c - a*d)*g*(b*f - a*g)^2*(d*f - c*g)^2 + 2*(b*c - a*d)*g*(b*f
- a*g)*(-(d*f) + c*g)*(-2*b*d*f + b*c*g + a*d*g)*(f + g*x) - 2*b^3*(d*f - c
*g)^3*(f + g*x)^2*Log[a + b*x] + 2*d^3*(b*f - a*g)^3*(f + g*x)^2*Log[c + d*
x] - 2*(b*c - a*d)*g*(a^2*d^2*g^2 + a*b*d*g*(-3*d*f + c*g) + b^2*(3*d^2*f^2
 - 3*c*d*f*g + c^2*g^2))*(f + g*x)^2*Log[f + g*x]) + 3*b^4*B*(d*f - c*g)^4*
n*(f + g*x)^3*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)
]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) - 3*B*d^4*(b*f - a*g)^4*n*
(f + g*x)^3*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d
*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]) - 6*B*(b*c - a*d)*g*(-2*b*d*
f + b*c*g + a*d*g)*(-2*a*b*d^2*f*g + a^2*d^2*g^2 + b^2*(2*d^2*f^2 - 2*c*d*f
*g + c^2*g^2))*n*(f + g*x)^3*((Log[(g*(a + b*x))/(-(b*f) + a*g)] - Log[(g*(
c + d*x))/(-(d*f) + c*g)])*Log[f + g*x] + PolyLog[2, (b*(f + g*x))/(b*f - a
*g)] - PolyLog[2, (d*(f + g*x))/(d*f - c*g)])))/((b*f - a*g)^4*(d*f - c*g)^
4))/(g*(f + g*x)^4)
```

Maple [**F**]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)\right)^{2}}{\left(gx+f\right)^{5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(g*x+f)^5,x)
```

```
[Out] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(g*x+f)^5,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(g*x+f)^5,x, algorithm="maxima
")
```

```
[Out] 1/12*(6*b^4*log(b*x + a)/(b^4*f^4*g - 4*a*b^3*f^3*g^2 + 6*a^2*b^2*f^2*g^3 -
 4*a^3*b*f*g^4 + a^4*g^5) - 6*d^4*log(d*x + c)/(d^4*f^4*g - 4*c*d^3*f^3*g^2
 + 6*c^2*d^2*f^2*g^3 - 4*c^3*d*f*g^4 + c^4*g^5) + 6*(4*(b^4*c*d^3 - a*b^3*d
^4)*f^3 - 6*(b^4*c^2*d^2 - a^2*b^2*d^4)*f^2*g + 4*(b^4*c^3*d - a^3*b*d^4)*f
*g^2 - (b^4*c^4 - a^4*d^4)*g^3)*log(g*x + f)/(b^4*d^4*f^8 + a^4*c^4*g^8 - 4
*(b^4*c*d^3 + a*b^3*d^4)*f^7*g + 2*(3*b^4*c^2*d^2 + 8*a*b^3*c*d^3 + 3*a^2*b
^2*d^4)*f^6*g^2 - 4*(b^4*c^3*d + 6*a*b^3*c^2*d^2 + 6*a^2*b^2*c*d^3 + a^3*b*
d^4)*f^5*g^3 + (b^4*c^4 + 16*a*b^3*c^3*d + 36*a^2*b^2*c^2*d^2 + 16*a^3*b*c*
d^3 + a^4*d^4)*f^4*g^4 - 4*(a*b^3*c^4 + 6*a^2*b^2*c^3*d + 6*a^3*b*c^2*d^2 +
 a^4*c*d^3)*f^3*g^5 + 2*(3*a^2*b^2*c^4 + 8*a^3*b*c^3*d + 3*a^4*c^2*d^2)*f^2
*g^6 - 4*(a^3*b*c^4 + a^4*c^3*d)*f*g^7) - (26*(b^3*c*d^2 - a*b^2*d^3)*f^4 -
 31*(b^3*c^2*d - a^2*b*d^3)*f^3*g + (11*b^3*c^3 + 15*a*b^2*c^2*d - 15*a^2*b
*c*d^2 - 11*a^3*d^3)*f^2*g^2 - 7*(a*b^2*c^3 - a^3*c*d^2)*f*g^3 + 2*(a^2*b*c
^3 - a^3*c^2*d)*g^4 + 6*(3*(b^3*c*d^2 - a*b^2*d^3)*f^2*g^2 - 3*(b^3*c^2*d -
 a^2*b*d^3)*f*g^3 + (b^3*c^3 - a^3*d^3)*g^4)*x^2 + 3*(14*(b^3*c*d^2 - a*b^2
*d^3)*f^3*g - 15*(b^3*c^2*d - a^2*b*d^3)*f^2*g^2 + (5*b^3*c^3 + 3*a*b^2*c^2
*d - 3*a^2*b*c*d^2 - 5*a^3*d^3)*f*g^3 - (a*b^2*c^3 - a^3*c*d^2)*g^4)*x)/(b^
3*d^3*f^9 + a^3*c^3*f^3*g^6 - 3*(b^3*c*d^2 + a*b^2*d^3)*f^8*g + 3*(b^3*c^2*
d + 3*a*b^2*c*d^2 + a^2*b*d^3)*f^7*g^2 - (b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2*b
*c*d^2 + a^3*d^3)*f^6*g^3 + 3*(a*b^2*c^3 + 3*a^2*b*c^2*d + a^3*c*d^2)*f^5*g
^4 - 3*(a^2*b*c^3 + a^3*c^2*d)*f^4*g^5 + (b^3*d^3*f^6*g^3 + a^3*c^3*g^9 - 3
*(b^3*c*d^2 + a*b^2*d^3)*f^5*g^4 + 3*(b^3*c^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3
)*f^4*g^5 - (b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*f^3*g^6 + 3
*(a*b^2*c^3 + 3*a^2*b*c^2*d + a^3*c*d^2)*f^2*g^7 - 3*(a^2*b*c^3 + a^3*c^2*d
)*f*g^8)*x^3 + 3*(b^3*d^3*f^7*g^2 + a^3*c^3*f*g^8 - 3*(b^3*c*d^2 + a*b^2*d^
3)*f^6*g^3 + 3*(b^3*c^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3)*f^5*g^4 - (b^3*c^3 +
 9*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*f^4*g^5 + 3*(a*b^2*c^3 + 3*a^2*b*
c^2*d + a^3*c*d^2)*f^3*g^6 - 3*(a^2*b*c^3 + a^3*c^2*d)*f^2*g^7)*x^2 + 3*(b^
3*d^3*f^8*g + a^3*c^3*f^2*g^7 - 3*(b^3*c*d^2 + a*b^2*d^3)*f^7*g^2 + 3*(b^3*
c^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3)*f^6*g^3 - (b^3*c^3 + 9*a*b^2*c^2*d + 9*a
^2*b*c*d^2 + a^3*d^3)*f^5*g^4 + 3*(a*b^2*c^3 + 3*a^2*b*c^2*d + a^3*c*d^2)*f
^4*g^5 - 3*(a^2*b*c^3 + a^3*c^2*d)*f^3*g^6)*x))*A*B*n - 1/4*B^2*(log((d*x +
 c)^n)^2/(g^5*x^4 + 4*f*g^4*x^3 + 6*f^2*g^3*x^2 + 4*f^3*g^2*x + f^4*g) + 4*
integrate(-1/2*(2*d*g*x + 2*(d*g*x + c*g)*log((b*x + a)^n)^2 + 2*c*g + 4*(d
*g*x + c*g)*log((b*x + a)^n) + (d*g*(n - 4)*x + d*f*n - 4*c*g - 4*(d*g*x +
c*g)*log((b*x + a)^n))*log((d*x + c)^n))/(d*g^6*x^6 + c*f^5*g + (5*d*f*g^5
+ c*g^6)*x^5 + 5*(2*d*f^2*g^4 + c*f*g^5)*x^4 + 10*(d*f^3*g^3 + c*f^2*g^4)*x
^3 + 5*(d*f^4*g^2 + 2*c*f^3*g^3)*x^2 + (d*f^5*g + 5*c*f^4*g^2)*x), x)) - 1/
2*A*B*log((b*x/(d*x + c) + a/(d*x + c))^n*e)/(g^5*x^4 + 4*f*g^4*x^3 + 6*f^2
*g^3*x^2 + 4*f^3*g^2*x + f^4*g) - 1/4*A^2/(g^5*x^4 + 4*f*g^4*x^3 + 6*f^2*g^
```

```
3*x^2 + 4*f^3*g^2*x + f^4*g)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(g*x+f)^5,x, algorithm="fricas
")
```

```
[Out] integral((B^2*log(((b*x + a)/(d*x + c))^n*e)^2 + 2*A*B*log(((b*x + a)/(d*x
+ c))^n*e) + A^2)/(g^5*x^5 + 5*f*g^4*x^4 + 10*f^2*g^3*x^3 + 10*f^3*g^2*x^2
+ 5*f^4*g*x + f^5), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(g*x+f)**5,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(g*x+f)^5,x, algorithm="giac")
```

```
[Out] integrate((B*log(((b*x + a)/(d*x + c))^n*e) + A)^2/(g*x + f)^5, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\right)^2}{{\left(f+g\,x\right)}^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/(f + g*x)^5,x)
```

```
[Out] int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/(f + g*x)^5, x)
```

3.76 $\int \frac{(f+gx)^2}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$

Optimal. Leaf size=35

$$\mathrm{Int}\left(\frac{(f+gx)^2}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)},x\right)$$

```
[Out] Unintegrable((g*x+f)^2/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(f+gx)^2}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(f + g*x)^2/(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
[Out] Defer[Int][(f + g*x)^2/(A + B*Log[e*((a + b*x)/(c + d*x))^n]), x]
```

Rubi steps

$$\begin{aligned}\int \frac{(f+gx)^2}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx &= \int \left(\frac{f^2}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}+\frac{2fgx}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}+\frac{g^2x^2}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\right.\\ &= f^2\int \frac{1}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx+(2fg)\int \frac{x}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx+g^2\int \frac{}{A}\end{aligned}$$

Mathematica [A]
time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{(f+gx)^2}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(f + g*x)^2/(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
[Out] Integrate[(f + g*x)^2/(A + B*Log[e*((a + b*x)/(c + d*x))^n]), x]
```

Maple [A]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{(gx+f)^2}{A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
[Out] int((g*x+f)^2/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="maxima")
[Out] integrate((g*x + f)^2/(B*log(((b*x + a)/(d*x + c))^n*e) + A), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fricas")
[Out] integral((g^2*x^2 + 2*f*g*x + f^2)/(B*log(((b*x + a)/(d*x + c))^n*e) + A),
x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(f+gx)^2}{A+B\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2/(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
[Out] Integral((f + g*x)**2/(A + B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="giac")
[Out] integrate((g*x + f)^2/(B*log(((b*x + a)/(d*x + c))^n*e) + A), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{(f+g\,x)^2}{A+B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^2/(A + B*log(e*((a + b*x)/(c + d*x))^n)),x)
[Out] int((f + g*x)^2/(A + B*log(e*((a + b*x)/(c + d*x))^n)), x)
```

3.77 $\int \frac{f+gx}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$

Optimal. Leaf size=33

$$\text{Int}\left(\frac{f+gx}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)},x\right)$$

```
[Out] Unintegrable((g*x+f)/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{f+gx}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(f + g*x)/(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
[Out] Defer[Int][(f + g*x)/(A + B*Log[e*((a + b*x)/(c + d*x))^n]), x]
```

Rubi steps

$$\begin{aligned}\int \frac{f+gx}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx &= \int \left(\frac{f}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}+\frac{gx}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\right)dx\\ &= f\int \frac{1}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx+g\int \frac{x}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx\end{aligned}$$

Mathematica [A]
time = 0.19, size = 0, normalized size = 0.00

$$\int \frac{f+gx}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(f + g*x)/(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
[Out] Integrate[(f + g*x)/(A + B*Log[e*((a + b*x)/(c + d*x))^n]), x]
```

Maple [A]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{gx+f}{A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
[Out] int((g*x+f)/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="maxima")
[Out] integrate((g*x + f)/(B*log(((b*x + a)/(d*x + c))^n*e) + A), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fricas")
[Out] integral((g*x + f)/(B*log(((b*x + a)/(d*x + c))^n*e) + A), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{f + gx}{A + B\log\left(e\left(\frac{a}{c+dx} + \frac{bx}{c+dx}\right)^n\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)/(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
[Out] Integral((f + g*x)/(A + B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="giac")
[Out] integrate((g*x + f)/(B*log(((b*x + a)/(d*x + c))^n*e) + A), x)
```

<u>**Mupad**</u> [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{f+g\,x}{A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)/(A + B*log(e*((a + b*x)/(c + d*x))^n)),x)
[Out] int((f + g*x)/(A + B*log(e*((a + b*x)/(c + d*x))^n)), x)
```

3.78 $\int \frac{1}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{1}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)},x\right)$$

```
[Out] Unintegrable(1/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

Rubi [A]
time = 0.00, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^(-1),x]
```

```
[Out] Defer[Int][(A + B*Log[e*((a + b*x)/(c + d*x))^n])^(-1), x]
```

Rubi steps

$$\int \frac{1}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx=\int \frac{1}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$$

Mathematica [A]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{1}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^(-1),x]
```

```
[Out] Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^(-1), x]
```

Maple [A]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{1}{A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

```
[Out] int(1/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="maxima")
```

```
[Out] integrate(1/(B*log(((b*x + a)/(d*x + c))^n*e) + A), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fricas")
```

```
[Out] integral(1/(B*log(((b*x + a)/(d*x + c))^n*e) + A), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

```
[Out] Integral(1/(A + B*log(e*((a + b*x)/(c + d*x))**n)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="giac")
```

```
[Out] integrate(1/(B*log(((b*x + a)/(d*x + c))^n*e) + A), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{A + B \ln \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(A + B*log(e*((a + b*x)/(c + d*x))^n)),x)
[Out] int(1/(A + B*log(e*((a + b*x)/(c + d*x))^n)), x)
```

3.79 $\int \frac{1}{(f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx$

Optimal. Leaf size=35

$$\text{Int}\left(\frac{1}{(f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)},x\right)$$

```
[Out] Unintegrable(1/(g*x+f)/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((f + g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])),x]
[Out] Defer[Int][1/((f + g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])), x]
```

Rubi steps

$$\int \frac{1}{(f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx = \int \frac{1}{(f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx$$

Mathematica [A]
time = 0.37, size = 0, normalized size = 0.00

$$\int \frac{1}{(f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((f + g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])),x]
[Out] Integrate[1/((f + g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])), x]
```

Maple [A]
time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{1}{(gx+f)\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(g*x+f)/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
[Out] int(1/(g*x+f)/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="maxima")
[Out] integrate(1/((g*x + f)*(B*log(((b*x + a)/(d*x + c))^n*e) + A)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fricas")
[Out] integral(1/(A*g*x + A*f + (B*g*x + B*f)*log(((b*x + a)/(d*x + c))^n*e)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(A + B \log \left(e \left(\frac{a}{c+dx} + \frac{bx}{c+dx}\right)^{n}\right)\right) (f + gx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)/(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
[Out] Integral(1/((A + B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n))*(f + g*x)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="giac")
[Out] integrate(1/((g*x + f)*(B*log(((b*x + a)/(d*x + c))^n*e) + A)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{(f+g\,x)\,\left(A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((f + g*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))),x)
[Out] int(1/((f + g*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))), x)
```

3.80 $\int \frac{1}{(f+gx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx$

Optimal. Leaf size=35

$$\text{Int}\left(\frac{1}{(f+gx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)},x\right)$$

```
[Out] Unintegrable(1/(g*x+f)^2/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(f+gx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((f + g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])),x]
[Out] Defer[Int][1/((f + g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])), x]
```

Rubi steps

$$\int \frac{1}{(f+gx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx=\int \frac{1}{(f+gx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx$$

Mathematica [A]
time = 0.57, size = 0, normalized size = 0.00

$$\int \frac{1}{(f+gx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((f + g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])),x]
[Out] Integrate[1/((f + g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])), x]
```

Maple [A]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{1}{(gx+f)^2\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(g*x+f)^2/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

```
[Out] int(1/(g*x+f)^2/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="maxima
")
```

```
[Out] integrate(1/((g*x + f)^2*(B*log(((b*x + a)/(d*x + c))^n*e) + A)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fricas
")
```

```
[Out] integral(1/(A*g^2*x^2 + 2*A*f*g*x + A*f^2 + (B*g^2*x^2 + 2*B*f*g*x + B*f^2)
*log(((b*x + a)/(d*x + c))^n*e)), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)**2/(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="giac")
```

```
[Out] integrate(1/((g*x + f)^2*(B*log(((b*x + a)/(d*x + c))^n*e) + A)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{{\left(f+g\,x\right)}^2\,\left(A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((f + g*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))),x)
[Out] int(1/((f + g*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))), x)
```

3.81 $\int \frac{1}{(f+gx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx$

Optimal. Leaf size=35

$$\text{Int}\left(\frac{1}{(f+gx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)},x\right)$$

```
[Out] Unintegrable(1/(g*x+f)^3/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(f+gx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((f + g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])),x]
[Out] Defer[Int][1/((f + g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])), x]
```

Rubi steps

$$\int \frac{1}{(f+gx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx = \int \frac{1}{(f+gx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx$$

Mathematica [A]
time = 6.41, size = 0, normalized size = 0.00

$$\int \frac{1}{(f+gx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((f + g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])),x]
[Out] Integrate[1/((f + g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])), x]
```

Maple [A]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{1}{(gx+f)^3\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(g*x+f)^3/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

```
[Out] int(1/(g*x+f)^3/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)^3/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="maxima
")
```

```
[Out] integrate(1/((g*x + f)^3*(B*log(((b*x + a)/(d*x + c))^n*e) + A)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)^3/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fricas
")
```

```
[Out] integral(1/(A*g^3*x^3 + 3*A*f*g^2*x^2 + 3*A*f^2*g*x + A*f^3 + (B*g^3*x^3 +
3*B*f*g^2*x^2 + 3*B*f^2*g*x + B*f^3)*log(((b*x + a)/(d*x + c))^n*e)), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)**3/(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)^3/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="giac")
```

```
[Out] integrate(1/((g*x + f)^3*(B*log(((b*x + a)/(d*x + c))^n*e) + A)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{{\left(f+g\,x\right)}^3\,\left(A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((f + g*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))),x)
[Out] int(1/((f + g*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))), x)
```

3.82 $\int \frac{(f+gx)^2}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$

Optimal. Leaf size=35

$$\text{Int}\left(\frac{(f+gx)^2}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2},x\right)$$

[Out] Unintegrable((g*x+f)^2/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(f+gx)^2}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$

Verification is not applicable to the result.

[In] Int[(f + g*x)^2/(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]

[Out] Defer[Int][(f + g*x)^2/(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2, x]

Rubi steps

$$\begin{aligned}\int \frac{(f+gx)^2}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx &= \int \left(\frac{f^2}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}+\frac{2fgx}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}+\frac{g^2}{\left(A+B\log\left(\right.\right.}\right.\\ &= f^2\int \frac{1}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx+(2fg)\int \frac{x}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx\end{aligned}$$

Mathematica [A]
time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{(f+gx)^2}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$

Verification is not applicable to the result.

[In] Integrate[(f + g*x)^2/(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]

[Out] Integrate[(f + g*x)^2/(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2, x]

Maple [A]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{(gx+f)^2}{\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

```
[Out] int((g*x+f)^2/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="maxima
")
```

```
[Out] -(b*d*g^2*x^4 + a*c*f^2 + (a*d*g^2 + (2*d*f*g + c*g^2)*b)*x^3 + ((2*d*f*g +
 c*g^2)*a + (d*f^2 + 2*c*f*g)*b)*x^2 + (b*c*f^2 + (d*f^2 + 2*c*f*g)*a)*x)/(
(b*c*n - a*d*n)*B^2*log((b*x + a)^n) - (b*c*n - a*d*n)*B^2*log((d*x + c)^n)
 + (b*c*n - a*d*n)*A*B + (b*c*n - a*d*n)*B^2) + integrate((4*b*d*g^2*x^3 +
b*c*f^2 + 3*(a*d*g^2 + (2*d*f*g + c*g^2)*b)*x^2 + (d*f^2 + 2*c*f*g)*a + 2*(
(2*d*f*g + c*g^2)*a + (d*f^2 + 2*c*f*g)*b)*x)/((b*c*n - a*d*n)*B^2*log((b*x
 + a)^n) - (b*c*n - a*d*n)*B^2*log((d*x + c)^n) + (b*c*n - a*d*n)*A*B + (b*
c*n - a*d*n)*B^2), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="fricas
")
```

```
[Out] integral((g^2*x^2 + 2*f*g*x + f^2)/(B^2*log(((b*x + a)/(d*x + c))^n*e)^2 +
2*A*B*log(((b*x + a)/(d*x + c))^n*e) + A^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(f+gx)^2}{\left(A+B\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2/(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
```

```
[Out] Integral((f + g*x)**2/(A + B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n))**2, x
)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="giac")
```

```
[Out] integrate((g*x + f)^2/(B*log(((b*x + a)/(d*x + c))^n*e) + A)^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{{\left(f+g\,x\right)}^2}{{\left(A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^2/(A + B*log(e*((a + b*x)/(c + d*x))^n))^2,x)
```

```
[Out] int((f + g*x)^2/(A + B*log(e*((a + b*x)/(c + d*x))^n))^2, x)
```

3.83 $\int \frac{f+gx}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$

Optimal. Leaf size=33

$$\text{Int}\left(\frac{f+gx}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2},x\right)$$

```
[Out] Unintegrable((g*x+f)/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

Rubi [A]

time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{f+gx}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(f + g*x)/(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

```
[Out] Defer[Int][(f + g*x)/(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{f+gx}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx &= \int \left(\frac{f}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}+\frac{gx}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\right)dx\\
&= f\int \frac{1}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx+g\int \frac{x}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx
\end{aligned}$$

Mathematica [A]

time = 0.46, size = 0, normalized size = 0.00

$$\int \frac{f+gx}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(f + g*x)/(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

```
[Out] Integrate[(f + g*x)/(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2, x]
```

Maple [A]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{gx+f}{\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

```
[Out] int((g*x+f)/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="maxima")
```

```
[Out] -(b*d*g*x^3 + a*c*f + (a*d*g + (d*f + c*g)*b)*x^2 + (b*c*f + (d*f + c*g)*a)
*x)/((b*c*n - a*d*n)*B^2*log((b*x + a)^n) - (b*c*n - a*d*n)*B^2*log((d*x +
c)^n) + (b*c*n - a*d*n)*A*B + (b*c*n - a*d*n)*B^2) + integrate((3*b*d*g*x^2
 + b*c*f + (d*f + c*g)*a + 2*(a*d*g + (d*f + c*g)*b)*x)/((b*c*n - a*d*n)*B^
2*log((b*x + a)^n) - (b*c*n - a*d*n)*B^2*log((d*x + c)^n) + (b*c*n - a*d*n)
*A*B + (b*c*n - a*d*n)*B^2), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="fricas")
```

```
[Out] integral((g*x + f)/(B^2*log(((b*x + a)/(d*x + c))^n*e)^2 + 2*A*B*log(((b*x
+ a)/(d*x + c))^n*e) + A^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{f+gx}{\left(A+B\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^{n}\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)/(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
```

```
[Out] Integral((f + g*x)/(A + B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n))**2, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="giac")
[Out] integrate((g*x + f)/(B*log(((b*x + a)/(d*x + c))^n*e) + A)^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{f+g\,x}{{\left(A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)/(A + B*log(e*((a + b*x)/(c + d*x))^n))^2,x)
[Out] int((f + g*x)/(A + B*log(e*((a + b*x)/(c + d*x))^n))^2, x)
```

3.84 $\int \frac{1}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{1}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2},x\right)$$

```
[Out] Unintegrable(1/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

Rubi [A]
time = 0.00, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^(-2),x]
[Out] Defer[Int][(A + B*Log[e*((a + b*x)/(c + d*x))^n])^(-2), x]
```

Rubi steps

$$\int \frac{1}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{1}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$

Mathematica [A]
time = 0.41, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^(-2),x]
[Out] Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^(-2), x]
```

Maple [A]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

```
[Out] int(1/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="maxima")
```

```
[Out] -(b*d*x^2 + a*c + (b*c + a*d)*x)/((b*c*n - a*d*n)*B^2*log((b*x + a)^n) - (b
*c*n - a*d*n)*B^2*log((d*x + c)^n) + (b*c*n - a*d*n)*A*B + (b*c*n - a*d*n)*
B^2) + integrate((2*b*d*x + b*c + a*d)/((b*c*n - a*d*n)*B^2*log((b*x + a)^n
) - (b*c*n - a*d*n)*B^2*log((d*x + c)^n) + (b*c*n - a*d*n)*A*B + (b*c*n - a
*d*n)*B^2), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="fricas")
```

```
[Out] integral(1/(B^2*log(((b*x + a)/(d*x + c))^n*e)^2 + 2*A*B*log(((b*x + a)/(d*
x + c))^n*e) + A^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^{n}\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
```

```
[Out] Integral((A + B*log(e*((a + b*x)/(c + d*x))**n))**(-2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="giac")
```

```
[Out] integrate((B*log(((b*x + a)/(d*x + c))^n*e) + A)^(-2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\left(A+B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(A + B*log(e*((a + b*x)/(c + d*x))^n))^2,x)
```

```
[Out] int(1/(A + B*log(e*((a + b*x)/(c + d*x))^n))^2, x)
```

3.85 $\int \frac{1}{(f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$

Optimal. Leaf size=35

$$\text{Int}\left(\frac{1}{(f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2},x\right)$$

```
[Out] Unintegrable(1/(g*x+f)/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((f + g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2),x]
[Out] Defer[Int][1/((f + g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2), x]
```

Rubi steps

$$\int \frac{1}{(f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{1}{(f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$

Mathematica [A]
time = 0.95, size = 0, normalized size = 0.00

$$\int \frac{1}{(f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((f + g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2),x]
[Out] Integrate[1/((f + g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2), x]
```

Maple [A]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{1}{(gx+f)\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(g*x+f)/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

```
[Out] int(1/(g*x+f)/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="maxima
")
```

```
[Out] -(b*d*x^2 + a*c + (b*c + a*d)*x)/((b*c*f*n - a*d*f*n)*A*B + (b*c*f*n - a*d*
f*n)*B^2 + ((b*c*g*n - a*d*g*n)*A*B + (b*c*g*n - a*d*g*n)*B^2)*x + ((b*c*g*
n - a*d*g*n)*B^2*x + (b*c*f*n - a*d*f*n)*B^2)*log((b*x + a)^n) - ((b*c*g*n
- a*d*g*n)*B^2*x + (b*c*f*n - a*d*f*n)*B^2)*log((d*x + c)^n)) + integrate((
b*d*g*x^2 + 2*b*d*f*x + b*c*f + (d*f - c*g)*a)/((b*c*f^2*n - a*d*f^2*n)*A*B
 + (b*c*f^2*n - a*d*f^2*n)*B^2 + ((b*c*g^2*n - a*d*g^2*n)*A*B + (b*c*g^2*n
- a*d*g^2*n)*B^2)*x^2 + 2*((b*c*f*g*n - a*d*f*g*n)*A*B + (b*c*f*g*n - a*d*f
*g*n)*B^2)*x + ((b*c*g^2*n - a*d*g^2*n)*B^2*x^2 + 2*(b*c*f*g*n - a*d*f*g*n)
*B^2*x + (b*c*f^2*n - a*d*f^2*n)*B^2)*log((b*x + a)^n) - ((b*c*g^2*n - a*d*
g^2*n)*B^2*x^2 + 2*(b*c*f*g*n - a*d*f*g*n)*B^2*x + (b*c*f^2*n - a*d*f^2*n)*
B^2)*log((d*x + c)^n)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="fricas
")
```

```
[Out] integral(1/(A^2*g*x + A^2*f + (B^2*g*x + B^2*f)*log(((b*x + a)/(d*x + c))^n
*e)^2 + 2*(A*B*g*x + A*B*f)*log(((b*x + a)/(d*x + c))^n*e)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(A + B\log\left(e\left(\frac{a}{c+dx} + \frac{bx}{c+dx}\right)^{n}\right)\right)^{2}(f + gx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)/(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
```

```
[Out] Integral(1/((A + B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n))**2*(f + g*x)),
x)
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((g*x + f)*(B*log(((b*x + a)/(d*x + c))^n*e) + A)^2), x)
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{(f+g\,x)\,\left(A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((f + g*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2),x)
```

```
[Out] int(1/((f + g*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2), x)
```

3.86 $\int \frac{1}{(f+gx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$

Optimal. Leaf size=35

$$\text{Int}\left(\frac{1}{(f+gx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2},x\right)$$

```
[Out] Unintegrable(1/(g*x+f)^2/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(f+gx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((f + g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2),x]
[Out] Defer[Int][1/((f + g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2), x]
```

Rubi steps

$$\int \frac{1}{(f+gx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = \int \frac{1}{(f+gx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$

Mathematica [A]
time = 1.84, size = 0, normalized size = 0.00

$$\int \frac{1}{(f+gx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((f + g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2),x]
[Out] Integrate[1/((f + g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2), x]
```

Maple [A]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(gx+f\right)^{2}\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(g*x+f)^2/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

```
[Out] int(1/(g*x+f)^2/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="maxi
ma")
```

```
[Out] -(b*d*x^2 + a*c + (b*c + a*d)*x)/((b*c*f^2*n - a*d*f^2*n)*A*B + (b*c*f^2*n
- a*d*f^2*n)*B^2 + ((b*c*g^2*n - a*d*g^2*n)*A*B + (b*c*g^2*n - a*d*g^2*n)*B
^2)*x^2 + 2*((b*c*f*g*n - a*d*f*g*n)*A*B + (b*c*f*g*n - a*d*f*g*n)*B^2)*x +
 ((b*c*g^2*n - a*d*g^2*n)*B^2*x^2 + 2*(b*c*f*g*n - a*d*f*g*n)*B^2*x + (b*c*
f^2*n - a*d*f^2*n)*B^2)*log((b*x + a)^n) - ((b*c*g^2*n - a*d*g^2*n)*B^2*x^2
 + 2*(b*c*f*g*n - a*d*f*g*n)*B^2*x + (b*c*f^2*n - a*d*f^2*n)*B^2)*log((d*x
+ c)^n)) - integrate(-(b*c*f + (d*f - 2*c*g)*a - (a*d*g - (2*d*f - c*g)*b)*
x)/(((b*c*g^3*n - a*d*g^3*n)*A*B + (b*c*g^3*n - a*d*g^3*n)*B^2)*x^3 + (b*c*
f^3*n - a*d*f^3*n)*A*B + (b*c*f^3*n - a*d*f^3*n)*B^2 + 3*((b*c*f*g^2*n - a*
d*f*g^2*n)*A*B + (b*c*f*g^2*n - a*d*f*g^2*n)*B^2)*x^2 + 3*((b*c*f^2*g*n - a
*d*f^2*g*n)*A*B + (b*c*f^2*g*n - a*d*f^2*g*n)*B^2)*x + ((b*c*g^3*n - a*d*g^
3*n)*B^2*x^3 + 3*(b*c*f*g^2*n - a*d*f*g^2*n)*B^2*x^2 + 3*(b*c*f^2*g*n - a*d
*f^2*g*n)*B^2*x + (b*c*f^3*n - a*d*f^3*n)*B^2)*log((b*x + a)^n) - ((b*c*g^3
*n - a*d*g^3*n)*B^2*x^3 + 3*(b*c*f*g^2*n - a*d*f*g^2*n)*B^2*x^2 + 3*(b*c*f^
2*g*n - a*d*f^2*g*n)*B^2*x + (b*c*f^3*n - a*d*f^3*n)*B^2)*log((d*x + c)^n))
, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="fric
as")
```

```
[Out] integral(1/(A^2*g^2*x^2 + 2*A^2*f*g*x + A^2*f^2 + (B^2*g^2*x^2 + 2*B^2*f*g*
x + B^2*f^2)*log(((b*x + a)/(d*x + c))^n*e)^2 + 2*(A*B*g^2*x^2 + 2*A*B*f*g*
x + A*B*f^2)*log(((b*x + a)/(d*x + c))^n*e)), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)**2/(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="giac
")
```

```
[Out] integrate(1/((g*x + f)^2*(B*log(((b*x + a)/(d*x + c))^n*e) + A)^2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{{\left(f+g\,x\right)}^2\,{\left(A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((f + g*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2),x)
```

```
[Out] int(1/((f + g*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2), x)
```

3.87 $\int \frac{1}{(f+gx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$

Optimal. Leaf size=35

$$\text{Int}\left(\frac{1}{(f+gx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2},x\right)$$

```
[Out] Unintegrable(1/(g*x+f)^3/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(f+gx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((f + g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2),x]
[Out] Defer[Int][1/((f + g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2), x]
```

Rubi steps

$$\int \frac{1}{(f+gx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{1}{(f+gx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$

Mathematica [A]
time = 20.88, size = 0, normalized size = 0.00

$$\int \frac{1}{(f+gx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((f + g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2),x]
[Out] Integrate[1/((f + g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2), x]
```

Maple [A]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(gx+f\right)^{3}\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(g*x+f)^3/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

```
[Out] int(1/(g*x+f)^3/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)^3/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="maxi
ma")
```

```
[Out] -(b*d*x^2 + a*c + (b*c + a*d)*x)/(((b*c*g^3*n - a*d*g^3*n)*A*B + (b*c*g^3*n
 - a*d*g^3*n)*B^2)*x^3 + (b*c*f^3*n - a*d*f^3*n)*A*B + (b*c*f^3*n - a*d*f^3
*n)*B^2 + 3*((b*c*f*g^2*n - a*d*f*g^2*n)*A*B + (b*c*f*g^2*n - a*d*f*g^2*n)*
B^2)*x^2 + 3*((b*c*f^2*g*n - a*d*f^2*g*n)*A*B + (b*c*f^2*g*n - a*d*f^2*g*n)
*B^2)*x + ((b*c*g^3*n - a*d*g^3*n)*B^2*x^3 + 3*(b*c*f*g^2*n - a*d*f*g^2*n)*
B^2*x^2 + 3*(b*c*f^2*g*n - a*d*f^2*g*n)*B^2*x + (b*c*f^3*n - a*d*f^3*n)*B^2
)*log((b*x + a)^n) - ((b*c*g^3*n - a*d*g^3*n)*B^2*x^3 + 3*(b*c*f*g^2*n - a*
d*f*g^2*n)*B^2*x^2 + 3*(b*c*f^2*g*n - a*d*f^2*g*n)*B^2*x + (b*c*f^3*n - a*d
*f^3*n)*B^2)*log((d*x + c)^n)) - integrate((b*d*g*x^2 - b*c*f - (d*f - 3*c*
g)*a + 2*(a*d*g - (d*f - c*g)*b)*x)/(((b*c*g^4*n - a*d*g^4*n)*A*B + (b*c*g^
4*n - a*d*g^4*n)*B^2)*x^4 + 4*((b*c*f*g^3*n - a*d*f*g^3*n)*A*B + (b*c*f*g^3
*n - a*d*f*g^3*n)*B^2)*x^3 + (b*c*f^4*n - a*d*f^4*n)*A*B + (b*c*f^4*n - a*d
*f^4*n)*B^2 + 6*((b*c*f^2*g^2*n - a*d*f^2*g^2*n)*A*B + (b*c*f^2*g^2*n - a*d
*f^2*g^2*n)*B^2)*x^2 + 4*((b*c*f^3*g*n - a*d*f^3*g*n)*A*B + (b*c*f^3*g*n -
a*d*f^3*g*n)*B^2)*x + ((b*c*g^4*n - a*d*g^4*n)*B^2*x^4 + 4*(b*c*f*g^3*n - a
*d*f*g^3*n)*B^2*x^3 + 6*(b*c*f^2*g^2*n - a*d*f^2*g^2*n)*B^2*x^2 + 4*(b*c*f^
3*g*n - a*d*f^3*g*n)*B^2*x + (b*c*f^4*n - a*d*f^4*n)*B^2)*log((b*x + a)^n)
- ((b*c*g^4*n - a*d*g^4*n)*B^2*x^4 + 4*(b*c*f*g^3*n - a*d*f*g^3*n)*B^2*x^3
+ 6*(b*c*f^2*g^2*n - a*d*f^2*g^2*n)*B^2*x^2 + 4*(b*c*f^3*g*n - a*d*f^3*g*n)
*B^2*x + (b*c*f^4*n - a*d*f^4*n)*B^2)*log((d*x + c)^n)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)^3/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="fric
as")
```

```
[Out] integral(1/(A^2*g^3*x^3 + 3*A^2*f*g^2*x^2 + 3*A^2*f^2*g*x + A^2*f^3 + (B^2*
g^3*x^3 + 3*B^2*f*g^2*x^2 + 3*B^2*f^2*g*x + B^2*f^3)*log(((b*x + a)/(d*x +
```

```
c))^n*e)^2 + 2*(A*B*g^3*x^3 + 3*A*B*f*g^2*x^2 + 3*A*B*f^2*g*x + A*B*f^3)*lo
g(((b*x + a)/(d*x + c))^n*e)), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)**3/(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)^3/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="giac
")
```

```
[Out] integrate(1/((g*x + f)^3*(B*log(((b*x + a)/(d*x + c))^n*e) + A)^2), x)
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{{\left(f+g\,x\right)}^3\,{\left(A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((f + g*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2),x)
```

```
[Out] int(1/((f + g*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2), x)
```

3.88 $\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$

Optimal. Leaf size=180

$$\frac{B(bc-ad)^4g^4x}{5d^4}-\frac{B(bc-ad)^3g^4(a+bx)^2}{10bd^3}+\frac{B(bc-ad)^2g^4(a+bx)^3}{15bd^2}-\frac{B(bc-ad)g^4(a+bx)^4}{20bd}+\frac{g^4(a+bx)^5\left(\right.}{}$$

```
[Out] 1/5*B*(-a*d+b*c)^4*g^4*x/d^4-1/10*B*(-a*d+b*c)^3*g^4*(b*x+a)^2/b/d^3+1/15*B
*(-a*d+b*c)^2*g^4*(b*x+a)^3/b/d^2-1/20*B*(-a*d+b*c)*g^4*(b*x+a)^4/b/d+1/5*g
^4*(b*x+a)^5*(A+B*ln(e*(b*x+a)/(d*x+c)))/b-1/5*B*(-a*d+b*c)^5*g^4*ln(d*x+c)
/b/d^5
```

Rubi [A]
time = 0.09, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2548, 21, 45}

$$\frac{g^4(a+bx)^5\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{5b}-\frac{Bg^4(bc-ad)^5\log(c+dx)}{5bd^5}+\frac{Bg^4x(bc-ad)^4}{5d^4}-\frac{Bg^4(a+bx)^2(bc-ad)^3}{10bd^3}+\frac{Bg^4(a+bx)^3(bc-ad)^2}{15bd^2}-\frac{Bg^4(a+bx)^4(bc-ad)}{20bd}$$

Antiderivative was successfully verified.

```
[In] Int[(a*g + b*g*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
```

```
[Out] (B*(b*c - a*d)^4*g^4*x)/(5*d^4) - (B*(b*c - a*d)^3*g^4*(a + b*x)^2)/(10*b*d
^3) + (B*(b*c - a*d)^2*g^4*(a + b*x)^3)/(15*b*d^2) - (B*(b*c - a*d)*g^4*(a
+ b*x)^4)/(20*b*d) + (g^4*(a + b*x)^5*(A + B*Log[(e*(a + b*x))/(c + d*x)]))
/(5*b) - (B*(b*c - a*d)^5*g^4*Log[c + d*x])/(5*b*d^5)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2548

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
```

```
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Dist[B*n*((b*c
- a*d)/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /;
 FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c -
 a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Rubi steps

$$\int (ag+bgx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)dx = \frac{g^4(a+bx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{5b} - \frac{B\int\frac{(bc-ad)g^5(a+bx)^4}{c+dx}\,dx}{5bg}$$

$$= \frac{g^4(a+bx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{5b} - \frac{(B(bc-ad)g^4)\int\frac{(a+}{c+}}{5b}$$

$$= \frac{g^4(a+bx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{5b} - \frac{(B(bc-ad)g^4)\int\left(-\right.}{}$$

$$= \frac{B(bc-ad)^4g^4x}{5d^4} - \frac{B(bc-ad)^3g^4(a+bx)^2}{10bd^3} + \frac{B(bc-ad)^2g}{15b}$$

Mathematica [A]
time = 0.07, size = 142, normalized size = 0.79

$$\frac{g^4\left((a+bx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)-\frac{B(bc-ad)\left(-12bd(bc-ad)^3x+6d^2(bc-ad)^2(a+bx)^2+4d^3(-bc+ad)(a+bx)^3+3d^4(a+bx)^4+12(bc-ad)^4\log(c+dx)\right)}{12d^5}\right)}{5b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*g + b*g*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
```

```
[Out] (g^4*((a + b*x)^5*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - (B*(b*c - a*d)*(-1
2*b*d*(b*c - a*d)^3*x + 6*d^2*(b*c - a*d)^2*(a + b*x)^2 + 4*d^3*(-(b*c) + a
*d)*(a + b*x)^3 + 3*d^4*(a + b*x)^4 + 12*(b*c - a*d)^4*Log[c + d*x]))/(12*d
^5)))/(5*b)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 9744 vs. $2(168) = 336$.
time = 0.34, size = 9745, normalized size = 54.14

method	result
risch	$\frac{g^4B\ln(dx+c)a^5}{5b}+\frac{(bx+a)^5g^4B\ln\left(\frac{e(bx+a)}{dx+c}\right)}{5b}+g^4b^3Aa\,x^4+\frac{g^4b^3Ba\,x^4}{20}-\frac{g^4b^4Bc\,x^4}{20d}+\frac{g^4b^4A\,x^5}{5}-\frac{2g^4bB}{d}$
derivativedivides	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*g*x+a*g)^4*(A+B*ln(e*(b*x+a)/(d*x+c))),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 633 vs. $2(169) = 338$.
time = 0.30, size = 633, normalized size = 3.52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^4*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="maxima")
```

```
[Out] 1/5*A*b^4*g^4*x^5 + A*a*b^3*g^4*x^4 + 2*A*a^2*b^2*g^4*x^3 + 2*A*a^3*b*g^4*x
^2 + (x*log(b*x*e/(d*x + c) + a*e/(d*x + c)) + a*log(b*x + a)/b - c*log(d*x
 + c)/d)*B*a^4*g^4 + 2*(x^2*log(b*x*e/(d*x + c) + a*e/(d*x + c)) - a^2*log(
b*x + a)/b^2 + c^2*log(d*x + c)/d^2 - (b*c - a*d)*x/(b*d))*B*a^3*b*g^4 + (2
*x^3*log(b*x*e/(d*x + c) + a*e/(d*x + c)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*
log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2
*d^2))*B*a^2*b^2*g^4 + 1/6*(6*x^4*log(b*x*e/(d*x + c) + a*e/(d*x + c)) - 6*
a^4*log(b*x + a)/b^4 + 6*c^4*log(d*x + c)/d^4 - (2*(b^3*c*d^2 - a*b^2*d^3)*
x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3))*B
*a*b^3*g^4 + 1/60*(12*x^5*log(b*x*e/(d*x + c) + a*e/(d*x + c)) + 12*a^5*log
(b*x + a)/b^5 - 12*c^5*log(d*x + c)/d^5 - (3*(b^4*c*d^3 - a*b^3*d^4)*x^4 -
4*(b^4*c^2*d^2 - a^2*b^2*d^4)*x^3 + 6*(b^4*c^3*d - a^3*b*d^4)*x^2 - 12*(b^4
*c^4 - a^4*d^4)*x)/(b^4*d^4))*B*b^4*g^4 + A*a^4*g^4*x
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 430 vs. $2(169) = 338$.
time = 0.44, size = 430, normalized size = 2.39

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^4*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="fricas")
```

```
[Out] 1/60*(12*A*b^5*d^5*g^4*x^5 + 12*B*a^5*d^5*g^4*log(b*x + a) - 3*(B*b^5*c*d^4
 - (20*A + B)*a*b^4*d^5)*g^4*x^4 + 4*(B*b^5*c^2*d^3 - 5*B*a*b^4*c*d^4 + 2*(
15*A + 2*B)*a^2*b^3*d^5)*g^4*x^3 - 6*(B*b^5*c^3*d^2 - 5*B*a*b^4*c^2*d^3 + 1
0*B*a^2*b^3*c*d^4 - 2*(10*A + 3*B)*a^3*b^2*d^5)*g^4*x^2 + 12*(B*b^5*c^4*d -
 5*B*a*b^4*c^3*d^2 + 10*B*a^2*b^3*c^2*d^3 - 10*B*a^3*b^2*c*d^4 + (5*A + 4*B
)*a^4*b*d^5)*g^4*x - 12*(B*b^5*c^5 - 5*B*a*b^4*c^4*d + 10*B*a^2*b^3*c^3*d^2
```

```
 - 10*B*a^3*b^2*c^2*d^3 + 5*B*a^4*b*c*d^4)*g^4*log(d*x + c) + 12*(B*b^5*d^5
*g^4*x^5 + 5*B*a*b^4*d^5*g^4*x^4 + 10*B*a^2*b^3*d^5*g^4*x^3 + 10*B*a^3*b^2*
d^5*g^4*x^2 + 5*B*a^4*b*d^5*g^4*x)*log((b*x + a)*e/(d*x + c)))/(b*d^5)
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 969 vs. $2(155) = 310$.
time = 4.80, size = 969, normalized size = 5.38

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)**4*(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

```
[Out] A*b**4*g**4*x**5/5 + B*a**5*g**4*log(x + (B*a**6*d**5*g**4/b + 5*B*a**5*c*d
**4*g**4 - 10*B*a**4*b*c**2*d**3*g**4 + 10*B*a**3*b**2*c**3*d**2*g**4 - 5*B
*a**2*b**3*c**4*d*g**4 + B*a*b**4*c**5*g**4)/(B*a**5*d**5*g**4 + 5*B*a**4*b
*c*d**4*g**4 - 10*B*a**3*b**2*c**2*d**3*g**4 + 10*B*a**2*b**3*c**3*d**2*g**
4 - 5*B*a*b**4*c**4*d*g**4 + B*b**5*c**5*g**4))/(5*b) - B*c*g**4*(5*a**4*d*
*4 - 10*a**3*b*c*d**3 + 10*a**2*b**2*c**2*d**2 - 5*a*b**3*c**3*d + b**4*c**
4)*log(x + (6*B*a**5*c*d**4*g**4 - 10*B*a**4*b*c**2*d**3*g**4 + 10*B*a**3*b
**2*c**3*d**2*g**4 - 5*B*a**2*b**3*c**4*d*g**4 + B*a*b**4*c**5*g**4 - B*a*c
*g**4*(5*a**4*d**4 - 10*a**3*b*c*d**3 + 10*a**2*b**2*c**2*d**2 - 5*a*b**3*c
**3*d + b**4*c**4) + B*b*c**2*g**4*(5*a**4*d**4 - 10*a**3*b*c*d**3 + 10*a**
2*b**2*c**2*d**2 - 5*a*b**3*c**3*d + b**4*c**4)/d)/(B*a**5*d**5*g**4 + 5*B*
a**4*b*c*d**4*g**4 - 10*B*a**3*b**2*c**2*d**3*g**4 + 10*B*a**2*b**3*c**3*d*
*2*g**4 - 5*B*a*b**4*c**4*d*g**4 + B*b**5*c**5*g**4))/(5*d**5) + x**4*(A*a*
b**3*g**4 + B*a*b**3*g**4/20 - B*b**4*c*g**4/(20*d)) + x**3*(2*A*a**2*b**2*
g**4 + 4*B*a**2*b**2*g**4/15 - B*a*b**3*c*g**4/(3*d) + B*b**4*c**2*g**4/(15
*d**2)) + x**2*(2*A*a**3*b*g**4 + 3*B*a**3*b*g**4/5 - B*a**2*b**2*c*g**4/d
+ B*a*b**3*c**2*g**4/(2*d**2) - B*b**4*c**3*g**4/(10*d**3)) + x*(A*a**4*g**
4 + 4*B*a**4*g**4/5 - 2*B*a**3*b*c*g**4/d + 2*B*a**2*b**2*c**2*g**4/d**2 -
B*a*b**3*c**3*g**4/d**3 + B*b**4*c**4*g**4/(5*d**4)) + (B*a**4*g**4*x + 2*B
*a**3*b*g**4*x**2 + 2*B*a**2*b**2*g**4*x**3 + B*a*b**3*g**4*x**4 + B*b**4*g
**4*x**5/5)*log(e*(a + b*x)/(c + d*x))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 5428 vs. $2(169) = 338$.
time = 4.18, size = 5428, normalized size = 30.16

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^4*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="giac")
```

```
[Out] 1/60*(12*B*b^11*c^6*g^4*e^6*log(-b*e + (b*x*e + a*e)*d/(d*x + c)) - 72*B*a*
b^10*c^5*d*g^4*e^6*log(-b*e + (b*x*e + a*e)*d/(d*x + c)) + 180*B*a^2*b^9*c^
```

4*d^2*g^4*e^6*log(-b*e + (b*x*e + a*e)*d/(d*x + c)) - 240*B*a^3*b^8*c^3*d^3
*g^4*e^6*log(-b*e + (b*x*e + a*e)*d/(d*x + c)) + 180*B*a^4*b^7*c^2*d^4*g^4*
e^6*log(-b*e + (b*x*e + a*e)*d/(d*x + c)) - 72*B*a^5*b^6*c*d^5*g^4*e^6*log(
-b*e + (b*x*e + a*e)*d/(d*x + c)) + 12*B*a^6*b^5*d^6*g^4*e^6*log(-b*e + (b*
x*e + a*e)*d/(d*x + c)) - 60*(b*x*e + a*e)*B*b^10*c^6*d*g^4*e^5*log(-b*e +
(b*x*e + a*e)*d/(d*x + c))/(d*x + c) + 360*(b*x*e + a*e)*B*a*b^9*c^5*d^2*g^
4*e^5*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c) - 900*(b*x*e + a*e)*B
*a^2*b^8*c^4*d^3*g^4*e^5*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c) +
1200*(b*x*e + a*e)*B*a^3*b^7*c^3*d^4*g^4*e^5*log(-b*e + (b*x*e + a*e)*d/(d*
x + c))/(d*x + c) - 900*(b*x*e + a*e)*B*a^4*b^6*c^2*d^5*g^4*e^5*log(-b*e +
(b*x*e + a*e)*d/(d*x + c))/(d*x + c) + 360*(b*x*e + a*e)*B*a^5*b^5*c*d^6*g^
4*e^5*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c) - 60*(b*x*e + a*e)*B*
a^6*b^4*d^7*g^4*e^5*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c) + 120*(
b*x*e + a*e)^2*B*b^9*c^6*d^2*g^4*e^4*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/
(d*x + c)^2 - 720*(b*x*e + a*e)^2*B*a*b^8*c^5*d^3*g^4*e^4*log(-b*e + (b*x*e
 + a*e)*d/(d*x + c))/(d*x + c)^2 + 1800*(b*x*e + a*e)^2*B*a^2*b^7*c^4*d^4*g
^4*e^4*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^2 - 2400*(b*x*e + a*
e)^2*B*a^3*b^6*c^3*d^5*g^4*e^4*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x +
 c)^2 + 1800*(b*x*e + a*e)^2*B*a^4*b^5*c^2*d^6*g^4*e^4*log(-b*e + (b*x*e +
a*e)*d/(d*x + c))/(d*x + c)^2 - 720*(b*x*e + a*e)^2*B*a^5*b^4*c*d^7*g^4*e^4
*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^2 + 120*(b*x*e + a*e)^2*B*
a^6*b^3*d^8*g^4*e^4*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^2 - 120
*(b*x*e + a*e)^3*B*b^8*c^6*d^3*g^4*e^3*log(-b*e + (b*x*e + a*e)*d/(d*x + c)
)/(d*x + c)^3 + 720*(b*x*e + a*e)^3*B*a*b^7*c^5*d^4*g^4*e^3*log(-b*e + (b*x
*e + a*e)*d/(d*x + c))/(d*x + c)^3 - 1800*(b*x*e + a*e)^3*B*a^2*b^6*c^4*d^5
*g^4*e^3*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^3 + 2400*(b*x*e +
a*e)^3*B*a^3*b^5*c^3*d^6*g^4*e^3*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x
 + c)^3 - 1800*(b*x*e + a*e)^3*B*a^4*b^4*c^2*d^7*g^4*e^3*log(-b*e + (b*x*e
+ a*e)*d/(d*x + c))/(d*x + c)^3 + 720*(b*x*e + a*e)^3*B*a^5*b^3*c*d^8*g^4*e
^3*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^3 - 120*(b*x*e + a*e)^3*
B*a^6*b^2*d^9*g^4*e^3*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^3 + 6
0*(b*x*e + a*e)^4*B*b^7*c^6*d^4*g^4*e^2*log(-b*e + (b*x*e + a*e)*d/(d*x + c
))/(d*x + c)^4 - 360*(b*x*e + a*e)^4*B*a*b^6*c^5*d^5*g^4*e^2*log(-b*e + (b*
x*e + a*e)*d/(d*x + c))/(d*x + c)^4 + 900*(b*x*e + a*e)^4*B*a^2*b^5*c^4*d^6
*g^4*e^2*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^4 - 1200*(b*x*e +
a*e)^4*B*a^3*b^4*c^3*d^7*g^4*e^2*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x
 + c)^4 + 900*(b*x*e + a*e)^4*B*a^4*b^3*c^2*d^8*g^4*e^2*log(-b*e + (b*x*e +
 a*e)*d/(d*x + c))/(d*x + c)^4 - 360*(b*x*e + a*e)^4*B*a^5*b^2*c*d^9*g^4*e^
2*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^4 + 60*(b*x*e + a*e)^4*B*
a^6*b*d^10*g^4*e^2*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^4 - 12*(
b*x*e + a*e)^5*B*b^6*c^6*d^5*g^4*e*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d
x + c)^5 + 72(b*x*e + a*e)^5*B*a*b^5*c^5*d^6*g^4*e*log(-b*e + (b*x*e + a*
e)*d/(d*x + c))/(d*x + c)^5 - 180*(b*x*e + a*e)^5*B*a^2*b^4*c^4*d^7*g^4*e*l
og(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^5 + 240*(b*x*e + a*e)^5*B*a^
3*b^3*c^3*d^8*g^4*e*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^5 - 180

```
*(b*x*e + a*e)^5*B*a^4*b^2*c^2*d^9*g^4*e*log(-b*e + (b*x*e + a*e)*d/(d*x +
c))/(d*x + c)^5 + 72*(b*x*e + a*e)^5*B*a^5*b*c*d^10*g^4*e*log(-b*e + (b*x*e
 + a*e)*d/(d*x + c))/(d*x + c)^5 - 12*(b*x*e + a*e)^5*B*a^6*d^11*g^4*e*log(
-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^5 + 12*(b*x*e + a*e)^5*B*b^6*c^
6*d^5*g^4*e*log((b*x*e + a*e)/(d*x + c))/(d*x + c)^5 - 72*(b*x*e + a*e)^5*B
*a*b^5*c^5*d^6*g^4*e*log((b*x*e + a*e)/(d*x + c))/(d*x + c)^5 + 180*(b*x*e
+ a*e)^5*B*a^2*b^4*c^4*d^7*g^4*e*log((b*x*e + a*e)/(d*x + c))/(d*x + c)^5 -
 240*(b*x*e + a*e)^5*B*a^3*b^3*c^3*d^8*g^4*e*log((b*x*e + a*e)/(d*x + c))/(
d*x + c)^5 + 180*(b*x*e + a*e)^5*B*a^4*b^2*c^2*d^9*g^4*e*log((b*x*e + a*e)/
(d*x + c))/(d*x + c)^5 - 72*(b*x*e + a*e)^5*B*a^5*b*c*d^10*g^4*e*log((b*x*e
 + a*e)/(d*x + c))/(d*x + c)^5 + 12*(b*x*e + a*e)^5*B*a^6*d^11*g^4*e*log((b
*x*e + a*e)/(d*x + c))/(d*x + c)^5 + 12*A*b^11*c^6*g^4*e^6 + 25*B*b^11*c^6*
g^4*e^6 - 72*A*a*b^10*c^5*d*g^4*e^6 - 150*B*a*b^10*c^5*d*g^4*e^6 + 180*A*a^
2*b^9*c^4*d^2*g^4*e^6 + 375*B*a^2*b^9*c^4*d^2*g^4*e^6 - 240*A*a^3*b^8*c^3*d
^3*g^4*e^6 - 500*B*a^3*b^8*c^3*d^3*g^4*e^6 + 180*A*a^4*b^7*c^2*d^4*g^4*e^6
+ 375*B*a^4*b^7*c^2*d^4*g^4*e^6 - 72*A*a^5*b^6*c*d^5*g^4*e^6 - 150*B*a^5*b^
6*c*d^5*g^4*e^6 + 12*A*a^6*b^5*d^6*g^4*e^6 + 25*B*a^6*b^5*d^6*g^4*e^6 - 60*
(b*x*e + a*e)*A*b^10*c^6*d*g^4*e^5/(d*x + c) - 113*(b*x*e + a*e)*B*b^10*c^6
*d*g^4*e^5/(d*x + c) + 360*(b*x*e + a*e)*A*a*b^9*c^5*d^2*g^4*e^5/(d*x + c)
+ 678*(b*x*e + a*e)*B*a*b^9*c^5*d^2*g^4*e^5/(d*...
```

Mupad [B]
time = 4.78, size = 1009, normalized size = 5.61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*g + b*g*x)^4*(A + B*log((e*(a + b*x))/(c + d*x))),x)
```

```
[Out] log((e*(a + b*x))/(c + d*x))*((B*b^4*g^4*x^5)/5 + B*a^4*g^4*x + 2*B*a^3*b*g
^4*x^2 + B*a*b^3*g^4*x^4 + 2*B*a^2*b^2*g^4*x^3) - x^3*((((b^3*g^4*(25*A*a*d
 + 5*A*b*c + B*a*d - B*b*c))/(5*d) - (A*b^3*g^4*(5*a*d + 5*b*c))/(5*d))*(5*
a*d + 5*b*c))/(15*b*d) - (a*b^2*g^4*(10*A*a*d + 5*A*b*c + B*a*d - B*b*c))/(
3*d) + (A*a*b^3*c*g^4)/(3*d)) + x^2*(((5*a*d + 5*b*c)*((((b^3*g^4*(25*A*a*d
 + 5*A*b*c + B*a*d - B*b*c))/(5*d) - (A*b^3*g^4*(5*a*d + 5*b*c))/(5*d))*(5*
a*d + 5*b*c))/(5*b*d) - (a*b^2*g^4*(10*A*a*d + 5*A*b*c + B*a*d - B*b*c))/d
+ (A*a*b^3*c*g^4)/d))/(10*b*d) + (a^2*b*g^4*(5*A*a*d + 5*A*b*c + B*a*d - B*
b*c))/d - (a*c*((b^3*g^4*(25*A*a*d + 5*A*b*c + B*a*d - B*b*c))/(5*d) - (A*b
^3*g^4*(5*a*d + 5*b*c))/(5*d)))/(2*b*d)) + x*((a^3*g^4*(5*A*a*d + 10*A*b*c
+ 2*B*a*d - 2*B*b*c))/d - ((5*a*d + 5*b*c)*(((5*a*d + 5*b*c)*((((b^3*g^4*(2
5*A*a*d + 5*A*b*c + B*a*d - B*b*c))/(5*d) - (A*b^3*g^4*(5*a*d + 5*b*c))/(5*
d))*(5*a*d + 5*b*c))/(5*b*d) - (a*b^2*g^4*(10*A*a*d + 5*A*b*c + B*a*d - B*b
*c))/d + (A*a*b^3*c*g^4)/d))/(5*b*d) + (2*a^2*b*g^4*(5*A*a*d + 5*A*b*c + B*
a*d - B*b*c))/d - (a*c*((b^3*g^4*(25*A*a*d + 5*A*b*c + B*a*d - B*b*c))/(5*d
) - (A*b^3*g^4*(5*a*d + 5*b*c))/(5*d)))/(b*d)))/(5*b*d) + (a*c*((((b^3*g^4*
```

```
(25*A*a*d + 5*A*b*c + B*a*d - B*b*c))/(5*d) - (A*b^3*g^4*(5*a*d + 5*b*c))/(
5*d))*(5*a*d + 5*b*c))/(5*b*d) - (a*b^2*g^4*(10*A*a*d + 5*A*b*c + B*a*d - B
*b*c))/d + (A*a*b^3*c*g^4)/d))/(b*d)) + x^4*((b^3*g^4*(25*A*a*d + 5*A*b*c +
 B*a*d - B*b*c))/(20*d) - (A*b^3*g^4*(5*a*d + 5*b*c))/(20*d)) - (log(c + d*
x)*(B*b^4*c^5*g^4 + 5*B*a^4*c*d^4*g^4 - 10*B*a^3*b*c^2*d^3*g^4 + 10*B*a^2*b
^2*c^3*d^2*g^4 - 5*B*a*b^3*c^4*d*g^4))/(5*d^5) + (A*b^4*g^4*x^5)/5 + (B*a^5
*g^4*log(a + b*x))/(5*b)
```

3.89 $\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$

Optimal. Leaf size=149

$$-\frac{B(bc-ad)^3g^3x}{4d^3}+\frac{B(bc-ad)^2g^3(a+bx)^2}{8bd^2}-\frac{B(bc-ad)g^3(a+bx)^3}{12bd}+\frac{g^3(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4b}+$$

```
[Out] -1/4*B*(-a*d+b*c)^3*g^3*x/d^3+1/8*B*(-a*d+b*c)^2*g^3*(b*x+a)^2/b/d^2-1/12*B
*(-a*d+b*c)*g^3*(b*x+a)^3/b/d+1/4*g^3*(b*x+a)^4*(A+B*ln(e*(b*x+a)/(d*x+c)))
/b+1/4*B*(-a*d+b*c)^4*g^3*ln(d*x+c)/b/d^4
```

Rubi [A]
time = 0.07, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2548, 21, 45}

$$\frac{g^3(a+bx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{4b}+\frac{Bg^3(bc-ad)^4\log(c+dx)}{4bd^4}-\frac{Bg^3x(bc-ad)^3}{4d^3}+\frac{Bg^3(a+bx)^2(bc-ad)^2}{8bd^2}-\frac{Bg^3(a+bx)^3(bc-ad)}{12bd}$$

Antiderivative was successfully verified.

```
[In] Int[(a*g + b*g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
```

```
[Out] -1/4*(B*(b*c - a*d)^3*g^3*x)/d^3 + (B*(b*c - a*d)^2*g^3*(a + b*x)^2)/(8*b*d
^2) - (B*(b*c - a*d)*g^3*(a + b*x)^3)/(12*b*d) + (g^3*(a + b*x)^4*(A + B*Lo
g[(e*(a + b*x))/(c + d*x)]))/(4*b) + (B*(b*c - a*d)^4*g^3*Log[c + d*x])/(4*
b*d^4)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2548

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Dist[B*n*((b*c
```

```
- a*d)/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /;
 FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c -
 a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Rubi steps

$$\int (ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)dx=\frac{g^3(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4b}-\frac{B\int\frac{(bc-ad)g^4(a+bx)^3}{c+dx}dx}{4bg}$$

$$=\frac{g^3(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4b}-\frac{(B(bc-ad)g^3)\int\frac{(a+bx}{c+d}$$

$$=\frac{g^3(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4b}-\frac{(B(bc-ad)g^3)\int\left(\frac{b(bc}{}\right.}{}$$

$$=-\frac{B(bc-ad)^3g^3x}{4d^3}+\frac{B(bc-ad)^2g^3(a+bx)^2}{8bd^2}-\frac{B(bc-ad)g}{12b}$$

Mathematica [A]
time = 0.07, size = 120, normalized size = 0.81

$$\frac{g^3\left((a+bx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)-\frac{B(bc-ad)\left(6bd(bc-ad)^2x+3d^2(-bc+ad)(a+bx)^2+2d^3(a+bx)^3-6(bc-ad)^3\log(c+dx)\right)}{6d^4}\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*g + b*g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
```

```
[Out] (g^3*((a + b*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - (B*(b*c - a*d)*(6*
b*d*(b*c - a*d)^2*x + 3*d^2*(-(b*c) + a*d)*(a + b*x)^2 + 2*d^3*(a + b*x)^3
- 6*(b*c - a*d)^3*Log[c + d*x]))/(6*d^4)))/(4*b)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 5349 vs. $2(139) = 278$.
time = 0.28, size = 5350, normalized size = 35.91

method	result
risch	$\frac{(bx+a)^4g^3B\ln\left(\frac{e(bx+a)}{dx+c}\right)}{4b}+\frac{g^3b^3Ax^4}{4}+g^3b^2Aax^3+\frac{g^3b^2Bax^3}{12}-\frac{g^3b^3Bcx^3}{12d}+\frac{3g^3bAa^2x^2}{2}+\frac{3g^3bBa^2x^2}{8}$
derivativedivides	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*g*x+a*g)^3*(A+B*ln(e*(b*x+a)/(d*x+c))),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 447 vs. $2(140) = 280$.
time = 0.29, size = 447, normalized size = 3.00

$$\frac{1}{4}\,Ab^3g^3x^4 + Aab^2g^3x^3 + \frac{3}{2}\,Aa^2bg^3x^2 + \left(x\log\left(\frac{bxe}{dx+c} + \frac{ae}{dx+c}\right) + \frac{a\log\left(bx+a\right)}{b} - \frac{c\log\left(dx+c\right)}{d}\right)Ba^3g^3 + \frac{3}{2}\left(x^2\log\left(\frac{bxe}{dx+c} + \frac{ae}{dx+c}\right) - \frac{a^2\log\left(bx+a\right)}{b^2} + \frac{c^2\log\left(dx+c\right)}{d^2} - \frac{(bc-ad)x}{bd}\right)Ba^2bg^3 + \frac{1}{2}\left(2\,x^3\log\left(\frac{bxe}{dx+c} + \frac{ae}{dx+c}\right) + \frac{2\,a^3\log\left(bx+a\right)}{b^3} - \frac{2\,c^3\log\left(dx+c\right)}{d^3} - \frac{(b^2cd - abd^2)x^2 - 2\,(b^2c^2 - a^2d^2)x}{b^2d^2}\right)Bab^2g^3 + \frac{1}{24}\left(6\,x^4\log\left(\frac{bxe}{dx+c} + \frac{ae}{dx+c}\right) - \frac{6\,a^4\log\left(bx+a\right)}{b^4} + \frac{6\,c^4\log\left(dx+c\right)}{d^4} - \frac{2\,(b^3cd^2 - ab^2d^3)x^3 - 3\,(b^3c^2d - a^2bd^3)x^2 + 6\,(b^3c^3 - a^3d^3)x}{b^3d^3}\right)Bb^3g^3 + Aa^3g^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^3*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="maxima")
```

```
[Out] 1/4*A*b^3*g^3*x^4 + A*a*b^2*g^3*x^3 + 3/2*A*a^2*b*g^3*x^2 + (x*log(b*x*e/(d
*x + c) + a*e/(d*x + c)) + a*log(b*x + a)/b - c*log(d*x + c)/d)*B*a^3*g^3 +
 3/2*(x^2*log(b*x*e/(d*x + c) + a*e/(d*x + c)) - a^2*log(b*x + a)/b^2 + c^2
*log(d*x + c)/d^2 - (b*c - a*d)*x/(b*d))*B*a^2*b*g^3 + 1/2*(2*x^3*log(b*x*e
/(d*x + c) + a*e/(d*x + c)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d
^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*B*a*b^2
*g^3 + 1/24*(6*x^4*log(b*x*e/(d*x + c) + a*e/(d*x + c)) - 6*a^4*log(b*x + a
)/b^4 + 6*c^4*log(d*x + c)/d^4 - (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^
2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3))*B*b^3*g^3 + A*a^
3*g^3*x
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 317 vs. $2(140) = 280$.
time = 0.39, size = 317, normalized size = 2.13

$$\frac{6\,Ab^4d^4g^3x^4 + 6\,Ba^4d^4g^3\log\left(bx+a\right) - 2\,(Bb^4cd^3 - (12\,A+B)ab^3d^4)g^3x^3 + 3\,(Bb^4c^2d^2 - 4\,Bab^3cd^3 + 3\,(4\,A+B)a^2b^2d^4)g^3x^2 - 6\,(Bb^4c^3d - 4\,Bab^3c^2d^2 + 6\,Ba^2b^2cd^3 - (4\,A+3\,B)a^3bd^4)g^3x + 6\,(Bb^4c^4 - 4\,Bab^3c^3d + 6\,Ba^2b^2c^2d^2 - 4\,Ba^3bcd^3)g^3\log\left(dx+c\right) + 6\,(Bb^4d^4g^3x^4 + 4\,Bab^3d^4g^3x^3 + 6\,Ba^2b^2d^4g^3x^2 + 4\,Ba^3bd^4g^3x)\log\left(\frac{(bx+a)e}{dx+c}\right)}{24\,bd^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^3*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="fricas")
```

```
[Out] 1/24*(6*A*b^4*d^4*g^3*x^4 + 6*B*a^4*d^4*g^3*log(b*x + a) - 2*(B*b^4*c*d^3 -
 (12*A + B)*a*b^3*d^4)*g^3*x^3 + 3*(B*b^4*c^2*d^2 - 4*B*a*b^3*c*d^3 + 3*(4*
A + B)*a^2*b^2*d^4)*g^3*x^2 - 6*(B*b^4*c^3*d - 4*B*a*b^3*c^2*d^2 + 6*B*a^2*
b^2*c*d^3 - (4*A + 3*B)*a^3*b*d^4)*g^3*x + 6*(B*b^4*c^4 - 4*B*a*b^3*c^3*d +
 6*B*a^2*b^2*c^2*d^2 - 4*B*a^3*b*c*d^3)*g^3*log(d*x + c) + 6*(B*b^4*d^4*g^3
*x^4 + 4*B*a*b^3*d^4*g^3*x^3 + 6*B*a^2*b^2*d^4*g^3*x^2 + 4*B*a^3*b*d^4*g^3*
x)*log((b*x + a)*e/(d*x + c)))/(b*d^4)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 706 vs. $2(128) = 256$.
time = 2.44, size = 706, normalized size = 4.74

$$\frac{Ab^3g^3x^4}{4} + \frac{Ba^4g^3\log\left(x + \frac{\frac{Ba^5d^4g^3}{b} + 4Ba^4cd^3g^3 - 6Ba^3bc^2d^2g^3 + 4Ba^2b^2c^3dg^3 - Bab^3c^4g^3}{Ba^4d^4g^3 + 4Ba^3bcd^3g^3 - 6Ba^2b^2c^2d^2g^3 + 4Bab^3c^3dg^3 - Bb^4c^4g^3}\right)}{4b} - \frac{Bcg^3\cdot(2ad-bc)\left(2a^2d^2 - 2abcd + b^2c^2\right)\log\left(x + \frac{5Ba^4cd^3g^3 - 6Ba^3bc^2d^2g^3 + 4Ba^2b^2c^3dg^3 - Bab^3c^4g^3 - Bacg^3\cdot(2ad-bc)\left(2a^2d^2-2abcd+b^2c^2\right) + \frac{Bbc^2g^3\cdot(2ad-bc)\left(2a^2d^2-2abcd+b^2c^2\right)}{d}}{Ba^4d^4g^3 + 4Ba^3bcd^3g^3 - 6Ba^2b^2c^2d^2g^3 + 4Bab^3c^3dg^3 - Bb^4c^4g^3}\right)}{4d^4} + x^3\left(Aab^2g^3 + \frac{Bab^2g^3}{12} - \frac{Bb^3cg^3}{12d}\right) + x^2\cdot\left(\frac{3Aa^2bg^3}{2} + \frac{3Ba^2bg^3}{8} - \frac{Bab^2cg^3}{2d} + \frac{Bb^3c^2g^3}{8d^2}\right) + x\left(Aa^3g^3 + \frac{3Ba^3g^3}{4} - \frac{3Ba^2bcg^3}{2d} + \frac{Bab^2c^2g^3}{d^2} - \frac{Bb^3c^3g^3}{4d^3}\right) + \left(Ba^3g^3x + \frac{3Ba^2bg^3x^2}{2} + Bab^2g^3x^3 + \frac{Bb^3g^3x^4}{4}\right)\log\left(\frac{e(a+bx)}{c+dx}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)**3*(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

```
[Out] A*b**3*g**3*x**4/4 + B*a**4*g**3*log(x + (B*a**5*d**4*g**3/b + 4*B*a**4*c*d
**3*g**3 - 6*B*a**3*b*c**2*d**2*g**3 + 4*B*a**2*b**2*c**3*d*g**3 - B*a*b**3
*c**4*g**3)/(B*a**4*d**4*g**3 + 4*B*a**3*b*c*d**3*g**3 - 6*B*a**2*b**2*c**2
*d**2*g**3 + 4*B*a*b**3*c**3*d*g**3 - B*b**4*c**4*g**3))/(4*b) - B*c*g**3*(
2*a*d - b*c)*(2*a**2*d**2 - 2*a*b*c*d + b**2*c**2)*log(x + (5*B*a**4*c*d**3
*g**3 - 6*B*a**3*b*c**2*d**2*g**3 + 4*B*a**2*b**2*c**3*d*g**3 - B*a*b**3*c*
*4*g**3 - B*a*c*g**3*(2*a*d - b*c)*(2*a**2*d**2 - 2*a*b*c*d + b**2*c**2) +
B*b*c**2*g**3*(2*a*d - b*c)*(2*a**2*d**2 - 2*a*b*c*d + b**2*c**2)/d)/(B*a**
4*d**4*g**3 + 4*B*a**3*b*c*d**3*g**3 - 6*B*a**2*b**2*c**2*d**2*g**3 + 4*B*a
*b**3*c**3*d*g**3 - B*b**4*c**4*g**3))/(4*d**4) + x**3*(A*a*b**2*g**3 + B*a
*b**2*g**3/12 - B*b**3*c*g**3/(12*d)) + x**2*(3*A*a**2*b*g**3/2 + 3*B*a**2*
b*g**3/8 - B*a*b**2*c*g**3/(2*d) + B*b**3*c**2*g**3/(8*d**2)) + x*(A*a**3*g
**3 + 3*B*a**3*g**3/4 - 3*B*a**2*b*c*g**3/(2*d) + B*a*b**2*c**2*g**3/d**2 -
 B*b**3*c**3*g**3/(4*d**3)) + (B*a**3*g**3*x + 3*B*a**2*b*g**3*x**2/2 + B*a
*b**2*g**3*x**3 + B*b**3*g**3*x**4/4)*log(e*(a + b*x)/(c + d*x))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 3795 vs. $2(140) = 280$.
time = 3.12, size = 3795, normalized size = 25.47

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^3*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="giac")
```

```
[Out] -1/24*(6*B*b^9*c^5*g^3*e^5*log(-b*e + (b*x*e + a*e)*d/(d*x + c)) - 30*B*a*b
^8*c^4*d*g^3*e^5*log(-b*e + (b*x*e + a*e)*d/(d*x + c)) + 60*B*a^2*b^7*c^3*d
^2*g^3*e^5*log(-b*e + (b*x*e + a*e)*d/(d*x + c)) - 60*B*a^3*b^6*c^2*d^3*g^3
*e^5*log(-b*e + (b*x*e + a*e)*d/(d*x + c)) + 30*B*a^4*b^5*c*d^4*g^3*e^5*log
(-b*e + (b*x*e + a*e)*d/(d*x + c)) - 6*B*a^5*b^4*d^5*g^3*e^5*log(-b*e + (b*
x*e + a*e)*d/(d*x + c)) - 24*(b*x*e + a*e)*B*b^8*c^5*d*g^3*e^4*log(-b*e + (
b*x*e + a*e)*d/(d*x + c))/(d*x + c) + 120*(b*x*e + a*e)*B*a*b^7*c^4*d^2*g^3
*e^4*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c) - 240*(b*x*e + a*e)*B*
a^2*b^6*c^3*d^3*g^3*e^4*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c) + 2
40*(b*x*e + a*e)*B*a^3*b^5*c^2*d^4*g^3*e^4*log(-b*e + (b*x*e + a*e)*d/(d*x
+ c))/(d*x + c) - 120*(b*x*e + a*e)*B*a^4*b^4*c*d^5*g^3*e^4*log(-b*e + (b*x
*e + a*e)*d/(d*x + c))/(d*x + c) + 24*(b*x*e + a*e)*B*a^5*b^3*d^6*g^3*e^4*l
og(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c) + 36*(b*x*e + a*e)^2*B*b^7*c
^5*d^2*g^3*e^3*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^2 - 180*(b*x
*e + a*e)^2*B*a*b^6*c^4*d^3*g^3*e^3*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(
d*x + c)^2 + 360*(b*x*e + a*e)^2*B*a^2*b^5*c^3*d^4*g^3*e^3*log(-b*e + (b*x*
e + a*e)*d/(d*x + c))/(d*x + c)^2 - 360*(b*x*e + a*e)^2*B*a^3*b^4*c^2*d^5*g
```

^3*e^3*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^2 + 180*(b*x*e + a*e
)^2*B*a^4*b^3*c*d^6*g^3*e^3*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)
^2 - 36*(b*x*e + a*e)^2*B*a^5*b^2*d^7*g^3*e^3*log(-b*e + (b*x*e + a*e)*d/(d
*x + c))/(d*x + c)^2 - 24*(b*x*e + a*e)^3*B*b^6*c^5*d^3*g^3*e^2*log(-b*e +
(b*x*e + a*e)*d/(d*x + c))/(d*x + c)^3 + 120*(b*x*e + a*e)^3*B*a*b^5*c^4*d^
4*g^3*e^2*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^3 - 240*(b*x*e +
a*e)^3*B*a^2*b^4*c^3*d^5*g^3*e^2*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x
 + c)^3 + 240*(b*x*e + a*e)^3*B*a^3*b^3*c^2*d^6*g^3*e^2*log(-b*e + (b*x*e +
 a*e)*d/(d*x + c))/(d*x + c)^3 - 120*(b*x*e + a*e)^3*B*a^4*b^2*c*d^7*g^3*e^
2*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^3 + 24*(b*x*e + a*e)^3*B*
a^5*b*d^8*g^3*e^2*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^3 + 6*(b*
x*e + a*e)^4*B*b^5*c^5*d^4*g^3*e*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x
 + c)^4 - 30*(b*x*e + a*e)^4*B*a*b^4*c^4*d^5*g^3*e*log(-b*e + (b*x*e + a*e)
*d/(d*x + c))/(d*x + c)^4 + 60*(b*x*e + a*e)^4*B*a^2*b^3*c^3*d^6*g^3*e*log(
-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^4 - 60*(b*x*e + a*e)^4*B*a^3*b^
2*c^2*d^7*g^3*e*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^4 + 30*(b*x
*e + a*e)^4*B*a^4*b*c*d^8*g^3*e*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x
+ c)^4 - 6*(b*x*e + a*e)^4*B*a^5*d^9*g^3*e*log(-b*e + (b*x*e + a*e)*d/(d*x
+ c))/(d*x + c)^4 - 6*(b*x*e + a*e)^4*B*b^5*c^5*d^4*g^3*e*log((b*x*e + a*e)
/(d*x + c))/(d*x + c)^4 + 30*(b*x*e + a*e)^4*B*a*b^4*c^4*d^5*g^3*e*log((b*x
*e + a*e)/(d*x + c))/(d*x + c)^4 - 60*(b*x*e + a*e)^4*B*a^2*b^3*c^3*d^6*g^3
*e*log((b*x*e + a*e)/(d*x + c))/(d*x + c)^4 + 60*(b*x*e + a*e)^4*B*a^3*b^2*
c^2*d^7*g^3*e*log((b*x*e + a*e)/(d*x + c))/(d*x + c)^4 - 30*(b*x*e + a*e)^4
*B*a^4*b*c*d^8*g^3*e*log((b*x*e + a*e)/(d*x + c))/(d*x + c)^4 + 6*(b*x*e +
a*e)^4*B*a^5*d^9*g^3*e*log((b*x*e + a*e)/(d*x + c))/(d*x + c)^4 + 6*A*b^9*c
^5*g^3*e^5 + 11*B*b^9*c^5*g^3*e^5 - 30*A*a*b^8*c^4*d*g^3*e^5 - 55*B*a*b^8*c
^4*d*g^3*e^5 + 60*A*a^2*b^7*c^3*d^2*g^3*e^5 + 110*B*a^2*b^7*c^3*d^2*g^3*e^5
 - 60*A*a^3*b^6*c^2*d^3*g^3*e^5 - 110*B*a^3*b^6*c^2*d^3*g^3*e^5 + 30*A*a^4*
b^5*c*d^4*g^3*e^5 + 55*B*a^4*b^5*c*d^4*g^3*e^5 - 6*A*a^5*b^4*d^5*g^3*e^5 -
11*B*a^5*b^4*d^5*g^3*e^5 - 24*(b*x*e + a*e)*A*b^8*c^5*d*g^3*e^4/(d*x + c) -
 38*(b*x*e + a*e)*B*b^8*c^5*d*g^3*e^4/(d*x + c) + 120*(b*x*e + a*e)*A*a*b^7
*c^4*d^2*g^3*e^4/(d*x + c) + 190*(b*x*e + a*e)*B*a*b^7*c^4*d^2*g^3*e^4/(d*x
 + c) - 240*(b*x*e + a*e)*A*a^2*b^6*c^3*d^3*g^3*e^4/(d*x + c) - 380*(b*x*e
+ a*e)*B*a^2*b^6*c^3*d^3*g^3*e^4/(d*x + c) + 240*(b*x*e + a*e)*A*a^3*b^5*c^
2*d^4*g^3*e^4/(d*x + c) + 380*(b*x*e + a*e)*B*a^3*b^5*c^2*d^4*g^3*e^4/(d*x
+ c) - 120*(b*x*e + a*e)*A*a^4*b^4*c*d^5*g^3*e^4/(d*x + c) - 190*(b*x*e + a
*e)*B*a^4*b^4*c*d^5*g^3*e^4/(d*x + c) + 24*(b*x*e + a*e)*A*a^5*b^3*d^6*g^3*
e^4/(d*x + c) + 38*(b*x*e + a*e)*B*a^5*b^3*d^6*g^3*e^4/(d*x + c) + 36*(b*x*
e + a*e)^2*A*b^7*c^5*d^2*g^3*e^3/(d*x + c)^2 + 45*(b*x*e + a*e)^2*B*b^7*c^5
*d^2*g^3*e^3/(d*x + c)^2 - 180*(b*x*e + a*e)^2*A*a*b^6*c^4*d^3*g^3*e^3/(d*x
 + c)^2 - 225*(b*x*e + a*e)^2*B*a*b^6*c^4*d^3*g^3*e^3/(d*x + c)^2 + 360*(b*
x*e + a*e)^2*A*a^2*b^5*c^3*d^4*g^3*e^3/(d*x + c)^2 + 450*(b*x*e + a*e)^2*B*
a^2*b^5*c^3*d^4*g^3*e^3/(d*x + c)^2 - 360*(b*x*e + a*e)^2*A*a^3*b^4*c^2*d^5
*g^3*e^3/(d*x + c)^2 - 450*(b*x*e + a*e)^2*B*a^3*b^4*c^2*d^5*g^3*e^3/(d*x +
 c)^2 + 180*(b*x*e + a*e)^2*A*a^4*b^3*c*d^6*g^3*e^3/(d*x + c)^2 + 225*(b*x*

```
e + a*e)^2*B*a^4*b^3*c*d^6*g^3*e^3/(d*x + c)^2 - 36*(b*x*e + a*e)^2*A*a^5*b
^2*d^7*g^3*e^3/(d*x + c)^2 - 45*(b*x*e + a*e)^2*B*a^5*b^2*d^7*g^3*e^3/(d*x
+ c)^2 - 24*(b*x*e + a*e)^3*A*b^6*c^5*d^3*g^3*e^2/(d*x + c)^3 - 18*(b*x*e +
 a*e)^3*B*b^6*c^5*d^3*g^3*e^2/(d*x + c)^3 + 120*(b*x*e + a*e)^3*A*a*b^5*c^4
*d^4*g^3*e^2/(d*x + c)^3 + 90*(b*x*e + a*e)^3*B...
```

Mupad [B]
time = 4.64, size = 566, normalized size = 3.80

$$x\left(\frac{(4ad+4bc)\left(\frac{\left(\frac{b^2g^3(16Aad+4Abc+Bad-Bbc)}{4d}-\frac{Ab^2g^3(4ad+4bc)}{4d}\right)(4ad+4bc)}{4bd}-\frac{abg^3(6Aad+4Abc+Bad-Bbc)}{d}+\frac{Aab^2cg^3}{d}\right)}{4bd}+\frac{a^2g^3(8Aad+12Abc+3Bad-3Bbc)}{2d}-\frac{ac\left(\frac{b^2g^3(16Aad+4Abc+Bad-Bbc)}{4d}-\frac{Ab^2g^3(4ad+4bc)}{4d}\right)}{bd}\right)-x^2\left(\frac{\left(\frac{b^2g^3(16Aad+4Abc+Bad-Bbc)}{4d}-\frac{Ab^2g^3(4ad+4bc)}{4d}\right)(4ad+4bc)}{8bd}-\frac{abg^3(6Aad+4Abc+Bad-Bbc)}{2d}+\frac{Aab^2cg^3}{2d}\right)+\ln\left(\frac{e(a+bx)}{c+dx}\right)\left(Ba^3g^3x+\frac{3Ba^2bg^3x^2}{2}+Bab^2g^3x^3+\frac{Bb^3g^3x^4}{4}\right)+x^3\left(\frac{b^2g^3(16Aad+4Abc+Bad-Bbc)}{12d}-\frac{Ab^2g^3(4ad+4bc)}{12d}\right)+\frac{\ln(c+dx)\left(-4Ba^3cd^3g^3+6Ba^2bc^2d^2g^3-4Bab^2c^3dg^3+Bb^3c^4g^3\right)}{4d^4}+\frac{Ab^3g^3x^4}{4}+\frac{Ba^4g^3\ln(a+bx)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*g + b*g*x)^3*(A + B*log((e*(a + b*x))/(c + d*x))),x)
```

```
[Out] x*(((4*a*d + 4*b*c)*((((b^2*g^3*(16*A*a*d + 4*A*b*c + B*a*d - B*b*c))/(4*d)
 - (A*b^2*g^3*(4*a*d + 4*b*c))/(4*d))*(4*a*d + 4*b*c))/(4*b*d) - (a*b*g^3*(
6*A*a*d + 4*A*b*c + B*a*d - B*b*c))/d + (A*a*b^2*c*g^3)/d))/(4*b*d) + (a^2*
g^3*(8*A*a*d + 12*A*b*c + 3*B*a*d - 3*B*b*c))/(2*d) - (a*c*((b^2*g^3*(16*A*
a*d + 4*A*b*c + B*a*d - B*b*c))/(4*d) - (A*b^2*g^3*(4*a*d + 4*b*c))/(4*d)))
/(b*d)) - x^2*((((b^2*g^3*(16*A*a*d + 4*A*b*c + B*a*d - B*b*c))/(4*d) - (A*
b^2*g^3*(4*a*d + 4*b*c))/(4*d))*(4*a*d + 4*b*c))/(8*b*d) - (a*b*g^3*(6*A*a*
d + 4*A*b*c + B*a*d - B*b*c))/(2*d) + (A*a*b^2*c*g^3)/(2*d)) + log((e*(a +
b*x))/(c + d*x))*((B*b^3*g^3*x^4)/4 + B*a^3*g^3*x + (3*B*a^2*b*g^3*x^2)/2 +
 B*a*b^2*g^3*x^3) + x^3*((b^2*g^3*(16*A*a*d + 4*A*b*c + B*a*d - B*b*c))/(12
*d) - (A*b^2*g^3*(4*a*d + 4*b*c))/(12*d)) + (log(c + d*x)*(B*b^3*c^4*g^3 -
4*B*a^3*c*d^3*g^3 + 6*B*a^2*b*c^2*d^2*g^3 - 4*B*a*b^2*c^3*d*g^3))/(4*d^4) +
 (A*b^3*g^3*x^4)/4 + (B*a^4*g^3*log(a + b*x))/(4*b)
```

3.90 $\int (ag+bgx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$

Optimal. Leaf size=118

$$\frac{B(bc-ad)^2g^2x}{3d^2}-\frac{B(bc-ad)g^2(a+bx)^2}{6bd}+\frac{g^2(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3b}-\frac{B(bc-ad)^3g^2\log(c+dx)}{3bd^3}$$

```
[Out] 1/3*B*(-a*d+b*c)^2*g^2*x/d^2-1/6*B*(-a*d+b*c)*g^2*(b*x+a)^2/b/d+1/3*g^2*(b*
x+a)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))/b-1/3*B*(-a*d+b*c)^3*g^2*ln(d*x+c)/b/d^3
```

Rubi [A]
time = 0.05, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2548, 21, 45}

$$\frac{g^2(a+bx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3b}-\frac{Bg^2(bc-ad)^3\log(c+dx)}{3bd^3}+\frac{Bg^2x(bc-ad)^2}{3d^2}-\frac{Bg^2(a+bx)^2(bc-ad)}{6bd}$$

Antiderivative was successfully verified.

```
[In] Int[(a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
```

```
[Out] (B*(b*c - a*d)^2*g^2*x)/(3*d^2) - (B*(b*c - a*d)*g^2*(a + b*x)^2)/(6*b*d) +
 (g^2*(a + b*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(3*b) - (B*(b*c - a
*d)^3*g^2*Log[c + d*x])/(3*b*d^3)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2548

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Dist[B*n*((b*c
- a*d)/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /;
```

```
FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c -
a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Rubi steps

$$\int (ag+bgx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)dx = \frac{g^2(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3b} - \frac{B\int \frac{(bc-ad)g^3(a+bx)^2}{c+dx}dx}{3bg}$$

$$= \frac{g^2(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3b} - \frac{(B(bc-ad)g^2)\int \frac{(a+ba}{c+d}}{3b}$$

$$= \frac{g^2(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3b} - \frac{(B(bc-ad)g^2)\int \left(-\frac{b}{}\right.}{}$$

$$= \frac{B(bc-ad)^2g^2x}{3d^2} - \frac{B(bc-ad)g^2(a+bx)^2}{6bd} + \frac{g^2(a+bx)^3\left(A\right.}{}$$

Mathematica [A]
time = 0.04, size = 99, normalized size = 0.84

$$\frac{g^2\left((a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)+\frac{B(-bc+ad)\left(d\left(a^2d+4abdx+b^2x(-2c+dx)\right)+2(bc-ad)^2\log(c+dx)\right)}{2d^3}\right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
```

```
[Out] (g^2*((a + b*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + (B*(-(b*c) + a*d)*
(d*(a^2*d + 4*a*b*d*x + b^2*x*(-2*c + d*x)) + 2*(b*c - a*d)^2*Log[c + d*x])
)/(2*d^3)))/(3*b)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2511 vs. 2(110) = 220.
time = 0.27, size = 2512, normalized size = 21.29

method	result
risch	$\frac{(bx+a)^3g^2B\ln\left(\frac{e(bx+a)}{dx+c}\right)}{3b}+\frac{g^2b^2Ax^3}{3}+g^2bAax^2+\frac{g^2bBax^2}{6}-\frac{g^2b^2Bcx^2}{6d}+g^2Aa^2x+\frac{g^2B\ln(dx+c)a^3}{3b}$
derivativedivides	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*g*x+a*g)^2*(A+B*ln(e*(b*x+a)/(d*x+c))),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d^2*e*(a*d-b*c)*(B*g^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*(b*e/d+(a*d-b*c)*
e/d/(d*x+c))*b/e/(b*e-(b*e/d+(a*d-b*c)*e/d/(d*x+c))*d)*c^2+B*g^2*b^3*e*ln(b
*e/d+(a*d-b*c)*e/d/(d*x+c))*(b*e/d+(a*d-b*c)*e/d/(d*x+c))/(b*e-(b*e/d+(a*d-
b*c)*e/d/(d*x+c))*d)^3*c^2-2*B*d*g^2*b^2*e*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*
(b*e/d+(a*d-b*c)*e/d/(d*x+c))/(b*e-(b*e/d+(a*d-b*c)*e/d/(d*x+c))*d)^3*a*c-2
/3*B*g^2/e*ln(b*e-(b*e/d+(a*d-b*c)*e/d/(d*x+c))*d)*a*c-B*d*g^2*ln(b*e/d+(a*
d-b*c)*e/d/(d*x+c))*(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2/(b*e-(b*e/d+(a*d-b*c)*e
/d/(d*x+c))*d)^3*b^2*c^2+1/3*B*g^2*b^2*e/(b*e-(b*e/d+(a*d-b*c)*e/d/(d*x+c))
*d)^2*a*c-2*B*g^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*(b*e/d+(a*d-b*c)*e/d/(d*x
+c))/(b*e-(b*e/d+(a*d-b*c)*e/d/(d*x+c))*d)^2*b^2*c^2-2*B*d^2*g^2*ln(b*e/d+(
a*d-b*c)*e/d/(d*x+c))*(b*e/d+(a*d-b*c)*e/d/(d*x+c))/(b*e-(b*e/d+(a*d-b*c)*e
/d/(d*x+c))*d)^2*a^2+1/3*B*d*g^2/b/e*ln(b*e-(b*e/d+(a*d-b*c)*e/d/(d*x+c))*d
)*a^2+1/3*B/d*g^2*b/e*ln(b*e-(b*e/d+(a*d-b*c)*e/d/(d*x+c))*d)*c^2-B*d^3*g^2
*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2/(b*e-(b*e/
d+(a*d-b*c)*e/d/(d*x+c))*d)^3*a^2-4/3*B*g^2/(b*e-(b*e/d+(a*d-b*c)*e/d/(d*x+
c))*d)*a*b*c+2/3*B/d*g^2/(b*e-(b*e/d+(a*d-b*c)*e/d/(d*x+c))*d)*b^2*c^2-1/6*
B/d*g^2*b^3*e/(b*e-(b*e/d+(a*d-b*c)*e/d/(d*x+c))*d)^2*c^2+A*d^2*g^2*(a^2*d^
2-2*a*b*c*d+b^2*c^2)*(1/3*b^2*e^2/d^3/(b*e-(b*e/d+(a*d-b*c)*e/d/(d*x+c))*d)
^3-b*e/d^3/(b*e-(b*e/d+(a*d-b*c)*e/d/(d*x+c))*d)^2+1/d^3/(b*e-(b*e/d+(a*d-b
*c)*e/d/(d*x+c))*d))+2/3*B*d*g^2/(b*e-(b*e/d+(a*d-b*c)*e/d/(d*x+c))*d)*a^2-
1/6*B*d*g^2*b*e/(b*e-(b*e/d+(a*d-b*c)*e/d/(d*x+c))*d)^2*a^2+B*d*g^2*b/e*ln(
b*e/d+(a*d-b*c)*e/d/(d*x+c))*(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2/(b*e-(b*e/d+(a
*d-b*c)*e/d/(d*x+c))*d)^2*c^2+B*d^2*g^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*(b*
e/d+(a*d-b*c)*e/d/(d*x+c))/b/e/(b*e-(b*e/d+(a*d-b*c)*e/d/(d*x+c))*d)*a^2+2*
B*d^2*g^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2/(
b*e-(b*e/d+(a*d-b*c)*e/d/(d*x+c))*d)^3*a*b*c+4*B*d*g^2*ln(b*e/d+(a*d-b*c)*e
/d/(d*x+c))*(b*e/d+(a*d-b*c)*e/d/(d*x+c))/(b*e-(b*e/d+(a*d-b*c)*e/d/(d*x+c)
)*d)^2*a*b*c+1/3*B*d^4*g^2/b/e*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*(b*e/d+(a*d-
b*c)*e/d/(d*x+c))^3/(b*e-(b*e/d+(a*d-b*c)*e/d/(d*x+c))*d)^3*a^2+B*d^3*g^2/b
/e*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2/(b*e-(b*
e/d+(a*d-b*c)*e/d/(d*x+c))*d)^2*a^2-2/3*B*d^3*g^2/e*ln(b*e/d+(a*d-b*c)*e/d/
(d*x+c))*(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3/(b*e-(b*e/d+(a*d-b*c)*e/d/(d*x+c))
*d)^3*a*c-2*B*d^2*g^2/e*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*(b*e/d+(a*d-b*c)*e/
d/(d*x+c))^2/(b*e-(b*e/d+(a*d-b*c)*e/d/(d*x+c))*d)^2*a*c-2*B*d*g^2*ln(b*e/d
+(a*d-b*c)*e/d/(d*x+c))*(b*e/d+(a*d-b*c)*e/d/(d*x+c))/e/(b*e-(b*e/d+(a*d-b*
c)*e/d/(d*x+c))*d)*a*c+1/3*B*d^2*g^2*b/e*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*(b
*e/d+(a*d-b*c)*e/d/(d*x+c))^3/(b*e-(b*e/d+(a*d-b*c)*e/d/(d*x+c))*d)^3*c^2+B
*d^2*g^2*b*e*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*(b*e/d+(a*d-b*c)*e/d/(d*x+c))/
(b*e-(b*e/d+(a*d-b*c)*e/d/(d*x+c))*d)^3*a^2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 286 vs. $2(111) = 222$.

time = 0.30, size = 286, normalized size = 2.42

$$\frac{1}{3}Ab^2g^2x^3 + Aabg^2x^2 + \left(x\log\left(\frac{bxe}{dx+c} + \frac{ae}{dx+c}\right) + \frac{a\log(bx+a)}{b} - \frac{c\log(dx+c)}{d}\right)Ba^2g^2 + \left(x^2\log\left(\frac{bxe}{dx+c} + \frac{ae}{dx+c}\right) - \frac{a^2\log(bx+a)}{b^2} + \frac{c^2\log(dx+c)}{d^2} - \frac{(bc-ad)x}{bd}\right)Babg^2 + \frac{1}{6}\left(2x^3\log\left(\frac{bxe}{dx+c} + \frac{ae}{dx+c}\right) + \frac{2a^3\log(bx+a)}{b^3} - \frac{2c^3\log(dx+c)}{d^3} - \frac{(b^2cd-abd^2)x^2 - 2(b^2c^2-a^2d^2)x}{b^2d^2}\right)Bb^2g^2 + Aa^2g^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^2*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="maxima")
```

```
[Out] 1/3*A*b^2*g^2*x^3 + A*a*b*g^2*x^2 + (x*log(b*x*e/(d*x + c) + a*e/(d*x + c))
 + a*log(b*x + a)/b - c*log(d*x + c)/d)*B*a^2*g^2 + (x^2*log(b*x*e/(d*x + c
) + a*e/(d*x + c)) - a^2*log(b*x + a)/b^2 + c^2*log(d*x + c)/d^2 - (b*c - a
*d)*x/(b*d))*B*a*b*g^2 + 1/6*(2*x^3*log(b*x*e/(d*x + c) + a*e/(d*x + c)) +
2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2
- 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*B*b^2*g^2 + A*a^2*g^2*x
```

Fricas [A]
time = 0.37, size = 221, normalized size = 1.87

$$\frac{2\,Ab^{3}d^{3}g^{2}x^{3} + 2\,Ba^{3}d^{3}g^{2}\log\left(bx+a\right) - \left(Bb^{3}cd^{2} - \left(6\,A+B\right)ab^{2}d^{3}\right)g^{2}x^{2} + 2\,\left(Bb^{3}c^{2}d - 3\,Bab^{2}cd^{2} + \left(3\,A+2\,B\right)a^{2}bd^{3}\right)g^{2}x - 2\,\left(Bb^{3}c^{3} - 3\,Bab^{2}c^{2}d + 3\,Ba^{2}bcd^{2}\right)g^{2}\log\left(dx+c\right) + 2\,\left(Bb^{3}d^{3}g^{2}x^{3} + 3\,Bab^{2}d^{3}g^{2}x^{2} + 3\,Ba^{2}bd^{3}g^{2}x\right)\log\left(\frac{(bx+a)e}{dx+c}\right)}{6\,bd^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^2*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="fricas")
```

```
[Out] 1/6*(2*A*b^3*d^3*g^2*x^3 + 2*B*a^3*d^3*g^2*log(b*x + a) - (B*b^3*c*d^2 - (6
*A + B)*a*b^2*d^3)*g^2*x^2 + 2*(B*b^3*c^2*d - 3*B*a*b^2*c*d^2 + (3*A + 2*B)
*a^2*b*d^3)*g^2*x - 2*(B*b^3*c^3 - 3*B*a*b^2*c^2*d + 3*B*a^2*b*c*d^2)*g^2*l
og(d*x + c) + 2*(B*b^3*d^3*g^2*x^3 + 3*B*a*b^2*d^3*g^2*x^2 + 3*B*a^2*b*d^3*
g^2*x)*log((b*x + a)*e/(d*x + c)))/(b*d^3)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 491 vs. $2(100) = 200$.
time = 1.57, size = 491, normalized size = 4.16

$$\frac{Ab^{2}g^{2}x^{3}}{3} + \frac{Ba^{3}g^{2}\log\left(x + \frac{\frac{Ba^{4}d^{3}g^{2}}{b} + 3Ba^{3}cd^{2}g^{2} - 3Ba^{2}bc^{2}dg^{2} + Bab^{2}c^{3}g^{2}}{Ba^{3}d^{3}g^{2} + 3Ba^{2}bcd^{2}g^{2} - 3Bab^{2}c^{2}dg^{2} + Bb^{3}c^{3}g^{2}}\right)}{3b} - \frac{Bcg^{2}\cdot\left(3a^{2}d^{2} - 3abcd + b^{2}c^{2}\right)\log\left(x + \frac{4Ba^{3}cd^{2}g^{2} - 3Ba^{2}bc^{2}dg^{2} + Bab^{2}c^{3}g^{2} - Bacg^{2}\cdot\left(3a^{2}d^{2} - 3abcd + b^{2}c^{2}\right) + \frac{Bbc^{2}g^{2}\cdot\left(3a^{2}d^{2} - 3abcd + b^{2}c^{2}\right)}{d}}{Ba^{3}d^{3}g^{2} + 3Ba^{2}bcd^{2}g^{2} - 3Bab^{2}c^{2}dg^{2} + Bb^{3}c^{3}g^{2}}\right)}{3d^{3}} + x^{2}\left(Aabg^{2} + \frac{Babg^{2}}{6} - \frac{Bb^{2}cg^{2}}{6d}\right) + x\left(Aa^{2}g^{2} + \frac{2Ba^{2}g^{2}}{3} - \frac{Babcg^{2}}{d} + \frac{Bb^{2}c^{2}g^{2}}{3d^{2}}\right) + \left(Ba^{2}g^{2}x + Babg^{2}x^{2} + \frac{Bb^{2}g^{2}x^{3}}{3}\right)\log\left(\frac{e(a+bx)}{c+dx}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)**2*(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

```
[Out] A*b**2*g**2*x**3/3 + B*a**3*g**2*log(x + (B*a**4*d**3*g**2/b + 3*B*a**3*c*d
**2*g**2 - 3*B*a**2*b*c**2*d*g**2 + B*a*b**2*c**3*g**2)/(B*a**3*d**3*g**2 +
 3*B*a**2*b*c*d**2*g**2 - 3*B*a*b**2*c**2*d*g**2 + B*b**3*c**3*g**2))/(3*b)
 - B*c*g**2*(3*a**2*d**2 - 3*a*b*c*d + b**2*c**2)*log(x + (4*B*a**3*c*d**2*
g**2 - 3*B*a**2*b*c**2*d*g**2 + B*a*b**2*c**3*g**2 - B*a*c*g**2*(3*a**2*d**
2 - 3*a*b*c*d + b**2*c**2) + B*b*c**2*g**2*(3*a**2*d**2 - 3*a*b*c*d + b**2*
c**2)/d)/(B*a**3*d**3*g**2 + 3*B*a**2*b*c*d**2*g**2 - 3*B*a*b**2*c**2*d*g**
2 + B*b**3*c**3*g**2))/(3*d**3) + x**2*(A*a*b*g**2 + B*a*b*g**2/6 - B*b**2*
c*g**2/(6*d)) + x*(A*a**2*g**2 + 2*B*a**2*g**2/3 - B*a*b*c*g**2/d + B*b**2*
c**2*g**2/(3*d**2)) + (B*a**2*g**2*x + B*a*b*g**2*x**2 + B*b**2*g**2*x**3/3
)*log(e*(a + b*x)/(c + d*x))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 2450 vs. $2(111) = 222$.
time = 4.06, size = 2450, normalized size = 20.76

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^2*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="giac")
```

```
[Out] 1/6*(2*B*b^7*c^4*g^2*e^4*log(-b*e + (b*x*e + a*e)*d/(d*x + c)) - 8*B*a*b^6*
c^3*d*g^2*e^4*log(-b*e + (b*x*e + a*e)*d/(d*x + c)) + 12*B*a^2*b^5*c^2*d^2*
g^2*e^4*log(-b*e + (b*x*e + a*e)*d/(d*x + c)) - 8*B*a^3*b^4*c*d^3*g^2*e^4*l
og(-b*e + (b*x*e + a*e)*d/(d*x + c)) + 2*B*a^4*b^3*d^4*g^2*e^4*log(-b*e + (
b*x*e + a*e)*d/(d*x + c)) - 6*(b*x*e + a*e)*B*b^6*c^4*d*g^2*e^3*log(-b*e +
(b*x*e + a*e)*d/(d*x + c))/(d*x + c) + 24*(b*x*e + a*e)*B*a*b^5*c^3*d^2*g^2
*e^3*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c) - 36*(b*x*e + a*e)*B*a
^2*b^4*c^2*d^3*g^2*e^3*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c) + 24
*(b*x*e + a*e)*B*a^3*b^3*c*d^4*g^2*e^3*log(-b*e + (b*x*e + a*e)*d/(d*x + c)
)/(d*x + c) - 6*(b*x*e + a*e)*B*a^4*b^2*d^5*g^2*e^3*log(-b*e + (b*x*e + a*e
)*d/(d*x + c))/(d*x + c) + 6*(b*x*e + a*e)^2*B*b^5*c^4*d^2*g^2*e^2*log(-b*e
 + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^2 - 24*(b*x*e + a*e)^2*B*a*b^4*c^3*
d^3*g^2*e^2*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^2 + 36*(b*x*e +
 a*e)^2*B*a^2*b^3*c^2*d^4*g^2*e^2*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*
x + c)^2 - 24*(b*x*e + a*e)^2*B*a^3*b^2*c*d^5*g^2*e^2*log(-b*e + (b*x*e + a
*e)*d/(d*x + c))/(d*x + c)^2 + 6*(b*x*e + a*e)^2*B*a^4*b*d^6*g^2*e^2*log(-b
*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^2 - 2*(b*x*e + a*e)^3*B*b^4*c^4*d
^3*g^2*e*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^3 + 8*(b*x*e + a*e
)^3*B*a*b^3*c^3*d^4*g^2*e*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^3
 - 12*(b*x*e + a*e)^3*B*a^2*b^2*c^2*d^5*g^2*e*log(-b*e + (b*x*e + a*e)*d/(d
*x + c))/(d*x + c)^3 + 8*(b*x*e + a*e)^3*B*a^3*b*c*d^6*g^2*e*log(-b*e + (b*
x*e + a*e)*d/(d*x + c))/(d*x + c)^3 - 2*(b*x*e + a*e)^3*B*a^4*d^7*g^2*e*log
(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^3 + 2*(b*x*e + a*e)^3*B*b^4*c^
4*d^3*g^2*e*log((b*x*e + a*e)/(d*x + c))/(d*x + c)^3 - 8*(b*x*e + a*e)^3*B*
a*b^3*c^3*d^4*g^2*e*log((b*x*e + a*e)/(d*x + c))/(d*x + c)^3 + 12*(b*x*e +
a*e)^3*B*a^2*b^2*c^2*d^5*g^2*e*log((b*x*e + a*e)/(d*x + c))/(d*x + c)^3 - 8
*(b*x*e + a*e)^3*B*a^3*b*c*d^6*g^2*e*log((b*x*e + a*e)/(d*x + c))/(d*x + c)
^3 + 2*(b*x*e + a*e)^3*B*a^4*d^7*g^2*e*log((b*x*e + a*e)/(d*x + c))/(d*x +
c)^3 + 2*A*b^7*c^4*g^2*e^4 + 3*B*b^7*c^4*g^2*e^4 - 8*A*a*b^6*c^3*d*g^2*e^4
- 12*B*a*b^6*c^3*d*g^2*e^4 + 12*A*a^2*b^5*c^2*d^2*g^2*e^4 + 18*B*a^2*b^5*c^
2*d^2*g^2*e^4 - 8*A*a^3*b^4*c*d^3*g^2*e^4 - 12*B*a^3*b^4*c*d^3*g^2*e^4 + 2*
A*a^4*b^3*d^4*g^2*e^4 + 3*B*a^4*b^3*d^4*g^2*e^4 - 6*(b*x*e + a*e)*A*b^6*c^4
*d*g^2*e^3/(d*x + c) - 7*(b*x*e + a*e)*B*b^6*c^4*d*g^2*e^3/(d*x + c) + 24*(
b*x*e + a*e)*A*a*b^5*c^3*d^2*g^2*e^3/(d*x + c) + 28*(b*x*e + a*e)*B*a*b^5*c
^3*d^2*g^2*e^3/(d*x + c) - 36*(b*x*e + a*e)*A*a^2*b^4*c^2*d^3*g^2*e^3/(d*x
+ c) - 42*(b*x*e + a*e)*B*a^2*b^4*c^2*d^3*g^2*e^3/(d*x + c) + 24*(b*x*e + a
```

```
*e)*A*a^3*b^3*c*d^4*g^2*e^3/(d*x + c) + 28*(b*x*e + a*e)*B*a^3*b^3*c*d^4*g^
2*e^3/(d*x + c) - 6*(b*x*e + a*e)*A*a^4*b^2*d^5*g^2*e^3/(d*x + c) - 7*(b*x*
e + a*e)*B*a^4*b^2*d^5*g^2*e^3/(d*x + c) + 6*(b*x*e + a*e)^2*A*b^5*c^4*d^2*
g^2*e^2/(d*x + c)^2 + 4*(b*x*e + a*e)^2*B*b^5*c^4*d^2*g^2*e^2/(d*x + c)^2 -
 24*(b*x*e + a*e)^2*A*a*b^4*c^3*d^3*g^2*e^2/(d*x + c)^2 - 16*(b*x*e + a*e)^
2*B*a*b^4*c^3*d^3*g^2*e^2/(d*x + c)^2 + 36*(b*x*e + a*e)^2*A*a^2*b^3*c^2*d^
4*g^2*e^2/(d*x + c)^2 + 24*(b*x*e + a*e)^2*B*a^2*b^3*c^2*d^4*g^2*e^2/(d*x +
 c)^2 - 24*(b*x*e + a*e)^2*A*a^3*b^2*c*d^5*g^2*e^2/(d*x + c)^2 - 16*(b*x*e
+ a*e)^2*B*a^3*b^2*c*d^5*g^2*e^2/(d*x + c)^2 + 6*(b*x*e + a*e)^2*A*a^4*b*d^
6*g^2*e^2/(d*x + c)^2 + 4*(b*x*e + a*e)^2*B*a^4*b*d^6*g^2*e^2/(d*x + c)^2)*
(b*c/((b*c*e - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))/(b^
4*d^3*e^3 - 3*(b*x*e + a*e)*b^3*d^4*e^2/(d*x + c) + 3*(b*x*e + a*e)^2*b^2*d
^5*e/(d*x + c)^2 - (b*x*e + a*e)^3*b*d^6/(d*x + c)^3)
```

Mupad [B]
time = 4.48, size = 290, normalized size = 2.46

$$x^2\left(\frac{b\,g^2\,(9\,A\,a\,d+3\,A\,b\,c+B\,a\,d-B\,b\,c)}{6\,d}-\frac{A\,b\,g^2\,(3\,a\,d+3\,b\,c)}{6\,d}\right)-x\left(\frac{(3\,a\,d+3\,b\,c)\,\left(\frac{b\,g^2\,(9\,A\,a\,d+3\,A\,b\,c+B\,a\,d-B\,b\,c)}{3\,d}-\frac{A\,b\,g^2\,(3\,a\,d+3\,b\,c)}{3\,d}\right)}{3\,b\,d}-\frac{a\,g^2\,(3\,A\,a\,d+3\,A\,b\,c+B\,a\,d-B\,b\,c)}{d}+\frac{A\,a\,b\,c\,g^2}{d}\right)+\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\,\left(B\,a^2\,g^2\,x+B\,a\,b\,g^2\,x^2+\frac{B\,b^2\,g^2\,x^3}{3}\right)-\frac{\ln(c+d\,x)\,(3\,B\,a^2\,c\,d^2\,g^2-3\,B\,a\,b\,c^2\,d\,g^2+B\,b^2\,c^3\,g^2)}{3\,d^3}+\frac{A\,b^2\,g^2\,x^3}{3}+\frac{B\,a^3\,g^2\,\ln(a+b\,x)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*g + b*g*x)^2*(A + B*log((e*(a + b*x))/(c + d*x))),x)
```

```
[Out] x^2*((b*g^2*(9*A*a*d + 3*A*b*c + B*a*d - B*b*c))/(6*d) - (A*b*g^2*(3*a*d +
3*b*c))/(6*d)) - x*(((3*a*d + 3*b*c)*((b*g^2*(9*A*a*d + 3*A*b*c + B*a*d - B
*b*c))/(3*d) - (A*b*g^2*(3*a*d + 3*b*c))/(3*d)))/(3*b*d) - (a*g^2*(3*A*a*d
+ 3*A*b*c + B*a*d - B*b*c))/d + (A*a*b*c*g^2)/d) + log((e*(a + b*x))/(c + d
*x))*((B*b^2*g^2*x^3)/3 + B*a^2*g^2*x + B*a*b*g^2*x^2) - (log(c + d*x)*(B*b
^2*c^3*g^2 + 3*B*a^2*c*d^2*g^2 - 3*B*a*b*c^2*d*g^2))/(3*d^3) + (A*b^2*g^2*x
^3)/3 + (B*a^3*g^2*log(a + b*x))/(3*b)
```

3.91 $\int (ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$

Optimal. Leaf size=81

$$-\frac{B(bc-ad)gx}{2d}+\frac{g(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2b}+\frac{B(bc-ad)^2g\log(c+dx)}{2bd^2}$$

```
[Out] -1/2*B*(-a*d+b*c)*g*x/d+1/2*g*(b*x+a)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/b+1/2*B
*(-a*d+b*c)^2*g*ln(d*x+c)/b/d^2
```

Rubi [A]
time = 0.03, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {2548, 21, 45}

$$\frac{g(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2b}+\frac{Bg(bc-ad)^2\log(c+dx)}{2bd^2}-\frac{Bgx(bc-ad)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a*g + b*g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
```

```
[Out] -1/2*(B*(b*c - a*d)*g*x)/d + (g*(a + b*x)^2*(A + B*Log[(e*(a + b*x))/(c + d
*x)]))/(2*b) + (B*(b*c - a*d)^2*g*Log[c + d*x])/(2*b*d^2)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2548

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Dist[B*n*((b*c
- a*d)/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /;
 FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c -
```

```
a*d, 0] && NeQ[m, -1] && !(EqQ[m, -2] && IntegerQ[n])
```

Rubi steps

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)dx = \frac{g(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2b} - \frac{B\int\frac{(bc-ad)g^2(a+bx)}{c+dx}dx}{2bg}$$
$$= \frac{g(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2b} - \frac{(B(bc-ad)g)\int\frac{a+bx}{c+dx}dx}{2b}$$
$$= \frac{g(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2b} - \frac{(B(bc-ad)g)\int\left(\frac{b}{d}+\frac{-}{d(}\right.}{2b}$$
$$= -\frac{B(bc-ad)gx}{2d} + \frac{g(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2b} + \frac{B(bc}{}$$

Mathematica [A]
time = 0.03, size = 69, normalized size = 0.85

$$\frac{g\left((a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)+\frac{B(-bc+ad)(bdx+(-bc+ad)\log(c+dx))}{d^2}\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*g + b*g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
```

```
[Out] (g*((a + b*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + (B*(-(b*c) + a*d)*(b
*d*x + (-(b*c) + a*d)*Log[c + d*x]))/d^2))/(2*b)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 900 vs. 2(75) = 150.
time = 0.25, size = 901, normalized size = 11.12 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*g*x+a*g)*(A+B*ln(e*(b*x+a)/(d*x+c))),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d^2*e*(a*d-b*c)*(-A*d^2*g*(a*d-b*c)*(-1/d^2/(b*e-(b*e/d+(a*d-b*c)*e/d/(d
*x+c))*d)+1/2*b*e/d^2/(b*e-(b*e/d+(a*d-b*c)*e/d/(d*x+c))*d)^2)+1/2*B*d*g/b/
e*ln(b*e-(b*e/d+(a*d-b*c)*e/d/(d*x+c))*d)*a-1/2*B*g/e*ln(b*e-(b*e/d+(a*d-b*
c)*e/d/(d*x+c))*d)*c+B*d^2*g*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*(b*e/d+(a*d-b*
c)*e/d/(d*x+c))/b/e/(b*e-(b*e/d+(a*d-b*c)*e/d/(d*x+c))*d)*a-B*d*g*ln(b*e/d+
(a*d-b*c)*e/d/(d*x+c))*(b*e/d+(a*d-b*c)*e/d/(d*x+c))/e/(b*e-(b*e/d+(a*d-b*c
)*e/d/(d*x+c))*d)*c+1/2*B*d*g/(b*e-(b*e/d+(a*d-b*c)*e/d/(d*x+c))*d)*a-1/2*B
*g/(b*e-(b*e/d+(a*d-b*c)*e/d/(d*x+c))*d)*b*c-B*d^2*g*ln(b*e/d+(a*d-b*c)*e/d
```

```
/(d*x+c))*(b*e/d+(a*d-b*c)*e/d/(d*x+c))/(b*e-(b*e/d+(a*d-b*c)*e/d/(d*x+c))*
d)^2*a+B*d*g*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*(b*e/d+(a*d-b*c)*e/d/(d*x+c))/
(b*e-(b*e/d+(a*d-b*c)*e/d/(d*x+c))*d)^2*b*c+1/2*B*d^3*g/b/e*ln(b*e/d+(a*d-b
*c)*e/d/(d*x+c))*(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2/(b*e-(b*e/d+(a*d-b*c)*e/d/
(d*x+c))*d)^2*a-1/2*B*d^2*g/e*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*(b*e/d+(a*d-b
*c)*e/d/(d*x+c))^2/(b*e-(b*e/d+(a*d-b*c)*e/d/(d*x+c))*d)^2*c)
```

Maxima [A]
time = 0.27, size = 148, normalized size = 1.83

$$\frac{1}{2}\,Abgx^2 + \left(x\log\left(\frac{bxe}{dx+c} + \frac{ae}{dx+c}\right) + \frac{a\log\left(bx+a\right)}{b} - \frac{c\log\left(dx+c\right)}{d}\right)Bag + \frac{1}{2}\left(x^2\log\left(\frac{bxe}{dx+c} + \frac{ae}{dx+c}\right) - \frac{a^2\log\left(bx+a\right)}{b^2} + \frac{c^2\log\left(dx+c\right)}{d^2} - \frac{(bc-ad)x}{bd}\right)Bbg + Aagx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="maxima")
```

```
[Out] 1/2*A*b*g*x^2 + (x*log(b*x*e/(d*x + c) + a*e/(d*x + c)) + a*log(b*x + a)/b
- c*log(d*x + c)/d)*B*a*g + 1/2*(x^2*log(b*x*e/(d*x + c) + a*e/(d*x + c)) -
 a^2*log(b*x + a)/b^2 + c^2*log(d*x + c)/d^2 - (b*c - a*d)*x/(b*d))*B*b*g +
 A*a*g*x
```

Fricas [A]
time = 0.35, size = 124, normalized size = 1.53

$$\frac{Ab^2d^2gx^2 + Ba^2d^2g\log\left(bx+a\right) - (Bb^2cd - (2\,A+B)abd^2)gx + (Bb^2c^2 - 2\,Babcd)g\log\left(dx+c\right) + (Bb^2d^2gx^2 + 2\,Babd^2gx)\log\left(\frac{(bx+a)e}{dx+c}\right)}{2\,bd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="fricas")
```

```
[Out] 1/2*(A*b^2*d^2*g*x^2 + B*a^2*d^2*g*log(b*x + a) - (B*b^2*c*d - (2*A + B)*a*
b*d^2)*g*x + (B*b^2*c^2 - 2*B*a*b*c*d)*g*log(d*x + c) + (B*b^2*d^2*g*x^2 +
2*B*a*b*d^2*g*x)*log((b*x + a)*e/(d*x + c)))/(b*d^2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 253 vs. $2(68) = 136$.
time = 1.03, size = 253, normalized size = 3.12

$$\frac{Abgx^2}{2} + \frac{Ba^2g\log\left(x + \frac{\frac{Ba^3d^2g}{b} + 2Ba^2cdg - Babc^2g}{Ba^2d^2g + 2Babcdg - Bb^2c^2g}\right)}{2b} - \frac{Bcg(2ad-bc)\log\left(x + \frac{3Ba^2cdg - Babc^2g - Bacg(2ad-bc) + \frac{Bbc^2g(2ad-bc)}{d}}{Ba^2d^2g + 2Babcdg - Bb^2c^2g}\right)}{2d^2} + x\left(Aag + \frac{Bag}{2} - \frac{Bbcg}{2d}\right) + \left(Bagx + \frac{Bbgx^2}{2}\right)\log\left(\frac{e(a+bx)}{c+dx}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)*(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

```
[Out] A*b*g*x**2/2 + B*a**2*g*log(x + (B*a**3*d**2*g/b + 2*B*a**2*c*d*g - B*a*b*c
**2*g)/(B*a**2*d**2*g + 2*B*a*b*c*d*g - B*b**2*c**2*g))/(2*b) - B*c*g*(2*a*
d - b*c)*log(x + (3*B*a**2*c*d*g - B*a*b*c**2*g - B*a*c*g*(2*a*d - b*c) + B
*b*c**2*g*(2*a*d - b*c)/d)/(B*a**2*d**2*g + 2*B*a*b*c*d*g - B*b**2*c**2*g))
```

```
/(2*d**2) + x*(A*a*g + B*a*g/2 - B*b*c*g/(2*d)) + (B*a*g*x + B*b*g*x**2/2)*
log(e*(a + b*x)/(c + d*x))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1319 vs. $2(76) = 152$.
time = 2.72, size = 1319, normalized size = 16.28

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="giac")
```

```
[Out] -1/2*(B*b^5*c^3*g*e^3*log(-b*e + (b*x*e + a*e)*d/(d*x + c)) - 3*B*a*b^4*c^2
*d*g*e^3*log(-b*e + (b*x*e + a*e)*d/(d*x + c)) + 3*B*a^2*b^3*c*d^2*g*e^3*lo
g(-b*e + (b*x*e + a*e)*d/(d*x + c)) - B*a^3*b^2*d^3*g*e^3*log(-b*e + (b*x*e
 + a*e)*d/(d*x + c)) - 2*(b*x*e + a*e)*B*b^4*c^3*d*g*e^2*log(-b*e + (b*x*e
+ a*e)*d/(d*x + c))/(d*x + c) + 6*(b*x*e + a*e)*B*a*b^3*c^2*d^2*g*e^2*log(-
b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c) - 6*(b*x*e + a*e)*B*a^2*b^2*c*d^
3*g*e^2*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c) + 2*(b*x*e + a*e)*B
*a^3*b*d^4*g*e^2*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c) + (b*x*e +
 a*e)^2*B*b^3*c^3*d^2*g*e*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^2
 - 3*(b*x*e + a*e)^2*B*a*b^2*c^2*d^3*g*e*log(-b*e + (b*x*e + a*e)*d/(d*x +
c))/(d*x + c)^2 + 3*(b*x*e + a*e)^2*B*a^2*b*c*d^4*g*e*log(-b*e + (b*x*e + a
*e)*d/(d*x + c))/(d*x + c)^2 - (b*x*e + a*e)^2*B*a^3*d^5*g*e*log(-b*e + (b*
x*e + a*e)*d/(d*x + c))/(d*x + c)^2 - (b*x*e + a*e)^2*B*b^3*c^3*d^2*g*e*log
((b*x*e + a*e)/(d*x + c))/(d*x + c)^2 + 3*(b*x*e + a*e)^2*B*a*b^2*c^2*d^3*g
*e*log((b*x*e + a*e)/(d*x + c))/(d*x + c)^2 - 3*(b*x*e + a*e)^2*B*a^2*b*c*d
^4*g*e*log((b*x*e + a*e)/(d*x + c))/(d*x + c)^2 + (b*x*e + a*e)^2*B*a^3*d^5
*g*e*log((b*x*e + a*e)/(d*x + c))/(d*x + c)^2 + A*b^5*c^3*g*e^3 + B*b^5*c^3
*g*e^3 - 3*A*a*b^4*c^2*d*g*e^3 - 3*B*a*b^4*c^2*d*g*e^3 + 3*A*a^2*b^3*c*d^2*
g*e^3 + 3*B*a^2*b^3*c*d^2*g*e^3 - A*a^3*b^2*d^3*g*e^3 - B*a^3*b^2*d^3*g*e^3
 - 2*(b*x*e + a*e)*A*b^4*c^3*d*g*e^2/(d*x + c) - (b*x*e + a*e)*B*b^4*c^3*d*
g*e^2/(d*x + c) + 6*(b*x*e + a*e)*A*a*b^3*c^2*d^2*g*e^2/(d*x + c) + 3*(b*x*
e + a*e)*B*a*b^3*c^2*d^2*g*e^2/(d*x + c) - 6*(b*x*e + a*e)*A*a^2*b^2*c*d^3*
g*e^2/(d*x + c) - 3*(b*x*e + a*e)*B*a^2*b^2*c*d^3*g*e^2/(d*x + c) + 2*(b*x*
e + a*e)*A*a^3*b*d^4*g*e^2/(d*x + c) + (b*x*e + a*e)*B*a^3*b*d^4*g*e^2/(d*x
 + c))*(b*c/((b*c*e - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d
)))/(b^3*d^2*e^2 - 2*(b*x*e + a*e)*b^2*d^3*e/(d*x + c) + (b*x*e + a*e)^2*b*
d^4/(d*x + c)^2)
```

Mupad [B]
time = 4.30, size = 126, normalized size = 1.56

$$x\left(\frac{g\,(4\,A\,a\,d+2\,A\,b\,c+B\,a\,d-B\,b\,c)}{2\,d}-\frac{A\,g\,(2\,a\,d+2\,b\,c)}{2\,d}\right)+\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\left(\frac{B\,b\,g\,x^2}{2}+B\,a\,g\,x\right)+\frac{\ln(c+d\,x)\,(B\,b\,c^2\,g-2\,B\,a\,c\,d\,g)}{2\,d^2}+\frac{A\,b\,g\,x^2}{2}+\frac{B\,a^2\,g\,\ln(a+b\,x)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*g + b*g*x)*(A + B*log((e*(a + b*x))/(c + d*x))),x)

[Out] x*((g*(4*A*a*d + 2*A*b*c + B*a*d - B*b*c))/(2*d) - (A*g*(2*a*d + 2*b*c))/(2*d)) + log((e*(a + b*x))/(c + d*x))*((B*b*g*x^2)/2 + B*a*g*x) + (log(c + d*x)*(B*b*c^2*g - 2*B*a*c*d*g))/(2*d^2) + (A*b*g*x^2)/2 + (B*a^2*g*log(a + b*x))/(2*b)

3.92 $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{ag+bgx}\,dx$

Optimal. Leaf size=80

$$-\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{bg}+\frac{B\text{Li}_2\left(1+\frac{bc-ad}{d(a+bx)}\right)}{bg}$$

```
[Out] -ln((a*d-b*c)/d/(b*x+a))*(A+B*ln(e*(b*x+a)/(d*x+c)))/b/g+B*polylog(2,1+(-a*
d+b*c)/d/(b*x+a))/b/g
```

Rubi [A]
time = 0.14, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2542, 2458, 2378, 2370, 2352}

$$\frac{B\text{PolyLog}\left(2,\frac{bc-ad}{d(a+bx)}+1\right)}{bg}-\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{bg}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])/(a*g + b*g*x),x]
```

```
[Out] -((Log[-((b*c - a*d)/(d*(a + b*x)))]*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/
(b*g)) + (B*PolyLog[2, 1 + (b*c - a*d)/(d*(a + b*x))])/(b*g)
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2370

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)/(x_))^(q_.)*(
x_)^(m_.), x_Symbol] :> Int[(e + d*x)^q*(a + b*Log[c*x^n])^p, x] /; FreeQ[{
a, b, c, d, e, m, n, p}, x] && EqQ[m, q] && IntegerQ[q]
```

Rule 2378

```
Int[((a_.) + Log[(c_.)*(x_)^(n_)]*(b_.))/((x_)*((d_) + (e_.)*(x_)^(r_.))),
x_Symbol] :> Dist[1/n, Subst[Int[(a + b*Log[c*x])/(x*(d + e*x^(r/n))), x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IntegerQ[r/n]
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
```

```
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2542

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))/((f_.) + (g_.)*(x_)), x_Symbol] :> Simp[(-Log[-(b*c - a*d)/(d*(a
+ b*x))])*((A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/g), x] + Dist[B*n*((b*c
 - a*d)/g), Int[Log[-(b*c - a*d)/(d*(a + b*x))]/((a + b*x)*(c + d*x)), x],
x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
 - a*d, 0] && EqQ[b*f - a*g, 0]
```

Rubi steps

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{ag+bgx}\,dx = \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log(ag+bgx)}{bg} - \frac{B\int \frac{(c+dx)\left(-\frac{de(a+bx)}{(c+dx)^2}+\frac{be}{c+dx}\right)\log(ag+bgx)}{e(a+bx)}\,dx}{bg}$$

$$= \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log(ag+bgx)}{bg} - \frac{B\int \frac{(c+dx)\left(-\frac{de(a+bx)}{(c+dx)^2}+\frac{be}{c+dx}\right)\log(ag+bgx)}{a+bx}\,dx}{beg}$$

$$= \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log(ag+bgx)}{bg} - \frac{B\int \left(\frac{be\log(ag+bgx)}{a+bx}-\frac{de\log(ag+bgx)}{c+dx}\right)dx}{beg}$$

$$= \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log(ag+bgx)}{bg} - \frac{B\int \frac{\log(ag+bgx)}{a+bx}\,dx}{g} + \frac{(Bd)\int \frac{\log(ag+bgx)}{c+dx}}{bg}$$

$$= \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log(ag+bgx)}{bg} + \frac{B\log\left(\frac{b(c+dx)}{bc-ad}\right)\log(ag+bgx)}{bg} - B\int$$

$$= \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log(ag+bgx)}{bg} + \frac{B\log\left(\frac{b(c+dx)}{bc-ad}\right)\log(ag+bgx)}{bg} - \frac{B\mathrm{Sul}}{}$$

$$= -\frac{B\log^2(g(a+bx))}{2bg} + \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log(ag+bgx)}{bg} + \frac{B\log\left(\frac{b(c+dx)}{bc-ad}\right)}{b}$$

Mathematica [A]
time = 0.03, size = 95, normalized size = 1.19

$$\frac{\log(g(a+bx))\left(-B\log(g(a+bx))+2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)+B\log\left(\frac{b(c+dx)}{bc-ad}\right)\right)\right)+2B\mathrm{Li}_2\left(\frac{d(a+bx)}{-bc+ad}\right)}{2bg}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])/(a*g + b*g*x),x]
```

```
[Out] (Log[g*(a + b*x)]*(-(B*Log[g*(a + b*x)]) + 2*(A + B*Log[(e*(a + b*x))/(c +
d*x)] + B*Log[(b*(c + d*x))/(b*c - a*d)])) + 2*B*PolyLog[2, (d*(a + b*x))/(
-(b*c) + a*d)])/(2*b*g)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 344 vs. $2(79) = 158$.
time = 0.60, size = 345, normalized size = 4.31

method	result
derivativedivides	$-\frac{e(ad-cb)\left(\frac{d^2 A\ln\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)}{g(ad-cb)be}-\frac{d^2 A\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{g(ad-cb)be}+\frac{d^2 B\,\mathrm{dilog}\left(-\frac{-be+\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d}{be}\right)}{g(ad-cb)be}+\frac{d^2 B\ln\left(\frac{be}{d}+\cdots\right.}{\cdots}\right.}{d^2}$
default	$-\frac{e(ad-cb)\left(\frac{d^2 A\ln\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)}{g(ad-cb)be}-\frac{d^2 A\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{g(ad-cb)be}+\frac{d^2 B\,\mathrm{dilog}\left(-\frac{-be+\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d}{be}\right)}{g(ad-cb)be}+\frac{d^2 B\ln\left(\frac{be}{d}+\cdots\right.}{\cdots}\right.}{d^2}$
risch	$\frac{A\ln(bx+a)}{gb}+\frac{Bd\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2 a}{2g(ad-cb)b}-\frac{B\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2 c}{2g(ad-cb)}-\frac{Bd\,\mathrm{dilog}\left(-\frac{-be+\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d}{be}\right)a}{g(ad-cb)b}+\frac{B\,\mathrm{dilo}\cdots}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d^2*e*(a*d-b*c)*(d^2/g/(a*d-b*c)*A/b/e*ln(b*e-(b*e/d+(a*d-b*c)*e/d/(d*x+
c))*d)-d^2/g/(a*d-b*c)*A/b/e*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))+d^2/g/(a*d-b*c
)*B/b/e*dilog(-(-b*e+(b*e/d+(a*d-b*c)*e/d/(d*x+c))*d)/b/e)+d^2/g/(a*d-b*c)*
B/b/e*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(-(-b*e+(b*e/d+(a*d-b*c)*e/d/(d*x+c
))*d)/b/e)-1/2*d^2/g/(a*d-b*c)*B*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2/b/e)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g),x, algorithm="maxima")
```

```
[Out] -B*(log(b*x + a)*log(d*x + c)/(b*g) - integrate((b*d*x + b*c + (2*b*d*x + b
*c + a*d)*log(b*x + a))/(b^2*d*g*x^2 + a*b*c*g + (b^2*c*g + a*b*d*g)*x), x)
) + A*log(b*g*x + a*g)/(b*g)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g),x, algorithm="fricas")
[Out] integral((B*log((b*x + a)*e/(d*x + c)) + A)/(b*g*x + a*g), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A}{a+bx}\,dx + \int \frac{B\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}{a+bx}\,dx}{g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g),x)
[Out] (Integral(A/(a + b*x), x) + Integral(B*log(a*e/(c + d*x) + b*e*x/(c + d*x))
/(a + b*x), x))/g
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g),x, algorithm="giac")
[Out] integrate((B*log((b*x + a)*e/(d*x + c)) + A)/(b*g*x + a*g), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)}{a\,g+b\,g\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x))/(c + d*x)))/(a*g + b*g*x),x)
[Out] int((A + B*log((e*(a + b*x))/(c + d*x)))/(a*g + b*g*x), x)
```

3.93 $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^2}\,dx$

Optimal. Leaf size=63

$$-\frac{B}{bg^2(a+bx)}-\frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)g^2(a+bx)}$$

```
[Out] -B/b/g^2/(b*x+a)-(d*x+c)*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*c)/g^2/(b*x+a)
```

Rubi [A]

time = 0.03, antiderivative size = 75, normalized size of antiderivative = 1.19, number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2550, 2341}

$$-\frac{(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{g^2(a+bx)(bc-ad)}-\frac{B(c+dx)}{g^2(a+bx)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])/(a*g + b*g*x)^2,x]
```

```
[Out] -((B*(c + d*x))/((b*c - a*d)*g^2*(a + b*x))) - ((c + d*x)*(A + B*Log[(e*(a
+ b*x))/(c + d*x)]))/((b*c - a*d)*g^2*(a + b*x))
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2550

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(
m + 1)*(g/b)^m, Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x],
 x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] &&
 EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && Eq
Q[b*f - a*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Rubi steps

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^2}\,dx = -\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{bg^2(a+bx)} + \frac{B\int \frac{bc-ad}{g(a+bx)^2(c+dx)}\,dx}{bg}$$

$$= -\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{bg^2(a+bx)} + \frac{(B(bc-ad))\int \frac{1}{(a+bx)^2(c+dx)}\,dx}{bg^2}$$

$$= -\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{bg^2(a+bx)} + \frac{(B(bc-ad))\int \left(\frac{b}{(bc-ad)(a+bx)^2} - \frac{bd}{(bc-ad)^2(a+bx)} + \frac{}{(bc-ad}\right.}{bg^2}$$

$$= -\frac{B}{bg^2(a+bx)} - \frac{Bd\log(a+bx)}{b(bc-ad)g^2} - \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{bg^2(a+bx)} + \frac{Bd\log(c+dx)}{b(bc-ad)g^2}$$

Mathematica [A]
time = 0.04, size = 105, normalized size = 1.67

$$\frac{-Abc-bBc+aAd+aBd-Bd(a+bx)\log(a+bx)+(-bBc+aBd)\log\left(\frac{e(a+bx)}{c+dx}\right)+aBd\log(c+dx)+bBdx\log(c+dx)}{b(bc-ad)g^2(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])/(a*g + b*g*x)^2,x]
```

```
[Out] (-(A*b*c) - b*B*c + a*A*d + a*B*d - B*d*(a + b*x)*Log[a + b*x] + (-(b*B*c)
+ a*B*d)*Log[(e*(a + b*x))/(c + d*x)] + a*B*d*Log[c + d*x] + b*B*d*x*Log[c
+ d*x])/(b*(b*c - a*d)*g^2*(a + b*x))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 172 vs. 2(63) = 126.
time = 0.25, size = 173, normalized size = 2.75

method	result	si
norman	$\frac{\frac{(A+B)x}{ga}+\frac{cB\ln\left(\frac{e(bx+a)}{dx+c}\right)}{(ad-cb)g}+\frac{Bdx\ln\left(\frac{e(bx+a)}{dx+c}\right)}{(ad-cb)g}}{g(bx+a)}$	8
risch	$-\frac{B\ln\left(\frac{e(bx+a)}{dx+c}\right)}{b\,g^2(bx+a)}-\frac{B\ln(dx+c)bdx-B\ln(-bx-a)bdx+B\ln(dx+c)ad-B\ln(-bx-a)ad+Aad-Abc+Bad-Bbc}{g^2(bx+a)b(ad-cb)}$	1
derivativedivides	$-\frac{e(ad-cb)\left(-\frac{d^2A}{(ad-cb)^2g^2\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}+\frac{d^2B\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}}-\frac{1}{\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}}\right)}{(ad-cb)^2g^2}\right)}{d^2}$	1

default	$-\frac{e(ad-cb)\left(-\frac{d^{2}A}{(ad-cb)^{2}g^{2}\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}+\frac{d^{2}B\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}}-\frac{1}{\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}}\right)}{(ad-cb)^{2}g^{2}}\right)}{d^{2}}$	173

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/d^2*e*(a*d-b*c)*(-d^2/(a*d-b*c)^2/g^2*A/(b*e/d+(a*d-b*c)*e/d/(d*x+c))+d^
2/(a*d-b*c)^2/g^2*B*(-1/(b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/
d/(d*x+c))-1/(b*e/d+(a*d-b*c)*e/d/(d*x+c))))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(64) = 128$.
time = 0.26, size = 134, normalized size = 2.13

$$-B\left(\frac{\log\left(\frac{bxe}{dx+c}+\frac{ae}{dx+c}\right)}{b^2g^2x+abg^2}+\frac{1}{b^2g^2x+abg^2}+\frac{d\log\left(bx+a\right)}{(b^2c-abd)g^2}-\frac{d\log\left(dx+c\right)}{(b^2c-abd)g^2}\right)-\frac{A}{b^2g^2x+abg^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^2,x, algorithm="maxima")
```

```
[Out] -B*(log(b*x*e/(d*x + c) + a*e/(d*x + c))/(b^2*g^2*x + a*b*g^2) + 1/(b^2*g^2
*x + a*b*g^2) + d*log(b*x + a)/((b^2*c - a*b*d)*g^2) - d*log(d*x + c)/((b^2
*c - a*b*d)*g^2)) - A/(b^2*g^2*x + a*b*g^2)
```

Fricas [A]
time = 0.35, size = 82, normalized size = 1.30

$$-\frac{(A+B)bc-(A+B)ad+(Bbdx+Bbc)\log\left(\frac{(bx+a)e}{dx+c}\right)}{(b^3c-ab^2d)g^2x+(ab^2c-a^2bd)g^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^2,x, algorithm="fricas")
```

```
[Out] -((A + B)*b*c - (A + B)*a*d + (B*b*d*x + B*b*c)*log((b*x + a)*e/(d*x + c)))
/((b^3*c - a*b^2*d)*g^2*x + (a*b^2*c - a^2*b*d)*g^2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 233 vs. $2(49) = 98$.
time = 0.72, size = 233, normalized size = 3.70

$$-\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)}{abg^2+b^2g^2x}-\frac{Bd\log\left(x+\frac{-\frac{Ba^2d^3}{ad-bc}+\frac{2Babcd^2}{ad-bc}+Bad^2-\frac{Bb^2c^2d}{ad-bc}+Bbcd}{2Bbd^2}\right)}{bg^2\left(ad-bc\right)}+\frac{Bd\log\left(x+\frac{\frac{Ba^2d^3}{ad-bc}-\frac{2Babcd^2}{ad-bc}+Bad^2+\frac{Bb^2c^2d}{ad-bc}+Bbcd}{2Bbd^2}\right)}{bg^2\left(ad-bc\right)}+\frac{-A-B}{abg^2+b^2g^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)**2,x)
```

```
[Out] -B*log(e*(a + b*x)/(c + d*x))/(a*b*g**2 + b**2*g**2*x) - B*d*log(x + (-B*a*
*2*d**3/(a*d - b*c) + 2*B*a*b*c*d**2/(a*d - b*c) + B*a*d**2 - B*b**2*c**2*d
/(a*d - b*c) + B*b*c*d)/(2*B*b*d**2))/(b*g**2*(a*d - b*c)) + B*d*log(x + (B
*a**2*d**3/(a*d - b*c) - 2*B*a*b*c*d**2/(a*d - b*c) + B*a*d**2 + B*b**2*c**
2*d/(a*d - b*c) + B*b*c*d)/(2*B*b*d**2))/(b*g**2*(a*d - b*c)) + (-A - B)/(a
*b*g**2 + b**2*g**2*x)
```

Giac [A]
time = 2.84, size = 110, normalized size = 1.75

$$-\frac{\left(Be^{2}\log\left(\frac{bxe+ae}{dx+c}\right)+Ae^{2}+Be^{2}\right)(dx+c)\left(\frac{bc}{(bce-ade)(bc-ad)}-\frac{ad}{(bce-ade)(bc-ad)}\right)}{(bxe+ae)g^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^2,x, algorithm="giac")
```

```
[Out] -(B*e^2*log((b*x*e + a*e)/(d*x + c)) + A*e^2 + B*e^2)*(d*x + c)*(b*c/((b*c*
e - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))/((b*x*e + a*e)
*g^2)
```

Mupad [B]
time = 5.02, size = 104, normalized size = 1.65

$$-\frac{A+B}{x\,b^2\,g^2+a\,b\,g^2}-\frac{B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)}{b^2\,g^2\,\left(x+\frac{a}{b}\right)}-\frac{B\,d\,\mathrm{atan}\left(\frac{b\,c\,2\mathrm{i}+b\,d\,x\,2\mathrm{i}}{a\,d-b\,c}+1\mathrm{i}\right)\,2\mathrm{i}}{b\,g^2\,\left(a\,d-b\,c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x))/(c + d*x)))/(a*g + b*g*x)^2,x)
```

```
[Out] - (A + B)/(b^2*g^2*x + a*b*g^2) - (B*log((e*(a + b*x))/(c + d*x)))/(b^2*g^2
*(x + a/b)) - (B*d*atan((b*c*2i + b*d*x*2i)/(a*d - b*c) + 1i)*2i)/(b*g^2*(a
*d - b*c))
```

3.94 $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^3}\,dx$

Optimal. Leaf size=144

$$-\frac{B}{4bg^3(a+bx)^2}+\frac{Bd}{2b(bc-ad)g^3(a+bx)}+\frac{Bd^2\log(a+bx)}{2b(bc-ad)^2g^3}-\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{2bg^3(a+bx)^2}-\frac{Bd^2\log(c+dx)}{2b(bc-ad)^2g^3}$$

```
[Out] -1/4*B/b/g^3/(b*x+a)^2+1/2*B*d/b/(-a*d+b*c)/g^3/(b*x+a)+1/2*B*d^2*ln(b*x+a)
/b/(-a*d+b*c)^2/g^3+1/2*(-A-B*ln(e*(b*x+a)/(d*x+c)))/b/g^3/(b*x+a)^2-1/2*B*
d^2*ln(d*x+c)/b/(-a*d+b*c)^2/g^3
```

Rubi [A]
time = 0.08, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2548, 21, 46}

$$-\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{2bg^3(a+bx)^2}+\frac{Bd^2\log(a+bx)}{2bg^3(bc-ad)^2}-\frac{Bd^2\log(c+dx)}{2bg^3(bc-ad)^2}+\frac{Bd}{2bg^3(a+bx)(bc-ad)}-\frac{B}{4bg^3(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])/(a*g + b*g*x)^3,x]
```

```
[Out] -1/4*B/(b*g^3*(a + b*x)^2) + (B*d)/(2*b*(b*c - a*d)*g^3*(a + b*x)) + (B*d^2
*Log[a + b*x])/(2*b*(b*c - a*d)^2*g^3) - (A + B*Log[(e*(a + b*x))/(c + d*x)
])/(2*b*g^3*(a + b*x)^2) - (B*d^2*Log[c + d*x])/(2*b*(b*c - a*d)^2*g^3)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2548

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
```

```
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Dist[B*n*((b*c
- a*d)/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /;
 FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c -
 a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Rubi steps

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^3}\,dx = -\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{2bg^3(a+bx)^2}+\frac{B\int \frac{bc-ad}{g^2(a+bx)^3(c+dx)}\,dx}{2bg}$$

$$= -\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{2bg^3(a+bx)^2}+\frac{(B(bc-ad))\int \frac{1}{(a+bx)^3(c+dx)}\,dx}{2bg^3}$$

$$= -\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{2bg^3(a+bx)^2}+\frac{(B(bc-ad))\int \left(\frac{b}{(bc-ad)(a+bx)^3}-\frac{bd}{(bc-ad)^2(a+bx)^2}+\frac{}{(bc-a}\right.}{2bg^3}$$

$$= -\frac{B}{4bg^3(a+bx)^2}+\frac{Bd}{2b(bc-ad)g^3(a+bx)}+\frac{Bd^2\log(a+bx)}{2b(bc-ad)^2g^3}-\frac{A+B\log\left(\frac{e(a}{c-}\right.}{2bg^3(a+bx}$$

Mathematica [A]
time = 0.09, size = 110, normalized size = 0.76

$$-\frac{2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)+\frac{B\left((bc-ad)(-3ad+b(c-2dx))-2d^2(a+bx)^2\log(a+bx)+2d^2(a+bx)^2\log(c+dx)\right)}{(bc-ad)^2}}{4bg^3(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])/(a*g + b*g*x)^3,x]
```

```
[Out] -1/4*(2*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + (B*((b*c - a*d)*(-3*a*d + b*
(c - 2*d*x)) - 2*d^2*(a + b*x)^2*Log[a + b*x] + 2*d^2*(a + b*x)^2*Log[c + d
*x]))/(b*c - a*d)^2)/(b*g^3*(a + b*x)^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 334 vs. $2(137) = 274$.
time = 0.31, size = 335, normalized size = 2.33

method	result
norman	$\frac{\frac{B a\,d^2 x \ln\left(\frac{e(bx+a)}{dx+c}\right)}{\left(a^2d^2-2abcd+b^2c^2\right)g}-\frac{2Aabd-2A\,b^2c+3Babd-B\,b^2c}{4g\,b^2(ad-cb)}-\frac{Bdx}{2g(ad-cb)}+\frac{Bc(2ad-cb)\ln\left(\frac{e(bx+a)}{dx+c}\right)}{2g\left(a^2d^2-2abcd+b^2c^2\right)}+\frac{B\,d^2 b\,x^2\ln\left(\frac{e(bx+a)}{dx+c}\right)}{2\left(a^2d^2-2abcd+b^2c^2\right)g}}{g^2(bx+a)^2}$

risch	$-\frac{B\ln\left(\frac{e(bx+a)}{dx+c}\right)}{2b\,g^3(bx+a)^2}-\frac{2B\ln(dx+c)b^2d^2x^2-2B\ln(-bx-a)b^2d^2x^2+4B\ln(dx+c)ab\,d^2x-4B\ln(-bx-a)ab\,d^2x+2B\ln(dx+}{4(a^2d^2-2ab}$
derivativedivides	$-\frac{e(ad-cb)\left(\frac{d^2Abe}{2(ad-cb)^3g^3\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}-\frac{d^3A}{(ad-cb)^3g^3\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}-\frac{d^2Bbe\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{2\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}-\frac{1}{4\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}\right.}{(ad-cb)^3g^3}\right.}{d^2}$
default	$-\frac{e(ad-cb)\left(\frac{d^2Abe}{2(ad-cb)^3g^3\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}-\frac{d^3A}{(ad-cb)^3g^3\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}-\frac{d^2Bbe\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{2\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}-\frac{1}{4\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}\right.}{(ad-cb)^3g^3}\right.}{d^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/d^2*e*(a*d-b*c)*(1/2*d^2/(a*d-b*c)^3/g^3*A*b*e/(b*e/d+(a*d-b*c)*e/d/(d*x
+c))^2-d^3/(a*d-b*c)^3/g^3*A/(b*e/d+(a*d-b*c)*e/d/(d*x+c))-d^2/(a*d-b*c)^3/
g^3*B*b*e*(-1/2/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x
+c))-1/4/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2)+d^3/(a*d-b*c)^3/g^3*B*(-1/(b*e/d+
(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/(b*e/d+(a*d-b*c)*e
/d/(d*x+c))))
```

Maxima [A]
time = 0.32, size = 257, normalized size = 1.78

$$\frac{1}{4}B\left(\frac{2bdx-bc+3ad}{(b^4c-ab^3d)g^3x^2+2(ab^3c-a^2b^2d)g^3x+(a^2b^2c-a^3bd)g^3}-\frac{2\log\left(\frac{bxe}{dx+c}+\frac{ae}{dx+c}\right)}{b^3g^3x^2+2ab^2g^3x+a^2bg^3}+\frac{2d^2\log(bx+a)}{(b^3c^2-2ab^2cd+a^2bd^2)g^3}-\frac{2d^2\log(dx+c)}{(b^3c^2-2ab^2cd+a^2bd^2)g^3}\right)-\frac{A}{2(b^3g^3x^2+2ab^2g^3x+a^2bg^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^3,x, algorithm="maxima")
```

```
[Out] 1/4*B*((2*b*d*x - b*c + 3*a*d)/((b^4*c - a*b^3*d)*g^3*x^2 + 2*(a*b^3*c - a^
2*b^2*d)*g^3*x + (a^2*b^2*c - a^3*b*d)*g^3) - 2*log(b*x*e/(d*x + c) + a*e/(
d*x + c))/(b^3*g^3*x^2 + 2*a*b^2*g^3*x + a^2*b*g^3) + 2*d^2*log(b*x + a)/((
b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*g^3) - 2*d^2*log(d*x + c)/((b^3*c^2 - 2*
a*b^2*c*d + a^2*b*d^2)*g^3)) - 1/2*A/(b^3*g^3*x^2 + 2*a*b^2*g^3*x + a^2*b*g
^3)
```

Fricas [A]
time = 0.36, size = 216, normalized size = 1.50

$$-\frac{(2A+B)b^2c^2-4(A+B)abcd+(2A+3B)a^2d^2-2(Bb^2cd-Babd^2)x-2(Bb^2d^2x^2+2Babd^2x-Bb^2c^2+2Babcd)\log\left(\frac{(bx+a)e}{dx+c}\right)}{4((b^5c^2-2ab^4cd+a^2b^3d^2)g^3x^2+2(ab^4c^2-2a^2b^3cd+a^3b^2d^2)g^3x+(a^2b^3c^2-2a^3b^2cd+a^4bd^2)g^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^3,x, algorithm="fricas")
```

```
[Out] -1/4*((2*A + B)*b^2*c^2 - 4*(A + B)*a*b*c*d + (2*A + 3*B)*a^2*d^2 - 2*(B*b^
2*c*d - B*a*b*d^2)*x - 2*(B*b^2*d^2*x^2 + 2*B*a*b*d^2*x - B*b^2*c^2 + 2*B*a
*b*c*d)*log((b*x + a)*e/(d*x + c)))/((b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*
g^3*x^2 + 2*(a*b^4*c^2 - 2*a^2*b^3*c*d + a^3*b^2*d^2)*g^3*x + (a^2*b^3*c^2
- 2*a^3*b^2*c*d + a^4*b*d^2)*g^3)
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 422 vs. $2(124) = 248$.
time = 1.28, size = 422, normalized size = 2.93

$$-\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)}{2a^2bg^3+4ab^2g^3x+2b^3g^3x^2}-\frac{Bd^2\log\left(x+\frac{-\frac{Ba^3d^5}{(ad-bc)^2}+\frac{3Ba^2bcd^4}{(ad-bc)^2}-\frac{3Bab^2c^2d^3}{(ad-bc)^2}+Bad^3+\frac{Bb^3c^3d^2}{(ad-bc)^2}+Bbcd^2}{2Bbd^3}\right)}{2bg^3\left(ad-bc\right)^2}+\frac{Bd^2\log\left(x+\frac{\frac{Ba^3d^5}{(ad-bc)^2}-\frac{3Ba^2bcd^4}{(ad-bc)^2}+\frac{3Bab^2c^2d^3}{(ad-bc)^2}+Bad^3-\frac{Bb^3c^3d^2}{(ad-bc)^2}+Bbcd^2}{2Bbd^3}\right)}{2bg^3\left(ad-bc\right)^2}+\frac{-2Aad+2Abc-3Bad+Bbc-2Bbdx}{4a^3bdg^3-4a^2b^2cg^3+x^2\cdot\left(4ab^3dg^3-4b^4cg^3\right)+x\left(8a^2b^2dg^3-8ab^3cg^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)**3,x)
```

```
[Out] -B*log(e*(a + b*x)/(c + d*x))/(2*a**2*b*g**3 + 4*a*b**2*g**3*x + 2*b**3*g**
3*x**2) - B*d**2*log(x + (-B*a**3*d**5/(a*d - b*c)**2 + 3*B*a**2*b*c*d**4/(
a*d - b*c)**2 - 3*B*a*b**2*c**2*d**3/(a*d - b*c)**2 + B*a*d**3 + B*b**3*c**
3*d**2/(a*d - b*c)**2 + B*b*c*d**2)/(2*B*b*d**3))/(2*b*g**3*(a*d - b*c)**2)
 + B*d**2*log(x + (B*a**3*d**5/(a*d - b*c)**2 - 3*B*a**2*b*c*d**4/(a*d - b*
c)**2 + 3*B*a*b**2*c**2*d**3/(a*d - b*c)**2 + B*a*d**3 - B*b**3*c**3*d**2/(
a*d - b*c)**2 + B*b*c*d**2)/(2*B*b*d**3))/(2*b*g**3*(a*d - b*c)**2) + (-2*A
*a*d + 2*A*b*c - 3*B*a*d + B*b*c - 2*B*b*d*x)/(4*a**3*b*d*g**3 - 4*a**2*b**
2*c*g**3 + x**2*(4*a*b**3*d*g**3 - 4*b**4*c*g**3) + x*(8*a**2*b**2*d*g**3 -
 8*a*b**3*c*g**3))
```

Giac [**A**]
time = 2.54, size = 237, normalized size = 1.65

$$-\frac{\left(2\,Bbe^3\log\left(\frac{bxe+ae}{dx+c}\right)-\frac{4\,(bxe+ae)Bde^2\log\left(\frac{bxe+ae}{dx+c}\right)}{dx+c}+2\,Abe^3+Bbe^3-\frac{4\,(bxe+ae)Ade^2}{dx+c}-\frac{4\,(bxe+ae)Bde^2}{dx+c}\right)\left(\frac{bc}{(bce-ade)(bc-ad)}-\frac{ad}{(bce-ade)(bc-ad)}\right)}{4\left(\frac{(bxe+ae)^2bcg^3}{(dx+c)^2}-\frac{(bxe+ae)^2adg^3}{(dx+c)^2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^3,x, algorithm="giac")
```

```
[Out] -1/4*(2*B*b*e^3*log((b*x*e + a*e)/(d*x + c)) - 4*(b*x*e + a*e)*B*d*e^2*log(
(b*x*e + a*e)/(d*x + c))/(d*x + c) + 2*A*b*e^3 + B*b*e^3 - 4*(b*x*e + a*e)*
A*d*e^2/(d*x + c) - 4*(b*x*e + a*e)*B*d*e^2/(d*x + c))*(b*c/((b*c*e - a*d*e
)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))/((b*x*e + a*e)^2*b*c*g^
3/(d*x + c)^2 - (b*x*e + a*e)^2*a*d*g^3/(d*x + c)^2)
```

Mupad [**B**]
time = 5.04, size = 209, normalized size = 1.45

$$-\frac{\frac{2\,A\,a\,d-2\,A\,b\,c+3\,B\,a\,d-B\,b\,c}{2\,(a\,d-b\,c)}+\frac{B\,b\,d\,x}{a\,d-b\,c}}{2\,a^2\,b\,g^3+4\,a\,b^2\,g^3\,x+2\,b^3\,g^3\,x^2}-\frac{B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)}{2\,b^2\,g^3\,\left(2\,a\,x+b\,x^2+\frac{a^2}{b}\right)}-\frac{B\,d^2\,\mathrm{atanh}\left(\frac{2\,b^3\,c^2\,g^3-2\,a^2\,b\,d^2\,g^3}{2\,b\,g^3\,{\left(a\,d-b\,c\right)}^2}-\frac{2\,b\,d\,x}{a\,d-b\,c}\right)}{b\,g^3\,{\left(a\,d-b\,c\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x))/(c + d*x)))/(a*g + b*g*x)^3,x)
```

```
[Out] - ((2*A*a*d - 2*A*b*c + 3*B*a*d - B*b*c)/(2*(a*d - b*c)) + (B*b*d*x)/(a*d -
 b*c))/(2*a^2*b*g^3 + 2*b^3*g^3*x^2 + 4*a*b^2*g^3*x) - (B*log((e*(a + b*x))
/(c + d*x)))/(2*b^2*g^3*(2*a*x + b*x^2 + a^2/b)) - (B*d^2*atanh((2*b^3*c^2*
g^3 - 2*a^2*b*d^2*g^3)/(2*b*g^3*(a*d - b*c)^2) - (2*b*d*x)/(a*d - b*c)))/(b
*g^3*(a*d - b*c)^2)
```

3.95 $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^4}\,dx$

Optimal. Leaf size=175

$$-\frac{B}{9bg^4(a+bx)^3}+\frac{Bd}{6b(bc-ad)g^4(a+bx)^2}-\frac{Bd^2}{3b(bc-ad)^2g^4(a+bx)}-\frac{Bd^3\log(a+bx)}{3b(bc-ad)^3g^4}-\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{3bg^4(a+bx)^3}$$

```
[Out] -1/9*B/b/g^4/(b*x+a)^3+1/6*B*d/b/(-a*d+b*c)/g^4/(b*x+a)^2-1/3*B*d^2/b/(-a*d
+b*c)^2/g^4/(b*x+a)-1/3*B*d^3*ln(b*x+a)/b/(-a*d+b*c)^3/g^4+1/3*(-A-B*ln(e*(
b*x+a)/(d*x+c)))/b/g^4/(b*x+a)^3+1/3*B*d^3*ln(d*x+c)/b/(-a*d+b*c)^3/g^4
```

Rubi [A]

time = 0.10, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2548, 21, 46}

$$-\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{3bg^4(a+bx)^3}-\frac{Bd^3\log(a+bx)}{3bg^4(bc-ad)^3}+\frac{Bd^3\log(c+dx)}{3bg^4(bc-ad)^3}-\frac{Bd^2}{3bg^4(a+bx)(bc-ad)^2}+\frac{Bd}{6bg^4(a+bx)^2(bc-ad)}-\frac{B}{9bg^4(a+bx)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])/(a*g + b*g*x)^4,x]
```

```
[Out] -1/9*B/(b*g^4*(a + b*x)^3) + (B*d)/(6*b*(b*c - a*d)*g^4*(a + b*x)^2) - (B*d
^2)/(3*b*(b*c - a*d)^2*g^4*(a + b*x)) - (B*d^3*Log[a + b*x])/(3*b*(b*c - a*
d)^3*g^4) - (A + B*Log[(e*(a + b*x))/(c + d*x)])/(3*b*g^4*(a + b*x)^3) + (B
*d^3*Log[c + d*x])/(3*b*(b*c - a*d)^3*g^4)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2548

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
```

```
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Dist[B*n*((b*c
- a*d)/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /;
 FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c -
 a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Rubi steps

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^4}\,dx = -\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{3bg^4(a+bx)^3}+\frac{B\int\frac{bc-ad}{g^3(a+bx)^4(c+dx)}\,dx}{3bg}$$

$$= -\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{3bg^4(a+bx)^3}+\frac{(B(bc-ad))\int\frac{1}{(a+bx)^4(c+dx)}\,dx}{3bg^4}$$

$$= -\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{3bg^4(a+bx)^3}+\frac{(B(bc-ad))\int\left(\frac{b}{(bc-ad)(a+bx)^4}-\frac{bd}{(bc-ad)^2(a+bx)^3}+\frac{b}{(bc-ad)}\right.}{3bg^4}$$

$$= -\frac{B}{9bg^4(a+bx)^3}+\frac{Bd}{6b(bc-ad)g^4(a+bx)^2}-\frac{Bd^2}{3b(bc-ad)^2g^4(a+bx)}-\frac{Bd^3\log}{3b(bc-}$$

Mathematica [**A**]
time = 0.11, size = 141, normalized size = 0.81

$$-\frac{6\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)+\frac{B\left((bc-ad)\left(11a^2d^2+abd(-7c+15dx)+b^2\left(2c^2-3cdx+6d^2x^2\right)\right)+6d^3(a+bx)^3\log(a+bx)-6d^3(a+bx)^3\log(c+dx)\right)}{(bc-ad)^3}}{18bg^4(a+bx)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])/(a*g + b*g*x)^4,x]
```

```
[Out] -1/18*(6*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + (B*((b*c - a*d)*(11*a^2*d^2
 + a*b*d*(-7*c + 15*d*x) + b^2*(2*c^2 - 3*c*d*x + 6*d^2*x^2)) + 6*d^3*(a +
b*x)^3*Log[a + b*x] - 6*d^3*(a + b*x)^3*Log[c + d*x]))/(b*c - a*d)^3)/(b*g^
4*(a + b*x)^3)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 502 vs. $2(166) = 332$.
time = 0.32, size = 503, normalized size = 2.87 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/d^2*e*(a*d-b*c)*(-1/3*d^2/(a*d-b*c)^4/g^4*A*b^2*e^2/(b*e/d+(a*d-b*c)*e/d
/(d*x+c))^3+d^3/(a*d-b*c)^4/g^4*A*b*e/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2-d^4/(
```

```
a*d-b*c)^4/g^4*A/(b*e/d+(a*d-b*c)*e/d/(d*x+c))+d^2/(a*d-b*c)^4/g^4*B*b^2*e^
2*(-1/3/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/9
/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3)-2*d^3/(a*d-b*c)^4/g^4*B*b*e*(-1/2/(b*e/d+
(a*d-b*c)*e/d/(d*x+c))^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/4/(b*e/d+(a*d-b*
c)*e/d/(d*x+c))^2)+d^4/(a*d-b*c)^4/g^4*B*(-1/(b*e/d+(a*d-b*c)*e/d/(d*x+c))*
ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/(b*e/d+(a*d-b*c)*e/d/(d*x+c))))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 430 vs. $2(164) = 328$.
time = 0.31, size = 430, normalized size = 2.46

$$-\frac{1}{18}\,B\left(\frac{6\,b^2d^2x^2+2\,b^2c^2-7\,abcd+11\,a^2d^2-3\,(b^2cd-5\,abd^2)x}{(b^6c^2-2\,ab^5cd+a^2b^4d^2)g^4x^3+3\,(ab^5c^2-2\,a^2b^4cd+a^3b^3d^2)g^4x^2+3\,(a^2b^4c^2-2\,a^3b^3cd+a^4b^2d^2)g^4x+(a^3b^3c^2-2\,a^4b^2cd+a^5bd^2)g^4}+\frac{6\,\log\left(\frac{bxe}{dx+c}+\frac{ae}{dx+c}\right)}{b^4g^4x^3+3\,ab^3g^4x^2+3\,a^2b^2g^4x+a^3bg^4}+\frac{6\,d^3\log\left(bx+a\right)}{(b^4c^3-3\,ab^3c^2d+3\,a^2b^2cd^2-a^3bd^3)g^4}-\frac{6\,d^3\log\left(dx+c\right)}{(b^4c^3-3\,ab^3c^2d+3\,a^2b^2cd^2-a^3bd^3)g^4}\right)-\frac{A}{3\,(b^4g^4x^3+3\,ab^3g^4x^2+3\,a^2b^2g^4x+a^3bg^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^4,x, algorithm="maxima")
```

```
[Out] -1/18*B*((6*b^2*d^2*x^2 + 2*b^2*c^2 - 7*a*b*c*d + 11*a^2*d^2 - 3*(b^2*c*d -
 5*a*b*d^2)*x)/((b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2)*g^4*x^3 + 3*(a*b^5*c^
2 - 2*a^2*b^4*c*d + a^3*b^3*d^2)*g^4*x^2 + 3*(a^2*b^4*c^2 - 2*a^3*b^3*c*d +
 a^4*b^2*d^2)*g^4*x + (a^3*b^3*c^2 - 2*a^4*b^2*c*d + a^5*b*d^2)*g^4) + 6*lo
g(b*x*e/(d*x + c) + a*e/(d*x + c))/(b^4*g^4*x^3 + 3*a*b^3*g^4*x^2 + 3*a^2*b
^2*g^4*x + a^3*b*g^4) + 6*d^3*log(b*x + a)/((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^
2*b^2*c*d^2 - a^3*b*d^3)*g^4) - 6*d^3*log(d*x + c)/((b^4*c^3 - 3*a*b^3*c^2*
d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*g^4)) - 1/3*A/(b^4*g^4*x^3 + 3*a*b^3*g^4*x
^2 + 3*a^2*b^2*g^4*x + a^3*b*g^4)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 405 vs. $2(164) = 328$.
time = 0.39, size = 405, normalized size = 2.31

$$-\frac{2\,(3\,A+B)b^3c^3-9\,(2\,A+B)ab^2c^2d+18\,(A+B)a^2bcd^2-(6\,A+11\,B)a^3d^3+6\,(Bb^3cd^2-Bab^2d^3)x^2-3\,(Bb^3c^2d-6\,Bab^2cd^2+5\,Ba^2bd^3)x+6\,(Bb^3d^3x^3+3\,Bab^2d^3x^2+3\,Ba^2bd^3x+Bb^3c^3-3\,Bab^2c^2d+3\,Ba^2bcd^2)\log\left(\frac{(bx+a)e}{dx+c}\right)}{18\,((b^7c^3-3\,ab^6c^2d+3\,a^2b^5cd^2-a^3b^4d^3)g^4x^3+3\,(ab^6c^3-3\,a^2b^5c^2d+3\,a^3b^4cd^2-a^4b^3d^3)g^4x^2+3\,(a^2b^5c^3-3\,a^3b^4c^2d+3\,a^4b^3cd^2-a^5b^2d^3)g^4x+(a^3b^4c^3-3\,a^4b^3c^2d+3\,a^5b^2cd^2-a^6bd^3)g^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^4,x, algorithm="fricas")
```

```
[Out] -1/18*(2*(3*A + B)*b^3*c^3 - 9*(2*A + B)*a*b^2*c^2*d + 18*(A + B)*a^2*b*c*d
^2 - (6*A + 11*B)*a^3*d^3 + 6*(B*b^3*c*d^2 - B*a*b^2*d^3)*x^2 - 3*(B*b^3*c^
2*d - 6*B*a*b^2*c*d^2 + 5*B*a^2*b*d^3)*x + 6*(B*b^3*d^3*x^3 + 3*B*a*b^2*d^3
*x^2 + 3*B*a^2*b*d^3*x + B*b^3*c^3 - 3*B*a*b^2*c^2*d + 3*B*a^2*b*c*d^2)*log
((b*x + a)*e/(d*x + c)))/((b^7*c^3 - 3*a*b^6*c^2*d + 3*a^2*b^5*c*d^2 - a^3*
b^4*d^3)*g^4*x^3 + 3*(a*b^6*c^3 - 3*a^2*b^5*c^2*d + 3*a^3*b^4*c*d^2 - a^4*b
^3*d^3)*g^4*x^2 + 3*(a^2*b^5*c^3 - 3*a^3*b^4*c^2*d + 3*a^4*b^3*c*d^2 - a^5*
b^2*d^3)*g^4*x + (a^3*b^4*c^3 - 3*a^4*b^3*c^2*d + 3*a^5*b^2*c*d^2 - a^6*b*d
^3)*g^4)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 656 vs. $2(151) = 302$.
time = 2.01, size = 656, normalized size = 3.75

$$-\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)}{3a^3bg^4+9a^2b^2g^4x+9ab^3g^4x^2+3b^4g^4x^3}-\frac{Bd^3\log\left(x+\frac{-\frac{Ba^4d^7}{(ad-bc)^3}+\frac{4Ba^3bcd^6}{(ad-bc)^3}-\frac{6Ba^2b^2c^2d^5}{(ad-bc)^3}+\frac{4Bab^3c^3d^4}{(ad-bc)^3}+Bad^4-\frac{Bb^4c^4d^3}{(ad-bc)^3}+Bbcd^3}{2Bbd^4}\right)}{3bg^4(ad-bc)^3}+\frac{Bd^3\log\left(x+\frac{\frac{Ba^4d^7}{(ad-bc)^3}-\frac{4Ba^3bcd^6}{(ad-bc)^3}+\frac{6Ba^2b^2c^2d^5}{(ad-bc)^3}-\frac{4Bab^3c^3d^4}{(ad-bc)^3}+Bad^4+\frac{Bb^4c^4d^3}{(ad-bc)^3}+Bbcd^3}{2Bbd^4}\right)}{3bg^4(ad-bc)^3}+\frac{-6Aa^2d^2+12Aabcd-6Ab^2c^2-11Ba^2d^2+7Babcd-2Bb^2c^2-6Bb^2d^2x^2+x(-15Babd^2+3Bb^2cd)}{18a^5bd^2g^4-36a^4b^2cdg^4+18a^3b^3c^2g^4+x^3\cdot(18a^2b^4d^2g^4-36ab^5cdg^4+18b^6c^2g^4)+x^2\cdot(54a^3b^3d^2g^4-108a^2b^4cdg^4+54ab^5c^2g^4)+x(54a^4b^2d^2g^4-108a^3b^3cdg^4+54a^2b^4c^2g^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)**4,x)
```

```
[Out] -B*log(e*(a + b*x)/(c + d*x))/(3*a**3*b*g**4 + 9*a**2*b**2*g**4*x + 9*a*b**
3*g**4*x**2 + 3*b**4*g**4*x**3) - B*d**3*log(x + (-B*a**4*d**7/(a*d - b*c)*
*3 + 4*B*a**3*b*c*d**6/(a*d - b*c)**3 - 6*B*a**2*b**2*c**2*d**5/(a*d - b*c)
**3 + 4*B*a*b**3*c**3*d**4/(a*d - b*c)**3 + B*a*d**4 - B*b**4*c**4*d**3/(a*
d - b*c)**3 + B*b*c*d**3)/(2*B*b*d**4))/(3*b*g**4*(a*d - b*c)**3) + B*d**3*
log(x + (B*a**4*d**7/(a*d - b*c)**3 - 4*B*a**3*b*c*d**6/(a*d - b*c)**3 + 6*
B*a**2*b**2*c**2*d**5/(a*d - b*c)**3 - 4*B*a*b**3*c**3*d**4/(a*d - b*c)**3
+ B*a*d**4 + B*b**4*c**4*d**3/(a*d - b*c)**3 + B*b*c*d**3)/(2*B*b*d**4))/(3
*b*g**4*(a*d - b*c)**3) + (-6*A*a**2*d**2 + 12*A*a*b*c*d - 6*A*b**2*c**2 -
11*B*a**2*d**2 + 7*B*a*b*c*d - 2*B*b**2*c**2 - 6*B*b**2*d**2*x**2 + x*(-15*
B*a*b*d**2 + 3*B*b**2*c*d))/(18*a**5*b*d**2*g**4 - 36*a**4*b**2*c*d*g**4 +
18*a**3*b**3*c**2*g**4 + x**3*(18*a**2*b**4*d**2*g**4 - 36*a*b**5*c*d*g**4
+ 18*b**6*c**2*g**4) + x**2*(54*a**3*b**3*d**2*g**4 - 108*a**2*b**4*c*d*g**
4 + 54*a*b**5*c**2*g**4) + x*(54*a**4*b**2*d**2*g**4 - 108*a**3*b**3*c*d*g*
*4 + 54*a**2*b**4*c**2*g**4))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 382 vs. $2(164) = 328$.
time = 2.56, size = 382, normalized size = 2.18

$$-\frac{\left(6\,Bb^2e^4\log\left(\frac{bxe+ae}{dx+c}\right)-\frac{18\,(bxe+ae)Bbde^3\log\left(\frac{bxe+ae}{dx+c}\right)}{dx+c}+\frac{18\,(bxe+ae)^2Bd^2e^2\log\left(\frac{bxe+ae}{dx+c}\right)}{(dx+c)^2}+6\,Ab^2e^4+2\,Bb^2e^4-\frac{18\,(bxe+ae)Abde^3}{dx+c}-\frac{9\,(bxe+ae)Bbde^3}{dx+c}+\frac{18\,(bxe+ae)^2Ad^2e^2}{(dx+c)^2}+\frac{18\,(bxe+ae)^2Bd^2e^2}{(dx+c)^2}\right)\left(\frac{bc}{(bce-ade)(bc-ad)}-\frac{ad}{(bce-ade)(bc-ad)}\right)}{18\left(\frac{(bxe+ae)^3b^2c^2g^4}{(dx+c)^3}-\frac{2\,(bxe+ae)^3abcdg^4}{(dx+c)^3}+\frac{(bxe+ae)^3a^2d^2g^4}{(dx+c)^3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^4,x, algorithm="giac")
```

```
[Out] -1/18*(6*B*b^2*e^4*log((b*x*e + a*e)/(d*x + c)) - 18*(b*x*e + a*e)*B*b*d*e^
3*log((b*x*e + a*e)/(d*x + c))/(d*x + c) + 18*(b*x*e + a*e)^2*B*d^2*e^2*log
((b*x*e + a*e)/(d*x + c))/(d*x + c)^2 + 6*A*b^2*e^4 + 2*B*b^2*e^4 - 18*(b*x
*e + a*e)*A*b*d*e^3/(d*x + c) - 9*(b*x*e + a*e)*B*b*d*e^3/(d*x + c) + 18*(b
*x*e + a*e)^2*A*d^2*e^2/(d*x + c)^2 + 18*(b*x*e + a*e)^2*B*d^2*e^2/(d*x + c
)^2)*(b*c/((b*c*e - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d))
)/((b*x*e + a*e)^3*b^2*c^2*g^4/(d*x + c)^3 - 2*(b*x*e + a*e)^3*a*b*c*d*g^4/
(d*x + c)^3 + (b*x*e + a*e)^3*a^2*d^2*g^4/(d*x + c)^3)
```

Mupad [B]
time = 5.58, size = 339, normalized size = 1.94

$$\frac{2\,A\,a\,c\,d}{3\,g^4\,(a\,d-b\,c)^2\,(a+b\,x)^3}-\frac{A\,b\,c^2}{3\,g^4\,(a\,d-b\,c)^2\,(a+b\,x)^3}-\frac{B\,b\,c^2}{9\,g^4\,(a\,d-b\,c)^2\,(a+b\,x)^3}-\frac{A\,a^2\,d^2}{3\,b\,g^4\,(a\,d-b\,c)^2\,(a+b\,x)^3}-\frac{11\,B\,a^2\,d^2}{18\,b\,g^4\,(a\,d-b\,c)^2\,(a+b\,x)^3}-\frac{5\,B\,a\,d^2\,x}{6\,g^4\,(a\,d-b\,c)^2\,(a+b\,x)^3}-\frac{B\,b\,d^2\,x^2}{3\,g^4\,(a\,d-b\,c)^2\,(a+b\,x)^3}-\frac{B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)}{3\,b\,g^4\,(a+b\,x)^3}+\frac{7\,B\,a\,c\,d}{18\,g^4\,(a\,d-b\,c)^2\,(a+b\,x)^3}+\frac{B\,b\,c\,d\,x}{6\,g^4\,(a\,d-b\,c)^2\,(a+b\,x)^3}-\frac{B\,d^3\,\mathrm{atan}\left(\frac{a\,d\,1\mathrm{i}+b\,c\,1\mathrm{i}+b\,d\,x\,2\mathrm{i}}{a\,d-b\,c}\right)\,2\mathrm{i}}{3\,b\,g^4\,(a\,d-b\,c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x))/(c + d*x)))/(a*g + b*g*x)^4,x)
```

```
[Out] (2*A*a*c*d)/(3*g^4*(a*d - b*c)^2*(a + b*x)^3) - (B*d^3*atan((a*d*1i + b*c*1
i + b*d*x*2i)/(a*d - b*c))*2i)/(3*b*g^4*(a*d - b*c)^3) - (A*b*c^2)/(3*g^4*(
a*d - b*c)^2*(a + b*x)^3) - (B*b*c^2)/(9*g^4*(a*d - b*c)^2*(a + b*x)^3) - (
A*a^2*d^2)/(3*b*g^4*(a*d - b*c)^2*(a + b*x)^3) - (11*B*a^2*d^2)/(18*b*g^4*(
a*d - b*c)^2*(a + b*x)^3) - (5*B*a*d^2*x)/(6*g^4*(a*d - b*c)^2*(a + b*x)^3)
 - (B*b*d^2*x^2)/(3*g^4*(a*d - b*c)^2*(a + b*x)^3) - (B*log((e*(a + b*x))/(
c + d*x)))/(3*b*g^4*(a + b*x)^3) + (7*B*a*c*d)/(18*g^4*(a*d - b*c)^2*(a + b
*x)^3) + (B*b*c*d*x)/(6*g^4*(a*d - b*c)^2*(a + b*x)^3)
```

3.96 $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^5}\,dx$

Optimal. Leaf size=206

$$-\frac{B}{16bg^5(a+bx)^4}+\frac{Bd}{12b(bc-ad)g^5(a+bx)^3}-\frac{Bd^2}{8b(bc-ad)^2g^5(a+bx)^2}+\frac{Bd^3}{4b(bc-ad)^3g^5(a+bx)}+\frac{Bd^4\log(a+}{4b(bc-ad)}$$

```
[Out] -1/16*B/b/g^5/(b*x+a)^4+1/12*B*d/b/(-a*d+b*c)/g^5/(b*x+a)^3-1/8*B*d^2/b/(-a
*d+b*c)^2/g^5/(b*x+a)^2+1/4*B*d^3/b/(-a*d+b*c)^3/g^5/(b*x+a)+1/4*B*d^4*ln(b
*x+a)/b/(-a*d+b*c)^4/g^5+1/4*(-A-B*ln(e*(b*x+a)/(d*x+c)))/b/g^5/(b*x+a)^4-1
/4*B*d^4*ln(d*x+c)/b/(-a*d+b*c)^4/g^5
```

Rubi [A]
time = 0.12, antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2548, 21, 46}

$$-\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{4bg^5(a+bx)^4}+\frac{Bd^4\log(a+bx)}{4bg^5(bc-ad)^4}-\frac{Bd^4\log(c+dx)}{4bg^5(bc-ad)^4}+\frac{Bd^3}{4bg^5(a+bx)(bc-ad)^3}-\frac{Bd^2}{8bg^5(a+bx)^2(bc-ad)^2}+\frac{Bd}{12bg^5(a+bx)^3(bc-ad)}-\frac{B}{16bg^5(a+bx)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])/(a*g + b*g*x)^5,x]
```

```
[Out] -1/16*B/(b*g^5*(a + b*x)^4) + (B*d)/(12*b*(b*c - a*d)*g^5*(a + b*x)^3) - (B
*d^2)/(8*b*(b*c - a*d)^2*g^5*(a + b*x)^2) + (B*d^3)/(4*b*(b*c - a*d)^3*g^5*
(a + b*x)) + (B*d^4*Log[a + b*x])/(4*b*(b*c - a*d)^4*g^5) - (A + B*Log[(e*(
a + b*x))/(c + d*x)])/(4*b*g^5*(a + b*x)^4) - (B*d^4*Log[c + d*x])/(4*b*(b*
c - a*d)^4*g^5)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2548

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Dist[B*n*((b*c
- a*d)/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /;
 FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c -
 a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Rubi steps

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^5}\,dx = -\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{4bg^5(a+bx)^4}+\frac{B\int \frac{bc-ad}{g^4(a+bx)^5(c+dx)}\,dx}{4bg}$$

$$= -\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{4bg^5(a+bx)^4}+\frac{(B(bc-ad))\int \frac{1}{(a+bx)^5(c+dx)}\,dx}{4bg^5}$$

$$= -\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{4bg^5(a+bx)^4}+\frac{(B(bc-ad))\int \left(\frac{b}{(bc-ad)(a+bx)^5}-\frac{bd}{(bc-ad)^2(a+bx)^4}+\frac{\ldots}{(bc-\ldots}\right.}{\ldots}$$

$$= -\frac{B}{16bg^5(a+bx)^4}+\frac{Bd}{12b(bc-ad)g^5(a+bx)^3}-\frac{Bd^2}{8b(bc-ad)^2g^5(a+bx)^2}+\frac{\ldots}{4b(\ldots}$$

Mathematica [A]
time = 0.14, size = 158, normalized size = 0.77

$$\frac{-\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(a+bx)^4}+\frac{B\left(-\frac{3(bc-ad)^4}{(a+bx)^4}+\frac{4d(bc-ad)^3}{(a+bx)^3}-\frac{6d^2(bc-ad)^2}{(a+bx)^2}+\frac{12d^3(bc-ad)}{a+bx}+12d^4\log(a+bx)-12d^4\log(c+dx)\right)}{12(bc-ad)^4}}{4bg^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])/(a*g + b*g*x)^5,x]
```

```
[Out] (-((A + B*Log[(e*(a + b*x))/(c + d*x)])/(a + b*x)^4) + (B*((-3*(b*c - a*d)^
4)/(a + b*x)^4 + (4*d*(b*c - a*d)^3)/(a + b*x)^3 - (6*d^2*(b*c - a*d)^2)/(a
 + b*x)^2 + (12*d^3*(b*c - a*d))/(a + b*x) + 12*d^4*Log[a + b*x] - 12*d^4*L
og[c + d*x]))/(12*(b*c - a*d)^4))/(4*b*g^5)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 674 vs. $2(195) = 390$.
time = 0.37, size = 675, normalized size = 3.28 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/d^2*e*(a*d-b*c)*(1/4*d^2/(a*d-b*c)^5/g^5*A*b^3*e^3/(b*e/d+(a*d-b*c)*e/d/
(d*x+c))^4-d^3/(a*d-b*c)^5/g^5*A*b^2*e^2/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3+3/
2*d^4/(a*d-b*c)^5/g^5*A*b*e/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2-d^5/(a*d-b*c)^5
/g^5*A/(b*e/d+(a*d-b*c)*e/d/(d*x+c))-d^2/(a*d-b*c)^5/g^5*B*b^3*e^3*(-1/4/(b
*e/d+(a*d-b*c)*e/d/(d*x+c))^4*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/16/(b*e/d+(
a*d-b*c)*e/d/(d*x+c))^4)+3*d^3/(a*d-b*c)^5/g^5*B*b^2*e^2*(-1/3/(b*e/d+(a*d-
b*c)*e/d/(d*x+c))^3*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/9/(b*e/d+(a*d-b*c)*e/
d/(d*x+c))^3)-3*d^4/(a*d-b*c)^5/g^5*B*b*e*(-1/2/(b*e/d+(a*d-b*c)*e/d/(d*x+c
))^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/4/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2)+d
^5/(a*d-b*c)^5/g^5*B*(-1/(b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e
/d/(d*x+c))-1/(b*e/d+(a*d-b*c)*e/d/(d*x+c))))
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 649 vs. $2(193) = 386$.
time = 0.32, size = 649, normalized size = 3.15

$$\frac{1}{48}\,B\left(\frac{12\,b^3d^3x^3-3\,b^3c^3+13\,ab^2c^2d-23\,a^2bcd^2+25\,a^3d^3-6\,(b^3cd^2-7\,ab^2d^3)x^2+4\,(b^3c^2d-5\,ab^2cd^2+13\,a^2bd^3)x}{(b^8c^3-3\,ab^7c^2d+3\,a^2b^6cd^2-a^3b^5d^3)g^5x^4+4\,(ab^7c^3-3\,a^2b^6c^2d+3\,a^3b^5cd^2-a^4b^4d^3)g^5x^3+6\,(a^2b^6c^3-3\,a^3b^5c^2d+3\,a^4b^4cd^2-a^5b^3d^3)g^5x^2+4\,(a^3b^5c^3-3\,a^4b^4c^2d+3\,a^5b^3cd^2-a^6b^2d^3)g^5x+(a^4b^4c^3-3\,a^5b^3c^2d+3\,a^6b^2cd^2-a^7bd^3)g^5}-\frac{12\,\log\left(\frac{bxe}{dx+c}+\frac{ae}{dx+c}\right)}{b^5g^5x^4+4\,ab^4g^5x^3+6\,a^2b^3g^5x^2+4\,a^3b^2g^5x+a^4bg^5}+\frac{12\,d^4\log\left(bx+a\right)}{(b^5c^4-4\,ab^4c^3d+6\,a^2b^3c^2d^2-4\,a^3b^2cd^3+a^4bd^4)g^5}-\frac{12\,d^4\log\left(dx+c\right)}{(b^5c^4-4\,ab^4c^3d+6\,a^2b^3c^2d^2-4\,a^3b^2cd^3+a^4bd^4)g^5}\right)-\frac{A}{4\,(b^5g^5x^4+4\,ab^4g^5x^3+6\,a^2b^3g^5x^2+4\,a^3b^2g^5x+a^4bg^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^5,x, algorithm="maxima")
```

```
[Out] 1/48*B*((12*b^3*d^3*x^3 - 3*b^3*c^3 + 13*a*b^2*c^2*d - 23*a^2*b*c*d^2 + 25*
a^3*d^3 - 6*(b^3*c*d^2 - 7*a*b^2*d^3)*x^2 + 4*(b^3*c^2*d - 5*a*b^2*c*d^2 +
13*a^2*b*d^3)*x)/((b^8*c^3 - 3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 - a^3*b^5*d^3)
*g^5*x^4 + 4*(a*b^7*c^3 - 3*a^2*b^6*c^2*d + 3*a^3*b^5*c*d^2 - a^4*b^4*d^3)*
g^5*x^3 + 6*(a^2*b^6*c^3 - 3*a^3*b^5*c^2*d + 3*a^4*b^4*c*d^2 - a^5*b^3*d^3)
*g^5*x^2 + 4*(a^3*b^5*c^3 - 3*a^4*b^4*c^2*d + 3*a^5*b^3*c*d^2 - a^6*b^2*d^3
)*g^5*x + (a^4*b^4*c^3 - 3*a^5*b^3*c^2*d + 3*a^6*b^2*c*d^2 - a^7*b*d^3)*g^5
) - 12*log(b*x*e/(d*x + c) + a*e/(d*x + c))/(b^5*g^5*x^4 + 4*a*b^4*g^5*x^3
+ 6*a^2*b^3*g^5*x^2 + 4*a^3*b^2*g^5*x + a^4*b*g^5) + 12*d^4*log(b*x + a)/((
b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*
g^5) - 12*d^4*log(d*x + c)/((b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 -
4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5)) - 1/4*A/(b^5*g^5*x^4 + 4*a*b^4*g^5*x^3 +
 6*a^2*b^3*g^5*x^2 + 4*a^3*b^2*g^5*x + a^4*b*g^5)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 628 vs. $2(193) = 386$.
time = 0.37, size = 628, normalized size = 3.05

$$-\frac{3\,(4\,A+B)b^4c^4-16\,(3\,A+B)ab^3c^3d+36\,(2\,A+B)a^2b^2c^2d^2-48\,(A+B)a^3bcd^3+(12\,A+25\,B)a^4d^4-12\,(Bb^4cd^3-Bab^3d^4)x^3+6\,(Bb^4c^2d^2-8\,Bab^3cd^3+7\,Ba^2b^2d^4)x^2-4\,(Bb^4c^3d-6\,Bab^3c^2d^2+18\,Ba^2b^2cd^3-13\,Ba^3bd^4)x-12\,(Bb^4d^4x^4+4\,Bab^3d^4x^3+6\,Ba^2b^2d^4x^2+4\,Ba^3bd^4x-Bb^4c^4+4\,Bab^3c^3d-6\,Ba^2b^2c^2d^2+4\,Ba^3bcd^3)\log\left(\frac{(bx+a)e}{dx+c}\right)}{48\,((b^9c^4-4\,ab^8c^3d+6\,a^2b^7c^2d^2-4\,a^3b^6cd^3+a^4b^5d^4)g^5x^4+4\,(ab^8c^4-4\,a^2b^7c^3d+6\,a^3b^6c^2d^2-4\,a^4b^5cd^3+a^5b^4d^4)g^5x^3+6\,(a^2b^7c^4-4\,a^3b^6c^3d+6\,a^4b^5c^2d^2-4\,a^5b^4cd^3+a^6b^3d^4)g^5x^2+4\,(a^3b^6c^4-4\,a^4b^5c^3d+6\,a^5b^4c^2d^2-4\,a^6b^3cd^3+a^7b^2d^4)g^5x+(a^4b^5c^4-4\,a^5b^4c^3d+6\,a^6b^3c^2d^2-4\,a^7b^2cd^3+a^8bd^4)g^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^5,x, algorithm="fricas")
```

```
[Out] -1/48*(3*(4*A + B)*b^4*c^4 - 16*(3*A + B)*a*b^3*c^3*d + 36*(2*A + B)*a^2*b^
2*c^2*d^2 - 48*(A + B)*a^3*b*c*d^3 + (12*A + 25*B)*a^4*d^4 - 12*(B*b^4*c*d^
```

```
3 - B*a*b^3*d^4)*x^3 + 6*(B*b^4*c^2*d^2 - 8*B*a*b^3*c*d^3 + 7*B*a^2*b^2*d^4
)*x^2 - 4*(B*b^4*c^3*d - 6*B*a*b^3*c^2*d^2 + 18*B*a^2*b^2*c*d^3 - 13*B*a^3*
b*d^4)*x - 12*(B*b^4*d^4*x^4 + 4*B*a*b^3*d^4*x^3 + 6*B*a^2*b^2*d^4*x^2 + 4*
B*a^3*b*d^4*x - B*b^4*c^4 + 4*B*a*b^3*c^3*d - 6*B*a^2*b^2*c^2*d^2 + 4*B*a^3
*b*c*d^3)*log((b*x + a)*e/(d*x + c)))/((b^9*c^4 - 4*a*b^8*c^3*d + 6*a^2*b^7
*c^2*d^2 - 4*a^3*b^6*c*d^3 + a^4*b^5*d^4)*g^5*x^4 + 4*(a*b^8*c^4 - 4*a^2*b^
7*c^3*d + 6*a^3*b^6*c^2*d^2 - 4*a^4*b^5*c*d^3 + a^5*b^4*d^4)*g^5*x^3 + 6*(a
^2*b^7*c^4 - 4*a^3*b^6*c^3*d + 6*a^4*b^5*c^2*d^2 - 4*a^5*b^4*c*d^3 + a^6*b^
3*d^4)*g^5*x^2 + 4*(a^3*b^6*c^4 - 4*a^4*b^5*c^3*d + 6*a^5*b^4*c^2*d^2 - 4*a
^6*b^3*c*d^3 + a^7*b^2*d^4)*g^5*x + (a^4*b^5*c^4 - 4*a^5*b^4*c^3*d + 6*a^6*
b^3*c^2*d^2 - 4*a^7*b^2*c*d^3 + a^8*b*d^4)*g^5)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 944 vs. $2(178) = 356$.
time = 2.95, size = 944, normalized size = 4.58

$$-\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)}{4a^4bg^5+16a^3b^2g^5x+24a^2b^3g^5x^2+16ab^4g^5x^3+4b^5g^5x^4}-\frac{Bd^4\log\left(x+\frac{-\frac{Ba^5d^9}{(ad-bc)^4}+\frac{5Ba^4bcd^8}{(ad-bc)^4}-\frac{10Ba^3b^2c^2d^7}{(ad-bc)^4}+\frac{10Ba^2b^3c^3d^6}{(ad-bc)^4}-\frac{5Bab^4c^4d^5}{(ad-bc)^4}+Bad^5+\frac{Bb^5c^5d^4}{(ad-bc)^4}+Bbcd^4}{2Bbd^5}\right)}{4bg^5(ad-bc)^4}+\frac{Bd^4\log\left(x+\frac{\frac{Ba^5d^9}{(ad-bc)^4}-\frac{5Ba^4bcd^8}{(ad-bc)^4}+\frac{10Ba^3b^2c^2d^7}{(ad-bc)^4}-\frac{10Ba^2b^3c^3d^6}{(ad-bc)^4}+\frac{5Bab^4c^4d^5}{(ad-bc)^4}+Bad^5-\frac{Bb^5c^5d^4}{(ad-bc)^4}+Bbcd^4}{2Bbd^5}\right)}{4bg^5(ad-bc)^4}+\frac{-12Aa^3d^3+36Aa^2bcd^2-36Aab^2c^2d+12Ab^3c^3-25Ba^3d^3+23Ba^2bcd^2-13Bab^2c^2d+3Bb^3c^3-12Bb^3d^3x^3+x^2(-42Bab^2d^3+6Bb^3cd^2)+x(-52Ba^2bd^3+20Bab^2cd^2-4Bb^3c^2d)}{48a^7bd^3g^5-144a^6b^2cd^2g^5+144a^5b^3c^2dg^5-48a^4b^4c^3g^5+x^4(48a^3b^5d^3g^5-144a^2b^6cd^2g^5+144ab^7c^2dg^5-48b^8c^3g^5)+x^3(192a^4b^4d^3g^5-576a^3b^5cd^2g^5+576a^2b^6c^2dg^5-192ab^7c^3g^5)+x^2(288a^5b^3d^3g^5-864a^4b^4cd^2g^5+864a^3b^5c^2dg^5-288a^2b^6c^3g^5)+x(192a^6b^2d^3g^5-576a^5b^3cd^2g^5+576a^4b^4c^2dg^5-192a^3b^5c^3g^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)**5,x)
```

```
[Out] -B*log(e*(a + b*x)/(c + d*x))/(4*a**4*b*g**5 + 16*a**3*b**2*g**5*x + 24*a**
2*b**3*g**5*x**2 + 16*a*b**4*g**5*x**3 + 4*b**5*g**5*x**4) - B*d**4*log(x +
 (-B*a**5*d**9/(a*d - b*c)**4 + 5*B*a**4*b*c*d**8/(a*d - b*c)**4 - 10*B*a**
3*b**2*c**2*d**7/(a*d - b*c)**4 + 10*B*a**2*b**3*c**3*d**6/(a*d - b*c)**4 -
 5*B*a*b**4*c**4*d**5/(a*d - b*c)**4 + B*a*d**5 + B*b**5*c**5*d**4/(a*d - b
*c)**4 + B*b*c*d**4)/(2*B*b*d**5))/(4*b*g**5*(a*d - b*c)**4) + B*d**4*log(x
 + (B*a**5*d**9/(a*d - b*c)**4 - 5*B*a**4*b*c*d**8/(a*d - b*c)**4 + 10*B*a*
*3*b**2*c**2*d**7/(a*d - b*c)**4 - 10*B*a**2*b**3*c**3*d**6/(a*d - b*c)**4
+ 5*B*a*b**4*c**4*d**5/(a*d - b*c)**4 + B*a*d**5 - B*b**5*c**5*d**4/(a*d -
b*c)**4 + B*b*c*d**4)/(2*B*b*d**5))/(4*b*g**5*(a*d - b*c)**4) + (-12*A*a**3
*d**3 + 36*A*a**2*b*c*d**2 - 36*A*a*b**2*c**2*d + 12*A*b**3*c**3 - 25*B*a**
3*d**3 + 23*B*a**2*b*c*d**2 - 13*B*a*b**2*c**2*d + 3*B*b**3*c**3 - 12*B*b**
3*d**3*x**3 + x**2*(-42*B*a*b**2*d**3 + 6*B*b**3*c*d**2) + x*(-52*B*a**2*b*
d**3 + 20*B*a*b**2*c*d**2 - 4*B*b**3*c**2*d))/(48*a**7*b*d**3*g**5 - 144*a*
*6*b**2*c*d**2*g**5 + 144*a**5*b**3*c**2*d*g**5 - 48*a**4*b**4*c**3*g**5 +
x**4*(48*a**3*b**5*d**3*g**5 - 144*a**2*b**6*c*d**2*g**5 + 144*a*b**7*c**2*
d*g**5 - 48*b**8*c**3*g**5) + x**3*(192*a**4*b**4*d**3*g**5 - 576*a**3*b**5
*c*d**2*g**5 + 576*a**2*b**6*c**2*d*g**5 - 192*a*b**7*c**3*g**5) + x**2*(28
8*a**5*b**3*d**3*g**5 - 864*a**4*b**4*c*d**2*g**5 + 864*a**3*b**5*c**2*d*g*
*5 - 288*a**2*b**6*c**3*g**5) + x*(192*a**6*b**2*d**3*g**5 - 576*a**5*b**3*
c*d**2*g**5 + 576*a**4*b**4*c**2*d*g**5 - 192*a**3*b**5*c**3*g**5))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 528 vs. $2(193) = 386$.

time = 3.13, size = 528, normalized size = 2.56

$$-\frac{\left(12\,Bb^3e^5\log\left(\frac{bxe+ae}{dx+c}\right)-\frac{48\,(bxe+ae)Bb^2de^4\log\left(\frac{bxe+ae}{dx+c}\right)}{dx+c}+\frac{72\,(bxe+ae)^2Bbd^2e^3\log\left(\frac{bxe+ae}{dx+c}\right)}{(dx+c)^2}-\frac{48\,(bxe+ae)^3Bd^3e^2\log\left(\frac{bxe+ae}{dx+c}\right)}{(dx+c)^3}+12\,Ab^3e^5+3\,Bb^3e^5-\frac{48\,(bxe+ae)Ab^2de^4}{dx+c}-\frac{16\,(bxe+ae)Bb^2de^4}{dx+c}+\frac{72\,(bxe+ae)^2Abd^2e^3}{(dx+c)^2}+\frac{36\,(bxe+ae)^2Bbd^2e^3}{(dx+c)^2}-\frac{48\,(bxe+ae)^3Ad^3e^2}{(dx+c)^3}-\frac{48\,(bxe+ae)^3Bd^3e^2}{(dx+c)^3}\right)\left(\frac{bc}{(bce-ade)(bc-ad)}-\frac{ad}{(bce-ade)(bc-ad)}\right)}{48\left(\frac{(bxe+ae)^4b^3c^3g^5}{(dx+c)^4}-\frac{3\,(bxe+ae)^4ab^2c^2dg^5}{(dx+c)^4}+\frac{3\,(bxe+ae)^4a^2bcd^2g^5}{(dx+c)^4}-\frac{(bxe+ae)^4a^3d^3g^5}{(dx+c)^4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^5,x, algorithm="giac")
```

```
[Out] -1/48*(12*B*b^3*e^5*log((b*x*e + a*e)/(d*x + c)) - 48*(b*x*e + a*e)*B*b^2*d
*e^4*log((b*x*e + a*e)/(d*x + c))/(d*x + c) + 72*(b*x*e + a*e)^2*B*b*d^2*e^
3*log((b*x*e + a*e)/(d*x + c))/(d*x + c)^2 - 48*(b*x*e + a*e)^3*B*d^3*e^2*l
og((b*x*e + a*e)/(d*x + c))/(d*x + c)^3 + 12*A*b^3*e^5 + 3*B*b^3*e^5 - 48*(
b*x*e + a*e)*A*b^2*d*e^4/(d*x + c) - 16*(b*x*e + a*e)*B*b^2*d*e^4/(d*x + c)
 + 72*(b*x*e + a*e)^2*A*b*d^2*e^3/(d*x + c)^2 + 36*(b*x*e + a*e)^2*B*b*d^2*
e^3/(d*x + c)^2 - 48*(b*x*e + a*e)^3*A*d^3*e^2/(d*x + c)^3 - 48*(b*x*e + a*
e)^3*B*d^3*e^2/(d*x + c)^3)*(b*c/((b*c*e - a*d*e)*(b*c - a*d)) - a*d/((b*c*
e - a*d*e)*(b*c - a*d)))/((b*x*e + a*e)^4*b^3*c^3*g^5/(d*x + c)^4 - 3*(b*x*
e + a*e)^4*a*b^2*c^2*d*g^5/(d*x + c)^4 + 3*(b*x*e + a*e)^4*a^2*b*c*d^2*g^5/
(d*x + c)^4 - (b*x*e + a*e)^4*a^3*d^3*g^5/(d*x + c)^4)
```

Mupad [B]
time = 6.17, size = 577, normalized size = 2.80

$$-\frac{\frac{12\,A\,a^3\,d^3-12\,A\,b^3\,c^3+25\,B\,a^3\,d^3-3\,B\,b^3\,c^3+36\,A\,a\,b^2\,c^2\,d-36\,A\,a^2\,b\,c\,d^2+13\,B\,a\,b^2\,c^2\,d-23\,B\,a^2\,b\,c\,d^2}{12\,(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3)}-\frac{d^2\,x^2\,(B\,b^3\,c-7\,B\,a\,b^2\,d)}{2\,(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3)}+\frac{d\,x\,(13\,B\,a^2\,b\,d^2-5\,B\,a\,b^2\,c\,d+B\,b^3\,c^2)}{3\,(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3)}+\frac{B\,b^3\,d^3\,x^3}{a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3}}{4\,a^4\,b\,g^5+16\,a^3\,b^2\,g^5\,x+24\,a^2\,b^3\,g^5\,x^2+16\,a\,b^4\,g^5\,x^3+4\,b^5\,g^5\,x^4}-\frac{B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)}{4\,b^2\,g^5\,\left(4\,a^3\,x+\frac{a^4}{b}+b^3\,x^4+6\,a^2\,b\,x^2+4\,a\,b^2\,x^3\right)}-\frac{B\,d^4\,\mathrm{atanh}\left(\frac{-4\,a^4\,b\,d^4\,g^5+8\,a^3\,b^2\,c\,d^3\,g^5-8\,a\,b^4\,c^3\,d\,g^5+4\,b^5\,c^4\,g^5}{4\,b\,g^5\,(a\,d-b\,c)^4}-\frac{2\,b\,d\,x\,(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3)}{(a\,d-b\,c)^4}\right)}{2\,b\,g^5\,(a\,d-b\,c)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x))/(c + d*x)))/(a*g + b*g*x)^5,x)
```

```
[Out] - ((12*A*a^3*d^3 - 12*A*b^3*c^3 + 25*B*a^3*d^3 - 3*B*b^3*c^3 + 36*A*a*b^2*c
^2*d - 36*A*a^2*b*c*d^2 + 13*B*a*b^2*c^2*d - 23*B*a^2*b*c*d^2)/(12*(a^3*d^3
 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) - (d^2*x^2*(B*b^3*c - 7*B*a*b^
2*d))/(2*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) + (d*x*(B*b^3
*c^2 + 13*B*a^2*b*d^2 - 5*B*a*b^2*c*d))/(3*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2
*d - 3*a^2*b*c*d^2)) + (B*b^3*d^3*x^3)/(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d -
 3*a^2*b*c*d^2))/(4*a^4*b*g^5 + 4*b^5*g^5*x^4 + 16*a^3*b^2*g^5*x + 16*a*b^4
*g^5*x^3 + 24*a^2*b^3*g^5*x^2) - (B*log((e*(a + b*x))/(c + d*x)))/(4*b^2*g^
5*(4*a^3*x + a^4/b + b^3*x^4 + 6*a^2*b*x^2 + 4*a*b^2*x^3)) - (B*d^4*atanh((
4*b^5*c^4*g^5 - 4*a^4*b*d^4*g^5 - 8*a*b^4*c^3*d*g^5 + 8*a^3*b^2*c*d^3*g^5)/
(4*b*g^5*(a*d - b*c)^4) - (2*b*d*x*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a
^2*b*c*d^2))/(a*d - b*c)^4))/(2*b*g^5*(a*d - b*c)^4)
```

3.97 $\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$

Optimal. Leaf size=365

$$-\frac{B(bc-ad)g^4(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{10bd}+\frac{g^4(a+bx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{5b}+\frac{B(bc-ad)^2g^4(a+$$

```
[Out] -1/10*B*(-a*d+b*c)*g^4*(b*x+a)^4*(A+B*ln(e*(b*x+a)/(d*x+c)))/b/d+1/5*g^4*(b
*x+a)^5*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/b+1/30*B*(-a*d+b*c)^2*g^4*(b*x+a)^3*(
4*A+B+4*B*ln(e*(b*x+a)/(d*x+c)))/b/d^2-1/60*B*(-a*d+b*c)^3*g^4*(b*x+a)^2*(1
2*A+7*B+12*B*ln(e*(b*x+a)/(d*x+c)))/b/d^3+1/30*B*(-a*d+b*c)^4*g^4*(b*x+a)*(
12*A+13*B+12*B*ln(e*(b*x+a)/(d*x+c)))/b/d^4+1/30*B*(-a*d+b*c)^5*g^4*ln((-a*
d+b*c)/b/(d*x+c))*(12*A+25*B+12*B*ln(e*(b*x+a)/(d*x+c)))/b/d^5+2/5*B^2*(-a*
d+b*c)^5*g^4*polylog(2,d*(b*x+a)/b/(d*x+c))/b/d^5
```

Rubi [A]

time = 0.35, antiderivative size = 365, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {2550, 2381, 2384, 2354, 2438}

$$\frac{2B^2g^4(bc-ad)^5\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{5bd^5}+\frac{Bg^4(bc-ad)^5\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(12B\log\left(\frac{e(a+bx)}{c+dx}\right)+12A+25B\right)}{30bd^5}+\frac{Bg^4(a+bx)(bc-ad)^4\left(12B\log\left(\frac{e(a+bx)}{c+dx}\right)+12A+13B\right)}{30bd^4}-\frac{Bg^4(a+bx)^2(bc-ad)^3\left(12B\log\left(\frac{e(a+bx)}{c+dx}\right)+12A+7B\right)}{60bd^3}+\frac{Bg^4(a+bx)^3(bc-ad)^2\left(4B\log\left(\frac{e(a+bx)}{c+dx}\right)+4A+B\right)}{30bd^2}-\frac{Bg^4(a+bx)^4(bc-ad)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{10bd}+\frac{g^4(a+bx)^5\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[(a*g + b*g*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]
```

```
[Out] -1/10*(B*(b*c - a*d)*g^4*(a + b*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/
(b*d) + (g^4*(a + b*x)^5*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(5*b) + (B
*(b*c - a*d)^2*g^4*(a + b*x)^3*(4*A + B + 4*B*Log[(e*(a + b*x))/(c + d*x)])
)/(30*b*d^2) - (B*(b*c - a*d)^3*g^4*(a + b*x)^2*(12*A + 7*B + 12*B*Log[(e*(
a + b*x))/(c + d*x)]))/(60*b*d^3) + (B*(b*c - a*d)^4*g^4*(a + b*x)*(12*A +
13*B + 12*B*Log[(e*(a + b*x))/(c + d*x)]))/(30*b*d^4) + (B*(b*c - a*d)^5*g^
4*Log[(b*c - a*d)/(b*(c + d*x))]*(12*A + 25*B + 12*B*Log[(e*(a + b*x))/(c +
 d*x)]))/(30*b*d^5) + (2*B^2*(b*c - a*d)^5*g^4*PolyLog[2, (d*(a + b*x))/(b*
(c + d*x))])/(5*b*d^5)
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2381

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
```

```
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + Dist[b*n*(p/(d*(q + 1))), Int[(f*x)^
m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, d
, e, f, m, n, q}, x] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 2384

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Dist[f/(e*(q + 1)), Int[(f*x)^(m - 1)*(d + e*x)^(q + 1
)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x
] && ILtQ[q, -1] && GtQ[m, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2550

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(
m + 1)*(g/b)^m, Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x],
 x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] &&
 EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && Eq
Q[b*f - a*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Rubi steps

$$\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \frac{g^4(a+bx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{5b} - \frac{(2B)\int \frac{(bc-ad)g^5(a+b}{}}{}$$

$$= \frac{g^4(a+bx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{5b} - \frac{(2B(bc-ad)g^4)\int}{}$$

$$= \frac{g^4(a+bx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{5b} - \frac{(2B(bc-ad)g^4)\int}{}$$

$$= \frac{g^4(a+bx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{5b} - \frac{(2B(bc-ad)g^4)\int(}{}$$

$$= \frac{2AB(bc-ad)^4g^4x}{5d^4} - \frac{B(bc-ad)^3g^4(a+bx)^2\left(A+B\log\right.}{5bd^3}$$

$$= \frac{2AB(bc-ad)^4g^4x}{5d^4} + \frac{2B^2(bc-ad)^4g^4(a+bx)\log\left(\frac{e(a+bx}{c+dx}\right.}{5bd^4}$$

$$= \frac{2AB(bc-ad)^4g^4x}{5d^4} + \frac{2B^2(bc-ad)^4g^4(a+bx)\log\left(\frac{e(a+bx}{c+dx}\right.}{5bd^4}$$

$$= \frac{2AB(bc-ad)^4g^4x}{5d^4} + \frac{13B^2(bc-ad)^4g^4x}{30d^4} - \frac{7B^2(bc-ad)}{60b}$$

$$= \frac{2AB(bc-ad)^4g^4x}{5d^4} + \frac{13B^2(bc-ad)^4g^4x}{30d^4} - \frac{7B^2(bc-ad)}{60b}$$

$$= \frac{2AB(bc-ad)^4g^4x}{5d^4} + \frac{13B^2(bc-ad)^4g^4x}{30d^4} - \frac{7B^2(bc-ad)}{60b}$$

$$= \frac{2AB(bc-ad)^4g^4x}{5d^4} + \frac{13B^2(bc-ad)^4g^4x}{30d^4} - \frac{7B^2(bc-ad)}{60b}$$

Mathematica [A]
time = 0.33, size = 511, normalized size = 1.40

$$\frac{g^4\left((a+bx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2+\frac{B(bc-ad)\left(24Abd(bc-ad)^3x+24Bd(bc-ad)^3(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)-12d^2(bc-ad)^2(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)+8d^3(bc-ad)(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)-6d^4(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)-24B(bc-ad)^4\log(c+dx)-24(bc-ad)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log(c+dx)+4B(bc-ad)^2\left(2bd(bc-ad)x-d^2(a+bx)^2-2(bc-ad)^2\log(c+dx)\right)+B(bc-ad)\left(6bd(bc-ad)^2x+3d^2(-bc+ad)(a+bx)^2+2d^3(a+bx)^3-6(bc-ad)^3\log(c+dx)\right)+12B(bc-ad)^3\left(bdx+(-bc+ad)\log(c+dx)\right)+12B(bc-ad)^4\left(\left(2\log\left(\frac{d(a+bx)}{-bc+ad}\right)-\log(c+dx)\right)\log(c+dx)+2\text{Li}_2\left(\frac{b(c+dx)}{bc-ad}\right)\right)\right)}{12d^5}\right)}{5b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*g + b*g*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]
[Out] (g^4*((a + b*x)^5*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 + (B*(b*c - a*d)*(
24*A*b*d*(b*c - a*d)^3*x + 24*B*d*(b*c - a*d)^3*(a + b*x)*Log[(e*(a + b*x))
```

```
/(c + d*x)] - 12*d^2*(b*c - a*d)^2*(a + b*x)^2*(A + B*Log[(e*(a + b*x))/(c
+ d*x)]) + 8*d^3*(b*c - a*d)*(a + b*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)
]) - 6*d^4*(a + b*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - 24*B*(b*c - a
*d)^4*Log[c + d*x] - 24*(b*c - a*d)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)])*
Log[c + d*x] + 4*B*(b*c - a*d)^2*(2*b*d*(b*c - a*d)*x - d^2*(a + b*x)^2 - 2
*(b*c - a*d)^2*Log[c + d*x]) + B*(b*c - a*d)*(6*b*d*(b*c - a*d)^2*x + 3*d^2
*(-(b*c) + a*d)*(a + b*x)^2 + 2*d^3*(a + b*x)^3 - 6*(b*c - a*d)^3*Log[c + d
*x]) + 12*B*(b*c - a*d)^3*(b*d*x + (-(b*c) + a*d)*Log[c + d*x]) + 12*B*(b*c
 - a*d)^4*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x
] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(12*d^5)))/(5*b)
```

Maple [F]
time = 0.31, size = 0, normalized size = 0.00

$$\int (bgx+ag)^4 \left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*g*x+a*g)^4*(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

```
[Out] int((b*g*x+a*g)^4*(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 2108 vs. $2(356) = 712$.
time = 0.40, size = 2108, normalized size = 5.78

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^4*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="maxima
")
```

```
[Out] 1/5*A^2*b^4*g^4*x^5 + A^2*a*b^3*g^4*x^4 + 2*A^2*a^2*b^2*g^4*x^3 + 2*A^2*a^3
*b*g^4*x^2 + 2*(x*log(b*x*e/(d*x + c) + a*e/(d*x + c)) + a*log(b*x + a)/b -
 c*log(d*x + c)/d)*A*B*a^4*g^4 + 4*(x^2*log(b*x*e/(d*x + c) + a*e/(d*x + c)
) - a^2*log(b*x + a)/b^2 + c^2*log(d*x + c)/d^2 - (b*c - a*d)*x/(b*d))*A*B*
a^3*b*g^4 + 2*(2*x^3*log(b*x*e/(d*x + c) + a*e/(d*x + c)) + 2*a^3*log(b*x +
 a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 -
a^2*d^2)*x)/(b^2*d^2))*A*B*a^2*b^2*g^4 + 1/3*(6*x^4*log(b*x*e/(d*x + c) + a
*e/(d*x + c)) - 6*a^4*log(b*x + a)/b^4 + 6*c^4*log(d*x + c)/d^4 - (2*(b^3*c
*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^
3)*x)/(b^3*d^3))*A*B*a*b^3*g^4 + 1/30*(12*x^5*log(b*x*e/(d*x + c) + a*e/(d*
x + c)) + 12*a^5*log(b*x + a)/b^5 - 12*c^5*log(d*x + c)/d^5 - (3*(b^4*c*d^3
 - a*b^3*d^4)*x^4 - 4*(b^4*c^2*d^2 - a^2*b^2*d^4)*x^3 + 6*(b^4*c^3*d - a^3*
b*d^4)*x^2 - 12*(b^4*c^4 - a^4*d^4)*x)/(b^4*d^4))*A*B*b^4*g^4 + A^2*a^4*g^4
*x - 1/30*(37*b^4*c^5*g^4 - 173*a*b^3*c^4*d*g^4 + 316*a^2*b^2*c^3*d^2*g^4 -
```

```
276*a^3*b*c^2*d^3*g^4 + 108*a^4*c*d^4*g^4)*B^2*log(d*x + c)/d^5 - 2/5*(b^5
*c^5*g^4 - 5*a*b^4*c^4*d*g^4 + 10*a^2*b^3*c^3*d^2*g^4 - 10*a^3*b^2*c^2*d^3*
g^4 + 5*a^4*b*c*d^4*g^4 - a^5*d^5*g^4)*(log(b*x + a)*log((b*d*x + a*d)/(b*c
 - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B^2/(b*d^5) + 1/60*(12*B^
2*b^5*d^5*g^4*x^5 - 6*(b^5*c*d^4*g^4 - 11*a*b^4*d^5*g^4)*B^2*x^4 + 2*(5*b^5
*c^2*d^3*g^4 - 22*a*b^4*c*d^4*g^4 + 77*a^2*b^3*d^5*g^4)*B^2*x^3 - (19*b^5*c
^3*d^2*g^4 - 87*a*b^4*c^2*d^3*g^4 + 153*a^2*b^3*c*d^4*g^4 - 205*a^3*b^2*d^5
*g^4)*B^2*x^2 + 2*(25*b^5*c^4*d*g^4 - 119*a*b^4*c^3*d^2*g^4 + 222*a^2*b^3*c
^2*d^3*g^4 - 199*a^3*b^2*c*d^4*g^4 + 101*a^4*b*d^5*g^4)*B^2*x + 12*(B^2*b^5
*d^5*g^4*x^5 + 5*B^2*a*b^4*d^5*g^4*x^4 + 10*B^2*a^2*b^3*d^5*g^4*x^3 + 10*B^
2*a^3*b^2*d^5*g^4*x^2 + 5*B^2*a^4*b*d^5*g^4*x + B^2*a^5*d^5*g^4)*log(b*x +
a)^2 + 12*(B^2*b^5*d^5*g^4*x^5 + 5*B^2*a*b^4*d^5*g^4*x^4 + 10*B^2*a^2*b^3*d
^5*g^4*x^3 + 10*B^2*a^3*b^2*d^5*g^4*x^2 + 5*B^2*a^4*b*d^5*g^4*x + (b^5*c^5*
g^4 - 5*a*b^4*c^4*d*g^4 + 10*a^2*b^3*c^3*d^2*g^4 - 10*a^3*b^2*c^2*d^3*g^4 +
 5*a^4*b*c*d^4*g^4)*B^2)*log(d*x + c)^2 + 2*(12*B^2*b^5*d^5*g^4*x^5 - 3*(b^
5*c*d^4*g^4 - 21*a*b^4*d^5*g^4)*B^2*x^4 + 4*(b^5*c^2*d^3*g^4 - 5*a*b^4*c*d^
4*g^4 + 34*a^2*b^3*d^5*g^4)*B^2*x^3 - 6*(b^5*c^3*d^2*g^4 - 5*a*b^4*c^2*d^3*
g^4 + 10*a^2*b^3*c*d^4*g^4 - 26*a^3*b^2*d^5*g^4)*B^2*x^2 + 12*(b^5*c^4*d*g^
4 - 5*a*b^4*c^3*d^2*g^4 + 10*a^2*b^3*c^2*d^3*g^4 - 10*a^3*b^2*c*d^4*g^4 + 9
*a^4*b*d^5*g^4)*B^2*x + (12*a*b^4*c^4*d*g^4 - 54*a^2*b^3*c^3*d^2*g^4 + 94*a
^3*b^2*c^2*d^3*g^4 - 77*a^4*b*c*d^4*g^4 + 37*a^5*d^5*g^4)*B^2)*log(b*x + a)
 - 2*(12*B^2*b^5*d^5*g^4*x^5 - 3*(b^5*c*d^4*g^4 - 21*a*b^4*d^5*g^4)*B^2*x^4
 + 4*(b^5*c^2*d^3*g^4 - 5*a*b^4*c*d^4*g^4 + 34*a^2*b^3*d^5*g^4)*B^2*x^3 - 6
*(b^5*c^3*d^2*g^4 - 5*a*b^4*c^2*d^3*g^4 + 10*a^2*b^3*c*d^4*g^4 - 26*a^3*b^2
*d^5*g^4)*B^2*x^2 + 12*(b^5*c^4*d*g^4 - 5*a*b^4*c^3*d^2*g^4 + 10*a^2*b^3*c^
2*d^3*g^4 - 10*a^3*b^2*c*d^4*g^4 + 9*a^4*b*d^5*g^4)*B^2*x + 12*(B^2*b^5*d^5
*g^4*x^5 + 5*B^2*a*b^4*d^5*g^4*x^4 + 10*B^2*a^2*b^3*d^5*g^4*x^3 + 10*B^2*a^
3*b^2*d^5*g^4*x^2 + 5*B^2*a^4*b*d^5*g^4*x + B^2*a^5*d^5*g^4)*log(b*x + a))*
log(d*x + c))/(b*d^5)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^4*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="fricas
")
```

```
[Out] integral(A^2*b^4*g^4*x^4 + 4*A^2*a*b^3*g^4*x^3 + 6*A^2*a^2*b^2*g^4*x^2 + 4*
A^2*a^3*b*g^4*x + A^2*a^4*g^4 + (B^2*b^4*g^4*x^4 + 4*B^2*a*b^3*g^4*x^3 + 6*
B^2*a^2*b^2*g^4*x^2 + 4*B^2*a^3*b*g^4*x + B^2*a^4*g^4)*log((b*x + a)*e/(d*x
 + c))^2 + 2*(A*B*b^4*g^4*x^4 + 4*A*B*a*b^3*g^4*x^3 + 6*A*B*a^2*b^2*g^4*x^2
 + 4*A*B*a^3*b*g^4*x + A*B*a^4*g^4)*log((b*x + a)*e/(d*x + c)), x)
```

Sympy [**F(-1)**] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)**4*(A+B*ln(e*(b*x+a)/(d*x+c)))**2,x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^4*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="giac")
[Out] integrate((b*g*x + a*g)^4*(B*log((b*x + a)*e/(d*x + c)) + A)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a\,g+b\,g\,x)^4\,\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*g + b*g*x)^4*(A + B*log((e*(a + b*x))/(c + d*x)))^2,x)
[Out] int((a*g + b*g*x)^4*(A + B*log((e*(a + b*x))/(c + d*x)))^2, x)
```

3.98 $\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$

Optimal. Leaf size=309

$$-\frac{B(bc-ad)g^3(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{6bd}+\frac{g^3(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{4b}+\frac{B(bc-ad)^2g^3(a+$$

```
[Out] -1/6*B*(-a*d+b*c)*g^3*(b*x+a)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))/b/d+1/4*g^3*(b*
x+a)^4*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/b+1/12*B*(-a*d+b*c)^2*g^3*(b*x+a)^2*(3
*A+B+3*B*ln(e*(b*x+a)/(d*x+c)))/b/d^2-1/12*B*(-a*d+b*c)^3*g^3*(b*x+a)*(6*A+
5*B+6*B*ln(e*(b*x+a)/(d*x+c)))/b/d^3-1/12*B*(-a*d+b*c)^4*g^3*ln((-a*d+b*c)/
b/(d*x+c))*(6*A+11*B+6*B*ln(e*(b*x+a)/(d*x+c)))/b/d^4-1/2*B^2*(-a*d+b*c)^4*
g^3*polylog(2,d*(b*x+a)/b/(d*x+c))/b/d^4
```

Rubi [A]
time = 0.26, antiderivative size = 309, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {2550, 2381, 2384, 2354, 2438}

$$-\frac{B^2g^3(bc-ad)^4\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{2bd^4}-\frac{Bg^3(bc-ad)^4\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(6B\log\left(\frac{e(a+bx)}{c+dx}\right)+6A+11B\right)}{12bd^4}-\frac{Bg^3(a+bx)(bc-ad)^3\left(6B\log\left(\frac{e(a+bx)}{c+dx}\right)+6A+5B\right)}{12bd^3}+\frac{Bg^3(a+bx)^2(bc-ad)^2\left(3B\log\left(\frac{e(a+bx)}{c+dx}\right)+3A+B\right)}{12bd^2}-\frac{Bg^3(a+bx)^3(bc-ad)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{6bd}+\frac{g^3(a+bx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(a*g + b*g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]
```

```
[Out] -1/6*(B*(b*c - a*d)*g^3*(a + b*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(
b*d) + (g^3*(a + b*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(4*b) + (B*
(b*c - a*d)^2*g^3*(a + b*x)^2*(3*A + B + 3*B*Log[(e*(a + b*x))/(c + d*x)]))
/(12*b*d^2) - (B*(b*c - a*d)^3*g^3*(a + b*x)*(6*A + 5*B + 6*B*Log[(e*(a + b
*x))/(c + d*x)]))/(12*b*d^3) - (B*(b*c - a*d)^4*g^3*Log[(b*c - a*d)/(b*(c +
 d*x))]*(6*A + 11*B + 6*B*Log[(e*(a + b*x))/(c + d*x)]))/(12*b*d^4) - (B^2*
(b*c - a*d)^4*g^3*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/(2*b*d^4)
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2381

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + Dist[b*n*(p/(d*(q + 1))), Int[(f*x)^
m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, d
```

```
, e, f, m, n, q}, x] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 2384

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Dist[f/(e*(q + 1)), Int[(f*x)^(m - 1)*(d + e*x)^(q + 1
)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x
] && ILtQ[q, -1] && GtQ[m, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2550

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(
m + 1)*(g/b)^m, Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x],
 x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] &&
 EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && Eq
Q[b*f - a*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Rubi steps

$$\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \frac{g^3(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{4b} - \frac{B\int \frac{(bc-ad)g^4(a+bx)^3 (}{c-}}{2b}$$

$$= \frac{g^3(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{4b} - \frac{(B(bc-ad)g^3)\int \frac{(}{}}{}$$

$$= \frac{g^3(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{4b} - \frac{(B(bc-ad)g^3)\int \big(}{}$$

$$= \frac{g^3(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{4b} - \frac{(B(bc-ad)g^3)\int (a}{}$$

$$= -\frac{AB(bc-ad)^3g^3x}{2d^3} + \frac{B(bc-ad)^2g^3(a+bx)^2\left(A+B\log\right.}{4bd^2}$$

$$= -\frac{AB(bc-ad)^3g^3x}{2d^3} - \frac{B^2(bc-ad)^3g^3(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right.}{2bd^3}$$

$$= -\frac{AB(bc-ad)^3g^3x}{2d^3} - \frac{B^2(bc-ad)^3g^3(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right.}{2bd^3}$$

$$= -\frac{AB(bc-ad)^3g^3x}{2d^3} - \frac{5B^2(bc-ad)^3g^3x}{12d^3} + \frac{B^2(bc-ad)^2g}{12bd}$$

$$= -\frac{AB(bc-ad)^3g^3x}{2d^3} - \frac{5B^2(bc-ad)^3g^3x}{12d^3} + \frac{B^2(bc-ad)^2g}{12bd}$$

$$= -\frac{AB(bc-ad)^3g^3x}{2d^3} - \frac{5B^2(bc-ad)^3g^3x}{12d^3} + \frac{B^2(bc-ad)^2g}{12bd}$$

$$= -\frac{AB(bc-ad)^3g^3x}{2d^3} - \frac{5B^2(bc-ad)^3g^3x}{12d^3} + \frac{B^2(bc-ad)^2g}{12bd}$$

Mathematica [A]

time = 0.22, size = 391, normalized size = 1.27

$$\frac{g^3\left((a+bx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 - \frac{B(bc-ad)\left(6Abd(bc-ad)^2x+6Bd(bc-ad)^2(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)+3d^2(-bc+ad)(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)+2d^3(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)-6B(bc-ad)^3\log(c+dx)-6(bc-ad)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log(c+dx)+B(bc-ad)\left(2bd(bc-ad)x-d^2(a+bx)^2-2(bc-ad)^2\log(c+dx)\right)+3B(bc-ad)^2(bdx+(-bc+ad)\log(c+dx))+3B(bc-ad)^3\left(\left(2\log\left(\frac{d(a+bx)}{-bc+ad}\right)-\log(c+dx)\right)\log(c+dx)+2\text{Li}_2\left(\frac{b(c+dx)}{bc-ad}\right)\right)\right)}{3d^4}\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*g + b*g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]
```

```
[Out] (g^3*((a + b*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 - (B*(b*c - a*d)*(
6*A*b*d*(b*c - a*d)^2*x + 6*B*d*(b*c - a*d)^2*(a + b*x)*Log[(e*(a + b*x))/(
```

```
c + d*x)] + 3*d^2*(-(b*c) + a*d)*(a + b*x)^2*(A + B*Log[(e*(a + b*x))/(c +
d*x)]) + 2*d^3*(a + b*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - 6*B*(b*c
- a*d)^3*Log[c + d*x] - 6*(b*c - a*d)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]
)*Log[c + d*x] + B*(b*c - a*d)*(2*b*d*(b*c - a*d)*x - d^2*(a + b*x)^2 - 2*(
b*c - a*d)^2*Log[c + d*x]) + 3*B*(b*c - a*d)^2*(b*d*x + (-(b*c) + a*d)*Log[
c + d*x]) + 3*B*(b*c - a*d)^3*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c
 + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(3*d^4))
)/(4*b)
```

Maple [**F**]
time = 0.24, size = 0, normalized size = 0.00

$$\int (bgx + ag)^3 \left(A + B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*g*x+a*g)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

```
[Out] int((b*g*x+a*g)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1539 vs. $2(301) = 602$.
time = 0.39, size = 1539, normalized size = 4.98

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="maxima
")
```

```
[Out] 1/4*A^2*b^3*g^3*x^4 + A^2*a*b^2*g^3*x^3 + 3/2*A^2*a^2*b*g^3*x^2 + 2*(x*log(
b*x*e/(d*x + c) + a*e/(d*x + c)) + a*log(b*x + a)/b - c*log(d*x + c)/d)*A*B
*a^3*g^3 + 3*(x^2*log(b*x*e/(d*x + c) + a*e/(d*x + c)) - a^2*log(b*x + a)/b
^2 + c^2*log(d*x + c)/d^2 - (b*c - a*d)*x/(b*d))*A*B*a^2*b*g^3 + (2*x^3*log
(b*x*e/(d*x + c) + a*e/(d*x + c)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x
+ c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*A
*B*a*b^2*g^3 + 1/12*(6*x^4*log(b*x*e/(d*x + c) + a*e/(d*x + c)) - 6*a^4*log
(b*x + a)/b^4 + 6*c^4*log(d*x + c)/d^4 - (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3
*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3))*A*B*b^3*
g^3 + A^2*a^3*g^3*x + 1/12*(17*b^3*c^4*g^3 - 62*a*b^2*c^3*d*g^3 + 81*a^2*b*
c^2*d^2*g^3 - 42*a^3*c*d^3*g^3)*B^2*log(d*x + c)/d^4 + 1/2*(b^4*c^4*g^3 - 4
*a*b^3*c^3*d*g^3 + 6*a^2*b^2*c^2*d^2*g^3 - 4*a^3*b*c*d^3*g^3 + a^4*d^4*g^3)
*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b
*c - a*d)))*B^2/(b*d^4) + 1/12*(3*B^2*b^4*d^4*g^3*x^4 - 2*(b^4*c*d^3*g^3 -
7*a*b^3*d^4*g^3)*B^2*x^3 + 2*(2*b^4*c^2*d^2*g^3 - 7*a*b^3*c*d^3*g^3 + 14*a^
2*b^2*d^4*g^3)*B^2*x^2 - (11*b^4*c^3*d*g^3 - 41*a*b^3*c^2*d^2*g^3 + 55*a^2*
```

```
b^2*c*d^3*g^3 - 37*a^3*b*d^4*g^3)*B^2*x + 3*(B^2*b^4*d^4*g^3*x^4 + 4*B^2*a*
b^3*d^4*g^3*x^3 + 6*B^2*a^2*b^2*d^4*g^3*x^2 + 4*B^2*a^3*b*d^4*g^3*x + B^2*a
^4*d^4*g^3)*log(b*x + a)^2 + 3*(B^2*b^4*d^4*g^3*x^4 + 4*B^2*a*b^3*d^4*g^3*x
^3 + 6*B^2*a^2*b^2*d^4*g^3*x^2 + 4*B^2*a^3*b*d^4*g^3*x - (b^4*c^4*g^3 - 4*a
*b^3*c^3*d*g^3 + 6*a^2*b^2*c^2*d^2*g^3 - 4*a^3*b*c*d^3*g^3)*B^2)*log(d*x +
c)^2 + (6*B^2*b^4*d^4*g^3*x^4 - 2*(b^4*c*d^3*g^3 - 13*a*b^3*d^4*g^3)*B^2*x^
3 + 3*(b^4*c^2*d^2*g^3 - 4*a*b^3*c*d^3*g^3 + 15*a^2*b^2*d^4*g^3)*B^2*x^2 -
6*(b^4*c^3*d*g^3 - 4*a*b^3*c^2*d^2*g^3 + 6*a^2*b^2*c*d^3*g^3 - 7*a^3*b*d^4*
g^3)*B^2*x - (6*a*b^3*c^3*d*g^3 - 21*a^2*b^2*c^2*d^2*g^3 + 26*a^3*b*c*d^3*g
^3 - 17*a^4*d^4*g^3)*B^2)*log(b*x + a) - (6*B^2*b^4*d^4*g^3*x^4 - 2*(b^4*c*
d^3*g^3 - 13*a*b^3*d^4*g^3)*B^2*x^3 + 3*(b^4*c^2*d^2*g^3 - 4*a*b^3*c*d^3*g^
3 + 15*a^2*b^2*d^4*g^3)*B^2*x^2 - 6*(b^4*c^3*d*g^3 - 4*a*b^3*c^2*d^2*g^3 +
6*a^2*b^2*c*d^3*g^3 - 7*a^3*b*d^4*g^3)*B^2*x + 6*(B^2*b^4*d^4*g^3*x^4 + 4*B
^2*a*b^3*d^4*g^3*x^3 + 6*B^2*a^2*b^2*d^4*g^3*x^2 + 4*B^2*a^3*b*d^4*g^3*x +
B^2*a^4*d^4*g^3)*log(b*x + a))*log(d*x + c))/(b*d^4)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="fricas
")
```

```
[Out] integral(A^2*b^3*g^3*x^3 + 3*A^2*a*b^2*g^3*x^2 + 3*A^2*a^2*b*g^3*x + A^2*a^
3*g^3 + (B^2*b^3*g^3*x^3 + 3*B^2*a*b^2*g^3*x^2 + 3*B^2*a^2*b*g^3*x + B^2*a^
3*g^3)*log((b*x + a)*e/(d*x + c))^2 + 2*(A*B*b^3*g^3*x^3 + 3*A*B*a*b^2*g^3*
x^2 + 3*A*B*a^2*b*g^3*x + A*B*a^3*g^3)*log((b*x + a)*e/(d*x + c)), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)**3*(A+B*ln(e*(b*x+a)/(d*x+c)))**2,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="giac")
```

```
[Out] integrate((b*g*x + a*g)^3*(B*log((b*x + a)*e/(d*x + c)) + A)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a\,g+b\,g\,x)^3\,\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*g + b*g*x)^3*(A + B*log((e*(a + b*x))/(c + d*x)))^2,x)
```

```
[Out] int((a*g + b*g*x)^3*(A + B*log((e*(a + b*x))/(c + d*x)))^2, x)
```

3.99 $\int (ag+bgx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$

Optimal. Leaf size=253

$$-\frac{B(bc-ad)g^2(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3bd}+\frac{g^2(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3b}+\frac{B(bc-ad)^2g^2(a+$$

```
[Out] -1/3*B*(-a*d+b*c)*g^2*(b*x+a)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/b/d+1/3*g^2*(b*
x+a)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/b+1/3*B*(-a*d+b*c)^2*g^2*(b*x+a)*(2*A+
B+2*B*ln(e*(b*x+a)/(d*x+c)))/b/d^2+1/3*B*(-a*d+b*c)^3*g^2*ln((-a*d+b*c)/b/(
d*x+c))*(2*A+3*B+2*B*ln(e*(b*x+a)/(d*x+c)))/b/d^3+2/3*B^2*(-a*d+b*c)^3*g^2*
polylog(2,d*(b*x+a)/b/(d*x+c))/b/d^3
```

Rubi [A]
time = 0.20, antiderivative size = 253, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {2550, 2381, 2384, 2354, 2438}

$$\frac{2B^2g^2(bc-ad)^3\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{3bd^3}+\frac{Bg^2(bc-ad)^3\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(2B\log\left(\frac{e(a+bx)}{c+dx}\right)+2A+3B\right)}{3bd^3}+\frac{Bg^2(a+bx)(bc-ad)^2\left(2B\log\left(\frac{e(a+bx)}{c+dx}\right)+2A+B\right)}{3bd^2}-\frac{Bg^2(a+bx)^2(bc-ad)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3bd}+\frac{g^2(a+bx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]
```

```
[Out] -1/3*(B*(b*c - a*d)*g^2*(a + b*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(
b*d) + (g^2*(a + b*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(3*b) + (B*
(b*c - a*d)^2*g^2*(a + b*x)*(2*A + B + 2*B*Log[(e*(a + b*x))/(c + d*x)]))/(
3*b*d^2) + (B*(b*c - a*d)^3*g^2*Log[(b*c - a*d)/(b*(c + d*x))]*(2*A + 3*B +
 2*B*Log[(e*(a + b*x))/(c + d*x)]))/(3*b*d^3) + (2*B^2*(b*c - a*d)^3*g^2*Po
lyLog[2, (d*(a + b*x))/(b*(c + d*x))])/(3*b*d^3)
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2381

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + Dist[b*n*(p/(d*(q + 1))), Int[(f*x)^
m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, d
, e, f, m, n, q}, x] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 2384

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Dist[f/(e*(q + 1)), Int[(f*x)^(m - 1)*(d + e*x)^(q + 1
)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x
] && ILtQ[q, -1] && GtQ[m, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2550

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(
m + 1)*(g/b)^m, Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x],
 x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] &&
 EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && Eq
Q[b*f - a*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Rubi steps

$$\int (ag+bgx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \frac{g^2(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3b} - (2B)\int \frac{(bc-ad)g^3(a+b\cdots}{\cdots}$$

$$= \frac{g^2(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3b} - \frac{(2B(bc-ad)g^2)\int\cdots}{\cdots}$$

$$= \frac{g^2(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3b} - \frac{(2B(bc-ad)g^2)\int\cdots}{\cdots}$$

$$= \frac{g^2(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3b} - \frac{(2B(bc-ad)g^2)\int(\cdots}{\cdots}$$

$$= \frac{2AB(bc-ad)^2g^2x}{3d^2} - \frac{B(bc-ad)g^2(a+bx)^2\left(A+B\log\left(\cdots\right.\right.}{3bd}$$

$$= \frac{2AB(bc-ad)^2g^2x}{3d^2} + \frac{2B^2(bc-ad)^2g^2(a+bx)\log\left(\frac{e(a+bx}{c+dx}\cdots\right.}{3bd^2}$$

$$= \frac{2AB(bc-ad)^2g^2x}{3d^2} + \frac{2B^2(bc-ad)^2g^2(a+bx)\log\left(\frac{e(a+bx}{c+dx}\cdots\right.}{3bd^2}$$

$$= \frac{2AB(bc-ad)^2g^2x}{3d^2} + \frac{B^2(bc-ad)^2g^2x}{3d^2} + \frac{2B^2(bc-ad)^2g\cdots}{\cdots}$$

$$= \frac{2AB(bc-ad)^2g^2x}{3d^2} + \frac{B^2(bc-ad)^2g^2x}{3d^2} + \frac{2B^2(bc-ad)^2g\cdots}{\cdots}$$

$$= \frac{2AB(bc-ad)^2g^2x}{3d^2} + \frac{B^2(bc-ad)^2g^2x}{3d^2} + \frac{2B^2(bc-ad)^2g\cdots}{\cdots}$$

$$= \frac{2AB(bc-ad)^2g^2x}{3d^2} + \frac{B^2(bc-ad)^2g^2x}{3d^2} + \frac{2B^2(bc-ad)^2g\cdots}{\cdots}$$

<u>Mathematica</u> [**A**]

time = 0.15, size = 287, normalized size = 1.13

$$\frac{g^2\left((a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 + \frac{B(bc-ad)\left(2Abd(bc-ad)x+2Bd(bc-ad)(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)-d^2(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)-2B(bc-ad)^2\log(c+dx)-2(bc-ad)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log(c+dx)+B(bc-ad)(bdx+(-bc+ad)\log(c+dx))+B(bc-ad)^2\left(\left(2\log\left(\frac{d(a+bx)}{-bc+ad}\right)-\log(c+dx)\right)\log(c+dx)+2\text{Li}_2\left(\frac{b(c+dx)}{bc-ad}\right)\right)\right)}{d^3}\right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]
```

```
[Out] (g^2*((a + b*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 + (B*(b*c - a*d)*(
2*A*b*d*(b*c - a*d)*x + 2*B*d*(b*c - a*d)*(a + b*x)*Log[(e*(a + b*x))/(c +
```

```
d*x)] - d^2*(a + b*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - 2*B*(b*c - a
*d)^2*Log[c + d*x] - 2*(b*c - a*d)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])*L
og[c + d*x] + B*(b*c - a*d)*(b*d*x + (-(b*c) + a*d)*Log[c + d*x]) + B*(b*c
- a*d)^2*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x]
 + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/d^3))/(3*b)
```

Maple [F]
time = 0.17, size = 0, normalized size = 0.00

$$\int (bgx+ag)^2 \left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*g*x+a*g)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

```
[Out] int((b*g*x+a*g)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1045 vs. 2(246) = 492.
time = 0.39, size = 1045, normalized size = 4.13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="maxima
")
```

```
[Out] 1/3*A^2*b^2*g^2*x^3 + A^2*a*b*g^2*x^2 + 2*(x*log(b*x*e/(d*x + c) + a*e/(d*x
 + c)) + a*log(b*x + a)/b - c*log(d*x + c)/d)*A*B*a^2*g^2 + 2*(x^2*log(b*x*
e/(d*x + c) + a*e/(d*x + c)) - a^2*log(b*x + a)/b^2 + c^2*log(d*x + c)/d^2
- (b*c - a*d)*x/(b*d))*A*B*a*b*g^2 + 1/3*(2*x^3*log(b*x*e/(d*x + c) + a*e/(
d*x + c)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a
*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*A*B*b^2*g^2 + A^2*a^2*g^2
*x - 1/3*(5*b^2*c^3*g^2 - 13*a*b*c^2*d*g^2 + 10*a^2*c*d^2*g^2)*B^2*log(d*x
+ c)/d^3 - 2/3*(b^3*c^3*g^2 - 3*a*b^2*c^2*d*g^2 + 3*a^2*b*c*d^2*g^2 - a^3*d
^3*g^2)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x +
a*d)/(b*c - a*d)))*B^2/(b*d^3) + 1/3*(B^2*b^3*d^3*g^2*x^3 - (b^3*c*d^2*g^2
- 4*a*b^2*d^3*g^2)*B^2*x^2 + (3*b^3*c^2*d*g^2 - 8*a*b^2*c*d^2*g^2 + 8*a^2*b
*d^3*g^2)*B^2*x + (B^2*b^3*d^3*g^2*x^3 + 3*B^2*a*b^2*d^3*g^2*x^2 + 3*B^2*a^
2*b*d^3*g^2*x + B^2*a^3*d^3*g^2)*log(b*x + a)^2 + (B^2*b^3*d^3*g^2*x^3 + 3*
B^2*a*b^2*d^3*g^2*x^2 + 3*B^2*a^2*b*d^3*g^2*x + (b^3*c^3*g^2 - 3*a*b^2*c^2*
d*g^2 + 3*a^2*b*c*d^2*g^2)*B^2)*log(d*x + c)^2 + (2*B^2*b^3*d^3*g^2*x^3 - (
b^3*c*d^2*g^2 - 7*a*b^2*d^3*g^2)*B^2*x^2 + 2*(b^3*c^2*d*g^2 - 3*a*b^2*c*d^2
*g^2 + 5*a^2*b*d^3*g^2)*B^2*x + (2*a*b^2*c^2*d*g^2 - 5*a^2*b*c*d^2*g^2 + 5*
a^3*d^3*g^2)*B^2)*log(b*x + a) - (2*B^2*b^3*d^3*g^2*x^3 - (b^3*c*d^2*g^2 -
7*a*b^2*d^3*g^2)*B^2*x^2 + 2*(b^3*c^2*d*g^2 - 3*a*b^2*c*d^2*g^2 + 5*a^2*b*d
```

```
^3*g^2)*B^2*x + 2*(B^2*b^3*d^3*g^2*x^3 + 3*B^2*a*b^2*d^3*g^2*x^2 + 3*B^2*a^
2*b*d^3*g^2*x + B^2*a^3*d^3*g^2)*log(b*x + a))*log(d*x + c))/(b*d^3)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="fricas
")
```

```
[Out] integral(A^2*b^2*g^2*x^2 + 2*A^2*a*b*g^2*x + A^2*a^2*g^2 + (B^2*b^2*g^2*x^2
 + 2*B^2*a*b*g^2*x + B^2*a^2*g^2)*log((b*x + a)*e/(d*x + c))^2 + 2*(A*B*b^2
*g^2*x^2 + 2*A*B*a*b*g^2*x + A*B*a^2*g^2)*log((b*x + a)*e/(d*x + c)), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)**2*(A+B*ln(e*(b*x+a)/(d*x+c)))**2,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="giac")
```

```
[Out] integrate((b*g*x + a*g)^2*(B*log((b*x + a)*e/(d*x + c)) + A)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a\,g+b\,g\,x)^2\,{\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)}^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*g + b*g*x)^2*(A + B*log((e*(a + b*x))/(c + d*x)))^2,x)
```

```
[Out] int((a*g + b*g*x)^2*(A + B*log((e*(a + b*x))/(c + d*x)))^2, x)
```

3.100 $\int (ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx$

Optimal. Leaf size=180

$$-\frac{B(bc-ad)g(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{bd}+\frac{g(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2b}-\frac{B(bc-ad)^2g\log\left(\frac{bc-ad}{b(c+dx)}\right.}{}$$

[Out] -B*(-a*d+b*c)*g*(b*x+a)*(A+B*ln(e*(b*x+a)/(d*x+c)))/b/d+1/2*g*(b*x+a)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/b-B*(-a*d+b*c)^2*g*ln((-a*d+b*c)/b/(d*x+c))*(A+B+B*ln(e*(b*x+a)/(d*x+c)))/b/d^2-B^2*(-a*d+b*c)^2*g*polylog(2,d*(b*x+a)/b/(d*x+c))/b/d^2

Rubi [A]
time = 0.12, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2550, 2381, 2384, 2354, 2438}

$$-\frac{B^2g(bc-ad)^2\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{bd^2}-\frac{Bg(bc-ad)^2\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A+B\right)}{bd^2}-\frac{Bg(a+bx)(bc-ad)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{bd}+\frac{g(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{2b}$$

Antiderivative was successfully verified.

[In] Int[(a*g + b*g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]

[Out] -((B*(b*c - a*d)*g*(a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(b*d)) + (g*(a + b*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(2*b) - (B*(b*c - a*d)^2*g*Log[(b*c - a*d)/(b*(c + d*x))]*(A + B + B*Log[(e*(a + b*x))/(c + d*x)]))/(b*d^2) - (B^2*(b*c - a*d)^2*g*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/(b*d^2)

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e), Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2381

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(d*f*(q + 1))), x] + Dist[b*n*(p/(d*(q + 1))), Int[(f*x)^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, q}, x] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 2384

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Dist[f/(e*(q + 1)), Int[(f*x)^(m - 1)*(d + e*x)^(q + 1
)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x
] && ILtQ[q, -1] && GtQ[m, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2550

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(
m + 1)*(g/b)^m, Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x],
 x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] &&
 EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && Eq
Q[b*f - a*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Rubi steps

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \frac{g(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2b} - \frac{B\int \frac{(bc-ad)g^2(a+bx)\left(A+B\right.}{c+dx}}{bg}$$

$$= \frac{g(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2b} - \frac{(B(bc-ad)g)\int \frac{(a+bx)}{}}{b}$$

$$= \frac{g(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2b} - \frac{(B(bc-ad)g)\int \left(\frac{b\left(A\right.}{}\right.}{}$$

$$= \frac{g(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2b} - \frac{(B(bc-ad)g)\int \left(A+\right.}{d}$$

$$= -\frac{AB(bc-ad)gx}{d} + \frac{g(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2b} + \frac{B}{}$$

$$= -\frac{AB(bc-ad)gx}{d} - \frac{B^2(bc-ad)g(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{bd} + \frac{g}{}$$

$$= -\frac{AB(bc-ad)gx}{d} - \frac{B^2(bc-ad)g(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{bd} + \frac{g}{}$$

$$= -\frac{AB(bc-ad)gx}{d} - \frac{B^2(bc-ad)g(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{bd} + \frac{g}{}$$

$$= -\frac{AB(bc-ad)gx}{d} - \frac{B^2(bc-ad)g(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{bd} + \frac{g}{}$$

$$= -\frac{AB(bc-ad)gx}{d} - \frac{B^2(bc-ad)g(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{bd} + \frac{g}{}$$

$$= -\frac{AB(bc-ad)gx}{d} - \frac{B^2(bc-ad)g(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{bd} + \frac{g}{}$$

Mathematica [A]
time = 0.12, size = 203, normalized size = 1.13

$$\frac{g\left((a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 - \frac{B(bc-ad)\left(2Abdx+2Bd(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)-2B(bc-ad)\log(c+dx)-2(bc-ad)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log(c+dx)+B(bc-ad)\left(\left(2\log\left(\frac{d(a+bx)}{-bc+ad}\right)-\log(c+dx)\right)\log(c+dx)+2\text{Li}_2\left(\frac{b(c+dx)}{bc-ad}\right)\right)\right)}{d^2}\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*g + b*g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]
[Out] (g*((a + b*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 - (B*(b*c - a*d)*(2*
A*b*d*x + 2*B*d*(a + b*x)*Log[(e*(a + b*x))/(c + d*x)] - 2*B*(b*c - a*d)*Lo
```

```
g[c + d*x] - 2*(b*c - a*d)*(A + B*Log[(e*(a + b*x))/(c + d*x)])*Log[c + d*x
] + B*(b*c - a*d)*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log
[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/d^2))/(2*b)
```

Maple [F]
time = 0.10, size = 0, normalized size = 0.00

$$\int (bgx + ag) \left(A + B\ln\left(\frac{e(bx + a)}{dx + c}\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*g*x+a*g)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

```
[Out] int((b*g*x+a*g)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 571 vs. $2(180) = 360$.
time = 0.36, size = 571, normalized size = 3.17

$$\frac{1}{2}A^2bgx^2 + 2\left(x\log\left(\frac{bxe}{dx+c} + \frac{ae}{dx+c}\right) + \frac{a\log(bx+a)}{b} - \frac{c\log(dx+c)}{d}\right)ABag + \left(x^2\log\left(\frac{bxe}{dx+c} + \frac{ae}{dx+c}\right) - \frac{a^2\log(bx+a)}{b^2} + \frac{c^2\log(dx+c)}{d^2} - \frac{(bc-ad)x}{bd}\right)ABbg + A^2agx + \frac{(2bc^2g - 3acdg)B^2\log(dx+c)}{d^2} + \frac{(b^2c^2g - 2abcdg + a^2d^2g)\left(\log(bx+a)\log\left(\frac{bdx+ad}{bc-ad} + 1\right) + \mathrm{Li}_2\left(-\frac{bdx+ad}{bc-ad}\right)\right)B^2}{bd^2} + \frac{B^2b^2d^2gx^2 - 2(b^2cdg - 2abd^2g)B^2x + (B^2b^2d^2gx^2 + 2B^2abd^2gx + B^2a^2d^2g)\log(bx+a)^2 + (B^2b^2d^2gx^2 + 2B^2abd^2gx - (b^2c^2g - 2abcdg)B^2)\log(dx+c)^2 + 2(B^2b^2d^2gx^2 - (b^2cdg - 3abd^2g)B^2x - (abcdg - 2a^2d^2g)B^2)\log(bx+a) - 2(B^2b^2d^2gx^2 - (b^2cdg - 3abd^2g)B^2x + (B^2b^2d^2gx^2 + 2B^2abd^2gx + B^2a^2d^2g)\log(bx+a))\log(dx+c)}{2bd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="maxima")
```

```
[Out] 1/2*A^2*b*g*x^2 + 2*(x*log(b*x*e/(d*x + c) + a*e/(d*x + c)) + a*log(b*x + a
)/b - c*log(d*x + c)/d)*A*B*a*g + (x^2*log(b*x*e/(d*x + c) + a*e/(d*x + c))
 - a^2*log(b*x + a)/b^2 + c^2*log(d*x + c)/d^2 - (b*c - a*d)*x/(b*d))*A*B*b
*g + A^2*a*g*x + (2*b*c^2*g - 3*a*c*d*g)*B^2*log(d*x + c)/d^2 + (b^2*c^2*g
- 2*a*b*c*d*g + a^2*d^2*g)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1)
 + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B^2/(b*d^2) + 1/2*(B^2*b^2*d^2*g*x^2
- 2*(b^2*c*d*g - 2*a*b*d^2*g)*B^2*x + (B^2*b^2*d^2*g*x^2 + 2*B^2*a*b*d^2*g*
x + B^2*a^2*d^2*g)*log(b*x + a)^2 + (B^2*b^2*d^2*g*x^2 + 2*B^2*a*b*d^2*g*x
- (b^2*c^2*g - 2*a*b*c*d*g)*B^2)*log(d*x + c)^2 + 2*(B^2*b^2*d^2*g*x^2 - (b
^2*c*d*g - 3*a*b*d^2*g)*B^2*x - (a*b*c*d*g - 2*a^2*d^2*g)*B^2)*log(b*x + a)
 - 2*(B^2*b^2*d^2*g*x^2 - (b^2*c*d*g - 3*a*b*d^2*g)*B^2*x + (B^2*b^2*d^2*g*
x^2 + 2*B^2*a*b*d^2*g*x + B^2*a^2*d^2*g)*log(b*x + a))*log(d*x + c))/(b*d^2
)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="fricas")
```

```
[Out] integral(A^2*b*g*x + A^2*a*g + (B^2*b*g*x + B^2*a*g)*log((b*x + a)*e/(d*x +
 c))^2 + 2*(A*B*b*g*x + A*B*a*g)*log((b*x + a)*e/(d*x + c)), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)*(A+B*ln(e*(b*x+a)/(d*x+c)))**2,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="giac")
```

```
[Out] integrate((b*g*x + a*g)*(B*log((b*x + a)*e/(d*x + c)) + A)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a\,g+b\,g\,x)\,\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*g + b*g*x)*(A + B*log((e*(a + b*x))/(c + d*x)))^2,x)
```

```
[Out] int((a*g + b*g*x)*(A + B*log((e*(a + b*x))/(c + d*x)))^2, x)
```

3.101 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ag+bgx}\,dx$

Optimal. Leaf size=128

$$-\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{bg}+\frac{2B\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\text{Li}_2\left(\frac{b(c+dx)}{d(a+bx)}\right)}{bg}+\frac{2B^2\text{Li}_3\left(\frac{b(c+dx)}{d(a+bx)}\right)}{bg}$$

```
[Out] -(A+B*ln(e*(b*x+a)/(d*x+c)))^2*ln(1-b*(d*x+c)/d/(b*x+a))/b/g+2*B*(A+B*ln(e*
(b*x+a)/(d*x+c)))*polylog(2,b*(d*x+c)/d/(b*x+a))/b/g+2*B^2*polylog(3,b*(d*x
+c)/d/(b*x+a))/b/g
```

Rubi [A]
time = 0.11, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2550, 2379, 2421, 6724}

$$\frac{2B\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{bg}+\frac{2B^2\text{PolyLog}\left(3,\frac{b(c+dx)}{d(a+bx)}\right)}{bg}-\frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{bg}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(a*g + b*g*x),x]
```

```
[Out] -(((A + B*Log[(e*(a + b*x))/(c + d*x)])^2*Log[1 - (b*(c + d*x))/(d*(a + b*x
))])/(b*g)) + (2*B*(A + B*Log[(e*(a + b*x))/(c + d*x)])*PolyLog[2, (b*(c +
d*x))/(d*(a + b*x))])/(b*g) + (2*B^2*PolyLog[3, (b*(c + d*x))/(d*(a + b*x))
])/(b*g)
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2550

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(
m + 1)*(g/b)^m, Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x],
 x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] &&
 EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && Eq
Q[b*f - a*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ag+bgx}\,dx = \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\log(ag+bgx)}{bg} - \frac{(2B)\int \frac{(c+dx)\left(-\frac{de(a+bx)}{(c+dx)^2}+\frac{be}{c+dx}\right)\left(A+B\right.}{e(a+bx}}{bg}$$

$$= \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\log(ag+bgx)}{bg} - \frac{(2B)\int \frac{(c+dx)\left(-\frac{de(a+bx)}{(c+dx)^2}+\frac{be}{c+dx}\right)\left(A+B\right.}{a+bx}}{beg}$$

$$= \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\log(ag+bgx)}{bg} - \frac{(2B)\int \frac{(bc-ad)e\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log}{(a+bx)(c+dx)}}{beg}$$

$$= \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\log(ag+bgx)}{bg} - \frac{(2B(bc-ad))\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)(c+}}{bg}$$

$$= \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\log(ag+bgx)}{bg} - \frac{(2B(bc-ad))\int \left(\frac{d\left(-A-B\log\left(\frac{e(a+}{c+}\right.\right.}{(bc-ad)}\right.}{}$$

$$= \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\log(ag+bgx)}{bg} - \frac{(2B)\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log(ag+bgx}{a+bx}}{g}$$

$$= \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\log(ag+bgx)}{bg} - \frac{(2B)\int \left(\frac{A\log(ag+bgx)}{a+bx} + \frac{B\log\left(\frac{e(a+b}{c+dx}\right.}{a}\right.}{g}$$

$$= \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\log(ag+bgx)}{bg} - \frac{(2AB)\int \frac{\log(ag+bgx)}{a+bx}\,dx}{g} - \frac{(2B^2)\int}{}$$

$$= \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\log(ag+bgx)}{bg} + \frac{2AB\log\left(\frac{b(c+dx)}{bc-ad}\right)\log(ag+bgx)}{bg}$$

$$= \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\log(ag+bgx)}{bg} + \frac{2AB\log\left(\frac{b(c+dx)}{bc-ad}\right)\log(ag+bgx)}{bg}$$

$$= -\frac{AB\log^2(g(a+bx))}{bg} + \frac{2B^2\log(a+bx)\log(g(a+bx))\log(-c-dx)}{bg} - \frac{B}{}$$

$$= -\frac{AB\log^2(g(a+bx))}{bg} + \frac{2B^2\log(a+bx)\log(g(a+bx))\log(-c-dx)}{bg} + \frac{B}{}$$

$$= -\frac{AB\log^2(g(a+bx))}{bg} + \frac{B^2\log^3(g(a+bx))}{3bg} - \frac{B^2\log^2(a+bx)\log(-c-d}{bg}$$

$$= -\frac{AB\log^2(g(a+bx))}{bg} + \frac{B^2\log^3(g(a+bx))}{3bg} - \frac{B^2\log^2(a+bx)\log(-c-d}{bg}$$

$$= -\frac{AB\log^2(g(a+bx))}{bg} + \frac{B^2\log^3(g(a+bx))}{3bg} - \frac{B^2\log^2(a+bx)\log(-c-d}{bg}$$

Mathematica [A]
time = 0.33, size = 250, normalized size = 1.95

$$\frac{AB\log^2\left(\frac{a}{b}+x\right)+A^2\log(a+bx)-2AB\log\left(\frac{a}{b}+x\right)\log(a+bx)+2AB\log\left(\frac{c}{d}+x\right)\log(a+bx)-2AB\log\left(\frac{c}{d}+x\right)\log\left(\frac{d(a+bx)}{-bc+ad}\right)+2AB\log(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)-B^2\log\left(\frac{-bc+ad}{d(a+bx)}\right)\log^2\left(\frac{e(a+bx)}{c+dx}\right)-2AB\text{Li}_2\left(\frac{b(c+dx)}{bc-ad}\right)+2B^2\log\left(\frac{e(a+bx)}{c+dx}\right)\text{Li}_2\left(\frac{b(c+dx)}{d(a+bx)}\right)+2B^2\text{Li}_3\left(\frac{b(c+dx)}{d(a+bx)}\right)}{bg}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(a*g + b*g*x),x]
```

```
[Out] (A*B*Log[a/b + x]^2 + A^2*Log[a + b*x] - 2*A*B*Log[a/b + x]*Log[a + b*x] +
2*A*B*Log[c/d + x]*Log[a + b*x] - 2*A*B*Log[c/d + x]*Log[(d*(a + b*x))/(-(b
*c) + a*d)] + 2*A*B*Log[a + b*x]*Log[(e*(a + b*x))/(c + d*x)] - B^2*Log[(-(
b*c) + a*d)/(d*(a + b*x))]*Log[(e*(a + b*x))/(c + d*x)]^2 - 2*A*B*PolyLog[2
, (b*(c + d*x))/(b*c - a*d)] + 2*B^2*Log[(e*(a + b*x))/(c + d*x)]*PolyLog[2
, (b*(c + d*x))/(d*(a + b*x))] + 2*B^2*PolyLog[3, (b*(c + d*x))/(d*(a + b*x
))])/(b*g)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 661 vs. $2(128) = 256$.
time = 0.62, size = 662, normalized size = 5.17 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d^2*e*(a*d-b*c)*(d^2/g/(a*d-b*c)*A^2/b/e*ln(b*e-(b*e/d+(a*d-b*c)*e/d/(d*
x+c))*d)-d^2/g/(a*d-b*c)*A^2/b/e*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))+d^2/g/(a*d
-b*c)*B^2/b/e*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*ln(1-1/b/e*d*(b*e/d+(a*d-b*
c)*e/d/(d*x+c)))+2*d^2/g/(a*d-b*c)*B^2/b/e*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*
polylog(2,1/b/e*d*(b*e/d+(a*d-b*c)*e/d/(d*x+c)))-2*d^2/g/(a*d-b*c)*B^2/b/e*
polylog(3,1/b/e*d*(b*e/d+(a*d-b*c)*e/d/(d*x+c)))-1/3*d^2/g/(a*d-b*c)*B^2/b/
e*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3+2*d^2/g/(a*d-b*c)*A*B/b/e*dilog(-(-b*e+
(b*e/d+(a*d-b*c)*e/d/(d*x+c))*d)/b/e)+2*d^2/g/(a*d-b*c)*A*B/b/e*ln(b*e/d+(a
*d-b*c)*e/d/(d*x+c))*ln(-(-b*e+(b*e/d+(a*d-b*c)*e/d/(d*x+c))*d)/b/e)-d^2/g/
(a*d-b*c)*A*B*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2/b/e)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g),x, algorithm="maxima")
```

```
[Out] B^2*log(b*x + a)*log(d*x + c)^2/(b*g) + A^2*log(b*g*x + a*g)/(b*g) - integr
ate(-(2*A*B*b*c + B^2*b*c + (B^2*b*d*x + B^2*b*c)*log(b*x + a)^2 + (2*A*B*b
*d + B^2*b*d)*x + 2*(A*B*b*c + B^2*b*c + (A*B*b*d + B^2*b*d)*x)*log(b*x + a
) - 2*(A*B*b*c + B^2*b*c + (A*B*b*d + B^2*b*d)*x + (2*B^2*b*d*x + (b*c + a*
```

```
d)*B^2)*log(b*x + a))*log(d*x + c))/(b^2*d*g*x^2 + a*b*c*g + (b^2*c*g + a*b
*d*g)*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g),x, algorithm="fricas")
[Out] integral((B^2*log((b*x + a)*e/(d*x + c))^2 + 2*A*B*log((b*x + a)*e/(d*x + c
)) + A^2)/(b*g*x + a*g), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A^2}{a+bx}\,dx + \int \frac{B^2\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)^2}{a+bx}\,dx + \int \frac{2AB\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}{a+bx}\,dx}{g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(b*x+a)/(d*x+c)))**2/(b*g*x+a*g),x)
[Out] (Integral(A**2/(a + b*x), x) + Integral(B**2*log(a*e/(c + d*x) + b*e*x/(c +
 d*x))**2/(a + b*x), x) + Integral(2*A*B*log(a*e/(c + d*x) + b*e*x/(c + d*x
))/(a + b*x), x))/g
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g),x, algorithm="giac")
[Out] integrate((B*log((b*x + a)*e/(d*x + c)) + A)^2/(b*g*x + a*g), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(A+B\,\ln\left(\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right)\right)}^2}{a\,g+b\,g\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x))/(c + d*x)))^2/(a*g + b*g*x),x)
[Out] int((A + B*log((e*(a + b*x))/(c + d*x)))^2/(a*g + b*g*x), x)
```

3.102 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2}\,dx$

Optimal. Leaf size=126

$$-\frac{2B^2(c+dx)}{(bc-ad)g^2(a+bx)}-\frac{2B(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)g^2(a+bx)}-\frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)g^2(a+bx)}$$

```
[Out] -2*B^2*(d*x+c)/(-a*d+b*c)/g^2/(b*x+a)-2*B*(d*x+c)*(A+B*ln(e*(b*x+a)/(d*x+c)
))/(-a*d+b*c)/g^2/(b*x+a)-(d*x+c)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(-a*d+b*c)/
g^2/(b*x+a)
```

Rubi [A]
time = 0.06, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {2550, 2342, 2341}

$$-\frac{2B(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{g^2(a+bx)(bc-ad)}-\frac{(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{g^2(a+bx)(bc-ad)}-\frac{2B^2(c+dx)}{g^2(a+bx)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(a*g + b*g*x)^2,x]
```

```
[Out] (-2*B^2*(c + d*x))/((b*c - a*d)*g^2*(a + b*x)) - (2*B*(c + d*x)*(A + B*Log[
(e*(a + b*x))/(c + d*x)]))/((b*c - a*d)*g^2*(a + b*x)) - ((c + d*x)*(A + B*
Log[(e*(a + b*x))/(c + d*x)])^2)/((b*c - a*d)*g^2*(a + b*x))
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2550

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(
m + 1)*(g/b)^m, Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x],
```

```
 x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] &&
 EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && Eq
Q[b*f - a*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Rubi steps

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2}\,dx = -\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{bg^2(a+bx)} + \frac{(2B)\int \frac{(bc-ad)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{g(a+bx)^2(c+dx)}\,dx}{bg}$$

$$= -\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{bg^2(a+bx)} + \frac{(2B(bc-ad))\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(a+bx)^2(c+dx)}\,dx}{bg^2}$$

$$= -\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{bg^2(a+bx)} + \frac{(2B(bc-ad))\int \left(\frac{b\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)(a+bx)^2} - \frac{bd\left(A+B\right.}{(bc-}\right.}{bg^2}$$

$$= -\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{bg^2(a+bx)} + \frac{(2B)\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(a+bx)^2}\,dx}{g^2} - \frac{(2Bd)\int \frac{A+B\log\left(\right.}{a+b}}{(bc-ad)}$$

$$= -\frac{2B\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{bg^2(a+bx)} - \frac{2Bd\log(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b(bc-ad)g^2} -$$

$$= -\frac{2B\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{bg^2(a+bx)} - \frac{2Bd\log(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b(bc-ad)g^2} -$$

$$= -\frac{2B\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{bg^2(a+bx)} - \frac{2Bd\log(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b(bc-ad)g^2} -$$

$$= -\frac{2B^2}{bg^2(a+bx)} - \frac{2B^2d\log(a+bx)}{b(bc-ad)g^2} - \frac{2B\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{bg^2(a+bx)} - \frac{2Bd\log}{}$$

$$= -\frac{2B^2}{bg^2(a+bx)} - \frac{2B^2d\log(a+bx)}{b(bc-ad)g^2} - \frac{2B\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{bg^2(a+bx)} - \frac{2Bd\log}{}$$

$$= -\frac{2B^2}{bg^2(a+bx)} - \frac{2B^2d\log(a+bx)}{b(bc-ad)g^2} + \frac{B^2d\log^2(a+bx)}{b(bc-ad)g^2} - \frac{2B\left(A+B\log\left(\right.\right.}{bg^2(a+b}$$

$$= -\frac{2B^2}{bg^2(a+bx)} - \frac{2B^2d\log(a+bx)}{b(bc-ad)g^2} + \frac{B^2d\log^2(a+bx)}{b(bc-ad)g^2} - \frac{2B\left(A+B\log\left(\right.\right.}{bg^2(a+b}$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

time = 0.31, size = 314, normalized size = 2.49

$$-\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2+\frac{B\left(2(bc-ad)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)+2d(a+bx)\log(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)-2d(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log(c+dx)+2B(bc-ad+d(a+bx)\log(a+bx)-d(a+bx)\log(c+dx))-Bd(a+bx)\left(\log(a+bx)\left(\log(a+bx)-2\log\left(\frac{b(c+dx)}{bc-ad}\right)\right)-2\mathrm{Li}_2\left(\frac{d(a+bx)}{-bc+ad}\right)\right)+Bd(a+bx)\left(\left(2\log\left(\frac{d(a+bx)}{-bc+ad}\right)-\log(c+dx)\right)\log(c+dx)+2\mathrm{Li}_2\left(\frac{b(c+dx)}{bc-ad}\right)\right)\right)}{bc-ad}}{bg^2(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(a*g + b*g*x)^2,x]
```

```
[Out] -(((A + B*Log[(e*(a + b*x))/(c + d*x)])^2 + (B*(2*(b*c - a*d)*(A + B*Log[(e
*(a + b*x))/(c + d*x)]) + 2*d*(a + b*x)*Log[a + b*x]*(A + B*Log[(e*(a + b*x
))/(c + d*x)]) - 2*d*(a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])*Log[c +
 d*x] + 2*B*(b*c - a*d + d*(a + b*x)*Log[a + b*x] - d*(a + b*x)*Log[c + d*x
]) - B*d*(a + b*x)*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c -
 a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) + B*d*(a + b*x)*((2*L
og[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2
, (b*(c + d*x))/(b*c - a*d)])))/(b*c - a*d))/(b*g^2*(a + b*x)))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 348 vs. $2(126) = 252$.
time = 0.25, size = 349, normalized size = 2.77

method	result
norman	$\frac{\frac{(A^2+2BA+2B^2)x}{ga}+\frac{B^2c\ln\left(\frac{e(bx+a)}{dx+c}\right)^2}{g(ad-cb)}+\frac{B^2dx\ln\left(\frac{e(bx+a)}{dx+c}\right)^2}{g(ad-cb)}+\frac{2cB(A+B)\ln\left(\frac{e(bx+a)}{dx+c}\right)}{g(ad-cb)}+\frac{2Bd(A+B)x\ln\left(\frac{e(bx+a)}{dx+c}\right)}{g(ad-cb)}}{g(bx+a)}$
derivativedivides	$-\frac{e(ad-cb)\left(-\frac{d^2A^2}{(ad-cb)^2g^2\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}+\frac{2d^2AB\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}}-\frac{1}{\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}}\right)}{(ad-cb)^2g^2}+\frac{d^2B^2\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}{\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}}-\cdots\right.}{(\cdots}\right.}{d^2}\cdots$
default	$-\frac{e(ad-cb)\left(-\frac{d^2A^2}{(ad-cb)^2g^2\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}+\frac{2d^2AB\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}}-\frac{1}{\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}}\right)}{(ad-cb)^2g^2}+\frac{d^2B^2\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}{\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}}-\cdots\right.}{(\cdots}\right.}{d^2}\cdots$
risch	$-\frac{A^2}{g^2(bx+a)b}+\frac{B^2e\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}{g^2(ad-cb)\left(\frac{be}{d}+\frac{ea}{dx+c}-\frac{ecb}{d(dx+c)}\right)}+\frac{2B^2e\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{g^2(ad-cb)\left(\frac{be}{d}+\frac{ea}{dx+c}-\frac{ecb}{d(dx+c)}\right)}+\frac{2B^2e}{g^2(ad-cb)\left(\frac{be}{d}+\frac{ea}{dx+c}-\cdots\right.}\cdots$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/d^2*e*(a*d-b*c)*(-d^2/(a*d-b*c)^2/g^2*A^2/(b*e/d+(a*d-b*c)*e/d/(d*x+c))+
2*d^2/(a*d-b*c)^2/g^2*A*B*(-1/(b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b
*c)*e/d/(d*x+c))-1/(b*e/d+(a*d-b*c)*e/d/(d*x+c)))+d^2/(a*d-b*c)^2/g^2*B^2*(
-1/(b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2-2/(b*e/d
```

```
+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-2/(b*e/d+(a*d-b*c)*
e/d/(d*x+c))))
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 422 vs. $2(128) = 256$.
time = 0.31, size = 422, normalized size = 3.35

$$-\left(2\left(\frac{1}{b^2g^2x+abg^2}+\frac{d\log(bx+a)}{(b^2c-abd)g^2}-\frac{d\log(dx+c)}{(b^2c-abd)g^2}\right)\log\left(\frac{bxe}{dx+c}+\frac{ae}{dx+c}\right)-\frac{(bdx+ad)\log(bx+a)^2+(bdx+ad)\log(dx+c)^2-2\,bc+2\,ad-2\,(bdx+ad)\log(bx+a)+2\,(bdx+ad-(bdx+ad)\log(bx+a))\log(dx+c)}{ab^2cg^2-a^2bdg^2+(b^3cg^2-ab^2dg^2)x}\right)B^2-2\,AB\left(\frac{\log\left(\frac{bxe}{dx+c}+\frac{ae}{dx+c}\right)}{b^2g^2x+abg^2}+\frac{1}{b^2g^2x+abg^2}+\frac{d\log(bx+a)}{(b^2c-abd)g^2}-\frac{d\log(dx+c)}{(b^2c-abd)g^2}\right)-\frac{B^2\log\left(\frac{bxe}{dx+c}+\frac{ae}{dx+c}\right)^2}{b^2g^2x+abg^2}-\frac{A^2}{b^2g^2x+abg^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^2,x, algorithm="maxima
")
```

```
[Out] -(2*(1/(b^2*g^2*x + a*b*g^2) + d*log(b*x + a)/((b^2*c - a*b*d)*g^2) - d*log
(d*x + c)/((b^2*c - a*b*d)*g^2))*log(b*x*e/(d*x + c) + a*e/(d*x + c)) - ((b
*d*x + a*d)*log(b*x + a)^2 + (b*d*x + a*d)*log(d*x + c)^2 - 2*b*c + 2*a*d -
 2*(b*d*x + a*d)*log(b*x + a) + 2*(b*d*x + a*d - (b*d*x + a*d)*log(b*x + a)
)*log(d*x + c))/(a*b^2*c*g^2 - a^2*b*d*g^2 + (b^3*c*g^2 - a*b^2*d*g^2)*x))*
B^2 - 2*A*B*(log(b*x*e/(d*x + c) + a*e/(d*x + c))/(b^2*g^2*x + a*b*g^2) + 1
/(b^2*g^2*x + a*b*g^2) + d*log(b*x + a)/((b^2*c - a*b*d)*g^2) - d*log(d*x +
 c)/((b^2*c - a*b*d)*g^2)) - B^2*log(b*x*e/(d*x + c) + a*e/(d*x + c))^2/(b^
2*g^2*x + a*b*g^2) - A^2/(b^2*g^2*x + a*b*g^2)
```

Fricas [**A**]
time = 0.37, size = 148, normalized size = 1.17

$$-\frac{(A^2+2\,AB+2\,B^2)bc-(A^2+2\,AB+2\,B^2)ad+(B^2bdx+B^2bc)\log\left(\frac{(bx+a)e}{dx+c}\right)^2+2\,((AB+B^2)bdx+(AB+B^2)bc)\log\left(\frac{(bx+a)e}{dx+c}\right)}{(b^3c-ab^2d)g^2x+(ab^2c-a^2bd)g^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^2,x, algorithm="fricas
")
```

```
[Out] -((A^2 + 2*A*B + 2*B^2)*b*c - (A^2 + 2*A*B + 2*B^2)*a*d + (B^2*b*d*x + B^2*
b*c)*log((b*x + a)*e/(d*x + c))^2 + 2*((A*B + B^2)*b*d*x + (A*B + B^2)*b*c)
*log((b*x + a)*e/(d*x + c)))/((b^3*c - a*b^2*d)*g^2*x + (a*b^2*c - a^2*b*d)
*g^2)
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 434 vs. $2(105) = 210$.
time = 1.34, size = 434, normalized size = 3.44

$$-\frac{2Bd(A+B)\log\left(x+\frac{2ABad^2+2ABbcd+2B^2ad^2+2B^2bcd-\frac{2Ba^2d^3(A+B)}{ad-bc}+\frac{4Babcd^2(A+B)}{ad-bc}-\frac{2Bb^2c^2d(A+B)}{ad-bc}}{4ABbd^2+4B^2bd^2}\right)}{bg^2\,(ad-bc)}+\frac{2Bd(A+B)\log\left(x+\frac{2ABad^2+2ABbcd+2B^2ad^2+2B^2bcd+\frac{2Ba^2d^3(A+B)}{ad-bc}-\frac{4Babcd^2(A+B)}{ad-bc}+\frac{2Bb^2c^2d(A+B)}{ad-bc}}{4ABbd^2+4B^2bd^2}\right)}{bg^2\,(ad-bc)}+\frac{(-2AB-2B^2)\log\left(\frac{e(a+bx)}{c+dx}\right)}{abg^2+b^2g^2x}+\frac{(B^2c+B^2dx)\log\left(\frac{e(a+bx)}{c+dx}\right)^2}{a^2dg^2-abcg^2+abdg^2x-b^2cg^2x}+\frac{-A^2-2AB-2B^2}{abg^2+b^2g^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(b*x+a)/(d*x+c)))**2/(b*g*x+a*g)**2,x)
```

```
[Out] -2*B*d*(A + B)*log(x + (2*A*B*a*d**2 + 2*A*B*b*c*d + 2*B**2*a*d**2 + 2*B**2
*b*c*d - 2*B*a**2*d**3*(A + B)/(a*d - b*c) + 4*B*a*b*c*d**2*(A + B)/(a*d -
b*c) - 2*B*b**2*c**2*d*(A + B)/(a*d - b*c))/(4*A*B*b*d**2 + 4*B**2*b*d**2))
/(b*g**2*(a*d - b*c)) + 2*B*d*(A + B)*log(x + (2*A*B*a*d**2 + 2*A*B*b*c*d +
 2*B**2*a*d**2 + 2*B**2*b*c*d + 2*B*a**2*d**3*(A + B)/(a*d - b*c) - 4*B*a*b
*c*d**2*(A + B)/(a*d - b*c) + 2*B*b**2*c**2*d*(A + B)/(a*d - b*c))/(4*A*B*b
*d**2 + 4*B**2*b*d**2))/(b*g**2*(a*d - b*c)) + (-2*A*B - 2*B**2)*log(e*(a +
 b*x)/(c + d*x))/(a*b*g**2 + b**2*g**2*x) + (B**2*c + B**2*d*x)*log(e*(a +
b*x)/(c + d*x))**2/(a**2*d*g**2 - a*b*c*g**2 + a*b*d*g**2*x - b**2*c*g**2*x
) + (-A**2 - 2*A*B - 2*B**2)/(a*b*g**2 + b**2*g**2*x)
```

Giac [A]
time = 3.07, size = 176, normalized size = 1.40

$$-\frac{\left(B^2e^2\log\left(\frac{bxe+ae}{dx+c}\right)^2+2\,ABe^2\log\left(\frac{bxe+ae}{dx+c}\right)+2\,B^2e^2\log\left(\frac{bxe+ae}{dx+c}\right)+A^2e^2+2\,ABe^2+2\,B^2e^2\right)(dx+c)\left(\frac{bc}{(bce-ade)(bc-ad)}-\frac{ad}{(bce-ade)(bc-ad)}\right)}{(bxe+ae)g^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^2,x, algorithm="giac")
```

```
[Out] -(B^2*e^2*log((b*x*e + a*e)/(d*x + c))^2 + 2*A*B*e^2*log((b*x*e + a*e)/(d*x
 + c)) + 2*B^2*e^2*log((b*x*e + a*e)/(d*x + c)) + A^2*e^2 + 2*A*B*e^2 + 2*B
^2*e^2)*(d*x + c)*(b*c/((b*c*e - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)
*(b*c - a*d)))/((b*x*e + a*e)*g^2)
```

Mupad [B]
time = 5.26, size = 222, normalized size = 1.76

$$-\frac{A^2+2\,A\,B+2\,B^2}{x\,b^2\,g^2+a\,b\,g^2}-\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)^2\left(\frac{B^2}{b^2\,g^2\,\left(x+\frac{a}{b}\right)}-\frac{B^2\,d}{b\,g^2\,(a\,d-b\,c)}\right)-\frac{\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\,\left(\frac{2\,B^2}{b^2\,d\,g^2}+\frac{2\,A\,B}{b^2\,d\,g^2}\right)}{\frac{x}{d}+\frac{a}{b\,d}}-\frac{B\,d\,\mathrm{atan}\left(\frac{\left(2\,b\,d\,x+\frac{c\,b^2\,g^2+a\,d\,b\,g^2}{b\,g^2}\right)\,1{}\mathrm{i}}{a\,d-b\,c}\right)\,(A+B)\,4{}\mathrm{i}}{b\,g^2\,(a\,d-b\,c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x))/(c + d*x)))^2/(a*g + b*g*x)^2,x)
```

```
[Out] - (A^2 + 2*B^2 + 2*A*B)/(b^2*g^2*x + a*b*g^2) - log((e*(a + b*x))/(c + d*x)
)^2*(B^2/(b^2*g^2*(x + a/b)) - (B^2*d)/(b*g^2*(a*d - b*c))) - (log((e*(a +
b*x))/(c + d*x))*((2*B^2)/(b^2*d*g^2) + (2*A*B)/(b^2*d*g^2)))/(x/d + a/(b*d
)) - (B*d*atan(((2*b*d*x + (b^2*c*g^2 + a*b*d*g^2)/(b*g^2))*1i)/(a*d - b*c)
)*(A + B)*4i)/(b*g^2*(a*d - b*c))
```

3.103 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3}\,dx$

Optimal. Leaf size=268

$$\frac{2B^2d(c+dx)}{(bc-ad)^2g^3(a+bx)}-\frac{bB^2(c+dx)^2}{4(bc-ad)^2g^3(a+bx)^2}+\frac{2Bd(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^2g^3(a+bx)}-\frac{bB(c+dx)^2\left(A+B\right.}{2(bc-ad)^2g^3(}$$

```
[Out] 2*B^2*d*(d*x+c)/(-a*d+b*c)^2/g^3/(b*x+a)-1/4*b*B^2*(d*x+c)^2/(-a*d+b*c)^2/g
^3/(b*x+a)^2+2*B*d*(d*x+c)*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*c)^2/g^3/(b*
x+a)-1/2*b*B*(d*x+c)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*c)^2/g^3/(b*x+a)
^2+d*(d*x+c)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(-a*d+b*c)^2/g^3/(b*x+a)-1/2*b*(
d*x+c)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(-a*d+b*c)^2/g^3/(b*x+a)^2
```

Rubi [A]
time = 0.14, antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2550, 2395, 2342, 2341}

$$-\frac{bB(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2g^3(a+bx)^2(bc-ad)^2}+\frac{2Bd(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{g^3(a+bx)(bc-ad)^2}-\frac{b(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{2g^3(a+bx)^2(bc-ad)^2}+\frac{d(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{g^3(a+bx)(bc-ad)^2}-\frac{bB^2(c+dx)^2}{4g^3(a+bx)^2(bc-ad)^2}+\frac{2B^2d(c+dx)}{g^3(a+bx)(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(a*g + b*g*x)^3,x]
```

```
[Out] (2*B^2*d*(c + d*x))/((b*c - a*d)^2*g^3*(a + b*x)) - (b*B^2*(c + d*x)^2)/(4*
(b*c - a*d)^2*g^3*(a + b*x)^2) + (2*B*d*(c + d*x)*(A + B*Log[(e*(a + b*x))/
(c + d*x)]))/((b*c - a*d)^2*g^3*(a + b*x)) - (b*B*(c + d*x)^2*(A + B*Log[(e
*(a + b*x))/(c + d*x)]))/(2*(b*c - a*d)^2*g^3*(a + b*x)^2) + (d*(c + d*x)*(
A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/((b*c - a*d)^2*g^3*(a + b*x)) - (b*(
c + d*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(2*(b*c - a*d)^2*g^3*(a
+ b*x)^2)
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2550

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(
m + 1)*(g/b)^m, Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x],
 x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] &&
 EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && Eq
Q[b*f - a*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Rubi steps

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3}\,dx = -\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2bg^3(a+bx)^2} + \frac{B\int \frac{(bc-ad)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{g^2(a+bx)^3(c+dx)}\,dx}{bg}$$

$$= -\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2bg^3(a+bx)^2} + \frac{(B(bc-ad))\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(a+bx)^3(c+dx)}\,dx}{bg^3}$$

$$= -\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2bg^3(a+bx)^2} + \frac{(B(bc-ad))\int \left(\frac{b\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)(a+bx)^3} - \frac{bd\left(A+B\right.}{(bc-a}\right.}{}$$

$$= -\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2bg^3(a+bx)^2} + \frac{B\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(a+bx)^3}\,dx}{g^3} + \frac{(Bd^2)\int \frac{A+B\log\left(\frac{e(a+}{c+d}\right.}{a+bx}}{(bc-ad)^2g^3}$$

$$= -\frac{B\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2bg^3(a+bx)^2} + \frac{Bd\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b(bc-ad)g^3(a+bx)} + \frac{Bd^2\log(a+bx}{b(}$$

$$= -\frac{B\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2bg^3(a+bx)^2} + \frac{Bd\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b(bc-ad)g^3(a+bx)} + \frac{Bd^2\log(a+bx}{b(}$$

$$= -\frac{B\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2bg^3(a+bx)^2} + \frac{Bd\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b(bc-ad)g^3(a+bx)} + \frac{Bd^2\log(a+bx}{b(}$$

$$= -\frac{B^2}{4bg^3(a+bx)^2} + \frac{3B^2d}{2b(bc-ad)g^3(a+bx)} + \frac{3B^2d^2\log(a+bx)}{2b(bc-ad)^2g^3} - \frac{B\left(A+\right.}{2bg}$$

$$= -\frac{B^2}{4bg^3(a+bx)^2} + \frac{3B^2d}{2b(bc-ad)g^3(a+bx)} + \frac{3B^2d^2\log(a+bx)}{2b(bc-ad)^2g^3} - \frac{B\left(A+\right.}{2bg}$$

$$= -\frac{B^2}{4bg^3(a+bx)^2} + \frac{3B^2d}{2b(bc-ad)g^3(a+bx)} + \frac{3B^2d^2\log(a+bx)}{2b(bc-ad)^2g^3} - \frac{B^2d^2\log}{2b(bc-}$$

$$= -\frac{B^2}{4bg^3(a+bx)^2} + \frac{3B^2d}{2b(bc-ad)g^3(a+bx)} + \frac{3B^2d^2\log(a+bx)}{2b(bc-ad)^2g^3} - \frac{B^2d^2\log}{2b(bc-}$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 0.32, size = 443, normalized size = 1.65

$$-\frac{2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2+\frac{B\left(2(bc-ad)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)+4d(-bc+ad)(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)-4d^2(a+bx)^2\log(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)+4d^2(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log(c+dx)-4Bd(a+bx)(bc-ad+d(a+bx)\log(a+bx)-d(a+bx)\log(c+dx))+B\left((bc-ad)^2+2d(-bc+ad)(a+bx)-2d^2(a+bx)^2\log(a+bx)+2d^2(a+bx)^2\log(c+dx)\right)+2Bd^2(a+bx)^2\left(\log(a+bx)\left(\log(a+bx)-2\log\left(\frac{b(c+dx)}{bc-ad}\right)\right)-2\mathrm{Li}_2\left(\frac{d(a+bx)}{-bc+ad}\right)\right)-2Bd^2(a+bx)^2\left(\left(2\log\left(\frac{d(a+bx)}{-bc+ad}\right)-\log(c+dx)\right)\log(c+dx)+2\mathrm{Li}_2\left(\frac{b(c+dx)}{bc-ad}\right)\right)\right)}{(bc-ad)^2}}{4bg^3(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(a*g + b*g*x)^3,x]
```

```
[Out] -1/4*(2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 + (B*(2*(b*c - a*d)^2*(A + B
*Log[(e*(a + b*x))/(c + d*x)]) + 4*d*(-(b*c) + a*d)*(a + b*x)*(A + B*Log[(e
*(a + b*x))/(c + d*x)]) - 4*d^2*(a + b*x)^2*Log[a + b*x]*(A + B*Log[(e*(a +
 b*x))/(c + d*x)]) + 4*d^2*(a + b*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])
*Log[c + d*x] - 4*B*d*(a + b*x)*(b*c - a*d + d*(a + b*x)*Log[a + b*x] - d*(
a + b*x)*Log[c + d*x]) + B*((b*c - a*d)^2 + 2*d*(-(b*c) + a*d)*(a + b*x) -
2*d^2*(a + b*x)^2*Log[a + b*x] + 2*d^2*(a + b*x)^2*Log[c + d*x]) + 2*B*d^2*
(a + b*x)^2*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)])
 - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) - 2*B*d^2*(a + b*x)^2*((2*Lo
g[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2,
 (b*(c + d*x))/(b*c - a*d)])))/(b*c - a*d)^2)/(b*g^3*(a + b*x)^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 688 vs. $2(262) = 524$.
time = 0.32, size = 689, normalized size = 2.57 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/d^2*e*(a*d-b*c)*(1/2*d^2/(a*d-b*c)^3/g^3*A^2*b*e/(b*e/d+(a*d-b*c)*e/d/(d
*x+c))^2-d^3/(a*d-b*c)^3/g^3*A^2/(b*e/d+(a*d-b*c)*e/d/(d*x+c))-2*d^2/(a*d-b
*c)^3/g^3*A*B*b*e*(-1/2/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*ln(b*e/d+(a*d-b*c)*
e/d/(d*x+c))-1/4/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2)+2*d^3/(a*d-b*c)^3/g^3*A*B
*(-1/(b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/(b*e/d
+(a*d-b*c)*e/d/(d*x+c)))-d^2/(a*d-b*c)^3/g^3*B^2*b*e*(-1/2/(b*e/d+(a*d-b*c)
*e/d/(d*x+c))^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2-1/2/(b*e/d+(a*d-b*c)*e/d/
(d*x+c))^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/4/(b*e/d+(a*d-b*c)*e/d/(d*x+c)
)^2)+d^3/(a*d-b*c)^3/g^3*B^2*(-1/(b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*
d-b*c)*e/d/(d*x+c))^2-2/(b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/
d/(d*x+c))-2/(b*e/d+(a*d-b*c)*e/d/(d*x+c))))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 854 vs. $2(266) = 532$.
time = 0.36, size = 854, normalized size = 3.19

$$\frac{1}{4}\left(2\left(\frac{2\,bdx - bc + 3\,ad}{(b^4c - ab^3d)g^3x^2 + 2\,(ab^3c - a^2b^2d)g^3x + (a^2b^2c - a^3bd)g^3} + \frac{2\,d^2\log(bx+a)}{(b^3c^2 - 2\,ab^2cd + a^2bd^2)g^3} - \frac{2\,d^2\log(dx+c)}{(b^3c^2 - 2\,ab^2cd + a^2bd^2)g^3}\right)\log\left(\frac{bxe}{dx+c} + \frac{ae}{dx+c}\right) - \frac{b^2c^2 - 8\,abcd + 7\,a^2d^2 + 2\,(b^2d^2x^2 + 2\,abd^2x + a^2d^2)\log(bx+a)^2 + 2\,(b^2d^2x^2 + 2\,abd^2x + a^2d^2)\log(dx+c)^2 - 6\,(b^2cd - abd^2)x - 6\,(b^2d^2x^2 + 2\,abd^2x + a^2d^2)\log(bx+a) + 2\,(3\,b^2d^2x^2 + 6\,abd^2x + 3\,a^2d^2 - 2\,(b^2d^2x^2 + 2\,abd^2x + a^2d^2)\log(bx+a))\log(dx+c)}{a^2b^3c^2g^3 - 2\,a^3b^2cdg^3 + a^4bd^2g^3 + (b^5c^2g^3 - 2\,ab^4cdg^3 + a^2b^3d^2g^3)x^2 + 2\,(ab^4c^2g^3 - 2\,a^2b^3cdg^3 + a^3b^2d^2g^3)x}\right)B^2 + \frac{1}{2}\,AB\left(\frac{2\,bdx - bc + 3\,ad}{(b^4c - ab^3d)g^3x^2 + 2\,(ab^3c - a^2b^2d)g^3x + (a^2b^2c - a^3bd)g^3} - \frac{2\,\log\left(\frac{bxe}{dx+c} + \frac{ae}{dx+c}\right)}{b^3g^3x^2 + 2\,ab^2g^3x + a^2bg^3} + \frac{2\,d^2\log(bx+a)}{(b^3c^2 - 2\,ab^2cd + a^2bd^2)g^3} - \frac{2\,d^2\log(dx+c)}{(b^3c^2 - 2\,ab^2cd + a^2bd^2)g^3}\right) - \frac{B^2\log\left(\frac{bxe}{dx+c} + \frac{ae}{dx+c}\right)^2}{2\,(b^3g^3x^2 + 2\,ab^2g^3x + a^2bg^3)} - \frac{A^2}{2\,(b^3g^3x^2 + 2\,ab^2g^3x + a^2bg^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^3,x, algorithm="maxima
")
```

```
[Out] 1/4*(2*((2*b*d*x - b*c + 3*a*d)/((b^4*c - a*b^3*d)*g^3*x^2 + 2*(a*b^3*c - a
^2*b^2*d)*g^3*x + (a^2*b^2*c - a^3*b*d)*g^3) + 2*d^2*log(b*x + a)/((b^3*c^2
 - 2*a*b^2*c*d + a^2*b*d^2)*g^3) - 2*d^2*log(d*x + c)/((b^3*c^2 - 2*a*b^2*c
*d + a^2*b*d^2)*g^3))*log(b*x*e/(d*x + c) + a*e/(d*x + c)) - (b^2*c^2 - 8*a
```

```
*b*c*d + 7*a^2*d^2 + 2*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*log(b*x + a)^2
 + 2*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*log(d*x + c)^2 - 6*(b^2*c*d - a*
b*d^2)*x - 6*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*log(b*x + a) + 2*(3*b^2*
d^2*x^2 + 6*a*b*d^2*x + 3*a^2*d^2 - 2*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)
*log(b*x + a))*log(d*x + c))/(a^2*b^3*c^2*g^3 - 2*a^3*b^2*c*d*g^3 + a^4*b*d
^2*g^3 + (b^5*c^2*g^3 - 2*a*b^4*c*d*g^3 + a^2*b^3*d^2*g^3)*x^2 + 2*(a*b^4*c
^2*g^3 - 2*a^2*b^3*c*d*g^3 + a^3*b^2*d^2*g^3)*x))*B^2 + 1/2*A*B*((2*b*d*x -
 b*c + 3*a*d)/((b^4*c - a*b^3*d)*g^3*x^2 + 2*(a*b^3*c - a^2*b^2*d)*g^3*x +
(a^2*b^2*c - a^3*b*d)*g^3) - 2*log(b*x*e/(d*x + c) + a*e/(d*x + c))/(b^3*g^
3*x^2 + 2*a*b^2*g^3*x + a^2*b*g^3) + 2*d^2*log(b*x + a)/((b^3*c^2 - 2*a*b^2
*c*d + a^2*b*d^2)*g^3) - 2*d^2*log(d*x + c)/((b^3*c^2 - 2*a*b^2*c*d + a^2*b
*d^2)*g^3)) - 1/2*B^2*log(b*x*e/(d*x + c) + a*e/(d*x + c))^2/(b^3*g^3*x^2 +
 2*a*b^2*g^3*x + a^2*b*g^3) - 1/2*A^2/(b^3*g^3*x^2 + 2*a*b^2*g^3*x + a^2*b*
g^3)
```

Fricas [**A**]
time = 0.37, size = 365, normalized size = 1.36

$$-\frac{(2A^2+2AB+B^2)b^2c^2-4(A^2+2AB+2B^2)abcd+(2A^2+6AB+7B^2)a^2d^2-2(B^2b^2d^2x^2+2B^2abd^2x-B^2b^2c^2+2B^2abcd)\log\left(\frac{(bx+a)e}{dx+c}\right)^2-2((2AB+3B^2)b^2cd-(2AB+3B^2)abd^2)x-2((2AB+3B^2)b^2d^2x^2-(2AB+B^2)b^2c^2+4(AB+B^2)abcd+2(B^2b^2cd+2(AB+B^2)abd^2)x)\log\left(\frac{(bx+a)e}{dx+c}\right)}{4((b^5c^2-2ab^4cd+a^2b^3d^2)g^3x^2+2(ab^4c^2-2a^2b^3cd+a^3b^2d^2)g^3x+(a^2b^3c^2-2a^3b^2cd+a^4bd^2)g^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^3,x, algorithm="fricas
")
```

```
[Out] -1/4*((2*A^2 + 2*A*B + B^2)*b^2*c^2 - 4*(A^2 + 2*A*B + 2*B^2)*a*b*c*d + (2*
A^2 + 6*A*B + 7*B^2)*a^2*d^2 - 2*(B^2*b^2*d^2*x^2 + 2*B^2*a*b*d^2*x - B^2*b
^2*c^2 + 2*B^2*a*b*c*d)*log((b*x + a)*e/(d*x + c))^2 - 2*((2*A*B + 3*B^2)*b
^2*c*d - (2*A*B + 3*B^2)*a*b*d^2)*x - 2*((2*A*B + 3*B^2)*b^2*d^2*x^2 - (2*A
*B + B^2)*b^2*c^2 + 4*(A*B + B^2)*a*b*c*d + 2*(B^2*b^2*c*d + 2*(A*B + B^2)*
a*b*d^2)*x)*log((b*x + a)*e/(d*x + c)))/((b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d
^2)*g^3*x^2 + 2*(a*b^4*c^2 - 2*a^2*b^3*c*d + a^3*b^2*d^2)*g^3*x + (a^2*b^3*
c^2 - 2*a^3*b^2*c*d + a^4*b*d^2)*g^3)
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 894 vs. $2(241) = 482$.
time = 2.62, size = 894, normalized size = 3.34

$$-\frac{Bd^2\cdot(2A+3B)\log\left(x+\frac{2ABad^3+2ABbcd^2+3B^2ad^3+3B^2bcd^2-\frac{Ba^3d^5\cdot(2A+3B)}{(ad-bc)^2}+\frac{3Ba^2bcd^4\cdot(2A+3B)}{(ad-bc)^2}-\frac{3Bab^2c^2d^3\cdot(2A+3B)}{(ad-bc)^2}+\frac{Bb^3c^3d^2\cdot(2A+3B)}{(ad-bc)^2}}{4ABbd^3+6B^2bd^3}\right)}{2bg^3(ad-bc)^2}+\frac{Bd^2\cdot(2A+3B)\log\left(x+\frac{2ABad^3+2ABbcd^2+3B^2ad^3+3B^2bcd^2+\frac{Ba^3d^5\cdot(2A+3B)}{(ad-bc)^2}-\frac{3Ba^2bcd^4\cdot(2A+3B)}{(ad-bc)^2}+\frac{3Bab^2c^2d^3\cdot(2A+3B)}{(ad-bc)^2}-\frac{Bb^3c^3d^2\cdot(2A+3B)}{(ad-bc)^2}}{4ABbd^3+6B^2bd^3}\right)}{2bg^3(ad-bc)^2}+\frac{(2B^2acd+2B^2ad^2x-B^2bc^2+B^2bd^2x^2)\log\left(\frac{e(a+bx)}{c+dx}\right)^2}{2a^4d^2g^3-4a^3bcdg^3+4a^3bd^2g^3x+2a^2b^2c^2g^3-8a^2b^2cdg^3x+2a^2b^2d^2g^3x^2+4ab^3c^2g^3x-4ab^3cdg^3x^2+2b^4c^2g^3x^2}+\frac{(-2ABad+2ABbc-3B^2ad+B^2bc-2B^2bdx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{2a^3bdg^3-2a^2b^2cg^3+4a^2b^2dg^3x-4ab^3cg^3x+2ab^3dg^3x^2-2b^4cg^3x^2}+\frac{-2A^2ad+2A^2bc-6ABad+2ABbc-7B^2ad+B^2bc+x(-4ABbd-6B^2bd)}{4a^3bdg^3-4a^2b^2cg^3+x^2\cdot(4ab^3dg^3-4b^4cg^3)+x(8a^2b^2dg^3-8ab^3cg^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(b*x+a)/(d*x+c)))**2/(b*g*x+a*g)**3,x)
```

```
[Out] -B*d**2*(2*A + 3*B)*log(x + (2*A*B*a*d**3 + 2*A*B*b*c*d**2 + 3*B**2*a*d**3
+ 3*B**2*b*c*d**2 - B*a**3*d**5*(2*A + 3*B)/(a*d - b*c)**2 + 3*B*a**2*b*c*d
**4*(2*A + 3*B)/(a*d - b*c)**2 - 3*B*a*b**2*c**2*d**3*(2*A + 3*B)/(a*d - b*
c)**2 + B*b**3*c**3*d**2*(2*A + 3*B)/(a*d - b*c)**2)/(4*A*B*b*d**3 + 6*B**2
```

```
*b*d**3))/(2*b*g**3*(a*d - b*c)**2) + B*d**2*(2*A + 3*B)*log(x + (2*A*B*a*d
**3 + 2*A*B*b*c*d**2 + 3*B**2*a*d**3 + 3*B**2*b*c*d**2 + B*a**3*d**5*(2*A +
 3*B)/(a*d - b*c)**2 - 3*B*a**2*b*c*d**4*(2*A + 3*B)/(a*d - b*c)**2 + 3*B*a
*b**2*c**2*d**3*(2*A + 3*B)/(a*d - b*c)**2 - B*b**3*c**3*d**2*(2*A + 3*B)/(
a*d - b*c)**2)/(4*A*B*b*d**3 + 6*B**2*b*d**3))/(2*b*g**3*(a*d - b*c)**2) +
(2*B**2*a*c*d + 2*B**2*a*d**2*x - B**2*b*c**2 + B**2*b*d**2*x**2)*log(e*(a
+ b*x)/(c + d*x))**2/(2*a**4*d**2*g**3 - 4*a**3*b*c*d*g**3 + 4*a**3*b*d**2*
g**3*x + 2*a**2*b**2*c**2*g**3 - 8*a**2*b**2*c*d*g**3*x + 2*a**2*b**2*d**2*
g**3*x**2 + 4*a*b**3*c**2*g**3*x - 4*a*b**3*c*d*g**3*x**2 + 2*b**4*c**2*g**
3*x**2) + (-2*A*B*a*d + 2*A*B*b*c - 3*B**2*a*d + B**2*b*c - 2*B**2*b*d*x)*l
og(e*(a + b*x)/(c + d*x))/(2*a**3*b*d*g**3 - 2*a**2*b**2*c*g**3 + 4*a**2*b*
*2*d*g**3*x - 4*a*b**3*c*g**3*x + 2*a*b**3*d*g**3*x**2 - 2*b**4*c*g**3*x**2
) + (-2*A**2*a*d + 2*A**2*b*c - 6*A*B*a*d + 2*A*B*b*c - 7*B**2*a*d + B**2*b
*c + x*(-4*A*B*b*d - 6*B**2*b*d))/(4*a**3*b*d*g**3 - 4*a**2*b**2*c*g**3 + x
**2*(4*a*b**3*d*g**3 - 4*b**4*c*g**3) + x*(8*a**2*b**2*d*g**3 - 8*a*b**3*c*
g**3))
```

Giac [A]
time = 3.60, size = 424, normalized size = 1.58

$$-\frac{\left(2\,B^2be^3\log\left(\frac{bxe+ae}{dx+c}\right)^2-\frac{4\,(bxe+ae)B^2de^2\log\left(\frac{bxe+ae}{dx+c}\right)^2}{dx+c}+4\,ABbe^3\log\left(\frac{bxe+ae}{dx+c}\right)+2\,B^2be^3\log\left(\frac{bxe+ae}{dx+c}\right)-\frac{8\,(bxe+ae)ABde^2\log\left(\frac{bxe+ae}{dx+c}\right)}{dx+c}-\frac{8\,(bxe+ae)B^2de^2\log\left(\frac{bxe+ae}{dx+c}\right)}{dx+c}+2\,A^2be^3+2\,ABbe^3+B^2be^3-\frac{4\,(bxe+ae)A^2de^2}{dx+c}-\frac{8\,(bxe+ae)ABde^2}{dx+c}-\frac{8\,(bxe+ae)B^2de^2}{dx+c}\right)\left(\frac{bc}{(bce-ade)(bc-ad)}-\frac{ad}{(bce-ade)(bc-ad)}\right)}{4\left(\frac{(bxe+ae)^2bcg^3}{(dx+c)^2}-\frac{(bxe+ae)^2adg^3}{(dx+c)^2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^3,x, algorithm="giac")
```

```
[Out] -1/4*(2*B^2*b*e^3*log((b*x*e + a*e)/(d*x + c))^2 - 4*(b*x*e + a*e)*B^2*d*e^
2*log((b*x*e + a*e)/(d*x + c))^2/(d*x + c) + 4*A*B*b*e^3*log((b*x*e + a*e)/
(d*x + c)) + 2*B^2*b*e^3*log((b*x*e + a*e)/(d*x + c)) - 8*(b*x*e + a*e)*A*B
*d*e^2*log((b*x*e + a*e)/(d*x + c))/(d*x + c) - 8*(b*x*e + a*e)*B^2*d*e^2*l
og((b*x*e + a*e)/(d*x + c))/(d*x + c) + 2*A^2*b*e^3 + 2*A*B*b*e^3 + B^2*b*e
^3 - 4*(b*x*e + a*e)*A^2*d*e^2/(d*x + c) - 8*(b*x*e + a*e)*A*B*d*e^2/(d*x +
 c) - 8*(b*x*e + a*e)*B^2*d*e^2/(d*x + c))*(b*c/((b*c*e - a*d*e)*(b*c - a*d
)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))/((b*x*e + a*e)^2*b*c*g^3/(d*x + c)^
2 - (b*x*e + a*e)^2*a*d*g^3/(d*x + c)^2)
```

Mupad [B]
time = 5.85, size = 507, normalized size = 1.89

$$-\frac{\frac{2\,A^2\,a\,d-2\,A^2\,b\,c+7\,B^2\,a\,d-B^2\,b\,c+6\,A\,B\,a\,d-2\,A\,B\,b\,c}{2\,(a\,d-b\,c)}+\frac{x\,(3\,b\,d\,B^2+2\,A\,b\,d\,B)}{a\,d-b\,c}}{2\,a^2\,b\,g^3+4\,a\,b^2\,g^3\,x+2\,b^3\,g^3\,x^2}-\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)^2\left(\frac{B^2}{2\,b^2\,g^3\,\left(2\,a\,x+b\,x^2+\frac{a^2}{b}\right)}-\frac{B^2\,d^2}{2\,b\,g^3\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}\right)-\frac{\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\,\left(\frac{A\,B}{b^2\,d\,g^3}+\frac{B^2\,x\,(a\,d-b\,c)}{b\,g^3\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}+\frac{B^2\,d^2\,\left(\frac{2\,a^2\,d^2-3\,a\,b\,c\,d+b^2\,c^2}{2\,b\,d^3}+\frac{a\,(a\,d-b\,c)}{2\,b\,d^2}\right)}{b\,g^3\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}\right)}{\frac{b\,x^2}{d}+\frac{a^2}{b\,d}+\frac{2\,a\,x}{d}}-\frac{B\,d^2\,\mathrm{atan}\left(\frac{B\,d^2\,\left(2\,b\,d\,x-\frac{b^3\,c^2\,g^3-a^2\,b\,d^2\,g^3}{b\,g^3\,(a\,d-b\,c)}\right)\,(2\,A+3\,B)\,1{}\mathrm{i}}{(a\,d-b\,c)\,(3\,B^2\,d^2+2\,A\,B\,d^2)}\right)\,(2\,A+3\,B)\,1{}\mathrm{i}}{b\,g^3\,{(a\,d-b\,c)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x))/(c + d*x)))^2/(a*g + b*g*x)^3,x)
```

```
[Out] - ((2*A^2*a*d - 2*A^2*b*c + 7*B^2*a*d - B^2*b*c + 6*A*B*a*d - 2*A*B*b*c)/(2
*(a*d - b*c)) + (x*(3*B^2*b*d + 2*A*B*b*d))/(a*d - b*c))/(2*a^2*b*g^3 + 2*b
```

```
^3*g^3*x^2 + 4*a*b^2*g^3*x) - log((e*(a + b*x))/(c + d*x))^2*(B^2/(2*b^2*g^
3*(2*a*x + b*x^2 + a^2/b)) - (B^2*d^2)/(2*b*g^3*(a^2*d^2 + b^2*c^2 - 2*a*b*
c*d))) - (log((e*(a + b*x))/(c + d*x))*((A*B)/(b^2*d*g^3) + (B^2*x*(a*d - b
*c))/(b*g^3*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (B^2*d^2*((2*a^2*d^2 + b^2*c
^2 - 3*a*b*c*d)/(2*b*d^3) + (a*(a*d - b*c))/(2*b*d^2)))/(b*g^3*(a^2*d^2 + b
^2*c^2 - 2*a*b*c*d))))/((b*x^2)/d + a^2/(b*d) + (2*a*x)/d) - (B*d^2*atan((B
*d^2*(2*b*d*x - (b^3*c^2*g^3 - a^2*b*d^2*g^3)/(b*g^3*(a*d - b*c)))*(2*A + 3
*B)*1i)/((a*d - b*c)*(3*B^2*d^2 + 2*A*B*d^2)))*(2*A + 3*B)*1i)/(b*g^3*(a*d
- b*c)^2)
```

3.104 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4}\,dx$

Optimal. Leaf size=418

$$-\frac{2B^2d^2(c+dx)}{(bc-ad)^3g^4(a+bx)}+\frac{bB^2d(c+dx)^2}{2(bc-ad)^3g^4(a+bx)^2}-\frac{2b^2B^2(c+dx)^3}{27(bc-ad)^3g^4(a+bx)^3}-\frac{2Bd^2(c+dx)\left(A+B\log\left(\frac{e(a+b}{c+d}\right.\right.}{(bc-ad)^3g^4(a+bx)}$$

```
[Out] -2*B^2*d^2*(d*x+c)/(-a*d+b*c)^3/g^4/(b*x+a)+1/2*b*B^2*d*(d*x+c)^2/(-a*d+b*c
)^3/g^4/(b*x+a)^2-2/27*b^2*B^2*(d*x+c)^3/(-a*d+b*c)^3/g^4/(b*x+a)^3-2*B*d^2
*(d*x+c)*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*c)^3/g^4/(b*x+a)+b*B*d*(d*x+c)
^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*c)^3/g^4/(b*x+a)^2-2/9*b^2*B*(d*x+c)
^3*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*c)^3/g^4/(b*x+a)^3-d^2*(d*x+c)*(A+B*
ln(e*(b*x+a)/(d*x+c)))^2/(-a*d+b*c)^3/g^4/(b*x+a)+b*d*(d*x+c)^2*(A+B*ln(e*(
b*x+a)/(d*x+c)))^2/(-a*d+b*c)^3/g^4/(b*x+a)^2-1/3*b^2*(d*x+c)^3*(A+B*ln(e*(
b*x+a)/(d*x+c)))^2/(-a*d+b*c)^3/g^4/(b*x+a)^3
```

Rubi [A]

time = 0.21, antiderivative size = 418, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2550, 2395, 2342, 2341}

$$-\frac{b^2(c+dx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{3g^4(a+bx)^3(bc-ad)^3}-\frac{2b^2B(c+dx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{9g^4(a+bx)^3(bc-ad)^3}-\frac{d^2(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{g^4(a+bx)(bc-ad)^3}-\frac{2Bd^2(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{g^4(a+bx)(bc-ad)^3}+\frac{bd(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{g^4(a+bx)^2(bc-ad)^3}+\frac{bBd(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{g^4(a+bx)^2(bc-ad)^3}-\frac{2b^2B^2(c+dx)^3}{27g^4(a+bx)^3(bc-ad)^3}-\frac{2B^2d^2(c+dx)}{g^4(a+bx)(bc-ad)^3}+\frac{bB^2d(c+dx)^2}{2g^4(a+bx)^2(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(a*g + b*g*x)^4,x]
```

```
[Out] (-2*B^2*d^2*(c + d*x))/((b*c - a*d)^3*g^4*(a + b*x)) + (b*B^2*d*(c + d*x)^2
)/(2*(b*c - a*d)^3*g^4*(a + b*x)^2) - (2*b^2*B^2*(c + d*x)^3)/(27*(b*c - a*
d)^3*g^4*(a + b*x)^3) - (2*B*d^2*(c + d*x)*(A + B*Log[(e*(a + b*x))/(c + d*
x)]))/((b*c - a*d)^3*g^4*(a + b*x)) + (b*B*d*(c + d*x)^2*(A + B*Log[(e*(a +
 b*x))/(c + d*x)]))/((b*c - a*d)^3*g^4*(a + b*x)^2) - (2*b^2*B*(c + d*x)^3*
(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(9*(b*c - a*d)^3*g^4*(a + b*x)^3) - (
d^2*(c + d*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/((b*c - a*d)^3*g^4*(a
 + b*x)) + (b*d*(c + d*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/((b*c -
 a*d)^3*g^4*(a + b*x)^2) - (b^2*(c + d*x)^3*(A + B*Log[(e*(a + b*x))/(c + d
*x)])^2)/(3*(b*c - a*d)^3*g^4*(a + b*x)^3)
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2550

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(
m + 1)*(g/b)^m, Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x],
 x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] &&
 EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && Eq
Q[b*f - a*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Rubi steps

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4}\,dx = -\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3bg^4(a+bx)^3} + \frac{(2B)\int \frac{(bc-ad)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{g^3(a+bx)^4(c+dx)}\,dx}{3bg}$$

$$= -\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3bg^4(a+bx)^3} + \frac{(2B(bc-ad))\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(a+bx)^4(c+dx)}\,dx}{3bg^4}$$

$$= -\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3bg^4(a+bx)^3} + \frac{(2B(bc-ad))\int \left(\frac{b\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)(a+bx)^4} - \frac{bd\left(A+B\,\mathrm{l}\ldots\right.}{(bc-ad\ldots}\right.}{\ldots}$$

$$= -\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3bg^4(a+bx)^3} + \frac{(2B)\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(a+bx)^4}\,dx}{3g^4} - \frac{(2Bd^3)\int \frac{A+B\log\left(\ldots\right)}{a+b\ldots}}{3(bc-ad)^3\ldots}$$

$$= -\frac{2B\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{9bg^4(a+bx)^3} + \frac{Bd\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3b(bc-ad)g^4(a+bx)^2} - \frac{2Bd^2\left(A+B\,\mathrm{lo}\ldots\right.}{3b(bc-ad)^2\ldots}$$

$$= -\frac{2B\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{9bg^4(a+bx)^3} + \frac{Bd\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3b(bc-ad)g^4(a+bx)^2} - \frac{2Bd^2\left(A+B\,\mathrm{lo}\ldots\right.}{3b(bc-ad)^2\ldots}$$

$$= -\frac{2B\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{9bg^4(a+bx)^3} + \frac{Bd\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3b(bc-ad)g^4(a+bx)^2} - \frac{2Bd^2\left(A+B\,\mathrm{lo}\ldots\right.}{3b(bc-ad)^2\ldots}$$

$$= -\frac{2B^2}{27bg^4(a+bx)^3} + \frac{5B^2d}{18b(bc-ad)g^4(a+bx)^2} - \frac{11B^2d^2}{9b(bc-ad)^2g^4(a+bx)} - \frac{11\ldots}{\ldots}$$

$$= -\frac{2B^2}{27bg^4(a+bx)^3} + \frac{5B^2d}{18b(bc-ad)g^4(a+bx)^2} - \frac{11B^2d^2}{9b(bc-ad)^2g^4(a+bx)} - \frac{11\ldots}{\ldots}$$

$$= -\frac{2B^2}{27bg^4(a+bx)^3} + \frac{5B^2d}{18b(bc-ad)g^4(a+bx)^2} - \frac{11B^2d^2}{9b(bc-ad)^2g^4(a+bx)} - \frac{11\ldots}{\ldots}$$

$$= -\frac{2B^2}{27bg^4(a+bx)^3} + \frac{5B^2d}{18b(bc-ad)g^4(a+bx)^2} - \frac{11B^2d^2}{9b(bc-ad)^2g^4(a+bx)} - \frac{11\ldots}{\ldots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 0.45, size = 585, normalized size = 1.40

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(a*g + b*g*x)^4,x]
```

```
[Out] -1/54*(18*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 + (B*(12*(b*c - a*d)^3*(A
+ B*Log[(e*(a + b*x))/(c + d*x)]) - 18*d*(b*c - a*d)^2*(a + b*x)*(A + B*Log
[(e*(a + b*x))/(c + d*x)]) + 36*d^2*(b*c - a*d)*(a + b*x)^2*(A + B*Log[(e*(
a + b*x))/(c + d*x)]) + 36*d^3*(a + b*x)^3*Log[a + b*x]*(A + B*Log[(e*(a +
b*x))/(c + d*x)]) - 36*d^3*(a + b*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])
*Log[c + d*x] + 36*B*d^2*(a + b*x)^2*(b*c - a*d + d*(a + b*x)*Log[a + b*x]
- d*(a + b*x)*Log[c + d*x]) - 9*B*d*(a + b*x)*((b*c - a*d)^2 + 2*d*(-(b*c)
+ a*d)*(a + b*x) - 2*d^2*(a + b*x)^2*Log[a + b*x] + 2*d^2*(a + b*x)^2*Log[c
 + d*x]) + 2*B*(2*(b*c - a*d)^3 - 3*d*(b*c - a*d)^2*(a + b*x) + 6*d^2*(b*c
- a*d)*(a + b*x)^2 + 6*d^3*(a + b*x)^3*Log[a + b*x] - 6*d^3*(a + b*x)^3*Log
[c + d*x]) - 18*B*d^3*(a + b*x)^3*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c
 + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) + 18*B
*d^3*(a + b*x)^3*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[
c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(b*c - a*d)^3)/(b*g^4
*(a + b*x)^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1038 vs. $2(410) = 820$.
time = 0.40, size = 1039, normalized size = 2.49 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/d^2*e*(a*d-b*c)*(-1/3*d^2/(a*d-b*c)^4/g^4*A^2*b^2*e^2/(b*e/d+(a*d-b*c)*e
/d/(d*x+c))^3+d^3/(a*d-b*c)^4/g^4*A^2*b*e/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2-d
^4/(a*d-b*c)^4/g^4*A^2/(b*e/d+(a*d-b*c)*e/d/(d*x+c))+2*d^2/(a*d-b*c)^4/g^4*
A*B*b^2*e^2*(-1/3/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3*ln(b*e/d+(a*d-b*c)*e/d/(d
*x+c))-1/9/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3)-4*d^3/(a*d-b*c)^4/g^4*A*B*b*e*(
-1/2/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/4/(b
*e/d+(a*d-b*c)*e/d/(d*x+c))^2)+2*d^4/(a*d-b*c)^4/g^4*A*B*(-1/(b*e/d+(a*d-b*
c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/(b*e/d+(a*d-b*c)*e/d/(d*x
+c)))+d^2/(a*d-b*c)^4/g^4*B^2*b^2*e^2*(-1/3/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3
*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2-2/9/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3*ln(b
*e/d+(a*d-b*c)*e/d/(d*x+c))-2/27/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3)-2*d^3/(a*
d-b*c)^4/g^4*B^2*b*e*(-1/2/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*ln(b*e/d+(a*d-b*
c)*e/d/(d*x+c))^2-1/2/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*ln(b*e/d+(a*d-b*c)*e/
d/(d*x+c))-1/4/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2)+d^4/(a*d-b*c)^4/g^4*B^2*(-1
/(b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2-2/(b*e/d+(
a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-2/(b*e/d+(a*d-b*c)*e/
d/(d*x+c))))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1425 vs. $2(416) = 832$.
time = 0.45, size = 1425, normalized size = 3.41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^4,x, algorithm="maxima
")
```

```
[Out] -1/54*(6*((6*b^2*d^2*x^2 + 2*b^2*c^2 - 7*a*b*c*d + 11*a^2*d^2 - 3*(b^2*c*d
- 5*a*b*d^2)*x)/((b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2)*g^4*x^3 + 3*(a*b^5*c
^2 - 2*a^2*b^4*c*d + a^3*b^3*d^2)*g^4*x^2 + 3*(a^2*b^4*c^2 - 2*a^3*b^3*c*d
+ a^4*b^2*d^2)*g^4*x + (a^3*b^3*c^2 - 2*a^4*b^2*c*d + a^5*b*d^2)*g^4) + 6*d
^3*log(b*x + a)/((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*g^
4) - 6*d^3*log(d*x + c)/((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b
*d^3)*g^4))*log(b*x*e/(d*x + c) + a*e/(d*x + c)) + (4*b^3*c^3 - 27*a*b^2*c^
2*d + 108*a^2*b*c*d^2 - 85*a^3*d^3 + 66*(b^3*c*d^2 - a*b^2*d^3)*x^2 - 18*(b
^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(b*x + a)^2 - 18
*(b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(d*x + c)^2 -
 3*(5*b^3*c^2*d - 54*a*b^2*c*d^2 + 49*a^2*b*d^3)*x + 66*(b^3*d^3*x^3 + 3*a*
b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(b*x + a) - 6*(11*b^3*d^3*x^3 + 3
3*a*b^2*d^3*x^2 + 33*a^2*b*d^3*x + 11*a^3*d^3 - 6*(b^3*d^3*x^3 + 3*a*b^2*d^
3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(b*x + a))*log(d*x + c))/(a^3*b^4*c^3*g
^4 - 3*a^4*b^3*c^2*d*g^4 + 3*a^5*b^2*c*d^2*g^4 - a^6*b*d^3*g^4 + (b^7*c^3*g
^4 - 3*a*b^6*c^2*d*g^4 + 3*a^2*b^5*c*d^2*g^4 - a^3*b^4*d^3*g^4)*x^3 + 3*(a*
b^6*c^3*g^4 - 3*a^2*b^5*c^2*d*g^4 + 3*a^3*b^4*c*d^2*g^4 - a^4*b^3*d^3*g^4)*
x^2 + 3*(a^2*b^5*c^3*g^4 - 3*a^3*b^4*c^2*d*g^4 + 3*a^4*b^3*c*d^2*g^4 - a^5*
b^2*d^3*g^4)*x))*B^2 - 1/9*A*B*((6*b^2*d^2*x^2 + 2*b^2*c^2 - 7*a*b*c*d + 11
*a^2*d^2 - 3*(b^2*c*d - 5*a*b*d^2)*x)/((b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2
)*g^4*x^3 + 3*(a*b^5*c^2 - 2*a^2*b^4*c*d + a^3*b^3*d^2)*g^4*x^2 + 3*(a^2*b^
4*c^2 - 2*a^3*b^3*c*d + a^4*b^2*d^2)*g^4*x + (a^3*b^3*c^2 - 2*a^4*b^2*c*d +
 a^5*b*d^2)*g^4) + 6*log(b*x*e/(d*x + c) + a*e/(d*x + c))/(b^4*g^4*x^3 + 3*
a*b^3*g^4*x^2 + 3*a^2*b^2*g^4*x + a^3*b*g^4) + 6*d^3*log(b*x + a)/((b^4*c^3
 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*g^4) - 6*d^3*log(d*x + c)/(
(b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*g^4)) - 1/3*B^2*log
(b*x*e/(d*x + c) + a*e/(d*x + c))^2/(b^4*g^4*x^3 + 3*a*b^3*g^4*x^2 + 3*a^2*
b^2*g^4*x + a^3*b*g^4) - 1/3*A^2/(b^4*g^4*x^3 + 3*a*b^3*g^4*x^2 + 3*a^2*b^2
*g^4*x + a^3*b*g^4)
```

<u>Fricas</u> [A]
time = 0.38, size = 670, normalized size = 1.60

$$-\frac{2\,(9\,A^2+6\,AB+2\,B^2)b^3c^3-27\,(2\,A^2+2\,AB+B^2)ab^2c^2d+54\,(A^2+2\,AB+2\,B^2)a^2bcd^2-(18\,A^2+66\,AB+85\,B^2)a^3d^3+6\,((6\,AB+11\,B^2)b^3cd^2-(6\,AB+11\,B^2)ab^2d^3)x^2+18\,(B^2b^3d^3x^3+3\,B^2ab^2d^3x^2+3\,B^2a^2bd^3x+B^2b^3c^3-3\,B^2ab^2c^2d+3\,B^2a^2bcd^2)\log\left(\frac{(bx+a)e}{dx+c}\right)^2-3\,((6\,AB+5\,B^2)b^3c^2d-18\,(2\,AB+3\,B^2)ab^2cd^2+(30\,AB+49\,B^2)a^2bd^3)x+6\,((6\,AB+11\,B^2)b^3d^3x^3+2\,(3\,AB+B^2)b^3c^3-9\,(2\,AB+B^2)ab^2c^2d+18\,(AB+B^2)a^2bcd^2+3\,(2\,B^2b^3cd^2+3\,(2\,AB+3\,B^2)ab^2d^3)x^2-3\,(B^2b^3c^2d-6\,B^2ab^2cd^2-6\,(AB+B^2)a^2bd^3)x)\log\left(\frac{(bx+a)e}{dx+c}\right)}{54\,((b^7c^3-3\,ab^6c^2d+3\,a^2b^5cd^2-a^3b^4d^3)g^4x^3+3\,(ab^6c^3-3\,a^2b^5c^2d+3\,a^3b^4cd^2-a^4b^3d^3)g^4x^2+3\,(a^2b^5c^3-3\,a^3b^4c^2d+3\,a^4b^3cd^2-a^5b^2d^3)g^4x+(a^3b^4c^3-3\,a^4b^3c^2d+3\,a^5b^2cd^2-a^6bd^3)g^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^4,x, algorithm="fricas
")
```

```
[Out] -1/54*(2*(9*A^2 + 6*A*B + 2*B^2)*b^3*c^3 - 27*(2*A^2 + 2*A*B + B^2)*a*b^2*c
^2*d + 54*(A^2 + 2*A*B + 2*B^2)*a^2*b*c*d^2 - (18*A^2 + 66*A*B + 85*B^2)*a^
```

```
3*d^3 + 6*((6*A*B + 11*B^2)*b^3*c*d^2 - (6*A*B + 11*B^2)*a*b^2*d^3)*x^2 + 1
8*(B^2*b^3*d^3*x^3 + 3*B^2*a*b^2*d^3*x^2 + 3*B^2*a^2*b*d^3*x + B^2*b^3*c^3
- 3*B^2*a*b^2*c^2*d + 3*B^2*a^2*b*c*d^2)*log((b*x + a)*e/(d*x + c))^2 - 3*(
(6*A*B + 5*B^2)*b^3*c^2*d - 18*(2*A*B + 3*B^2)*a*b^2*c*d^2 + (30*A*B + 49*B
^2)*a^2*b*d^3)*x + 6*((6*A*B + 11*B^2)*b^3*d^3*x^3 + 2*(3*A*B + B^2)*b^3*c^
3 - 9*(2*A*B + B^2)*a*b^2*c^2*d + 18*(A*B + B^2)*a^2*b*c*d^2 + 3*(2*B^2*b^3
*c*d^2 + 3*(2*A*B + 3*B^2)*a*b^2*d^3)*x^2 - 3*(B^2*b^3*c^2*d - 6*B^2*a*b^2*
c*d^2 - 6*(A*B + B^2)*a^2*b*d^3)*x)*log((b*x + a)*e/(d*x + c)))/((b^7*c^3 -
 3*a*b^6*c^2*d + 3*a^2*b^5*c*d^2 - a^3*b^4*d^3)*g^4*x^3 + 3*(a*b^6*c^3 - 3*
a^2*b^5*c^2*d + 3*a^3*b^4*c*d^2 - a^4*b^3*d^3)*g^4*x^2 + 3*(a^2*b^5*c^3 - 3
*a^3*b^4*c^2*d + 3*a^4*b^3*c*d^2 - a^5*b^2*d^3)*g^4*x + (a^3*b^4*c^3 - 3*a^
4*b^3*c^2*d + 3*a^5*b^2*c*d^2 - a^6*b*d^3)*g^4)
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1544 vs. $2(384) = 768$.
time = 16.43, size = 1544, normalized size = 3.69

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(b*x+a)/(d*x+c)))**2/(b*g*x+a*g)**4,x)
```

```
[Out] -B*d**3*(6*A + 11*B)*log(x + (6*A*B*a*d**4 + 6*A*B*b*c*d**3 + 11*B**2*a*d**
4 + 11*B**2*b*c*d**3 - B*a**4*d**7*(6*A + 11*B)/(a*d - b*c)**3 + 4*B*a**3*b
*c*d**6*(6*A + 11*B)/(a*d - b*c)**3 - 6*B*a**2*b**2*c**2*d**5*(6*A + 11*B)/
(a*d - b*c)**3 + 4*B*a*b**3*c**3*d**4*(6*A + 11*B)/(a*d - b*c)**3 - B*b**4*
c**4*d**3*(6*A + 11*B)/(a*d - b*c)**3)/(12*A*B*b*d**4 + 22*B**2*b*d**4))/(9
*b*g**4*(a*d - b*c)**3) + B*d**3*(6*A + 11*B)*log(x + (6*A*B*a*d**4 + 6*A*B
*b*c*d**3 + 11*B**2*a*d**4 + 11*B**2*b*c*d**3 + B*a**4*d**7*(6*A + 11*B)/(a
*d - b*c)**3 - 4*B*a**3*b*c*d**6*(6*A + 11*B)/(a*d - b*c)**3 + 6*B*a**2*b**
2*c**2*d**5*(6*A + 11*B)/(a*d - b*c)**3 - 4*B*a*b**3*c**3*d**4*(6*A + 11*B)
/(a*d - b*c)**3 + B*b**4*c**4*d**3*(6*A + 11*B)/(a*d - b*c)**3)/(12*A*B*b*d
**4 + 22*B**2*b*d**4))/(9*b*g**4*(a*d - b*c)**3) + (3*B**2*a**2*c*d**2 + 3*
B**2*a**2*d**3*x - 3*B**2*a*b*c**2*d + 3*B**2*a*b*d**3*x**2 + B**2*b**2*c**
3 + B**2*b**2*d**3*x**3)*log(e*(a + b*x)/(c + d*x))**2/(3*a**6*d**3*g**4 -
9*a**5*b*c*d**2*g**4 + 9*a**5*b*d**3*g**4*x + 9*a**4*b**2*c**2*d*g**4 - 27*
a**4*b**2*c*d**2*g**4*x + 9*a**4*b**2*d**3*g**4*x**2 - 3*a**3*b**3*c**3*g**
4 + 27*a**3*b**3*c**2*d*g**4*x - 27*a**3*b**3*c*d**2*g**4*x**2 + 3*a**3*b**
3*d**3*g**4*x**3 - 9*a**2*b**4*c**3*g**4*x + 27*a**2*b**4*c**2*d*g**4*x**2
- 9*a**2*b**4*c*d**2*g**4*x**3 - 9*a*b**5*c**3*g**4*x**2 + 9*a*b**5*c**2*d*
g**4*x**3 - 3*b**6*c**3*g**4*x**3) + (-6*A*B*a**2*d**2 + 12*A*B*a*b*c*d - 6
*A*B*b**2*c**2 - 11*B**2*a**2*d**2 + 7*B**2*a*b*c*d - 15*B**2*a*b*d**2*x -
2*B**2*b**2*c**2 + 3*B**2*b**2*c*d*x - 6*B**2*b**2*d**2*x**2)*log(e*(a + b*
x)/(c + d*x))/(9*a**5*b*d**2*g**4 - 18*a**4*b**2*c*d*g**4 + 27*a**4*b**2*d*
*2*g**4*x + 9*a**3*b**3*c**2*g**4 - 54*a**3*b**3*c*d*g**4*x + 27*a**3*b**3*
```

```
d**2*g**4*x**2 + 27*a**2*b**4*c**2*g**4*x - 54*a**2*b**4*c*d*g**4*x**2 + 9*
a**2*b**4*d**2*g**4*x**3 + 27*a*b**5*c**2*g**4*x**2 - 18*a*b**5*c*d*g**4*x*
*3 + 9*b**6*c**2*g**4*x**3) - (18*A**2*a**2*d**2 - 36*A**2*a*b*c*d + 18*A**
2*b**2*c**2 + 66*A*B*a**2*d**2 - 42*A*B*a*b*c*d + 12*A*B*b**2*c**2 + 85*B**
2*a**2*d**2 - 23*B**2*a*b*c*d + 4*B**2*b**2*c**2 + x**2*(36*A*B*b**2*d**2 +
 66*B**2*b**2*d**2) + x*(90*A*B*a*b*d**2 - 18*A*B*b**2*c*d + 147*B**2*a*b*d
**2 - 15*B**2*b**2*c*d))/(54*a**5*b*d**2*g**4 - 108*a**4*b**2*c*d*g**4 + 54
*a**3*b**3*c**2*g**4 + x**3*(54*a**2*b**4*d**2*g**4 - 108*a*b**5*c*d*g**4 +
 54*b**6*c**2*g**4) + x**2*(162*a**3*b**3*d**2*g**4 - 324*a**2*b**4*c*d*g**
4 + 162*a*b**5*c**2*g**4) + x*(162*a**4*b**2*d**2*g**4 - 324*a**3*b**3*c*d*
g**4 + 162*a**2*b**4*c**2*g**4))
```

Giac [A]
time = 4.06, size = 709, normalized size = 1.70

$$-\frac{\left(18\,B^2b^2e^4\log\left(\frac{bxe+ae}{dx+c}\right)^2-\frac{54\,(bxe+ae)B^2bde^3\log\left(\frac{bxe+ae}{dx+c}\right)^2}{dx+c}+\frac{54\,(bxe+ae)^2B^2d^2e^2\log\left(\frac{bxe+ae}{dx+c}\right)^2}{(dx+c)^2}+36\,ABb^2e^4\log\left(\frac{bxe+ae}{dx+c}\right)+12\,B^2b^2e^4\log\left(\frac{bxe+ae}{dx+c}\right)-\frac{108\,(bxe+ae)ABbde^3\log\left(\frac{bxe+ae}{dx+c}\right)}{dx+c}-\frac{54\,(bxe+ae)B^2bde^3\log\left(\frac{bxe+ae}{dx+c}\right)}{dx+c}+\frac{108\,(bxe+ae)^2ABd^2e^2\log\left(\frac{bxe+ae}{dx+c}\right)}{(dx+c)^2}+\frac{108\,(bxe+ae)^2B^2d^2e^2\log\left(\frac{bxe+ae}{dx+c}\right)}{(dx+c)^2}+18\,A^2b^2e^4+12\,ABb^2e^4+4\,B^2b^2e^4-\frac{54\,(bxe+ae)A^2bde^3}{dx+c}-\frac{54\,(bxe+ae)ABbde^3}{dx+c}-\frac{27\,(bxe+ae)B^2bde^3}{dx+c}+\frac{54\,(bxe+ae)^2A^2d^2e^2}{(dx+c)^2}+\frac{108\,(bxe+ae)^2ABd^2e^2}{(dx+c)^2}+\frac{108\,(bxe+ae)^2B^2d^2e^2}{(dx+c)^2}\right)\left(\frac{bc}{(bce-ade)(bc-ad)}-\frac{ad}{(bce-ade)(bc-ad)}\right)}{54\left(\frac{(bxe+ae)^3b^2c^2g^4}{(dx+c)^3}-\frac{2\,(bxe+ae)^3abcdg^4}{(dx+c)^3}+\frac{(bxe+ae)^3a^2d^2g^4}{(dx+c)^3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^4,x, algorithm="giac")
```

```
[Out] -1/54*(18*B^2*b^2*e^4*log((b*x*e + a*e)/(d*x + c))^2 - 54*(b*x*e + a*e)*B^2
*b*d*e^3*log((b*x*e + a*e)/(d*x + c))^2/(d*x + c) + 54*(b*x*e + a*e)^2*B^2*
d^2*e^2*log((b*x*e + a*e)/(d*x + c))^2/(d*x + c)^2 + 36*A*B*b^2*e^4*log((b*
x*e + a*e)/(d*x + c)) + 12*B^2*b^2*e^4*log((b*x*e + a*e)/(d*x + c)) - 108*(
b*x*e + a*e)*A*B*b*d*e^3*log((b*x*e + a*e)/(d*x + c))/(d*x + c) - 54*(b*x*e
 + a*e)*B^2*b*d*e^3*log((b*x*e + a*e)/(d*x + c))/(d*x + c) + 108*(b*x*e + a
*e)^2*A*B*d^2*e^2*log((b*x*e + a*e)/(d*x + c))/(d*x + c)^2 + 108*(b*x*e + a
*e)^2*B^2*d^2*e^2*log((b*x*e + a*e)/(d*x + c))/(d*x + c)^2 + 18*A^2*b^2*e^4
 + 12*A*B*b^2*e^4 + 4*B^2*b^2*e^4 - 54*(b*x*e + a*e)*A^2*b*d*e^3/(d*x + c)
- 54*(b*x*e + a*e)*A*B*b*d*e^3/(d*x + c) - 27*(b*x*e + a*e)*B^2*b*d*e^3/(d*
x + c) + 54*(b*x*e + a*e)^2*A^2*d^2*e^2/(d*x + c)^2 + 108*(b*x*e + a*e)^2*A
*B*d^2*e^2/(d*x + c)^2 + 108*(b*x*e + a*e)^2*B^2*d^2*e^2/(d*x + c)^2)*(b*c/
((b*c*e - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))/((b*x*e
+ a*e)^3*b^2*c^2*g^4/(d*x + c)^3 - 2*(b*x*e + a*e)^3*a*b*c*d*g^4/(d*x + c)^
3 + (b*x*e + a*e)^3*a^2*d^2*g^4/(d*x + c)^3)
```

Mupad [B]
time = 7.41, size = 1064, normalized size = 2.55

$$\frac{\frac{18\,A^2a^2d^2-36\,A^2abcd+18\,A^2b^2c^2+66\,ABa^2d^2-42\,ABabcd+12\,ABb^2c^2+85\,B^2a^2d^2-23\,B^2abcd+4\,B^2b^2c^2}{6\,(ad-bc)}+\frac{x\,(-5\,cB^2b^2d+49\,aB^2bd^2-6\,AcBb^2d+30\,AaBbd^2)}{2\,(ad-bc)}+\frac{dx^2\,(11\,dB^2b^2+6\,AdBb^2)}{ad-bc}}{x\,(27\,a^2b^3cg^4-27\,a^3b^2dg^4)-x^2\,(27\,a^2b^3dg^4-27\,ab^4cg^4)+x^3\,(9\,b^5cg^4-9\,ab^4dg^4)+9\,a^3b^2cg^4-9\,a^4bdg^4}-\ln\left(\frac{e\,(a+bx)}{c+dx}\right)^2\left(\frac{B^2}{3\,b^2g^4\,\left(3\,a^2x+\frac{a^3}{b}+b^2x^3+3\,abx^2\right)}-\frac{B^2d^3}{3\,bg^4\,(a^3d^3-3\,a^2bcd^2+3\,ab^2c^2d-b^3c^3)}\right)-\frac{\ln\left(\frac{e\,(a+bx)}{c+dx}\right)\left(\frac{2\,AB}{3\,b^2dg^4}+\frac{2\,B^2d^3\left(a\left(\frac{3\,a^2d^2-4\,abcd+b^2c^2}{6\,bd^3}+\frac{a\,(ad-bc)}{3\,bd^2}\right)+\frac{3\,a^3d^3-6\,a^2bcd^2+4\,ab^2c^2d-b^3c^3}{3\,bd^4}\right)}{3\,bg^4\,(a^3d^3-3\,a^2bcd^2+3\,ab^2c^2d-b^3c^3)}-\frac{2\,B^2d^3x^2\left(\frac{b^2c-abd}{3\,d^2}-\frac{2\,b\,(ad-bc)}{3\,d^2}\right)}{3\,bg^4\,(a^3d^3-3\,a^2bcd^2+3\,ab^2c^2d-b^3c^3)}+\frac{2\,B^2d^3x\left(b\left(\frac{3\,a^2d^2-4\,abcd+b^2c^2}{6\,bd^3}+\frac{a\,(ad-bc)}{3\,bd^2}\right)+\frac{3\,a^2d^2-4\,abcd+b^2c^2}{3\,d^3}+\frac{2\,a\,(ad-bc)}{3\,d^2}\right)}{3\,bg^4\,(a^3d^3-3\,a^2bcd^2+3\,ab^2c^2d-b^3c^3)}\right)}{\frac{3\,a^2x}{d}+\frac{a^3}{bd}+\frac{b^2x^3}{d}+\frac{3\,abx^2}{d}}-\frac{B\,d^3\,\mathrm{atan}\left(\frac{B\,d^3\left(\frac{a^3bd^2g^4-a^2b^2cd^2g^4-ab^3c^2dg^4+b^4c^3g^4}{a^2bd^2g^4-2\,ab^2cdg^4+b^3c^2g^4}+2\,bdx\right)(6\,A+11\,B)\,(a^2bd^2g^4-2\,ab^2cdg^4+b^3c^2g^4)\,1{}\mathrm{i}}{bg^4\,(ad-bc)^3\,(11\,B^2d^3+6\,ABd^3)}\right)(6\,A+11\,B)\,2{}\mathrm{i}}{9\,bg^4\,(ad-bc)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x))/(c + d*x)))^2/(a*g + b*g*x)^4,x)
```

```
[Out] ((18*A^2*a^2*d^2 + 18*A^2*b^2*c^2 + 85*B^2*a^2*d^2 + 4*B^2*b^2*c^2 + 66*A*B
*a^2*d^2 + 12*A*B*b^2*c^2 - 36*A^2*a*b*c*d - 23*B^2*a*b*c*d - 42*A*B*a*b*c*
```

```
d)/(6*(a*d - b*c)) + (x*(49*B^2*a*b*d^2 - 5*B^2*b^2*c*d + 30*A*B*a*b*d^2 -
6*A*B*b^2*c*d))/(2*(a*d - b*c)) + (d*x^2*(11*B^2*b^2*d + 6*A*B*b^2*d))/(a*d
 - b*c))/(x*(27*a^2*b^3*c*g^4 - 27*a^3*b^2*d*g^4) - x^2*(27*a^2*b^3*d*g^4 -
 27*a*b^4*c*g^4) + x^3*(9*b^5*c*g^4 - 9*a*b^4*d*g^4) + 9*a^3*b^2*c*g^4 - 9*
a^4*b*d*g^4) - log((e*(a + b*x))/(c + d*x))^2*(B^2/(3*b^2*g^4*(3*a^2*x + a^
3/b + b^2*x^3 + 3*a*b*x^2)) - (B^2*d^3)/(3*b*g^4*(a^3*d^3 - b^3*c^3 + 3*a*b
^2*c^2*d - 3*a^2*b*c*d^2))) - (log((e*(a + b*x))/(c + d*x))*((2*A*B)/(3*b^2
*d*g^4) + (2*B^2*d^3*(a*((3*a^2*d^2 + b^2*c^2 - 4*a*b*c*d)/(6*b*d^3) + (a*(
a*d - b*c))/(3*b*d^2)) + (3*a^3*d^3 - b^3*c^3 + 4*a*b^2*c^2*d - 6*a^2*b*c*d
^2)/(3*b*d^4)))/(3*b*g^4*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2
)) - (2*B^2*d^3*x^2*((b^2*c - a*b*d)/(3*d^2) - (2*b*(a*d - b*c))/(3*d^2)))/
(3*b*g^4*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) + (2*B^2*d^3*
x*(b*((3*a^2*d^2 + b^2*c^2 - 4*a*b*c*d)/(6*b*d^3) + (a*(a*d - b*c))/(3*b*d^
2)) + (3*a^2*d^2 + b^2*c^2 - 4*a*b*c*d)/(3*d^3) + (2*a*(a*d - b*c))/(3*d^2)
))/(3*b*g^4*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))))/((3*a^2*
x)/d + a^3/(b*d) + (b^2*x^3)/d + (3*a*b*x^2)/d) - (B*d^3*atan((B*d^3*((b^4*
c^3*g^4 + a^3*b*d^3*g^4 - a*b^3*c^2*d*g^4 - a^2*b^2*c*d^2*g^4)/(b^3*c^2*g^4
 + a^2*b*d^2*g^4 - 2*a*b^2*c*d*g^4) + 2*b*d*x)*(6*A + 11*B)*(b^3*c^2*g^4 +
a^2*b*d^2*g^4 - 2*a*b^2*c*d*g^4)*1i)/(b*g^4*(a*d - b*c)^3*(11*B^2*d^3 + 6*A
*B*d^3)))*(6*A + 11*B)*2i)/(9*b*g^4*(a*d - b*c)^3)
```

3.105 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^5}\,dx$

Optimal. Leaf size=575

$$\frac{2B^2d^3(c+dx)}{(bc-ad)^4g^5(a+bx)}-\frac{3bB^2d^2(c+dx)^2}{4(bc-ad)^4g^5(a+bx)^2}+\frac{2b^2B^2d(c+dx)^3}{9(bc-ad)^4g^5(a+bx)^3}-\frac{b^3B^2(c+dx)^4}{32(bc-ad)^4g^5(a+bx)^4}+\frac{2Bd^3(c+\cdots}{(\cdots}$$

```
[Out] 2*B^2*d^3*(d*x+c)/(-a*d+b*c)^4/g^5/(b*x+a)-3/4*b*B^2*d^2*(d*x+c)^2/(-a*d+b*
c)^4/g^5/(b*x+a)^2+2/9*b^2*B^2*d*(d*x+c)^3/(-a*d+b*c)^4/g^5/(b*x+a)^3-1/32*
b^3*B^2*(d*x+c)^4/(-a*d+b*c)^4/g^5/(b*x+a)^4+2*B*d^3*(d*x+c)*(A+B*ln(e*(b*x
+a)/(d*x+c)))/(-a*d+b*c)^4/g^5/(b*x+a)-3/2*b*B*d^2*(d*x+c)^2*(A+B*ln(e*(b*x
+a)/(d*x+c)))/(-a*d+b*c)^4/g^5/(b*x+a)^2+2/3*b^2*B*d*(d*x+c)^3*(A+B*ln(e*(b
*x+a)/(d*x+c)))/(-a*d+b*c)^4/g^5/(b*x+a)^3-1/8*b^3*B*(d*x+c)^4*(A+B*ln(e*(b
*x+a)/(d*x+c)))/(-a*d+b*c)^4/g^5/(b*x+a)^4+d^3*(d*x+c)*(A+B*ln(e*(b*x+a)/(d
*x+c)))^2/(-a*d+b*c)^4/g^5/(b*x+a)-3/2*b*d^2*(d*x+c)^2*(A+B*ln(e*(b*x+a)/(d
*x+c)))^2/(-a*d+b*c)^4/g^5/(b*x+a)^2+b^2*d*(d*x+c)^3*(A+B*ln(e*(b*x+a)/(d*x
+c)))^2/(-a*d+b*c)^4/g^5/(b*x+a)^3-1/4*b^3*(d*x+c)^4*(A+B*ln(e*(b*x+a)/(d*x
+c)))^2/(-a*d+b*c)^4/g^5/(b*x+a)^4
```

Rubi [A]
time = 0.27, antiderivative size = 575, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2550, 2395, 2342, 2341}

$$-\frac{b^3(c+dx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{4g^5(a+bx)^4(bc-ad)^4}-\frac{b^3B(c+dx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{8g^5(a+bx)^4(bc-ad)^4}+\frac{b^2d(c+dx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{g^5(a+bx)^3(bc-ad)^4}+\frac{2b^2Bd(c+dx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3g^5(a+bx)^3(bc-ad)^4}+\frac{d^3(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{g^5(a+bx)(bc-ad)^4}+\frac{2Bd^3(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{g^5(a+bx)(bc-ad)^4}-\frac{3bd^2(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{2g^5(a+bx)^2(bc-ad)^4}-\frac{3bBd^2(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2g^5(a+bx)^2(bc-ad)^4}-\frac{b^3B^2(c+dx)^4}{32g^5(a+bx)^4(bc-ad)^4}+\frac{2b^2B^2d(c+dx)^3}{9g^5(a+bx)^3(bc-ad)^4}+\frac{2B^2d^3(c+dx)}{g^5(a+bx)(bc-ad)^4}-\frac{3bB^2d^2(c+dx)^2}{4g^5(a+bx)^2(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(a*g + b*g*x)^5,x]
```

```
[Out] (2*B^2*d^3*(c + d*x))/((b*c - a*d)^4*g^5*(a + b*x)) - (3*b*B^2*d^2*(c + d*x
)^2)/(4*(b*c - a*d)^4*g^5*(a + b*x)^2) + (2*b^2*B^2*d*(c + d*x)^3)/(9*(b*c
- a*d)^4*g^5*(a + b*x)^3) - (b^3*B^2*(c + d*x)^4)/(32*(b*c - a*d)^4*g^5*(a
+ b*x)^4) + (2*B*d^3*(c + d*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/((b*c
- a*d)^4*g^5*(a + b*x)) - (3*b*B*d^2*(c + d*x)^2*(A + B*Log[(e*(a + b*x))/(
c + d*x)]))/(2*(b*c - a*d)^4*g^5*(a + b*x)^2) + (2*b^2*B*d*(c + d*x)^3*(A +
 B*Log[(e*(a + b*x))/(c + d*x)]))/(3*(b*c - a*d)^4*g^5*(a + b*x)^3) - (b^3*
B*(c + d*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(8*(b*c - a*d)^4*g^5*(a
 + b*x)^4) + (d^3*(c + d*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/((b*c -
 a*d)^4*g^5*(a + b*x)) - (3*b*d^2*(c + d*x)^2*(A + B*Log[(e*(a + b*x))/(c +
 d*x)])^2)/(2*(b*c - a*d)^4*g^5*(a + b*x)^2) + (b^2*d*(c + d*x)^3*(A + B*Lo
g[(e*(a + b*x))/(c + d*x)])^2)/((b*c - a*d)^4*g^5*(a + b*x)^3) - (b^3*(c +
d*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(4*(b*c - a*d)^4*g^5*(a + b*
x)^4)
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2550

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(
m + 1)*(g/b)^m, Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x],
 x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] &&
 EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && Eq
Q[b*f - a*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Rubi steps

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^5}\,dx = -\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{4bg^5(a+bx)^4} + \frac{B\int \frac{(bc-ad)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{g^4(a+bx)^5(c+dx)}\,dx}{2bg}$$

$$= -\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{4bg^5(a+bx)^4} + \frac{(B(bc-ad))\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(a+bx)^5(c+dx)}\,dx}{2bg^5}$$

$$= -\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{4bg^5(a+bx)^4} + \frac{(B(bc-ad))\int \left(\frac{b\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)(a+bx)^5} - \frac{bd\left(A+B\log\right.}{(bc-ad)}\right.}{}$$

$$= -\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{4bg^5(a+bx)^4} + \frac{B\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(a+bx)^5}\,dx}{2g^5} + \frac{(Bd^4)\int \frac{A+B\log\left(\frac{e(a+bx}{c+dx}\right.}{a+bx}}{2(bc-ad)^4g^5}$$

$$= -\frac{B\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{8bg^5(a+bx)^4} + \frac{Bd\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{6b(bc-ad)g^5(a+bx)^3} - \frac{Bd^2\left(A+B\log\right.}{4b(bc-ad)^2g^5}$$

$$= -\frac{B\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{8bg^5(a+bx)^4} + \frac{Bd\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{6b(bc-ad)g^5(a+bx)^3} - \frac{Bd^2\left(A+B\log\right.}{4b(bc-ad)^2g^5}$$

$$= -\frac{B\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{8bg^5(a+bx)^4} + \frac{Bd\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{6b(bc-ad)g^5(a+bx)^3} - \frac{Bd^2\left(A+B\log\right.}{4b(bc-ad)^2g^5}$$

$$= -\frac{B^2}{32bg^5(a+bx)^4} + \frac{7B^2d}{72b(bc-ad)g^5(a+bx)^3} - \frac{13B^2d^2}{48b(bc-ad)^2g^5(a+bx)^2} +$$

$$= -\frac{B^2}{32bg^5(a+bx)^4} + \frac{7B^2d}{72b(bc-ad)g^5(a+bx)^3} - \frac{13B^2d^2}{48b(bc-ad)^2g^5(a+bx)^2} +$$

$$= -\frac{B^2}{32bg^5(a+bx)^4} + \frac{7B^2d}{72b(bc-ad)g^5(a+bx)^3} - \frac{13B^2d^2}{48b(bc-ad)^2g^5(a+bx)^2} +$$

$$= -\frac{B^2}{32bg^5(a+bx)^4} + \frac{7B^2d}{72b(bc-ad)g^5(a+bx)^3} - \frac{13B^2d^2}{48b(bc-ad)^2g^5(a+bx)^2} +$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 0.63, size = 748, normalized size = 1.30

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(a*g + b*g*x)^5,x]
```

```
[Out] -1/288*(72*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 + (B*(36*(b*c - a*d)^4*(A
 + B*Log[(e*(a + b*x))/(c + d*x)]) + 48*d*(-(b*c) + a*d)^3*(a + b*x)*(A + B
*Log[(e*(a + b*x))/(c + d*x)]) + 72*d^2*(b*c - a*d)^2*(a + b*x)^2*(A + B*Lo
g[(e*(a + b*x))/(c + d*x)]) + 144*d^3*(-(b*c) + a*d)*(a + b*x)^3*(A + B*Log
[(e*(a + b*x))/(c + d*x)]) - 144*d^4*(a + b*x)^4*Log[a + b*x]*(A + B*Log[(e
*(a + b*x))/(c + d*x)]) + 144*d^4*(a + b*x)^4*(A + B*Log[(e*(a + b*x))/(c +
 d*x)])*Log[c + d*x] - 144*B*d^3*(a + b*x)^3*(b*c - a*d + d*(a + b*x)*Log[a
 + b*x] - d*(a + b*x)*Log[c + d*x]) + 36*B*d^2*(a + b*x)^2*((b*c - a*d)^2 +
 2*d*(-(b*c) + a*d)*(a + b*x) - 2*d^2*(a + b*x)^2*Log[a + b*x] + 2*d^2*(a +
 b*x)^2*Log[c + d*x]) - 8*B*d*(a + b*x)*(2*(b*c - a*d)^3 - 3*d*(b*c - a*d)^
2*(a + b*x) + 6*d^2*(b*c - a*d)*(a + b*x)^2 + 6*d^3*(a + b*x)^3*Log[a + b*x
] - 6*d^3*(a + b*x)^3*Log[c + d*x]) + 3*B*(3*(b*c - a*d)^4 + 4*d*(-(b*c) +
a*d)^3*(a + b*x) + 6*d^2*(b*c - a*d)^2*(a + b*x)^2 + 12*d^3*(-(b*c) + a*d)*
(a + b*x)^3 - 12*d^4*(a + b*x)^4*Log[a + b*x] + 12*d^4*(a + b*x)^4*Log[c +
d*x]) + 72*B*d^4*(a + b*x)^4*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*
x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) - 72*B*d^4*
(a + b*x)^4*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d
*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(b*c - a*d)^4)/(b*g^5*(a +
 b*x)^4)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1392 vs. $2(559) = 1118$.
time = 0.44, size = 1393, normalized size = 2.42

method	result	size
derivativedivides	Expression too large to display	1393
default	Expression too large to display	1393
norman	Expression too large to display	1796
risch	Expression too large to display	3080

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/d^2*e*(a*d-b*c)*(1/4*d^2/(a*d-b*c)^5/g^5*A^2*b^3*e^3/(b*e/d+(a*d-b*c)*e/
d/(d*x+c))^4-d^3/(a*d-b*c)^5/g^5*A^2*b^2*e^2/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^
3+3/2*d^4/(a*d-b*c)^5/g^5*A^2*b*e/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2-d^5/(a*d-
b*c)^5/g^5*A^2/(b*e/d+(a*d-b*c)*e/d/(d*x+c))-2*d^2/(a*d-b*c)^5/g^5*A*B*b^3*
e^3*(-1/4/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^4*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1
/16/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^4)+6*d^3/(a*d-b*c)^5/g^5*A*B*b^2*e^2*(-1/
3/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/9/(b*e/
d+(a*d-b*c)*e/d/(d*x+c))^3)-6*d^4/(a*d-b*c)^5/g^5*A*B*b*e*(-1/2/(b*e/d+(a*d
-b*c)*e/d/(d*x+c))^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/4/(b*e/d+(a*d-b*c)*e
/d/(d*x+c))^2)+2*d^5/(a*d-b*c)^5/g^5*A*B*(-1/(b*e/d+(a*d-b*c)*e/d/(d*x+c))*
```

```
ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/(b*e/d+(a*d-b*c)*e/d/(d*x+c)))-d^2/(a*d-b
*c)^5/g^5*B^2*b^3*e^3*(-1/4/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^4*ln(b*e/d+(a*d-b
*c)*e/d/(d*x+c))^2-1/8/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^4*ln(b*e/d+(a*d-b*c)*e
/d/(d*x+c))-1/32/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^4)+3*d^3/(a*d-b*c)^5/g^5*B^2
*b^2*e^2*(-1/3/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3*ln(b*e/d+(a*d-b*c)*e/d/(d*x+
c))^2-2/9/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-2
/27/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3)-3*d^4/(a*d-b*c)^5/g^5*B^2*b*e*(-1/2/(b
*e/d+(a*d-b*c)*e/d/(d*x+c))^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2-1/2/(b*e/d+
(a*d-b*c)*e/d/(d*x+c))^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/4/(b*e/d+(a*d-b*
c)*e/d/(d*x+c))^2)+d^5/(a*d-b*c)^5/g^5*B^2*(-1/(b*e/d+(a*d-b*c)*e/d/(d*x+c)
)*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2-2/(b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/
d+(a*d-b*c)*e/d/(d*x+c))-2/(b*e/d+(a*d-b*c)*e/d/(d*x+c))))
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2129 vs. $2(567) = 1134$.
time = 0.51, size = 2129, normalized size = 3.70

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^5,x, algorithm="maxima
")
```

```
[Out] 1/288*(12*((12*b^3*d^3*x^3 - 3*b^3*c^3 + 13*a*b^2*c^2*d - 23*a^2*b*c*d^2 +
25*a^3*d^3 - 6*(b^3*c*d^2 - 7*a*b^2*d^3)*x^2 + 4*(b^3*c^2*d - 5*a*b^2*c*d^2
 + 13*a^2*b*d^3)*x)/((b^8*c^3 - 3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 - a^3*b^5*d
^3)*g^5*x^4 + 4*(a*b^7*c^3 - 3*a^2*b^6*c^2*d + 3*a^3*b^5*c*d^2 - a^4*b^4*d^
3)*g^5*x^3 + 6*(a^2*b^6*c^3 - 3*a^3*b^5*c^2*d + 3*a^4*b^4*c*d^2 - a^5*b^3*d
^3)*g^5*x^2 + 4*(a^3*b^5*c^3 - 3*a^4*b^4*c^2*d + 3*a^5*b^3*c*d^2 - a^6*b^2*
d^3)*g^5*x + (a^4*b^4*c^3 - 3*a^5*b^3*c^2*d + 3*a^6*b^2*c*d^2 - a^7*b*d^3)*
g^5) + 12*d^4*log(b*x + a)/((b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 -
4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5) - 12*d^4*log(d*x + c)/((b^5*c^4 - 4*a*b^4
*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5))*log(b*x*e/(
d*x + c) + a*e/(d*x + c)) - (9*b^4*c^4 - 64*a*b^3*c^3*d + 216*a^2*b^2*c^2*d
^2 - 576*a^3*b*c*d^3 + 415*a^4*d^4 - 300*(b^4*c*d^3 - a*b^3*d^4)*x^3 + 6*(1
3*b^4*c^2*d^2 - 176*a*b^3*c*d^3 + 163*a^2*b^2*d^4)*x^2 + 72*(b^4*d^4*x^4 +
4*a*b^3*d^4*x^3 + 6*a^2*b^2*d^4*x^2 + 4*a^3*b*d^4*x + a^4*d^4)*log(b*x + a)
^2 + 72*(b^4*d^4*x^4 + 4*a*b^3*d^4*x^3 + 6*a^2*b^2*d^4*x^2 + 4*a^3*b*d^4*x
+ a^4*d^4)*log(d*x + c)^2 - 4*(7*b^4*c^3*d - 60*a*b^3*c^2*d^2 + 324*a^2*b^2
*c*d^3 - 271*a^3*b*d^4)*x - 300*(b^4*d^4*x^4 + 4*a*b^3*d^4*x^3 + 6*a^2*b^2*
d^4*x^2 + 4*a^3*b*d^4*x + a^4*d^4)*log(b*x + a) + 12*(25*b^4*d^4*x^4 + 100*
a*b^3*d^4*x^3 + 150*a^2*b^2*d^4*x^2 + 100*a^3*b*d^4*x + 25*a^4*d^4 - 12*(b^
4*d^4*x^4 + 4*a*b^3*d^4*x^3 + 6*a^2*b^2*d^4*x^2 + 4*a^3*b*d^4*x + a^4*d^4)*
log(b*x + a))*log(d*x + c))/(a^4*b^5*c^4*g^5 - 4*a^5*b^4*c^3*d*g^5 + 6*a^6*
b^3*c^2*d^2*g^5 - 4*a^7*b^2*c*d^3*g^5 + a^8*b*d^4*g^5 + (b^9*c^4*g^5 - 4*a*
```

```
b^8*c^3*d*g^5 + 6*a^2*b^7*c^2*d^2*g^5 - 4*a^3*b^6*c*d^3*g^5 + a^4*b^5*d^4*g
^5)*x^4 + 4*(a*b^8*c^4*g^5 - 4*a^2*b^7*c^3*d*g^5 + 6*a^3*b^6*c^2*d^2*g^5 -
4*a^4*b^5*c*d^3*g^5 + a^5*b^4*d^4*g^5)*x^3 + 6*(a^2*b^7*c^4*g^5 - 4*a^3*b^6
*c^3*d*g^5 + 6*a^4*b^5*c^2*d^2*g^5 - 4*a^5*b^4*c*d^3*g^5 + a^6*b^3*d^4*g^5)
*x^2 + 4*(a^3*b^6*c^4*g^5 - 4*a^4*b^5*c^3*d*g^5 + 6*a^5*b^4*c^2*d^2*g^5 - 4
*a^6*b^3*c*d^3*g^5 + a^7*b^2*d^4*g^5)*x))*B^2 + 1/24*A*B*((12*b^3*d^3*x^3 -
 3*b^3*c^3 + 13*a*b^2*c^2*d - 23*a^2*b*c*d^2 + 25*a^3*d^3 - 6*(b^3*c*d^2 -
7*a*b^2*d^3)*x^2 + 4*(b^3*c^2*d - 5*a*b^2*c*d^2 + 13*a^2*b*d^3)*x)/((b^8*c^
3 - 3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 - a^3*b^5*d^3)*g^5*x^4 + 4*(a*b^7*c^3 -
 3*a^2*b^6*c^2*d + 3*a^3*b^5*c*d^2 - a^4*b^4*d^3)*g^5*x^3 + 6*(a^2*b^6*c^3
- 3*a^3*b^5*c^2*d + 3*a^4*b^4*c*d^2 - a^5*b^3*d^3)*g^5*x^2 + 4*(a^3*b^5*c^3
 - 3*a^4*b^4*c^2*d + 3*a^5*b^3*c*d^2 - a^6*b^2*d^3)*g^5*x + (a^4*b^4*c^3 -
3*a^5*b^3*c^2*d + 3*a^6*b^2*c*d^2 - a^7*b*d^3)*g^5) - 12*log(b*x*e/(d*x + c
) + a*e/(d*x + c))/(b^5*g^5*x^4 + 4*a*b^4*g^5*x^3 + 6*a^2*b^3*g^5*x^2 + 4*a
^3*b^2*g^5*x + a^4*b*g^5) + 12*d^4*log(b*x + a)/((b^5*c^4 - 4*a*b^4*c^3*d +
 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5) - 12*d^4*log(d*x + c
)/((b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d
^4)*g^5)) - 1/4*B^2*log(b*x*e/(d*x + c) + a*e/(d*x + c))^2/(b^5*g^5*x^4 + 4
*a*b^4*g^5*x^3 + 6*a^2*b^3*g^5*x^2 + 4*a^3*b^2*g^5*x + a^4*b*g^5) - 1/4*A^2
/(b^5*g^5*x^4 + 4*a*b^4*g^5*x^3 + 6*a^2*b^3*g^5*x^2 + 4*a^3*b^2*g^5*x + a^4
*b*g^5)
```

Fricas [A]
time = 0.37, size = 1033, normalized size = 1.80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^5,x, algorithm="fricas
")
```

```
[Out] -1/288*(9*(8*A^2 + 4*A*B + B^2)*b^4*c^4 - 32*(9*A^2 + 6*A*B + 2*B^2)*a*b^3*
c^3*d + 216*(2*A^2 + 2*A*B + B^2)*a^2*b^2*c^2*d^2 - 288*(A^2 + 2*A*B + 2*B^
2)*a^3*b*c*d^3 + (72*A^2 + 300*A*B + 415*B^2)*a^4*d^4 - 12*((12*A*B + 25*B^
2)*b^4*c*d^3 - (12*A*B + 25*B^2)*a*b^3*d^4)*x^3 + 6*((12*A*B + 13*B^2)*b^4*
c^2*d^2 - 16*(6*A*B + 11*B^2)*a*b^3*c*d^3 + (84*A*B + 163*B^2)*a^2*b^2*d^4)
*x^2 - 72*(B^2*b^4*d^4*x^4 + 4*B^2*a*b^3*d^4*x^3 + 6*B^2*a^2*b^2*d^4*x^2 +
4*B^2*a^3*b*d^4*x - B^2*b^4*c^4 + 4*B^2*a*b^3*c^3*d - 6*B^2*a^2*b^2*c^2*d^2
 + 4*B^2*a^3*b*c*d^3)*log((b*x + a)*e/(d*x + c))^2 - 4*((12*A*B + 7*B^2)*b^
4*c^3*d - 12*(6*A*B + 5*B^2)*a*b^3*c^2*d^2 + 108*(2*A*B + 3*B^2)*a^2*b^2*c*
d^3 - (156*A*B + 271*B^2)*a^3*b*d^4)*x - 12*((12*A*B + 25*B^2)*b^4*d^4*x^4
- 3*(4*A*B + B^2)*b^4*c^4 + 16*(3*A*B + B^2)*a*b^3*c^3*d - 36*(2*A*B + B^2)
*a^2*b^2*c^2*d^2 + 48*(A*B + B^2)*a^3*b*c*d^3 + 4*(3*B^2*b^4*c*d^3 + 2*(6*A
*B + 11*B^2)*a*b^3*d^4)*x^3 - 6*(B^2*b^4*c^2*d^2 - 8*B^2*a*b^3*c*d^3 - 6*(2
*A*B + 3*B^2)*a^2*b^2*d^4)*x^2 + 4*(B^2*b^4*c^3*d - 6*B^2*a*b^3*c^2*d^2 + 1
```

```
8*B^2*a^2*b^2*c*d^3 + 12*(A*B + B^2)*a^3*b*d^4)*x)*log((b*x + a)*e/(d*x + c
)))/((b^9*c^4 - 4*a*b^8*c^3*d + 6*a^2*b^7*c^2*d^2 - 4*a^3*b^6*c*d^3 + a^4*b
^5*d^4)*g^5*x^4 + 4*(a*b^8*c^4 - 4*a^2*b^7*c^3*d + 6*a^3*b^6*c^2*d^2 - 4*a^
4*b^5*c*d^3 + a^5*b^4*d^4)*g^5*x^3 + 6*(a^2*b^7*c^4 - 4*a^3*b^6*c^3*d + 6*a
^4*b^5*c^2*d^2 - 4*a^5*b^4*c*d^3 + a^6*b^3*d^4)*g^5*x^2 + 4*(a^3*b^6*c^4 -
4*a^4*b^5*c^3*d + 6*a^5*b^4*c^2*d^2 - 4*a^6*b^3*c*d^3 + a^7*b^2*d^4)*g^5*x
+ (a^4*b^5*c^4 - 4*a^5*b^4*c^3*d + 6*a^6*b^3*c^2*d^2 - 4*a^7*b^2*c*d^3 + a^
8*b*d^4)*g^5)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(b*x+a)/(d*x+c)))**2/(b*g*x+a*g)**5,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 3.64, size = 995, normalized size = 1.73

$$-\frac{\left(72\,B^2b^3e^5\log\left(\frac{bxe+ae}{dx+c}\right)^2-\frac{288\,(bxe+ae)B^2b^2de^4\log\left(\frac{bxe+ae}{dx+c}\right)^2}{dx+c}+\frac{432\,(bxe+ae)^2B^2bd^2e^3\log\left(\frac{bxe+ae}{dx+c}\right)^2}{(dx+c)^2}-\frac{288\,(bxe+ae)^3B^2d^3e^2\log\left(\frac{bxe+ae}{dx+c}\right)^2}{(dx+c)^3}+144\,ABb^3e^5\log\left(\frac{bxe+ae}{dx+c}\right)+36\,B^2b^3e^5\log\left(\frac{bxe+ae}{dx+c}\right)-\frac{576\,(bxe+ae)ABb^2de^4\log\left(\frac{bxe+ae}{dx+c}\right)}{dx+c}-\frac{192\,(bxe+ae)B^2b^2de^4\log\left(\frac{bxe+ae}{dx+c}\right)}{dx+c}+\frac{864\,(bxe+ae)^2ABbd^2e^3\log\left(\frac{bxe+ae}{dx+c}\right)}{(dx+c)^2}+\frac{432\,(bxe+ae)^2B^2bd^2e^3\log\left(\frac{bxe+ae}{dx+c}\right)}{(dx+c)^2}-\frac{576\,(bxe+ae)^3ABd^3e^2\log\left(\frac{bxe+ae}{dx+c}\right)}{(dx+c)^3}-\frac{576\,(bxe+ae)^3B^2d^3e^2\log\left(\frac{bxe+ae}{dx+c}\right)}{(dx+c)^3}+72\,A^2b^3e^5+36\,ABb^3e^5+9\,B^2b^3e^5-\frac{288\,(bxe+ae)A^2b^2de^4}{dx+c}-\frac{192\,(bxe+ae)ABb^2de^4}{dx+c}-\frac{64\,(bxe+ae)B^2b^2de^4}{dx+c}+\frac{432\,(bxe+ae)^2A^2bd^2e^3}{(dx+c)^2}+\frac{432\,(bxe+ae)^2ABbd^2e^3}{(dx+c)^2}+\frac{216\,(bxe+ae)^2B^2bd^2e^3}{(dx+c)^2}-\frac{288\,(bxe+ae)^3A^2d^3e^2}{(dx+c)^3}-\frac{576\,(bxe+ae)^3ABd^3e^2}{(dx+c)^3}-\frac{576\,(bxe+ae)^3B^2d^3e^2}{(dx+c)^3}\right)\left(\frac{bc}{(bce-ade)(bc-ad)}-\frac{ad}{(bce-ade)(bc-ad)}\right)}{288\left(\frac{(bxe+ae)^4b^3c^3g^5}{(dx+c)^4}-\frac{3\,(bxe+ae)^4ab^2c^2dg^5}{(dx+c)^4}+\frac{3\,(bxe+ae)^4a^2bcd^2g^5}{(dx+c)^4}-\frac{(bxe+ae)^4a^3d^3g^5}{(dx+c)^4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^5,x, algorithm="giac")
```

```
[Out] -1/288*(72*B^2*b^3*e^5*log((b*x*e + a*e)/(d*x + c))^2 - 288*(b*x*e + a*e)*B
^2*b^2*d*e^4*log((b*x*e + a*e)/(d*x + c))^2/(d*x + c) + 432*(b*x*e + a*e)^2
*B^2*b*d^2*e^3*log((b*x*e + a*e)/(d*x + c))^2/(d*x + c)^2 - 288*(b*x*e + a*
e)^3*B^2*d^3*e^2*log((b*x*e + a*e)/(d*x + c))^2/(d*x + c)^3 + 144*A*B*b^3*e
^5*log((b*x*e + a*e)/(d*x + c)) + 36*B^2*b^3*e^5*log((b*x*e + a*e)/(d*x + c
)) - 576*(b*x*e + a*e)*A*B*b^2*d*e^4*log((b*x*e + a*e)/(d*x + c))/(d*x + c)
 - 192*(b*x*e + a*e)*B^2*b^2*d*e^4*log((b*x*e + a*e)/(d*x + c))/(d*x + c) +
 864*(b*x*e + a*e)^2*A*B*b*d^2*e^3*log((b*x*e + a*e)/(d*x + c))/(d*x + c)^2
 + 432*(b*x*e + a*e)^2*B^2*b*d^2*e^3*log((b*x*e + a*e)/(d*x + c))/(d*x + c)
^2 - 576*(b*x*e + a*e)^3*A*B*d^3*e^2*log((b*x*e + a*e)/(d*x + c))/(d*x + c)
^3 - 576*(b*x*e + a*e)^3*B^2*d^3*e^2*log((b*x*e + a*e)/(d*x + c))/(d*x + c)
^3 + 72*A^2*b^3*e^5 + 36*A*B*b^3*e^5 + 9*B^2*b^3*e^5 - 288*(b*x*e + a*e)*A^
2*b^2*d*e^4/(d*x + c) - 192*(b*x*e + a*e)*A*B*b^2*d*e^4/(d*x + c) - 64*(b*x
*e + a*e)*B^2*b^2*d*e^4/(d*x + c) + 432*(b*x*e + a*e)^2*A^2*b*d^2*e^3/(d*x
+ c)^2 + 432*(b*x*e + a*e)^2*A*B*b*d^2*e^3/(d*x + c)^2 + 216*(b*x*e + a*e)^
2*B^2*b*d^2*e^3/(d*x + c)^2 - 288*(b*x*e + a*e)^3*A^2*d^3*e^2/(d*x + c)^3 -
 576*(b*x*e + a*e)^3*A*B*d^3*e^2/(d*x + c)^3 - 576*(b*x*e + a*e)^3*B^2*d^3*
e^2/(d*x + c)^3)*(b*c/((b*c*e - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*
(b*c - a*d)))/((b*x*e + a*e)^4*b^3*c^3*g^5/(d*x + c)^4 - 3*(b*x*e + a*e)^4*
```

```
a*b^2*c^2*d*g^5/(d*x + c)^4 + 3*(b*x*e + a*e)^4*a^2*b*c*d^2*g^5/(d*x + c)^4
 - (b*x*e + a*e)^4*a^3*d^3*g^5/(d*x + c)^4)
```

Mupad [B]
time = 10.30, size = 1881, normalized size = 3.27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x))/(c + d*x)))^2/(a*g + b*g*x)^5,x)
```

```
[Out] (B*d^4*atan((B*d^4*(12*A + 25*B)*(24*b^5*c^4*g^5 - 24*a^4*b*d^4*g^5 - 48*a*
b^4*c^3*d*g^5 + 48*a^3*b^2*c*d^3*g^5)*1i)/(24*b*g^5*(a*d - b*c)^4*(25*B^2*d
^4 + 12*A*B*d^4)) + (B*d^5*x*(12*A + 25*B)*(b^4*c^3*g^5 - a^3*b*d^3*g^5 - 3
*a*b^3*c^2*d*g^5 + 3*a^2*b^2*c*d^2*g^5)*2i)/(g^5*(a*d - b*c)^4*(25*B^2*d^4
+ 12*A*B*d^4)))*(12*A + 25*B)*1i)/(12*b*g^5*(a*d - b*c)^4) - log((e*(a + b*
x))/(c + d*x))^2*(B^2/(4*b^2*g^5*(4*a^3*x + a^4/b + b^3*x^4 + 6*a^2*b*x^2 +
 4*a*b^2*x^3)) - (B^2*d^4)/(4*b*g^5*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2
- 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))) - (log((e*(a + b*x))/(c + d*x))*((A*B)/(
2*b^2*d*g^5) + (B^2*d^4*(a*(a*((4*a^2*d^2 + b^2*c^2 - 5*a*b*c*d)/(12*b*d^3)
 + (a*(a*d - b*c))/(4*b*d^2)) + (6*a^3*d^3 - b^3*c^3 + 5*a*b^2*c^2*d - 10*a
^2*b*c*d^2)/(12*b*d^4)) + (4*a^4*d^4 + b^4*c^4 + 10*a^2*b^2*c^2*d^2 - 5*a*b
^3*c^3*d - 10*a^3*b*c*d^3)/(4*b*d^5)))/(2*b*g^5*(a^4*d^4 + b^4*c^4 + 6*a^2*
b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)) + (B^2*d^4*x^2*(b*(b*((4*a^2*
d^2 + b^2*c^2 - 5*a*b*c*d)/(12*b*d^3) + (a*(a*d - b*c))/(4*b*d^2)) + (4*a^2
*d^2 + b^2*c^2 - 5*a*b*c*d)/(6*d^3) + (a*(a*d - b*c))/(2*d^2)) - a*((b^2*c
- a*b*d)/(4*d^2) - (b*(a*d - b*c))/(2*d^2)) + (b^3*c^2 + 4*a^2*b*d^2 - 5*a*
b^2*c*d)/(4*d^3)))/(2*b*g^5*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^
3*c^3*d - 4*a^3*b*c*d^3)) - (B^2*d^4*x^3*(b*((b^2*c - a*b*d)/(4*d^2) - (b*(
a*d - b*c))/(2*d^2)) + (b^3*c - a*b^2*d)/(4*d^2)))/(2*b*g^5*(a^4*d^4 + b^4*
c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)) + (B^2*d^4*x*(b*(
a*((4*a^2*d^2 + b^2*c^2 - 5*a*b*c*d)/(12*b*d^3) + (a*(a*d - b*c))/(4*b*d^2)
) + (6*a^3*d^3 - b^3*c^3 + 5*a*b^2*c^2*d - 10*a^2*b*c*d^2)/(12*b*d^4)) + a*
(b*((4*a^2*d^2 + b^2*c^2 - 5*a*b*c*d)/(12*b*d^3) + (a*(a*d - b*c))/(4*b*d^2
)) + (4*a^2*d^2 + b^2*c^2 - 5*a*b*c*d)/(6*d^3) + (a*(a*d - b*c))/(2*d^2)) +
 (6*a^3*d^3 - b^3*c^3 + 5*a*b^2*c^2*d - 10*a^2*b*c*d^2)/(4*d^4)))/(2*b*g^5*
(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))))/
((4*a^3*x)/d + a^4/(b*d) + (b^3*x^4)/d + (6*a^2*b*x^2)/d + (4*a*b^2*x^3)/d)
 - ((72*A^2*a^3*d^3 - 72*A^2*b^3*c^3 + 415*B^2*a^3*d^3 - 9*B^2*b^3*c^3 + 30
0*A*B*a^3*d^3 - 36*A*B*b^3*c^3 + 216*A^2*a*b^2*c^2*d - 216*A^2*a^2*b*c*d^2
+ 55*B^2*a*b^2*c^2*d - 161*B^2*a^2*b*c*d^2 + 156*A*B*a*b^2*c^2*d - 276*A*B*
a^2*b*c*d^2)/(12*(a*d - b*c)) + (x^2*(163*B^2*a*b^2*d^3 - 13*B^2*b^3*c*d^2
+ 84*A*B*a*b^2*d^3 - 12*A*B*b^3*c*d^2))/(2*(a*d - b*c)) + (x*(271*B^2*a^2*b
*d^3 + 7*B^2*b^3*c^2*d - 53*B^2*a*b^2*c*d^2 + 156*A*B*a^2*b*d^3 + 12*A*B*b^
3*c^2*d - 60*A*B*a*b^2*c*d^2))/(3*(a*d - b*c)) + (d*x^3*(25*B^2*b^3*d^2 + 1
```

```
2*A*B*b^3*d^2))/(a*d - b*c))/(x*(96*a^3*b^4*c^2*g^5 + 96*a^5*b^2*d^2*g^5 -
192*a^4*b^3*c*d*g^5) + x^3*(96*a*b^6*c^2*g^5 + 96*a^3*b^4*d^2*g^5 - 192*a^2
*b^5*c*d*g^5) + x^4*(24*b^7*c^2*g^5 + 24*a^2*b^5*d^2*g^5 - 48*a*b^6*c*d*g^5
) + x^2*(144*a^2*b^5*c^2*g^5 + 144*a^4*b^3*d^2*g^5 - 288*a^3*b^4*c*d*g^5) +
 24*a^6*b*d^2*g^5 + 24*a^4*b^3*c^2*g^5 - 48*a^5*b^2*c*d*g^5)
```

3.106 $\int \frac{\log\left(\frac{d(a+bx)}{b(c+dx)}\right)}{cf+dfx}\,dx$

Optimal. Leaf size=28

$$\frac{\mathrm{Li}_2\left(\frac{bc-ad}{b(c+dx)}\right)}{df}$$

```
[Out] polylog(2,(-a*d+b*c)/b/(d*x+c))/d/f
```

Rubi [A]
time = 0.02, antiderivative size = 29, normalized size of antiderivative = 1.04, number of steps used = 1, number of rules used = 1, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.034$, Rules used = $\{2497\}$

$$\frac{\mathrm{PolyLog}\left(2, 1-\frac{d(a+bx)}{b(c+dx)}\right)}{df}$$

Antiderivative was successfully verified.

```
[In] Int[Log[(d*(a + b*x))/(b*(c + d*x))]/(c*f + d*f*x),x]
```

```
[Out] PolyLog[2, 1 - (d*(a + b*x))/(b*(c + d*x))]/(d*f)
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rubi steps

$$\int \frac{\log\left(\frac{d(a+bx)}{b(c+dx)}\right)}{cf+dfx}\,dx = \frac{\mathrm{Li}_2\left(1-\frac{d(a+bx)}{b(c+dx)}\right)}{df}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 114 vs. 2(28) = 56.
time = 0.04, size = 114, normalized size = 4.07

$$\frac{\log\left(\frac{bc-ad}{bc+bdx}\right)\left(2\log\left(\frac{d(a+bx)}{-bc+ad}\right)-2\log\left(\frac{d(a+bx)}{b(c+dx)}\right)+\log\left(\frac{bc-ad}{bc+bdx}\right)\right)-2\mathrm{Li}_2\left(\frac{b(c+dx)}{bc-ad}\right)}{2df}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[(d*(a + b*x))/(b*(c + d*x))]/(c*f + d*f*x),x]
```

```
[Out] (Log[(b*c - a*d)/(b*c + b*d*x)]*(2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - 2*Lo
g[(d*(a + b*x))/(b*(c + d*x))] + Log[(b*c - a*d)/(b*c + b*d*x)]) - 2*PolyLo
g[2, (b*(c + d*x))/(b*c - a*d)])/(2*d*f)
```

Maple [A]
time = 0.23, size = 30, normalized size = 1.07

method	result	size
derivativedivides	$\frac{\operatorname{dilog}\left(1+\frac{ad-cb}{b(dx+c)}\right)}{df}$	30
default	$\frac{\operatorname{dilog}\left(1+\frac{ad-cb}{b(dx+c)}\right)}{df}$	30
risch	$\frac{\operatorname{dilog}\left(1+\frac{ad-cb}{b(dx+c)}\right)}{df}$	30

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(d*(b*x+a)/b/(d*x+c))/(d*f*x+c*f),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*dilog(1+(a*d-b*c)/b/(d*x+c))/f
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 158 vs. 2(27) = 54.
time = 0.28, size = 158, normalized size = 5.64

$$-\frac{b\left(\frac{\log(dx+c)^2}{bf}-\frac{2\left(\log(bx+a)\log\left(\frac{bdx+ad}{bc-ad}+1\right)+\mathrm{Li}_2\left(-\frac{bdx+ad}{bc-ad}\right)\right)}{bf}\right)}{2\,d}-\frac{b\left(\frac{d\log(bx+a)}{b}-\frac{d\log(dx+c)}{b}\right)\log\left(dfx+cf\right)}{d^2f}+\frac{\log\left(dfx+cf\right)\log\left(\frac{(bx+a)d}{(dx+c)b}\right)}{df}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(b*x+a)/b/(d*x+c))/(d*f*x+c*f),x, algorithm="maxima")
```

```
[Out] -1/2*b*(log(d*x + c)^2/(b*f) - 2*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d
) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))/(b*f))/d - b*(d*log(b*x + a)/b
- d*log(d*x + c)/b)*log(d*f*x + c*f)/(d^2*f) + log(d*f*x + c*f)*log((b*x +
a)*d/((d*x + c)*b))/(d*f)
```

Fricas [A]
time = 0.35, size = 30, normalized size = 1.07

$$\frac{\mathrm{Li}_2\left(-\frac{bdx+ad}{bdx+bc}+1\right)}{df}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(b*x+a)/b/(d*x+c))/(d*f*x+c*f),x, algorithm="fricas")
```

```
[Out] dilog(-(b*d*x + a*d)/(b*d*x + b*c) + 1)/(d*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\log\left(\frac{ad}{bc+bdx}+\frac{bdx}{bc+bdx}\right)}{c+dx}\,dx}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(d*(b*x+a)/b/(d*x+c))/(d*f*x+c*f),x)
```

```
[Out] Integral(log(a*d/(b*c + b*d*x) + b*d*x/(b*c + b*d*x))/(c + d*x), x)/f
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1203 vs. $2(27) = 54$.
time = 35.39, size = 1203, normalized size = 42.96

$$-\frac{1}{2}\left(\frac{b^2cd}{(bc-ad)^2}-\frac{abd^2}{(bc-ad)^2}\right)^2\left((b^3c^3-3ab^2c^2d+3a^2bcd^2-a^3d^3)\left(\frac{\log\left(\frac{|bdx+ad|}{|bdx+bc|}\right)}{b^3d^4f}-\frac{\log\left(\left|\frac{bdx+ad}{bdx+bc}-1\right|\right)}{b^3d^4f}-\frac{1}{b^3d^4f\left(\frac{bdx+ad}{bdx+bc}-1\right)}\right)-\frac{(b^3c^3-3ab^2c^2d+3a^2bcd^2-a^3d^3)\log\left(\frac{\left(a+\frac{b\left(\frac{\left(ad-\frac{b\left(\frac{(bdx+ad)bc}{(bdx+bc)(bc-ad)}-\frac{ad}{bc-ad}\right)d}{\frac{(bdx+ad)bd}{(bdx+bc)(bc-ad)}-\frac{bd}{bc-ad}}\right)bc}{(bc-ad)\left(bc-\frac{b\left(\frac{(bdx+ad)bc}{(bdx+bc)(bc-ad)}-\frac{ad}{bc-ad}\right)d}{\frac{(bdx+ad)bd}{(bdx+bc)(bc-ad)}-\frac{bd}{bc-ad}}\right)}-\frac{ad}{bc-ad}\right)}{\frac{bd}{bc-ad}-\frac{\left(ad-\frac{b\left(\frac{(bdx+ad)bc}{(bdx+bc)(bc-ad)}-\frac{ad}{bc-ad}\right)d}{\frac{(bdx+ad)bd}{(bdx+bc)(bc-ad)}-\frac{bd}{bc-ad}}\right)bd}{(bc-ad)\left(bc-\frac{b\left(\frac{(bdx+ad)bc}{(bdx+bc)(bc-ad)}-\frac{ad}{bc-ad}\right)d}{\frac{(bdx+ad)bd}{(bdx+bc)(bc-ad)}-\frac{bd}{bc-ad}}\right)}}\right)d}{b\left(c+\frac{\left(\frac{\left(ad-\frac{b\left(\frac{(bdx+ad)bc}{(bdx+bc)(bc-ad)}-\frac{ad}{bc-ad}\right)d}{\frac{(bdx+ad)bd}{(bdx+bc)(bc-ad)}-\frac{bd}{bc-ad}}\right)bc}{(bc-ad)\left(bc-\frac{b\left(\frac{(bdx+ad)bc}{(bdx+bc)(bc-ad)}-\frac{ad}{bc-ad}\right)d}{\frac{(bdx+ad)bd}{(bdx+bc)(bc-ad)}-\frac{bd}{bc-ad}}\right)}-\frac{ad}{bc-ad}\right)d}{\frac{bd}{bc-ad}-\frac{\left(ad-\frac{b\left(\frac{(bdx+ad)bc}{(bdx+bc)(bc-ad)}-\frac{ad}{bc-ad}\right)d}{\frac{(bdx+ad)bd}{(bdx+bc)(bc-ad)}-\frac{bd}{bc-ad}}\right)bd}{(bc-ad)\left(bc-\frac{b\left(\frac{(bdx+ad)bc}{(bdx+bc)(bc-ad)}-\frac{ad}{bc-ad}\right)d}{\frac{(bdx+ad)bd}{(bdx+bc)(bc-ad)}-\frac{bd}{bc-ad}}\right)}}\right)}\right)}{b^3d^4f\left(\frac{bdx+ad}{bdx+bc}-1\right)^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(b*x+a)/b/(d*x+c))/(d*f*x+c*f),x, algorithm="giac")
```

```
[Out] -1/2*(b^2*c*d/(b*c - a*d)^2 - a*b*d^2/(b*c - a*d)^2)^2*((b^3*c^3 - 3*a*b^2*
c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*(log(abs(b*d*x + a*d)/abs(b*d*x + b*c))/(b
^3*d^4*f) - log(abs((b*d*x + a*d)/(b*d*x + b*c) - 1))/(b^3*d^4*f) - 1/(b^3*
d^4*f*((b*d*x + a*d)/(b*d*x + b*c) - 1))) - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^
2*b*c*d^2 - a^3*d^3)*log((a + b*((a*d - b*((b*d*x + a*d)*b*c/((b*d*x + b*c)
*(b*c - a*d)) - a*d/(b*c - a*d))*d/((b*d*x + a*d)*b*d/((b*d*x + b*c)*(b*c -
 a*d)) - b*d/(b*c - a*d)))*b*c/((b*c - a*d)*(b*c - b*((b*d*x + a*d)*b*c/((b
*d*x + b*c)*(b*c - a*d)) - a*d/(b*c - a*d))*d/((b*d*x + a*d)*b*d/((b*d*x +
b*c)*(b*c - a*d)) - b*d/(b*c - a*d)))) - a*d/(b*c - a*d))/(b*d/(b*c - a*d)
- (a*d - b*((b*d*x + a*d)*b*c/((b*d*x + b*c)*(b*c - a*d)) - a*d/(b*c - a*d)
)*d/((b*d*x + a*d)*b*d/((b*d*x + b*c)*(b*c - a*d)) - b*d/(b*c - a*d)))*b*d/
```

```
((b*c - a*d)*(b*c - b*((b*d*x + a*d)*b*c/((b*d*x + b*c)*(b*c - a*d)) - a*d/
(b*c - a*d))*d/((b*d*x + a*d)*b*d/((b*d*x + b*c)*(b*c - a*d)) - b*d/(b*c -
a*d))))))*d/(b*(c + ((a*d - b*((b*d*x + a*d)*b*c/((b*d*x + b*c)*(b*c - a*d)
) - a*d/(b*c - a*d))*d/((b*d*x + a*d)*b*d/((b*d*x + b*c)*(b*c - a*d)) - b*d
/(b*c - a*d)))*b*c/((b*c - a*d)*(b*c - b*((b*d*x + a*d)*b*c/((b*d*x + b*c)*
(b*c - a*d)) - a*d/(b*c - a*d))*d/((b*d*x + a*d)*b*d/((b*d*x + b*c)*(b*c -
a*d)) - b*d/(b*c - a*d)))) - a*d/(b*c - a*d))*d/(b*d/(b*c - a*d) - (a*d - b
*((b*d*x + a*d)*b*c/((b*d*x + b*c)*(b*c - a*d)) - a*d/(b*c - a*d))*d/((b*d*
x + a*d)*b*d/((b*d*x + b*c)*(b*c - a*d)) - b*d/(b*c - a*d)))*b*d/((b*c - a*
d)*(b*c - b*((b*d*x + a*d)*b*c/((b*d*x + b*c)*(b*c - a*d)) - a*d/(b*c - a*d
))*d/((b*d*x + a*d)*b*d/((b*d*x + b*c)*(b*c - a*d)) - b*d/(b*c - a*d))))))
)/(b^3*d^4*f*((b*d*x + a*d)/(b*d*x + b*c) - 1)^2))
```

Mupad [B]
time = 4.25, size = 25, normalized size = 0.89

$$\frac{\mathrm{Li}_2\left(\frac{d\,(a+b\,x)}{b\,(c+d\,x)}\right)}{d\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log((d*(a + b*x))/(b*(c + d*x)))/(c*f + d*f*x),x)
[Out] dilog((d*(a + b*x))/(b*(c + d*x)))/(d*f)
```

3.107 $\int \frac{\log\left(1+\frac{1}{a+bx}\right)}{a+bx}\,dx$

Optimal. Leaf size=15

$$\frac{\text{Li}_2\left(-\frac{1}{a+bx}\right)}{b}$$

```
[Out] polylog(2,-1/(b*x+a))/b
```

Rubi [A]
time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {2497}

$$\frac{\text{PolyLog}\left(2,-\frac{1}{a+bx}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Log[1 + (a + b*x)^(-1)]/(a + b*x),x]
```

```
[Out] PolyLog[2, -(a + b*x)^(-1)]/b
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rubi steps

$$\int \frac{\log\left(1+\frac{1}{a+bx}\right)}{a+bx}\,dx = \frac{\text{Li}_2\left(-\frac{1}{a+bx}\right)}{b}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 140 vs. 2(15) = 30.
time = 0.01, size = 140, normalized size = 9.33

$$\frac{\log\left(\frac{b(-1-a-bx)}{(-1-a)b+ab}\right)\log\left(\frac{ab-(1+a)b}{b(a+bx)}\right)}{b}+\frac{\log^2\left(\frac{ab-(1+a)b}{b(a+bx)}\right)}{2b}-\frac{\log\left(\frac{ab-(1+a)b}{b(a+bx)}\right)\log\left(\frac{1+a+bx}{a+bx}\right)}{b}-\frac{\text{Li}_2(-a-bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[1 + (a + b*x)^(-1)]/(a + b*x),x]
```

```
[Out] (Log[(b*(-1 - a - b*x))/((-1 - a)*b + a*b)]*Log[(a*b - (1 + a)*b)/(b*(a + b
*x))])/b + Log[(a*b - (1 + a)*b)/(b*(a + b*x))]^2/(2*b) - (Log[(a*b - (1 +
a)*b)/(b*(a + b*x))]*Log[(1 + a + b*x)/(a + b*x)])/b - PolyLog[2, -a - b*x]
/b
```

Maple [A]
time = 0.17, size = 15, normalized size = 1.00

method	result	size
derivativedivides	$\frac{\operatorname{dilog}\left(1+\frac{1}{bx+a}\right)}{b}$	15
default	$\frac{\operatorname{dilog}\left(1+\frac{1}{bx+a}\right)}{b}$	15
risch	$\frac{\operatorname{dilog}\left(1+\frac{1}{bx+a}\right)}{b}$	15

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(1+1/(b*x+a))/(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*dilog(1+1/(b*x+a))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 61 vs. 2(14) = 28.
time = 0.27, size = 61, normalized size = 4.07

$$\frac{2\,\log\left(bx+a+1\right)\log\left(bx+a\right)-\log\left(bx+a\right)^{2}}{2\,b}-\frac{\log\left(bx+a+1\right)\log\left(bx+a\right)+\mathrm{Li}_{2}(-bx-a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1+1/(b*x+a))/(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*(2*log(b*x + a + 1)*log(b*x + a) - log(b*x + a)^2)/b - (log(b*x + a + 1
)*log(b*x + a) + dilog(-b*x - a))/b
```

Fricas [A]
time = 0.36, size = 22, normalized size = 1.47

$$\frac{\mathrm{Li}_{2}\left(-\frac{bx+a+1}{bx+a}+1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1+1/(b*x+a))/(b*x+a),x, algorithm="fricas")
```

```
[Out] dilog(-(b*x + a + 1)/(b*x + a) + 1)/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(1+\frac{1}{a+bx}\right)}{a+bx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(1+1/(b*x+a))/(b*x+a),x)
```

```
[Out] Integral(log(1 + 1/(a + b*x))/(a + b*x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 320 vs. $2(14) = 28$.
time = 7.67, size = 320, normalized size = 21.33

$$\frac{1}{2}\left((a+1)b-ab\right)^2\left(\frac{\log\left(\frac{|bx+a+1|}{|bx+a|}\right)}{b^4}-\frac{\log\left(\left|\frac{bx+a+1}{bx+a}-1\right|\right)}{b^4}-\frac{1}{b^4\left(\frac{bx+a+1}{bx+a}-1\right)}-\frac{\log\left(\frac{1}{a-\frac{\left(\frac{\left(a-\frac{\left(\frac{(bx+a+1)a}{bx+a}-a-1\right)b}{\frac{(bx+a+1)b}{bx+a}-b}+1\right)a}{a-\frac{\left(\frac{(bx+a+1)a}{bx+a}-a-1\right)b}{\frac{(bx+a+1)b}{bx+a}-b}}-a-1\right)b}{\frac{\left(a-\frac{\left(\frac{(bx+a+1)a}{bx+a}-a-1\right)b}{\frac{(bx+a+1)b}{bx+a}-b}+1\right)b}{a-\frac{\left(\frac{(bx+a+1)a}{bx+a}-a-1\right)b}{\frac{(bx+a+1)b}{bx+a}-b}}-b}}+1\right)}{b^4\left(\frac{bx+a+1}{bx+a}-1\right)^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1+1/(b*x+a))/(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*((a + 1)*b - a*b)^2*(log(abs(b*x + a + 1)/abs(b*x + a))/b^4 - log(abs((
b*x + a + 1)/(b*x + a) - 1))/b^4 - 1/(b^4*((b*x + a + 1)/(b*x + a) - 1)) -
```

```
log(1/(a - ((a - ((b*x + a + 1)*a/(b*x + a) - a - 1)*b/((b*x + a + 1)*b/(b*
x + a) - b) + 1)*a/(a - ((b*x + a + 1)*a/(b*x + a) - a - 1)*b/((b*x + a + 1
)*b/(b*x + a) - b)) - a - 1)*b/((a - ((b*x + a + 1)*a/(b*x + a) - a - 1)*b/
((b*x + a + 1)*b/(b*x + a) - b) + 1)*b/(a - ((b*x + a + 1)*a/(b*x + a) - a
- 1)*b/((b*x + a + 1)*b/(b*x + a) - b)) - b)) + 1)/(b^4*((b*x + a + 1)/(b*x
 + a) - 1)^2))
```

Mupad [B]
time = 4.03, size = 15, normalized size = 1.00

$$\frac{\mathrm{polylog}\left(2,-\frac{1}{a+b\,x}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(1/(a + b*x) + 1)/(a + b*x),x)
```

```
[Out] polylog(2, -1/(a + b*x))/b
```

3.108 $\int \frac{\log\left(1-\frac{1}{a+bx}\right)}{a+bx}\,dx$

Optimal. Leaf size=13

$$\frac{\text{Li}_2\left(\frac{1}{a+bx}\right)}{b}$$

```
[Out] polylog(2,1/(b*x+a))/b
```

Rubi [A]
time = 0.01, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = $\{2497\}$

$$\frac{\text{PolyLog}\left(2,\frac{1}{a+bx}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Log[1 - (a + b*x)^(-1)]/(a + b*x),x]
```

```
[Out] PolyLog[2, (a + b*x)^(-1)]/b
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rubi steps

$$\int \frac{\log\left(1-\frac{1}{a+bx}\right)}{a+bx}\,dx = \frac{\text{Li}_2\left(\frac{1}{a+bx}\right)}{b}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 133 vs. 2(13) = 26.
time = 0.01, size = 133, normalized size = 10.23

$$\frac{\log\left(\frac{b(-1+a+bx)}{(-1+a)b-ab}\right)\log\left(\frac{-((-1+a)b)+ab}{b(a+bx)}\right)}{b}+\frac{\log^2\left(\frac{-((-1+a)b)+ab}{b(a+bx)}\right)}{2b}-\frac{\log\left(\frac{-((-1+a)b)+ab}{b(a+bx)}\right)\log\left(\frac{-1+a+bx}{a+bx}\right)}{b}-\frac{\text{Li}_2(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[1 - (a + b*x)^(-1)]/(a + b*x),x]
```

```
[Out] (Log[(b*(-1 + a + b*x))/((-1 + a)*b - a*b)]*Log[(-((-1 + a)*b) + a*b)/(b*(a
 + b*x))])/b + Log[(-((-1 + a)*b) + a*b)/(b*(a + b*x))]^2/(2*b) - (Log[(-((
-1 + a)*b) + a*b)/(b*(a + b*x))]*Log[(-1 + a + b*x)/(a + b*x)])/b - PolyLog
[2, a + b*x]/b
```

Maple [A]
time = 0.17, size = 17, normalized size = 1.31

method	result	size
derivativedivides	$\frac{\operatorname{dilog}\left(1-\frac{1}{bx+a}\right)}{b}$	17
default	$\frac{\operatorname{dilog}\left(1-\frac{1}{bx+a}\right)}{b}$	17
risch	$\frac{\operatorname{dilog}\left(1-\frac{1}{bx+a}\right)}{b}$	17

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(1-1/(b*x+a))/(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*dilog(1-1/(b*x+a))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 59 vs. 2(12) = 24.
time = 0.27, size = 59, normalized size = 4.54

$$-\frac{\log\left(bx+a\right)^{2}-2\,\log\left(bx+a\right)\log\left(bx+a-1\right)}{2\,b}-\frac{\log\left(bx+a\right)\log\left(-bx-a+1\right)+\mathrm{Li}_{2}(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1-1/(b*x+a))/(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/2*(log(b*x + a)^2 - 2*log(b*x + a)*log(b*x + a - 1))/b - (log(b*x + a)*l
og(-b*x - a + 1) + dilog(b*x + a))/b
```

Fricas [A]
time = 0.33, size = 22, normalized size = 1.69

$$\frac{\mathrm{Li}_{2}\left(-\frac{bx+a-1}{bx+a}+1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1-1/(b*x+a))/(b*x+a),x, algorithm="fricas")
```

```
[Out] dilog(-(b*x + a - 1)/(b*x + a) + 1)/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(1-\frac{1}{a+bx}\right)}{a+bx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(1-1/(b*x+a))/(b*x+a),x)
```

```
[Out] Integral(log(1 - 1/(a + b*x))/(a + b*x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 322 vs. 2(12) = 24.
time = 7.00, size = 322, normalized size = 24.77

$$-\frac{1}{2}\left((a-1)b-ab\right)^2\left(\frac{\log\left(\frac{|bx+a-1|}{|bx+a|}\right)}{b^4}-\frac{\log\left(\left|\frac{bx+a-1}{bx+a}-1\right|\right)}{b^4}-\frac{1}{b^4\left(\frac{bx+a-1}{bx+a}-1\right)}-\frac{\log\left(-\frac{1}{a-\frac{\left(\frac{\left(a-\frac{\left(\frac{(bx+a-1)a}{bx+a}-a+1\right)b}{\frac{(bx+a-1)b}{bx+a}-b}-1\right)a}{a-\frac{\left(\frac{(bx+a-1)a}{bx+a}-a+1\right)b}{\frac{(bx+a-1)b}{bx+a}-b}}-a+1\right)b}{\frac{\left(a-\frac{\left(\frac{(bx+a-1)a}{bx+a}-a+1\right)b}{\frac{(bx+a-1)b}{bx+a}-b}-1\right)b}{a-\frac{\left(\frac{(bx+a-1)a}{bx+a}-a+1\right)b}{\frac{(bx+a-1)b}{bx+a}-b}}-b}}+1\right)}{b^4\left(\frac{bx+a-1}{bx+a}-1\right)^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1-1/(b*x+a))/(b*x+a),x, algorithm="giac")
```

```
[Out] -1/2*((a - 1)*b - a*b)^2*(log(abs(b*x + a - 1)/abs(b*x + a))/b^4 - log(abs(
(b*x + a - 1)/(b*x + a) - 1))/b^4 - 1/(b^4*((b*x + a - 1)/(b*x + a) - 1)) -
 log(-1/(a - ((a - ((b*x + a - 1)*a/(b*x + a) - a + 1)*b/((b*x + a - 1)*b/(
```

```
b*x + a) - b) - 1)*a/(a - ((b*x + a - 1)*a/(b*x + a) - a + 1)*b/((b*x + a -
 1)*b/(b*x + a) - b)) - a + 1)*b/((a - ((b*x + a - 1)*a/(b*x + a) - a + 1)*
b/((b*x + a - 1)*b/(b*x + a) - b) - 1)*b/(a - ((b*x + a - 1)*a/(b*x + a) -
a + 1)*b/((b*x + a - 1)*b/(b*x + a) - b)) - b)) + 1)/(b^4*((b*x + a - 1)/(b
*x + a) - 1)^2))
```

Mupad [B]
time = 4.23, size = 13, normalized size = 1.00

$$\frac{\mathrm{polylog}\left(2,\frac{1}{a+b\,x}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(1 - 1/(a + b*x))/(a + b*x),x)
```

```
[Out] polylog(2, 1/(a + b*x))/b
```

3.109 $\int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx$

Optimal. Leaf size=35

$$\text{Int}\left(\frac{(ag+bgx)^2}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)},x\right)$$

[Out] Unintegrable((b*g*x+a*g)^2/(A+B*ln(e*(b*x+a)/(d*x+c))),x)

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx$$

Verification is not applicable to the result.

[In] Int[(a*g + b*g*x)^2/(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]

[Out] Defer[Int][(a*g + b*g*x)^2/(A + B*Log[(e*(a + b*x))/(c + d*x)]), x]

Rubi steps

$$\begin{aligned}\int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx &= \int \left(\frac{a^2g^2}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}+\frac{2abg^2x}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}+\frac{b^2g^2x^2}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\right)dx\\ &= \left(a^2g^2\right)\int \frac{1}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx+\left(2abg^2\right)\int \frac{x}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx+\left(b^2g^2\right.\end{aligned}$$

Mathematica [A]
time = 0.57, size = 0, normalized size = 0.00

$$\int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx$$

Verification is not applicable to the result.

[In] Integrate[(a*g + b*g*x)^2/(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]

[Out] Integrate[(a*g + b*g*x)^2/(A + B*Log[(e*(a + b*x))/(c + d*x)]), x]

Maple [A]
time = 0.76, size = 0, normalized size = 0.00

$$\int \frac{(bgx + ag)^2}{A + B\ln\left(\frac{e(bx+a)}{dx+c}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*g*x+a*g)^2/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

```
[Out] int((b*g*x+a*g)^2/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^2/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="maxima")
```

```
[Out] integrate((b*g*x + a*g)^2/(B*log((b*x + a)*e/(d*x + c)) + A), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^2/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="fricas")
```

```
[Out] integral((b^2*g^2*x^2 + 2*a*b*g^2*x + a^2*g^2)/(B*log((b*x + a)*e/(d*x + c)
) + A), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$g^2\left(\int \frac{a^2}{A + B\log\left(\frac{ae}{c+dx} + \frac{bex}{c+dx}\right)}\, dx + \int \frac{b^2x^2}{A + B\log\left(\frac{ae}{c+dx} + \frac{bex}{c+dx}\right)}\, dx + \int \frac{2abx}{A + B\log\left(\frac{ae}{c+dx} + \frac{bex}{c+dx}\right)}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)**2/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

```
[Out] g**2*(Integral(a**2/(A + B*log(a*e/(c + d*x) + b*e*x/(c + d*x))), x) + Inte
gral(b**2*x**2/(A + B*log(a*e/(c + d*x) + b*e*x/(c + d*x))), x) + Integral(
2*a*b*x/(A + B*log(a*e/(c + d*x) + b*e*x/(c + d*x))), x))
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^2/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="giac")
[Out] integrate((b*g*x + a*g)^2/(B*log((b*x + a)*e/(d*x + c)) + A), x)
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{(a\,g+b\,g\,x)^2}{A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*g + b*g*x)^2/(A + B*log((e*(a + b*x))/(c + d*x))),x)
[Out] int((a*g + b*g*x)^2/(A + B*log((e*(a + b*x))/(c + d*x))), x)
```

3.110 $\int \frac{ag+bgx}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx$

Optimal. Leaf size=33

$$\text{Int}\left(\frac{ag+bgx}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)},x\right)$$

[Out] Unintegrable((b*g*x+a*g)/(A+B*ln(e*(b*x+a)/(d*x+c))),x)

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{ag+bgx}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx$$

Verification is not applicable to the result.

[In] Int[(a*g + b*g*x)/(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]

[Out] Defer[Int][(a*g + b*g*x)/(A + B*Log[(e*(a + b*x))/(c + d*x)]), x]

Rubi steps

$$\int \frac{ag+bgx}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx = \int \left(\frac{ag}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}+\frac{bgx}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\right)dx$$
$$=(ag)\int \frac{1}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx+(bg)\int \frac{x}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx$$

Mathematica [A]
time = 0.17, size = 0, normalized size = 0.00

$$\int \frac{ag+bgx}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx$$

Verification is not applicable to the result.

[In] Integrate[(a*g + b*g*x)/(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]

[Out] Integrate[(a*g + b*g*x)/(A + B*Log[(e*(a + b*x))/(c + d*x)]), x]

Maple [A]
time = 0.80, size = 0, normalized size = 0.00

$$\int \frac{bgx+ag}{A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*g*x+a*g)/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
[Out] int((b*g*x+a*g)/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="maxima")
[Out] integrate((b*g*x + a*g)/(B*log((b*x + a)*e/(d*x + c)) + A), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="fricas")
[Out] integral((b*g*x + a*g)/(B*log((b*x + a)*e/(d*x + c)) + A), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$g\left(\int \frac{a}{A+B\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}\,dx+\int \frac{bx}{A+B\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
[Out] g*(Integral(a/(A + B*log(a*e/(c + d*x) + b*e*x/(c + d*x))), x) + Integral(b
*x/(A + B*log(a*e/(c + d*x) + b*e*x/(c + d*x))), x))
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="giac")
[Out] integrate((b*g*x + a*g)/(B*log((b*x + a)*e/(d*x + c)) + A), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{a\,g+b\,g\,x}{A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*g + b*g*x)/(A + B*log((e*(a + b*x))/(c + d*x))),x)
[Out] int((a*g + b*g*x)/(A + B*log((e*(a + b*x))/(c + d*x))), x)
```

3.111 $\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx$

Optimal. Leaf size=35

$$\text{Int}\left(\frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)},x\right)$$

```
[Out] Unintegrable(1/(b*g*x+a*g)/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((a*g + b*g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])),x]
[Out] Defer[Int][1/((a*g + b*g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])), x]
```

Rubi steps

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx=\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx$$

Mathematica [A]
time = 0.21, size = 0, normalized size = 0.00

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((a*g + b*g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])),x]
[Out] Integrate[1/((a*g + b*g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])), x]
```

Maple [A]
time = 0.47, size = 0, normalized size = 0.00

$$\int \frac{1}{(bgx+ag)\left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*g*x+a*g)/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

```
[Out] int(1/(b*g*x+a*g)/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*g*x + a*g)*(B*log((b*x + a)*e/(d*x + c)) + A)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="fricas")
```

```
[Out] integral(1/(A*b*g*x + A*a*g + (B*b*g*x + B*a*g)*log((b*x + a)*e/(d*x + c)))
, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{Aa+Abx+Ba\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)+Bbx\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}\,dx}{g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

```
[Out] Integral(1/(A*a + A*b*x + B*a*log(a*e/(c + d*x) + b*e*x/(c + d*x)) + B*b*x*
log(a*e/(c + d*x) + b*e*x/(c + d*x))), x)/g
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="giac")
```

```
[Out] integrate(1/((b*g*x + a*g)*(B*log((b*x + a)*e/(d*x + c)) + A)), x)
```

Mupad [A]

time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{(a\,g+b\,g\,x)\,\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*g + b*g*x)*(A + B*log((e*(a + b*x))/(c + d*x)))),x)
[Out] int(1/((a*g + b*g*x)*(A + B*log((e*(a + b*x))/(c + d*x)))), x)
```

3.112 $\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx$

Optimal. Leaf size=50

$$\frac{ee^{A/B}\text{Ei}\left(-\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{B}\right)}{B(bc-ad)g^2}$$

```
[Out] e*exp(A/B)*Ei((-A-B*ln(e*(b*x+a)/(d*x+c)))/B)/B/(-a*d+b*c)/g^2
```

Rubi [A]
time = 0.07, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {2550, 2346, 2209}

$$\frac{ee^{A/B}\text{Ei}\left(-\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{B}\right)}{Bg^2(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])),x]
```

```
[Out] (e*E^(A/B)*ExpIntegralEi[-((A + B*Log[(e*(a + b*x))/(c + d*x)])/B)])/(B*(b*
c - a*d)*g^2)
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2346

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> Dist[1/c^
(m + 1), Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x] /; FreeQ
[{a, b, c, p}, x] && IntegerQ[m]
```

Rule 2550

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(
m + 1)*(g/b)^m, Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x],
 x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] &&
 EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && Eq
Q[b*f - a*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Rubi steps

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx=\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx$$

Mathematica [A]
time = 0.10, size = 52, normalized size = 1.04

$$\frac{ee^{A/B}\text{Ei}\left(-\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{B}\right)}{bBcg^2-aBdg^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])),x]
[Out] (e*E^(A/B)*ExpIntegralEi[-((A + B*Log[(e*(a + b*x))/(c + d*x)])/B)])/(b*B*c
*g^2 - a*B*d*g^2)
```

Maple [A]
time = 8.68, size = 61, normalized size = 1.22

method	result	size
derivativedivides	$\frac{e\,\mathrm{e}^{\frac{A}{B}}\,\mathrm{expIntegral}\left(1,\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)+\frac{A}{B}\right)}{(ad-cb)g^2B}$	61
default	$\frac{e\,\mathrm{e}^{\frac{A}{B}}\,\mathrm{expIntegral}\left(1,\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)+\frac{A}{B}\right)}{(ad-cb)g^2B}$	61
risch	$\frac{e\,\mathrm{e}^{\frac{A}{B}}\,\mathrm{expIntegral}\left(1,\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)+\frac{A}{B}\right)}{(ad-cb)g^2B}$	61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*g*x+a*g)^2/(A+B*ln(e*(b*x+a)/(d*x+c))),x,method=_RETURNVERBOSE)
[Out] e/(a*d-b*c)/g^2/B*exp(A/B)*Ei(1,ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))+A/B)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="maxima
")
```

[Out] integrate(1/((b*g*x + a*g)^2*(B*log((b*x + a)*e/(d*x + c)) + A)), x)

Fricas [A]
time = 0.34, size = 47, normalized size = 0.94

$$\frac{e^{\left(\frac{A}{B}+1\right)}\ \mathrm{log\_integral}\left(\frac{(dx+c)e^{\left(-\frac{A}{B}-1\right)}}{bx+a}\right)}{(Bbc-Bad)g^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="fricas
")

[Out] e^(A/B + 1)*log_integral((d*x + c)*e^(-A/B - 1)/(b*x + a))/((B*b*c - B*a*d)
*g^2)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{Aa^2+2Aabx+Ab^2x^2+Ba^2\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)+2Babx\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)+Bb^2x^2\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}\,dx}{g^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*g*x+a*g)**2/(A+B*ln(e*(b*x+a)/(d*x+c))),x)

[Out] Integral(1/(A*a**2 + 2*A*a*b*x + A*b**2*x**2 + B*a**2*log(a*e/(c + d*x) + b
*e*x/(c + d*x)) + 2*B*a*b*x*log(a*e/(c + d*x) + b*e*x/(c + d*x)) + B*b**2*x
2*log(a*e/(c + d*x) + b*e*x/(c + d*x))), x)/g2

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="giac")

[Out] integrate(1/((b*g*x + a*g)^2*(B*log((b*x + a)*e/(d*x + c)) + A)), x)

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{(a\,g+b\,g\,x)^2\,\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a*g + b*g*x)^2*(A + B*log((e*(a + b*x))/(c + d*x)))),x)

[Out] int(1/((a*g + b*g*x)^2*(A + B*log((e*(a + b*x))/(c + d*x)))), x)

3.113 $\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx$

Optimal. Leaf size=107

$$\frac{be^2e^{\frac{2A}{B}}\text{Ei}\left(-\frac{2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{B}\right)}{B(bc-ad)^2g^3}-\frac{dee^{A/B}\text{Ei}\left(-\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{B}\right)}{B(bc-ad)^2g^3}$$

```
[Out] b*e^2*exp(2*A/B)*Ei(-2*(A+B*ln(e*(b*x+a)/(d*x+c)))/B)/B/(-a*d+b*c)^2/g^3-d*
e*exp(A/B)*Ei((-A-B*ln(e*(b*x+a)/(d*x+c)))/B)/B/(-a*d+b*c)^2/g^3
```

Rubi [A]
time = 0.15, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2550, 2395, 2346, 2209}

$$\frac{be^2e^{\frac{2A}{B}}\text{Ei}\left(-\frac{2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{B}\right)}{Bg^3(bc-ad)^2}-\frac{dee^{A/B}\text{Ei}\left(-\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{B}\right)}{Bg^3(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a*g + b*g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])),x]
```

```
[Out] (b*e^2*E^((2*A)/B)*ExpIntegralEi[(-2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/
B])/(B*(b*c - a*d)^2*g^3) - (d*e*E^(A/B)*ExpIntegralEi[-((A + B*Log[(e*(a +
 b*x))/(c + d*x)])/B)])/(B*(b*c - a*d)^2*g^3)
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2346

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> Dist[1/c^
(m + 1), Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x] /; FreeQ
[{a, b, c, p}, x] && IntegerQ[m]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
```

```
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2550

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(
m + 1)*(g/b)^m, Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x],
 x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] &&
 EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && Eq
Q[b*f - a*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Rubi steps

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx=\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx$$

Mathematica [A]
time = 0.16, size = 89, normalized size = 0.83

$$\frac{ee^{A/B}\left(bee^{A/B}\text{Ei}\left(-\frac{2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{B}\right)-d\text{Ei}\left(-\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{B}\right)\right)}{B(bc-ad)^2g^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a*g + b*g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])),x]
```

```
[Out] (e*E^(A/B)*(b*e*E^(A/B)*ExpIntegralEi[(-2*(A + B*Log[(e*(a + b*x))/(c + d*x
)]))/B] - d*ExpIntegralEi[-((A + B*Log[(e*(a + b*x))/(c + d*x)])/B)]))/(B*(
b*c - a*d)^2*g^3)
```

Maple [A]
time = 9.96, size = 117, normalized size = 1.09

method	result	size
derivativedivides	$\frac{e\left(\frac{d\,\mathrm{e}^{\frac{A}{B}}\,\mathrm{expIntegral}\left(1,\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)+\frac{A}{B}\right)}{B}-\frac{be\,\mathrm{e}^{\frac{2A}{B}}\,\mathrm{expIntegral}\left(1,2\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)+\frac{2A}{B}\right)}{B}\right)}{(ad-cb)^2g^3}$	117
default	$\frac{e\left(\frac{d\,\mathrm{e}^{\frac{A}{B}}\,\mathrm{expIntegral}\left(1,\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)+\frac{A}{B}\right)}{B}-\frac{be\,\mathrm{e}^{\frac{2A}{B}}\,\mathrm{expIntegral}\left(1,2\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)+\frac{2A}{B}\right)}{B}\right)}{(ad-cb)^2g^3}$	117

risch	$\frac{ed\,\mathrm{e}^{\frac{A}{B}}\,\mathrm{expIntegral}\left(1,\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)+\frac{A}{B}\right)}{g^3(ad-cb)^2B}-\frac{e^2b\,\mathrm{e}^{\frac{2A}{B}}\,\mathrm{expIntegral}\left(1,2\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)+\frac{2A}{B}\right)}{g^3(ad-cb)^2B}$	131

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*g*x+a*g)^3/(A+B*ln(e*(b*x+a)/(d*x+c))),x,method=_RETURNVERBOSE)
```

```
[Out] e/(a*d-b*c)^2/g^3*(d/B*exp(A/B)*Ei(1,ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))+A/B)-b
*e/B*exp(2*A/B)*Ei(1,2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))+2*A/B))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)^3/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="maxima
")
```

```
[Out] integrate(1/((b*g*x + a*g)^3*(B*log((b*x + a)*e/(d*x + c)) + A)), x)
```

Fricas [A]
time = 0.37, size = 122, normalized size = 1.14

$$-\frac{de^{\left(\frac{A}{B}+1\right)}\operatorname{log\_integral}\left(\frac{(dx+c)e^{\left(-\frac{A}{B}-1\right)}}{bx+a}\right)-be^{\left(\frac{2A}{B}+2\right)}\operatorname{log\_integral}\left(\frac{(d^2x^2+2\,cdx+c^2)e^{\left(-\frac{2A}{B}-2\right)}}{b^2x^2+2\,abx+a^2}\right)}{(Bb^2c^2-2\,Babcd+Ba^2d^2)g^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)^3/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="fricas
")
```

```
[Out] -(d*e^(A/B + 1)*log_integral((d*x + c)*e^(-A/B - 1)/(b*x + a)) - b*e^(2*A/B
 + 2)*log_integral((d^2*x^2 + 2*c*d*x + c^2)*e^(-2*A/B - 2)/(b^2*x^2 + 2*a*
b*x + a^2)))/((B*b^2*c^2 - 2*B*a*b*c*d + B*a^2*d^2)*g^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{Aa^3+3Aa^2bx+3Aab^2x^2+Ab^3x^3+Ba^3\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)+3Ba^2bx\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)+3Bab^2x^2\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)+Bb^3x^3\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}\,dx}{g^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)**3/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

```
[Out] Integral(1/(A*a**3 + 3*A*a**2*b*x + 3*A*a*b**2*x**2 + A*b**3*x**3 + B*a**3*
log(a*e/(c + d*x) + b*e*x/(c + d*x)) + 3*B*a**2*b*x*log(a*e/(c + d*x) + b*e
```

```
*x/(c + d*x)) + 3*B*a*b**2*x**2*log(a*e/(c + d*x) + b*e*x/(c + d*x)) + B*b*
*3*x**3*log(a*e/(c + d*x) + b*e*x/(c + d*x))), x)/g**3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)^3/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="giac")
[Out] integrate(1/((b*g*x + a*g)^3*(B*log((b*x + a)*e/(d*x + c)) + A)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a\,g+b\,g\,x)^3\,\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*g + b*g*x)^3*(A + B*log((e*(a + b*x))/(c + d*x)))),x)
[Out] int(1/((a*g + b*g*x)^3*(A + B*log((e*(a + b*x))/(c + d*x)))), x)
```

3.114 $\int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$

Optimal. Leaf size=35

$$\text{Int}\left(\frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2},x\right)$$

```
[Out] Unintegrable((b*g*x+a*g)^2/(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a*g + b*g*x)^2/(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]
```

```
[Out] Defer[Int][(a*g + b*g*x)^2/(A + B*Log[(e*(a + b*x))/(c + d*x)])^2, x]
```

Rubi steps

$$\int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx = \int \left(\frac{a^2g^2}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}+\frac{2abg^2x}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}+\frac{b^2g^2x^2}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\right)dx$$
$$= \left(a^2g^2\right)\int \frac{1}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx+\left(2abg^2\right)\int \frac{x}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}$$

Mathematica [A]
time = 1.06, size = 0, normalized size = 0.00

$$\int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a*g + b*g*x)^2/(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]
```

```
[Out] Integrate[(a*g + b*g*x)^2/(A + B*Log[(e*(a + b*x))/(c + d*x)])^2, x]
```

Maple [A]
time = 0.67, size = 0, normalized size = 0.00

$$\int \frac{(bgx+ag)^2}{\left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*g*x+a*g)^2/(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

```
[Out] int((b*g*x+a*g)^2/(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^2/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="maxima
")
```

```
[Out] -(b^3*d*g^2*x^4 + a^3*c*g^2 + (b^3*c*g^2 + 3*a*b^2*d*g^2)*x^3 + 3*(a*b^2*c*
g^2 + a^2*b*d*g^2)*x^2 + (3*a^2*b*c*g^2 + a^3*d*g^2)*x)/((b*c - a*d)*B^2*lo
g(b*x + a) - (b*c - a*d)*B^2*log(d*x + c) + (b*c - a*d)*A*B + (b*c - a*d)*B
^2) + integrate((4*b^3*d*g^2*x^3 + 3*a^2*b*c*g^2 + a^3*d*g^2 + 3*(b^3*c*g^2
 + 3*a*b^2*d*g^2)*x^2 + 6*(a*b^2*c*g^2 + a^2*b*d*g^2)*x)/((b*c - a*d)*B^2*l
og(b*x + a) - (b*c - a*d)*B^2*log(d*x + c) + (b*c - a*d)*A*B + (b*c - a*d)*
B^2), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^2/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="fricas
")
```

```
[Out] integral((b^2*g^2*x^2 + 2*a*b*g^2*x + a^2*g^2)/(B^2*log((b*x + a)*e/(d*x +
c))^2 + 2*A*B*log((b*x + a)*e/(d*x + c)) + A^2), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)**2/(A+B*ln(e*(b*x+a)/(d*x+c)))**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^2/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="giac")
```

```
[Out] integrate((b*g*x + a*g)^2/(B*log((b*x + a)*e/(d*x + c)) + A)^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{(a\,g+b\,g\,x)^2}{\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*g + b*g*x)^2/(A + B*log((e*(a + b*x))/(c + d*x)))^2,x)
```

```
[Out] int((a*g + b*g*x)^2/(A + B*log((e*(a + b*x))/(c + d*x)))^2, x)
```

3.115 $\int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$

Optimal. Leaf size=33

$$\mathrm{Int}\left(\frac{ag+bgx}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2},x\right)$$

```
[Out] Unintegrable((b*g*x+a*g)/(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a*g + b*g*x)/(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]
[Out] Defer[Int][(a*g + b*g*x)/(A + B*Log[(e*(a + b*x))/(c + d*x)])^2, x]
```

Rubi steps

$$\begin{aligned}\int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx &= \int \left(\frac{ag}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}+\frac{bgx}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\right)dx\\ &= (ag)\int \frac{1}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx+(bg)\int \frac{x}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx\end{aligned}$$

Mathematica [A]
time = 0.62, size = 0, normalized size = 0.00

$$\int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a*g + b*g*x)/(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]
```

```
[Out] Integrate[(a*g + b*g*x)/(A + B*Log[(e*(a + b*x))/(c + d*x)])^2, x]
```

Maple [A]
time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{bgx + ag}{\left(A + B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*g*x+a*g)/(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

```
[Out] int((b*g*x+a*g)/(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="maxima")
```

```
[Out] -(b^2*d*g*x^3 + a^2*c*g + (b^2*c*g + 2*a*b*d*g)*x^2 + (2*a*b*c*g + a^2*d*g)
*x)/((b*c - a*d)*B^2*log(b*x + a) - (b*c - a*d)*B^2*log(d*x + c) + (b*c - a
*d)*A*B + (b*c - a*d)*B^2) + integrate((3*b^2*d*g*x^2 + 2*a*b*c*g + a^2*d*g
 + 2*(b^2*c*g + 2*a*b*d*g)*x)/((b*c - a*d)*B^2*log(b*x + a) - (b*c - a*d)*B
^2*log(d*x + c) + (b*c - a*d)*A*B + (b*c - a*d)*B^2), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="fricas")
```

```
[Out] integral((b*g*x + a*g)/(B^2*log((b*x + a)*e/(d*x + c))^2 + 2*A*B*log((b*x +
 a)*e/(d*x + c)) + A^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^2cg + a^2dgx + 2abcgx + 2abdgx^2 + b^2cgx^2 + b^2dgx^3}{ABad - ABbc + (B^2ad - B^2bc)\log\left(\frac{e(a+bx)}{c+dx}\right)} - \frac{g\left(\int \frac{a^2d}{A+B\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}\,dx + \int \frac{2abc}{A+B\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}\,dx + \int \frac{2b^2cx}{A+B\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}\,dx + \int \frac{3b^2dx^2}{A+B\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}\,dx + \int \frac{4abdx}{A+B\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}\,dx\right)}{B\,(ad - bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)/(A+B*ln(e*(b*x+a)/(d*x+c)))**2,x)
```

```
[Out] (a**2*c*g + a**2*d*g*x + 2*a*b*c*g*x + 2*a*b*d*g*x**2 + b**2*c*g*x**2 + b**
2*d*g*x**3)/(A*B*a*d - A*B*b*c + (B**2*a*d - B**2*b*c)*log(e*(a + b*x)/(c +
 d*x))) - g*(Integral(a**2*d/(A + B*log(a*e/(c + d*x) + b*e*x/(c + d*x))),
x) + Integral(2*a*b*c/(A + B*log(a*e/(c + d*x) + b*e*x/(c + d*x))), x) + In
tegral(2*b**2*c*x/(A + B*log(a*e/(c + d*x) + b*e*x/(c + d*x))), x) + Integr
al(3*b**2*d*x**2/(A + B*log(a*e/(c + d*x) + b*e*x/(c + d*x))), x) + Integra
l(4*a*b*d*x/(A + B*log(a*e/(c + d*x) + b*e*x/(c + d*x))), x))/(B*(a*d - b*c
))
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="giac")
```

```
[Out] integrate((b*g*x + a*g)/(B*log((b*x + a)*e/(d*x + c)) + A)^2, x)
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{a\,g+b\,g\,x}{{\left(A+B\,\ln\left(\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right)\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*g + b*g*x)/(A + B*log((e*(a + b*x))/(c + d*x)))^2,x)
```

```
[Out] int((a*g + b*g*x)/(A + B*log((e*(a + b*x))/(c + d*x)))^2, x)
```

3.116 $\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$

Optimal. Leaf size=35

$$\operatorname{Int}\left(\frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2},x\right)$$

```
[Out] Unintegrable(1/(b*g*x+a*g)/(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((a*g + b*g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2),x]
[Out] Defer[Int][1/((a*g + b*g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2), x]
```

Rubi steps

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx=\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$$

Mathematica [A]
time = 0.50, size = 0, normalized size = 0.00

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((a*g + b*g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2),x]
[Out] Integrate[1/((a*g + b*g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2), x]
```

Maple [A]
time = 1.28, size = 0, normalized size = 0.00

$$\int \frac{1}{(bgx+ag)\left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*g*x+a*g)/(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

```
[Out] int(1/(b*g*x+a*g)/(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="maxima
")
```

```
[Out] d*integrate(1/((b*c*g - a*d*g)*B^2*log(b*x + a) - (b*c*g - a*d*g)*B^2*log(d
*x + c) + (b*c*g - a*d*g)*A*B + (b*c*g - a*d*g)*B^2), x) - (d*x + c)/((b*c*
g - a*d*g)*B^2*log(b*x + a) - (b*c*g - a*d*g)*B^2*log(d*x + c) + (b*c*g - a
*d*g)*A*B + (b*c*g - a*d*g)*B^2)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="fricas
")
```

```
[Out] integral(1/(A^2*b*g*x + A^2*a*g + (B^2*b*g*x + B^2*a*g)*log((b*x + a)*e/(d*
x + c))^2 + 2*(A*B*b*g*x + A*B*a*g)*log((b*x + a)*e/(d*x + c))), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{c+dx}{ABadg-ABbcg+(B^{2}adg-B^{2}bcg)\log\left(\frac{e(a+bx)}{c+dx}\right)}-\frac{d\int\frac{1}{A+B\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}\,dx}{Bg\,(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)/(A+B*ln(e*(b*x+a)/(d*x+c)))**2,x)
```

```
[Out] (c + d*x)/(A*B*a*d*g - A*B*b*c*g + (B**2*a*d*g - B**2*b*c*g)*log(e*(a + b*x
)/(c + d*x))) - d*Integral(1/(A + B*log(a*e/(c + d*x) + b*e*x/(c + d*x))),
x)/(B*g*(a*d - b*c))
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((b*g*x + a*g)*(B*log((b*x + a)*e/(d*x + c)) + A)^2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{(a\,g+b\,g\,x)\,\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*g + b*g*x)*(A + B*log((e*(a + b*x))/(c + d*x)))^2),x)
```

```
[Out] int(1/((a*g + b*g*x)*(A + B*log((e*(a + b*x))/(c + d*x)))^2), x)
```

3.117 $\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$

Optimal. Leaf size=103

$$-\frac{ee^{A/B}\text{Ei}\left(-\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{B}\right)}{B^2(bc-ad)g^2}-\frac{c+dx}{B(bc-ad)g^2(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}$$

```
[Out] -e*exp(A/B)*Ei((-A-B*ln(e*(b*x+a)/(d*x+c)))/B)/B^2/(-a*d+b*c)/g^2+(-d*x-c)/
B/(-a*d+b*c)/g^2/(b*x+a)/(A+B*ln(e*(b*x+a)/(d*x+c)))
```

Rubi [A]
time = 0.09, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2550, 2343, 2346, 2209}

$$-\frac{ee^{A/B}\text{Ei}\left(-\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{B}\right)}{B^2g^2(bc-ad)}-\frac{c+dx}{Bg^2(a+bx)(bc-ad)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2),x]
```

```
[Out] -((e*E^(A/B)*ExpIntegralEi[-((A + B*Log[(e*(a + b*x))/(c + d*x)])/B)])/(B^2
*(b*c - a*d)*g^2)) - (c + d*x)/(B*(b*c - a*d)*g^2*(a + b*x)*(A + B*Log[(e*(
a + b*x))/(c + d*x)]))
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2343

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^(p + 1)/(b*d*n*(p + 1))), x] -
Dist[(m + 1)/(b*n*(p + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p + 1), x], x]
/; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]
```

Rule 2346

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> Dist[1/c^
(m + 1), Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x] /; FreeQ
```

```
[{a, b, c, p}, x] && IntegerQ[m]
```

Rule 2550

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(
m + 1)*(g/b)^m, Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x],
 x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] &&
 EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && Eq
Q[b*f - a*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Rubi steps

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx=\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$$

Mathematica [A]
time = 0.13, size = 87, normalized size = 0.84

$$\frac{ee^{A/B}\mathrm{Ei}\left(-\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{B}\right)+\frac{B(c+dx)}{(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}}{B^2(-bc+ad)g^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2),x]
[Out] (e*E^(A/B)*ExpIntegralEi[-((A + B*Log[(e*(a + b*x))/(c + d*x)])/B)] + (B*(c
 + d*x))/((a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])))/(B^2*(-(b*c) + a
*d)*g^2)
```

Maple [A]
time = 10.08, size = 132, normalized size = 1.28

method	result	size
risch	$\frac{dx+c}{(ad-cb)B(bx+a)g^2\left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)}-\frac{e\,\mathrm{e}^{\frac{A}{B}}\,\mathrm{expIntegral}\left(1,\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)+\frac{A}{B}\right)}{g^2B^2(ad-cb)}$	113
derivativedivides	$-\frac{e\left(-\frac{1}{\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)B\left(A+B\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)\right)}+\frac{\mathrm{e}^{\frac{A}{B}}\,\mathrm{expIntegral}\left(1,\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)+\frac{A}{B}\right)}{B^2}\right)}{(ad-cb)g^2}$	132

default	$-\frac{e\left(-\frac{1}{\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)B\left(A+B\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)\right)}+\frac{e^{\frac{A}{B}}\,\mathrm{expIntegral}\left(1,\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)+\frac{A}{B}\right)}{B^2}\right)}{(ad-cb)g^2}$	132

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*g*x+a*g)^2/(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -e/(a*d-b*c)/g^2*(-1/(b*e/d+(a*d-b*c)*e/d/(d*x+c))/B/(A+B*ln(b*e/d+(a*d-b*c
)*e/d/(d*x+c)))+1/B^2*exp(A/B)*Ei(1,ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))+A/B))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="maxi
ma")
```

```
[Out] -(d*x + c)/((a*b*c*g^2 - a^2*d*g^2)*A*B + (a*b*c*g^2 - a^2*d*g^2)*B^2 + ((b
^2*c*g^2 - a*b*d*g^2)*A*B + (b^2*c*g^2 - a*b*d*g^2)*B^2)*x + ((b^2*c*g^2 -
a*b*d*g^2)*B^2*x + (a*b*c*g^2 - a^2*d*g^2)*B^2)*log(b*x + a) - ((b^2*c*g^2
- a*b*d*g^2)*B^2*x + (a*b*c*g^2 - a^2*d*g^2)*B^2)*log(d*x + c)) + integrate
(-1/(A*B*a^2*g^2 + B^2*a^2*g^2 + (A*B*b^2*g^2 + B^2*b^2*g^2)*x^2 + 2*(A*B*a
*b*g^2 + B^2*a*b*g^2)*x + (B^2*b^2*g^2*x^2 + 2*B^2*a*b*g^2*x + B^2*a^2*g^2)
*log(b*x + a) - (B^2*b^2*g^2*x^2 + 2*B^2*a*b*g^2*x + B^2*a^2*g^2)*log(d*x +
 c)), x)
```

Fricas [A]
time = 0.38, size = 196, normalized size = 1.90

$$-\frac{Bdx+Bc+\left((Bbx+Ba)e^{\left(\frac{A}{B}+1\right)}\log\left(\frac{(bx+a)e}{dx+c}\right)+(Abx+Aa)e^{\left(\frac{A}{B}+1\right)}\right)\mathrm{log\_integral}\left(\frac{(dx+c)e^{\left(-\frac{A}{B}-1\right)}}{bx+a}\right)}{(AB^2b^2c-AB^2abd)g^2x+(AB^2abc-AB^2a^2d)g^2+((B^3b^2c-B^3abd)g^2x+(B^3abc-B^3a^2d)g^2)\log\left(\frac{(bx+a)e}{dx+c}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="fric
as")
```

```
[Out] -(B*d*x + B*c + ((B*b*x + B*a)*e^(A/B + 1)*log((b*x + a)*e/(d*x + c)) + (A*
b*x + A*a)*e^(A/B + 1))*log_integral((d*x + c)*e^(-A/B - 1)/(b*x + a)))/((A
*B^2*b^2*c - A*B^2*a*b*d)*g^2*x + (A*B^2*a*b*c - A*B^2*a^2*d)*g^2 + ((B^3*b
^2*c - B^3*a*b*d)*g^2*x + (B^3*a*b*c - B^3*a^2*d)*g^2)*log((b*x + a)*e/(d*x
 + c)))
```

<u>Sympy</u> [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{c+dx}{ABa^2dg^2 - ABabcg^2 + ABabdg^2x - ABb^2cg^2x + (B^2a^2dg^2 - B^2abcg^2 + B^2abdg^2x - B^2b^2cg^2x)\log\left(\frac{e(a+bx)}{c+dx}\right)} - \frac{\int \frac{1}{Aa^2+2Aabx+Ab^2x^2+Ba^2\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)+2Babx\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)+Bb^2x^2\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}\,dx}{Bg^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)**2/(A+B*ln(e*(b*x+a)/(d*x+c)))**2,x)
```

```
[Out] (c + d*x)/(A*B*a**2*d*g**2 - A*B*a*b*c*g**2 + A*B*a*b*d*g**2*x - A*B*b**2*c
*g**2*x + (B**2*a**2*d*g**2 - B**2*a*b*c*g**2 + B**2*a*b*d*g**2*x - B**2*b*
*2*c*g**2*x)*log(e*(a + b*x)/(c + d*x))) - Integral(1/(A*a**2 + 2*A*a*b*x +
 A*b**2*x**2 + B*a**2*log(a*e/(c + d*x) + b*e*x/(c + d*x)) + 2*B*a*b*x*log(
a*e/(c + d*x) + b*e*x/(c + d*x)) + B*b**2*x**2*log(a*e/(c + d*x) + b*e*x/(c
 + d*x))), x)/(B*g**2)
```

<u>Giac</u> [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="giac
")
```

```
[Out] integrate(1/((b*g*x + a*g)^2*(B*log((b*x + a)*e/(d*x + c)) + A)^2), x)
```

<u>Mupad</u> [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a\,g+b\,g\,x\right)}^2\,{\left(A+B\,\ln\left(\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*g + b*g*x)^2*(A + B*log((e*(a + b*x))/(c + d*x)))^2),x)
```

```
[Out] int(1/((a*g + b*g*x)^2*(A + B*log((e*(a + b*x))/(c + d*x)))^2), x)
```

3.118 $\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$

Optimal. Leaf size=212

$$-\frac{2be^2e^{\frac{2A}{B}}\text{Ei}\left(-\frac{2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{B}\right)}{B^2(bc-ad)^2g^3}+\frac{dee^{A/B}\text{Ei}\left(-\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{B}\right)}{B^2(bc-ad)^2g^3}+\frac{d(c+dx)}{B(bc-ad)^2g^3(a+bx)\left(A+B\log\left(\frac{e(a-}{c+}\right.\right.}$$

```
[Out] -2*b*e^2*exp(2*A/B)*Ei(-2*(A+B*ln(e*(b*x+a)/(d*x+c)))/B)/B^2/(-a*d+b*c)^2/g
^3+d*e*exp(A/B)*Ei((-A-B*ln(e*(b*x+a)/(d*x+c)))/B)/B^2/(-a*d+b*c)^2/g^3+d*(
d*x+c)/B/(-a*d+b*c)^2/g^3/(b*x+a)/(A+B*ln(e*(b*x+a)/(d*x+c)))-b*(d*x+c)^2/B
/(-a*d+b*c)^2/g^3/(b*x+a)^2/(A+B*ln(e*(b*x+a)/(d*x+c)))
```

Rubi [A]
time = 0.21, antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {2550, 2395, 2343, 2346, 2209}

$$-\frac{2be^2e^{\frac{2A}{B}}\text{Ei}\left(-\frac{2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{B}\right)}{B^2g^3(bc-ad)^2}+\frac{dee^{A/B}\text{Ei}\left(-\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{B}\right)}{B^2g^3(bc-ad)^2}-\frac{b(c+dx)^2}{Bg^3(a+bx)^2(bc-ad)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}+\frac{d(c+dx)}{Bg^3(a+bx)(bc-ad)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a*g + b*g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2),x]
```

```
[Out] (-2*b*e^2*E^((2*A)/B)*ExpIntegralEi[(-2*(A + B*Log[(e*(a + b*x))/(c + d*x)]
))/B])/(B^2*(b*c - a*d)^2*g^3) + (d*e*E^(A/B)*ExpIntegralEi[-((A + B*Log[(e
*(a + b*x))/(c + d*x)])/B)])/(B^2*(b*c - a*d)^2*g^3) + (d*(c + d*x))/(B*(b*
c - a*d)^2*g^3*(a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])) - (b*(c + d*
x)^2)/(B*(b*c - a*d)^2*g^3*(a + b*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])
)
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2343

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^(p + 1)/(b*d*n*(p + 1))), x] -
Dist[(m + 1)/(b*n*(p + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p + 1), x], x]
/; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]
```

Rule 2346

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> Dist[1/c^
(m + 1), Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x] /; FreeQ
[{a, b, c, p}, x] && IntegerQ[m]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2550

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(
m + 1)*(g/b)^m, Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x],
 x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] &&
 EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && Eq
Q[b*f - a*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Rubi steps

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx=\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$$

Mathematica [A]
time = 0.47, size = 136, normalized size = 0.64

$$\frac{-2be^2e^{\frac{2A}{B}}\mathrm{Ei}\left(-\frac{2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{B}\right)+dee^{A/B}\mathrm{Ei}\left(-\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{B}\right)-\frac{B(bc-ad)(c+dx)}{(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}}{B^2(bc-ad)^2g^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a*g + b*g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2),x]
```

```
[Out] (-2*b*e^2*E^((2*A)/B)*ExpIntegralEi[(-2*(A + B*Log[(e*(a + b*x))/(c + d*x)]
))/B] + d*e*E^(A/B)*ExpIntegralEi[-((A + B*Log[(e*(a + b*x))/(c + d*x)])/B)
] - (B*(b*c - a*d)*(c + d*x))/((a + b*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*
x)])))/(B^2*(b*c - a*d)^2*g^3)
```

Maple [A]
time = 13.54, size = 258, normalized size = 1.22

method	result
risch	$\frac{dx+c}{(ad-cb)B(bx+a)^2g^3\left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)}-\frac{ed\,e^{\frac{A}{B}}\,\mathrm{expIntegral}\left(1,\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)+\frac{A}{B}\right)}{g^3B^2(ad-cb)^2}+\frac{2e^2b\,e^{\frac{2A}{B}}\,\mathrm{expIntegral}\left(1,\cdots\right.}{g^3B^2(\cdots}$
derivativedivides	$\frac{e\left(-d\left(-\frac{1}{\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)B\left(A+B\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)\right)}+\frac{e^{\frac{A}{B}}\,\mathrm{expIntegral}\left(1,\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)+\frac{A}{B}\right)}{B^2}\right)+be\left(-\frac{\cdots}{\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right.\cdots}\right.\right.}{(ad-cb)^2g^3}$
default	$\frac{e\left(-d\left(-\frac{1}{\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)B\left(A+B\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)\right)}+\frac{e^{\frac{A}{B}}\,\mathrm{expIntegral}\left(1,\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)+\frac{A}{B}\right)}{B^2}\right)+be\left(-\frac{\cdots}{\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right.\cdots}\right.\right.}{(ad-cb)^2g^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*g*x+a*g)^3/(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x,method=_RETURNVERBOSE)
```

```
[Out] e/(a*d-b*c)^2/g^3*(-d*(-1/(b*e/d+(a*d-b*c)*e/d/(d*x+c))/B/(A+B*ln(b*e/d+(a*
d-b*c)*e/d/(d*x+c)))+1/B^2*exp(A/B)*Ei(1,ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))+A/
B))+b*e*(-1/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2/B/(A+B*ln(b*e/d+(a*d-b*c)*e/d/(
d*x+c)))+2/B^2*exp(2*A/B)*Ei(1,2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))+2*A/B)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)^3/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="maxi
ma")
```

```
[Out] -(d*x + c)/((a^2*b*c*g^3 - a^3*d*g^3)*A*B + (a^2*b*c*g^3 - a^3*d*g^3)*B^2 +
 ((b^3*c*g^3 - a*b^2*d*g^3)*A*B + (b^3*c*g^3 - a*b^2*d*g^3)*B^2)*x^2 + 2*((
a*b^2*c*g^3 - a^2*b*d*g^3)*A*B + (a*b^2*c*g^3 - a^2*b*d*g^3)*B^2)*x + ((b^3
*c*g^3 - a*b^2*d*g^3)*B^2*x^2 + 2*(a*b^2*c*g^3 - a^2*b*d*g^3)*B^2*x + (a^2*
b*c*g^3 - a^3*d*g^3)*B^2)*log(b*x + a) - ((b^3*c*g^3 - a*b^2*d*g^3)*B^2*x^2
 + 2*(a*b^2*c*g^3 - a^2*b*d*g^3)*B^2*x + (a^2*b*c*g^3 - a^3*d*g^3)*B^2)*log
(d*x + c)) - integrate((b*d*x + 2*b*c - a*d)/(((b^4*c*g^3 - a*b^3*d*g^3)*A*
B + (b^4*c*g^3 - a*b^3*d*g^3)*B^2)*x^3 + (a^3*b*c*g^3 - a^4*d*g^3)*A*B + (a
^3*b*c*g^3 - a^4*d*g^3)*B^2 + 3*((a*b^3*c*g^3 - a^2*b^2*d*g^3)*A*B + (a*b^3
*c*g^3 - a^2*b^2*d*g^3)*B^2)*x^2 + 3*((a^2*b^2*c*g^3 - a^3*b*d*g^3)*A*B + (
a^2*b^2*c*g^3 - a^3*b*d*g^3)*B^2)*x + ((b^4*c*g^3 - a*b^3*d*g^3)*B^2*x^3 +
3*(a*b^3*c*g^3 - a^2*b^2*d*g^3)*B^2*x^2 + 3*(a^2*b^2*c*g^3 - a^3*b*d*g^3)*B
^2*x + (a^3*b*c*g^3 - a^4*d*g^3)*B^2)*log(b*x + a) - ((b^4*c*g^3 - a*b^3*d*
```

```
g^3)*B^2*x^3 + 3*(a*b^3*c*g^3 - a^2*b^2*d*g^3)*B^2*x^2 + 3*(a^2*b^2*c*g^3 -
 a^3*b*d*g^3)*B^2*x + (a^3*b*c*g^3 - a^4*d*g^3)*B^2)*log(d*x + c)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 542 vs. $2(214) = 428$.
time = 0.35, size = 542, normalized size = 2.56

$$-\frac{Bbc^2 - Bacd + (Bbcd - Bad^2)x - \left((Bb^2dx^2 + 2\,Babdx + Ba^2d)e^{\left(\frac{A}{B}+1\right)}\log\left(\frac{(bx+a)e}{dx+c}\right) + (Ab^2dx^2 + 2\,Aabdx + Aa^2d)e^{\left(\frac{A}{B}+1\right)}\right)\mathrm{log\_integral}\left(\frac{(dx+c)e^{\left(-\frac{A}{B}-1\right)}}{bx+a}\right) + 2\left((Bb^3x^2 + 2\,Bab^2x + Ba^2b)e^{\left(\frac{2\,A}{B}+2\right)}\log\left(\frac{(bx+a)e}{dx+c}\right) + (Ab^3x^2 + 2\,Aab^2x + Aa^2b)e^{\left(\frac{2\,A}{B}+2\right)}\right)\mathrm{log\_integral}\left(\frac{(d^2x^2+2\,cdx+c^2)e^{\left(-\frac{2\,A}{B}-2\right)}}{b^2x^2+2\,abx+a^2}\right)}{(AB^2b^4c^2 - 2\,AB^2ab^3cd + AB^2a^2b^2d^2)g^3x^2 + 2\,(AB^2ab^3c^2 - 2\,AB^2a^2b^2cd + AB^2a^3bd^2)g^3x + (AB^2a^2b^2c^2 - 2\,AB^2a^3bcd + AB^2a^4d^2)g^3 + ((B^3b^4c^2 - 2\,B^3ab^3cd + B^3a^2b^2d^2)g^3x^2 + 2\,(B^3ab^3c^2 - 2\,B^3a^2b^2cd + B^3a^3bd^2)g^3x + (B^3a^2b^2c^2 - 2\,B^3a^3bcd + B^3a^4d^2)g^3)\log\left(\frac{(bx+a)e}{dx+c}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)^3/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="fric
as")
```

```
[Out] -(B*b*c^2 - B*a*c*d + (B*b*c*d - B*a*d^2)*x - ((B*b^2*d*x^2 + 2*B*a*b*d*x +
 B*a^2*d)*e^(A/B + 1)*log((b*x + a)*e/(d*x + c)) + (A*b^2*d*x^2 + 2*A*a*b*d
*x + A*a^2*d)*e^(A/B + 1))*log_integral((d*x + c)*e^(-A/B - 1)/(b*x + a)) +
 2*((B*b^3*x^2 + 2*B*a*b^2*x + B*a^2*b)*e^(2*A/B + 2)*log((b*x + a)*e/(d*x
+ c)) + (A*b^3*x^2 + 2*A*a*b^2*x + A*a^2*b)*e^(2*A/B + 2))*log_integral((d^
2*x^2 + 2*c*d*x + c^2)*e^(-2*A/B - 2)/(b^2*x^2 + 2*a*b*x + a^2)))/((A*B^2*b
^4*c^2 - 2*A*B^2*a*b^3*c*d + A*B^2*a^2*b^2*d^2)*g^3*x^2 + 2*(A*B^2*a*b^3*c^
2 - 2*A*B^2*a^2*b^2*c*d + A*B^2*a^3*b*d^2)*g^3*x + (A*B^2*a^2*b^2*c^2 - 2*A
*B^2*a^3*b*c*d + A*B^2*a^4*d^2)*g^3 + ((B^3*b^4*c^2 - 2*B^3*a*b^3*c*d + B^3
*a^2*b^2*d^2)*g^3*x^2 + 2*(B^3*a*b^3*c^2 - 2*B^3*a^2*b^2*c*d + B^3*a^3*b*d^
2)*g^3*x + (B^3*a^2*b^2*c^2 - 2*B^3*a^3*b*c*d + B^3*a^4*d^2)*g^3)*log((b*x
+ a)*e/(d*x + c)))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)**3/(A+B*ln(e*(b*x+a)/(d*x+c)))**2,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)^3/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="giac
")
```

```
[Out] integrate(1/((b*g*x + a*g)^3*(B*log((b*x + a)*e/(d*x + c)) + A)^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(a\,g+b\,g\,x)^3\,\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*g + b*g*x)^3*(A + B*log((e*(a + b*x))/(c + d*x)))^2),x)
[Out] int(1/((a*g + b*g*x)^3*(A + B*log((e*(a + b*x))/(c + d*x)))^2), x)
```

3.119 $\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right) dx$

Optimal. Leaf size=182

$$\frac{2B(bc-ad)^4g^4x}{5d^4}-\frac{B(bc-ad)^3g^4(a+bx)^2}{5bd^3}+\frac{2B(bc-ad)^2g^4(a+bx)^3}{15bd^2}-\frac{B(bc-ad)g^4(a+bx)^4}{10bd}+\frac{g^4(a+bx)}{}$$

```
[Out] 2/5*B*(-a*d+b*c)^4*g^4*x/d^4-1/5*B*(-a*d+b*c)^3*g^4*(b*x+a)^2/b/d^3+2/15*B*
(-a*d+b*c)^2*g^4*(b*x+a)^3/b/d^2-1/10*B*(-a*d+b*c)*g^4*(b*x+a)^4/b/d+1/5*g^
4*(b*x+a)^5*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/b-2/5*B*(-a*d+b*c)^5*g^4*ln(d*x
+c)/b/d^5
```

Rubi [A]

time = 0.07, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {2548, 21, 45}

$$\frac{g^4(a+bx)^5\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{5b}-\frac{2Bg^4(bc-ad)^5\log(c+dx)}{5bd^5}+\frac{2Bg^4x(bc-ad)^4}{5d^4}-\frac{Bg^4(a+bx)^2(bc-ad)^3}{5bd^3}+\frac{2Bg^4(a+bx)^3(bc-ad)^2}{15bd^2}-\frac{Bg^4(a+bx)^4(bc-ad)}{10bd}$$

Antiderivative was successfully verified.

```
[In] Int[(a*g + b*g*x)^4*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]),x]
```

```
[Out] (2*B*(b*c - a*d)^4*g^4*x)/(5*d^4) - (B*(b*c - a*d)^3*g^4*(a + b*x)^2)/(5*b*
d^3) + (2*B*(b*c - a*d)^2*g^4*(a + b*x)^3)/(15*b*d^2) - (B*(b*c - a*d)*g^4*
(a + b*x)^4)/(10*b*d) + (g^4*(a + b*x)^5*(A + B*Log[(e*(a + b*x)^2)/(c + d*
x)^2]))/(5*b) - (2*B*(b*c - a*d)^5*g^4*Log[c + d*x])/(5*b*d^5)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2548

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
```

```
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Dist[B*n*((b*c
- a*d)/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /;
 FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c -
 a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Rubi steps

$$\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right) dx = \frac{g^4(a+bx)^5\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{5b} - \frac{B\int \frac{2(bc-ad)g^5(a+bx)^4}{c+dx}}{5bg}$$

$$= \frac{g^4(a+bx)^5\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{5b} - \frac{(2B(bc-ad)g^4)\int \frac{(a}{}}{5b}$$

$$= \frac{g^4(a+bx)^5\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{5b} - \frac{(2B(bc-ad)g^4)\int \Big(}{}$$

$$= \frac{2B(bc-ad)^4g^4x}{5d^4} - \frac{B(bc-ad)^3g^4(a+bx)^2}{5bd^3} + \frac{2B(bc-ad)}{15}$$

Mathematica [A]
time = 0.07, size = 144, normalized size = 0.79

$$\frac{g^4\left((a+bx)^5\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)+\frac{B(bc-ad)\left(12bd(bc-ad)^3x-6d^2(bc-ad)^2(a+bx)^2+4d^3(bc-ad)(a+bx)^3-3d^4(a+bx)^4-12(bc-ad)^4\log(c+dx)\right)}{6d^5}\right)}{5b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*g + b*g*x)^4*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]),x]
```

```
[Out] (g^4*((a + b*x)^5*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]) + (B*(b*c - a*d)
*(12*b*d*(b*c - a*d)^3*x - 6*d^2*(b*c - a*d)^2*(a + b*x)^2 + 4*d^3*(b*c - a
*d)*(a + b*x)^3 - 3*d^4*(a + b*x)^4 - 12*(b*c - a*d)^4*Log[c + d*x]))/(6*d^
5)))/(5*b)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1957 vs. 2(170) = 340.
time = 0.34, size = 1958, normalized size = 10.76

method	result
risch	$\frac{8g^4B\,a^4x}{5}+\frac{2g^4b^4B\,c^4x}{5d^4}-\frac{2g^4b^4B\ln(dx+c)c^5}{5d^5}-\frac{2g^4B\ln(dx+c)a^4c}{d}+\frac{g^4b^3Ba\,x^4}{10}+g^4b^3Aa\,x^4-\frac{g^4b^4Bc\,x^4}{10d}$
derivativedivides	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*g*x+a*g)^4*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d*(-30*B*g^4/d^2*b^3/(a*d-b*c)*ln(a/(d*x+c)*d-b*c/(d*x+c)+b)*a^2*c^4+4*B
*g^4/d*(d*x+c)*ln(e*(a/(d*x+c)*d-b*c/(d*x+c)+b)^2/d^2)*a^3*b*c+6*B*g^4/d^2*
(d*x+c)^2*ln(e*(a/(d*x+c)*d-b*c/(d*x+c)+b)^2/d^2)*a^2*b^2*c+12*B*g^4/d^3/(a
*d-b*c)*ln(a/(d*x+c)*d-b*c/(d*x+c)+b)*a*c^5*b^4-6*B*g^4/d^2*(d*x+c)*ln(e*(a
/(d*x+c)*d-b*c/(d*x+c)+b)^2/d^2)*a^2*b^2*c^2+4*B*g^4/d^3*(d*x+c)*ln(e*(a/(d
*x+c)*d-b*c/(d*x+c)+b)^2/d^2)*a*b^3*c^3-6*B*g^4/d^3*(d*x+c)^2*ln(e*(a/(d*x+
c)*d-b*c/(d*x+c)+b)^2/d^2)*a*b^3*c^2+4*B*g^4/d^3*b^3*(d*x+c)^3*ln(e*(a/(d*x
+c)*d-b*c/(d*x+c)+b)^2/d^2)*a*c+40*B*g^4/d*b^2/(a*d-b*c)*ln(a/(d*x+c)*d-b*c
/(d*x+c)+b)*a^3*c^3+12*B*g^4*d/(a*d-b*c)*ln(a/(d*x+c)*d-b*c/(d*x+c)+b)*a^5*
c+4*B*g^4/d*b*a^3*ln(1/(d*x+c))*c^2-4*B*g^4/d^2*b^2*a^2*ln(1/(d*x+c))*c^3-8
/5*B*g^4/d^4*b^4*c^4*(d*x+c)-2/5*B*g^4/d^4*b^4*c^5*ln(1/(d*x+c))+1/10*B*g^4
/d^4*b^4*c*(d*x+c)^4-B*g^4*(d*x+c)*ln(e*(a/(d*x+c)*d-b*c/(d*x+c)+b)^2/d^2)*
a^4-8*B*g^4*a^4*ln(a/(d*x+c)*d-b*c/(d*x+c)+b)*c-2*B*g^4*a^4*ln(1/(d*x+c))*c
-8/15*B*g^4/d^4*b^4*c^2*(d*x+c)^3+6/5*B*g^4/d^4*b^4*c^3*(d*x+c)^2-1/5*B*g^4
/d^4*b^4*(d*x+c)^5*ln(e*(a/(d*x+c)*d-b*c/(d*x+c)+b)^2/d^2)-6/5*B*g^4/d*b*a^
3*(d*x+c)^2+2/5*B*g^4*d/b*a^5*ln(1/(d*x+c))-8/5*B*g^4/d^4*b^4*c^5*ln(a/(d*x
+c)*d-b*c/(d*x+c)+b)+8/5*B*g^4*d/b*a^5*ln(a/(d*x+c)*d-b*c/(d*x+c)+b)-1/10*B
*g^4/d^3*b^3*a*(d*x+c)^4-8/15*B*g^4/d^2*b^2*a^2*(d*x+c)^3-8/5*B*g^4*a^4*(d*
x+c)+A*g^4/d^4*(-1/5*b^4*(d*x+c)^5-2*b*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d
-b^3*c^3)*(d*x+c)^2-b^3*(a*d-b*c)*(d*x+c)^4-2*b^2*(a^2*d^2-2*a*b*c*d+b^2*c^
2)*(d*x+c)^3-(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4
)*(d*x+c))+B*g^4/d^4*b^4*(d*x+c)^4*ln(e*(a/(d*x+c)*d-b*c/(d*x+c)+b)^2/d^2)*
c+16/15*B*g^4/d^3*b^3*a*(d*x+c)^3*c+32/5*B*g^4/d*b*a^3*(d*x+c)*c+16*B*g^4/d
*b*a^3*ln(a/(d*x+c)*d-b*c/(d*x+c)+b)*c^2-16*B*g^4/d^2*b^2*a^2*ln(a/(d*x+c)*
d-b*c/(d*x+c)+b)*c^3-B*g^4/d^3*b^3*(d*x+c)^4*ln(e*(a/(d*x+c)*d-b*c/(d*x+c)+
b)^2/d^2)*a-2*B*g^4/d^4*b^4*(d*x+c)^3*ln(e*(a/(d*x+c)*d-b*c/(d*x+c)+b)^2/d^
2)*c^2+32/5*B*g^4/d^3*b^3*a*(d*x+c)*c^3-18/5*B*g^4/d^3*b^3*a*(d*x+c)^2*c^2-
48/5*B*g^4/d^2*b^2*a^2*(d*x+c)*c^2+2*B*g^4/d^3*b^3*a*ln(1/(d*x+c))*c^4-2*B*
g^4*d^2/b/(a*d-b*c)*ln(a/(d*x+c)*d-b*c/(d*x+c)+b)*a^6-30*B*g^4*b/(a*d-b*c)*
ln(a/(d*x+c)*d-b*c/(d*x+c)+b)*a^4*c^2-B*g^4/d^4*(d*x+c)*ln(e*(a/(d*x+c)*d-b
*c/(d*x+c)+b)^2/d^2)*b^4*c^4-2*B*g^4/d^4/(a*d-b*c)*ln(a/(d*x+c)*d-b*c/(d*x+
c)+b)*c^6*b^5+8*B*g^4/d^3*b^3*a*ln(a/(d*x+c)*d-b*c/(d*x+c)+b)*c^4+2*B*g^4/d
^4*(d*x+c)^2*ln(e*(a/(d*x+c)*d-b*c/(d*x+c)+b)^2/d^2)*b^4*c^3-2*B*g^4/d^2*b^
2*(d*x+c)^3*ln(e*(a/(d*x+c)*d-b*c/(d*x+c)+b)^2/d^2)*a^2+18/5*B*g^4/d^2*b^2*
a^2*(d*x+c)^2*c-2*B*g^4/d*b*(d*x+c)^2*ln(e*(a/(d*x+c)*d-b*c/(d*x+c)+b)^2/d^
2)*a^3)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 900 vs. $2(171) = 342$.

time = 0.33, size = 900, normalized size = 4.95

$$\frac{1}{5}\,Ab^4g^4x^5 + Aab^3g^4x^4 + 2\,Aa^2b^2g^4x^3 + 2\,Aa^3bg^4x^2 + \left(x\log\left(\frac{b^2x^2e}{d^2x^2+2\,cdx+c^2} + \frac{2\,abxe}{d^2x^2+2\,cdx+c^2} + \frac{a^2e}{d^2x^2+2\,cdx+c^2}\right) + \frac{2\,a\log\left(bx+a\right)}{b} - \frac{2\,c\log\left(dx+c\right)}{d}\right)Ba^4g^4 + 2\left(x^2\log\left(\frac{b^2x^2e}{d^2x^2+2\,cdx+c^2} + \frac{2\,abxe}{d^2x^2+2\,cdx+c^2} + \frac{a^2e}{d^2x^2+2\,cdx+c^2}\right) - \frac{2\,a^2\log\left(bx+a\right)}{b^2} + \frac{2\,c^2\log\left(dx+c\right)}{d^2} - \frac{2\,(bc-ad)x}{bd}\right)Ba^3bg^4 + 2\left(x^3\log\left(\frac{b^2x^2e}{d^2x^2+2\,cdx+c^2} + \frac{2\,abxe}{d^2x^2+2\,cdx+c^2} + \frac{a^2e}{d^2x^2+2\,cdx+c^2}\right) + \frac{2\,a^3\log\left(bx+a\right)}{b^3} - \frac{2\,c^3\log\left(dx+c\right)}{d^3} - \frac{(b^2cd-abd^2)x^2 - 2\,(b^2c^2-a^2d^2)x}{b^2d^2}\right)Ba^2b^2g^4 + \frac{1}{3}\left(3\,x^4\log\left(\frac{b^2x^2e}{d^2x^2+2\,cdx+c^2} + \frac{2\,abxe}{d^2x^2+2\,cdx+c^2} + \frac{a^2e}{d^2x^2+2\,cdx+c^2}\right) - \frac{6\,a^4\log\left(bx+a\right)}{b^4} + \frac{6\,c^4\log\left(dx+c\right)}{d^4} - \frac{2\,(b^3cd^2-ab^2d^3)x^3 - 3\,(b^3c^2d-a^2bd^3)x^2 + 6\,(b^3c^3-a^3d^3)x}{b^3d^3}\right)Bab^3g^4 + \frac{1}{30}\left(6\,x^5\log\left(\frac{b^2x^2e}{d^2x^2+2\,cdx+c^2} + \frac{2\,abxe}{d^2x^2+2\,cdx+c^2} + \frac{a^2e}{d^2x^2+2\,cdx+c^2}\right) + \frac{12\,a^5\log\left(bx+a\right)}{b^5} - \frac{12\,c^5\log\left(dx+c\right)}{d^5} - \frac{3\,(b^4cd^3-ab^3d^4)x^4 - 4\,(b^4c^2d^2-a^2b^2d^4)x^3 + 6\,(b^4c^3d-a^3bd^4)x^2 - 12\,(b^4c^4-a^4d^4)x}{b^4d^4}\right)Bb^4g^4 + Aa^4g^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^4*(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="maxi
ma")
```

```
[Out] 1/5*A*b^4*g^4*x^5 + A*a*b^3*g^4*x^4 + 2*A*a^2*b^2*g^4*x^3 + 2*A*a^3*b*g^4*x
^2 + (x*log(b^2*x^2*e/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*x*e/(d^2*x^2 + 2*c*
d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) + 2*a*log(b*x + a)/b - 2*c*lo
g(d*x + c)/d)*B*a^4*g^4 + 2*(x^2*log(b^2*x^2*e/(d^2*x^2 + 2*c*d*x + c^2) +
2*a*b*x*e/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) - 2*
a^2*log(b*x + a)/b^2 + 2*c^2*log(d*x + c)/d^2 - 2*(b*c - a*d)*x/(b*d))*B*a^
3*b*g^4 + 2*(x^3*log(b^2*x^2*e/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*x*e/(d^2*x
^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) + 2*a^3*log(b*x + a)
/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2
*d^2)*x)/(b^2*d^2))*B*a^2*b^2*g^4 + 1/3*(3*x^4*log(b^2*x^2*e/(d^2*x^2 + 2*c
*d*x + c^2) + 2*a*b*x*e/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*
x + c^2)) - 6*a^4*log(b*x + a)/b^4 + 6*c^4*log(d*x + c)/d^4 - (2*(b^3*c*d^2
 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x
)/(b^3*d^3))*B*a*b^3*g^4 + 1/30*(6*x^5*log(b^2*x^2*e/(d^2*x^2 + 2*c*d*x + c
^2) + 2*a*b*x*e/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)
) + 12*a^5*log(b*x + a)/b^5 - 12*c^5*log(d*x + c)/d^5 - (3*(b^4*c*d^3 - a*b
^3*d^4)*x^4 - 4*(b^4*c^2*d^2 - a^2*b^2*d^4)*x^3 + 6*(b^4*c^3*d - a^3*b*d^4)
*x^2 - 12*(b^4*c^4 - a^4*d^4)*x)/(b^4*d^4))*B*b^4*g^4 + A*a^4*g^4*x
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 452 vs. $2(171) = 342$.
time = 0.43, size = 452, normalized size = 2.48

$$\frac{6\,Ab^5d^5g^4x^5 + 12\,Ba^5d^5g^4\log\left(bx+a\right) - 3\,(Bb^5cd^4 - (10\,A+B)ab^4d^5)g^4x^4 + 4\,(Bb^5c^2d^3 - 5\,Bab^4cd^4 + (15\,A+4\,B)a^2b^3d^5)g^4x^3 - 6\,(Bb^5c^3d^2 - 5\,Bab^4c^2d^3 + 10\,Ba^2b^3cd^4 - 2\,(5\,A+3\,B)a^3b^2d^5)g^4x^2 + 6\,(2\,Bb^5c^4d - 10\,Bab^4c^3d^2 + 20\,Ba^2b^3c^2d^3 - 20\,Ba^3b^2cd^4 + (5\,A+8\,B)a^4bd^5)g^4x - 12\,(Bb^5c^5 - 5\,Bab^4c^4d + 10\,Ba^2b^3c^3d^2 - 10\,Ba^3b^2c^2d^3 + 5\,Ba^4bcd^4)g^4\log\left(dx+c\right) + 6\,(Bb^5d^5g^4x^5 + 5\,Bab^4d^5g^4x^4 + 10\,Ba^2b^3d^5g^4x^3 + 10\,Ba^3b^2d^5g^4x^2 + 5\,Ba^4bd^5g^4x)\log\left(\frac{(b^2x^2+2\,abx+a^2)e}{d^2x^2+2\,cdx+c^2}\right)}{30\,bd^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^4*(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="fric
as")
```

```
[Out] 1/30*(6*A*b^5*d^5*g^4*x^5 + 12*B*a^5*d^5*g^4*log(b*x + a) - 3*(B*b^5*c*d^4
- (10*A + B)*a*b^4*d^5)*g^4*x^4 + 4*(B*b^5*c^2*d^3 - 5*B*a*b^4*c*d^4 + (15*
A + 4*B)*a^2*b^3*d^5)*g^4*x^3 - 6*(B*b^5*c^3*d^2 - 5*B*a*b^4*c^2*d^3 + 10*B
*a^2*b^3*c*d^4 - 2*(5*A + 3*B)*a^3*b^2*d^5)*g^4*x^2 + 6*(2*B*b^5*c^4*d - 10
*B*a*b^4*c^3*d^2 + 20*B*a^2*b^3*c^2*d^3 - 20*B*a^3*b^2*c*d^4 + (5*A + 8*B)*
a^4*b*d^5)*g^4*x - 12*(B*b^5*c^5 - 5*B*a*b^4*c^4*d + 10*B*a^2*b^3*c^3*d^2 -
 10*B*a^3*b^2*c^2*d^3 + 5*B*a^4*b*c*d^4)*g^4*log(d*x + c) + 6*(B*b^5*d^5*g^
4*x^5 + 5*B*a*b^4*d^5*g^4*x^4 + 10*B*a^2*b^3*d^5*g^4*x^3 + 10*B*a^3*b^2*d^5
```

```
*g^4*x^2 + 5*B*a^4*b*d^5*g^4*x)*log((b^2*x^2 + 2*a*b*x + a^2)*e/(d^2*x^2 +
2*c*d*x + c^2)))/(b*d^5)
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 998 vs. $2(163) = 326$.
time = 4.44, size = 998, normalized size = 5.48

$$\frac{Ab^4g^4x^5}{5} + \frac{2Ba^5g^4\log\left(x + \frac{\frac{2Ba^6d^5g^4}{b} + 10Ba^5cd^4g^4 - 20Ba^4bc^2d^3g^4 + 20Ba^3b^2c^3d^2g^4 - 10Ba^2b^3c^4dg^4 + 2Bab^4c^5g^4}{2Ba^5d^5g^4 + 10Ba^4bcd^4g^4 - 20Ba^3b^2c^2d^3g^4 + 20Ba^2b^3c^3d^2g^4 - 10Bab^4c^4dg^4 + 2Bb^5c^5g^4}\right)}{5b} - \frac{2Bcg^4\cdot(5a^4d^4 - 10a^3bcd^3 + 10a^2b^2c^2d^2 - 5ab^3c^3d + b^4c^4)\log\left(x + \frac{12Ba^5cd^4g^4 - 20Ba^4bc^2d^3g^4 + 20Ba^3b^2c^3d^2g^4 - 10Ba^2b^3c^4dg^4 + 2Bab^4c^5g^4 - 2Bacg^4\cdot(5a^4d^4 - 10a^3bcd^3 + 10a^2b^2c^2d^2 - 5ab^3c^3d + b^4c^4) + \frac{2Bbc^2g^4\cdot(5a^4d^4 - 10a^3bcd^3 + 10a^2b^2c^2d^2 - 5ab^3c^3d + b^4c^4)}{d}}{2Ba^5d^5g^4 + 10Ba^4bcd^4g^4 - 20Ba^3b^2c^2d^3g^4 + 20Ba^2b^3c^3d^2g^4 - 10Bab^4c^4dg^4 + 2Bb^5c^5g^4}\right)}{5d^5} + x^4\left(Aab^3g^4 + \frac{Bab^3g^4}{10} - \frac{Bb^4cg^4}{10d}\right) + x^3\cdot\left(2Aa^2b^2g^4 + \frac{8Ba^2b^2g^4}{15} - \frac{2Bab^3cg^4}{3d} + \frac{2Bb^4c^2g^4}{15d^2}\right) + x^2\cdot\left(2Aa^3bg^4 + \frac{6Ba^3bg^4}{5} - \frac{2Ba^2b^2cg^4}{d} + \frac{Bab^3c^2g^4}{d^2} - \frac{Bb^4c^3g^4}{5d^3}\right) + x\left(Aa^4g^4 + \frac{8Ba^4g^4}{5} - \frac{4Ba^3bcg^4}{d} + \frac{4Ba^2b^2c^2g^4}{d^2} - \frac{2Bab^3c^3g^4}{d^3} + \frac{2Bb^4c^4g^4}{5d^4}\right) + \left(Ba^4g^4x + 2Ba^3bg^4x^2 + 2Ba^2b^2g^4x^3 + Bab^3g^4x^4 + \frac{Bb^4g^4x^5}{5}\right)\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)**4*(A+B*ln(e*(b*x+a)**2/(d*x+c)**2)),x)
```

```
[Out] A*b**4*g**4*x**5/5 + 2*B*a**5*g**4*log(x + (2*B*a**6*d**5*g**4/b + 10*B*a**
5*c*d**4*g**4 - 20*B*a**4*b*c**2*d**3*g**4 + 20*B*a**3*b**2*c**3*d**2*g**4
- 10*B*a**2*b**3*c**4*d*g**4 + 2*B*a*b**4*c**5*g**4)/(2*B*a**5*d**5*g**4 +
10*B*a**4*b*c*d**4*g**4 - 20*B*a**3*b**2*c**2*d**3*g**4 + 20*B*a**2*b**3*c*
*3*d**2*g**4 - 10*B*a*b**4*c**4*d*g**4 + 2*B*b**5*c**5*g**4))/(5*b) - 2*B*c
*g**4*(5*a**4*d**4 - 10*a**3*b*c*d**3 + 10*a**2*b**2*c**2*d**2 - 5*a*b**3*c
**3*d + b**4*c**4)*log(x + (12*B*a**5*c*d**4*g**4 - 20*B*a**4*b*c**2*d**3*g
**4 + 20*B*a**3*b**2*c**3*d**2*g**4 - 10*B*a**2*b**3*c**4*d*g**4 + 2*B*a*b*
*4*c**5*g**4 - 2*B*a*c*g**4*(5*a**4*d**4 - 10*a**3*b*c*d**3 + 10*a**2*b**2*
c**2*d**2 - 5*a*b**3*c**3*d + b**4*c**4) + 2*B*b*c**2*g**4*(5*a**4*d**4 - 1
0*a**3*b*c*d**3 + 10*a**2*b**2*c**2*d**2 - 5*a*b**3*c**3*d + b**4*c**4)/d)/
(2*B*a**5*d**5*g**4 + 10*B*a**4*b*c*d**4*g**4 - 20*B*a**3*b**2*c**2*d**3*g*
*4 + 20*B*a**2*b**3*c**3*d**2*g**4 - 10*B*a*b**4*c**4*d*g**4 + 2*B*b**5*c**
5*g**4))/(5*d**5) + x**4*(A*a*b**3*g**4 + B*a*b**3*g**4/10 - B*b**4*c*g**4/
(10*d)) + x**3*(2*A*a**2*b**2*g**4 + 8*B*a**2*b**2*g**4/15 - 2*B*a*b**3*c*g
**4/(3*d) + 2*B*b**4*c**2*g**4/(15*d**2)) + x**2*(2*A*a**3*b*g**4 + 6*B*a**
3*b*g**4/5 - 2*B*a**2*b**2*c*g**4/d + B*a*b**3*c**2*g**4/d**2 - B*b**4*c**3
*g**4/(5*d**3)) + x*(A*a**4*g**4 + 8*B*a**4*g**4/5 - 4*B*a**3*b*c*g**4/d +
4*B*a**2*b**2*c**2*g**4/d**2 - 2*B*a*b**3*c**3*g**4/d**3 + 2*B*b**4*c**4*g*
*4/(5*d**4)) + (B*a**4*g**4*x + 2*B*a**3*b*g**4*x**2 + 2*B*a**2*b**2*g**4*x
**3 + B*a*b**3*g**4*x**4 + B*b**4*g**4*x**5/5)*log(e*(a + b*x)**2/(c + d*x)
**2)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 496 vs. $2(171) = 342$.
time = 255.27, size = 496, normalized size = 2.73

$$\frac{2\,Ba^5g^4\log(bx+a)}{5\,b} + \frac{1}{5}\,(Ab^4g^4 + Bb^4g^4)x^5 - \frac{(Bb^4cg^4 - 10\,Aab^3dg^4 - 11\,Bab^3dg^4)x^4}{10\,d} + \frac{2\,(Bb^4c^2g^4 - 5\,Bab^3cdg^4 + 15\,Aa^2b^2d^2g^4 + 19\,Ba^2b^2d^2g^4)x^3}{15\,d^2} + \frac{1}{5}\,(Bb^4g^4x^5 + 5\,Bab^3g^4x^4 + 10\,Ba^2b^2g^4x^3 + 10\,Ba^3bg^4x^2 + 5\,Ba^4g^4x)\log\left(\frac{b^2x^2 + 2\,abx + a^2}{d^2x^2 + 2\,cdx + c^2}\right) - \frac{(Bb^4c^3g^4 - 5\,Bab^3c^2dg^4 + 10\,Ba^2b^2cd^2g^4 - 10\,Aa^3bd^3g^4 - 16\,Ba^3bd^3g^4)x^2}{5\,d^3} + \frac{(2\,Bb^4c^4g^4 - 10\,Bab^3c^3dg^4 + 20\,Ba^2b^2c^2d^2g^4 - 20\,Ba^3bcd^3g^4 + 5\,Aa^4d^4g^4 + 13\,Ba^4d^4g^4)x}{5\,d^4} - \frac{2\,(Bb^4c^5g^4 - 5\,Bab^3c^4dg^4 + 10\,Ba^2b^2c^3d^2g^4 - 10\,Ba^3bc^2d^3g^4 + 5\,Ba^4cd^4g^4)\log(-dx-c)}{5\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^4*(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="giac
")
```

```
[Out] 2/5*B*a^5*g^4*log(b*x + a)/b + 1/5*(A*b^4*g^4 + B*b^4*g^4)*x^5 - 1/10*(B*b^
4*c*g^4 - 10*A*a*b^3*d*g^4 - 11*B*a*b^3*d*g^4)*x^4/d + 2/15*(B*b^4*c^2*g^4
- 5*B*a*b^3*c*d*g^4 + 15*A*a^2*b^2*d^2*g^4 + 19*B*a^2*b^2*d^2*g^4)*x^3/d^2
+ 1/5*(B*b^4*g^4*x^5 + 5*B*a*b^3*g^4*x^4 + 10*B*a^2*b^2*g^4*x^3 + 10*B*a^3*
b*g^4*x^2 + 5*B*a^4*g^4*x)*log((b^2*x^2 + 2*a*b*x + a^2)/(d^2*x^2 + 2*c*d*x
 + c^2)) - 1/5*(B*b^4*c^3*g^4 - 5*B*a*b^3*c^2*d*g^4 + 10*B*a^2*b^2*c*d^2*g^
4 - 10*A*a^3*b*d^3*g^4 - 16*B*a^3*b*d^3*g^4)*x^2/d^3 + 1/5*(2*B*b^4*c^4*g^4
 - 10*B*a*b^3*c^3*d*g^4 + 20*B*a^2*b^2*c^2*d^2*g^4 - 20*B*a^3*b*c*d^3*g^4 +
 5*A*a^4*d^4*g^4 + 13*B*a^4*d^4*g^4)*x/d^4 - 2/5*(B*b^4*c^5*g^4 - 5*B*a*b^3
*c^4*d*g^4 + 10*B*a^2*b^2*c^3*d^2*g^4 - 10*B*a^3*b*c^2*d^3*g^4 + 5*B*a^4*c*
d^4*g^4)*log(-d*x - c)/d^5
```

Mupad [B]

time = 4.99, size = 1025, normalized size = 5.63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*g + b*g*x)^4*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2)),x)
```

```
[Out] x^2*(((5*a*d + 5*b*c)*((((b^3*g^4*(25*A*a*d + 5*A*b*c + 2*B*a*d - 2*B*b*c))
/(5*d) - (A*b^3*g^4*(5*a*d + 5*b*c))/(5*d))*(5*a*d + 5*b*c))/(5*b*d) - (a*b
^2*g^4*(10*A*a*d + 5*A*b*c + 2*B*a*d - 2*B*b*c))/d + (A*a*b^3*c*g^4)/d))/(1
0*b*d) + (a^2*b*g^4*(5*A*a*d + 5*A*b*c + 2*B*a*d - 2*B*b*c))/d - (a*c*((b^3
*g^4*(25*A*a*d + 5*A*b*c + 2*B*a*d - 2*B*b*c))/(5*d) - (A*b^3*g^4*(5*a*d +
5*b*c))/(5*d)))/(2*b*d)) - x^3*((((b^3*g^4*(25*A*a*d + 5*A*b*c + 2*B*a*d -
2*B*b*c))/(5*d) - (A*b^3*g^4*(5*a*d + 5*b*c))/(5*d))*(5*a*d + 5*b*c))/(15*b
*d) - (a*b^2*g^4*(10*A*a*d + 5*A*b*c + 2*B*a*d - 2*B*b*c))/(3*d) + (A*a*b^3
*c*g^4)/(3*d)) + x*((a^3*g^4*(5*A*a*d + 10*A*b*c + 4*B*a*d - 4*B*b*c))/d -
((5*a*d + 5*b*c)*(((5*a*d + 5*b*c)*((((b^3*g^4*(25*A*a*d + 5*A*b*c + 2*B*a*
d - 2*B*b*c))/(5*d) - (A*b^3*g^4*(5*a*d + 5*b*c))/(5*d))*(5*a*d + 5*b*c))/(
5*b*d) - (a*b^2*g^4*(10*A*a*d + 5*A*b*c + 2*B*a*d - 2*B*b*c))/d + (A*a*b^3*
c*g^4)/d))/(5*b*d) + (2*a^2*b*g^4*(5*A*a*d + 5*A*b*c + 2*B*a*d - 2*B*b*c))/
d - (a*c*((b^3*g^4*(25*A*a*d + 5*A*b*c + 2*B*a*d - 2*B*b*c))/(5*d) - (A*b^3
*g^4*(5*a*d + 5*b*c))/(5*d)))/(b*d)))/(5*b*d) + (a*c*((((b^3*g^4*(25*A*a*d
+ 5*A*b*c + 2*B*a*d - 2*B*b*c))/(5*d) - (A*b^3*g^4*(5*a*d + 5*b*c))/(5*d))*
(5*a*d + 5*b*c))/(5*b*d) - (a*b^2*g^4*(10*A*a*d + 5*A*b*c + 2*B*a*d - 2*B*b
*c))/d + (A*a*b^3*c*g^4)/d))/(b*d)) + log((e*(a + b*x)^2)/(c + d*x)^2)*((B*
b^4*g^4*x^5)/5 + B*a^4*g^4*x + 2*B*a^3*b*g^4*x^2 + B*a*b^3*g^4*x^4 + 2*B*a^
2*b^2*g^4*x^3) + x^4*((b^3*g^4*(25*A*a*d + 5*A*b*c + 2*B*a*d - 2*B*b*c))/(2
0*d) - (A*b^3*g^4*(5*a*d + 5*b*c))/(20*d)) - (log(c + d*x)*(2*B*b^4*c^5*g^4
 + 10*B*a^4*c*d^4*g^4 - 20*B*a^3*b*c^2*d^3*g^4 + 20*B*a^2*b^2*c^3*d^2*g^4 -
 10*B*a*b^3*c^4*d*g^4))/(5*d^5) + (A*b^4*g^4*x^5)/5 + (2*B*a^5*g^4*log(a +
b*x))/(5*b)
```

3.120 $\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx$

Optimal. Leaf size=151

$$-\frac{B(bc-ad)^3g^3x}{2d^3}+\frac{B(bc-ad)^2g^3(a+bx)^2}{4bd^2}-\frac{B(bc-ad)g^3(a+bx)^3}{6bd}+\frac{g^3(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{4b}$$

```
[Out] -1/2*B*(-a*d+b*c)^3*g^3*x/d^3+1/4*B*(-a*d+b*c)^2*g^3*(b*x+a)^2/b/d^2-1/6*B*
(-a*d+b*c)*g^3*(b*x+a)^3/b/d+1/4*g^3*(b*x+a)^4*(A+B*ln(e*(b*x+a)^2/(d*x+c)^
2))/b+1/2*B*(-a*d+b*c)^4*g^3*ln(d*x+c)/b/d^4
```

Rubi [A]
time = 0.06, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {2548, 21, 45}

$$\frac{g^3(a+bx)^4\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{4b}+\frac{Bg^3(bc-ad)^4\log(c+dx)}{2bd^4}-\frac{Bg^3x(bc-ad)^3}{2d^3}+\frac{Bg^3(a+bx)^2(bc-ad)^2}{4bd^2}-\frac{Bg^3(a+bx)^3(bc-ad)}{6bd}$$

Antiderivative was successfully verified.

```
[In] Int[(a*g + b*g*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]),x]
```

```
[Out] -1/2*(B*(b*c - a*d)^3*g^3*x)/d^3 + (B*(b*c - a*d)^2*g^3*(a + b*x)^2)/(4*b*d
^2) - (B*(b*c - a*d)*g^3*(a + b*x)^3)/(6*b*d) + (g^3*(a + b*x)^4*(A + B*Log
[(e*(a + b*x)^2)/(c + d*x)^2]))/(4*b) + (B*(b*c - a*d)^4*g^3*Log[c + d*x])/
(2*b*d^4)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2548

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Dist[B*n*((b*c
```

```
- a*d)/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /;
 FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c -
 a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Rubi steps

$$\int (ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)dx = \frac{g^3(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{4b} - \frac{B\int \frac{2(bc-ad)g^4(a+bx)^3}{c+dx}}{4bg}$$

$$= \frac{g^3(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{4b} - \frac{(B(bc-ad)g^3)\int \frac{(a+}{c+}}{2b}$$

$$= \frac{g^3(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{4b} - \frac{(B(bc-ad)g^3)\int \Big(\frac{b(}{}}{}$$

$$= -\frac{B(bc-ad)^3g^3x}{2d^3} + \frac{B(bc-ad)^2g^3(a+bx)^2}{4bd^2} - \frac{B(bc-ad)}{6l}$$

Mathematica [A]
time = 0.07, size = 122, normalized size = 0.81

$$\frac{g^3\left((a+bx)^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)-\frac{B(bc-ad)\left(6bd(bc-ad)^2x+3d^2(-bc+ad)(a+bx)^2+2d^3(a+bx)^3-6(bc-ad)^3\log(c+dx)\right)}{3d^4}\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*g + b*g*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]),x]
```

```
[Out] (g^3*((a + b*x)^4*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]) - (B*(b*c - a*d)
*(6*b*d*(b*c - a*d)^2*x + 3*d^2*(-(b*c) + a*d)*(a + b*x)^2 + 2*d^3*(a + b*x
)^3 - 6*(b*c - a*d)^3*Log[c + d*x]))/(3*d^4)))/(4*b)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1394 vs. $2(141) = 282$.
time = 0.32, size = 1395, normalized size = 9.24

method	result
risch	$\frac{g^3(bx+a)^4 B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)}{4b}+\frac{g^3b^3Ax^4}{4}+g^3b^2Aa\,x^3+\frac{g^3b^2Ba\,x^3}{6}-\frac{g^3b^3Bc\,x^3}{6d}+\frac{3g^3bA\,a^2x^2}{2}+\frac{3g^3bB\,a^2x}{4}$
derivativedivides	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*g*x+a*g)^3*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d*(-2*B*g^3*ln(1/(d*x+c))*a^3*c+20*B*g^3/d*b^2/(a*d-b*c)*ln(a/(d*x+c)*d-
b*c/(d*x+c)+b)*a^2*c^3-10*B*g^3/d^2/(a*d-b*c)*ln(a/(d*x+c)*d-b*c/(d*x+c)+b)
*a*c^4*b^3+3*B*g^3/d*(d*x+c)*ln(e*(a/(d*x+c)*d-b*c/(d*x+c)+b)^2/d^2)*a^2*b*
c+3*B*g^3/d^2*(d*x+c)^2*ln(e*(a/(d*x+c)*d-b*c/(d*x+c)+b)^2/d^2)*a*b^2*c-3*B
*g^3/d^2*(d*x+c)*ln(e*(a/(d*x+c)*d-b*c/(d*x+c)+b)^2/d^2)*a*b^2*c^2-B*g^3*(d
*x+c)*ln(e*(a/(d*x+c)*d-b*c/(d*x+c)+b)^2/d^2)*a^3-6*B*g^3*a^3*ln(a/(d*x+c)*
d-b*c/(d*x+c)+b)*c+1/6*B*g^3/d^3*b^3*c*(d*x+c)^3+3/2*B*g^3/d^3*b^3*c^4*ln(a
/(d*x+c)*d-b*c/(d*x+c)+b)+1/2*B*g^3/d^3*ln(1/(d*x+c))*c^4*b^3-1/4*B*g^3/d^3
*b^3*(d*x+c)^4*ln(e*(a/(d*x+c)*d-b*c/(d*x+c)+b)^2/d^2)+1/2*B*g^3*d/b*ln(1/(
d*x+c))*a^4+3/2*B*g^3*d/b*a^4*ln(a/(d*x+c)*d-b*c/(d*x+c)+b)-3/2*B*g^3*a^3*(
d*x+c)+A*g^3/d^3*(-(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)*(d*x+c)-b^
2*(a*d-b*c)*(d*x+c)^3-3/2*b*(a^2*d^2-2*a*b*c*d+b^2*c^2)*(d*x+c)^2-1/4*b^3*(
d*x+c)^4)-3/4*B*g^3/d*b*a^2*(d*x+c)^2-3/4*B*g^3/d^3*b^3*c^2*(d*x+c)^2+3/2*B
*g^3/d^3*b^3*c^3*(d*x+c)-1/6*B*g^3/d^2*b^2*a*(d*x+c)^3-2*B*g^3*d^2/b/(a*d-b
*c)*ln(a/(d*x+c)*d-b*c/(d*x+c)+b)*a^5+10*B*g^3*d/(a*d-b*c)*ln(a/(d*x+c)*d-b
*c/(d*x+c)+b)*a^4*c+B*g^3/d^3*(d*x+c)*ln(e*(a/(d*x+c)*d-b*c/(d*x+c)+b)^2/d^
2)*b^3*c^3+2*B*g^3/d^3/(a*d-b*c)*ln(a/(d*x+c)*d-b*c/(d*x+c)+b)*c^5*b^4-3/2*
B*g^3/d^3*(d*x+c)^2*ln(e*(a/(d*x+c)*d-b*c/(d*x+c)+b)^2/d^2)*b^3*c^2+9*B*g^3
/d*b*a^2*ln(a/(d*x+c)*d-b*c/(d*x+c)+b)*c^2-6*B*g^3/d^2*b^2*a*ln(a/(d*x+c)*d
-b*c/(d*x+c)+b)*c^3-B*g^3/d^2*b^2*(d*x+c)^3*ln(e*(a/(d*x+c)*d-b*c/(d*x+c)+b
)^2/d^2)*a-20*B*g^3*b/(a*d-b*c)*ln(a/(d*x+c)*d-b*c/(d*x+c)+b)*a^3*c^2-2*B*g
^3/d^2*b^2*ln(1/(d*x+c))*a*c^3+9/2*B*g^3/d*b*a^2*(d*x+c)*c-9/2*B*g^3/d^2*b^
2*a*(d*x+c)*c^2+3/2*B*g^3/d^2*b^2*a*(d*x+c)^2*c+B*g^3/d^3*b^3*(d*x+c)^3*ln(
e*(a/(d*x+c)*d-b*c/(d*x+c)+b)^2/d^2)*c-3/2*B*g^3/d*b*(d*x+c)^2*ln(e*(a/(d*x
+c)*d-b*c/(d*x+c)+b)^2/d^2)*a^2+3*B*g^3/d*b*ln(1/(d*x+c))*a^2*c^2)
```

<u>Maxima</u> [**B**] Leaf count of result is larger than twice the leaf count of optimal. 659 vs. $2(142) = 284$.
time = 0.32, size = 659, normalized size = 4.36

$$\frac{1}{4}Ab^3g^3x^4 + Aab^2g^3x^3 + \frac{3}{2}Aa^2bg^3x^2 + \left(x\log\left(\frac{b^2x^2e}{d^2x^2+2cdx+c^2} + \frac{2abxe}{d^2x^2+2cdx+c^2} + \frac{a^2e}{d^2x^2+2cdx+c^2}\right) + \frac{2a\log(bx+a)}{b} - \frac{2c\log(dx+c)}{d}\right)Ba^3g^3 + \frac{3}{2}\left(x^2\log\left(\frac{b^2x^2e}{d^2x^2+2cdx+c^2} + \frac{2abxe}{d^2x^2+2cdx+c^2} + \frac{a^2e}{d^2x^2+2cdx+c^2}\right) - \frac{2a^2\log(bx+a)}{b^2} + \frac{2c^2\log(dx+c)}{d^2} - \frac{2(bc-ad)x}{bd}\right)Ba^2bg^3 + \left(x^3\log\left(\frac{b^2x^2e}{d^2x^2+2cdx+c^2} + \frac{2abxe}{d^2x^2+2cdx+c^2} + \frac{a^2e}{d^2x^2+2cdx+c^2}\right) + \frac{2a^3\log(bx+a)}{b^3} - \frac{2c^3\log(dx+c)}{d^3} - \frac{(b^2cd-abd^2)x^2 - 2(b^2c^2-a^2d^2)x}{b^2d^2}\right)Bab^2g^3 + \frac{1}{12}\left(3x^4\log\left(\frac{b^2x^2e}{d^2x^2+2cdx+c^2} + \frac{2abxe}{d^2x^2+2cdx+c^2} + \frac{a^2e}{d^2x^2+2cdx+c^2}\right) - \frac{6a^4\log(bx+a)}{b^4} + \frac{6c^4\log(dx+c)}{d^4} - \frac{2(b^3cd^2-ab^2d^3)x^3 - 3(b^3c^2d-a^2bd^3)x^2 + 6(b^3c^3-a^3d^3)x}{b^3d^3}\right)Bb^3g^3 + Aa^3g^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^3*(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="maxi
ma")
```

```
[Out] 1/4*A*b^3*g^3*x^4 + A*a*b^2*g^3*x^3 + 3/2*A*a^2*b*g^3*x^2 + (x*log(b^2*x^2*
e/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*x*e/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(
d^2*x^2 + 2*c*d*x + c^2)) + 2*a*log(b*x + a)/b - 2*c*log(d*x + c)/d)*B*a^3*
g^3 + 3/2*(x^2*log(b^2*x^2*e/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*x*e/(d^2*x^2
 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) - 2*a^2*log(b*x + a)/b
^2 + 2*c^2*log(d*x + c)/d^2 - 2*(b*c - a*d)*x/(b*d))*B*a^2*b*g^3 + (x^3*log
(b^2*x^2*e/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*x*e/(d^2*x^2 + 2*c*d*x + c^2)
+ a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x
```

```
 + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*
B*a*b^2*g^3 + 1/12*(3*x^4*log(b^2*x^2*e/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*x
*e/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) - 6*a^4*log
(b*x + a)/b^4 + 6*c^4*log(d*x + c)/d^4 - (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3
*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3))*B*b^3*g^
3 + A*a^3*g^3*x
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 339 vs. 2(142) = 284.
time = 0.38, size = 339, normalized size = 2.25

$$\frac{3\,Ab^4d^4g^3x^4 + 6\,Ba^4d^4g^3\log\left(bx+a\right) - 2\,(Bb^4cd^3 - (6\,A+B)ab^3d^4)g^3x^3 + 3\,(Bb^4c^2d^2 - 4\,Bab^3cd^3 + 3\,(2\,A+B)a^2b^2d^4)g^3x^2 - 6\,(Bb^4c^3d - 4\,Bab^3c^2d^2 + 6\,Ba^2b^2cd^3 - (2\,A+3\,B)a^3bd^4)g^3x + 6\,(Bb^4c^4 - 4\,Bab^3c^3d + 6\,Ba^2b^2c^2d^2 - 4\,Ba^3bcd^3)g^3\log\left(dx+c\right) + 3\,(Bb^4d^4g^3x^4 + 4\,Bab^3d^4g^3x^3 + 6\,Ba^2b^2d^4g^3x^2 + 4\,Ba^3bd^4g^3x)\log\left(\frac{(b^2x^2+2\,abx+a^2)e}{d^2x^2+2\,cdx+c^2}\right)}{12\,bd^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^3*(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="fric
as")
```

```
[Out] 1/12*(3*A*b^4*d^4*g^3*x^4 + 6*B*a^4*d^4*g^3*log(b*x + a) - 2*(B*b^4*c*d^3 -
 (6*A + B)*a*b^3*d^4)*g^3*x^3 + 3*(B*b^4*c^2*d^2 - 4*B*a*b^3*c*d^3 + 3*(2*A
 + B)*a^2*b^2*d^4)*g^3*x^2 - 6*(B*b^4*c^3*d - 4*B*a*b^3*c^2*d^2 + 6*B*a^2*b
^2*c*d^3 - (2*A + 3*B)*a^3*b*d^4)*g^3*x + 6*(B*b^4*c^4 - 4*B*a*b^3*c^3*d +
6*B*a^2*b^2*c^2*d^2 - 4*B*a^3*b*c*d^3)*g^3*log(d*x + c) + 3*(B*b^4*d^4*g^3*
x^4 + 4*B*a*b^3*d^4*g^3*x^3 + 6*B*a^2*b^2*d^4*g^3*x^2 + 4*B*a^3*b*d^4*g^3*x
)*log((b^2*x^2 + 2*a*b*x + a^2)*e/(d^2*x^2 + 2*c*d*x + c^2)))/(b*d^4)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 707 vs. 2(131) = 262.
time = 2.55, size = 707, normalized size = 4.68

$$\frac{Ab^3g^3x^4}{4} + \frac{Ba^4g^3\log\left(x + \frac{\frac{Ba^5d^4g^3}{b} + 4Ba^4cd^3g^3 - 6Ba^3bc^2d^2g^3 + 4Ba^2b^2c^3dg^3 - Bab^3c^4g^3}{Ba^4d^4g^3 + 4Ba^3bcd^3g^3 - 6Ba^2b^2c^2d^2g^3 + 4Bab^3c^3dg^3 - Bb^4c^4g^3}\right)}{2b} - \frac{Bcg^3\cdot(2ad-bc)(2a^2d^2-2abcd+b^2c^2)\log\left(x + \frac{5Ba^4cd^3g^3 - 6Ba^3bc^2d^2g^3 + 4Ba^2b^2c^3dg^3 - Bab^3c^4g^3 - Bacg^3\cdot(2ad-bc)(2a^2d^2-2abcd+b^2c^2) + \frac{Bbc^2g^3\cdot(2ad-bc)(2a^2d^2-2abcd+b^2c^2)}{d}}{Ba^4d^4g^3 + 4Ba^3bcd^3g^3 - 6Ba^2b^2c^2d^2g^3 + 4Bab^3c^3dg^3 - Bb^4c^4g^3}\right)}{2d^4} + x^3\left(Aab^2g^3 + \frac{Bab^2g^3}{6} - \frac{Bb^3cg^3}{6d}\right) + x^2\cdot\left(\frac{3Aa^2bg^3}{2} + \frac{3Ba^2bg^3}{4} - \frac{Bab^2cg^3}{d} + \frac{Bb^3c^2g^3}{4d^2}\right) + x\left(Aa^3g^3 + \frac{3Ba^3g^3}{2} - \frac{3Ba^2bcg^3}{d} + \frac{2Bab^2c^2g^3}{d^2} - \frac{Bb^3c^3g^3}{2d^3}\right) + \left(Ba^3g^3x + \frac{3Ba^2bg^3x^2}{2} + Bab^2g^3x^3 + \frac{Bb^3g^3x^4}{4}\right)\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)**3*(A+B*ln(e*(b*x+a)**2/(d*x+c)**2)),x)
```

```
[Out] A*b**3*g**3*x**4/4 + B*a**4*g**3*log(x + (B*a**5*d**4*g**3/b + 4*B*a**4*c*d
**3*g**3 - 6*B*a**3*b*c**2*d**2*g**3 + 4*B*a**2*b**2*c**3*d*g**3 - B*a*b**3
*c**4*g**3)/(B*a**4*d**4*g**3 + 4*B*a**3*b*c*d**3*g**3 - 6*B*a**2*b**2*c**2
*d**2*g**3 + 4*B*a*b**3*c**3*d*g**3 - B*b**4*c**4*g**3))/(2*b) - B*c*g**3*(
2*a*d - b*c)*(2*a**2*d**2 - 2*a*b*c*d + b**2*c**2)*log(x + (5*B*a**4*c*d**3
*g**3 - 6*B*a**3*b*c**2*d**2*g**3 + 4*B*a**2*b**2*c**3*d*g**3 - B*a*b**3*c*
*4*g**3 - B*a*c*g**3*(2*a*d - b*c)*(2*a**2*d**2 - 2*a*b*c*d + b**2*c**2) +
B*b*c**2*g**3*(2*a*d - b*c)*(2*a**2*d**2 - 2*a*b*c*d + b**2*c**2)/d)/(B*a**
4*d**4*g**3 + 4*B*a**3*b*c*d**3*g**3 - 6*B*a**2*b**2*c**2*d**2*g**3 + 4*B*a
*b**3*c**3*d*g**3 - B*b**4*c**4*g**3))/(2*d**4) + x**3*(A*a*b**2*g**3 + B*a
*b**2*g**3/6 - B*b**3*c*g**3/(6*d)) + x**2*(3*A*a**2*b*g**3/2 + 3*B*a**2*b*
g**3/4 - B*a*b**2*c*g**3/d + B*b**3*c**2*g**3/(4*d**2)) + x*(A*a**3*g**3 +
```

```
3*B*a**3*g**3/2 - 3*B*a**2*b*c*g**3/d + 2*B*a*b**2*c**2*g**3/d**2 - B*b**3*
c**3*g**3/(2*d**3)) + (B*a**3*g**3*x + 3*B*a**2*b*g**3*x**2/2 + B*a*b**2*g*
*3*x**3 + B*b**3*g**3*x**4/4)*log(e*(a + b*x)**2/(c + d*x)**2)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 361 vs. 2(142) = 284.
time = 77.95, size = 361, normalized size = 2.39

$$\frac{Ba^4g^3\log(bx+a)}{2b}+\frac{1}{4}(Ab^3g^3+Bb^3g^3)x^4-\frac{(Bb^3cg^3-6Aab^2dg^3-7Bab^2dg^3)x^3}{6d}+\frac{1}{4}(Bb^3g^3x^4+4Bab^2g^3x^3+6Ba^2bg^3x^2+4Ba^3g^3x)\log\left(\frac{b^2x^2+2abx+a^2}{d^2x^2+2cdx+c^2}\right)+\frac{(Bb^3c^2g^3-4Bab^2cdg^3+6Aa^2bd^2g^3+9Ba^2bd^2g^3)x^2}{4d^2}-\frac{(Bb^3c^3g^3-4Bab^2c^2dg^3+6Ba^2bcd^2g^3-2Aa^3d^3g^3-5Ba^3d^3g^3)x}{2d^3}+\frac{(Bb^3c^4g^3-4Bab^2c^3dg^3+6Ba^2bc^2d^2g^3-4Ba^3cd^3g^3)\log(dx+c)}{2d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^3*(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="giac
")
```

```
[Out] 1/2*B*a^4*g^3*log(b*x + a)/b + 1/4*(A*b^3*g^3 + B*b^3*g^3)*x^4 - 1/6*(B*b^3
*c*g^3 - 6*A*a*b^2*d*g^3 - 7*B*a*b^2*d*g^3)*x^3/d + 1/4*(B*b^3*g^3*x^4 + 4*
B*a*b^2*g^3*x^3 + 6*B*a^2*b*g^3*x^2 + 4*B*a^3*g^3*x)*log((b^2*x^2 + 2*a*b*x
 + a^2)/(d^2*x^2 + 2*c*d*x + c^2)) + 1/4*(B*b^3*c^2*g^3 - 4*B*a*b^2*c*d*g^3
 + 6*A*a^2*b*d^2*g^3 + 9*B*a^2*b*d^2*g^3)*x^2/d^2 - 1/2*(B*b^3*c^3*g^3 - 4*
B*a*b^2*c^2*d*g^3 + 6*B*a^2*b*c*d^2*g^3 - 2*A*a^3*d^3*g^3 - 5*B*a^3*d^3*g^3
)*x/d^3 + 1/2*(B*b^3*c^4*g^3 - 4*B*a*b^2*c^3*d*g^3 + 6*B*a^2*b*c^2*d^2*g^3
- 4*B*a^3*c*d^3*g^3)*log(d*x + c)/d^4
```

Mupad [B]
time = 4.74, size = 567, normalized size = 3.75

$$\ln\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\left(Ba^3g^3x+\frac{3Ba^2bg^3x^2}{2}+Bab^2g^3x^3+\frac{Bb^3g^3x^4}{4}\right)-x^2\left(\frac{\left(\frac{b^2g^3(8Aad+2Abc+Bad-Bbc)}{2d}-\frac{Ab^2g^3(2ad+2bc)}{2d}\right)(2ad+2bc)}{4bd}-\frac{abg^3(3Aad+2Abc+Bad-Bbc)}{d}+\frac{Aab^2cg^3}{2d}\right)+x\left(\frac{(2ad+2bc)\left(\frac{\left(\frac{b^2g^3(8Aad+2Abc+Bad-Bbc)}{2d}-\frac{Ab^2g^3(2ad+2bc)}{2d}\right)(2ad+2bc)}{2bd}-\frac{2abg^3(3Aad+2Abc+Bad-Bbc)}{d}+\frac{Aab^2cg^3}{d}\right)}{2bd}+\frac{a^2g^3(4Aad+6Abc+3Bad-3Bbc)}{d}-\frac{ac\left(\frac{b^2g^3(8Aad+2Abc+Bad-Bbc)}{2d}-\frac{Ab^2g^3(2ad+2bc)}{2d}\right)}{bd}\right)+x^3\left(\frac{b^2g^3(8Aad+2Abc+Bad-Bbc)}{6d}-\frac{Ab^2g^3(2ad+2bc)}{6d}\right)+\frac{\ln(c+dx)(-4Ba^3cd^3g^3+6Ba^2bc^2d^2g^3-4Bab^2c^3dg^3+Bb^3c^4g^3)}{2d^4}+\frac{Ab^3g^3x^4}{4}+\frac{Ba^4g^3\ln(a+bx)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*g + b*g*x)^3*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2)),x)
```

```
[Out] log((e*(a + b*x)^2)/(c + d*x)^2)*((B*b^3*g^3*x^4)/4 + B*a^3*g^3*x + (3*B*a^
2*b*g^3*x^2)/2 + B*a*b^2*g^3*x^3) - x^2*((((b^2*g^3*(8*A*a*d + 2*A*b*c + B*
a*d - B*b*c))/(2*d) - (A*b^2*g^3*(2*a*d + 2*b*c))/(2*d))*(2*a*d + 2*b*c))/(
4*b*d) - (a*b*g^3*(3*A*a*d + 2*A*b*c + B*a*d - B*b*c))/d + (A*a*b^2*c*g^3)/
(2*d)) + x*(((2*a*d + 2*b*c)*((((b^2*g^3*(8*A*a*d + 2*A*b*c + B*a*d - B*b*c
))/(2*d) - (A*b^2*g^3*(2*a*d + 2*b*c))/(2*d))*(2*a*d + 2*b*c))/(2*b*d) - (2
*a*b*g^3*(3*A*a*d + 2*A*b*c + B*a*d - B*b*c))/d + (A*a*b^2*c*g^3)/d))/(2*b*
d) + (a^2*g^3*(4*A*a*d + 6*A*b*c + 3*B*a*d - 3*B*b*c))/d - (a*c*((b^2*g^3*(
8*A*a*d + 2*A*b*c + B*a*d - B*b*c))/(2*d) - (A*b^2*g^3*(2*a*d + 2*b*c))/(2*
d)))/(b*d)) + x^3*((b^2*g^3*(8*A*a*d + 2*A*b*c + B*a*d - B*b*c))/(6*d) - (A
*b^2*g^3*(2*a*d + 2*b*c))/(6*d)) + (log(c + d*x)*(B*b^3*c^4*g^3 - 4*B*a^3*c
*d^3*g^3 + 6*B*a^2*b*c^2*d^2*g^3 - 4*B*a*b^2*c^3*d*g^3))/(2*d^4) + (A*b^3*g
^3*x^4)/4 + (B*a^4*g^3*log(a + b*x))/(2*b)
```

3.121 $\int (ag+bgx)^2 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right) dx$

Optimal. Leaf size=120

$$\frac{2B(bc-ad)^2g^2x}{3d^2}-\frac{B(bc-ad)g^2(a+bx)^2}{3bd}+\frac{g^2(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{3b}-\frac{2B(bc-ad)^3g^2\log(c+dx)}{3bd^3}$$

```
[Out] 2/3*B*(-a*d+b*c)^2*g^2*x/d^2-1/3*B*(-a*d+b*c)*g^2*(b*x+a)^2/b/d+1/3*g^2*(b*
x+a)^3*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/b-2/3*B*(-a*d+b*c)^3*g^2*ln(d*x+c)/b
/d^3
```

Rubi [A]
time = 0.05, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {2548, 21, 45}

$$\frac{g^2(a+bx)^3\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{3b}-\frac{2Bg^2(bc-ad)^3\log(c+dx)}{3bd^3}+\frac{2Bg^2x(bc-ad)^2}{3d^2}-\frac{Bg^2(a+bx)^2(bc-ad)}{3bd}$$

Antiderivative was successfully verified.

```
[In] Int[(a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]),x]
```

```
[Out] (2*B*(b*c - a*d)^2*g^2*x)/(3*d^2) - (B*(b*c - a*d)*g^2*(a + b*x)^2)/(3*b*d)
 + (g^2*(a + b*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]))/(3*b) - (2*B*
(b*c - a*d)^3*g^2*Log[c + d*x])/(3*b*d^3)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2548

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Dist[B*n*((b*c
```

```
- a*d)/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /;
 FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c -
 a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Rubi steps

$$\int (ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)dx = \frac{g^2(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{3b} - \frac{B\int \frac{2(bc-ad)g^3(a+bx)^2}{c+dx}}{3bg}$$
$$= \frac{g^2(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{3b} - \frac{(2B(bc-ad)g^2)\int}{3b}$$
$$= \frac{g^2(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{3b} - \frac{(2B(bc-ad)g^2)\int}{}$$
$$= \frac{2B(bc-ad)^2g^2x}{3d^2} - \frac{B(bc-ad)g^2(a+bx)^2}{3bd} + \frac{g^2(a+bx)^3}{}$$

Mathematica [A]
time = 0.04, size = 98, normalized size = 0.82

$$\frac{g^2\left((a+bx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)+\frac{B(-bc+ad)\left(d\left(a^2d+4abdx+b^2x(-2c+dx)\right)+2(bc-ad)^2\log(c+dx)\right)}{d^3}\right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]),x]
```

```
[Out] (g^2*((a + b*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]) + (B*(-(b*c) + a
*d)*(d*(a^2*d + 4*a*b*d*x + b^2*x*(-2*c + d*x)) + 2*(b*c - a*d)^2*Log[c + d
*x]))/d^3))/(3*b)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 927 vs. $2(112) = 224$.
time = 0.29, size = 928, normalized size = 7.73 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*g*x+a*g)^2*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d*(8*B*g^2/d/(a*d-b*c)*ln(a/(d*x+c)*d-b*c/(d*x+c)+b)*a*c^3*b^2+2*B*g^2/d
*(d*x+c)*ln(e*(a/(d*x+c)*d-b*c/(d*x+c)+b)^2/d^2)*a*c*b+1/3*B*g^2/d^2*b^2*c*
(d*x+c)^2-2*B*g^2/d^2/(a*d-b*c)*ln(a/(d*x+c)*d-b*c/(d*x+c)+b)*c^4*b^3+4*B*g
^2/d*b*a*ln(a/(d*x+c)*d-b*c/(d*x+c)+b)*c^2+A*g^2/d^2*(-1/3*b^2*(d*x+c)^3-(a
^2*d^2-2*a*b*c*d+b^2*c^2)*(d*x+c)-b*(a*d-b*c)*(d*x+c)^2)-4/3*B*g^2*a^2*(d*x
```

```
+c)-1/3*B*g^2/d^2*b^2*(d*x+c)^3*ln(e*(a/(d*x+c)*d-b*c/(d*x+c)+b)^2/d^2)-1/3
*B*g^2/d*b*a*(d*x+c)^2-4/3*B*g^2/d^2*b^2*c^3*ln(a/(d*x+c)*d-b*c/(d*x+c)+b)-
2/3*B*g^2/d^2*b^2*c^3*ln(1/(d*x+c))-4/3*B*g^2/d^2*b^2*c^2*(d*x+c)+2*B*g^2/d
*b*a*ln(1/(d*x+c))*c^2+8/3*B*g^2/d*b*a*(d*x+c)*c-12*B*g^2*b/(a*d-b*c)*ln(a/
(d*x+c)*d-b*c/(d*x+c)+b)*a^2*c^2+B*g^2/d^2*(d*x+c)^2*ln(e*(a/(d*x+c)*d-b*c/
(d*x+c)+b)^2/d^2)*b^2*c-B*g^2/d*b*(d*x+c)^2*ln(e*(a/(d*x+c)*d-b*c/(d*x+c)+b
)^2/d^2)*a-2*B*g^2*d^2/b/(a*d-b*c)*ln(a/(d*x+c)*d-b*c/(d*x+c)+b)*a^4+8*B*g^
2*d/(a*d-b*c)*ln(a/(d*x+c)*d-b*c/(d*x+c)+b)*a^3*c-B*g^2/d^2*(d*x+c)*ln(e*(a
/(d*x+c)*d-b*c/(d*x+c)+b)^2/d^2)*c^2*b^2+4/3*B*g^2*d/b*a^3*ln(a/(d*x+c)*d-b
*c/(d*x+c)+b)+2/3*B*g^2*d/b*a^3*ln(1/(d*x+c))-2*B*g^2*a^2*ln(1/(d*x+c))*c-B
*g^2*(d*x+c)*ln(e*(a/(d*x+c)*d-b*c/(d*x+c)+b)^2/d^2)*a^2-4*B*g^2*a^2*ln(a/(
d*x+c)*d-b*c/(d*x+c)+b)*c)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 446 vs. $2(113) = 226$.
time = 0.31, size = 446, normalized size = 3.72

$$\frac{1}{3}\,Ab^2g^2x^3 + Aabg^2x^2 + \left(x\log\left(\frac{b^2x^2e}{d^2x^2 + 2\,cdx + c^2} + \frac{2\,abxe}{d^2x^2 + 2\,cdx + c^2} + \frac{a^2e}{d^2x^2 + 2\,cdx + c^2}\right) + \frac{2\,a\log\left(bx + a\right)}{b} - \frac{2\,c\log\left(dx + c\right)}{d}\right)Ba^2g^2 + \left(x^2\log\left(\frac{b^2x^2e}{d^2x^2 + 2\,cdx + c^2} + \frac{2\,abxe}{d^2x^2 + 2\,cdx + c^2} + \frac{a^2e}{d^2x^2 + 2\,cdx + c^2}\right) - \frac{2\,a^2\log\left(bx + a\right)}{b^2} + \frac{2\,c^2\log\left(dx + c\right)}{d^2} - \frac{2\,(bc - ad)x}{bd}\right)Babg^2 + \frac{1}{3}\left(x^3\log\left(\frac{b^2x^2e}{d^2x^2 + 2\,cdx + c^2} + \frac{2\,abxe}{d^2x^2 + 2\,cdx + c^2} + \frac{a^2e}{d^2x^2 + 2\,cdx + c^2}\right) + \frac{2\,a^3\log\left(bx + a\right)}{b^3} - \frac{2\,c^3\log\left(dx + c\right)}{d^3} - \frac{(b^2cd - abd^2)x^2 - 2\,(b^2c^2 - a^2d^2)x}{b^2d^2}\right)Bb^2g^2 + Aa^2g^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^2*(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="maxi
ma")
```

```
[Out] 1/3*A*b^2*g^2*x^3 + A*a*b*g^2*x^2 + (x*log(b^2*x^2*e/(d^2*x^2 + 2*c*d*x + c
^2) + 2*a*b*x*e/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)
) + 2*a*log(b*x + a)/b - 2*c*log(d*x + c)/d)*B*a^2*g^2 + (x^2*log(b^2*x^2*e
/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*x*e/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d
^2*x^2 + 2*c*d*x + c^2)) - 2*a^2*log(b*x + a)/b^2 + 2*c^2*log(d*x + c)/d^2
- 2*(b*c - a*d)*x/(b*d))*B*a*b*g^2 + 1/3*(x^3*log(b^2*x^2*e/(d^2*x^2 + 2*c*
d*x + c^2) + 2*a*b*x*e/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x
 + c^2)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*
b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*B*b^2*g^2 + A*a^2*g^2*x
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 241 vs. $2(113) = 226$.
time = 0.35, size = 241, normalized size = 2.01

$$\frac{Ab^3d^3g^2x^3 + 2\,Ba^3d^3g^2\log\left(bx + a\right) - (Bb^3cd^2 - (3\,A + B)ab^2d^3)g^2x^2 + (2\,Bb^3c^2d - 6\,Bab^2cd^2 + (3\,A + 4\,B)a^2bd^3)g^2x - 2\,(Bb^3c^3 - 3\,Bab^2c^2d + 3\,Ba^2bcd^2)g^2\log\left(dx + c\right) + (Bb^3d^3g^2x^3 + 3\,Bab^2d^3g^2x^2 + 3\,Ba^2bd^3g^2x)\log\left(\frac{(b^2x^2 + 2\,abx + a^2)e}{d^2x^2 + 2\,cdx + c^2}\right)}{3\,bd^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^2*(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="fric
as")
```

```
[Out] 1/3*(A*b^3*d^3*g^2*x^3 + 2*B*a^3*d^3*g^2*log(b*x + a) - (B*b^3*c*d^2 - (3*A
 + B)*a*b^2*d^3)*g^2*x^2 + (2*B*b^3*c^2*d - 6*B*a*b^2*c*d^2 + (3*A + 4*B)*a
^2*b*d^3)*g^2*x - 2*(B*b^3*c^3 - 3*B*a*b^2*c^2*d + 3*B*a^2*b*c*d^2)*g^2*log
```

```
(d*x + c) + (B*b^3*d^3*g^2*x^3 + 3*B*a*b^2*d^3*g^2*x^2 + 3*B*a^2*b*d^3*g^2*
x)*log((b^2*x^2 + 2*a*b*x + a^2)*e/(d^2*x^2 + 2*c*d*x + c^2)))/(b*d^3)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 517 vs. $2(107) = 214$.
time = 1.78, size = 517, normalized size = 4.31

$$\frac{Ab^2g^2x^3}{3} + \frac{2Ba^3g^2\log\left(x + \frac{\frac{2Ba^4d^3g^2}{b} + 6Ba^3cd^2g^2 - 6Ba^2bc^2dg^2 + 2Bab^2c^3g^2}{2Ba^3d^3g^2 + 6Ba^2bcd^2g^2 - 6Bab^2c^2dg^2 + 2Bb^3c^3g^2}\right)}{3b} - \frac{2Bcg^2\cdot(3a^2d^2 - 3abcd + b^2c^2)\log\left(x + \frac{8Ba^3cd^2g^2 - 6Ba^2bc^2dg^2 + 2Bab^2c^3g^2 - 2Bacg^2\cdot(3a^2d^2 - 3abcd + b^2c^2) + \frac{2Bbc^2g^2\cdot(3a^2d^2 - 3abcd + b^2c^2)}{d}}{2Ba^3d^3g^2 + 6Ba^2bcd^2g^2 - 6Bab^2c^2dg^2 + 2Bb^3c^3g^2}\right)}{3d^3} + x^2\left(Aabg^2 + \frac{Babg^2}{3} - \frac{Bb^2cg^2}{3d}\right) + x\left(Aa^2g^2 + \frac{4Ba^2g^2}{3} - \frac{2Babcg^2}{d} + \frac{2Bb^2c^2g^2}{3d^2}\right) + \left(Ba^2g^2x + Babg^2x^2 + \frac{Bb^2g^2x^3}{3}\right)\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)**2*(A+B*ln(e*(b*x+a)**2/(d*x+c)**2)),x)
```

```
[Out] A*b**2*g**2*x**3/3 + 2*B*a**3*g**2*log(x + (2*B*a**4*d**3*g**2/b + 6*B*a**3
*c*d**2*g**2 - 6*B*a**2*b*c**2*d*g**2 + 2*B*a*b**2*c**3*g**2)/(2*B*a**3*d**
3*g**2 + 6*B*a**2*b*c*d**2*g**2 - 6*B*a*b**2*c**2*d*g**2 + 2*B*b**3*c**3*g*
*2))/(3*b) - 2*B*c*g**2*(3*a**2*d**2 - 3*a*b*c*d + b**2*c**2)*log(x + (8*B*
a**3*c*d**2*g**2 - 6*B*a**2*b*c**2*d*g**2 + 2*B*a*b**2*c**3*g**2 - 2*B*a*c*
g**2*(3*a**2*d**2 - 3*a*b*c*d + b**2*c**2) + 2*B*b*c**2*g**2*(3*a**2*d**2 -
 3*a*b*c*d + b**2*c**2)/d)/(2*B*a**3*d**3*g**2 + 6*B*a**2*b*c*d**2*g**2 - 6
*B*a*b**2*c**2*d*g**2 + 2*B*b**3*c**3*g**2))/(3*d**3) + x**2*(A*a*b*g**2 +
B*a*b*g**2/3 - B*b**2*c*g**2/(3*d)) + x*(A*a**2*g**2 + 4*B*a**2*g**2/3 - 2*
B*a*b*c*g**2/d + 2*B*b**2*c**2*g**2/(3*d**2)) + (B*a**2*g**2*x + B*a*b*g**2
*x**2 + B*b**2*g**2*x**3/3)*log(e*(a + b*x)**2/(c + d*x)**2)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 252 vs. $2(113) = 226$.
time = 7.52, size = 252, normalized size = 2.10

$$\frac{2\,Ba^3g^2\log\left(bx+a\right)}{3\,b} + \frac{1}{3}\,(Ab^2g^2 + Bb^2g^2)x^3 - \frac{(Bb^2cg^2 - 3\,Aabdg^2 - 4\,Babdg^2)x^2}{3\,d} + \frac{1}{3}\,(Bb^2g^2x^3 + 3\,Babg^2x^2 + 3\,Ba^2g^2x)\log\left(\frac{b^2x^2 + 2\,abx + a^2}{d^2x^2 + 2\,cdx + c^2}\right) + \frac{(2\,Bb^2c^2g^2 - 6\,Babcdg^2 + 3\,Aa^2d^2g^2 + 7\,Ba^2d^2g^2)x}{3\,d^2} - \frac{2\,(Bb^2c^3g^2 - 3\,Babc^2dg^2 + 3\,Ba^2cd^2g^2)\log\left(-dx-c\right)}{3\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^2*(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="giac
")
```

```
[Out] 2/3*B*a^3*g^2*log(b*x + a)/b + 1/3*(A*b^2*g^2 + B*b^2*g^2)*x^3 - 1/3*(B*b^2
*c*g^2 - 3*A*a*b*d*g^2 - 4*B*a*b*d*g^2)*x^2/d + 1/3*(B*b^2*g^2*x^3 + 3*B*a*
b*g^2*x^2 + 3*B*a^2*g^2*x)*log((b^2*x^2 + 2*a*b*x + a^2)/(d^2*x^2 + 2*c*d*x
 + c^2)) + 1/3*(2*B*b^2*c^2*g^2 - 6*B*a*b*c*d*g^2 + 3*A*a^2*d^2*g^2 + 7*B*a
^2*d^2*g^2)*x/d^2 - 2/3*(B*b^2*c^3*g^2 - 3*B*a*b*c^2*d*g^2 + 3*B*a^2*c*d^2*
g^2)*log(-d*x - c)/d^3
```

Mupad [B]
time = 4.59, size = 296, normalized size = 2.47

$$x^2\left(\frac{b\,g^2\,(9\,A\,a\,d+3\,A\,b\,c+2\,B\,a\,d-2\,B\,b\,c)}{6\,d} - \frac{A\,b\,g^2\,(3\,a\,d+3\,b\,c)}{6\,d}\right) - x\left(\frac{(3\,a\,d+3\,b\,c)\,\left(\frac{b\,g^2\,(9\,A\,a\,d+3\,A\,b\,c+2\,B\,a\,d-2\,B\,b\,c)}{3\,d} - \frac{A\,b\,g^2\,(3\,a\,d+3\,b\,c)}{3\,d}\right)}{3\,b\,d} - \frac{a\,g^2\,(3\,A\,a\,d+3\,A\,b\,c+2\,B\,a\,d-2\,B\,b\,c)}{d} + \frac{A\,a\,b\,c\,g^2}{d}\right) + \ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)\,\left(B\,a^2\,g^2\,x + B\,a\,b\,g^2\,x^2 + \frac{B\,b^2\,g^2\,x^3}{3}\right) - \frac{\ln(c+d\,x)\,(6\,B\,a^2\,c\,d^2\,g^2 - 6\,B\,a\,b\,c^2\,d\,g^2 + 2\,B\,b^2\,c^3\,g^2)}{3\,d^3} + \frac{A\,b^2\,g^2\,x^3}{3} + \frac{2\,B\,a^3\,g^2\,\ln(a+b\,x)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*g + b*g*x)^2*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2)),x)
```

```
[Out] x^2*((b*g^2*(9*A*a*d + 3*A*b*c + 2*B*a*d - 2*B*b*c))/(6*d) - (A*b*g^2*(3*a*
d + 3*b*c))/(6*d)) - x*(((3*a*d + 3*b*c)*((b*g^2*(9*A*a*d + 3*A*b*c + 2*B*a
*d - 2*B*b*c))/(3*d) - (A*b*g^2*(3*a*d + 3*b*c))/(3*d)))/(3*b*d) - (a*g^2*(
3*A*a*d + 3*A*b*c + 2*B*a*d - 2*B*b*c))/d + (A*a*b*c*g^2)/d) + log((e*(a +
b*x)^2)/(c + d*x)^2)*((B*b^2*g^2*x^3)/3 + B*a^2*g^2*x + B*a*b*g^2*x^2) - (l
og(c + d*x)*(2*B*b^2*c^3*g^2 + 6*B*a^2*c*d^2*g^2 - 6*B*a*b*c^2*d*g^2))/(3*d
^3) + (A*b^2*g^2*x^3)/3 + (2*B*a^3*g^2*log(a + b*x))/(3*b)
```

3.122 $\int (ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx$

Optimal. Leaf size=78

$$-\frac{B(bc-ad)gx}{d}+\frac{g(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{2b}+\frac{B(bc-ad)^2g\log(c+dx)}{bd^2}$$

```
[Out] -B*(-a*d+b*c)*g*x/d+1/2*g*(b*x+a)^2*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/b+B*(-a
*d+b*c)^2*g*ln(d*x+c)/b/d^2
```

Rubi [A]
time = 0.03, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2548, 21, 45}

$$\frac{g(a+bx)^2\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{2b}+\frac{Bg(bc-ad)^2\log(c+dx)}{bd^2}-\frac{Bgx(bc-ad)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a*g + b*g*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]),x]
```

```
[Out] -((B*(b*c - a*d)*g*x)/d) + (g*(a + b*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d
*x)^2]))/(2*b) + (B*(b*c - a*d)^2*g*Log[c + d*x])/(b*d^2)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2548

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Dist[B*n*((b*c
- a*d)/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /;
 FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c -
```

```
a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Rubi steps

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)dx = \frac{g(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{2b} - \frac{B\int\frac{2(bc-ad)g^2(a+bx)}{c+dx}dx}{2bg}$$
$$= \frac{g(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{2b} - \frac{(B(bc-ad)g)\int\frac{a+bx}{c+dx}d}{b}$$
$$= \frac{g(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{2b} - \frac{(B(bc-ad)g)\int\left(\frac{b}{d}+\right.}{b}$$
$$= -\frac{B(bc-ad)gx}{d} + \frac{g(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{2b} + \frac{B(b}{}$$

Mathematica [A]
time = 0.03, size = 72, normalized size = 0.92

$$\frac{g\left((a+bx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)+\frac{2B(-bc+ad)(bdx+(-bc+ad)\log(c+dx))}{d^2}\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*g + b*g*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]),x]
[Out] (g*((a + b*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]) + (2*B*(-(b*c) + a
*d)*(b*d*x + (-(b*c) + a*d)*Log[c + d*x]))/d^2))/(2*b)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 520 vs. 2(76) = 152.
time = 0.24, size = 521, normalized size = 6.68

method	result
risch	$\frac{gBx(bx+2a)\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)}{2}+\frac{gbA\,x^2}{2}+gAax+\frac{B\,a^2g\ln(-bx-a)}{b}-\frac{2gB\ln(dx+c)ac}{d}+\frac{gbB\ln(dx+c)c^2}{d^2}+g$
derivativedivides	$-\frac{\frac{Ag\left(-(ad-cb)(dx+c)-\frac{b(dx+c)^2}{2}\right)}{d}-Bg(dx+c)\ln\left(\frac{e\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^2}{d^2}\right)a+\frac{Bg(dx+c)\ln\left(\frac{e\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^2}{d^2}\right)bc}{d}-\frac{2Bg\,d^2}{}}{}$

default	$-\frac{\frac{Ag\left(-(ad-cb)(dx+c)-\frac{b(dx+c)^2}{2}\right)}{d}-Bg(dx+c)\ln\left(\frac{e\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^2}{d^2}\right)a+\frac{Bg(dx+c)\ln\left(\frac{e\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^2}{d^2}\right)bc}{d}-\frac{2Bg}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*g*x+a*g)*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d*(A*g/d*(-(a*d-b*c)*(d*x+c)-1/2*b*(d*x+c)^2)-B*g*(d*x+c)*ln(e*(a/(d*x+c
)*d-b*c/(d*x+c)+b)^2/d^2)*a+B*g/d*(d*x+c)*ln(e*(a/(d*x+c)*d-b*c/(d*x+c)+b)^
2/d^2)*b*c-2*B*g*d^2/b/(a*d-b*c)*ln(a/(d*x+c)*d-b*c/(d*x+c)+b)*a^3+6*B*g*d/
(a*d-b*c)*ln(a/(d*x+c)*d-b*c/(d*x+c)+b)*a^2*c-6*B*g/(a*d-b*c)*ln(a/(d*x+c)*
d-b*c/(d*x+c)+b)*a*c^2*b+2*B*g/d/(a*d-b*c)*ln(a/(d*x+c)*d-b*c/(d*x+c)+b)*c^
3*b^2+B*g*d/b*ln(1/(d*x+c))*a^2-2*B*g*ln(1/(d*x+c))*a*c+B*g/d*ln(1/(d*x+c))
*c^2*b-1/2*B*g/d*b*(d*x+c)^2*ln(e*(a/(d*x+c)*d-b*c/(d*x+c)+b)^2/d^2)+B*g*d/
b*ln(a/(d*x+c)*d-b*c/(d*x+c)+b)*a^2-2*B*g*ln(a/(d*x+c)*d-b*c/(d*x+c)+b)*a*c
+B*g/d*b*ln(a/(d*x+c)*d-b*c/(d*x+c)+b)*c^2-B*g*(d*x+c)*a+B*g/d*b*(d*x+c)*c)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 256 vs. $2(77) = 154$.
time = 0.33, size = 256, normalized size = 3.28

$$\frac{1}{2}\,Abgx^2+\left(x\log\left(\frac{b^2x^2e}{d^2x^2+2\,cdx+c^2}+\frac{2\,abxe}{d^2x^2+2\,cdx+c^2}+\frac{a^2e}{d^2x^2+2\,cdx+c^2}\right)+\frac{2\,a\log\left(bx+a\right)}{b}-\frac{2\,c\log\left(dx+c\right)}{d}\right)Bag+\frac{1}{2}\left(x^2\log\left(\frac{b^2x^2e}{d^2x^2+2\,cdx+c^2}+\frac{2\,abxe}{d^2x^2+2\,cdx+c^2}+\frac{a^2e}{d^2x^2+2\,cdx+c^2}\right)-\frac{2\,a^2\log\left(bx+a\right)}{b^2}+\frac{2\,c^2\log\left(dx+c\right)}{d^2}-\frac{2\,(bc-ad)x}{bd}\right)Bbg+Aagx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)*(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="maxima
")
```

```
[Out] 1/2*A*b*g*x^2 + (x*log(b^2*x^2*e/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*x*e/(d^2
*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) + 2*a*log(b*x + a)
/b - 2*c*log(d*x + c)/d)*B*a*g + 1/2*(x^2*log(b^2*x^2*e/(d^2*x^2 + 2*c*d*x
+ c^2) + 2*a*b*x*e/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c
^2)) - 2*a^2*log(b*x + a)/b^2 + 2*c^2*log(d*x + c)/d^2 - 2*(b*c - a*d)*x/(b
*d))*B*b*g + A*a*g*x
```

Fricas [A]
time = 0.36, size = 146, normalized size = 1.87

$$\frac{Ab^2d^2gx^2+2\,Ba^2d^2g\log\left(bx+a\right)-2\left(Bb^2cd-(A+B)abd^2\right)gx+2\left(Bb^2c^2-2\,Babcd\right)g\log\left(dx+c\right)+\left(Bb^2d^2gx^2+2\,Babd^2gx\right)\log\left(\frac{(b^2x^2+2\,abx+a^2)e}{d^2x^2+2\,cdx+c^2}\right)}{2\,bd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)*(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="fricas
")
```

```
[Out] 1/2*(A*b^2*d^2*g*x^2 + 2*B*a^2*d^2*g*log(b*x + a) - 2*(B*b^2*c*d - (A + B)*
a*b*d^2)*g*x + 2*(B*b^2*c^2 - 2*B*a*b*c*d)*g*log(d*x + c) + (B*b^2*d^2*g*x^
```

```
2 + 2*B*a*b*d^2*g*x)*log((b^2*x^2 + 2*a*b*x + a^2)*e/(d^2*x^2 + 2*c*d*x + c
^2)))/(b*d^2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 250 vs. $2(68) = 136$.
time = 1.01, size = 250, normalized size = 3.21

$$\frac{Abgx^2}{2}+\frac{Ba^2g\log\left(x+\frac{\frac{Ba^3d^2g}{b}+2Ba^2cdg-Babc^2g}{Ba^2d^2g+2Babcdg-Bb^2c^2g}\right)}{b}-\frac{Bcg(2ad-bc)\log\left(x+\frac{3Ba^2cdg-Babc^2g-Bacg(2ad-bc)+\frac{Bbc^2g(2ad-bc)}{d}}{Ba^2d^2g+2Babcdg-Bb^2c^2g}\right)}{d^2}+x\left(Aag+Bag-\frac{Bbcg}{d}\right)+\left(Bagx+\frac{Bbgx^2}{2}\right)\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)*(A+B*ln(e*(b*x+a)**2/(d*x+c)**2)),x)
```

```
[Out] A*b*g*x**2/2 + B*a**2*g*log(x + (B*a**3*d**2*g/b + 2*B*a**2*c*d*g - B*a*b*c
**2*g)/(B*a**2*d**2*g + 2*B*a*b*c*d*g - B*b**2*c**2*g))/b - B*c*g*(2*a*d -
b*c)*log(x + (3*B*a**2*c*d*g - B*a*b*c**2*g - B*a*c*g*(2*a*d - b*c) + B*b*c
**2*g*(2*a*d - b*c)/d)/(B*a**2*d**2*g + 2*B*a*b*c*d*g - B*b**2*c**2*g))/d**
2 + x*(A*a*g + B*a*g - B*b*c*g/d) + (B*a*g*x + B*b*g*x**2/2)*log(e*(a + b*x
)**2/(c + d*x)**2)
```

Giac [A]
time = 3.20, size = 131, normalized size = 1.68

$$\frac{Ba^2g\log(bx+a)}{b}+\frac{1}{2}(Abg+Bbg)x^2+\frac{1}{2}\left(Bbgx^2+2\,Bagx\right)\log\left(\frac{b^2x^2+2\,abx+a^2}{d^2x^2+2\,cdx+c^2}\right)-\frac{(Bbcg-Aadg-2\,Badg)x}{d}+\frac{(Bbc^2g-2\,Bacdg)\log(dx+c)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)*(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="giac")
```

```
[Out] B*a^2*g*log(b*x + a)/b + 1/2*(A*b*g + B*b*g)*x^2 + 1/2*(B*b*g*x^2 + 2*B*a*g
*x)*log((b^2*x^2 + 2*a*b*x + a^2)/(d^2*x^2 + 2*c*d*x + c^2)) - (B*b*c*g - A
*a*d*g - 2*B*a*d*g)*x/d + (B*b*c^2*g - 2*B*a*c*d*g)*log(d*x + c)/d^2
```

Mupad [B]
time = 4.39, size = 120, normalized size = 1.54

$$x\left(\frac{g\,(2\,A\,a\,d+A\,b\,c+B\,a\,d-B\,b\,c)}{d}-\frac{A\,g\,(a\,d+b\,c)}{d}\right)+\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)\,\left(\frac{B\,b\,g\,x^2}{2}+B\,a\,g\,x\right)+\frac{A\,b\,g\,x^2}{2}+\frac{B\,a^2\,g\,\ln(a+b\,x)}{b}-\frac{B\,c\,g\,\ln(c+d\,x)\,(2\,a\,d-b\,c)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*g + b*g*x)*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2)),x)
```

```
[Out] x*((g*(2*A*a*d + A*b*c + B*a*d - B*b*c))/d - (A*g*(a*d + b*c))/d) + log((e*
(a + b*x)^2)/(c + d*x)^2)*((B*b*g*x^2)/2 + B*a*g*x) + (A*b*g*x^2)/2 + (B*a^
2*g*log(a + b*x))/b - (B*c*g*log(c + d*x)*(2*a*d - b*c))/d^2
```

3.123 $\int \frac{A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{ag+bgx} \, dx$

Optimal. Leaf size=83

$$-\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{bg}+\frac{2B\text{Li}_2\left(1+\frac{bc-ad}{d(a+bx)}\right)}{bg}$$

```
[Out] -ln((a*d-b*c)/d/(b*x+a))*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/b/g+2*B*polylog(2,
1+(-a*d+b*c)/d/(b*x+a))/b/g
```

Rubi [A]

time = 0.14, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {2542, 2458, 2378, 2370, 2352}

$$\frac{2B\text{PolyLog}\left(2,\frac{bc-ad}{d(a+bx)}+1\right)}{bg}-\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{bg}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(a*g + b*g*x),x]
```

```
[Out] -((Log[-((b*c - a*d)/(d*(a + b*x)))]*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2
]))/(b*g)) + (2*B*PolyLog[2, 1 + (b*c - a*d)/(d*(a + b*x))])/(b*g)
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2370

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)/(x_))^(q_.)*(
x_)^(m_.), x_Symbol] :> Int[(e + d*x)^q*(a + b*Log[c*x^n])^p, x] /; FreeQ[{
a, b, c, d, e, m, n, p}, x] && EqQ[m, q] && IntegerQ[q]
```

Rule 2378

```
Int[((a_.) + Log[(c_.)*(x_)^(n_)]*(b_.))/((x_)*((d_) + (e_.)*(x_)^(r_.))),
x_Symbol] :> Dist[1/n, Subst[Int[(a + b*Log[c*x])/(x*(d + e*x^(r/n))), x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IntegerQ[r/n]
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
```

```
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2542

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))/((f_.) + (g_.)*(x_)), x_Symbol] :> Simp[(-Log[-(b*c - a*d)/(d*(a
+ b*x))])*((A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/g), x] + Dist[B*n*((b*c
 - a*d)/g), Int[Log[-(b*c - a*d)/(d*(a + b*x))]/((a + b*x)*(c + d*x)), x],
x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
 - a*d, 0] && EqQ[b*f - a*g, 0]
```

Rubi steps

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{ag+bgx}\,dx = \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\log(ag+bgx)}{bg} - \frac{B\int \frac{(c+dx)^2\left(-\frac{2de(a+bx)^2}{(c+dx)^3}+\frac{2be(a+bx)}{(c+dx)^2}\right)\log(ag+}{e(a+bx)^2}}{bg}$$

$$= \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\log(ag+bgx)}{bg} - \frac{B\int \frac{(c+dx)^2\left(-\frac{2de(a+bx)^2}{(c+dx)^3}+\frac{2be(a+bx)}{(c+dx)^2}\right)\log(ag+}{(a+bx)^2}}{beg}$$

$$= \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\log(ag+bgx)}{bg} - \frac{B\int \left(\frac{2be\log(ag+bgx)}{a+bx}-\frac{2de\log(ag+bgx)}{c+dx}\right)\,d}{beg}$$

$$= \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\log(ag+bgx)}{bg} - \frac{(2B)\int \frac{\log(ag+bgx)}{a+bx}\,dx}{g} + \frac{(2Bd)\int \frac{\log(ag-}{c+}}{bg}$$

$$= \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\log(ag+bgx)}{bg} + \frac{2B\log\left(\frac{b(c+dx)}{bc-ad}\right)\log(ag+bgx)}{bg} - (2B$$

$$= \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\log(ag+bgx)}{bg} + \frac{2B\log\left(\frac{b(c+dx)}{bc-ad}\right)\log(ag+bgx)}{bg} - \frac{(2E}{}$$

$$= -\frac{B\log^2(g(a+bx))}{bg} + \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\log(ag+bgx)}{bg} + \frac{2B\log\left(\frac{b(c+dx)}{bc-ad}\right.}{}$$

Mathematica [A]
time = 0.03, size = 88, normalized size = 1.06

$$\frac{\log(a+bx)\left(A-B\log(a+bx)+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+2B\log\left(\frac{b(c+dx)}{bc-ad}\right)\right)+2B\text{Li}_2\left(\frac{d(a+bx)}{-bc+ad}\right)}{bg}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(a*g + b*g*x),x]
```

```
[Out] (Log[a + b*x]*(A - B*Log[a + b*x] + B*Log[(e*(a + b*x)^2)/(c + d*x)^2] + 2*
B*Log[(b*(c + d*x))/(b*c - a*d)]) + 2*B*PolyLog[2, (d*(a + b*x))/(-(b*c) +
a*d)])/(b*g)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 571 vs. $2(82) = 164$.
time = 0.51, size = 572, normalized size = 6.89

method	result
risch	$\frac{A\ln(bx+a)}{gb}+\frac{B\ln\left(\frac{ad-cb}{dx+c}+b\right)\ln\left(\frac{e\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^2}{d^2}\right)ad}{gb(ad-cb)}-\frac{B\ln\left(\frac{ad-cb}{dx+c}+b\right)\ln\left(\frac{e\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^2}{d^2}\right)c}{g(ad-cb)}-\frac{B\ln(}{g}$
derivativedivides	$-\frac{-\frac{d^2A\ln\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)a}{gb(ad-cb)}+\frac{dA\ln\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)c}{g(ad-cb)}+\frac{dA\ln\left(\frac{1}{dx+c}\right)}{gb}-\frac{d^2B\ln\left(\frac{ad-cb}{dx+c}+b\right)\ln\left(\frac{e\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^2}{d^2}\right)a}{gb(ad-cb)}+\frac{dI}{}}{}$
default	$-\frac{-\frac{d^2A\ln\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)a}{gb(ad-cb)}+\frac{dA\ln\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)c}{g(ad-cb)}+\frac{dA\ln\left(\frac{1}{dx+c}\right)}{gb}-\frac{d^2B\ln\left(\frac{ad-cb}{dx+c}+b\right)\ln\left(\frac{e\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^2}{d^2}\right)a}{gb(ad-cb)}+\frac{dI}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/(b*g*x+a*g),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d*(-d^2/g*A/b/(a*d-b*c)*ln(a/(d*x+c)*d-b*c/(d*x+c)+b)*a+d/g*A/(a*d-b*c)*
ln(a/(d*x+c)*d-b*c/(d*x+c)+b)*c+d/g*A/b*ln(1/(d*x+c))-d^2/g*B/b*ln((a*d-b*c
)/(d*x+c)+b)/(a*d-b*c)*ln(e*(a/(d*x+c)*d-b*c/(d*x+c)+b)^2/d^2)*a+d/g*B*ln((
a*d-b*c)/(d*x+c)+b)/(a*d-b*c)*ln(e*(a/(d*x+c)*d-b*c/(d*x+c)+b)^2/d^2)*c+d^2
/g*B/b/(a*d-b*c)*ln((a*d-b*c)/(d*x+c)+b)^2*a-d/g*B/(a*d-b*c)*ln((a*d-b*c)/(
d*x+c)+b)^2*c+d/g*B/b*ln(1/(d*x+c))*ln(e*(a/(d*x+c)*d-b*c/(d*x+c)+b)^2/d^2)
-2*d^2/g*B/b*dilog(((a*d-b*c)/(d*x+c)+b)/b)/(a*d-b*c)*a+2*d/g*B*dilog(((a*d
-b*c)/(d*x+c)+b)/b)/(a*d-b*c)*c-2*d^2/g*B/b*ln(1/(d*x+c))*ln(((a*d-b*c)/(d*
x+c)+b)/b)/(a*d-b*c)*a+2*d/g*B*ln(1/(d*x+c))*ln(((a*d-b*c)/(d*x+c)+b)/b)/(a
*d-b*c)*c)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(b*g*x+a*g),x, algorithm="maxima
")
```

```
[Out] -B*(2*log(b*x + a)*log(d*x + c)/(b*g) - integrate((b*d*x + b*c + 2*(2*b*d*x
 + b*c + a*d)*log(b*x + a))/(b^2*d*g*x^2 + a*b*c*g + (b^2*c*g + a*b*d*g)*x)
, x)) + A*log(b*g*x + a*g)/(b*g)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(b*g*x+a*g),x, algorithm="fricas
")
[Out] integral((B*log((b^2*x^2 + 2*a*b*x + a^2)*e/(d^2*x^2 + 2*c*d*x + c^2)) + A)
/(b*g*x + a*g), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A}{a+bx}\,dx + \int \frac{B\log\left(\frac{a^2e}{c^2+2cdx+d^2x^2}+\frac{2abex}{c^2+2cdx+d^2x^2}+\frac{b^2ex^2}{c^2+2cdx+d^2x^2}\right)}{a+bx}\,dx}{g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(b*x+a)**2/(d*x+c)**2))/(b*g*x+a*g),x)
[Out] (Integral(A/(a + b*x), x) + Integral(B*log(a**2*e/(c**2 + 2*c*d*x + d**2*x*
*2) + 2*a*b*e*x/(c**2 + 2*c*d*x + d**2*x**2) + b**2*e*x**2/(c**2 + 2*c*d*x
+ d**2*x**2))/(a + b*x), x))/g
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(b*g*x+a*g),x, algorithm="giac")
[Out] integrate((B*log((b*x + a)^2*e/(d*x + c)^2) + A)/(b*g*x + a*g), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\,\ln\left(\frac{e\,{\left(a+b\,x\right)}^2}{{\left(c+d\,x\right)}^2}\right)}{a\,g+b\,g\,x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))/(a*g + b*g*x),x)
[Out] int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))/(a*g + b*g*x), x)
```

3.124 $\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(ag+bgx)^2}\,dx$

Optimal. Leaf size=65

$$-\frac{2B}{bg^2(a+bx)}-\frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(bc-ad)g^2(a+bx)}$$

```
[Out] -2*B/b/g^2/(b*x+a)-(d*x+c)*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/(-a*d+b*c)/g^2/(
b*x+a)
```

Rubi [A]
time = 0.03, antiderivative size = 77, normalized size of antiderivative = 1.18, number of steps used = 2, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {2550, 2341}

$$-\frac{(c+dx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{g^2(a+bx)(bc-ad)}-\frac{2B(c+dx)}{g^2(a+bx)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(a*g + b*g*x)^2,x]
```

```
[Out] (-2*B*(c + d*x))/((b*c - a*d)*g^2*(a + b*x)) - ((c + d*x)*(A + B*Log[(e*(a
+ b*x)^2)/(c + d*x)^2]))/((b*c - a*d)*g^2*(a + b*x))
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2550

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(
m + 1)*(g/b)^m, Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x],
 x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] &&
 EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && Eq
Q[b*f - a*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Rubi steps

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(ag+bgx)^2}\,dx = -\frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{bg^2(a+bx)}+\frac{B\int\frac{2(bc-ad)}{g(a+bx)^2(c+dx)}\,dx}{bg}$$
$$= -\frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{bg^2(a+bx)}+\frac{(2B(bc-ad))\int\frac{1}{(a+bx)^2(c+dx)}\,dx}{bg^2}$$
$$= -\frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{bg^2(a+bx)}+\frac{(2B(bc-ad))\int\left(\frac{b}{(bc-ad)(a+bx)^2}-\frac{bd}{(bc-ad)^2(a+bx)}+\frac{}{(bc-}\right.}{bg^2}$$
$$= -\frac{2B}{bg^2(a+bx)}-\frac{2Bd\log(a+bx)}{b(bc-ad)g^2}-\frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{bg^2(a+bx)}+\frac{2Bd\log(c+dx)}{b(bc-ad)g^2}$$

Mathematica [A]
time = 0.04, size = 111, normalized size = 1.71

$$-\frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{bg^2(a+bx)}+\frac{2B(bc-ad)\left(-\frac{1}{(bc-ad)(a+bx)}-\frac{d\log(a+bx)}{(bc-ad)^2}+\frac{d\log(c+dx)}{(bc-ad)^2}\right)}{bg^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(a*g + b*g*x)^2,x]
[Out] -((A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(b*g^2*(a + b*x))) + (2*B*(b*c -
 a*d)*(-(1/((b*c - a*d)*(a + b*x))) - (d*Log[a + b*x])/(b*c - a*d)^2 + (d*L
og[c + d*x])/(b*c - a*d)^2))/(b*g^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 144 vs. 2(65) = 130.
time = 0.30, size = 145, normalized size = 2.23

method	result
norman	$\frac{\frac{(A+2B)x}{ga}+\frac{cB\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)}{(ad-cb)g}+\frac{Bdx\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)}{(ad-cb)g}}{g(bx+a)}$
risch	$-\frac{B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)}{b\,g^2(bx+a)}-\frac{-2B\ln(-bx-a)bdx+2B\ln(dx+c)bdx-2B\ln(-bx-a)ad+2B\ln(dx+c)ad+Aad-Abc+2Bad-2B}{g^2(bx+a)b(ad-cb)}$
derivativedivides	$-\frac{-\frac{d^2A}{g^2\left(\frac{ad-cb}{dx+c}+b\right)(ad-cb)}+\frac{\frac{2d^2B}{bg(dx+c)}-\frac{d^2B\ln\left(\frac{e\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^2}{d^2}\right)}{g(ad-cb)}}{g\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)}}{d}$

default	$-\frac{-\frac{d^{2} A}{g^{2}\left(\frac{ad-cb}{dx+c}+b\right)(ad-cb)}+\frac{\frac{2 d^{2} B}{b g(dx+c)}-\frac{d^{2} B \ln\left(\frac{e\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^{2}}{d^{2}}\right)}{g(ad-cb)}}{g\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)}}{d}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/(b*g*x+a*g)^2,x,method=_RETURNVERBOSE)

[Out] -1/d*(-d^2/g^2*A/((a*d-b*c)/(d*x+c)+b)/(a*d-b*c)+(2*d^2*B/b/g/(d*x+c)-d^2*B /g/(a*d-b*c)*ln(e*(a/(d*x+c)*d-b*c/(d*x+c)+b)^2/d^2))/g/(a/(d*x+c)*d-b*c/(d *x+c)+b))

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 190 vs. $2(66) = 132$.
time = 0.31, size = 190, normalized size = 2.92

$$-B\left(\frac{\log\left(\frac{b^{2}x^{2}e}{d^{2}x^{2}+2\,cdx+c^{2}}+\frac{2\,abxe}{d^{2}x^{2}+2\,cdx+c^{2}}+\frac{a^{2}e}{d^{2}x^{2}+2\,cdx+c^{2}}\right)}{b^{2}g^{2}x+abg^{2}}+\frac{2}{b^{2}g^{2}x+abg^{2}}+\frac{2\,d\log\left(bx+a\right)}{(b^{2}c-abd)g^{2}}-\frac{2\,d\log\left(dx+c\right)}{(b^{2}c-abd)g^{2}}\right)-\frac{A}{b^{2}g^{2}x+abg^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(b*g*x+a*g)^2,x, algorithm="maxima")

[Out] -B*(log(b^2*x^2*e/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*x*e/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2))/(b^2*g^2*x + a*b*g^2) + 2/(b^2*g^2*x + a*b*g^2) + 2*d*log(b*x + a)/((b^2*c - a*b*d)*g^2) - 2*d*log(d*x + c)/((b^2*c - a*b*d)*g^2)) - A/(b^2*g^2*x + a*b*g^2)

Fricas [A]
time = 0.34, size = 108, normalized size = 1.66

$$-\frac{(A+2\,B)bc-(A+2\,B)ad+(Bbdx+Bbc)\log\left(\frac{(b^{2}x^{2}+2\,abx+a^{2})e}{d^{2}x^{2}+2\,cdx+c^{2}}\right)}{(b^{3}c-ab^{2}d)g^{2}x+(ab^{2}c-a^{2}bd)g^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(b*g*x+a*g)^2,x, algorithm="fricas")

[Out] -((A + 2*B)*b*c - (A + 2*B)*a*d + (B*b*d*x + B*b*c)*log((b^2*x^2 + 2*a*b*x + a^2)*e/(d^2*x^2 + 2*c*d*x + c^2)))/((b^3*c - a*b^2*d)*g^2*x + (a*b^2*c - a^2*b*d)*g^2)

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 255 vs. $2(54) = 108$.
time = 1.04, size = 255, normalized size = 3.92

$$-\frac{B\log\left(\frac{e(a+bx)^{2}}{(c+dx)^{2}}\right)}{abg^{2}+b^{2}g^{2}x}-\frac{2Bd\log\left(x+\frac{-\frac{2Ba^{2}d^{3}}{ad-bc}+\frac{4Babcd^{2}}{ad-bc}+2Bad^{2}-\frac{2Bb^{2}c^{2}d}{ad-bc}+2Bbcd}{4Bbd^{2}}\right)}{bg^{2}\left(ad-bc\right)}+\frac{2Bd\log\left(x+\frac{\frac{2Ba^{2}d^{3}}{ad-bc}-\frac{4Babcd^{2}}{ad-bc}+2Bad^{2}+\frac{2Bb^{2}c^{2}d}{ad-bc}+2Bbcd}{4Bbd^{2}}\right)}{bg^{2}\left(ad-bc\right)}+\frac{-A-2B}{abg^{2}+b^{2}g^{2}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(b*x+a)**2/(d*x+c)**2))/(b*g*x+a*g)**2,x)
```

```
[Out] -B*log(e*(a + b*x)**2/(c + d*x)**2)/(a*b*g**2 + b**2*g**2*x) - 2*B*d*log(x
+ (-2*B*a**2*d**3/(a*d - b*c) + 4*B*a*b*c*d**2/(a*d - b*c) + 2*B*a*d**2 - 2
*B*b**2*c**2*d/(a*d - b*c) + 2*B*b*c*d)/(4*B*b*d**2))/(b*g**2*(a*d - b*c))
+ 2*B*d*log(x + (2*B*a**2*d**3/(a*d - b*c) - 4*B*a*b*c*d**2/(a*d - b*c) + 2
*B*a*d**2 + 2*B*b**2*c**2*d/(a*d - b*c) + 2*B*b*c*d)/(4*B*b*d**2))/(b*g**2*
(a*d - b*c)) + (-A - 2*B)/(a*b*g**2 + b**2*g**2*x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 188 vs. $2(66) = 132$.
time = 2.40, size = 188, normalized size = 2.89

$$\left(2\left(b^{2}cg^{2}-abdg^{2}\right)\left(\frac{d\log\left(\left|\frac{bcg}{bgx+ag}-\frac{adg}{bgx+ag}+d\right|\right)}{b^{4}c^{2}g^{4}-2\,ab^{3}cdg^{4}+a^{2}b^{2}d^{2}g^{4}}-\frac{1}{(b^{2}cg^{2}-abdg^{2})(bgx+ag)bg}\right)-\frac{\log\left(\frac{(bx+a)^{2}e}{(dx+c)^{2}}\right)}{(bgx+ag)bg}\right)B-\frac{A}{(bgx+ag)bg}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(b*g*x+a*g)^2,x, algorithm="giac
")
```

```
[Out] (2*(b^2*c*g^2 - a*b*d*g^2)*(d*log(abs(b*c*g/(b*g*x + a*g) - a*d*g/(b*g*x +
a*g) + d))/(b^4*c^2*g^4 - 2*a*b^3*c*d*g^4 + a^2*b^2*d^2*g^4) - 1/((b^2*c*g^
2 - a*b*d*g^2)*(b*g*x + a*g)*b*g)) - log((b*x + a)^2*e/(d*x + c)^2)/((b*g*x
 + a*g)*b*g))*B - A/((b*g*x + a*g)*b*g)
```

Mupad [**B**]
time = 5.25, size = 108, normalized size = 1.66

$$-\frac{A+2\,B}{x\,b^{2}\,g^{2}+a\,b\,g^{2}}-\frac{B\,\ln\left(\frac{e\,(a+b\,x)^{2}}{(c+d\,x)^{2}}\right)}{b^{2}\,g^{2}\,\left(x+\frac{a}{b}\right)}-\frac{B\,d\,\mathrm{atan}\left(\frac{b\,c\,2\mathrm{i}+b\,d\,x\,2\mathrm{i}}{a\,d-b\,c}+1\mathrm{i}\right)\,4\mathrm{i}}{b\,g^{2}\,\left(a\,d-b\,c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))/(a*g + b*g*x)^2,x)
```

```
[Out] - (A + 2*B)/(b^2*g^2*x + a*b*g^2) - (B*log((e*(a + b*x)^2)/(c + d*x)^2))/(b
^2*g^2*(x + a/b)) - (B*d*atan((b*c*2i + b*d*x*2i)/(a*d - b*c) + 1i)*4i)/(b*
g^2*(a*d - b*c))
```

3.125 $\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(ag+bgx)^3}\,dx$

Optimal. Leaf size=138

$$-\frac{B}{2bg^3(a+bx)^2}+\frac{Bd}{b(bc-ad)g^3(a+bx)}+\frac{Bd^2\log(a+bx)}{b(bc-ad)^2g^3}-\frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{2bg^3(a+bx)^2}-\frac{Bd^2\log(c+dx)}{b(bc-ad)^2g^3}$$

```
[Out] -1/2*B/b/g^3/(b*x+a)^2+B*d/b/(-a*d+b*c)/g^3/(b*x+a)+B*d^2*ln(b*x+a)/b/(-a*d
+b*c)^2/g^3+1/2*(-A-B*ln(e*(b*x+a)^2/(d*x+c)^2))/b/g^3/(b*x+a)^2-B*d^2*ln(d
*x+c)/b/(-a*d+b*c)^2/g^3
```

Rubi [A]
time = 0.07, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {2548, 21, 46}

$$-\frac{B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A}{2bg^3(a+bx)^2}+\frac{Bd^2\log(a+bx)}{bg^3(bc-ad)^2}-\frac{Bd^2\log(c+dx)}{bg^3(bc-ad)^2}+\frac{Bd}{bg^3(a+bx)(bc-ad)}-\frac{B}{2bg^3(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(a*g + b*g*x)^3,x]
```

```
[Out] -1/2*B/(b*g^3*(a + b*x)^2) + (B*d)/(b*(b*c - a*d)*g^3*(a + b*x)) + (B*d^2*L
og[a + b*x])/(b*(b*c - a*d)^2*g^3) - (A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2
])/(2*b*g^3*(a + b*x)^2) - (B*d^2*Log[c + d*x])/(b*(b*c - a*d)^2*g^3)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2548

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
```

```
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Dist[B*n*((b*c
- a*d)/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /;
 FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c -
 a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Rubi steps

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(ag+bgx)^3}\,dx = -\frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{2bg^3(a+bx)^2}+\frac{B\int\frac{2(bc-ad)}{g^2(a+bx)^3(c+dx)}\,dx}{2bg}$$
$$= -\frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{2bg^3(a+bx)^2}+\frac{(B(bc-ad))\int\frac{1}{(a+bx)^3(c+dx)}\,dx}{bg^3}$$
$$= -\frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{2bg^3(a+bx)^2}+\frac{(B(bc-ad))\int\left(\frac{b}{(bc-ad)(a+bx)^3}-\frac{bd}{(bc-ad)^2(a+bx)^2}+\frac{}{(bc-}\right.}{bg^3}$$
$$= -\frac{B}{2bg^3(a+bx)^2}+\frac{Bd}{b(bc-ad)g^3(a+bx)}+\frac{Bd^2\log(a+bx)}{b(bc-ad)^2g^3}-\frac{A+B\log\left(\frac{e(a+}{(c+d}\right.}{2bg^3(a+bx)}$$

Mathematica [A]
time = 0.09, size = 109, normalized size = 0.79

$$-\frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+\frac{B((bc-ad)(-3ad+b(c-2dx))-2d^2(a+bx)^2\log(a+bx)+2d^2(a+bx)^2\log(c+dx))}{(bc-ad)^2}}{2bg^3(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(a*g + b*g*x)^3,x]
```

```
[Out] -1/2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2] + (B*((b*c - a*d)*(-3*a*d + b*
(c - 2*d*x)) - 2*d^2*(a + b*x)^2*Log[a + b*x] + 2*d^2*(a + b*x)^2*Log[c + d
*x]))/(b*c - a*d)^2)/(b*g^3*(a + b*x)^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 285 vs. $2(137) = 274$.
time = 0.37, size = 286, normalized size = 2.07

method	result
risch	$-\frac{B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)}{2b\,g^3(bx+a)^2}-\frac{2B\ln(dx+c)b^2d^2x^2-2B\ln(-bx-a)b^2d^2x^2+4B\ln(dx+c)ab\,d^2x-4B\ln(-bx-a)ab\,d^2x+2B\ln(dx}{2(a^2d^2-2ab}$

norman	$\frac{\frac{(Aad-Abc+2Bad-Bbc)x}{ag(ad-cb)}+\frac{Ba\,d^2x\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)}{\left(a^2d^2-2abcd+b^2c^2\right)g}+\frac{Bc(2ad-cb)\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)}{2g\left(a^2d^2-2abcd+b^2c^2\right)}+\frac{(Aad-Abc+3Bad-Bbc)b\,x^2}{2a^2g(ad-cb)}+\frac{B\,d^2b\,x^2\ln\left(\frac{e(b}{(d}\right.}{2\left(a^2d^2-2abcd+\right.}}{g^2(bx+a)^2}$
derivativedivides	$-\frac{\frac{d^3A\left(-\frac{1}{(ad-cb)^2\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)}+\frac{b}{2(ad-cb)^2\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^2}\right)}{g^3}+\frac{\frac{B\,d^3}{g(ad-cb)(dx+c)}+\frac{3B\,d^3}{2bg(dx+c)^2}-\frac{bB\,d^3\ln\left(\frac{e\left(\frac{ad}{dx+c}-\right.}{d}\right.}{2g\left(a^2d^2-2abcd\right.}}{\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}\right.}}{d}$
default	$-\frac{\frac{d^3A\left(-\frac{1}{(ad-cb)^2\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)}+\frac{b}{2(ad-cb)^2\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^2}\right)}{g^3}+\frac{\frac{B\,d^3}{g(ad-cb)(dx+c)}+\frac{3B\,d^3}{2bg(dx+c)^2}-\frac{bB\,d^3\ln\left(\frac{e\left(\frac{ad}{dx+c}-\right.}{d}\right.}{2g\left(a^2d^2-2abcd\right.}}{\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}\right.}}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/(b*g*x+a*g)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/d*(d^3/g^3*A*(-1/(a*d-b*c)^2/(a/(d*x+c)*d-b*c/(d*x+c)+b)+1/2*b/(a*d-b*c)
^2/(a/(d*x+c)*d-b*c/(d*x+c)+b)^2)+(B*d^3/g/(a*d-b*c)/(d*x+c)+3/2*B/b*d^3/g/
(d*x+c)^2-1/2*b*B*d^3/g/(a^2*d^2-2*a*b*c*d+b^2*c^2)*ln(e*(a/(d*x+c)*d-b*c/(
d*x+c)+b)^2/d^2)-B*d^3/g/(a*d-b*c)/(d*x+c)*ln(e*(a/(d*x+c)*d-b*c/(d*x+c)+b)
^2/d^2))/(a/(d*x+c)*d-b*c/(d*x+c)+b)^2/g^2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 310 vs. $2(135) = 270$.
time = 0.29, size = 310, normalized size = 2.25

$$\frac{1}{2}B\left(\frac{2\,bdx-bc+3\,ad}{(b^4c-ab^3d)g^3x^2+2\,(ab^3c-a^2b^2d)g^3x+(a^2b^2c-a^3bd)g^3}-\frac{\log\left(\frac{b^2x^2e}{d^2x^2+2\,cdx+c^2}+\frac{2\,abxe}{d^2x^2+2\,cdx+c^2}+\frac{a^2e}{d^2x^2+2\,cdx+c^2}\right)}{b^3g^3x^2+2\,ab^2g^3x+a^2bg^3}+\frac{2\,d^2\log(bx+a)}{(b^3c^2-2\,ab^2cd+a^2bd^2)g^3}-\frac{2\,d^2\log(dx+c)}{(b^3c^2-2\,ab^2cd+a^2bd^2)g^3}\right)-\frac{A}{2\,(b^3g^3x^2+2\,ab^2g^3x+a^2bg^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(b*g*x+a*g)^3,x, algorithm="maxi
ma")
```

```
[Out] 1/2*B*((2*b*d*x - b*c + 3*a*d)/((b^4*c - a*b^3*d)*g^3*x^2 + 2*(a*b^3*c - a^
2*b^2*d)*g^3*x + (a^2*b^2*c - a^3*b*d)*g^3) - log(b^2*x^2*e/(d^2*x^2 + 2*c*
d*x + c^2) + 2*a*b*x*e/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x
 + c^2))/(b^3*g^3*x^2 + 2*a*b^2*g^3*x + a^2*b*g^3) + 2*d^2*log(b*x + a)/((b
^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*g^3) - 2*d^2*log(d*x + c)/((b^3*c^2 - 2*a
*b^2*c*d + a^2*b*d^2)*g^3)) - 1/2*A/(b^3*g^3*x^2 + 2*a*b^2*g^3*x + a^2*b*g^
3)
```

Fricas [A]
time = 0.37, size = 236, normalized size = 1.71

$$-\frac{(A+B)b^2c^2-2\,(A+2\,B)abcd+(A+3\,B)a^2d^2-2\,(Bb^2cd-Babd^2)x-(Bb^2d^2x^2+2\,Babd^2x-Bb^2c^2+2\,Babcd)\log\left(\frac{(b^2x^2+2\,abx+a^2)e}{d^2x^2+2\,cdx+c^2}\right)}{2\,((b^5c^2-2\,ab^4cd+a^2b^3d^2)g^3x^2+2\,(ab^4c^2-2\,a^2b^3cd+a^3b^2d^2)g^3x+(a^2b^3c^2-2\,a^3b^2cd+a^4bd^2)g^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(b*g*x+a*g)^3,x, algorithm="fric
as")
```

```
[Out] -1/2*((A + B)*b^2*c^2 - 2*(A + 2*B)*a*b*c*d + (A + 3*B)*a^2*d^2 - 2*(B*b^2*
c*d - B*a*b*d^2)*x - (B*b^2*d^2*x^2 + 2*B*a*b*d^2*x - B*b^2*c^2 + 2*B*a*b*c
*d)*log((b^2*x^2 + 2*a*b*x + a^2)*e/(d^2*x^2 + 2*c*d*x + c^2)))/((b^5*c^2 -
 2*a*b^4*c*d + a^2*b^3*d^2)*g^3*x^2 + 2*(a*b^4*c^2 - 2*a^2*b^3*c*d + a^3*b^
2*d^2)*g^3*x + (a^2*b^3*c^2 - 2*a^3*b^2*c*d + a^4*b*d^2)*g^3)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 418 vs. $2(122) = 244$.
time = 1.39, size = 418, normalized size = 3.03

$$-\frac{B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{2a^2bg^3+4ab^2g^3x+2b^3g^3x^2}-\frac{Bd^2\log\left(x+\frac{-\frac{Ba^3d^5}{(ad-bc)^2}+\frac{3Ba^2bcd^4}{(ad-bc)^2}-\frac{3Bab^2c^2d^3}{(ad-bc)^2}+Bad^3+\frac{Bb^3c^3d^2}{(ad-bc)^2}+Bbcd^2}{2Bbd^3}\right)}{bg^3\left(ad-bc\right)^2}+\frac{Bd^2\log\left(x+\frac{\frac{Ba^3d^5}{(ad-bc)^2}-\frac{3Ba^2bcd^4}{(ad-bc)^2}+\frac{3Bab^2c^2d^3}{(ad-bc)^2}+Bad^3-\frac{Bb^3c^3d^2}{(ad-bc)^2}+Bbcd^2}{2Bbd^3}\right)}{bg^3\left(ad-bc\right)^2}+\frac{-Aad+Abc-3Bad+Bbc-2Bbdx}{2a^3bdg^3-2a^2b^2cg^3+x^2\cdot(2ab^3dg^3-2b^4cg^3)+x\left(4a^2b^2dg^3-4ab^3cg^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(b*x+a)**2/(d*x+c)**2))/(b*g*x+a*g)**3,x)
```

```
[Out] -B*log(e*(a + b*x)**2/(c + d*x)**2)/(2*a**2*b*g**3 + 4*a*b**2*g**3*x + 2*b*
*3*g**3*x**2) - B*d**2*log(x + (-B*a**3*d**5/(a*d - b*c)**2 + 3*B*a**2*b*c*
d**4/(a*d - b*c)**2 - 3*B*a*b**2*c**2*d**3/(a*d - b*c)**2 + B*a*d**3 + B*b*
*3*c**3*d**2/(a*d - b*c)**2 + B*b*c*d**2)/(2*B*b*d**3))/(b*g**3*(a*d - b*c)
**2) + B*d**2*log(x + (B*a**3*d**5/(a*d - b*c)**2 - 3*B*a**2*b*c*d**4/(a*d
- b*c)**2 + 3*B*a*b**2*c**2*d**3/(a*d - b*c)**2 + B*a*d**3 - B*b**3*c**3*d*
*2/(a*d - b*c)**2 + B*b*c*d**2)/(2*B*b*d**3))/(b*g**3*(a*d - b*c)**2) + (-A
*a*d + A*b*c - 3*B*a*d + B*b*c - 2*B*b*d*x)/(2*a**3*b*d*g**3 - 2*a**2*b**2*
c*g**3 + x**2*(2*a*b**3*d*g**3 - 2*b**4*c*g**3) + x*(4*a**2*b**2*d*g**3 - 4
*a*b**3*c*g**3))
```

Giac [A]
time = 2.89, size = 264, normalized size = 1.91

$$\frac{Bd^2\log\left(bx+a\right)}{b^3c^2g^3-2\,ab^2cdg^3+a^2bd^2g^3}-\frac{Bd^2\log\left(dx+c\right)}{b^3c^2g^3-2\,ab^2cdg^3+a^2bd^2g^3}-\frac{B\log\left(\frac{b^2x^2+2\,abx+a^2}{d^2x^2+2\,cdx+c^2}\right)}{2\left(b^3g^3x^2+2\,ab^2g^3x+a^2bg^3\right)}+\frac{2\,Bbdx-Abc-2\,Bbc+Aad+4\,Bad}{2\left(b^4cg^3x^2-ab^3dg^3x^2+2\,ab^3cg^3x-2\,a^2b^2dg^3x+a^2b^2cg^3-a^3bdg^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(b*g*x+a*g)^3,x, algorithm="giac
")
```

```
[Out] B*d^2*log(b*x + a)/(b^3*c^2*g^3 - 2*a*b^2*c*d*g^3 + a^2*b*d^2*g^3) - B*d^2*
log(d*x + c)/(b^3*c^2*g^3 - 2*a*b^2*c*d*g^3 + a^2*b*d^2*g^3) - 1/2*B*log((b
^2*x^2 + 2*a*b*x + a^2)/(d^2*x^2 + 2*c*d*x + c^2))/(b^3*g^3*x^2 + 2*a*b^2*g
^3*x + a^2*b*g^3) + 1/2*(2*B*b*d*x - A*b*c - 2*B*b*c + A*a*d + 4*B*a*d)/(b^
4*c*g^3*x^2 - a*b^3*d*g^3*x^2 + 2*a*b^3*c*g^3*x - 2*a^2*b^2*d*g^3*x + a^2*b
^2*c*g^3 - a^3*b*d*g^3)
```

Mupad [B]
time = 5.14, size = 206, normalized size = 1.49

$$-\frac{\frac{A\,a\,d-A\,b\,c+3\,B\,a\,d-B\,b\,c}{2\,\left(a\,d-b\,c\right)}+\frac{B\,b\,d\,x}{a\,d-b\,c}}{a^2\,b\,g^3+2\,a\,b^2\,g^3\,x+b^3\,g^3\,x^2}-\frac{B\,\ln\left(\frac{e\,{\left(a+b\,x\right)}^2}{{\left(c+d\,x\right)}^2}\right)}{2\,b^2\,g^3\,\left(2\,a\,x+b\,x^2+\frac{a^2}{b}\right)}-\frac{2\,B\,d^2\,\mathrm{atanh}\left(\frac{b^3\,c^2\,g^3-a^2\,b\,d^2\,g^3}{b\,g^3\,{\left(a\,d-b\,c\right)}^2}-\frac{2\,b\,d\,x}{a\,d-b\,c}\right)}{b\,g^3\,{\left(a\,d-b\,c\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))/(a*g + b*g*x)^3,x)
```

```
[Out] - ((A*a*d - A*b*c + 3*B*a*d - B*b*c)/(2*(a*d - b*c)) + (B*b*d*x)/(a*d - b*c
))/(a^2*b*g^3 + b^3*g^3*x^2 + 2*a*b^2*g^3*x) - (B*log((e*(a + b*x)^2)/(c +
d*x)^2))/(2*b^2*g^3*(2*a*x + b*x^2 + a^2/b)) - (2*B*d^2*atanh((b^3*c^2*g^3
- a^2*b*d^2*g^3)/(b*g^3*(a*d - b*c)^2) - (2*b*d*x)/(a*d - b*c)))/(b*g^3*(a*
d - b*c)^2)
```

3.126 $\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(ag+bgx)^4}\,dx$

Optimal. Leaf size=177

$$-\frac{2B}{9bg^4(a+bx)^3}+\frac{Bd}{3b(bc-ad)g^4(a+bx)^2}-\frac{2Bd^2}{3b(bc-ad)^2g^4(a+bx)}-\frac{2Bd^3\log(a+bx)}{3b(bc-ad)^3g^4}-\frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{3bg^4(a+bx)^3}$$

```
[Out] -2/9*B/b/g^4/(b*x+a)^3+1/3*B*d/b/(-a*d+b*c)/g^4/(b*x+a)^2-2/3*B*d^2/b/(-a*d
+b*c)^2/g^4/(b*x+a)-2/3*B*d^3*ln(b*x+a)/b/(-a*d+b*c)^3/g^4+1/3*(-A-B*ln(e*(
b*x+a)^2/(d*x+c)^2))/b/g^4/(b*x+a)^3+2/3*B*d^3*ln(d*x+c)/b/(-a*d+b*c)^3/g^4
```

Rubi [A]

time = 0.09, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {2548, 21, 46}

$$-\frac{B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A}{3bg^4(a+bx)^3}-\frac{2Bd^3\log(a+bx)}{3bg^4(bc-ad)^3}+\frac{2Bd^3\log(c+dx)}{3bg^4(bc-ad)^3}-\frac{2Bd^2}{3bg^4(a+bx)(bc-ad)^2}+\frac{Bd}{3bg^4(a+bx)^2(bc-ad)}-\frac{2B}{9bg^4(a+bx)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(a*g + b*g*x)^4,x]
```

```
[Out] (-2*B)/(9*b*g^4*(a + b*x)^3) + (B*d)/(3*b*(b*c - a*d)*g^4*(a + b*x)^2) - (2
*B*d^2)/(3*b*(b*c - a*d)^2*g^4*(a + b*x)) - (2*B*d^3*Log[a + b*x])/(3*b*(b*
c - a*d)^3*g^4) - (A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(3*b*g^4*(a + b*
x)^3) + (2*B*d^3*Log[c + d*x])/(3*b*(b*c - a*d)^3*g^4)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2548

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
```

```
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Dist[B*n*((b*c
- a*d)/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /;
 FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c -
 a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Rubi steps

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(ag+bgx)^4}\,dx = -\frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{3bg^4(a+bx)^3}+\frac{B\int\frac{2(bc-ad)}{g^3(a+bx)^4(c+dx)}\,dx}{3bg}$$
$$= -\frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{3bg^4(a+bx)^3}+\frac{(2B(bc-ad))\int\frac{1}{(a+bx)^4(c+dx)}\,dx}{3bg^4}$$
$$= -\frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{3bg^4(a+bx)^3}+\frac{(2B(bc-ad))\int\left(\frac{b}{(bc-ad)(a+bx)^4}-\frac{bd}{(bc-ad)^2(a+bx)^3}+\frac{}{(b}\right.}{3bg^4}$$
$$= -\frac{2B}{9bg^4(a+bx)^3}+\frac{Bd}{3b(bc-ad)g^4(a+bx)^2}-\frac{2Bd^2}{3b(bc-ad)^2g^4(a+bx)}-\frac{2Bd^3}{3b(b}$$

Mathematica [A]
time = 0.08, size = 140, normalized size = 0.79

$$-\frac{3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)+\frac{B(2(bc-ad)^3-3d(bc-ad)^2(a+bx)+6d^2(bc-ad)(a+bx)^2+6d^3(a+bx)^3\log(a+bx)-6d^3(a+bx)^3\log(c+dx))}{(bc-ad)^3}}{9bg^4(a+bx)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(a*g + b*g*x)^4,x]
```

```
[Out] -1/9*(3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]) + (B*(2*(b*c - a*d)^3 - 3*
d*(b*c - a*d)^2*(a + b*x) + 6*d^2*(b*c - a*d)*(a + b*x)^2 + 6*d^3*(a + b*x)
^3*Log[a + b*x] - 6*d^3*(a + b*x)^3*Log[c + d*x]))/(b*c - a*d)^3)/(b*g^4*(a
 + b*x)^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 453 vs. $2(168) = 336$.
time = 0.47, size = 454, normalized size = 2.56

method	result
risch	$-\frac{B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)}{3b\,g^4(bx+a)^3}-\frac{6B\ln(dx+c)b^3d^3x^3-6B\ln(-bx-a)b^3d^3x^3+18B\ln(dx+c)a\,b^2d^3x^2-18B\ln(-bx-a)a\,b^2d^3x^2+}{}$

derivativedivides	$-\frac{\frac{d^4 A\left(-\frac{b^2}{3(ad-cb)^3\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^3}+\frac{b}{(ad-cb)^3\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^2}-\frac{1}{(ad-cb)^3\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)}\right)}{g^4}+\frac{\frac{11B\,d^4}{9bg(dx+c)^3}-\frac{b^2 B\,d^4 \ln}{3g\left(a^3d^3-\right.}}{}}{}$
default	$-\frac{\frac{d^4 A\left(-\frac{b^2}{3(ad-cb)^3\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^3}+\frac{b}{(ad-cb)^3\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^2}-\frac{1}{(ad-cb)^3\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)}\right)}{g^4}+\frac{\frac{11B\,d^4}{9bg(dx+c)^3}-\frac{b^2 B\,d^4 \ln}{3g\left(a^3d^3-\right.}}{}}{}$
norman	$\frac{\frac{B\,a^2d^3x\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)}{\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)g}+\frac{Bab\,d^3x^2\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)}{\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)g}+\frac{\left(3A\,a^2d^2-6Aabcd+3A\,b^2c^2+6B\,a^2d^2-6Babcd+2B\,b^2c^2\right.}{3ga\left(a^2d^2-2abcd+b^2c^2\right)}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/(b*g*x+a*g)^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/d*(d^4/g^4*A*(-1/3*b^2/(a*d-b*c)^3/(a/(d*x+c)*d-b*c/(d*x+c)+b)^3+b/(a*d-
b*c)^3/(a/(d*x+c)*d-b*c/(d*x+c)+b)^2-1/(a*d-b*c)^3/(a/(d*x+c)*d-b*c/(d*x+c)
+b))+(11/9*B/b*d^4/g/(d*x+c)^3-1/3*b^2*B*d^4/g/(a^3*d^3-3*a^2*b*c*d^2+3*a*b
^2*c^2*d-b^3*c^3)*ln(e*(a/(d*x+c)*d-b*c/(d*x+c)+b)^2/d^2)+2/3*B*b*d^4/g/(a^
2*d^2-2*a*b*c*d+b^2*c^2)/(d*x+c)+5/3*B*d^4/g/(a*d-b*c)/(d*x+c)^2-B*d^4/(a*d
-b*c)/g/(d*x+c)^2*ln(e*(a/(d*x+c)*d-b*c/(d*x+c)+b)^2/d^2)-B*d^4*b/g/(a^2*d^
2-2*a*b*c*d+b^2*c^2)/(d*x+c)*ln(e*(a/(d*x+c)*d-b*c/(d*x+c)+b)^2/d^2))/(a/(d
*x+c)*d-b*c/(d*x+c)+b)^3/g^3)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 483 vs. $2(166) = 332$.
time = 0.31, size = 483, normalized size = 2.73

$$-\frac{1}{9}B\left(\frac{6b^2d^2x^2+2b^2c^2-7abcd+11a^2d^2-3(b^2cd-5abd^2)x}{(b^6c^2-2ab^5cd+a^2b^4d^2)g^4x^3+3(ab^5c^2-2a^2b^4cd+a^3b^3d^2)g^4x^2+3(a^2b^4c^2-2a^3b^3cd+a^4b^2d^2)g^4x+(a^3b^3c^2-2a^4b^2cd+a^5bd^2)g^4}+\frac{3\log\left(\frac{b^2x^2e}{d^2x^2+2cdx+c^2}+\frac{2abxe}{d^2x^2+2cdx+c^2}+\frac{a^2e}{d^2x^2+2cdx+c^2}\right)}{b^4g^4x^3+3ab^3g^4x^2+3a^2b^2g^4x+a^3bg^4}+\frac{6d^3\log(bx+a)}{(b^4c^3-3ab^3c^2d+3a^2b^2cd^2-a^3bd^3)g^4}-\frac{6d^3\log(dx+c)}{(b^4c^3-3ab^3c^2d+3a^2b^2cd^2-a^3bd^3)g^4}\right)-\frac{A}{3(b^4g^4x^3+3ab^3g^4x^2+3a^2b^2g^4x+a^3bg^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(b*g*x+a*g)^4,x, algorithm="maxi
ma")
```

```
[Out] -1/9*B*((6*b^2*d^2*x^2 + 2*b^2*c^2 - 7*a*b*c*d + 11*a^2*d^2 - 3*(b^2*c*d -
5*a*b*d^2)*x)/((b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2)*g^4*x^3 + 3*(a*b^5*c^2
 - 2*a^2*b^4*c*d + a^3*b^3*d^2)*g^4*x^2 + 3*(a^2*b^4*c^2 - 2*a^3*b^3*c*d +
a^4*b^2*d^2)*g^4*x + (a^3*b^3*c^2 - 2*a^4*b^2*c*d + a^5*b*d^2)*g^4) + 3*log
(b^2*x^2*e/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*x*e/(d^2*x^2 + 2*c*d*x + c^2)
+ a^2*e/(d^2*x^2 + 2*c*d*x + c^2))/(b^4*g^4*x^3 + 3*a*b^3*g^4*x^2 + 3*a^2*b
^2*g^4*x + a^3*b*g^4) + 6*d^3*log(b*x + a)/((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^
2*b^2*c*d^2 - a^3*b*d^3)*g^4) - 6*d^3*log(d*x + c)/((b^4*c^3 - 3*a*b^3*c^2*
d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*g^4)) - 1/3*A/(b^4*g^4*x^3 + 3*a*b^3*g^4*x
^2 + 3*a^2*b^2*g^4*x + a^3*b*g^4)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 428 vs. $2(166) = 332$.
time = 0.36, size = 428, normalized size = 2.42

$$-\frac{(3\,A+2\,B)b^3c^3-9\,(A+B)ab^2c^2d+9\,(A+2\,B)a^2bcd^2-(3\,A+11\,B)a^3d^3+6\,(Bb^3cd^2-Bab^2d^3)x^2-3\,(Bb^3c^2d-6\,Bab^2cd^2+5\,Ba^2bd^3)x+3\,(Bb^3d^3x^3+3\,Bab^2d^3x^2+3\,Ba^2bd^3x+Bb^3c^3-3\,Bab^2c^2d+3\,Ba^2bcd^2)\log\left(\frac{(b^2x^2+2\,abx+a^2)e}{d^2x^2+2\,cdx+c^2}\right)}{9\,((b^7c^3-3\,ab^6c^2d+3\,a^2b^5cd^2-a^3b^4d^3)g^4x^3+3\,(ab^6c^3-3\,a^2b^5c^2d+3\,a^3b^4cd^2-a^4b^3d^3)g^4x^2+3\,(a^2b^5c^3-3\,a^3b^4c^2d+3\,a^4b^3cd^2-a^5b^2d^3)g^4x+(a^3b^4c^3-3\,a^4b^3c^2d+3\,a^5b^2cd^2-a^6bd^3)g^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(b*g*x+a*g)^4,x, algorithm="fric
as")
```

```
[Out] -1/9*((3*A + 2*B)*b^3*c^3 - 9*(A + B)*a*b^2*c^2*d + 9*(A + 2*B)*a^2*b*c*d^2
 - (3*A + 11*B)*a^3*d^3 + 6*(B*b^3*c*d^2 - B*a*b^2*d^3)*x^2 - 3*(B*b^3*c^2*
d - 6*B*a*b^2*c*d^2 + 5*B*a^2*b*d^3)*x + 3*(B*b^3*d^3*x^3 + 3*B*a*b^2*d^3*x
^2 + 3*B*a^2*b*d^3*x + B*b^3*c^3 - 3*B*a*b^2*c^2*d + 3*B*a^2*b*c*d^2)*log((
b^2*x^2 + 2*a*b*x + a^2)*e/(d^2*x^2 + 2*c*d*x + c^2)))/((b^7*c^3 - 3*a*b^6*
c^2*d + 3*a^2*b^5*c*d^2 - a^3*b^4*d^3)*g^4*x^3 + 3*(a*b^6*c^3 - 3*a^2*b^5*c
^2*d + 3*a^3*b^4*c*d^2 - a^4*b^3*d^3)*g^4*x^2 + 3*(a^2*b^5*c^3 - 3*a^3*b^4*
c^2*d + 3*a^4*b^3*c*d^2 - a^5*b^2*d^3)*g^4*x + (a^3*b^4*c^3 - 3*a^4*b^3*c^2
*d + 3*a^5*b^2*c*d^2 - a^6*b*d^3)*g^4)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 677 vs. $2(162) = 324$.
time = 2.18, size = 677, normalized size = 3.82

$$-\frac{B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{3a^3bg^4+9a^2b^2g^4x+9ab^3g^4x^2+3b^4g^4x^3}-\frac{2Bd^3\log\left(x+\frac{-\frac{2Ba^4d^7}{(ad-bc)^3}+\frac{8Ba^3bcd^6}{(ad-bc)^3}-\frac{12Ba^2b^2c^2d^5}{(ad-bc)^3}+\frac{8Bab^3c^3d^4}{(ad-bc)^3}+2Bad^4-\frac{2Bb^4c^4d^3}{(ad-bc)^3}+2Bbcd^3}{4Bbd^4}\right)}{3bg^4\,(ad-bc)^3}+\frac{2Bd^3\log\left(x+\frac{\frac{2Ba^4d^7}{(ad-bc)^3}-\frac{8Ba^3bcd^6}{(ad-bc)^3}+\frac{12Ba^2b^2c^2d^5}{(ad-bc)^3}-\frac{8Bab^3c^3d^4}{(ad-bc)^3}+2Bad^4+\frac{2Bb^4c^4d^3}{(ad-bc)^3}+2Bbcd^3}{4Bbd^4}\right)}{3bg^4\,(ad-bc)^3}+\frac{-3Aa^2d^2+6Aabcd-3Ab^2c^2-11Ba^2d^2+7Babcd-2Bb^2c^2-6Bb^2d^2x^2+x(-15Babd^2+3Bb^2cd)}{9a^5bd^2g^4-18a^4b^2cdg^4+9a^3b^3c^2g^4+x^3\cdot(9a^2b^4d^2g^4-18ab^5cdg^4+9b^6c^2g^4)+x^2\cdot(27a^3b^3d^2g^4-54a^2b^4cdg^4+27ab^5c^2g^4)+x\,(27a^4b^2d^2g^4-54a^3b^3cdg^4+27a^2b^4c^2g^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(b*x+a)**2/(d*x+c)**2))/(b*g*x+a*g)**4,x)
```

```
[Out] -B*log(e*(a + b*x)**2/(c + d*x)**2)/(3*a**3*b*g**4 + 9*a**2*b**2*g**4*x + 9
*a*b**3*g**4*x**2 + 3*b**4*g**4*x**3) - 2*B*d**3*log(x + (-2*B*a**4*d**7/(a
*d - b*c)**3 + 8*B*a**3*b*c*d**6/(a*d - b*c)**3 - 12*B*a**2*b**2*c**2*d**5/
(a*d - b*c)**3 + 8*B*a*b**3*c**3*d**4/(a*d - b*c)**3 + 2*B*a*d**4 - 2*B*b**
4*c**4*d**3/(a*d - b*c)**3 + 2*B*b*c*d**3)/(4*B*b*d**4))/(3*b*g**4*(a*d - b
*c)**3) + 2*B*d**3*log(x + (2*B*a**4*d**7/(a*d - b*c)**3 - 8*B*a**3*b*c*d**
6/(a*d - b*c)**3 + 12*B*a**2*b**2*c**2*d**5/(a*d - b*c)**3 - 8*B*a*b**3*c**
3*d**4/(a*d - b*c)**3 + 2*B*a*d**4 + 2*B*b**4*c**4*d**3/(a*d - b*c)**3 + 2*
B*b*c*d**3)/(4*B*b*d**4))/(3*b*g**4*(a*d - b*c)**3) + (-3*A*a**2*d**2 + 6*A
*a*b*c*d - 3*A*b**2*c**2 - 11*B*a**2*d**2 + 7*B*a*b*c*d - 2*B*b**2*c**2 - 6
*B*b**2*d**2*x**2 + x*(-15*B*a*b*d**2 + 3*B*b**2*c*d))/(9*a**5*b*d**2*g**4
- 18*a**4*b**2*c*d*g**4 + 9*a**3*b**3*c**2*g**4 + x**3*(9*a**2*b**4*d**2*g*
*4 - 18*a*b**5*c*d*g**4 + 9*b**6*c**2*g**4) + x**2*(27*a**3*b**3*d**2*g**4
- 54*a**2*b**4*c*d*g**4 + 27*a*b**5*c**2*g**4) + x*(27*a**4*b**2*d**2*g**4
- 54*a**3*b**3*c*d*g**4 + 27*a**2*b**4*c**2*g**4))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 473 vs. 2(166) = 332.
time = 3.39, size = 473, normalized size = 2.67

$$-\frac{2\,Bd^{3}\log\left(bx+a\right)}{3\,\left(b^{4}c^{3}g^{4}-3\,ab^{3}c^{2}dg^{4}+3\,a^{2}b^{2}cd^{2}g^{4}-a^{3}bd^{3}g^{4}\right)}+\frac{2\,Bd^{3}\log\left(dx+c\right)}{3\,\left(b^{4}c^{3}g^{4}-3\,ab^{3}c^{2}dg^{4}+3\,a^{2}b^{2}cd^{2}g^{4}-a^{3}bd^{3}g^{4}\right)}-\frac{B\log\left(\frac{b^{2}x^{2}+2\,abx+a^{2}}{d^{2}x^{2}+2\,cdx+c^{2}}\right)}{3\,\left(b^{4}g^{4}x^{3}+3\,ab^{3}g^{4}x^{2}+3\,a^{2}b^{2}g^{4}x+a^{3}bg^{4}\right)}-\frac{6\,Bb^{2}d^{2}x^{2}-3\,Bb^{2}cdx+15\,Babd^{2}x+3\,Ab^{2}c^{2}+5\,Bb^{2}c^{2}-6\,Aabcd-13\,Babcd+3\,Aa^{2}d^{2}+14\,Ba^{2}d^{2}}{9\,\left(b^{6}c^{2}g^{4}x^{3}-2\,ab^{5}cdg^{4}x^{3}+a^{2}b^{4}d^{2}g^{4}x^{3}+3\,ab^{5}c^{2}g^{4}x^{2}-6\,a^{2}b^{4}cdg^{4}x^{2}+3\,a^{3}b^{3}d^{2}g^{4}x^{2}+3\,a^{2}b^{4}c^{2}g^{4}x-6\,a^{3}b^{3}cdg^{4}x+3\,a^{4}b^{2}d^{2}g^{4}x+a^{3}b^{3}c^{2}g^{4}-2\,a^{4}b^{2}cdg^{4}+a^{5}bd^{2}g^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(b*g*x+a*g)^4,x, algorithm="giac
")
```

```
[Out] -2/3*B*d^3*log(b*x + a)/(b^4*c^3*g^4 - 3*a*b^3*c^2*d*g^4 + 3*a^2*b^2*c*d^2*
g^4 - a^3*b*d^3*g^4) + 2/3*B*d^3*log(d*x + c)/(b^4*c^3*g^4 - 3*a*b^3*c^2*d*
g^4 + 3*a^2*b^2*c*d^2*g^4 - a^3*b*d^3*g^4) - 1/3*B*log((b^2*x^2 + 2*a*b*x +
 a^2)/(d^2*x^2 + 2*c*d*x + c^2))/(b^4*g^4*x^3 + 3*a*b^3*g^4*x^2 + 3*a^2*b^2
*g^4*x + a^3*b*g^4) - 1/9*(6*B*b^2*d^2*x^2 - 3*B*b^2*c*d*x + 15*B*a*b*d^2*x
 + 3*A*b^2*c^2 + 5*B*b^2*c^2 - 6*A*a*b*c*d - 13*B*a*b*c*d + 3*A*a^2*d^2 + 1
4*B*a^2*d^2)/(b^6*c^2*g^4*x^3 - 2*a*b^5*c*d*g^4*x^3 + a^2*b^4*d^2*g^4*x^3 +
 3*a*b^5*c^2*g^4*x^2 - 6*a^2*b^4*c*d*g^4*x^2 + 3*a^3*b^3*d^2*g^4*x^2 + 3*a^
2*b^4*c^2*g^4*x - 6*a^3*b^3*c*d*g^4*x + 3*a^4*b^2*d^2*g^4*x + a^3*b^3*c^2*g
^4 - 2*a^4*b^2*c*d*g^4 + a^5*b*d^2*g^4)
```

Mupad [B]
time = 5.80, size = 341, normalized size = 1.93

$$\frac{2\,A\,a\,c\,d}{3\,g^4\,{\left(a\,d-b\,c\right)}^2\,{\left(a+b\,x\right)}^3}-\frac{A\,b\,c^2}{3\,g^4\,{\left(a\,d-b\,c\right)}^2\,{\left(a+b\,x\right)}^3}-\frac{2\,B\,b\,c^2}{9\,g^4\,{\left(a\,d-b\,c\right)}^2\,{\left(a+b\,x\right)}^3}-\frac{A\,a^2\,d^2}{3\,b\,g^4\,{\left(a\,d-b\,c\right)}^2\,{\left(a+b\,x\right)}^3}-\frac{11\,B\,a^2\,d^2}{9\,b\,g^4\,{\left(a\,d-b\,c\right)}^2\,{\left(a+b\,x\right)}^3}-\frac{5\,B\,a\,d^2\,x}{3\,g^4\,{\left(a\,d-b\,c\right)}^2\,{\left(a+b\,x\right)}^3}-\frac{2\,B\,b\,d^2\,x^2}{3\,g^4\,{\left(a\,d-b\,c\right)}^2\,{\left(a+b\,x\right)}^3}-\frac{B\,\ln\left(\frac{e\,{\left(a+b\,x\right)}^2}{{\left(c+d\,x\right)}^2}\right)}{3\,b\,g^4\,{\left(a+b\,x\right)}^3}+\frac{7\,B\,a\,c\,d}{9\,g^4\,{\left(a\,d-b\,c\right)}^2\,{\left(a+b\,x\right)}^3}+\frac{B\,b\,c\,d\,x}{3\,g^4\,{\left(a\,d-b\,c\right)}^2\,{\left(a+b\,x\right)}^3}-\frac{B\,d^3\,\mathrm{atan}\left(\frac{a\,d\,1{}\mathrm{i}+b\,c\,1{}\mathrm{i}+b\,d\,x\,2{}\mathrm{i}}{a\,d-b\,c}\right)\,4{}\mathrm{i}}{3\,b\,g^4\,{\left(a\,d-b\,c\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))/(a*g + b*g*x)^4,x)
```

```
[Out] (2*A*a*c*d)/(3*g^4*(a*d - b*c)^2*(a + b*x)^3) - (B*d^3*atan((a*d*1i + b*c*1
i + b*d*x*2i)/(a*d - b*c))*4i)/(3*b*g^4*(a*d - b*c)^3) - (A*b*c^2)/(3*g^4*(
a*d - b*c)^2*(a + b*x)^3) - (2*B*b*c^2)/(9*g^4*(a*d - b*c)^2*(a + b*x)^3) -
 (A*a^2*d^2)/(3*b*g^4*(a*d - b*c)^2*(a + b*x)^3) - (11*B*a^2*d^2)/(9*b*g^4*
(a*d - b*c)^2*(a + b*x)^3) - (5*B*a*d^2*x)/(3*g^4*(a*d - b*c)^2*(a + b*x)^3
) - (2*B*b*d^2*x^2)/(3*g^4*(a*d - b*c)^2*(a + b*x)^3) - (B*log((e*(a + b*x)
^2)/(c + d*x)^2))/(3*b*g^4*(a + b*x)^3) + (7*B*a*c*d)/(9*g^4*(a*d - b*c)^2*
(a + b*x)^3) + (B*b*c*d*x)/(3*g^4*(a*d - b*c)^2*(a + b*x)^3)
```

3.127 $\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(ag+bgx)^5}\,dx$

Optimal. Leaf size=208

$$-\frac{B}{8bg^5(a+bx)^4}+\frac{Bd}{6b(bc-ad)g^5(a+bx)^3}-\frac{Bd^2}{4b(bc-ad)^2g^5(a+bx)^2}+\frac{Bd^3}{2b(bc-ad)^3g^5(a+bx)}+\frac{Bd^4\log(a+}{2b(bc-ad)^4}$$

```
[Out] -1/8*B/b/g^5/(b*x+a)^4+1/6*B*d/b/(-a*d+b*c)/g^5/(b*x+a)^3-1/4*B*d^2/b/(-a*d
+b*c)^2/g^5/(b*x+a)^2+1/2*B*d^3/b/(-a*d+b*c)^3/g^5/(b*x+a)+1/2*B*d^4*ln(b*x
+a)/b/(-a*d+b*c)^4/g^5+1/4*(-A-B*ln(e*(b*x+a)^2/(d*x+c)^2))/b/g^5/(b*x+a)^4
-1/2*B*d^4*ln(d*x+c)/b/(-a*d+b*c)^4/g^5
```

Rubi [A]
time = 0.11, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {2548, 21, 46}

$$-\frac{B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A}{4bg^5(a+bx)^4}+\frac{Bd^4\log(a+bx)}{2bg^5(bc-ad)^4}-\frac{Bd^4\log(c+dx)}{2bg^5(bc-ad)^4}+\frac{Bd^3}{2bg^5(a+bx)(bc-ad)^3}-\frac{Bd^2}{4bg^5(a+bx)^2(bc-ad)^2}+\frac{Bd}{6bg^5(a+bx)^3(bc-ad)}-\frac{B}{8bg^5(a+bx)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(a*g + b*g*x)^5,x]
```

```
[Out] -1/8*B/(b*g^5*(a + b*x)^4) + (B*d)/(6*b*(b*c - a*d)*g^5*(a + b*x)^3) - (B*d
^2)/(4*b*(b*c - a*d)^2*g^5*(a + b*x)^2) + (B*d^3)/(2*b*(b*c - a*d)^3*g^5*(a
 + b*x)) + (B*d^4*Log[a + b*x])/(2*b*(b*c - a*d)^4*g^5) - (A + B*Log[(e*(a
+ b*x)^2)/(c + d*x)^2])/(4*b*g^5*(a + b*x)^4) - (B*d^4*Log[c + d*x])/(2*b*(
b*c - a*d)^4*g^5)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2548

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Dist[B*n*((b*c
- a*d)/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /;
 FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c -
 a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Rubi steps

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(ag+bgx)^5}\,dx = -\frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{4bg^5(a+bx)^4}+\frac{B\int\frac{2(bc-ad)}{g^4(a+bx)^5(c+dx)}\,dx}{4bg}$$

$$= -\frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{4bg^5(a+bx)^4}+\frac{(B(bc-ad))\int\frac{1}{(a+bx)^5(c+dx)}\,dx}{2bg^5}$$

$$= -\frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{4bg^5(a+bx)^4}+\frac{(B(bc-ad))\int\left(\frac{b}{(bc-ad)(a+bx)^5}-\frac{bd}{(bc-ad)^2(a+bx)^4}+\frac{\cdots}{(bc-\cdots}\right.}{\cdots}$$

$$= -\frac{B}{8bg^5(a+bx)^4}+\frac{Bd}{6b(bc-ad)g^5(a+bx)^3}-\frac{Bd^2}{4b(bc-ad)^2g^5(a+bx)^2}+\frac{\cdots}{2b(bc\cdots}$$

Mathematica [A]
time = 0.13, size = 162, normalized size = 0.78

$$-\frac{6\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)+\frac{B\left(3(bc-ad)^4+4d(-bc+ad)^3(a+bx)+6d^2(bc-ad)^2(a+bx)^2+12d^3(-bc+ad)(a+bx)^3-12d^4(a+bx)^4\log(a+bx)+12d^4(a+bx)^4\log(c+dx)\right)}{(bc-ad)^4}}{24bg^5(a+bx)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(a*g + b*g*x)^5,x]
```

```
[Out] -1/24*(6*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]) + (B*(3*(b*c - a*d)^4 + 4
*d*(-(b*c) + a*d)^3*(a + b*x) + 6*d^2*(b*c - a*d)^2*(a + b*x)^2 + 12*d^3*(-
(b*c) + a*d)*(a + b*x)^3 - 12*d^4*(a + b*x)^4*Log[a + b*x] + 12*d^4*(a + b*
x)^4*Log[c + d*x]))/(b*c - a*d)^4)/(b*g^5*(a + b*x)^4)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 652 vs. $2(197) = 394$.
time = 0.54, size = 653, normalized size = 3.14 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/(b*g*x+a*g)^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/d*(d^5/g^5*A*(3/2*b/(a*d-b*c)^4/(a/(d*x+c)*d-b*c/(d*x+c)+b)^2-b^2/(a*d-b
*c)^4/(a/(d*x+c)*d-b*c/(d*x+c)+b)^3-1/(a*d-b*c)^4/(a/(d*x+c)*d-b*c/(d*x+c)+
```

```
b)+1/4*b^3/(a*d-b*c)^4/(a/(d*x+c)*d-b*c/(d*x+c)+b)^4)+(25/24*B/b*d^5/g/(d*x
+c)^4-1/4*b^3*B*d^5/g/(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*
d+b^4*c^4)*ln(e*(a/(d*x+c)*d-b*c/(d*x+c)+b)^2/d^2)+1/2*B*b^2*d^5/g/(a^3*d^3
-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/(d*x+c)+7/4*B*b*d^5/g/(a^2*d^2-2*a*b*
c*d+b^2*c^2)/(d*x+c)^2+13/6*B*d^5/g/(a*d-b*c)/(d*x+c)^3-B*d^5/(a*d-b*c)/g/(
d*x+c)^3*ln(e*(a/(d*x+c)*d-b*c/(d*x+c)+b)^2/d^2)-3/2*b*B*d^5/g/(a^2*d^2-2*a
*b*c*d+b^2*c^2)/(d*x+c)^2*ln(e*(a/(d*x+c)*d-b*c/(d*x+c)+b)^2/d^2)-B*d^5*b^2
/g/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/(d*x+c)*ln(e*(a/(d*x+c)*d-
b*c/(d*x+c)+b)^2/d^2))/(a/(d*x+c)*d-b*c/(d*x+c)+b)^4/g^4)
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 702 vs. $2(195) = 390$.
time = 0.32, size = 702, normalized size = 3.38

$$\frac{1}{24}\,B\left(\frac{12\,b^3d^3x^3-3\,b^3c^3+13\,ab^2c^2d-23\,a^2bcd^2+25\,a^3d^3-6\,(b^3cd^2-7\,ab^2d^3)x^2+4\,(b^3c^2d-5\,ab^2cd^2+13\,a^2bd^3)x}{(b^8c^3-3\,ab^7c^2d+3\,a^2b^6cd^2-a^3b^5d^3)g^5x^4+4\,(ab^7c^3-3\,a^2b^6c^2d+3\,a^3b^5cd^2-a^4b^4d^3)g^5x^3+6\,(a^2b^6c^3-3\,a^3b^5c^2d+3\,a^4b^4cd^2-a^5b^3d^3)g^5x^2+4\,(a^3b^5c^3-3\,a^4b^4c^2d+3\,a^5b^3cd^2-a^6b^2d^3)g^5x+(a^4b^4c^3-3\,a^5b^3c^2d+3\,a^6b^2cd^2-a^7bd^3)g^5}-\frac{6\,\log\left(\frac{b^2x^2e}{d^2x^2+2\,cdx+c^2}+\frac{2\,abxe}{d^2x^2+2\,cdx+c^2}+\frac{a^2e}{d^2x^2+2\,cdx+c^2}\right)}{b^5g^5x^4+4\,ab^4g^5x^3+6\,a^2b^3g^5x^2+4\,a^3b^2g^5x+a^4bg^5}+\frac{12\,d^4\log(bx+a)}{(b^5c^4-4\,ab^4c^3d+6\,a^2b^3c^2d^2-4\,a^3b^2cd^3+a^4bd^4)g^5}-\frac{12\,d^4\log(dx+c)}{(b^5c^4-4\,ab^4c^3d+6\,a^2b^3c^2d^2-4\,a^3b^2cd^3+a^4bd^4)g^5}\right)-\frac{A}{4\,(b^5g^5x^4+4\,ab^4g^5x^3+6\,a^2b^3g^5x^2+4\,a^3b^2g^5x+a^4bg^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(b*g*x+a*g)^5,x, algorithm="maxi
ma")
```

```
[Out] 1/24*B*((12*b^3*d^3*x^3 - 3*b^3*c^3 + 13*a*b^2*c^2*d - 23*a^2*b*c*d^2 + 25*
a^3*d^3 - 6*(b^3*c*d^2 - 7*a*b^2*d^3)*x^2 + 4*(b^3*c^2*d - 5*a*b^2*c*d^2 +
13*a^2*b*d^3)*x)/((b^8*c^3 - 3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 - a^3*b^5*d^3)
*g^5*x^4 + 4*(a*b^7*c^3 - 3*a^2*b^6*c^2*d + 3*a^3*b^5*c*d^2 - a^4*b^4*d^3)*
g^5*x^3 + 6*(a^2*b^6*c^3 - 3*a^3*b^5*c^2*d + 3*a^4*b^4*c*d^2 - a^5*b^3*d^3)
*g^5*x^2 + 4*(a^3*b^5*c^3 - 3*a^4*b^4*c^2*d + 3*a^5*b^3*c*d^2 - a^6*b^2*d^3
)*g^5*x + (a^4*b^4*c^3 - 3*a^5*b^3*c^2*d + 3*a^6*b^2*c*d^2 - a^7*b*d^3)*g^5
) - 6*log(b^2*x^2*e/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*x*e/(d^2*x^2 + 2*c*d*
x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2))/(b^5*g^5*x^4 + 4*a*b^4*g^5*x^3
+ 6*a^2*b^3*g^5*x^2 + 4*a^3*b^2*g^5*x + a^4*b*g^5) + 12*d^4*log(b*x + a)/((
b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*
g^5) - 12*d^4*log(d*x + c)/((b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 -
4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5)) - 1/4*A/(b^5*g^5*x^4 + 4*a*b^4*g^5*x^3 +
 6*a^2*b^3*g^5*x^2 + 4*a^3*b^2*g^5*x + a^4*b*g^5)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 652 vs. $2(195) = 390$.
time = 0.36, size = 652, normalized size = 3.13

$$-\frac{3\,(2\,A+B)b^4c^4-8\,(3\,A+2\,B)ab^3c^3d+36\,(A+B)a^2b^2c^2d^2-24\,(A+2\,B)a^3bcd^3+(6\,A+25\,B)a^4d^4-12\,(Bb^4cd^3-Bab^3d^4)x^3+6\,(Bb^4c^2d^2-8\,Bab^3cd^3+7\,Ba^2b^2d^4)x^2-4\,(Bb^4c^3d-6\,Bab^3c^2d^2+18\,Ba^2b^2cd^3-13\,Ba^3bd^4)x-6\,(Bb^4d^4x^4+4\,Bab^3d^4x^3+6\,Ba^2b^2d^4x^2+4\,Ba^3bd^4x-Bb^4c^4+4\,Bab^3c^3d-6\,Ba^2b^2c^2d^2+4\,Ba^3bcd^3)\log\left(\frac{(b^2x^2+2\,abx+a^2)e}{d^2x^2+2\,cdx+c^2}\right)}{24\,((b^9c^4-4\,ab^8c^3d+6\,a^2b^7c^2d^2-4\,a^3b^6cd^3+a^4b^5d^4)g^5x^4+4\,(ab^8c^4-4\,a^2b^7c^3d+6\,a^3b^6c^2d^2-4\,a^4b^5cd^3+a^5b^4d^4)g^5x^3+6\,(a^2b^7c^4-4\,a^3b^6c^3d+6\,a^4b^5c^2d^2-4\,a^5b^4cd^3+a^6b^3d^4)g^5x^2+4\,(a^3b^6c^4-4\,a^4b^5c^3d+6\,a^5b^4c^2d^2-4\,a^6b^3cd^3+a^7b^2d^4)g^5x+(a^4b^5c^4-4\,a^5b^4c^3d+6\,a^6b^3c^2d^2-4\,a^7b^2cd^3+a^8bd^4)g^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(b*g*x+a*g)^5,x, algorithm="fric
as")
```

```
[Out] -1/24*(3*(2*A + B)*b^4*c^4 - 8*(3*A + 2*B)*a*b^3*c^3*d + 36*(A + B)*a^2*b^2
*c^2*d^2 - 24*(A + 2*B)*a^3*b*c*d^3 + (6*A + 25*B)*a^4*d^4 - 12*(B*b^4*c*d^
3 - B*a*b^3*d^4)*x^3 + 6*(B*b^4*c^2*d^2 - 8*B*a*b^3*c*d^3 + 7*B*a^2*b^2*d^4
)*x^2 - 4*(B*b^4*c^3*d - 6*B*a*b^3*c^2*d^2 + 18*B*a^2*b^2*c*d^3 - 13*B*a^3*
b*d^4)*x - 6*(B*b^4*d^4*x^4 + 4*B*a*b^3*d^4*x^3 + 6*B*a^2*b^2*d^4*x^2 + 4*B
*a^3*b*d^4*x - B*b^4*c^4 + 4*B*a*b^3*c^3*d - 6*B*a^2*b^2*c^2*d^2 + 4*B*a^3*
b*c*d^3)*log((b^2*x^2 + 2*a*b*x + a^2)*e/(d^2*x^2 + 2*c*d*x + c^2)))/((b^9*
c^4 - 4*a*b^8*c^3*d + 6*a^2*b^7*c^2*d^2 - 4*a^3*b^6*c*d^3 + a^4*b^5*d^4)*g^
5*x^4 + 4*(a*b^8*c^4 - 4*a^2*b^7*c^3*d + 6*a^3*b^6*c^2*d^2 - 4*a^4*b^5*c*d^
3 + a^5*b^4*d^4)*g^5*x^3 + 6*(a^2*b^7*c^4 - 4*a^3*b^6*c^3*d + 6*a^4*b^5*c^2
*d^2 - 4*a^5*b^4*c*d^3 + a^6*b^3*d^4)*g^5*x^2 + 4*(a^3*b^6*c^4 - 4*a^4*b^5*
c^3*d + 6*a^5*b^4*c^2*d^2 - 4*a^6*b^3*c*d^3 + a^7*b^2*d^4)*g^5*x + (a^4*b^5
*c^4 - 4*a^5*b^4*c^3*d + 6*a^6*b^3*c^2*d^2 - 4*a^7*b^2*c*d^3 + a^8*b*d^4)*g
^5)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 947 vs. $2(182) = 364$.
time = 3.06, size = 947, normalized size = 4.55

$$-\frac{B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{4a^4bg^5+16a^3b^2g^5x+24a^2b^3g^5x^2+16ab^4g^5x^3+4b^5g^5x^4}-\frac{Bd^4\log\left(x+\frac{-\frac{Ba^5d^9}{(ad-bc)^4}+\frac{5Ba^4bcd^8}{(ad-bc)^4}-\frac{10Ba^3b^2c^2d^7}{(ad-bc)^4}+\frac{10Ba^2b^3c^3d^6}{(ad-bc)^4}-\frac{5Bab^4c^4d^5}{(ad-bc)^4}+Bad^5+\frac{Bb^5c^5d^4}{(ad-bc)^4}+Bbcd^4}{2Bbd^5}\right)}{2bg^5(ad-bc)^4}+\frac{Bd^4\log\left(x+\frac{\frac{Ba^5d^9}{(ad-bc)^4}-\frac{5Ba^4bcd^8}{(ad-bc)^4}+\frac{10Ba^3b^2c^2d^7}{(ad-bc)^4}-\frac{10Ba^2b^3c^3d^6}{(ad-bc)^4}+\frac{5Bab^4c^4d^5}{(ad-bc)^4}+Bad^5-\frac{Bb^5c^5d^4}{(ad-bc)^4}+Bbcd^4}{2Bbd^5}\right)}{2bg^5(ad-bc)^4}+\frac{-6Aa^3d^3+18Aa^2bcd^2-18Aab^2c^2d+6Ab^3c^3-25Ba^3d^3+23Ba^2bcd^2-13Bab^2c^2d+3Bb^3c^3-12Bb^3d^3x^3+x^2(-42Bab^2d^3+6Bb^3cd^2)+x(-52Ba^2bd^3+20Bab^2cd^2-4Bb^3c^2d)}{24a^7bd^3g^5-72a^6b^2cd^2g^5+72a^5b^3c^2dg^5-24a^4b^4c^3g^5+x^4\cdot(24a^3b^5d^3g^5-72a^2b^6cd^2g^5+72ab^7c^2dg^5-24b^8c^3g^5)+x^3\cdot(96a^4b^4d^3g^5-288a^3b^5cd^2g^5+288a^2b^6c^2dg^5-96ab^7c^3g^5)+x^2\cdot(144a^5b^3d^3g^5-432a^4b^4cd^2g^5+432a^3b^5c^2dg^5-144a^2b^6c^3g^5)+x(96a^6b^2d^3g^5-288a^5b^3cd^2g^5+288a^4b^4c^2dg^5-96a^3b^5c^3g^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(b*x+a)**2/(d*x+c)**2))/(b*g*x+a*g)**5,x)
```

```
[Out] -B*log(e*(a + b*x)**2/(c + d*x)**2)/(4*a**4*b*g**5 + 16*a**3*b**2*g**5*x +
24*a**2*b**3*g**5*x**2 + 16*a*b**4*g**5*x**3 + 4*b**5*g**5*x**4) - B*d**4*l
og(x + (-B*a**5*d**9/(a*d - b*c)**4 + 5*B*a**4*b*c*d**8/(a*d - b*c)**4 - 10
*B*a**3*b**2*c**2*d**7/(a*d - b*c)**4 + 10*B*a**2*b**3*c**3*d**6/(a*d - b*c
)**4 - 5*B*a*b**4*c**4*d**5/(a*d - b*c)**4 + B*a*d**5 + B*b**5*c**5*d**4/(a
*d - b*c)**4 + B*b*c*d**4)/(2*B*b*d**5))/(2*b*g**5*(a*d - b*c)**4) + B*d**4
*log(x + (B*a**5*d**9/(a*d - b*c)**4 - 5*B*a**4*b*c*d**8/(a*d - b*c)**4 + 1
0*B*a**3*b**2*c**2*d**7/(a*d - b*c)**4 - 10*B*a**2*b**3*c**3*d**6/(a*d - b*
c)**4 + 5*B*a*b**4*c**4*d**5/(a*d - b*c)**4 + B*a*d**5 - B*b**5*c**5*d**4/(
a*d - b*c)**4 + B*b*c*d**4)/(2*B*b*d**5))/(2*b*g**5*(a*d - b*c)**4) + (-6*A
*a**3*d**3 + 18*A*a**2*b*c*d**2 - 18*A*a*b**2*c**2*d + 6*A*b**3*c**3 - 25*B
*a**3*d**3 + 23*B*a**2*b*c*d**2 - 13*B*a*b**2*c**2*d + 3*B*b**3*c**3 - 12*B
*b**3*d**3*x**3 + x**2*(-42*B*a*b**2*d**3 + 6*B*b**3*c*d**2) + x*(-52*B*a**
2*b*d**3 + 20*B*a*b**2*c*d**2 - 4*B*b**3*c**2*d))/(24*a**7*b*d**3*g**5 - 72
*a**6*b**2*c*d**2*g**5 + 72*a**5*b**3*c**2*d*g**5 - 24*a**4*b**4*c**3*g**5
+ x**4*(24*a**3*b**5*d**3*g**5 - 72*a**2*b**6*c*d**2*g**5 + 72*a*b**7*c**2*
d*g**5 - 24*b**8*c**3*g**5) + x**3*(96*a**4*b**4*d**3*g**5 - 288*a**3*b**5*
c*d**2*g**5 + 288*a**2*b**6*c**2*d*g**5 - 96*a*b**7*c**3*g**5) + x**2*(144*
a**5*b**3*d**3*g**5 - 432*a**4*b**4*c*d**2*g**5 + 432*a**3*b**5*c**2*d*g**5
 - 144*a**2*b**6*c**3*g**5) + x*(96*a**6*b**2*d**3*g**5 - 288*a**5*b**3*c*d
**2*g**5 + 288*a**4*b**4*c**2*d*g**5 - 96*a**3*b**5*c**3*g**5))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 419 vs. 2(195) = 390.

time = 3.49, size = 419, normalized size = 2.01

$$-\frac{Bd^4\log\left(-\frac{bcg}{bgx+ag}+\frac{adg}{bgx+ag}-d\right)}{2\left(b^5c^4g^5-4\,ab^4c^3dg^5+6\,a^2b^3c^2d^2g^5-4\,a^3b^2cd^3g^5+a^4bd^4g^5\right)}+\frac{Bd^3}{2\left(b^3c^3g^3-3\,ab^2c^2dg^3+3\,a^2bcd^2g^3-a^3d^3g^3\right)(bgx+ag)bg}-\frac{Bd^2}{4\left(b^2c^2g-2\,abcdg+a^2d^2g\right)(bgx+ag)^2bg^2}-\frac{B\log\left(\frac{b^2}{\frac{b^2c^2g^2}{(bgx+ag)^2}-\frac{2\,abcdg^2}{(bgx+ag)^2}+\frac{a^2d^2g^2}{(bgx+ag)^2}+\frac{2\,bcdg}{bgx+ag}-\frac{2\,ad^2g}{bgx+ag}+d^2}\right)}{4\,(bgx+ag)^4bg}+\frac{Bd}{6\,(bgx+ag)^3(bc-ad)bg^2}-\frac{2\,Ab^3g^3+3\,Bb^3g^3}{8\,(bgx+ag)^4b^4g^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(b*g*x+a*g)^5,x, algorithm="giac
")
```

```
[Out] -1/2*B*d^4*log(-b*c*g/(b*g*x + a*g) + a*d*g/(b*g*x + a*g) - d)/(b^5*c^4*g^5
 - 4*a*b^4*c^3*d*g^5 + 6*a^2*b^3*c^2*d^2*g^5 - 4*a^3*b^2*c*d^3*g^5 + a^4*b*
d^4*g^5) + 1/2*B*d^3/((b^3*c^3*g^3 - 3*a*b^2*c^2*d*g^3 + 3*a^2*b*c*d^2*g^3
- a^3*d^3*g^3)*(b*g*x + a*g)*b*g) - 1/4*B*d^2/((b^2*c^2*g - 2*a*b*c*d*g + a
^2*d^2*g)*(b*g*x + a*g)^2*b*g^2) - 1/4*B*log(b^2/(b^2*c^2*g^2/(b*g*x + a*g)
^2 - 2*a*b*c*d*g^2/(b*g*x + a*g)^2 + a^2*d^2*g^2/(b*g*x + a*g)^2 + 2*b*c*d*
g/(b*g*x + a*g) - 2*a*d^2*g/(b*g*x + a*g) + d^2))/((b*g*x + a*g)^4*b*g) + 1
/6*B*d/((b*g*x + a*g)^3*(b*c - a*d)*b*g^2) - 1/8*(2*A*b^3*g^3 + 3*B*b^3*g^3
)/((b*g*x + a*g)^4*b^4*g^4)
```

Mupad [B]

time = 6.51, size = 579, normalized size = 2.78

$$-\frac{\frac{6\,A\,a^3\,d^3-6\,A\,b^3\,c^3+25\,B\,a^3\,d^3-3\,B\,b^3\,c^3+18\,A\,a\,b^2\,c^2\,d-18\,A\,a^2\,b\,c\,d^2+13\,B\,a\,b^2\,c^2\,d-23\,B\,a^2\,b\,c\,d^2}{12\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}-\frac{d^2\,x^2\,\left(B\,b^3\,c-7\,B\,a\,b^2\,d\right)}{2\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}+\frac{d\,x\,\left(13\,B\,a^2\,b\,d^2-5\,B\,a\,b^2\,c\,d+B\,b^3\,c^2\right)}{3\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}+\frac{B\,b^3\,d^3\,x^3}{a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3}}{2\,a^4\,b\,g^5+8\,a^3\,b^2\,g^5\,x+12\,a^2\,b^3\,g^5\,x^2+8\,a\,b^4\,g^5\,x^3+2\,b^5\,g^5\,x^4}-\frac{B\,\ln\left(\frac{e\,{\left(a+b\,x\right)}^2}{{\left(c+d\,x\right)}^2}\right)}{4\,b^2\,g^5\,\left(4\,a^3\,x+\frac{a^4}{b}+b^3\,x^4+6\,a^2\,b\,x^2+4\,a\,b^2\,x^3\right)}-\frac{B\,d^4\,\mathrm{atanh}\left(\frac{-2\,a^4\,b\,d^4\,g^5+4\,a^3\,b^2\,c\,d^3\,g^5-4\,a\,b^4\,c^3\,d\,g^5+2\,b^5\,c^4\,g^5}{2\,b\,g^5\,{\left(a\,d-b\,c\right)}^4}-\frac{2\,b\,d\,x\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}{{\left(a\,d-b\,c\right)}^4}\right)}{b\,g^5\,{\left(a\,d-b\,c\right)}^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))/(a*g + b*g*x)^5,x)
```

```
[Out] - ((6*A*a^3*d^3 - 6*A*b^3*c^3 + 25*B*a^3*d^3 - 3*B*b^3*c^3 + 18*A*a*b^2*c^2
*d - 18*A*a^2*b*c*d^2 + 13*B*a*b^2*c^2*d - 23*B*a^2*b*c*d^2)/(12*(a^3*d^3 -
 b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) - (d^2*x^2*(B*b^3*c - 7*B*a*b^2*
d))/(2*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) + (d*x*(B*b^3*c
^2 + 13*B*a^2*b*d^2 - 5*B*a*b^2*c*d))/(3*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d
 - 3*a^2*b*c*d^2)) + (B*b^3*d^3*x^3)/(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3
*a^2*b*c*d^2))/(2*a^4*b*g^5 + 2*b^5*g^5*x^4 + 8*a^3*b^2*g^5*x + 8*a*b^4*g^5
*x^3 + 12*a^2*b^3*g^5*x^2) - (B*log((e*(a + b*x)^2)/(c + d*x)^2))/(4*b^2*g^
5*(4*a^3*x + a^4/b + b^3*x^4 + 6*a^2*b*x^2 + 4*a*b^2*x^3)) - (B*d^4*atanh((
2*b^5*c^4*g^5 - 2*a^4*b*d^4*g^5 - 4*a*b^4*c^3*d*g^5 + 4*a^3*b^2*c*d^3*g^5)/
(2*b*g^5*(a*d - b*c)^4) - (2*b*d*x*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a
^2*b*c*d^2))/(a*d - b*c)^4))/(b*g^5*(a*d - b*c)^4)
```

3.128 $\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx$

Optimal. Leaf size=377

$$-\frac{B(bc-ad)g^4(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{5bd}+\frac{g^4(a+bx)^5\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{5b}+\frac{2B(bc-ad)^2g^4(a+$$

```
[Out] -1/5*B*(-a*d+b*c)*g^4*(b*x+a)^4*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/b/d+1/5*g^4
*(b*x+a)^5*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/b+2/15*B*(-a*d+b*c)^2*g^4*(b*x
+a)^3*(2*A+B+2*B*ln(e*(b*x+a)^2/(d*x+c)^2))/b/d^2-1/15*B*(-a*d+b*c)^3*g^4*(
b*x+a)^2*(6*A+7*B+6*B*ln(e*(b*x+a)^2/(d*x+c)^2))/b/d^3+2/15*B*(-a*d+b*c)^4*
g^4*(b*x+a)*(6*A+13*B+6*B*ln(e*(b*x+a)^2/(d*x+c)^2))/b/d^4+2/15*B*(-a*d+b*c
)^5*g^4*(6*A+25*B+6*B*ln(e*(b*x+a)^2/(d*x+c)^2))*ln((-a*d+b*c)/b/(d*x+c))/b
/d^5+8/5*B^2*(-a*d+b*c)^5*g^4*polylog(2,d*(b*x+a)/b/(d*x+c))/b/d^5
```

Rubi [A]
time = 0.36, antiderivative size = 377, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.147$, Rules used = {2550, 2381, 2384, 2354, 2438}

$$\frac{8B^2g^4(bc-ad)^5\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{5bd^5}+\frac{2Bg^4(bc-ad)^5\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(6B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+6A+25B\right)}{15bd^5}+\frac{2Bg^4(a+bx)(bc-ad)^4\left(6B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+6A+13B\right)}{15bd^4}-\frac{Bg^4(a+bx)^2(bc-ad)^3\left(6B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+6A+7B\right)}{15bd^3}+\frac{2Bg^4(a+bx)^3(bc-ad)^2\left(2B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+2A+B\right)}{15bd^2}-\frac{Bg^4(a+bx)^4(bc-ad)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{5bd}+\frac{g^4(a+bx)^5\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[(a*g + b*g*x)^4*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2,x]
```

```
[Out] -1/5*(B*(b*c - a*d)*g^4*(a + b*x)^4*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]
))/(b*d) + (g^4*(a + b*x)^5*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2)/(5*
b) + (2*B*(b*c - a*d)^2*g^4*(a + b*x)^3*(2*A + B + 2*B*Log[(e*(a + b*x)^2)/
(c + d*x)^2]))/(15*b*d^2) - (B*(b*c - a*d)^3*g^4*(a + b*x)^2*(6*A + 7*B + 6
*B*Log[(e*(a + b*x)^2)/(c + d*x)^2]))/(15*b*d^3) + (2*B*(b*c - a*d)^4*g^4*(
a + b*x)*(6*A + 13*B + 6*B*Log[(e*(a + b*x)^2)/(c + d*x)^2]))/(15*b*d^4) +
(2*B*(b*c - a*d)^5*g^4*(6*A + 25*B + 6*B*Log[(e*(a + b*x)^2)/(c + d*x)^2])*
Log[(b*c - a*d)/(b*(c + d*x))])/(15*b*d^5) + (8*B^2*(b*c - a*d)^5*g^4*PolyL
og[2, (d*(a + b*x))/(b*(c + d*x))])/(5*b*d^5)
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2381

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
```

```
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + Dist[b*n*(p/(d*(q + 1))), Int[(f*x)^
m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, d
, e, f, m, n, q}, x] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 2384

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Dist[f/(e*(q + 1)), Int[(f*x)^(m - 1)*(d + e*x)^(q + 1
)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x
] && ILtQ[q, -1] && GtQ[m, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2550

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(
m + 1)*(g/b)^m, Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x],
 x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] &&
 EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && Eq
Q[b*f - a*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Rubi steps

$$\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx = \frac{g^4(a+bx)^5\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{5b} - (2B)\int \frac{2(bc-ad)g^5(a+ \cdots}{\cdots}$$

$$= \frac{g^4(a+bx)^5\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{5b} - \frac{(4B(bc-ad)g^4)\int \cdots}{\cdots}$$

$$= \frac{g^4(a+bx)^5\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{5b} - \frac{(4B(bc-ad)g^4)\int \cdots}{\cdots}$$

$$= \frac{g^4(a+bx)^5\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{5b} - \frac{(4B(bc-ad)g^4)\int \cdots}{\cdots}$$

$$= \frac{4AB(bc-ad)^4g^4x}{5d^4} - \frac{2B(bc-ad)^3g^4(a+bx)^2\left(A+B\log \cdots\right.}{5bd^3}$$

$$= \frac{4AB(bc-ad)^4g^4x}{5d^4} + \frac{4B^2(bc-ad)^4g^4(a+bx)\log\left(\frac{e(a+bx)}{(c+dx)^{\cdots}}\cdots\right.}{5bd^4}$$

$$= \frac{4AB(bc-ad)^4g^4x}{5d^4} + \frac{4B^2(bc-ad)^4g^4(a+bx)\log\left(\frac{e(a+bx)}{(c+dx)^{\cdots}}\cdots\right.}{5bd^4}$$

$$= \frac{4AB(bc-ad)^4g^4x}{5d^4} + \frac{26B^2(bc-ad)^4g^4x}{15d^4} - \frac{7B^2(bc-ad)^{3}\cdots}{15b\cdots}$$

$$= \frac{4AB(bc-ad)^4g^4x}{5d^4} + \frac{26B^2(bc-ad)^4g^4x}{15d^4} - \frac{7B^2(bc-ad)^{3}\cdots}{15b\cdots}$$

$$= \frac{4AB(bc-ad)^4g^4x}{5d^4} + \frac{26B^2(bc-ad)^4g^4x}{15d^4} - \frac{7B^2(bc-ad)^{3}\cdots}{15b\cdots}$$

$$= \frac{4AB(bc-ad)^4g^4x}{5d^4} + \frac{26B^2(bc-ad)^4g^4x}{15d^4} - \frac{7B^2(bc-ad)^{3}\cdots}{15b\cdots}$$

Mathematica [A]
time = 0.31, size = 523, normalized size = 1.39

$$\frac{g^4\left((a+bx)^5\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2+\frac{B(bc-ad)\left(12Abd(bc-ad)^3x+12Bd(bc-ad)^3(a+bx)\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)-6d^2(bc-ad)^2(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)+4d^3(bc-ad)(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)-3d^4(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)-24B(bc-ad)^4\log(c+dx)-12(bc-ad)^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\log(c+dx)+4B(bc-ad)^2\left(2bd(bc-ad)x-d^2(a+bx)^2-2(bc-ad)^2\log(c+dx)\right)+B(bc-ad)\left(6bd(bc-ad)^2x+3d^2(-bc+ad)(a+bx)^2+2d^3(a+bx)^3-6(bc-ad)^3\log(c+dx)\right)+12B(bc-ad)^3(bdx+(-bc+ad)\log(c+dx))+12B(bc-ad)^4\left(\left(2\log\left(\frac{d(a+bx)}{-bc+ad}\right)-\log(c+dx)\right)\log(c+dx)+2\text{Li}_2\left(\frac{b(c+dx)}{bc-ad}\right)\right)\right)}{3d^5}\right)}{5b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*g + b*g*x)^4*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2,x]
```

```
[Out] (g^4*((a + b*x)^5*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2 + (B*(b*c - a*
d)*(12*A*b*d*(b*c - a*d)^3*x + 12*B*d*(b*c - a*d)^3*(a + b*x)*Log[(e*(a + b
*x)^2)/(c + d*x)^2] - 6*d^2*(b*c - a*d)^2*(a + b*x)^2*(A + B*Log[(e*(a + b*
x)^2)/(c + d*x)^2]) + 4*d^3*(b*c - a*d)*(a + b*x)^3*(A + B*Log[(e*(a + b*x)
^2)/(c + d*x)^2]) - 3*d^4*(a + b*x)^4*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^
2]) - 24*B*(b*c - a*d)^4*Log[c + d*x] - 12*(b*c - a*d)^4*(A + B*Log[(e*(a +
 b*x)^2)/(c + d*x)^2])*Log[c + d*x] + 4*B*(b*c - a*d)^2*(2*b*d*(b*c - a*d)*
x - d^2*(a + b*x)^2 - 2*(b*c - a*d)^2*Log[c + d*x]) + B*(b*c - a*d)*(6*b*d*
(b*c - a*d)^2*x + 3*d^2*(-(b*c) + a*d)*(a + b*x)^2 + 2*d^3*(a + b*x)^3 - 6*
(b*c - a*d)^3*Log[c + d*x]) + 12*B*(b*c - a*d)^3*(b*d*x + (-(b*c) + a*d)*Lo
g[c + d*x]) + 12*B*(b*c - a*d)^4*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Lo
g[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(3*d^
5)))/(5*b)
```

Maple [F]
time = 0.21, size = 0, normalized size = 0.00

$$\int (bgx + ag)^4 \left(A + B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*g*x+a*g)^4*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

```
[Out] int((b*g*x+a*g)^4*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 2374 vs. 2(368) = 736.
time = 0.45, size = 2374, normalized size = 6.30

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^4*(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="ma
xima")
```

```
[Out] 1/5*A^2*b^4*g^4*x^5 + A^2*a*b^3*g^4*x^4 + 2*A^2*a^2*b^2*g^4*x^3 + 2*A^2*a^3
*b*g^4*x^2 + 2*(x*log(b^2*x^2*e/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*x*e/(d^2*
x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) + 2*a*log(b*x + a)/
b - 2*c*log(d*x + c)/d)*A*B*a^4*g^4 + 4*(x^2*log(b^2*x^2*e/(d^2*x^2 + 2*c*d
*x + c^2) + 2*a*b*x*e/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x
+ c^2)) - 2*a^2*log(b*x + a)/b^2 + 2*c^2*log(d*x + c)/d^2 - 2*(b*c - a*d)*x
/(b*d))*A*B*a^3*b*g^4 + 4*(x^3*log(b^2*x^2*e/(d^2*x^2 + 2*c*d*x + c^2) + 2*
a*b*x*e/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) + 2*a^
3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*
(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*A*B*a^2*b^2*g^4 + 2/3*(3*x^4*log(b^2*x^2*
```

```
e/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*x*e/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(
d^2*x^2 + 2*c*d*x + c^2)) - 6*a^4*log(b*x + a)/b^4 + 6*c^4*log(d*x + c)/d^4
 - (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*
c^3 - a^3*d^3)*x)/(b^3*d^3))*A*B*a*b^3*g^4 + 1/15*(6*x^5*log(b^2*x^2*e/(d^2
*x^2 + 2*c*d*x + c^2) + 2*a*b*x*e/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^
2 + 2*c*d*x + c^2)) + 12*a^5*log(b*x + a)/b^5 - 12*c^5*log(d*x + c)/d^5 - (
3*(b^4*c*d^3 - a*b^3*d^4)*x^4 - 4*(b^4*c^2*d^2 - a^2*b^2*d^4)*x^3 + 6*(b^4*
c^3*d - a^3*b*d^4)*x^2 - 12*(b^4*c^4 - a^4*d^4)*x)/(b^4*d^4))*A*B*b^4*g^4 +
 A^2*a^4*g^4*x - 2/15*(31*b^4*c^5*g^4 - 143*a*b^3*c^4*d*g^4 + 256*a^2*b^2*c
^3*d^2*g^4 - 216*a^3*b*c^2*d^3*g^4 + 78*a^4*c*d^4*g^4)*B^2*log(d*x + c)/d^5
 - 8/5*(b^5*c^5*g^4 - 5*a*b^4*c^4*d*g^4 + 10*a^2*b^3*c^3*d^2*g^4 - 10*a^3*b
^2*c^2*d^3*g^4 + 5*a^4*b*c*d^4*g^4 - a^5*d^5*g^4)*(log(b*x + a)*log((b*d*x
+ a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B^2/(b*d^5) +
1/15*(3*B^2*b^5*d^5*g^4*x^5 - 3*(b^5*c*d^4*g^4 - 6*a*b^4*d^5*g^4)*B^2*x^4 +
 6*(b^5*c^2*d^3*g^4 - 4*a*b^4*c*d^4*g^4 + 8*a^2*b^3*d^5*g^4)*B^2*x^3 - (13*
b^5*c^3*d^2*g^4 - 57*a*b^4*c^2*d^3*g^4 + 93*a^2*b^3*c*d^4*g^4 - 79*a^3*b^2*
d^5*g^4)*B^2*x^2 + (38*b^5*c^4*d*g^4 - 178*a*b^4*c^3*d^2*g^4 + 324*a^2*b^3*
c^2*d^3*g^4 - 278*a^3*b^2*c*d^4*g^4 + 109*a^4*b*d^5*g^4)*B^2*x + 12*(B^2*b^
5*d^5*g^4*x^5 + 5*B^2*a*b^4*d^5*g^4*x^4 + 10*B^2*a^2*b^3*d^5*g^4*x^3 + 10*B
^2*a^3*b^2*d^5*g^4*x^2 + 5*B^2*a^4*b*d^5*g^4*x + B^2*a^5*d^5*g^4)*log(b*x +
 a)^2 + 12*(B^2*b^5*d^5*g^4*x^5 + 5*B^2*a*b^4*d^5*g^4*x^4 + 10*B^2*a^2*b^3*
d^5*g^4*x^3 + 10*B^2*a^3*b^2*d^5*g^4*x^2 + 5*B^2*a^4*b*d^5*g^4*x + (b^5*c^5
*g^4 - 5*a*b^4*c^4*d*g^4 + 10*a^2*b^3*c^3*d^2*g^4 - 10*a^3*b^2*c^2*d^3*g^4
+ 5*a^4*b*c*d^4*g^4)*B^2)*log(d*x + c)^2 + 2*(6*B^2*b^5*d^5*g^4*x^5 - 3*(b^
5*c*d^4*g^4 - 11*a*b^4*d^5*g^4)*B^2*x^4 + 4*(b^5*c^2*d^3*g^4 - 5*a*b^4*c*d^
4*g^4 + 19*a^2*b^3*d^5*g^4)*B^2*x^3 - 6*(b^5*c^3*d^2*g^4 - 5*a*b^4*c^2*d^3*
g^4 + 10*a^2*b^3*c*d^4*g^4 - 16*a^3*b^2*d^5*g^4)*B^2*x^2 + 6*(2*b^5*c^4*d*g
^4 - 10*a*b^4*c^3*d^2*g^4 + 20*a^2*b^3*c^2*d^3*g^4 - 20*a^3*b^2*c*d^4*g^4 +
 13*a^4*b*d^5*g^4)*B^2*x + (12*a*b^4*c^4*d*g^4 - 54*a^2*b^3*c^3*d^2*g^4 + 9
4*a^3*b^2*c^2*d^3*g^4 - 77*a^4*b*c*d^4*g^4 + 31*a^5*d^5*g^4)*B^2)*log(b*x +
 a) - 2*(6*B^2*b^5*d^5*g^4*x^5 - 3*(b^5*c*d^4*g^4 - 11*a*b^4*d^5*g^4)*B^2*x
^4 + 4*(b^5*c^2*d^3*g^4 - 5*a*b^4*c*d^4*g^4 + 19*a^2*b^3*d^5*g^4)*B^2*x^3 -
 6*(b^5*c^3*d^2*g^4 - 5*a*b^4*c^2*d^3*g^4 + 10*a^2*b^3*c*d^4*g^4 - 16*a^3*b
^2*d^5*g^4)*B^2*x^2 + 6*(2*b^5*c^4*d*g^4 - 10*a*b^4*c^3*d^2*g^4 + 20*a^2*b^
3*c^2*d^3*g^4 - 20*a^3*b^2*c*d^4*g^4 + 13*a^4*b*d^5*g^4)*B^2*x + 12*(B^2*b^
5*d^5*g^4*x^5 + 5*B^2*a*b^4*d^5*g^4*x^4 + 10*B^2*a^2*b^3*d^5*g^4*x^3 + 10*B
^2*a^3*b^2*d^5*g^4*x^2 + 5*B^2*a^4*b*d^5*g^4*x + B^2*a^5*d^5*g^4)*log(b*x +
 a))*log(d*x + c))/(b*d^5)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^4*(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="fr
icas")
```

```
[Out] integral(A^2*b^4*g^4*x^4 + 4*A^2*a*b^3*g^4*x^3 + 6*A^2*a^2*b^2*g^4*x^2 + 4*
A^2*a^3*b*g^4*x + A^2*a^4*g^4 + (B^2*b^4*g^4*x^4 + 4*B^2*a*b^3*g^4*x^3 + 6*
B^2*a^2*b^2*g^4*x^2 + 4*B^2*a^3*b*g^4*x + B^2*a^4*g^4)*log((b^2*x^2 + 2*a*b
*x + a^2)*e/(d^2*x^2 + 2*c*d*x + c^2))^2 + 2*(A*B*b^4*g^4*x^4 + 4*A*B*a*b^3
*g^4*x^3 + 6*A*B*a^2*b^2*g^4*x^2 + 4*A*B*a^3*b*g^4*x + A*B*a^4*g^4)*log((b^
2*x^2 + 2*a*b*x + a^2)*e/(d^2*x^2 + 2*c*d*x + c^2)), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)**4*(A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^4*(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="gi
ac")
```

```
[Out] integrate((b*g*x + a*g)^4*(B*log((b*x + a)^2*e/(d*x + c)^2) + A)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a\,g+b\,g\,x)^4\,\left(A+B\,\ln\left(\frac{e\,{(a+b\,x)}^2}{{(c+d\,x)}^2}\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*g + b*g*x)^4*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2,x)
```

```
[Out] int((a*g + b*g*x)^4*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2, x)
```

3.129 $\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx$

Optimal. Leaf size=319

$$-\frac{B(bc-ad)g^3(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{3bd}+\frac{g^3(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{4b}+\frac{B(bc-ad)^2g^3(a+$$

```
[Out] -1/3*B*(-a*d+b*c)*g^3*(b*x+a)^3*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/b/d+1/4*g^3
*(b*x+a)^4*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/b+1/6*B*(-a*d+b*c)^2*g^3*(b*x+
a)^2*(3*A+2*B+3*B*ln(e*(b*x+a)^2/(d*x+c)^2))/b/d^2-1/3*B*(-a*d+b*c)^3*g^3*(
b*x+a)*(3*A+5*B+3*B*ln(e*(b*x+a)^2/(d*x+c)^2))/b/d^3-1/3*B*(-a*d+b*c)^4*g^3
*(3*A+11*B+3*B*ln(e*(b*x+a)^2/(d*x+c)^2))*ln((-a*d+b*c)/b/(d*x+c))/b/d^4-2*
B^2*(-a*d+b*c)^4*g^3*polylog(2,d*(b*x+a)/b/(d*x+c))/b/d^4
```

Rubi [A]
time = 0.27, antiderivative size = 319, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.147$, Rules used = {2550, 2381, 2384, 2354, 2438}

$$-\frac{2B^2g^3(bc-ad)^4\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{bd^4}-\frac{Bg^3(bc-ad)^4\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(3B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+3A+11B\right)}{3bd^4}-\frac{Bg^3(a+bx)(bc-ad)^3\left(3B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+3A+5B\right)}{3bd^3}+\frac{Bg^3(a+bx)^2(bc-ad)^2\left(3B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+3A+2B\right)}{6bd^2}-\frac{Bg^3(a+bx)^3(bc-ad)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{3bd}+\frac{g^3(a+bx)^4\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(a*g + b*g*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2,x]
```

```
[Out] -1/3*(B*(b*c - a*d)*g^3*(a + b*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]
))/(b*d) + (g^3*(a + b*x)^4*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2)/(4*
b) + (B*(b*c - a*d)^2*g^3*(a + b*x)^2*(3*A + 2*B + 3*B*Log[(e*(a + b*x)^2)/
(c + d*x)^2]))/(6*b*d^2) - (B*(b*c - a*d)^3*g^3*(a + b*x)*(3*A + 5*B + 3*B*
Log[(e*(a + b*x)^2)/(c + d*x)^2]))/(3*b*d^3) - (B*(b*c - a*d)^4*g^3*(3*A +
11*B + 3*B*Log[(e*(a + b*x)^2)/(c + d*x)^2])*Log[(b*c - a*d)/(b*(c + d*x))]
)/(3*b*d^4) - (2*B^2*(b*c - a*d)^4*g^3*PolyLog[2, (d*(a + b*x))/(b*(c + d*x
))])/(b*d^4)
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2381

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + Dist[b*n*(p/(d*(q + 1))), Int[(f*x)^
```

```
m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, d
, e, f, m, n, q}, x] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 2384

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Dist[f/(e*(q + 1)), Int[(f*x)^(m - 1)*(d + e*x)^(q + 1
)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x
] && ILtQ[q, -1] && GtQ[m, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2550

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(
m + 1)*(g/b)^m, Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x],
 x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] &&
 EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && Eq
Q[b*f - a*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Rubi steps

$$\int (ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2\,dx=\frac{g^3(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{4b}-\frac{B\int\frac{2(bc-ad)g^4(a+bx)\cdots}{\cdots}}{\cdots}$$

$$=\frac{g^3(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{4b}-\frac{(B(bc-ad)g^3)\int\cdots}{\cdots}$$

$$=\frac{g^3(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{4b}-\frac{(B(bc-ad)g^3)\int\cdots}{\cdots}$$

$$=\frac{g^3(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{4b}-\frac{(B(bc-ad)g^3)\int(\cdots}{\cdots}$$

$$=-\frac{AB(bc-ad)^3g^3x}{d^3}+\frac{B(bc-ad)^2g^3(a+bx)^2\left(A+B\log\cdots\right.}{2bd^2}\cdots$$

$$=-\frac{AB(bc-ad)^3g^3x}{d^3}-\frac{B^2(bc-ad)^3g^3(a+bx)\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\cdots\right.}{bd^3}\cdots$$

$$=-\frac{AB(bc-ad)^3g^3x}{d^3}-\frac{B^2(bc-ad)^3g^3(a+bx)\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\cdots\right.}{bd^3}\cdots$$

$$=-\frac{AB(bc-ad)^3g^3x}{d^3}-\frac{5B^2(bc-ad)^3g^3x}{3d^3}+\frac{B^2(bc-ad)^2g\cdots}{3bd^2\cdots}\cdots$$

$$=-\frac{AB(bc-ad)^3g^3x}{d^3}-\frac{5B^2(bc-ad)^3g^3x}{3d^3}+\frac{B^2(bc-ad)^2g\cdots}{3bd^2\cdots}\cdots$$

$$=-\frac{AB(bc-ad)^3g^3x}{d^3}-\frac{5B^2(bc-ad)^3g^3x}{3d^3}+\frac{B^2(bc-ad)^2g\cdots}{3bd^2\cdots}\cdots$$

$$=-\frac{AB(bc-ad)^3g^3x}{d^3}-\frac{5B^2(bc-ad)^3g^3x}{3d^3}+\frac{B^2(bc-ad)^2g\cdots}{3bd^2\cdots}\cdots$$

Mathematica [A]
time = 0.21, size = 402, normalized size = 1.26

$$\frac{g^3\left((a+bx)^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2-\frac{2B(bc-ad)\left(6Abd(bc-ad)^2x+6Bd(bc-ad)^2(a+bx)\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+3d^2(-bc+ad)(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)+2d^3(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)-12B(bc-ad)^3\log(c+dx)-6(bc-ad)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\log(c+dx)+2B(bc-ad)(2bd(bc-ad)x-d^2(a+bx)^2-2(bc-ad)^2\log(c+dx))+6B(bc-ad)^2(bdx+(-bc+ad)\log(c+dx))+6B(bc-ad)^3\left(\left(2\log\left(\frac{d(a+bx)}{-bc+ad}\right)-\log(c+dx)\right)\log(c+dx)+2\text{Li}_2\left(\frac{b(c+dx)}{bc-ad}\right)\right)\right)}{3d^4}\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*g + b*g*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2,x]
```

```
[Out] (g^3*((a + b*x)^4*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2 - (2*B*(b*c -
a*d)*(6*A*b*d*(b*c - a*d)^2*x + 6*B*d*(b*c - a*d)^2*(a + b*x)*Log[(e*(a + b
*x)^2)/(c + d*x)^2] + 3*d^2*(-(b*c) + a*d)*(a + b*x)^2*(A + B*Log[(e*(a + b
*x)^2)/(c + d*x)^2]) + 2*d^3*(a + b*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*
x)^2]) - 12*B*(b*c - a*d)^3*Log[c + d*x] - 6*(b*c - a*d)^3*(A + B*Log[(e*(a
 + b*x)^2)/(c + d*x)^2])*Log[c + d*x] + 2*B*(b*c - a*d)*(2*b*d*(b*c - a*d)*
x - d^2*(a + b*x)^2 - 2*(b*c - a*d)^2*Log[c + d*x]) + 6*B*(b*c - a*d)^2*(b*
d*x + (-(b*c) + a*d)*Log[c + d*x]) + 6*B*(b*c - a*d)^3*((2*Log[(d*(a + b*x)
)/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))
/(b*c - a*d)])))/(3*d^4)))/(4*b)
```

Maple [F]
time = 0.18, size = 0, normalized size = 0.00

$$\int (bgx+ag)^3 \left(A+B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*g*x+a*g)^3*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

```
[Out] int((b*g*x+a*g)^3*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1754 vs. 2(313) = 626.
time = 0.44, size = 1754, normalized size = 5.50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^3*(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="ma
xima")
```

```
[Out] 1/4*A^2*b^3*g^3*x^4 + A^2*a*b^2*g^3*x^3 + 3/2*A^2*a^2*b*g^3*x^2 + 2*(x*log(
b^2*x^2*e/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*x*e/(d^2*x^2 + 2*c*d*x + c^2) +
 a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) + 2*a*log(b*x + a)/b - 2*c*log(d*x + c)/d
)*A*B*a^3*g^3 + 3*(x^2*log(b^2*x^2*e/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*x*e/
(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) - 2*a^2*log(b*
x + a)/b^2 + 2*c^2*log(d*x + c)/d^2 - 2*(b*c - a*d)*x/(b*d))*A*B*a^2*b*g^3
+ 2*(x^3*log(b^2*x^2*e/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*x*e/(d^2*x^2 + 2*c
*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) + 2*a^3*log(b*x + a)/b^3 - 2
*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)
/(b^2*d^2))*A*B*a*b^2*g^3 + 1/6*(3*x^4*log(b^2*x^2*e/(d^2*x^2 + 2*c*d*x + c
^2) + 2*a*b*x*e/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)
) - 6*a^4*log(b*x + a)/b^4 + 6*c^4*log(d*x + c)/d^4 - (2*(b^3*c*d^2 - a*b^2
*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d
```

```
^3))*A*B*b^3*g^3 + A^2*a^3*g^3*x + 1/3*(14*b^3*c^4*g^3 - 50*a*b^2*c^3*d*g^3
 + 63*a^2*b*c^2*d^2*g^3 - 30*a^3*c*d^3*g^3)*B^2*log(d*x + c)/d^4 + 2*(b^4*c
^4*g^3 - 4*a*b^3*c^3*d*g^3 + 6*a^2*b^2*c^2*d^2*g^3 - 4*a^3*b*c*d^3*g^3 + a^
4*d^4*g^3)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x
 + a*d)/(b*c - a*d)))*B^2/(b*d^4) + 1/12*(3*B^2*b^4*d^4*g^3*x^4 - 4*(b^4*c*
d^3*g^3 - 4*a*b^3*d^4*g^3)*B^2*x^3 + 2*(5*b^4*c^2*d^2*g^3 - 16*a*b^3*c*d^3*
g^3 + 20*a^2*b^2*d^4*g^3)*B^2*x^2 - 4*(8*b^4*c^3*d*g^3 - 29*a*b^3*c^2*d^2*g
^3 + 37*a^2*b^2*c*d^3*g^3 - 19*a^3*b*d^4*g^3)*B^2*x + 12*(B^2*b^4*d^4*g^3*x
^4 + 4*B^2*a*b^3*d^4*g^3*x^3 + 6*B^2*a^2*b^2*d^4*g^3*x^2 + 4*B^2*a^3*b*d^4*
g^3*x + B^2*a^4*d^4*g^3)*log(b*x + a)^2 + 12*(B^2*b^4*d^4*g^3*x^4 + 4*B^2*a
*b^3*d^4*g^3*x^3 + 6*B^2*a^2*b^2*d^4*g^3*x^2 + 4*B^2*a^3*b*d^4*g^3*x - (b^4
*c^4*g^3 - 4*a*b^3*c^3*d*g^3 + 6*a^2*b^2*c^2*d^2*g^3 - 4*a^3*b*c*d^3*g^3)*B
^2)*log(d*x + c)^2 + 4*(3*B^2*b^4*d^4*g^3*x^4 - 2*(b^4*c*d^3*g^3 - 7*a*b^3*
d^4*g^3)*B^2*x^3 + 3*(b^4*c^2*d^2*g^3 - 4*a*b^3*c*d^3*g^3 + 9*a^2*b^2*d^4*g
^3)*B^2*x^2 - 6*(b^4*c^3*d*g^3 - 4*a*b^3*c^2*d^2*g^3 + 6*a^2*b^2*c*d^3*g^3
- 5*a^3*b*d^4*g^3)*B^2*x - (6*a*b^3*c^3*d*g^3 - 21*a^2*b^2*c^2*d^2*g^3 + 26
*a^3*b*c*d^3*g^3 - 14*a^4*d^4*g^3)*B^2)*log(b*x + a) - 4*(3*B^2*b^4*d^4*g^3
*x^4 - 2*(b^4*c*d^3*g^3 - 7*a*b^3*d^4*g^3)*B^2*x^3 + 3*(b^4*c^2*d^2*g^3 - 4
*a*b^3*c*d^3*g^3 + 9*a^2*b^2*d^4*g^3)*B^2*x^2 - 6*(b^4*c^3*d*g^3 - 4*a*b^3*
c^2*d^2*g^3 + 6*a^2*b^2*c*d^3*g^3 - 5*a^3*b*d^4*g^3)*B^2*x + 6*(B^2*b^4*d^4
*g^3*x^4 + 4*B^2*a*b^3*d^4*g^3*x^3 + 6*B^2*a^2*b^2*d^4*g^3*x^2 + 4*B^2*a^3*
b*d^4*g^3*x + B^2*a^4*d^4*g^3)*log(b*x + a))*log(d*x + c))/(b*d^4)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^3*(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="fr
icas")
```

```
[Out] integral(A^2*b^3*g^3*x^3 + 3*A^2*a*b^2*g^3*x^2 + 3*A^2*a^2*b*g^3*x + A^2*a^
3*g^3 + (B^2*b^3*g^3*x^3 + 3*B^2*a*b^2*g^3*x^2 + 3*B^2*a^2*b*g^3*x + B^2*a^
3*g^3)*log((b^2*x^2 + 2*a*b*x + a^2)*e/(d^2*x^2 + 2*c*d*x + c^2))^2 + 2*(A*
B*b^3*g^3*x^3 + 3*A*B*a*b^2*g^3*x^2 + 3*A*B*a^2*b*g^3*x + A*B*a^3*g^3)*log(
(b^2*x^2 + 2*a*b*x + a^2)*e/(d^2*x^2 + 2*c*d*x + c^2)), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)**3*(A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^3*(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="gi
ac")
```

```
[Out] integrate((b*g*x + a*g)^3*(B*log((b*x + a)^2*e/(d*x + c)^2) + A)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a\,g+b\,g\,x)^3\,\left(A+B\,\ln\left(\frac{e\,{(a+b\,x)}^2}{{(c+d\,x)}^2}\right)\right)^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*g + b*g*x)^3*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2,x)
```

```
[Out] int((a*g + b*g*x)^3*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2, x)
```

3.130 $\int (ag+bgx)^2 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx$

Optimal. Leaf size=255

$$-\frac{2B(bc-ad)g^2(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{3bd}+\frac{g^2(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{3b}+\frac{4B(bc-ad)^2g^2(a$$

```
[Out] -2/3*B*(-a*d+b*c)*g^2*(b*x+a)^2*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/b/d+1/3*g^2
*(b*x+a)^3*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/b+4/3*B*(-a*d+b*c)^2*g^2*(b*x+
a)*(A+B+B*ln(e*(b*x+a)^2/(d*x+c)^2))/b/d^2+4/3*B*(-a*d+b*c)^3*g^2*(A+3*B+B*
ln(e*(b*x+a)^2/(d*x+c)^2))*ln((-a*d+b*c)/b/(d*x+c))/b/d^3+8/3*B^2*(-a*d+b*c
)^3*g^2*polylog(2,d*(b*x+a)/b/(d*x+c))/b/d^3
```

Rubi [A]
time = 0.21, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.147$, Rules used = {2550, 2381, 2384, 2354, 2438}

$$\frac{8B^2g^2(bc-ad)^3\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{3bd^3}+\frac{4Bg^2(bc-ad)^3\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A+3B\right)}{3bd^3}+\frac{4Bg^2(a+bx)(bc-ad)^2\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A+B\right)}{3bd^2}-\frac{2Bg^2(a+bx)^2(bc-ad)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{3bd}+\frac{g^2(a+bx)^3\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2,x]
```

```
[Out] (-2*B*(b*c - a*d)*g^2*(a + b*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]))
/(3*b*d) + (g^2*(a + b*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2)/(3*
b) + (4*B*(b*c - a*d)^2*g^2*(a + b*x)*(A + B + B*Log[(e*(a + b*x)^2)/(c + d
*x)^2]))/(3*b*d^2) + (4*B*(b*c - a*d)^3*g^2*(A + 3*B + B*Log[(e*(a + b*x)^2
)/(c + d*x)^2])*Log[(b*c - a*d)/(b*(c + d*x))])/(3*b*d^3) + (8*B^2*(b*c - a
*d)^3*g^2*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/(3*b*d^3)
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2381

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + Dist[b*n*(p/(d*(q + 1))), Int[(f*x)^
m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, d
, e, f, m, n, q}, x] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 2384

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Dist[f/(e*(q + 1)), Int[(f*x)^(m - 1)*(d + e*x)^(q + 1
)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x
] && ILtQ[q, -1] && GtQ[m, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2550

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(
m + 1)*(g/b)^m, Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x],
 x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] &&
 EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && Eq
Q[b*f - a*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Rubi steps

$$\int (ag+bgx)^2 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx = \frac{g^2(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{3b} - (2B)\int \frac{2(bc-ad)g^3(a+}{}$$

$$= \frac{g^2(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{3b} - \frac{(4B(bc-ad)g^2)\int}{}$$

$$= \frac{g^2(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{3b} - \frac{(4B(bc-ad)g^2)\int}{}$$

$$= \frac{g^2(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{3b} - \frac{(4B(bc-ad)g^2)\int}{}$$

$$= \frac{4AB(bc-ad)^2g^2x}{3d^2} - \frac{2B(bc-ad)g^2(a+bx)^2\left(A+B\log\right.}{3bd}$$

$$= \frac{4AB(bc-ad)^2g^2x}{3d^2} + \frac{4B^2(bc-ad)^2g^2(a+bx)\log\left(\frac{e(a+bx)}{(c+dx)}\right.}{3bd^2}$$

$$= \frac{4AB(bc-ad)^2g^2x}{3d^2} + \frac{4B^2(bc-ad)^2g^2(a+bx)\log\left(\frac{e(a+bx)}{(c+dx)}\right.}{3bd^2}$$

$$= \frac{4AB(bc-ad)^2g^2x}{3d^2} + \frac{4B^2(bc-ad)^2g^2x}{3d^2} + \frac{4B^2(bc-ad)^2g}{}$$

$$= \frac{4AB(bc-ad)^2g^2x}{3d^2} + \frac{4B^2(bc-ad)^2g^2x}{3d^2} + \frac{4B^2(bc-ad)^2g}{}$$

$$= \frac{4AB(bc-ad)^2g^2x}{3d^2} + \frac{4B^2(bc-ad)^2g^2x}{3d^2} + \frac{4B^2(bc-ad)^2g}{}$$

$$= \frac{4AB(bc-ad)^2g^2x}{3d^2} + \frac{4B^2(bc-ad)^2g^2x}{3d^2} + \frac{4B^2(bc-ad)^2g}{}$$

Mathematica [A]
time = 0.15, size = 298, normalized size = 1.17

$$\frac{g^2\left((a+bx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2+\frac{2B(bc-ad)\left(2Abd(bc-ad)x+2Bd(bc-ad)(a+bx)\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)-d^2(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)-4B(bc-ad)^2\log(c+dx)-2(bc-ad)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\log(c+dx)+2B(bc-ad)(bdx+(-bc+ad)\log(c+dx))+2B(bc-ad)^2\left(\left(2\log\left(\frac{d(a+bx)}{-bc+ad}\right)-\log(c+dx)\right)\log(c+dx)+2\text{Li}_2\left(\frac{b(c+dx)}{bc-ad}\right)\right)\right)}{d^3}\right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2,x]
```

```
[Out] (g^2*((a + b*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2 + (2*B*(b*c -
a*d)*(2*A*b*d*(b*c - a*d)*x + 2*B*d*(b*c - a*d)*(a + b*x)*Log[(e*(a + b*x)^
2)/(c + d*x)^2] - d^2*(a + b*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])
- 4*B*(b*c - a*d)^2*Log[c + d*x] - 2*(b*c - a*d)^2*(A + B*Log[(e*(a + b*x)^
2)/(c + d*x)^2])*Log[c + d*x] + 2*B*(b*c - a*d)*(b*d*x + (-(b*c) + a*d)*Log
[c + d*x]) + 2*B*(b*c - a*d)^2*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[
c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/d^3))/(
3*b)
```

Maple [F]
time = 0.14, size = 0, normalized size = 0.00

$$\int \left(bgx+ag\right)^{2} \left(A+B\ln\left(\frac{e \left(bx+a\right)^{2}}{\left(dx+c\right)^{2}}\right)\right)^{2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*g*x+a*g)^2*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

```
[Out] int((b*g*x+a*g)^2*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1207 vs. $2(248) = 496$.
time = 0.42, size = 1207, normalized size = 4.73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^2*(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="ma
xima")
```

```
[Out] 1/3*A^2*b^2*g^2*x^3 + A^2*a*b*g^2*x^2 + 2*(x*log(b^2*x^2*e/(d^2*x^2 + 2*c*d
*x + c^2) + 2*a*b*x*e/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x
+ c^2)) + 2*a*log(b*x + a)/b - 2*c*log(d*x + c)/d)*A*B*a^2*g^2 + 2*(x^2*log
(b^2*x^2*e/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*x*e/(d^2*x^2 + 2*c*d*x + c^2)
+ a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) - 2*a^2*log(b*x + a)/b^2 + 2*c^2*log(d*x
 + c)/d^2 - 2*(b*c - a*d)*x/(b*d))*A*B*a*b*g^2 + 2/3*(x^3*log(b^2*x^2*e/(d^
2*x^2 + 2*c*d*x + c^2) + 2*a*b*x*e/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x
^2 + 2*c*d*x + c^2)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((
b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*A*B*b^2*g^2 +
A^2*a^2*g^2*x - 4/3*(4*b^2*c^3*g^2 - 10*a*b*c^2*d*g^2 + 7*a^2*c*d^2*g^2)*B^
2*log(d*x + c)/d^3 - 8/3*(b^3*c^3*g^2 - 3*a*b^2*c^2*d*g^2 + 3*a^2*b*c*d^2*g
^2 - a^3*d^3*g^2)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(
-(b*d*x + a*d)/(b*c - a*d)))*B^2/(b*d^3) + 1/3*(B^2*b^3*d^3*g^2*x^3 - (2*b^
3*c*d^2*g^2 - 5*a*b^2*d^3*g^2)*B^2*x^2 + (8*b^3*c^2*d*g^2 - 20*a*b^2*c*d^2*
g^2 + 15*a^2*b*d^3*g^2)*B^2*x + 4*(B^2*b^3*d^3*g^2*x^3 + 3*B^2*a*b^2*d^3*g^
```

```
2*x^2 + 3*B^2*a^2*b*d^3*g^2*x + B^2*a^3*d^3*g^2)*log(b*x + a)^2 + 4*(B^2*b^
3*d^3*g^2*x^3 + 3*B^2*a*b^2*d^3*g^2*x^2 + 3*B^2*a^2*b*d^3*g^2*x + (b^3*c^3*
g^2 - 3*a*b^2*c^2*d*g^2 + 3*a^2*b*c*d^2*g^2)*B^2)*log(d*x + c)^2 + 4*(B^2*b
^3*d^3*g^2*x^3 - (b^3*c*d^2*g^2 - 4*a*b^2*d^3*g^2)*B^2*x^2 + (2*b^3*c^2*d*g
^2 - 6*a*b^2*c*d^2*g^2 + 7*a^2*b*d^3*g^2)*B^2*x + (2*a*b^2*c^2*d*g^2 - 5*a^
2*b*c*d^2*g^2 + 4*a^3*d^3*g^2)*B^2)*log(b*x + a) - 4*(B^2*b^3*d^3*g^2*x^3 -
 (b^3*c*d^2*g^2 - 4*a*b^2*d^3*g^2)*B^2*x^2 + (2*b^3*c^2*d*g^2 - 6*a*b^2*c*d
^2*g^2 + 7*a^2*b*d^3*g^2)*B^2*x + 2*(B^2*b^3*d^3*g^2*x^3 + 3*B^2*a*b^2*d^3*
g^2*x^2 + 3*B^2*a^2*b*d^3*g^2*x + B^2*a^3*d^3*g^2)*log(b*x + a))*log(d*x +
c))/(b*d^3)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^2*(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="fr
icas")
```

```
[Out] integral(A^2*b^2*g^2*x^2 + 2*A^2*a*b*g^2*x + A^2*a^2*g^2 + (B^2*b^2*g^2*x^2
 + 2*B^2*a*b*g^2*x + B^2*a^2*g^2)*log((b^2*x^2 + 2*a*b*x + a^2)*e/(d^2*x^2
+ 2*c*d*x + c^2))^2 + 2*(A*B*b^2*g^2*x^2 + 2*A*B*a*b*g^2*x + A*B*a^2*g^2)*l
og((b^2*x^2 + 2*a*b*x + a^2)*e/(d^2*x^2 + 2*c*d*x + c^2)), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)**2*(A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^2*(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="gi
ac")
```

```
[Out] integrate((b*g*x + a*g)^2*(B*log((b*x + a)^2*e/(d*x + c)^2) + A)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a\,g+b\,g\,x)^2\,\left(A+B\,\ln\left(\frac{e\,{(a+b\,x)}^2}{{(c+d\,x)}^2}\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*g + b*g*x)^2*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2,x)
[Out] int((a*g + b*g*x)^2*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2, x)
```

3.131 $\int (ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx$

Optimal. Leaf size=188

$$-\frac{2B(bc-ad)g(a+bx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{bd}+\frac{g(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{2b}-\frac{2B(bc-ad)^2g\left(A+\right.}{}$$

```
[Out] -2*B*(-a*d+b*c)*g*(b*x+a)*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/b/d+1/2*g*(b*x+a)
^2*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/b-2*B*(-a*d+b*c)^2*g*(A+2*B+B*ln(e*(b*
x+a)^2/(d*x+c)^2))*ln((-a*d+b*c)/b/(d*x+c))/b/d^2-4*B^2*(-a*d+b*c)^2*g*poly
log(2,d*(b*x+a)/b/(d*x+c))/b/d^2
```

Rubi [A]

time = 0.13, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {2550, 2381, 2384, 2354, 2438}

$$-\frac{4B^2g(bc-ad)^2\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{bd^2}-\frac{2Bg(bc-ad)^2\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A+2B\right)}{bd^2}-\frac{2Bg(a+bx)(bc-ad)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{bd}+\frac{g(a+bx)^2\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(a*g + b*g*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2,x]
```

```
[Out] (-2*B*(b*c - a*d)*g*(a + b*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]))/(b*
d) + (g*(a + b*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2)/(2*b) - (2*
B*(b*c - a*d)^2*g*(A + 2*B + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])*Log[(b*c -
 a*d)/(b*(c + d*x))])/(b*d^2) - (4*B^2*(b*c - a*d)^2*g*PolyLog[2, (d*(a + b
*x))/(b*(c + d*x))])/(b*d^2)
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2381

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + Dist[b*n*(p/(d*(q + 1))), Int[(f*x)^
m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, d
, e, f, m, n, q}, x] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 2384

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Dist[f/(e*(q + 1)), Int[(f*x)^(m - 1)*(d + e*x)^(q + 1
)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x
] && ILtQ[q, -1] && GtQ[m, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2550

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(
m + 1)*(g/b)^m, Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x],
 x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] &&
 EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && Eq
Q[b*f - a*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Rubi steps

$$
\begin{aligned}
\int (ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx &= \frac{g(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{2b} - \frac{B\int \frac{2(bc-ad)g^2(a+bx)}{}}{} \\
&= \frac{g(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{2b} - (2B(bc-ad)g)\int \\
&= \frac{g(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{2b} - (2B(bc-ad)g)\int \\
&= \frac{g(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{2b} - (2B(bc-ad)g)\int \\
&= -\frac{2AB(bc-ad)gx}{d} + \frac{g(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{2b} + \\
&= -\frac{2AB(bc-ad)gx}{d} - \frac{2B^2(bc-ad)g(a+bx)\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{bd} \\
&= -\frac{2AB(bc-ad)gx}{d} - \frac{2B^2(bc-ad)g(a+bx)\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{bd} \\
&= -\frac{2AB(bc-ad)gx}{d} - \frac{2B^2(bc-ad)g(a+bx)\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{bd} \\
&= -\frac{2AB(bc-ad)gx}{d} - \frac{2B^2(bc-ad)g(a+bx)\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{bd} \\
&= -\frac{2AB(bc-ad)gx}{d} - \frac{2B^2(bc-ad)g(a+bx)\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{bd} \\
&= -\frac{2AB(bc-ad)gx}{d} - \frac{2B^2(bc-ad)g(a+bx)\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{bd}
\end{aligned}
$$

Mathematica [A]
time = 0.12, size = 207, normalized size = 1.10

$$
\frac{g\left((a+bx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2-\frac{4B(bc-ad)\left(Abdx+Bd(a+bx)\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)-2B(bc-ad)\log(c+dx)-(bc-ad)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\log(c+dx)+B(bc-ad)\left(\left(2\log\left(\frac{d(a+bx)}{-bc+ad}\right)-\log(c+dx)\right)\log(c+dx)+2\text{Li}_2\left(\frac{b(c+dx)}{bc-ad}\right)\right)\right)}{d^2}\right)}{2b}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*g + b*g*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2,x]
```

```
[Out] (g*((a + b*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2 - (4*B*(b*c - a*
d)*(A*b*d*x + B*d*(a + b*x)*Log[(e*(a + b*x)^2)/(c + d*x)^2] - 2*B*(b*c - a
*d)*Log[c + d*x] - (b*c - a*d)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])*Log
[c + d*x] + B*(b*c - a*d)*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d
*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/d^2))/(2*b)
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int \left(bgx+ag\right) \left(A+B\ln \left(\frac{e \left(bx+a\right)^{2}}{\left(dx+c\right)^{2}}\right)\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*g*x+a*g)*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

```
[Out] int((b*g*x+a*g)*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 686 vs. 2(188) = 376.
time = 0.43, size = 686, normalized size = 3.65

$$\frac{1}{2}A^{2}bgx^{2}+2\left(x\log\left(\frac{b^{2}x^{2}e}{d^{2}x^{2}+2cdx+c^{2}}+\frac{2abxe}{d^{2}x^{2}+2cdx+c^{2}}+\frac{a^{2}e}{d^{2}x^{2}+2cdx+c^{2}}\right)+\frac{2a\log\left(bx+a\right)}{b}-\frac{2c\log\left(dx+c\right)}{d}\right)ABag+\left(x^{2}\log\left(\frac{b^{2}x^{2}e}{d^{2}x^{2}+2cdx+c^{2}}+\frac{2abxe}{d^{2}x^{2}+2cdx+c^{2}}+\frac{a^{2}e}{d^{2}x^{2}+2cdx+c^{2}}\right)-\frac{2a^{2}\log\left(bx+a\right)}{b^{2}}+\frac{2c^{2}\log\left(dx+c\right)}{d^{2}}-\frac{2\left(bc-ad\right)x}{bd}\right)ABbg+A^{2}agx+\frac{2\left(3bc^{2}g-4acdg\right)B^{2}\log\left(dx+c\right)}{d^{2}}+\frac{4\left(b^{2}c^{2}g-2abcdg+a^{2}d^{2}g\right)\left(\log\left(bx+a\right)\log\left(\frac{bdx+ad}{bc-ad}+1\right)+\mathrm{Li}_2\left(-\frac{bdx+ad}{bc-ad}\right)\right)B^{2}}{bd^{2}}+\frac{B^{2}b^{2}d^{2}gx^{2}-2\left(2b^{2}cdg-3abd^{2}g\right)B^{2}x+4\left(B^{2}b^{2}d^{2}gx^{2}+2B^{2}abd^{2}gx+B^{2}a^{2}d^{2}g\right)\log\left(bx+a\right)^{2}+4\left(B^{2}b^{2}d^{2}gx^{2}+2B^{2}abd^{2}gx-\left(b^{2}c^{2}g-2abcdg\right)B^{2}\right)\log\left(dx+c\right)^{2}+4\left(B^{2}b^{2}d^{2}gx^{2}-2\left(b^{2}cdg-2abd^{2}g\right)B^{2}x-\left(2abcdg-3a^{2}d^{2}g\right)B^{2}\right)\log\left(bx+a\right)-4\left(B^{2}b^{2}d^{2}gx^{2}-2\left(b^{2}cdg-2abd^{2}g\right)B^{2}x+2\left(B^{2}b^{2}d^{2}gx^{2}+2B^{2}abd^{2}gx+B^{2}a^{2}d^{2}g\right)\log\left(bx+a\right)\right)\log\left(dx+c\right)}{2bd^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)*(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="maxi
ma")
```

```
[Out] 1/2*A^2*b*g*x^2 + 2*(x*log(b^2*x^2*e/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*x*e/
(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) + 2*a*log(b*x
+ a)/b - 2*c*log(d*x + c)/d)*A*B*a*g + (x^2*log(b^2*x^2*e/(d^2*x^2 + 2*c*d*
x + c^2) + 2*a*b*x*e/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x +
 c^2)) - 2*a^2*log(b*x + a)/b^2 + 2*c^2*log(d*x + c)/d^2 - 2*(b*c - a*d)*x/
(b*d))*A*B*b*g + A^2*a*g*x + 2*(3*b*c^2*g - 4*a*c*d*g)*B^2*log(d*x + c)/d^2
 + 4*(b^2*c^2*g - 2*a*b*c*d*g + a^2*d^2*g)*(log(b*x + a)*log((b*d*x + a*d)/
(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B^2/(b*d^2) + 1/2*(B^
2*b^2*d^2*g*x^2 - 2*(2*b^2*c*d*g - 3*a*b*d^2*g)*B^2*x + 4*(B^2*b^2*d^2*g*x^
2 + 2*B^2*a*b*d^2*g*x + B^2*a^2*d^2*g)*log(b*x + a)^2 + 4*(B^2*b^2*d^2*g*x^
2 + 2*B^2*a*b*d^2*g*x - (b^2*c^2*g - 2*a*b*c*d*g)*B^2)*log(d*x + c)^2 + 4*(
B^2*b^2*d^2*g*x^2 - 2*(b^2*c*d*g - 2*a*b*d^2*g)*B^2*x - (2*a*b*c*d*g - 3*a^
2*d^2*g)*B^2)*log(b*x + a) - 4*(B^2*b^2*d^2*g*x^2 - 2*(b^2*c*d*g - 2*a*b*d^
2*g)*B^2*x + 2*(B^2*b^2*d^2*g*x^2 + 2*B^2*a*b*d^2*g*x + B^2*a^2*d^2*g)*log(
b*x + a))*log(d*x + c))/(b*d^2)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)*(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="fric
as")
```

```
[Out] integral(A^2*b*g*x + A^2*a*g + (B^2*b*g*x + B^2*a*g)*log((b^2*x^2 + 2*a*b*x
 + a^2)*e/(d^2*x^2 + 2*c*d*x + c^2))^2 + 2*(A*B*b*g*x + A*B*a*g)*log((b^2*x
^2 + 2*a*b*x + a^2)*e/(d^2*x^2 + 2*c*d*x + c^2)), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)*(A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)*(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="giac
")
```

```
[Out] integrate((b*g*x + a*g)*(B*log((b*x + a)^2*e/(d*x + c)^2) + A)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a\,g+b\,g\,x)\,\left(A+B\,\ln\left(\frac{e\,{(a+b\,x)}^2}{{(c+d\,x)}^2}\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*g + b*g*x)*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2,x)
```

```
[Out] int((a*g + b*g*x)*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2, x)
```

3.132 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{ag+bgx}\,dx$

Optimal. Leaf size=132

$$-\frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{bg}+\frac{4B\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\mathrm{Li}_2\left(\frac{b(c+dx)}{d(a+bx)}\right)}{bg}+\frac{8B^2\mathrm{Li}_3\left(\frac{b(c+dx)}{d(a+bx)}\right)}{bg}$$

```
[Out] -(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2*ln(1-b*(d*x+c)/d/(b*x+a))/b/g+4*B*(A+B*l
n(e*(b*x+a)^2/(d*x+c)^2))*polylog(2,b*(d*x+c)/d/(b*x+a))/b/g+8*B^2*polylog(
3,b*(d*x+c)/d/(b*x+a))/b/g
```

Rubi [A]
time = 0.11, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2550, 2379, 2421, 6724}

$$\frac{4B\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{bg}+\frac{8B^2\text{PolyLog}\left(3,\frac{b(c+dx)}{d(a+bx)}\right)}{bg}-\frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{bg}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2/(a*g + b*g*x),x]
```

```
[Out] -(((A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2*Log[1 - (b*(c + d*x))/(d*(a +
 b*x))])/(b*g)) + (4*B*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])*PolyLog[2,
(b*(c + d*x))/(d*(a + b*x))])/(b*g) + (8*B^2*PolyLog[3, (b*(c + d*x))/(d*(a
 + b*x))])/(b*g)
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2550

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(
m + 1)*(g/b)^m, Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x],
 x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] &&
 EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && Eq
Q[b*f - a*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{ag+bgx}\,dx = \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2\log(ag+bgx)}{bg} - \frac{(2B)\int \frac{(c+dx)^2\left(-\frac{2de(a+bx)^2}{(c+dx)^3}+\frac{2be(a+b}{(c+dx)}\right.}{}}{}$$

$$= \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2\log(ag+bgx)}{bg} - \frac{(2B)\int \frac{(c+dx)^2\left(-\frac{2de(a+bx)^2}{(c+dx)^3}+\frac{2be(a+b}{(c+dx)}\right.}{}}{}$$

$$= \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2\log(ag+bgx)}{bg} - \frac{(2B)\int \frac{2(bc-ad)e\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right.\right.}{(a+bx)(c+dx)}}{beg}$$

$$= \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2\log(ag+bgx)}{bg} - \frac{(4B(bc-ad))\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{(c+dx)}\right.\right.}{(a+bx)(}}{bg}$$

$$= \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2\log(ag+bgx)}{bg} - \frac{(4B(bc-ad))\int\left(\frac{d\left(-A-B\log\left(\frac{e}{(}\right.\right.}{(bc-}\right.}{}$$

$$= \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2\log(ag+bgx)}{bg} - \frac{(4B)\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\log(ag-}{a+bx}}{g}$$

$$= \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2\log(ag+bgx)}{bg} - \frac{(4B)\int\left(\frac{A\log(ag+bgx)}{a+bx}+\frac{B\log\left(\frac{e(a}{(c-}\right.}{}\right.}{g}$$

$$= \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2\log(ag+bgx)}{bg} - \frac{(4AB)\int \frac{\log(ag+bgx)}{a+bx}\,dx}{g} - \frac{(4B^2)}{}$$

$$= \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2\log(ag+bgx)}{bg} + \frac{4AB\log\left(\frac{b(c+dx)}{bc-ad}\right)\log(ag+bgx}{bg}$$

$$= \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2\log(ag+bgx)}{bg} + \frac{4AB\log\left(\frac{b(c+dx)}{bc-ad}\right)\log(ag+bgx}{bg}$$

$$= -\frac{2AB\log^2(g(a+bx))}{bg} + \frac{4B^2\log(g(a+bx))\log\left((a+bx)^2\right)\log(-c-dx)}{bg}$$

$$= -\frac{2AB\log^2(g(a+bx))}{bg} + \frac{4B^2\log(g(a+bx))\log\left((a+bx)^2\right)\log(-c-dx)}{bg}$$

$$= -\frac{2AB\log^2(g(a+bx))}{bg} + \frac{4B^2\log^3(g(a+bx))}{3bg} - \frac{4B^2\log^2(g(a+bx))\log(}{bg}$$

$$= -\frac{2AB\log^2(g(a+bx))}{} + \frac{4B^2\log^3(g(a+bx))}{} - \frac{4B^2\log^2(g(a+bx))\log(}{}$$

Mathematica [A]
time = 0.16, size = 257, normalized size = 1.95

$$\frac{2AB\log^2\left(\frac{a}{b}+x\right)+A^2\log(a+bx)-4AB\log\left(\frac{a}{b}+x\right)\log(a+bx)+4AB\log\left(\frac{c}{d}+x\right)\log(a+bx)-4AB\log\left(\frac{c}{d}+x\right)\log\left(\frac{d(a+bx)}{-bc+ad}\right)+2AB\log(a+bx)\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)-B^2\log\left(\frac{-bc+ad}{d(a+bx)}\right)\log^2\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)-4AB\mathrm{Li}_2\left(\frac{b(c+dx)}{bc-ad}\right)+4B^2\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\mathrm{Li}_2\left(\frac{b(c+dx)}{d(a+bx)}\right)+8B^2\mathrm{Li}_3\left(\frac{b(c+dx)}{d(a+bx)}\right)}{bg}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2/(a*g + b*g*x),x]
```

```
[Out] (2*A*B*Log[a/b + x]^2 + A^2*Log[a + b*x] - 4*A*B*Log[a/b + x]*Log[a + b*x]
+ 4*A*B*Log[c/d + x]*Log[a + b*x] - 4*A*B*Log[c/d + x]*Log[(d*(a + b*x))/(-
(b*c) + a*d)] + 2*A*B*Log[a + b*x]*Log[(e*(a + b*x)^2)/(c + d*x)^2] - B^2*L
og[(-(b*c) + a*d)/(d*(a + b*x))]*Log[(e*(a + b*x)^2)/(c + d*x)^2]^2 - 4*A*B
*PolyLog[2, (b*(c + d*x))/(b*c - a*d)] + 4*B^2*Log[(e*(a + b*x)^2)/(c + d*x
)^2]*PolyLog[2, (b*(c + d*x))/(d*(a + b*x))] + 8*B^2*PolyLog[3, (b*(c + d*x
))/(d*(a + b*x))])/(b*g)
```

Maple [F]
time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{\left(A+B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)\right)^2}{bgx+ag}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/(b*g*x+a*g),x)
```

```
[Out] int((A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/(b*g*x+a*g),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(b*g*x+a*g),x, algorithm="maxi
ma")
```

```
[Out] 4*B^2*log(b*x + a)*log(d*x + c)^2/(b*g) + A^2*log(b*g*x + a*g)/(b*g) - inte
grate(-(2*A*B*b*c + B^2*b*c + 4*(B^2*b*d*x + B^2*b*c)*log(b*x + a)^2 + (2*A
*B*b*d + B^2*b*d)*x + 4*(A*B*b*c + B^2*b*c + (A*B*b*d + B^2*b*d)*x)*log(b*x
 + a) - 4*(A*B*b*c + B^2*b*c + (A*B*b*d + B^2*b*d)*x + 2*(2*B^2*b*d*x + (b*
c + a*d)*B^2)*log(b*x + a))*log(d*x + c))/(b^2*d*g*x^2 + a*b*c*g + (b^2*c*g
 + a*b*d*g)*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(b*g*x+a*g),x, algorithm="fric
as")
```

```
[Out] integral((B^2*log((b^2*x^2 + 2*a*b*x + a^2)*e/(d^2*x^2 + 2*c*d*x + c^2))^2
+ 2*A*B*log((b^2*x^2 + 2*a*b*x + a^2)*e/(d^2*x^2 + 2*c*d*x + c^2)) + A^2)/(
b*g*x + a*g), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A^2}{a+bx}\,dx + \int \frac{B^2\log\left(\frac{a^2e}{c^2+2cdx+d^2x^2}+\frac{2abex}{c^2+2cdx+d^2x^2}+\frac{b^2ex^2}{c^2+2cdx+d^2x^2}\right)^2}{a+bx}\,dx + \int \frac{2AB\log\left(\frac{a^2e}{c^2+2cdx+d^2x^2}+\frac{2abex}{c^2+2cdx+d^2x^2}+\frac{b^2ex^2}{c^2+2cdx+d^2x^2}\right)}{a+bx}\,dx}{g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2/(b*g*x+a*g),x)
```

```
[Out] (Integral(A**2/(a + b*x), x) + Integral(B**2*log(a**2*e/(c**2 + 2*c*d*x + d
**2*x**2) + 2*a*b*e*x/(c**2 + 2*c*d*x + d**2*x**2) + b**2*e*x**2/(c**2 + 2*
c*d*x + d**2*x**2))**2/(a + b*x), x) + Integral(2*A*B*log(a**2*e/(c**2 + 2*
c*d*x + d**2*x**2) + 2*a*b*e*x/(c**2 + 2*c*d*x + d**2*x**2) + b**2*e*x**2/(
c**2 + 2*c*d*x + d**2*x**2))/(a + b*x), x))/g
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(b*g*x+a*g),x, algorithm="giac
")
```

```
[Out] integrate((B*log((b*x + a)^2*e/(d*x + c)^2) + A)^2/(b*g*x + a*g), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A+B\,\ln\left(\frac{e\,{\left(a+b\,x\right)}^2}{{\left(c+d\,x\right)}^2}\right)\right)^2}{a\,g+b\,g\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2/(a*g + b*g*x),x)
```

```
[Out] int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2/(a*g + b*g*x), x)
```

3.133 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(ag+bgx)^2}\,dx$

Optimal. Leaf size=130

$$-\frac{8B^2(c+dx)}{(bc-ad)g^2(a+bx)}-\frac{4B(c+dx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(bc-ad)g^2(a+bx)}-\frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(bc-ad)g^2(a+bx)}$$

```
[Out] -8*B^2*(d*x+c)/(-a*d+b*c)/g^2/(b*x+a)-4*B*(d*x+c)*(A+B*ln(e*(b*x+a)^2/(d*x+
c)^2))/(-a*d+b*c)/g^2/(b*x+a)-(d*x+c)*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/(-a
*d+b*c)/g^2/(b*x+a)
```

Rubi [A]
time = 0.06, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.088$, Rules used = {2550, 2342, 2341}

$$-\frac{4B(c+dx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{g^2(a+bx)(bc-ad)}-\frac{(c+dx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{g^2(a+bx)(bc-ad)}-\frac{8B^2(c+dx)}{g^2(a+bx)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2/(a*g + b*g*x)^2,x]
```

```
[Out] (-8*B^2*(c + d*x))/((b*c - a*d)*g^2*(a + b*x)) - (4*B*(c + d*x)*(A + B*Log[
(e*(a + b*x)^2)/(c + d*x)^2]))/((b*c - a*d)*g^2*(a + b*x)) - ((c + d*x)*(A
+ B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2)/((b*c - a*d)*g^2*(a + b*x))
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2550

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(
```

```
m + 1)*(g/b)^m, Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x],
 x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] &&
 EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && Eq
Q[b*f - a*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Rubi steps

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(ag+bgx)^2}\,dx = -\frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{bg^2(a+bx)} + \frac{(2B)\int \frac{2(bc-ad)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{g(a+bx)^2(c+dx)}\,dx}{bg}$$

$$= -\frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{bg^2(a+bx)} + \frac{(4B(bc-ad))\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(a+bx)^2(c+dx)}\,dx}{bg^2}$$

$$= -\frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{bg^2(a+bx)} + \frac{(4B(bc-ad))\int \left(\frac{b\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(bc-ad)(a+bx)^2} - \frac{bd(}{}\right.}{b}$$

$$= -\frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{bg^2(a+bx)} + \frac{(4B)\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(a+bx)^2}\,dx}{g^2} - \frac{(4Bd)\int \frac{A+B\,\mathrm{l}}{}}{(bc-}$$

$$= -\frac{4B\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{bg^2(a+bx)} - \frac{4Bd\log(a+bx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{b(bc-ad)g^2}$$

$$= -\frac{4B\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{bg^2(a+bx)} - \frac{4Bd\log(a+bx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{b(bc-ad)g^2}$$

$$= -\frac{4B\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{bg^2(a+bx)} - \frac{4Bd\log(a+bx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{b(bc-ad)g^2}$$

$$= -\frac{8B^2}{bg^2(a+bx)} - \frac{8B^2d\log(a+bx)}{b(bc-ad)g^2} - \frac{4B\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{bg^2(a+bx)} - \frac{4Bd\,\mathrm{lo}}{}$$

$$= -\frac{8B^2}{bg^2(a+bx)} - \frac{8B^2d\log(a+bx)}{b(bc-ad)g^2} - \frac{4B\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{bg^2(a+bx)} - \frac{4Bd\,\mathrm{lo}}{}$$

$$= -\frac{8B^2}{bg^2(a+bx)} - \frac{8B^2d\log(a+bx)}{b(bc-ad)g^2} + \frac{4B^2d\log^2(a+bx)}{b(bc-ad)g^2} - \frac{4B\left(A+B\log\right.}{bg^2(a+}$$

$$= -\frac{8B^2}{bg^2(a+bx)} - \frac{8B^2d\log(a+bx)}{b(bc-ad)g^2} + \frac{4B^2d\log^2(a+bx)}{b(bc-ad)g^2} - \frac{4B\left(A+B\log\right.}{bg^2(a+}$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 0.29, size = 321, normalized size = 2.47

$$-\frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2+\frac{4B\left((bc-ad)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)+d(a+bx)\log(a+bx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)-d(a+bx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\log(c+dx)+2B(bc-ad+d(a+bx)\log(a+bx)-d(a+bx)\log(c+dx))-Bd(a+bx)\left(\log(a+bx)\left(\log(a+bx)-2\log\left(\frac{b(c+dx)}{bc-ad}\right)\right)-2\text{Li}_2\left(\frac{d(a+bx)}{-bc+ad}\right)\right)+Bd(a+bx)\left(\left(2\log\left(\frac{d(a+bx)}{-bc+ad}\right)-\log(c+dx)\right)\log(c+dx)+2\text{Li}_2\left(\frac{b(c+dx)}{bc-ad}\right)\right)\right)}{bc-ad}}{bg^2(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2/(a*g + b*g*x)^2,x]
```

```
[Out] -(((A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2 + (4*B*((b*c - a*d)*(A + B*Lo
g[(e*(a + b*x)^2)/(c + d*x)^2]) + d*(a + b*x)*Log[a + b*x]*(A + B*Log[(e*(a
 + b*x)^2)/(c + d*x)^2]) - d*(a + b*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)
^2])*Log[c + d*x] + 2*B*(b*c - a*d + d*(a + b*x)*Log[a + b*x] - d*(a + b*x)
*Log[c + d*x]) - B*d*(a + b*x)*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c +
d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) + B*d*(a
+ b*x)*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] +
 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(b*c - a*d))/(b*g^2*(a + b*x)))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 305 vs. 2(130) = 260.
time = 0.46, size = 306, normalized size = 2.35

method	result
norman	$\frac{\frac{(A^2+4BA+8B^2)x}{ga}+\frac{B^2c\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)^2}{g(ad-cb)}+\frac{B^2dx\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)^2}{g(ad-cb)}+\frac{2cB(A+2B)\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)}{g(ad-cb)}+\frac{2Bd(A+2B)x\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)}{g(ad-cb)}}{g(bx+a)}$
risch	$-\frac{A^2}{g^2(bx+a)b}+\frac{\frac{8B^2x}{ag}+\frac{B^2c\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)^2}{g(ad-cb)}+\frac{B^2dx\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)^2}{g(ad-cb)}+\frac{4B^2c\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)}{g(ad-cb)}+\frac{4B^2dx\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)}{g(ad-cb)}}{g(bx+a)}+\frac{4BA}{ag}\cdots$
derivativedivides	$-\frac{-\frac{d^2A^2}{g^2\left(\frac{ad-cb}{dx+c}+b\right)(ad-cb)}+\frac{\frac{8d^2B^2}{bg(dx+c)}-\frac{4d^2B^2\ln\left(\frac{e\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^2}{d^2}\right)}{g(ad-cb)}-\frac{d^2B^2\ln\left(\frac{e\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^2}{d^2}\right)^2}{g(ad-cb)}+\frac{4d^2AB}{bg(dx+c)}-\cdots}{g\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)}}{d}$
default	$-\frac{-\frac{d^2A^2}{g^2\left(\frac{ad-cb}{dx+c}+b\right)(ad-cb)}+\frac{\frac{8d^2B^2}{bg(dx+c)}-\frac{4d^2B^2\ln\left(\frac{e\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^2}{d^2}\right)}{g(ad-cb)}-\frac{d^2B^2\ln\left(\frac{e\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^2}{d^2}\right)^2}{g(ad-cb)}+\frac{4d^2AB}{bg(dx+c)}-\cdots}{g\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)}}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/(b*g*x+a*g)^2,x,method=_RETURNVERBOSE
)
```

```
[Out] -1/d*(-d^2/g^2*A^2/((a*d-b*c)/(d*x+c)+b)/(a*d-b*c)+(8*d^2*B^2/b/g/(d*x+c)-4
*d^2*B^2/g/(a*d-b*c)*ln(e*(a/(d*x+c)*d-b*c/(d*x+c)+b)^2/d^2)-d^2*B^2/g/(a*d
-b*c)*ln(e*(a/(d*x+c)*d-b*c/(d*x+c)+b)^2/d^2)^2)/g/(a/(d*x+c)*d-b*c/(d*x+c)
```

```
+b)+(4*d^2*A*B/b/g/(d*x+c)-2*d^2*A*B/g/(a*d-b*c)*ln(e*(a/(d*x+c)*d-b*c/(d*x
+c)+b)^2/d^2))/g/(a/(d*x+c)*d-b*c/(d*x+c)+b))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 583 vs. $2(132) = 264$.
time = 0.33, size = 583, normalized size = 4.48

$$-4\left(\left(\frac{1}{b^2g^2x+abg^2}+\frac{d\log(bx+a)}{(b^2c-abd)g^2}-\frac{d\log(dx+c)}{(b^2c-abd)g^2}\right)\log\left(\frac{b^2x^2e}{d^2x^2+2cdx+c^2}+\frac{2abxe}{d^2x^2+2cdx+c^2}+\frac{a^2e}{d^2x^2+2cdx+c^2}\right)-\frac{(bdx+ad)\log(bx+a)^2+(bdx+ad)\log(dx+c)^2-2bc+2ad-2(bdx+ad)\log(bx+a)+2(bdx+ad-(bdx+ad)\log(bx+a))\log(dx+c)}{ab^2cg^2-a^2bdg^2+(b^3cg^2-ab^2dg^2)x}\right)B^2-2AB\left(\frac{\log\left(\frac{b^2x^2e}{d^2x^2+2cdx+c^2}+\frac{2abxe}{d^2x^2+2cdx+c^2}+\frac{a^2e}{d^2x^2+2cdx+c^2}\right)}{b^2g^2x+abg^2}+\frac{2}{b^2g^2x+abg^2}+\frac{2d\log(bx+a)}{(b^2c-abd)g^2}-\frac{2d\log(dx+c)}{(b^2c-abd)g^2}\right)-\frac{B^2\log\left(\frac{b^2x^2e}{d^2x^2+2cdx+c^2}+\frac{2abxe}{d^2x^2+2cdx+c^2}+\frac{a^2e}{d^2x^2+2cdx+c^2}\right)^2}{b^2g^2x+abg^2}-\frac{A^2}{b^2g^2x+abg^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(b*g*x+a*g)^2,x, algorithm="ma
xima")
```

```
[Out] -4*((1/(b^2*g^2*x + a*b*g^2) + d*log(b*x + a)/((b^2*c - a*b*d)*g^2) - d*log
(d*x + c)/((b^2*c - a*b*d)*g^2))*log(b^2*x^2*e/(d^2*x^2 + 2*c*d*x + c^2) +
2*a*b*x*e/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) - ((
b*d*x + a*d)*log(b*x + a)^2 + (b*d*x + a*d)*log(d*x + c)^2 - 2*b*c + 2*a*d
- 2*(b*d*x + a*d)*log(b*x + a) + 2*(b*d*x + a*d - (b*d*x + a*d)*log(b*x + a
))*log(d*x + c))/(a*b^2*c*g^2 - a^2*b*d*g^2 + (b^3*c*g^2 - a*b^2*d*g^2)*x))
*B^2 - 2*A*B*(log(b^2*x^2*e/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*x*e/(d^2*x^2
+ 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2))/(b^2*g^2*x + a*b*g^2) +
 2/(b^2*g^2*x + a*b*g^2) + 2*d*log(b*x + a)/((b^2*c - a*b*d)*g^2) - 2*d*log
(d*x + c)/((b^2*c - a*b*d)*g^2)) - B^2*log(b^2*x^2*e/(d^2*x^2 + 2*c*d*x + c
^2) + 2*a*b*x*e/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)
)^2/(b^2*g^2*x + a*b*g^2) - A^2/(b^2*g^2*x + a*b*g^2)
```

Fricas [A]
time = 0.35, size = 196, normalized size = 1.51

$$-\frac{(A^2+4\,AB+8\,B^2)bc-(A^2+4\,AB+8\,B^2)ad+(B^2bdx+B^2bc)\log\left(\frac{(b^2x^2+2\,abx+a^2)e}{d^2x^2+2\,cdx+c^2}\right)^2+2\,((AB+2\,B^2)bdx+(AB+2\,B^2)bc)\log\left(\frac{(b^2x^2+2\,abx+a^2)e}{d^2x^2+2\,cdx+c^2}\right)}{(b^3c-ab^2d)g^2x+(ab^2c-a^2bd)g^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(b*g*x+a*g)^2,x, algorithm="fr
icas")
```

```
[Out] -((A^2 + 4*A*B + 8*B^2)*b*c - (A^2 + 4*A*B + 8*B^2)*a*d + (B^2*b*d*x + B^2*
b*c)*log((b^2*x^2 + 2*a*b*x + a^2)*e/(d^2*x^2 + 2*c*d*x + c^2))^2 + 2*((A*B
 + 2*B^2)*b*d*x + (A*B + 2*B^2)*b*c)*log((b^2*x^2 + 2*a*b*x + a^2)*e/(d^2*x
^2 + 2*c*d*x + c^2)))/((b^3*c - a*b^2*d)*g^2*x + (a*b^2*c - a^2*b*d)*g^2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 454 vs. $2(112) = 224$.
time = 1.42, size = 454, normalized size = 3.49

$$-\frac{4Bd(A+2B)\log\left(x+\frac{4ABad^2+4ABbcd+8B^2ad^2+8B^2bcd-\frac{4Ba^2d^3(A+2B)}{ad-bc}+\frac{8Babcd^2(A+2B)}{ad-bc}-\frac{4Bb^2c^2d(A+2B)}{ad-bc}}{8ABbd^2+16B^2bd^2}\right)}{bg^2\,(ad-bc)}+\frac{4Bd(A+2B)\log\left(x+\frac{4ABad^2+4ABbcd+8B^2ad^2+8B^2bcd+\frac{4Ba^2d^3(A+2B)}{ad-bc}-\frac{8Babcd^2(A+2B)}{ad-bc}+\frac{4Bb^2c^2d(A+2B)}{ad-bc}}{8ABbd^2+16B^2bd^2}\right)}{bg^2\,(ad-bc)}+\frac{(-2AB-4B^2)\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{abg^2+b^2g^2x}+\frac{(B^2c+B^2dx)\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)^2}{a^2dg^2-abcg^2+abdg^2x-b^2cg^2x}+\frac{-A^2-4AB-8B^2}{abg^2+b^2g^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2/(b*g*x+a*g)**2,x)
```

```
[Out] -4*B*d*(A + 2*B)*log(x + (4*A*B*a*d**2 + 4*A*B*b*c*d + 8*B**2*a*d**2 + 8*B*
*2*b*c*d - 4*B*a**2*d**3*(A + 2*B)/(a*d - b*c) + 8*B*a*b*c*d**2*(A + 2*B)/(
a*d - b*c) - 4*B*b**2*c**2*d*(A + 2*B)/(a*d - b*c))/(8*A*B*b*d**2 + 16*B**2
*b*d**2))/(b*g**2*(a*d - b*c)) + 4*B*d*(A + 2*B)*log(x + (4*A*B*a*d**2 + 4*
A*B*b*c*d + 8*B**2*a*d**2 + 8*B**2*b*c*d + 4*B*a**2*d**3*(A + 2*B)/(a*d - b
*c) - 8*B*a*b*c*d**2*(A + 2*B)/(a*d - b*c) + 4*B*b**2*c**2*d*(A + 2*B)/(a*d
 - b*c))/(8*A*B*b*d**2 + 16*B**2*b*d**2))/(b*g**2*(a*d - b*c)) + (-2*A*B -
4*B**2)*log(e*(a + b*x)**2/(c + d*x)**2)/(a*b*g**2 + b**2*g**2*x) + (B**2*c
 + B**2*d*x)*log(e*(a + b*x)**2/(c + d*x)**2)**2/(a**2*d*g**2 - a*b*c*g**2
+ a*b*d*g**2*x - b**2*c*g**2*x) + (-A**2 - 4*A*B - 8*B**2)/(a*b*g**2 + b**2
*g**2*x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 378 vs. 2(132) = 264.
time = 3.98, size = 378, normalized size = 2.91

$$-\left(\frac{B^2d}{b^2cg^2-abdg^2}+\frac{B^2}{(bgx+ag)bg}\right)\log\left(\frac{b^2}{\frac{b^2c^2g^2}{(bgx+ag)^2}-\frac{2\,abcdg^2}{(bgx+ag)^2}+\frac{a^2d^2g^2}{(bgx+ag)^2}+\frac{2\,bcdg}{bgx+ag}-\frac{2\,ad^2g}{bgx+ag}+d^2}\right)^2+\frac{4\,(ABd+3\,B^2d)\log\left(\frac{bcg}{bgx+ag}-\frac{adg}{bgx+ag}+d\right)}{b^2cg^2-abdg^2}-\frac{2\,(AB+3\,B^2)\log\left(\frac{b^2}{\frac{b^2c^2g^2}{(bgx+ag)^2}-\frac{2\,abcdg^2}{(bgx+ag)^2}+\frac{a^2d^2g^2}{(bgx+ag)^2}+\frac{2\,bcdg}{bgx+ag}-\frac{2\,ad^2g}{bgx+ag}+d^2}\right)}{(bgx+ag)bg}-\frac{A^2+6\,AB+13\,B^2}{(bgx+ag)bg}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(b*g*x+a*g)^2,x, algorithm="gi
ac")
```

```
[Out] -(B^2*d/(b^2*c*g^2 - a*b*d*g^2) + B^2/((b*g*x + a*g)*b*g))*log(b^2/(b^2*c^2
*g^2/(b*g*x + a*g)^2 - 2*a*b*c*d*g^2/(b*g*x + a*g)^2 + a^2*d^2*g^2/(b*g*x +
 a*g)^2 + 2*b*c*d*g/(b*g*x + a*g) - 2*a*d^2*g/(b*g*x + a*g) + d^2))^2 + 4*(
A*B*d + 3*B^2*d)*log(b*c*g/(b*g*x + a*g) - a*d*g/(b*g*x + a*g) + d)/(b^2*c*
g^2 - a*b*d*g^2) - 2*(A*B + 3*B^2)*log(b^2/(b^2*c^2*g^2/(b*g*x + a*g)^2 - 2
*a*b*c*d*g^2/(b*g*x + a*g)^2 + a^2*d^2*g^2/(b*g*x + a*g)^2 + 2*b*c*d*g/(b*g
*x + a*g) - 2*a*d^2*g/(b*g*x + a*g) + d^2))/((b*g*x + a*g)*b*g) - (A^2 + 6*
A*B + 13*B^2)/((b*g*x + a*g)*b*g)
```

Mupad [**B**]
time = 5.97, size = 228, normalized size = 1.75

$$-\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)^2\left(\frac{B^2}{b^2\,g^2\,\left(x+\frac{a}{b}\right)}-\frac{B^2\,d}{b\,g^2\,(a\,d-b\,c)}\right)-\frac{A^2+4\,A\,B+8\,B^2}{x\,b^2\,g^2+a\,b\,g^2}-\frac{\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)\,\left(\frac{4\,B^2}{b^2\,d\,g^2}+\frac{2\,A\,B}{b^2\,d\,g^2}\right)}{\frac{x}{d}+\frac{a}{b\,d}}-\frac{B\,d\,\mathrm{atan}\left(\frac{\left(2\,b\,d\,x+\frac{c\,b^2\,g^2+a\,d\,b\,g^2}{b\,g^2}\right)\,1{}\mathrm{i}}{a\,d-b\,c}\right)\,(A+2\,B)\,8{}\mathrm{i}}{b\,g^2\,(a\,d-b\,c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2/(a*g + b*g*x)^2,x)
```

```
[Out] - log((e*(a + b*x)^2)/(c + d*x)^2)^2*(B^2/(b^2*g^2*(x + a/b)) - (B^2*d)/(b*
g^2*(a*d - b*c))) - (A^2 + 8*B^2 + 4*A*B)/(b^2*g^2*x + a*b*g^2) - (log((e*(
```

```
a + b*x)^2)/(c + d*x)^2)*((4*B^2)/(b^2*d*g^2) + (2*A*B)/(b^2*d*g^2)))/(x/d
+ a/(b*d)) - (B*d*atan(((2*b*d*x + (b^2*c*g^2 + a*b*d*g^2)/(b*g^2))*1i)/(a*
d - b*c))*(A + 2*B)*8i)/(b*g^2*(a*d - b*c))
```

3.134 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(ag+bgx)^3}\,dx$

Optimal. Leaf size=272

$$\frac{8B^2d(c+dx)}{(bc-ad)^2g^3(a+bx)}-\frac{bB^2(c+dx)^2}{(bc-ad)^2g^3(a+bx)^2}+\frac{4Bd(c+dx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(bc-ad)^2g^3(a+bx)}-\frac{bB(c+dx)^2\left(A+B\,\mathrm{lo}\right.}{(bc-ad)^2g^3(a\,}$$

```
[Out] 8*B^2*d*(d*x+c)/(-a*d+b*c)^2/g^3/(b*x+a)-b*B^2*(d*x+c)^2/(-a*d+b*c)^2/g^3/(
b*x+a)^2+4*B*d*(d*x+c)*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/(-a*d+b*c)^2/g^3/(b*
x+a)-b*B*(d*x+c)^2*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/(-a*d+b*c)^2/g^3/(b*x+a)
^2+d*(d*x+c)*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/(-a*d+b*c)^2/g^3/(b*x+a)-1/2
*b*(d*x+c)^2*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/(-a*d+b*c)^2/g^3/(b*x+a)^2
```

Rubi [A]
time = 0.15, antiderivative size = 272, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2550, 2395, 2342, 2341}

$$-\frac{bB(c+dx)^2\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{g^3(a+bx)^2(bc-ad)^2}+\frac{4Bd(c+dx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{g^3(a+bx)(bc-ad)^2}-\frac{b(c+dx)^2\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{2g^3(a+bx)^2(bc-ad)^2}+\frac{d(c+dx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{g^3(a+bx)(bc-ad)^2}-\frac{bB^2(c+dx)^2}{g^3(a+bx)^2(bc-ad)^2}+\frac{8B^2d(c+dx)}{g^3(a+bx)(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2/(a*g + b*g*x)^3,x]
```

```
[Out] (8*B^2*d*(c + d*x))/((b*c - a*d)^2*g^3*(a + b*x)) - (b*B^2*(c + d*x)^2)/((b
*c - a*d)^2*g^3*(a + b*x)^2) + (4*B*d*(c + d*x)*(A + B*Log[(e*(a + b*x)^2)/
(c + d*x)^2]))/((b*c - a*d)^2*g^3*(a + b*x)) - (b*B*(c + d*x)^2*(A + B*Log[
(e*(a + b*x)^2)/(c + d*x)^2]))/((b*c - a*d)^2*g^3*(a + b*x)^2) + (d*(c + d*
x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2)/((b*c - a*d)^2*g^3*(a + b*x)
) - (b*(c + d*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2)/(2*(b*c - a*
d)^2*g^3*(a + b*x)^2)
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2550

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(
m + 1)*(g/b)^m, Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x],
 x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] &&
 EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && Eq
Q[b*f - a*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Rubi steps

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(ag+bgx)^3}\,dx = -\frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{2bg^3(a+bx)^2} + \frac{B\int \frac{2(bc-ad)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{g^2(a+bx)^3(c+dx)}\,dx}{bg}$$

$$= -\frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{2bg^3(a+bx)^2} + \frac{(2B(bc-ad))\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(a+bx)^3(c+dx)}\,dx}{bg^3}$$

$$= -\frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{2bg^3(a+bx)^2} + \frac{(2B(bc-ad))\int \left(\frac{b\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(bc-ad)(a+bx)^3} - \frac{bd\left(A \cdots\right.}{\cdots}\right.}{\cdots}$$

$$= -\frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{2bg^3(a+bx)^2} + \frac{(2B)\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(a+bx)^3}\,dx}{g^3} + \frac{(2Bd^2)\int \frac{A+B\log \cdots}{\cdots}}{(bc-a \cdots}$$

$$= -\frac{B\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{bg^3(a+bx)^2} + \frac{2Bd\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{b(bc-ad)g^3(a+bx)} + \frac{2Bd^2\log(a+ \cdots}{\cdots}$$

$$= -\frac{B\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{bg^3(a+bx)^2} + \frac{2Bd\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{b(bc-ad)g^3(a+bx)} + \frac{2Bd^2\log(a+ \cdots}{\cdots}$$

$$= -\frac{B\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{bg^3(a+bx)^2} + \frac{2Bd\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{b(bc-ad)g^3(a+bx)} + \frac{2Bd^2\log(a+ \cdots}{\cdots}$$

$$= -\frac{B^2}{bg^3(a+bx)^2} + \frac{6B^2d}{b(bc-ad)g^3(a+bx)} + \frac{6B^2d^2\log(a+bx)}{b(bc-ad)^2g^3} - \frac{B\left(A+B\log \cdots\right.}{bg^3(a \cdots}$$

$$= -\frac{B^2}{bg^3(a+bx)^2} + \frac{6B^2d}{b(bc-ad)g^3(a+bx)} + \frac{6B^2d^2\log(a+bx)}{b(bc-ad)^2g^3} - \frac{B\left(A+B\log \cdots\right.}{bg^3(a \cdots}$$

$$= -\frac{B^2}{bg^3(a+bx)^2} + \frac{6B^2d}{b(bc-ad)g^3(a+bx)} + \frac{6B^2d^2\log(a+bx)}{b(bc-ad)^2g^3} - \frac{2B^2d^2\log^2(\cdots}{b(bc-ad \cdots}$$

$$= -\frac{B^2}{bg^3(a+bx)^2} + \frac{6B^2d}{b(bc-ad)g^3(a+bx)} + \frac{6B^2d^2\log(a+bx)}{b(bc-ad)^2g^3} - \frac{2B^2d^2\log^2(\cdots}{b(bc-ad \cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

time = 0.30, size = 451, normalized size = 1.66

$$-\frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 + \frac{2B\left((bc-ad)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)+2d(-bc+ad)(a+bx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)-2d^2(a+bx)^2\log(a+bx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)+2d^2(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\log(c+dx)-4Bd(a+bx)(bc-ad+d(a+bx)\log(a+bx)-d(a+bx)\log(c+dx))+B\left((bc-ad)^2+2d(-bc+ad)(a+bx)-2d^2(a+bx)^2\log(a+bx)+2d^2(a+bx)^2\log(c+dx)\right)+2Bd^2(a+bx)^2\left(\log(a+bx)\left(\log(a+bx)-2\log\left(\frac{b(c+dx)}{bc-ad}\right)\right)-2\text{Li}_2\left(\frac{d(a+bx)}{-bc+ad}\right)\right)-2Bd^2(a+bx)^2\left(\left(2\log\left(\frac{d(a+bx)}{-bc+ad}\right)-\log(c+dx)\right)\log(c+dx)+2\text{Li}_2\left(\frac{b(c+dx)}{bc-ad}\right)\right)\right)}{(bc-ad)^2}}{2bg^3(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2/(a*g + b*g*x)^3,x]
```

```
[Out] -1/2*((A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2 + (2*B*((b*c - a*d)^2*(A +
 B*Log[(e*(a + b*x)^2)/(c + d*x)^2]) + 2*d*(-(b*c) + a*d)*(a + b*x)*(A + B*
Log[(e*(a + b*x)^2)/(c + d*x)^2]) - 2*d^2*(a + b*x)^2*Log[a + b*x]*(A + B*L
og[(e*(a + b*x)^2)/(c + d*x)^2]) + 2*d^2*(a + b*x)^2*(A + B*Log[(e*(a + b*x
)^2)/(c + d*x)^2])*Log[c + d*x] - 4*B*d*(a + b*x)*(b*c - a*d + d*(a + b*x)*
Log[a + b*x] - d*(a + b*x)*Log[c + d*x]) + B*((b*c - a*d)^2 + 2*d*(-(b*c) +
 a*d)*(a + b*x) - 2*d^2*(a + b*x)^2*Log[a + b*x] + 2*d^2*(a + b*x)^2*Log[c
+ d*x]) + 2*B*d^2*(a + b*x)^2*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d
*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) - 2*B*d^2*
(a + b*x)^2*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d
*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(b*c - a*d)^2)/(b*g^3*(a +
 b*x)^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 625 vs. $2(270) = 540$.
time = 0.69, size = 626, normalized size = 2.30 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/(b*g*x+a*g)^3,x,method=_RETURNVERBOSE
)
```

```
[Out] -1/d*(d^3/g^3*A^2*(-1/(a*d-b*c)^2/(a/(d*x+c)*d-b*c/(d*x+c)+b)+1/2*b/(a*d-b*
c)^2/(a/(d*x+c)*d-b*c/(d*x+c)+b)^2)+(7*B^2/b*d^3/g/(d*x+c)^2-3*b*B^2*d^3/g/
(a^2*d^2-2*a*b*c*d+b^2*c^2)*ln(e*(a/(d*x+c)*d-b*c/(d*x+c)+b)^2/d^2)+6*B^2*d
^3/g/(a*d-b*c)/(d*x+c)-4*B^2*d^3/g/(a*d-b*c)/(d*x+c)*ln(e*(a/(d*x+c)*d-b*c/
(d*x+c)+b)^2/d^2)-1/2*B^2*b*d^3/g/(a^2*d^2-2*a*b*c*d+b^2*c^2)*ln(e*(a/(d*x+
c)*d-b*c/(d*x+c)+b)^2/d^2)^2-B^2*d^3/g/(a*d-b*c)/(d*x+c)*ln(e*(a/(d*x+c)*d-
b*c/(d*x+c)+b)^2/d^2)^2)/(a/(d*x+c)*d-b*c/(d*x+c)+b)^2/g^2+(3*A*B/b*d^3/g/(
d*x+c)^2-b*A*B*d^3/g/(a^2*d^2-2*a*b*c*d+b^2*c^2)*ln(e*(a/(d*x+c)*d-b*c/(d*x
+c)+b)^2/d^2)+2*A*B*d^3/g/(a*d-b*c)/(d*x+c)-2*A*B*d^3/g/(a*d-b*c)/(d*x+c)*l
n(e*(a/(d*x+c)*d-b*c/(d*x+c)+b)^2/d^2))/(a/(d*x+c)*d-b*c/(d*x+c)+b)^2/g^2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1010 vs. $2(274) = 548$.
time = 0.37, size = 1010, normalized size = 3.71

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(b*g*x+a*g)^3,x, algorithm="ma
xima")
```

```
[Out] (((2*b*d*x - b*c + 3*a*d)/((b^4*c - a*b^3*d)*g^3*x^2 + 2*(a*b^3*c - a^2*b^2
*d)*g^3*x + (a^2*b^2*c - a^3*b*d)*g^3) + 2*d^2*log(b*x + a)/((b^3*c^2 - 2*a
```

```
*b^2*c*d + a^2*b*d^2)*g^3) - 2*d^2*log(d*x + c)/((b^3*c^2 - 2*a*b^2*c*d + a
^2*b*d^2)*g^3))*log(b^2*x^2*e/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*x*e/(d^2*x^
2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) - (b^2*c^2 - 8*a*b*c*
d + 7*a^2*d^2 + 2*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*log(b*x + a)^2 + 2*
(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*log(d*x + c)^2 - 6*(b^2*c*d - a*b*d^2
)*x - 6*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*log(b*x + a) + 2*(3*b^2*d^2*x
^2 + 6*a*b*d^2*x + 3*a^2*d^2 - 2*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*log(
b*x + a))*log(d*x + c))/(a^2*b^3*c^2*g^3 - 2*a^3*b^2*c*d*g^3 + a^4*b*d^2*g^
3 + (b^5*c^2*g^3 - 2*a*b^4*c*d*g^3 + a^2*b^3*d^2*g^3)*x^2 + 2*(a*b^4*c^2*g^
3 - 2*a^2*b^3*c*d*g^3 + a^3*b^2*d^2*g^3)*x))*B^2 + A*B*((2*b*d*x - b*c + 3*
a*d)/((b^4*c - a*b^3*d)*g^3*x^2 + 2*(a*b^3*c - a^2*b^2*d)*g^3*x + (a^2*b^2*
c - a^3*b*d)*g^3) - log(b^2*x^2*e/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*x*e/(d^
2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2))/(b^3*g^3*x^2 + 2*
a*b^2*g^3*x + a^2*b*g^3) + 2*d^2*log(b*x + a)/((b^3*c^2 - 2*a*b^2*c*d + a^2
*b*d^2)*g^3) - 2*d^2*log(d*x + c)/((b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*g^3)
) - 1/2*B^2*log(b^2*x^2*e/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*x*e/(d^2*x^2 +
2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2))^2/(b^3*g^3*x^2 + 2*a*b^2*
g^3*x + a^2*b*g^3) - 1/2*A^2/(b^3*g^3*x^2 + 2*a*b^2*g^3*x + a^2*b*g^3)
```

Fricas [**A**]
time = 0.42, size = 406, normalized size = 1.49

$$-\frac{(A^2+2\,AB+2\,B^2)b^2c^2-2\,(A^2+4\,AB+8\,B^2)abcd+(A^2+6\,AB+14\,B^2)a^2d^2-(B^2b^2d^2x^2+2\,B^2abd^2x-B^2b^2c^2+2\,B^2abcd)\log\left(\frac{(b^2x^2+2\,abx+a^2)e}{d^2x^2+2\,cdx+c^2}\right)^2-4\,((AB+3\,B^2)b^2cd-(AB+3\,B^2)abd^2)x-2\,((AB+3\,B^2)b^2d^2x^2-(AB+B^2)b^2c^2+2\,(AB+2\,B^2)abcd+2\,(B^2b^2cd+(AB+2\,B^2)abd^2)x)\log\left(\frac{(b^2x^2+2\,abx+a^2)e}{d^2x^2+2\,cdx+c^2}\right)}{2\,((b^5c^2-2\,ab^4cd+a^2b^3d^2)g^3x^2+2\,(ab^4c^2-2\,a^2b^3cd+a^3b^2d^2)g^3x+(a^2b^3c^2-2\,a^3b^2cd+a^4bd^2)g^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(b*g*x+a*g)^3,x, algorithm="fr
icas")
```

```
[Out] -1/2*((A^2 + 2*A*B + 2*B^2)*b^2*c^2 - 2*(A^2 + 4*A*B + 8*B^2)*a*b*c*d + (A^
2 + 6*A*B + 14*B^2)*a^2*d^2 - (B^2*b^2*d^2*x^2 + 2*B^2*a*b*d^2*x - B^2*b^2*
c^2 + 2*B^2*a*b*c*d)*log((b^2*x^2 + 2*a*b*x + a^2)*e/(d^2*x^2 + 2*c*d*x + c
^2))^2 - 4*((A*B + 3*B^2)*b^2*c*d - (A*B + 3*B^2)*a*b*d^2)*x - 2*((A*B + 3*
B^2)*b^2*d^2*x^2 - (A*B + B^2)*b^2*c^2 + 2*(A*B + 2*B^2)*a*b*c*d + 2*(B^2*b
^2*c*d + (A*B + 2*B^2)*a*b*d^2)*x)*log((b^2*x^2 + 2*a*b*x + a^2)*e/(d^2*x^2
 + 2*c*d*x + c^2)))/((b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*g^3*x^2 + 2*(a*b
^4*c^2 - 2*a^2*b^3*c*d + a^3*b^2*d^2)*g^3*x + (a^2*b^3*c^2 - 2*a^3*b^2*c*d
+ a^4*b*d^2)*g^3)
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 879 vs. $2(252) = 504$.
time = 2.69, size = 879, normalized size = 3.23

$$-\frac{2Bd^2(A+3B)\log\left(x+\frac{2ABad^3+2ABbcd^2+6B^2ad^3+6B^2bcd^2-\frac{2Ba^3d^5(A+3B)}{(ad-bc)^2}+\frac{6Ba^2bcd^4(A+3B)}{(ad-bc)^2}-\frac{6Bab^2c^2d^3(A+3B)}{(ad-bc)^2}+\frac{2Bb^3c^3d^2(A+3B)}{(ad-bc)^2}}{4ABbd^3+12B^2bd^3}\right)}{bg^3\,(ad-bc)^2}+\frac{2Bd^2(A+3B)\log\left(x+\frac{2ABad^3+2ABbcd^2+6B^2ad^3+6B^2bcd^2+\frac{2Ba^3d^5(A+3B)}{(ad-bc)^2}-\frac{6Ba^2bcd^4(A+3B)}{(ad-bc)^2}+\frac{6Bab^2c^2d^3(A+3B)}{(ad-bc)^2}-\frac{2Bb^3c^3d^2(A+3B)}{(ad-bc)^2}}{4ABbd^3+12B^2bd^3}\right)}{bg^3\,(ad-bc)^2}+\frac{(2B^2acd+2B^2ad^2x-B^2bc^2+B^2bd^2x^2)\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)^2}{2a^4d^2g^3-4a^3bcdg^3+4a^3bd^2g^3x+2a^2b^2c^2g^3-8a^2b^2cdg^3x+2a^2b^2d^2g^3x^2+4ab^3c^2g^3x-4ab^3cdg^3x^2+2b^4c^2g^3x^2}+\frac{(-ABad+ABbc-3B^2ad+B^2bc-2B^2bdx)\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{a^3bdg^3-a^2b^2cg^3+2a^2b^2dg^3x-2ab^3cg^3x+ab^3dg^3x^2-b^4cg^3x^2}+\frac{-A^2ad+A^2bc-6ABad+2ABbc-14B^2ad+2B^2bc+x(-4ABbd-12B^2bd)}{2a^3bdg^3-2a^2b^2cg^3+x^2\cdot(2ab^3dg^3-2b^4cg^3)+x\,(4a^2b^2dg^3-4ab^3cg^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2/(b*g*x+a*g)**3,x)
```

[Out] -2*B*d**2*(A + 3*B)*log(x + (2*A*B*a*d**3 + 2*A*B*b*c*d**2 + 6*B**2*a*d**3 + 6*B**2*b*c*d**2 - 2*B*a**3*d**5*(A + 3*B)/(a*d - b*c)**2 + 6*B*a**2*b*c*d**4*(A + 3*B)/(a*d - b*c)**2 - 6*B*a*b**2*c**2*d**3*(A + 3*B)/(a*d - b*c)**2 + 2*B*b**3*c**3*d**2*(A + 3*B)/(a*d - b*c)**2)/(4*A*B*b*d**3 + 12*B**2*b*d**3))/(b*g**3*(a*d - b*c)**2) + 2*B*d**2*(A + 3*B)*log(x + (2*A*B*a*d**3 + 2*A*B*b*c*d**2 + 6*B**2*a*d**3 + 6*B**2*b*c*d**2 + 2*B*a**3*d**5*(A + 3*B)/(a*d - b*c)**2 - 6*B*a**2*b*c*d**4*(A + 3*B)/(a*d - b*c)**2 + 6*B*a*b**2*c**2*d**3*(A + 3*B)/(a*d - b*c)**2 - 2*B*b**3*c**3*d**2*(A + 3*B)/(a*d - b*c)**2)/(4*A*B*b*d**3 + 12*B**2*b*d**3))/(b*g**3*(a*d - b*c)**2) + (2*B**2*a*c*d + 2*B**2*a*d**2*x - B**2*b*c**2 + B**2*b*d**2*x**2)*log(e*(a + b*x)**2/(c + d*x)**2)**2/(2*a**4*d**2*g**3 - 4*a**3*b*c*d*g**3 + 4*a**3*b*d**2*g**3*x + 2*a**2*b**2*c**2*g**3 - 8*a**2*b**2*c*d*g**3*x + 2*a**2*b**2*d**2*g**3*x**2 + 4*a*b**3*c**2*g**3*x - 4*a*b**3*c*d*g**3*x**2 + 2*b**4*c**2*g**3*x**2) + (-A*B*a*d + A*B*b*c - 3*B**2*a*d + B**2*b*c - 2*B**2*b*d*x)*log(e*(a + b*x)**2/(c + d*x)**2)/(a**3*b*d*g**3 - a**2*b**2*c*g**3 + 2*a**2*b**2*d*g**3*x - 2*a*b**3*c*g**3*x + a*b**3*d*g**3*x**2 - b**4*c*g**3*x**2) + (-A**2*a*d + A**2*b*c - 6*A*B*a*d + 2*A*B*b*c - 14*B**2*a*d + 2*B**2*b*c + x*(-4*A*B*b*d - 12*B**2*b*d))/(2*a**3*b*d*g**3 - 2*a**2*b**2*c*g**3 + x**2*(2*a*b**3*d*g**3 - 2*b**4*c*g**3) + x*(4*a**2*b**2*d*g**3 - 4*a*b**3*c*g**3))

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(b*g*x+a*g)^3,x, algorithm="giac")

[Out] integrate((B*log((b*x + a)^2*e/(d*x + c)^2) + A)^2/(b*g*x + a*g)^3, x)

Mupad [B]
time = 5.89, size = 503, normalized size = 1.85

$$-\frac{\frac{A^2\,a\,d-A^2\,b\,c+14\,B^2\,a\,d-2\,B^2\,b\,c+6\,A\,B\,a\,d-2\,A\,B\,b\,c}{2\,\left(a\,d-b\,c\right)}+\frac{2\,x\,\left(3\,b\,d\,B^2+A\,b\,d\,B\right)}{a\,d-b\,c}}{a^2\,b\,g^3+2\,a\,b^2\,g^3\,x+b^3\,g^3\,x^2}-{\ln\left(\frac{e\,{\left(a+b\,x\right)}^2}{{\left(c+d\,x\right)}^2}\right)}^2\,\left(\frac{B^2}{2\,b^2\,g^3\,\left(2\,a\,x+b\,x^2+\frac{a^2}{b}\right)}-\frac{B^2\,d^2}{2\,b\,g^3\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}\right)-\frac{\ln\left(\frac{e\,{\left(a+b\,x\right)}^2}{{\left(c+d\,x\right)}^2}\right)\,\left(\frac{A\,B}{b^2\,d\,g^3}+\frac{2\,B^2\,x\,\left(a\,d-b\,c\right)}{b\,g^3\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}+\frac{B^2\,d^2\,\left(\frac{2\,a^2\,d^2-3\,a\,b\,c\,d+b^2\,c^2}{b\,d^3}+\frac{a\,\left(a\,d-b\,c\right)}{b\,d^2}\right)}{b\,g^3\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}\right)}{\frac{b\,x^2}{d}+\frac{a^2}{b\,d}+\frac{2\,a\,x}{d}}-\frac{B\,d^2\,\mathrm{atan}\left(\frac{B\,d^2\,\left(2\,b\,d\,x-\frac{b^3\,c^2\,g^3-a^2\,b\,d^2\,g^3}{b\,g^3\,\left(a\,d-b\,c\right)}\right)\,\left(A+3\,B\right)\,2{}\mathrm{i}}{\left(a\,d-b\,c\right)\,\left(6\,B^2\,d^2+2\,A\,B\,d^2\right)}\right)\,\left(A+3\,B\right)\,4{}\mathrm{i}}{b\,g^3\,{\left(a\,d-b\,c\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2/(a*g + b*g*x)^3,x)

[Out] - ((A^2*a*d - A^2*b*c + 14*B^2*a*d - 2*B^2*b*c + 6*A*B*a*d - 2*A*B*b*c)/(2*(a*d - b*c)) + (2*x*(3*B^2*b*d + A*B*b*d))/(a*d - b*c))/(a^2*b*g^3 + b^3*g^3*x^2 + 2*a*b^2*g^3*x) - log((e*(a + b*x)^2)/(c + d*x)^2)^2*(B^2/(2*b^2*g^3*(2*a*x + b*x^2 + a^2/b)) - (B^2*d^2)/(2*b*g^3*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))) - (log((e*(a + b*x)^2)/(c + d*x)^2)*((A*B)/(b^2*d*g^3) + (2*B^2*x*(a*d - b*c))/(b*g^3*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (B^2*d^2*((2*a^2*d^2 +

```
b^2*c^2 - 3*a*b*c*d)/(b*d^3) + (a*(a*d - b*c))/(b*d^2)))/(b*g^3*(a^2*d^2 +
b^2*c^2 - 2*a*b*c*d))))/((b*x^2)/d + a^2/(b*d) + (2*a*x)/d) - (B*d^2*atan((
B*d^2*(2*b*d*x - (b^3*c^2*g^3 - a^2*b*d^2*g^3)/(b*g^3*(a*d - b*c)))*(A + 3*
B)*2i)/((a*d - b*c)*(6*B^2*d^2 + 2*A*B*d^2)))*(A + 3*B)*4i)/(b*g^3*(a*d - b
*c)^2)
```

3.135 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(ag+bgx)^4}\,dx$

Optimal. Leaf size=429

$$-\frac{8B^2d^2(c+dx)}{(bc-ad)^3g^4(a+bx)}+\frac{2bB^2d(c+dx)^2}{(bc-ad)^3g^4(a+bx)^2}-\frac{8b^2B^2(c+dx)^3}{27(bc-ad)^3g^4(a+bx)^3}-\frac{4Bd^2(c+dx)\left(A+B\log\left(\frac{e(a+b}{(c+d}\right.\right.}{(bc-ad)^3g^4(a+bx)}$$

```
[Out] -8*B^2*d^2*(d*x+c)/(-a*d+b*c)^3/g^4/(b*x+a)+2*b*B^2*d*(d*x+c)^2/(-a*d+b*c)^
3/g^4/(b*x+a)^2-8/27*b^2*B^2*(d*x+c)^3/(-a*d+b*c)^3/g^4/(b*x+a)^3-4*B*d^2*(
d*x+c)*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/(-a*d+b*c)^3/g^4/(b*x+a)+2*b*B*d*(d*
x+c)^2*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/(-a*d+b*c)^3/g^4/(b*x+a)^2-4/9*b^2*B
*(d*x+c)^3*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/(-a*d+b*c)^3/g^4/(b*x+a)^3-d^2*(
d*x+c)*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/(-a*d+b*c)^3/g^4/(b*x+a)+b*d*(d*x+
c)^2*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/(-a*d+b*c)^3/g^4/(b*x+a)^2-1/3*b^2*(
d*x+c)^3*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/(-a*d+b*c)^3/g^4/(b*x+a)^3
```

Rubi [A]

time = 0.22, antiderivative size = 429, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2550, 2395, 2342, 2341}

$$-\frac{b^2(c+dx)^3\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{3g^4(a+bx)^3(bc-ad)^3}-\frac{4b^2B(c+dx)^3\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{9g^4(a+bx)^3(bc-ad)^3}-\frac{d^2(c+dx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{g^4(a+bx)(bc-ad)^3}-\frac{4Bd^2(c+dx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{g^4(a+bx)(bc-ad)^3}+\frac{bd(c+dx)^2\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{g^4(a+bx)^2(bc-ad)^3}+\frac{2bBd(c+dx)^2\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{g^4(a+bx)^2(bc-ad)^3}-\frac{8b^2B^2(c+dx)^3}{27g^4(a+bx)^3(bc-ad)^3}-\frac{8B^2d^2(c+dx)}{g^4(a+bx)(bc-ad)^3}+\frac{2bB^2d(c+dx)^2}{g^4(a+bx)^2(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2/(a*g + b*g*x)^4,x]
```

```
[Out] (-8*B^2*d^2*(c + d*x))/((b*c - a*d)^3*g^4*(a + b*x)) + (2*b*B^2*d*(c + d*x)
^2)/((b*c - a*d)^3*g^4*(a + b*x)^2) - (8*b^2*B^2*(c + d*x)^3)/(27*(b*c - a*
d)^3*g^4*(a + b*x)^3) - (4*B*d^2*(c + d*x)*(A + B*Log[(e*(a + b*x)^2)/(c +
d*x)^2]))/((b*c - a*d)^3*g^4*(a + b*x)) + (2*b*B*d*(c + d*x)^2*(A + B*Log[(
e*(a + b*x)^2)/(c + d*x)^2]))/((b*c - a*d)^3*g^4*(a + b*x)^2) - (4*b^2*B*(c
 + d*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]))/(9*(b*c - a*d)^3*g^4*(a
 + b*x)^3) - (d^2*(c + d*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2)/((b
*c - a*d)^3*g^4*(a + b*x)) + (b*d*(c + d*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c
 + d*x)^2])^2)/((b*c - a*d)^3*g^4*(a + b*x)^2) - (b^2*(c + d*x)^3*(A + B*Lo
g[(e*(a + b*x)^2)/(c + d*x)^2])^2)/(3*(b*c - a*d)^3*g^4*(a + b*x)^3)
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2550

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(
m + 1)*(g/b)^m, Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x],
 x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] &&
 EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && Eq
Q[b*f - a*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Rubi steps

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(ag+bgx)^4}\,dx = -\frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{3bg^4(a+bx)^3} + \frac{(2B)\int \frac{2(bc-ad)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{g^3(a+bx)^4(c+dx)}dx}{3bg}$$

$$= -\frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{3bg^4(a+bx)^3} + \frac{(4B(bc-ad))\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(a+bx)^4(c+dx)}dx}{3bg^4}$$

$$= -\frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{3bg^4(a+bx)^3} + \frac{(4B(bc-ad))\int \left(\frac{b\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(bc-ad)(a+bx)^4} - \frac{bd(}{}\right.}{}$$

$$= -\frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{3bg^4(a+bx)^3} + \frac{(4B)\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(a+bx)^4}dx}{3g^4} - \frac{(4Bd^3)\int \frac{A+B}{}}{3(bc-}$$

$$= -\frac{4B\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{9bg^4(a+bx)^3} + \frac{2Bd\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{3b(bc-ad)g^4(a+bx)^2} - \frac{4Bd^2\left(A+\right.}{3b(bc-}$$

$$= -\frac{4B\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{9bg^4(a+bx)^3} + \frac{2Bd\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{3b(bc-ad)g^4(a+bx)^2} - \frac{4Bd^2\left(A+\right.}{3b(bc-}$$

$$= -\frac{4B\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{9bg^4(a+bx)^3} + \frac{2Bd\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{3b(bc-ad)g^4(a+bx)^2} - \frac{4Bd^2\left(A+\right.}{3b(bc-}$$

$$= -\frac{8B^2}{27bg^4(a+bx)^3} + \frac{10B^2d}{9b(bc-ad)g^4(a+bx)^2} - \frac{44B^2d^2}{9b(bc-ad)^2g^4(a+bx)} -$$

$$= -\frac{8B^2}{27bg^4(a+bx)^3} + \frac{10B^2d}{9b(bc-ad)g^4(a+bx)^2} - \frac{44B^2d^2}{9b(bc-ad)^2g^4(a+bx)} -$$

$$= -\frac{8B^2}{27bg^4(a+bx)^3} + \frac{10B^2d}{9b(bc-ad)g^4(a+bx)^2} - \frac{44B^2d^2}{9b(bc-ad)^2g^4(a+bx)} -$$

$$= -\frac{8B^2}{27bg^4(a+bx)^3} + \frac{10B^2d}{9b(bc-ad)g^4(a+bx)^2} - \frac{44B^2d^2}{9b(bc-ad)^2g^4(a+bx)} -$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 0.42, size = 598, normalized size = 1.39

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2/(a*g + b*g*x)^4,x]
```

```
[Out] -1/27*(9*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2 + (2*B*(6*(b*c - a*d)^3
*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]) - 9*d*(b*c - a*d)^2*(a + b*x)*(A
+ B*Log[(e*(a + b*x)^2)/(c + d*x)^2]) + 18*d^2*(b*c - a*d)*(a + b*x)^2*(A +
 B*Log[(e*(a + b*x)^2)/(c + d*x)^2]) + 18*d^3*(a + b*x)^3*Log[a + b*x]*(A +
 B*Log[(e*(a + b*x)^2)/(c + d*x)^2]) - 18*d^3*(a + b*x)^3*(A + B*Log[(e*(a
+ b*x)^2)/(c + d*x)^2])*Log[c + d*x] + 36*B*d^2*(a + b*x)^2*(b*c - a*d + d*
(a + b*x)*Log[a + b*x] - d*(a + b*x)*Log[c + d*x]) - 9*B*d*(a + b*x)*((b*c
- a*d)^2 + 2*d*(-(b*c) + a*d)*(a + b*x) - 2*d^2*(a + b*x)^2*Log[a + b*x] +
2*d^2*(a + b*x)^2*Log[c + d*x]) + 2*B*(2*(b*c - a*d)^3 - 3*d*(b*c - a*d)^2*
(a + b*x) + 6*d^2*(b*c - a*d)*(a + b*x)^2 + 6*d^3*(a + b*x)^3*Log[a + b*x]
- 6*d^3*(a + b*x)^3*Log[c + d*x]) - 18*B*d^3*(a + b*x)^3*(Log[a + b*x]*(Log
[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/
(-(b*c) + a*d)]) + 18*B*d^3*(a + b*x)^3*((2*Log[(d*(a + b*x))/(-(b*c) + a*d
)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]))
)/(b*c - a*d)^3)/(b*g^4*(a + b*x)^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1018 vs. $2(423) = 846$.

time = 0.90, size = 1019, normalized size = 2.38 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/(b*g*x+a*g)^4,x,method=_RETURNVERBOSE
)
```

```
[Out] -1/d*(d^4/g^4*A^2*(-1/3*b^2/(a*d-b*c)^3/(a/(d*x+c)*d-b*c/(d*x+c)+b)^3+b/(a*
d-b*c)^3/(a/(d*x+c)*d-b*c/(d*x+c)+b)^2-1/(a*d-b*c)^3/(a/(d*x+c)*d-b*c/(d*x+
c)+b))+(170/27*B^2/b*d^4/g/(d*x+c)^3-22/9*b^2*B^2*d^4/g/(a^3*d^3-3*a^2*b*c*
d^2+3*a*b^2*c^2*d-b^3*c^3)*ln(e*(a/(d*x+c)*d-b*c/(d*x+c)+b)^2/d^2)+44/9*B^2
*b*d^4/g/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(d*x+c)+98/9*B^2*d^4/g/(a*d-b*c)/(d*x+
c)^2-4*B^2*d^4/g/(a*d-b*c)/(d*x+c)^2*ln(e*(a/(d*x+c)*d-b*c/(d*x+c)+b)^2/d^2
)-1/3*B^2*b^2*d^4/g/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)*ln(e*(a/(
d*x+c)*d-b*c/(d*x+c)+b)^2/d^2)^2-B^2*d^4/g/(a*d-b*c)/(d*x+c)^2*ln(e*(a/(d*x
+c)*d-b*c/(d*x+c)+b)^2/d^2)^2-6*B^2*d^4*b/g/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(d*
x+c)*ln(e*(a/(d*x+c)*d-b*c/(d*x+c)+b)^2/d^2)-B^2*b*d^4/g/(a^2*d^2-2*a*b*c*d
+b^2*c^2)/(d*x+c)*ln(e*(a/(d*x+c)*d-b*c/(d*x+c)+b)^2/d^2)^2)/(a/(d*x+c)*d-b
*c/(d*x+c)+b)^3/g^3+(22/9*A*B/b*d^4/g/(d*x+c)^3-2/3*b^2*A*B*d^4/g/(a^3*d^3-
3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)*ln(e*(a/(d*x+c)*d-b*c/(d*x+c)+b)^2/d^2
)+4/3*A*B*b*d^4/g/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(d*x+c)+10/3*A*B*d^4/g/(a*d-b
*c)/(d*x+c)^2-2*A*B*d^4/g/(a*d-b*c)/(d*x+c)^2*ln(e*(a/(d*x+c)*d-b*c/(d*x+c)
+b)^2/d^2)-2*A*B*d^4*b/g/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(d*x+c)*ln(e*(a/(d*x+c
)*d-b*c/(d*x+c)+b)^2/d^2))/(a/(d*x+c)*d-b*c/(d*x+c)+b)^3/g^3)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1584 vs. $2(429) = 858$.

time = 0.45, size = 1584, normalized size = 3.69

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(b*g*x+a*g)^4,x, algorithm="ma
xima")
```

```
[Out] -2/27*(3*((6*b^2*d^2*x^2 + 2*b^2*c^2 - 7*a*b*c*d + 11*a^2*d^2 - 3*(b^2*c*d
- 5*a*b*d^2)*x)/((b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2)*g^4*x^3 + 3*(a*b^5*c
^2 - 2*a^2*b^4*c*d + a^3*b^3*d^2)*g^4*x^2 + 3*(a^2*b^4*c^2 - 2*a^3*b^3*c*d
+ a^4*b^2*d^2)*g^4*x + (a^3*b^3*c^2 - 2*a^4*b^2*c*d + a^5*b*d^2)*g^4) + 6*d
^3*log(b*x + a)/((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*g^
4) - 6*d^3*log(d*x + c)/((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b
*d^3)*g^4))*log(b^2*x^2*e/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*x*e/(d^2*x^2 +
2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) + (4*b^3*c^3 - 27*a*b^2*c
^2*d + 108*a^2*b*c*d^2 - 85*a^3*d^3 + 66*(b^3*c*d^2 - a*b^2*d^3)*x^2 - 18*(
b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(b*x + a)^2 - 1
8*(b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(d*x + c)^2
- 3*(5*b^3*c^2*d - 54*a*b^2*c*d^2 + 49*a^2*b*d^3)*x + 66*(b^3*d^3*x^3 + 3*a
*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(b*x + a) - 6*(11*b^3*d^3*x^3 +
33*a*b^2*d^3*x^2 + 33*a^2*b*d^3*x + 11*a^3*d^3 - 6*(b^3*d^3*x^3 + 3*a*b^2*d
^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(b*x + a))*log(d*x + c))/(a^3*b^4*c^3*
g^4 - 3*a^4*b^3*c^2*d*g^4 + 3*a^5*b^2*c*d^2*g^4 - a^6*b*d^3*g^4 + (b^7*c^3*
g^4 - 3*a*b^6*c^2*d*g^4 + 3*a^2*b^5*c*d^2*g^4 - a^3*b^4*d^3*g^4)*x^3 + 3*(a
*b^6*c^3*g^4 - 3*a^2*b^5*c^2*d*g^4 + 3*a^3*b^4*c*d^2*g^4 - a^4*b^3*d^3*g^4)
*x^2 + 3*(a^2*b^5*c^3*g^4 - 3*a^3*b^4*c^2*d*g^4 + 3*a^4*b^3*c*d^2*g^4 - a^5
*b^2*d^3*g^4)*x))*B^2 - 2/9*A*B*((6*b^2*d^2*x^2 + 2*b^2*c^2 - 7*a*b*c*d + 1
1*a^2*d^2 - 3*(b^2*c*d - 5*a*b*d^2)*x)/((b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^
2)*g^4*x^3 + 3*(a*b^5*c^2 - 2*a^2*b^4*c*d + a^3*b^3*d^2)*g^4*x^2 + 3*(a^2*b
^4*c^2 - 2*a^3*b^3*c*d + a^4*b^2*d^2)*g^4*x + (a^3*b^3*c^2 - 2*a^4*b^2*c*d
+ a^5*b*d^2)*g^4) + 3*log(b^2*x^2*e/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*x*e/(
d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2))/(b^4*g^4*x^3 +
3*a*b^3*g^4*x^2 + 3*a^2*b^2*g^4*x + a^3*b*g^4) + 6*d^3*log(b*x + a)/((b^4*c
^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*g^4) - 6*d^3*log(d*x + c)
/((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*g^4)) - 1/3*B^2*l
og(b^2*x^2*e/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*x*e/(d^2*x^2 + 2*c*d*x + c^2
) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2))^2/(b^4*g^4*x^3 + 3*a*b^3*g^4*x^2 + 3*a
^2*b^2*g^4*x + a^3*b*g^4) - 1/3*A^2/(b^4*g^4*x^3 + 3*a*b^3*g^4*x^2 + 3*a^2*
b^2*g^4*x + a^3*b*g^4)
```

Fricas [A]
time = 0.37, size = 715, normalized size = 1.67

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(b*g*x+a*g)^4,x, algorithm="fr
icas")
```

```
[Out] -1/27*((9*A^2 + 12*A*B + 8*B^2)*b^3*c^3 - 27*(A^2 + 2*A*B + 2*B^2)*a*b^2*c^
2*d + 27*(A^2 + 4*A*B + 8*B^2)*a^2*b*c*d^2 - (9*A^2 + 66*A*B + 170*B^2)*a^3
*d^3 + 12*((3*A*B + 11*B^2)*b^3*c*d^2 - (3*A*B + 11*B^2)*a*b^2*d^3)*x^2 + 9
*(B^2*b^3*d^3*x^3 + 3*B^2*a*b^2*d^3*x^2 + 3*B^2*a^2*b*d^3*x + B^2*b^3*c^3 -
 3*B^2*a*b^2*c^2*d + 3*B^2*a^2*b*c*d^2)*log((b^2*x^2 + 2*a*b*x + a^2)*e/(d^
2*x^2 + 2*c*d*x + c^2))^2 - 6*((3*A*B + 5*B^2)*b^3*c^2*d - 18*(A*B + 3*B^2)
*a*b^2*c*d^2 + (15*A*B + 49*B^2)*a^2*b*d^3)*x + 6*((3*A*B + 11*B^2)*b^3*d^3
*x^3 + (3*A*B + 2*B^2)*b^3*c^3 - 9*(A*B + B^2)*a*b^2*c^2*d + 9*(A*B + 2*B^2
)*a^2*b*c*d^2 + 3*(2*B^2*b^3*c*d^2 + 3*(A*B + 3*B^2)*a*b^2*d^3)*x^2 - 3*(B^
2*b^3*c^2*d - 6*B^2*a*b^2*c*d^2 - 3*(A*B + 2*B^2)*a^2*b*d^3)*x)*log((b^2*x^
2 + 2*a*b*x + a^2)*e/(d^2*x^2 + 2*c*d*x + c^2)))/((b^7*c^3 - 3*a*b^6*c^2*d
+ 3*a^2*b^5*c*d^2 - a^3*b^4*d^3)*g^4*x^3 + 3*(a*b^6*c^3 - 3*a^2*b^5*c^2*d +
 3*a^3*b^4*c*d^2 - a^4*b^3*d^3)*g^4*x^2 + 3*(a^2*b^5*c^3 - 3*a^3*b^4*c^2*d
+ 3*a^4*b^3*c*d^2 - a^5*b^2*d^3)*g^4*x + (a^3*b^4*c^3 - 3*a^4*b^3*c^2*d + 3
*a^5*b^2*c*d^2 - a^6*b*d^3)*g^4)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1561 vs. $2(406) = 812$.
time = 16.28, size = 1561, normalized size = 3.64

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2/(b*g*x+a*g)**4,x)
```

```
[Out] -4*B*d**3*(3*A + 11*B)*log(x + (12*A*B*a*d**4 + 12*A*B*b*c*d**3 + 44*B**2*a
*d**4 + 44*B**2*b*c*d**3 - 4*B*a**4*d**7*(3*A + 11*B)/(a*d - b*c)**3 + 16*B
*a**3*b*c*d**6*(3*A + 11*B)/(a*d - b*c)**3 - 24*B*a**2*b**2*c**2*d**5*(3*A
+ 11*B)/(a*d - b*c)**3 + 16*B*a*b**3*c**3*d**4*(3*A + 11*B)/(a*d - b*c)**3
- 4*B*b**4*c**4*d**3*(3*A + 11*B)/(a*d - b*c)**3)/(24*A*B*b*d**4 + 88*B**2*
b*d**4))/(9*b*g**4*(a*d - b*c)**3) + 4*B*d**3*(3*A + 11*B)*log(x + (12*A*B*
a*d**4 + 12*A*B*b*c*d**3 + 44*B**2*a*d**4 + 44*B**2*b*c*d**3 + 4*B*a**4*d**
7*(3*A + 11*B)/(a*d - b*c)**3 - 16*B*a**3*b*c*d**6*(3*A + 11*B)/(a*d - b*c)
**3 + 24*B*a**2*b**2*c**2*d**5*(3*A + 11*B)/(a*d - b*c)**3 - 16*B*a*b**3*c*
*3*d**4*(3*A + 11*B)/(a*d - b*c)**3 + 4*B*b**4*c**4*d**3*(3*A + 11*B)/(a*d
- b*c)**3)/(24*A*B*b*d**4 + 88*B**2*b*d**4))/(9*b*g**4*(a*d - b*c)**3) + (3
*B**2*a**2*c*d**2 + 3*B**2*a**2*d**3*x - 3*B**2*a*b*c**2*d + 3*B**2*a*b*d**
3*x**2 + B**2*b**2*c**3 + B**2*b**2*d**3*x**3)*log(e*(a + b*x)**2/(c + d*x)
**2)**2/(3*a**6*d**3*g**4 - 9*a**5*b*c*d**2*g**4 + 9*a**5*b*d**3*g**4*x + 9
*a**4*b**2*c**2*d*g**4 - 27*a**4*b**2*c*d**2*g**4*x + 9*a**4*b**2*d**3*g**4
*x**2 - 3*a**3*b**3*c**3*g**4 + 27*a**3*b**3*c**2*d*g**4*x - 27*a**3*b**3*c
```

```
*d**2*g**4*x**2 + 3*a**3*b**3*d**3*g**4*x**3 - 9*a**2*b**4*c**3*g**4*x + 27
*a**2*b**4*c**2*d*g**4*x**2 - 9*a**2*b**4*c*d**2*g**4*x**3 - 9*a*b**5*c**3*
g**4*x**2 + 9*a*b**5*c**2*d*g**4*x**3 - 3*b**6*c**3*g**4*x**3) + (-6*A*B*a*
*2*d**2 + 12*A*B*a*b*c*d - 6*A*B*b**2*c**2 - 22*B**2*a**2*d**2 + 14*B**2*a*
b*c*d - 30*B**2*a*b*d**2*x - 4*B**2*b**2*c**2 + 6*B**2*b**2*c*d*x - 12*B**2
*b**2*d**2*x**2)*log(e*(a + b*x)**2/(c + d*x)**2)/(9*a**5*b*d**2*g**4 - 18*
a**4*b**2*c*d*g**4 + 27*a**4*b**2*d**2*g**4*x + 9*a**3*b**3*c**2*g**4 - 54*
a**3*b**3*c*d*g**4*x + 27*a**3*b**3*d**2*g**4*x**2 + 27*a**2*b**4*c**2*g**4
*x - 54*a**2*b**4*c*d*g**4*x**2 + 9*a**2*b**4*d**2*g**4*x**3 + 27*a*b**5*c*
*2*g**4*x**2 - 18*a*b**5*c*d*g**4*x**3 + 9*b**6*c**2*g**4*x**3) - (9*A**2*a
**2*d**2 - 18*A**2*a*b*c*d + 9*A**2*b**2*c**2 + 66*A*B*a**2*d**2 - 42*A*B*a
*b*c*d + 12*A*B*b**2*c**2 + 170*B**2*a**2*d**2 - 46*B**2*a*b*c*d + 8*B**2*b
**2*c**2 + x**2*(36*A*B*b**2*d**2 + 132*B**2*b**2*d**2) + x*(90*A*B*a*b*d**
2 - 18*A*B*b**2*c*d + 294*B**2*a*b*d**2 - 30*B**2*b**2*c*d))/(27*a**5*b*d**
2*g**4 - 54*a**4*b**2*c*d*g**4 + 27*a**3*b**3*c**2*g**4 + x**3*(27*a**2*b**
4*d**2*g**4 - 54*a*b**5*c*d*g**4 + 27*b**6*c**2*g**4) + x**2*(81*a**3*b**3*
d**2*g**4 - 162*a**2*b**4*c*d*g**4 + 81*a*b**5*c**2*g**4) + x*(81*a**4*b**2
*d**2*g**4 - 162*a**3*b**3*c*d*g**4 + 81*a**2*b**4*c**2*g**4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(b*g*x+a*g)^4,x, algorithm="gi
ac")
```

```
[Out] integrate((B*log((b*x + a)^2*e/(d*x + c)^2) + A)^2/(b*g*x + a*g)^4, x)
```

Mupad [B]
time = 7.67, size = 1069, normalized size = 2.49

$$\frac{\frac{9A^2a^2d^2-18A^2abcd+9A^2b^2c^2+66ABa^2d^2-42ABabcd+12ABb^2c^2+170B^2a^2d^2-46B^2abcd+8B^2b^2c^2}{3(ad-bc)}+\frac{2x(-5cB^2b^2d+49aB^2bd^2-3AcBb^2d+15AaBbd^2)}{ad-bc}+\frac{4dx^2(11dB^2b^2+3AdBb^2)}{ad-bc}}{x(27a^2b^3cg^4-27a^3b^2dg^4)-x^2(27a^2b^3dg^4-27ab^4cg^4)+x^3(9b^5cg^4-9ab^4dg^4)+9a^3b^2cg^4-9a^4bdg^4}-\ln\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)^2\left(\frac{B^2}{3b^2g^4\left(3a^2x+\frac{a^3}{b}+b^2x^3+3abx^2\right)}-\frac{B^2d^3}{3bg^4(a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3)}\right)-\frac{\ln\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\left(\frac{2AB}{3b^2dg^4}+\frac{2B^2d^3\left(a\left(\frac{3a^2d^2-4abcd+b^2c^2}{3bd^3}+\frac{2a(ad-bc)}{3bd^2}\right)+\frac{2(3a^2d^3-6a^2bcd^2+4ab^2c^2d-b^3c^3)}{3bd^4}\right)}{3bg^4(a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3)}-\frac{2B^2d^3x^2\left(\frac{2(b^2c-abd)}{3d^2}-\frac{4b(ad-bc)}{3d^2}\right)}{3bg^4(a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3)}+\frac{2B^2d^3x\left(b\left(\frac{3a^2d^2-4abcd+b^2c^2}{3bd^3}+\frac{2a(ad-bc)}{3bd^2}\right)+\frac{2(3a^2d^2-4abcd+b^2c^2)}{3d^3}+\frac{4a(ad-bc)}{3d^2}\right)}{3bg^4(a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3)}\right)}{\frac{3a^2x}{d}+\frac{a^3}{bd}+\frac{b^2x^3}{d}+\frac{3abx^2}{d}}-\frac{Bd^3\,\mathrm{atan}\left(\frac{Bd^3\left(\frac{a^3bd^3g^4-a^2b^2cd^2g^4-ab^3c^2dg^4+b^4c^3g^4}{a^2bd^2g^4-2ab^2cdg^4+b^3c^2g^4}+2bdx\right)(3A+11B)(a^2bd^2g^4-2ab^2cdg^4+b^3c^2g^4)\,4\mathrm{i}}{bg^4(ad-bc)^3(44B^2d^3+12ABd^3)}\right)(3A+11B)\,8\mathrm{i}}{9bg^4(ad-bc)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2/(a*g + b*g*x)^4,x)
```

```
[Out] ((9*A^2*a^2*d^2 + 9*A^2*b^2*c^2 + 170*B^2*a^2*d^2 + 8*B^2*b^2*c^2 + 66*A*B*
a^2*d^2 + 12*A*B*b^2*c^2 - 18*A^2*a*b*c*d - 46*B^2*a*b*c*d - 42*A*B*a*b*c*d
)/(3*(a*d - b*c)) + (2*x*(49*B^2*a*b*d^2 - 5*B^2*b^2*c*d + 15*A*B*a*b*d^2 -
 3*A*B*b^2*c*d))/(a*d - b*c) + (4*d*x^2*(11*B^2*b^2*d + 3*A*B*b^2*d))/(a*d
- b*c))/(x*(27*a^2*b^3*c*g^4 - 27*a^3*b^2*d*g^4) - x^2*(27*a^2*b^3*d*g^4 -
27*a*b^4*c*g^4) + x^3*(9*b^5*c*g^4 - 9*a*b^4*d*g^4) + 9*a^3*b^2*c*g^4 - 9*a
^4*b*d*g^4) - log((e*(a + b*x)^2)/(c + d*x)^2)^2*(B^2/(3*b^2*g^4*(3*a^2*x +
 a^3/b + b^2*x^3 + 3*a*b*x^2)) - (B^2*d^3)/(3*b*g^4*(a^3*d^3 - b^3*c^3 + 3*
```

```
a*b^2*c^2*d - 3*a^2*b*c*d^2))) - (log((e*(a + b*x)^2)/(c + d*x)^2)*((2*A*B)
/(3*b^2*d*g^4) + (2*B^2*d^3*(a*((3*a^2*d^2 + b^2*c^2 - 4*a*b*c*d)/(3*b*d^3)
 + (2*a*(a*d - b*c))/(3*b*d^2)) + (2*(3*a^3*d^3 - b^3*c^3 + 4*a*b^2*c^2*d -
 6*a^2*b*c*d^2))/(3*b*d^4)))/(3*b*g^4*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d -
3*a^2*b*c*d^2)) - (2*B^2*d^3*x^2*((2*(b^2*c - a*b*d))/(3*d^2) - (4*b*(a*d -
 b*c))/(3*d^2)))/(3*b*g^4*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^
2)) + (2*B^2*d^3*x*(b*((3*a^2*d^2 + b^2*c^2 - 4*a*b*c*d)/(3*b*d^3) + (2*a*(
a*d - b*c))/(3*b*d^2)) + (2*(3*a^2*d^2 + b^2*c^2 - 4*a*b*c*d))/(3*d^3) + (4
*a*(a*d - b*c))/(3*d^2)))/(3*b*g^4*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a
^2*b*c*d^2))))/((3*a^2*x)/d + a^3/(b*d) + (b^2*x^3)/d + (3*a*b*x^2)/d) - (B
*d^3*atan((B*d^3*((b^4*c^3*g^4 + a^3*b*d^3*g^4 - a*b^3*c^2*d*g^4 - a^2*b^2*
c*d^2*g^4)/(b^3*c^2*g^4 + a^2*b*d^2*g^4 - 2*a*b^2*c*d*g^4) + 2*b*d*x)*(3*A
+ 11*B)*(b^3*c^2*g^4 + a^2*b*d^2*g^4 - 2*a*b^2*c*d*g^4)*4i)/(b*g^4*(a*d - b
*c)^3*(44*B^2*d^3 + 12*A*B*d^3)))*(3*A + 11*B)*8i)/(9*b*g^4*(a*d - b*c)^3)
```

3.136 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(ag+bgx)^5}\,dx$

Optimal. Leaf size=587

$$\frac{8B^2d^3(c+dx)}{(bc-ad)^4g^5(a+bx)}-\frac{3bB^2d^2(c+dx)^2}{(bc-ad)^4g^5(a+bx)^2}+\frac{8b^2B^2d(c+dx)^3}{9(bc-ad)^4g^5(a+bx)^3}-\frac{b^3B^2(c+dx)^4}{8(bc-ad)^4g^5(a+bx)^4}+\frac{4Bd^3(c+\cdots}{(\cdots}$$

```
[Out] 8*B^2*d^3*(d*x+c)/(-a*d+b*c)^4/g^5/(b*x+a)-3*b*B^2*d^2*(d*x+c)^2/(-a*d+b*c)
^4/g^5/(b*x+a)^2+8/9*b^2*B^2*d*(d*x+c)^3/(-a*d+b*c)^4/g^5/(b*x+a)^3-1/8*b^3
*B^2*(d*x+c)^4/(-a*d+b*c)^4/g^5/(b*x+a)^4+4*B*d^3*(d*x+c)*(A+B*ln(e*(b*x+a)
^2/(d*x+c)^2))/(-a*d+b*c)^4/g^5/(b*x+a)-3*b*B*d^2*(d*x+c)^2*(A+B*ln(e*(b*x+
a)^2/(d*x+c)^2))/(-a*d+b*c)^4/g^5/(b*x+a)^2+4/3*b^2*B*d*(d*x+c)^3*(A+B*ln(e
*(b*x+a)^2/(d*x+c)^2))/(-a*d+b*c)^4/g^5/(b*x+a)^3-1/4*b^3*B*(d*x+c)^4*(A+B*
ln(e*(b*x+a)^2/(d*x+c)^2))/(-a*d+b*c)^4/g^5/(b*x+a)^4+d^3*(d*x+c)*(A+B*ln(e
*(b*x+a)^2/(d*x+c)^2))^2/(-a*d+b*c)^4/g^5/(b*x+a)-3/2*b*d^2*(d*x+c)^2*(A+B*
ln(e*(b*x+a)^2/(d*x+c)^2))^2/(-a*d+b*c)^4/g^5/(b*x+a)^2+b^2*d*(d*x+c)^3*(A+
B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/(-a*d+b*c)^4/g^5/(b*x+a)^3-1/4*b^3*(d*x+c)^4
*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/(-a*d+b*c)^4/g^5/(b*x+a)^4
```

Rubi [A]

time = 0.27, antiderivative size = 587, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2550, 2395, 2342, 2341}

$$-\frac{b^3(c+dx)^4\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{4g^5(a+bx)^4(bc-ad)^4}-\frac{b^3B(c+dx)^4\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{4g^5(a+bx)^4(bc-ad)^4}+\frac{b^2d(c+dx)^3\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{g^5(a+bx)^3(bc-ad)^4}+\frac{4b^2Bd(c+dx)^3\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{3g^5(a+bx)^3(bc-ad)^4}+\frac{d^3(c+dx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{g^5(a+bx)(bc-ad)^4}+\frac{4Bd^3(c+dx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{g^5(a+bx)(bc-ad)^4}-\frac{3bd^2(c+dx)^2\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{2g^5(a+bx)^2(bc-ad)^4}-\frac{3bBd^2(c+dx)^2\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{g^5(a+bx)^2(bc-ad)^4}-\frac{b^3B^2(c+dx)^4}{8g^5(a+bx)^4(bc-ad)^4}+\frac{8b^2B^2d(c+dx)^3}{9g^5(a+bx)^3(bc-ad)^4}+\frac{8B^2d^3(c+dx)}{g^5(a+bx)(bc-ad)^4}-\frac{3bB^2d^2(c+dx)^2}{g^5(a+bx)^2(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2/(a*g + b*g*x)^5,x]
```

```
[Out] (8*B^2*d^3*(c + d*x))/((b*c - a*d)^4*g^5*(a + b*x)) - (3*b*B^2*d^2*(c + d*x
)^2)/((b*c - a*d)^4*g^5*(a + b*x)^2) + (8*b^2*B^2*d*(c + d*x)^3)/(9*(b*c -
a*d)^4*g^5*(a + b*x)^3) - (b^3*B^2*(c + d*x)^4)/(8*(b*c - a*d)^4*g^5*(a + b
*x)^4) + (4*B*d^3*(c + d*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]))/((b*c
 - a*d)^4*g^5*(a + b*x)) - (3*b*B*d^2*(c + d*x)^2*(A + B*Log[(e*(a + b*x)^2
)/(c + d*x)^2]))/((b*c - a*d)^4*g^5*(a + b*x)^2) + (4*b^2*B*d*(c + d*x)^3*(
A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]))/(3*(b*c - a*d)^4*g^5*(a + b*x)^3)
- (b^3*B*(c + d*x)^4*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]))/(4*(b*c - a*
d)^4*g^5*(a + b*x)^4) + (d^3*(c + d*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)
^2])^2)/((b*c - a*d)^4*g^5*(a + b*x)) - (3*b*d^2*(c + d*x)^2*(A + B*Log[(e*
(a + b*x)^2)/(c + d*x)^2])^2)/(2*(b*c - a*d)^4*g^5*(a + b*x)^2) + (b^2*d*(c
 + d*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2)/((b*c - a*d)^4*g^5*(a
 + b*x)^3) - (b^3*(c + d*x)^4*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2)/(
4*(b*c - a*d)^4*g^5*(a + b*x)^4)
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2550

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(
m + 1)*(g/b)^m, Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x],
 x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] &&
 EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && Eq
Q[b*f - a*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Rubi steps

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(ag+bgx)^5}\,dx = -\frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{4bg^5(a+bx)^4} + \frac{B\int \frac{2(bc-ad)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{g^4(a+bx)^5(c+dx)}\,dx}{2bg}$$

$$= -\frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{4bg^5(a+bx)^4} + \frac{(B(bc-ad))\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(a+bx)^5(c+dx)}\,dx}{bg^5}$$

$$= -\frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{4bg^5(a+bx)^4} + \frac{(B(bc-ad))\int \left(\frac{b\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(bc-ad)(a+bx)^5} - \frac{bd\left(A \cdots\right.}{\cdots}\right.}{\cdots}$$

$$= -\frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{4bg^5(a+bx)^4} + \frac{B\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(a+bx)^5}\,dx}{g^5} + \frac{(Bd^4)\int \frac{A+B\log\left(\cdots\right.}{a+\cdots}}{(bc-ad)\cdots}$$

$$= -\frac{B\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{4bg^5(a+bx)^4} + \frac{Bd\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{3b(bc-ad)g^5(a+bx)^3} - \frac{Bd^2\left(A+B\cdots\right.}{2b(bc-ad\cdots}$$

$$= -\frac{B\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{4bg^5(a+bx)^4} + \frac{Bd\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{3b(bc-ad)g^5(a+bx)^3} - \frac{Bd^2\left(A+B\cdots\right.}{2b(bc-ad\cdots}$$

$$= -\frac{B\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{4bg^5(a+bx)^4} + \frac{Bd\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{3b(bc-ad)g^5(a+bx)^3} - \frac{Bd^2\left(A+B\cdots\right.}{2b(bc-ad\cdots}$$

$$= -\frac{B^2}{8bg^5(a+bx)^4} + \frac{7B^2d}{18b(bc-ad)g^5(a+bx)^3} - \frac{13B^2d^2}{12b(bc-ad)^2g^5(a+bx)^2} + \cdots$$

$$= -\frac{B^2}{8bg^5(a+bx)^4} + \frac{7B^2d}{18b(bc-ad)g^5(a+bx)^3} - \frac{13B^2d^2}{12b(bc-ad)^2g^5(a+bx)^2} + \cdots$$

$$= -\frac{B^2}{8bg^5(a+bx)^4} + \frac{7B^2d}{18b(bc-ad)g^5(a+bx)^3} - \frac{13B^2d^2}{12b(bc-ad)^2g^5(a+bx)^2} + \cdots$$

$$= -\frac{B^2}{8bg^5(a+bx)^4} + \frac{7B^2d}{18b(bc-ad)g^5(a+bx)^3} - \frac{13B^2d^2}{12b(bc-ad)^2g^5(a+bx)^2} + \cdots$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 0.59, size = 762, normalized size = 1.30

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2/(a*g + b*g*x)^5,x]
```

```
[Out] -1/72*(18*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2 + (B*(18*(b*c - a*d)^4
*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]) + 24*d*(-(b*c) + a*d)^3*(a + b*x)
*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]) + 36*d^2*(b*c - a*d)^2*(a + b*x)^
2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]) + 72*d^3*(-(b*c) + a*d)*(a + b*x
)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]) - 72*d^4*(a + b*x)^4*Log[a + b
*x]*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]) + 72*d^4*(a + b*x)^4*(A + B*Lo
g[(e*(a + b*x)^2)/(c + d*x)^2])*Log[c + d*x] - 144*B*d^3*(a + b*x)^3*(b*c -
 a*d + d*(a + b*x)*Log[a + b*x] - d*(a + b*x)*Log[c + d*x]) + 36*B*d^2*(a +
 b*x)^2*((b*c - a*d)^2 + 2*d*(-(b*c) + a*d)*(a + b*x) - 2*d^2*(a + b*x)^2*L
og[a + b*x] + 2*d^2*(a + b*x)^2*Log[c + d*x]) - 8*B*d*(a + b*x)*(2*(b*c - a
*d)^3 - 3*d*(b*c - a*d)^2*(a + b*x) + 6*d^2*(b*c - a*d)*(a + b*x)^2 + 6*d^3
*(a + b*x)^3*Log[a + b*x] - 6*d^3*(a + b*x)^3*Log[c + d*x]) + 3*B*(3*(b*c -
 a*d)^4 + 4*d*(-(b*c) + a*d)^3*(a + b*x) + 6*d^2*(b*c - a*d)^2*(a + b*x)^2
+ 12*d^3*(-(b*c) + a*d)*(a + b*x)^3 - 12*d^4*(a + b*x)^4*Log[a + b*x] + 12*
d^4*(a + b*x)^4*Log[c + d*x]) + 72*B*d^4*(a + b*x)^4*(Log[a + b*x]*(Log[a +
 b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b
*c) + a*d)]) - 72*B*d^4*(a + b*x)^4*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] -
 Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(b
*c - a*d)^4)/(b*g^5*(a + b*x)^4)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1485 vs. $2(575) = 1150$.
time = 1.17, size = 1486, normalized size = 2.53

method	result	size
derivativedivides	Expression too large to display	1486
default	Expression too large to display	1486
norman	Expression too large to display	1816
risch	Expression too large to display	2235

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/(b*g*x+a*g)^5,x,method=_RETURNVERBOSE
)
```

```
[Out] -1/d*(d^5/g^5*A^2*(3/2*b/(a*d-b*c)^4/(a/(d*x+c)*d-b*c/(d*x+c)+b)^2-b^2/(a*d
-b*c)^4/(a/(d*x+c)*d-b*c/(d*x+c)+b)^3-1/(a*d-b*c)^4/(a/(d*x+c)*d-b*c/(d*x+c
)+b)+1/4*b^3/(a*d-b*c)^4/(a/(d*x+c)*d-b*c/(d*x+c)+b)^4)+(415/72*B^2/b*d^5/g
/(d*x+c)^4-25/12*b^3*B^2*d^5/g/(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a
*b^3*c^3*d+b^4*c^4)*ln(e*(a/(d*x+c)*d-b*c/(d*x+c)+b)^2/d^2)+25/6*B^2*b^2*d^
5/g/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/(d*x+c)+163/12*B^2*b*d^5/
g/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(d*x+c)^2+271/18*B^2*d^5/g/(a*d-b*c)/(d*x+c)^
```

3-4*B^2*d^5/g/(a*d-b*c)/(d*x+c)^3*ln(e*(a/(d*x+c)*d-b*c/(d*x+c)+b)^2/d^2)-1/4*B^2*b^3*d^5/g/(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)*ln(e*(a/(d*x+c)*d-b*c/(d*x+c)+b)^2/d^2)^2-B^2*d^5/g/(a*d-b*c)/(d*x+c)^3*ln(e*(a/(d*x+c)*d-b*c/(d*x+c)+b)^2/d^2)^2-9*B^2*d^5*b/g/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(d*x+c)^2*ln(e*(a/(d*x+c)*d-b*c/(d*x+c)+b)^2/d^2)-22/3*B^2*d^5*b^2/g/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/(d*x+c)*ln(e*(a/(d*x+c)*d-b*c/(d*x+c)+b)^2/d^2)-3/2*B^2*b*d^5/g/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(d*x+c)^2*ln(e*(a/(d*x+c)*d-b*c/(d*x+c)+b)^2/d^2)^2-B^2*b^2*d^5/g/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/(d*x+c)*ln(e*(a/(d*x+c)*d-b*c/(d*x+c)+b)^2/d^2)^2)/(a/(d*x+c)*d-b*c/(d*x+c)+b)^4/g^4+(A*B*b^2*d^5/g/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/(d*x+c)+25/12*A*B/b*d^5/g/(d*x+c)^4-1/2*b^3*A*B*d^5/g/(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)*ln(e*(a/(d*x+c)*d-b*c/(d*x+c)+b)^2/d^2)+7/2*A*B*b*d^5/g/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(d*x+c)^2+13/3*A*B*d^5/g/(a*d-b*c)/(d*x+c)^3-2*A*B*d^5/g/(a*d-b*c)/(d*x+c)^3*ln(e*(a/(d*x+c)*d-b*c/(d*x+c)+b)^2/d^2)-3*A*B*d^5*b/g/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(d*x+c)^2*ln(e*(a/(d*x+c)*d-b*c/(d*x+c)+b)^2/d^2)-2*A*B*d^5*b^2/g/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/(d*x+c)*ln(e*(a/(d*x+c)*d-b*c/(d*x+c)+b)^2/d^2))/(a/(d*x+c)*d-b*c/(d*x+c)+b)^4/g^4)

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 2288 vs. $2(583) = 1166$.
time = 0.57, size = 2288, normalized size = 3.90

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(b*g*x+a*g)^5,x, algorithm="maxima")
```

```
[Out] 1/72*(6*((12*b^3*d^3*x^3 - 3*b^3*c^3 + 13*a*b^2*c^2*d - 23*a^2*b*c*d^2 + 25*a^3*d^3 - 6*(b^3*c*d^2 - 7*a*b^2*d^3)*x^2 + 4*(b^3*c^2*d - 5*a*b^2*c*d^2 + 13*a^2*b*d^3)*x)/((b^8*c^3 - 3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 - a^3*b^5*d^3)*g^5*x^4 + 4*(a*b^7*c^3 - 3*a^2*b^6*c^2*d + 3*a^3*b^5*c*d^2 - a^4*b^4*d^3)*g^5*x^3 + 6*(a^2*b^6*c^3 - 3*a^3*b^5*c^2*d + 3*a^4*b^4*c*d^2 - a^5*b^3*d^3)*g^5*x^2 + 4*(a^3*b^5*c^3 - 3*a^4*b^4*c^2*d + 3*a^5*b^3*c*d^2 - a^6*b^2*d^3)*g^5*x + (a^4*b^4*c^3 - 3*a^5*b^3*c^2*d + 3*a^6*b^2*c*d^2 - a^7*b*d^3)*g^5) + 12*d^4*log(b*x + a)/((b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5) - 12*d^4*log(d*x + c)/((b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5))*log(b^2*x^2*e/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*x*e/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) - (9*b^4*c^4 - 64*a*b^3*c^3*d + 216*a^2*b^2*c^2*d^2 - 576*a^3*b*c*d^3 + 415*a^4*d^4 - 300*(b^4*c*d^3 - a*b^3*d^4)*x^3 + 6*(13*b^4*c^2*d^2 - 176*a*b^3*c*d^3 + 163*a^2*b^2*d^4)*x^2 + 72*(b^4*d^4*x^4 + 4*a*b^3*d^4*x^3 + 6*a^2*b^2*d^4*x^2 + 4*a^3*b*d^4*x + a^4*d^4)*log(b*x + a)^2 + 72*(b^4*d^4*x^4 + 4*a*b^3*d^4*x^3 + 6*a^2*b^2*d^4*x^2 + 4*a^3*b*d^4*x +
```

```
 a^4*d^4)*log(d*x + c)^2 - 4*(7*b^4*c^3*d - 60*a*b^3*c^2*d^2 + 324*a^2*b^2*
c*d^3 - 271*a^3*b*d^4)*x - 300*(b^4*d^4*x^4 + 4*a*b^3*d^4*x^3 + 6*a^2*b^2*d
^4*x^2 + 4*a^3*b*d^4*x + a^4*d^4)*log(b*x + a) + 12*(25*b^4*d^4*x^4 + 100*a
*b^3*d^4*x^3 + 150*a^2*b^2*d^4*x^2 + 100*a^3*b*d^4*x + 25*a^4*d^4 - 12*(b^4
*d^4*x^4 + 4*a*b^3*d^4*x^3 + 6*a^2*b^2*d^4*x^2 + 4*a^3*b*d^4*x + a^4*d^4)*l
og(b*x + a))*log(d*x + c))/(a^4*b^5*c^4*g^5 - 4*a^5*b^4*c^3*d*g^5 + 6*a^6*b
^3*c^2*d^2*g^5 - 4*a^7*b^2*c*d^3*g^5 + a^8*b*d^4*g^5 + (b^9*c^4*g^5 - 4*a*b
^8*c^3*d*g^5 + 6*a^2*b^7*c^2*d^2*g^5 - 4*a^3*b^6*c*d^3*g^5 + a^4*b^5*d^4*g^
5)*x^4 + 4*(a*b^8*c^4*g^5 - 4*a^2*b^7*c^3*d*g^5 + 6*a^3*b^6*c^2*d^2*g^5 - 4
*a^4*b^5*c*d^3*g^5 + a^5*b^4*d^4*g^5)*x^3 + 6*(a^2*b^7*c^4*g^5 - 4*a^3*b^6*
c^3*d*g^5 + 6*a^4*b^5*c^2*d^2*g^5 - 4*a^5*b^4*c*d^3*g^5 + a^6*b^3*d^4*g^5)*
x^2 + 4*(a^3*b^6*c^4*g^5 - 4*a^4*b^5*c^3*d*g^5 + 6*a^5*b^4*c^2*d^2*g^5 - 4*
a^6*b^3*c*d^3*g^5 + a^7*b^2*d^4*g^5)*x))*B^2 + 1/12*A*B*((12*b^3*d^3*x^3 -
3*b^3*c^3 + 13*a*b^2*c^2*d - 23*a^2*b*c*d^2 + 25*a^3*d^3 - 6*(b^3*c*d^2 - 7
*a*b^2*d^3)*x^2 + 4*(b^3*c^2*d - 5*a*b^2*c*d^2 + 13*a^2*b*d^3)*x)/((b^8*c^3
 - 3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 - a^3*b^5*d^3)*g^5*x^4 + 4*(a*b^7*c^3 -
3*a^2*b^6*c^2*d + 3*a^3*b^5*c*d^2 - a^4*b^4*d^3)*g^5*x^3 + 6*(a^2*b^6*c^3 -
 3*a^3*b^5*c^2*d + 3*a^4*b^4*c*d^2 - a^5*b^3*d^3)*g^5*x^2 + 4*(a^3*b^5*c^3
- 3*a^4*b^4*c^2*d + 3*a^5*b^3*c*d^2 - a^6*b^2*d^3)*g^5*x + (a^4*b^4*c^3 - 3
*a^5*b^3*c^2*d + 3*a^6*b^2*c*d^2 - a^7*b*d^3)*g^5) - 6*log(b^2*x^2*e/(d^2*x
^2 + 2*c*d*x + c^2) + 2*a*b*x*e/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2
+ 2*c*d*x + c^2))/(b^5*g^5*x^4 + 4*a*b^4*g^5*x^3 + 6*a^2*b^3*g^5*x^2 + 4*a^
3*b^2*g^5*x + a^4*b*g^5) + 12*d^4*log(b*x + a)/((b^5*c^4 - 4*a*b^4*c^3*d +
6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5) - 12*d^4*log(d*x + c)
/((b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^
4)*g^5)) - 1/4*B^2*log(b^2*x^2*e/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*x*e/(d^2
*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2))^2/(b^5*g^5*x^4 + 4
*a*b^4*g^5*x^3 + 6*a^2*b^3*g^5*x^2 + 4*a^3*b^2*g^5*x + a^4*b*g^5) - 1/4*A^2
/(b^5*g^5*x^4 + 4*a*b^4*g^5*x^3 + 6*a^2*b^3*g^5*x^2 + 4*a^3*b^2*g^5*x + a^4
*b*g^5)
```

Fricas [A]
time = 0.41, size = 1080, normalized size = 1.84

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(b*g*x+a*g)^5,x, algorithm="fr
icas")
```

```
[Out] -1/72*(9*(2*A^2 + 2*A*B + B^2)*b^4*c^4 - 8*(9*A^2 + 12*A*B + 8*B^2)*a*b^3*c
^3*d + 108*(A^2 + 2*A*B + 2*B^2)*a^2*b^2*c^2*d^2 - 72*(A^2 + 4*A*B + 8*B^2)
*a^3*b*c*d^3 + (18*A^2 + 150*A*B + 415*B^2)*a^4*d^4 - 12*((6*A*B + 25*B^2)*
b^4*c*d^3 - (6*A*B + 25*B^2)*a*b^3*d^4)*x^3 + 6*((6*A*B + 13*B^2)*b^4*c^2*d
^2 - 16*(3*A*B + 11*B^2)*a*b^3*c*d^3 + (42*A*B + 163*B^2)*a^2*b^2*d^4)*x^2
```

```
- 18*(B^2*b^4*d^4*x^4 + 4*B^2*a*b^3*d^4*x^3 + 6*B^2*a^2*b^2*d^4*x^2 + 4*B^2
*a^3*b*d^4*x - B^2*b^4*c^4 + 4*B^2*a*b^3*c^3*d - 6*B^2*a^2*b^2*c^2*d^2 + 4*
B^2*a^3*b*c*d^3)*log((b^2*x^2 + 2*a*b*x + a^2)*e/(d^2*x^2 + 2*c*d*x + c^2))
^2 - 4*((6*A*B + 7*B^2)*b^4*c^3*d - 12*(3*A*B + 5*B^2)*a*b^3*c^2*d^2 + 108*
(A*B + 3*B^2)*a^2*b^2*c*d^3 - (78*A*B + 271*B^2)*a^3*b*d^4)*x - 6*((6*A*B +
 25*B^2)*b^4*d^4*x^4 - 3*(2*A*B + B^2)*b^4*c^4 + 8*(3*A*B + 2*B^2)*a*b^3*c^
3*d - 36*(A*B + B^2)*a^2*b^2*c^2*d^2 + 24*(A*B + 2*B^2)*a^3*b*c*d^3 + 4*(3*
B^2*b^4*c*d^3 + 2*(3*A*B + 11*B^2)*a*b^3*d^4)*x^3 - 6*(B^2*b^4*c^2*d^2 - 8*
B^2*a*b^3*c*d^3 - 6*(A*B + 3*B^2)*a^2*b^2*d^4)*x^2 + 4*(B^2*b^4*c^3*d - 6*B
^2*a*b^3*c^2*d^2 + 18*B^2*a^2*b^2*c*d^3 + 6*(A*B + 2*B^2)*a^3*b*d^4)*x)*log
((b^2*x^2 + 2*a*b*x + a^2)*e/(d^2*x^2 + 2*c*d*x + c^2)))/((b^9*c^4 - 4*a*b^
8*c^3*d + 6*a^2*b^7*c^2*d^2 - 4*a^3*b^6*c*d^3 + a^4*b^5*d^4)*g^5*x^4 + 4*(a
*b^8*c^4 - 4*a^2*b^7*c^3*d + 6*a^3*b^6*c^2*d^2 - 4*a^4*b^5*c*d^3 + a^5*b^4*
d^4)*g^5*x^3 + 6*(a^2*b^7*c^4 - 4*a^3*b^6*c^3*d + 6*a^4*b^5*c^2*d^2 - 4*a^5
*b^4*c*d^3 + a^6*b^3*d^4)*g^5*x^2 + 4*(a^3*b^6*c^4 - 4*a^4*b^5*c^3*d + 6*a^
5*b^4*c^2*d^2 - 4*a^6*b^3*c*d^3 + a^7*b^2*d^4)*g^5*x + (a^4*b^5*c^4 - 4*a^5
*b^4*c^3*d + 6*a^6*b^3*c^2*d^2 - 4*a^7*b^2*c*d^3 + a^8*b*d^4)*g^5)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2/(b*g*x+a*g)**5,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 4.72, size = 874, normalized size = 1.49

$$\frac{1}{4}\left(\frac{B^2d^4}{b^5c^4g^5-4\,ab^4c^3dg^5+6\,a^2b^3c^2d^2g^5-4\,a^3b^2cd^3g^5+a^4bd^4g^5}-\frac{B^2}{(bgx+ag)^4bg}\right)\log\left(\frac{b^2}{\frac{b^2c^2g^2}{(bgx+ag)^2}-\frac{2\,abcdg^2}{(bgx+ag)^2}+\frac{a^2d^2g^2}{(bgx+ag)^2}+\frac{2\,bcdg}{bgx+ag}-\frac{2\,ad^2g}{bgx+ag}+d^2}\right)^2+\frac{1}{12}\left(\frac{12\,B^2d^3}{(b^3c^3g^3-3\,ab^2c^2dg^3+3\,a^2bcd^2g^3-a^3d^3g^3)(bgx+ag)bg}-\frac{6\,B^2d^2}{(b^2c^2g-2\,abcdg+a^2d^2g)(bgx+ag)^2bg^2}+\frac{4\,B^2d}{(bgx+ag)^3(bc-ad)bg^2}-\frac{3\,(2\,ABb^3g^3+3\,B^2b^3g^3)}{(bgx+ag)^4b^4g^4}\right)\log\left(\frac{b^2}{\frac{b^2c^2g^2}{(bgx+ag)^2}-\frac{2\,abcdg^2}{(bgx+ag)^2}+\frac{a^2d^2g^2}{(bgx+ag)^2}+\frac{2\,bcdg}{bgx+ag}-\frac{2\,ad^2g}{bgx+ag}+d^2}\right)-\frac{(6\,ABd^4+31\,B^2d^4)\log\left(-\frac{bcg}{bgx+ag}+\frac{adg}{bgx+ag}-d\right)}{6\,(b^5c^4g^5-4\,ab^4c^3dg^5+6\,a^2b^3c^2d^2g^5-4\,a^3b^2cd^3g^5+a^4bd^4g^5)}+\frac{6\,ABd^3+31\,B^2d^3}{6\,(b^3c^3g^3-3\,ab^2c^2dg^3+3\,a^2bcd^2g^3-a^3d^3g^3)(bgx+ag)bg}-\frac{6\,ABbd^2+19\,B^2bd^2}{12\,(b^2c^2g-2\,abcdg+a^2d^2g)(bgx+ag)^2b^2g^2}+\frac{6\,ABb^2dg+13\,B^2b^2dg}{18\,(bgx+ag)^3(bc-ad)b^3g^3}-\frac{2\,A^2b^3g^3+6\,ABb^3g^3+5\,B^2b^3g^3}{8\,(bgx+ag)^4b^4g^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(b*g*x+a*g)^5,x, algorithm="gi
ac")
```

```
[Out] 1/4*(B^2*d^4/(b^5*c^4*g^5 - 4*a*b^4*c^3*d*g^5 + 6*a^2*b^3*c^2*d^2*g^5 - 4*a
^3*b^2*c*d^3*g^5 + a^4*b*d^4*g^5) - B^2/((b*g*x + a*g)^4*b*g))*log(b^2/(b^2
*c^2*g^2/(b*g*x + a*g)^2 - 2*a*b*c*d*g^2/(b*g*x + a*g)^2 + a^2*d^2*g^2/(b*g
*x + a*g)^2 + 2*b*c*d*g/(b*g*x + a*g) - 2*a*d^2*g/(b*g*x + a*g) + d^2))^2 +
 1/12*(12*B^2*d^3/((b^3*c^3*g^3 - 3*a*b^2*c^2*d*g^3 + 3*a^2*b*c*d^2*g^3 - a
^3*d^3*g^3)*(b*g*x + a*g)*b*g) - 6*B^2*d^2/((b^2*c^2*g - 2*a*b*c*d*g + a^2*
d^2*g)*(b*g*x + a*g)^2*b*g^2) + 4*B^2*d/((b*g*x + a*g)^3*(b*c - a*d)*b*g^2)
 - 3*(2*A*B*b^3*g^3 + 3*B^2*b^3*g^3)/((b*g*x + a*g)^4*b^4*g^4))*log(b^2/(b^
2*c^2*g^2/(b*g*x + a*g)^2 - 2*a*b*c*d*g^2/(b*g*x + a*g)^2 + a^2*d^2*g^2/(b*
```

```
g*x + a*g)^2 + 2*b*c*d*g/(b*g*x + a*g) - 2*a*d^2*g/(b*g*x + a*g) + d^2)) -
1/6*(6*A*B*d^4 + 31*B^2*d^4)*log(-b*c*g/(b*g*x + a*g) + a*d*g/(b*g*x + a*g)
 - d)/(b^5*c^4*g^5 - 4*a*b^4*c^3*d*g^5 + 6*a^2*b^3*c^2*d^2*g^5 - 4*a^3*b^2*
c*d^3*g^5 + a^4*b*d^4*g^5) + 1/6*(6*A*B*d^3 + 31*B^2*d^3)/((b^3*c^3*g^3 - 3
*a*b^2*c^2*d*g^3 + 3*a^2*b*c*d^2*g^3 - a^3*d^3*g^3)*(b*g*x + a*g)*b*g) - 1/
12*(6*A*B*b*d^2 + 19*B^2*b*d^2)/((b^2*c^2*g - 2*a*b*c*d*g + a^2*d^2*g)*(b*g
*x + a*g)^2*b^2*g^2) + 1/18*(6*A*B*b^2*d*g + 13*B^2*b^2*d*g)/((b*g*x + a*g)
^3*(b*c - a*d)*b^3*g^3) - 1/8*(2*A^2*b^3*g^3 + 6*A*B*b^3*g^3 + 5*B^2*b^3*g^
3)/((b*g*x + a*g)^4*b^4*g^4)
```

Mupad [B]
time = 10.55, size = 1883, normalized size = 3.21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2/(a*g + b*g*x)^5,x)
```

```
[Out] (B*d^4*atan((B*d^4*(6*A + 25*B)*(6*b^5*c^4*g^5 - 6*a^4*b*d^4*g^5 - 12*a*b^4
*c^3*d*g^5 + 12*a^3*b^2*c*d^3*g^5)*1i)/(6*b*g^5*(a*d - b*c)^4*(25*B^2*d^4 +
 6*A*B*d^4)) + (B*d^5*x*(6*A + 25*B)*(b^4*c^3*g^5 - a^3*b*d^3*g^5 - 3*a*b^3
*c^2*d*g^5 + 3*a^2*b^2*c*d^2*g^5)*2i)/(g^5*(a*d - b*c)^4*(25*B^2*d^4 + 6*A*
B*d^4)))*(6*A + 25*B)*1i)/(3*b*g^5*(a*d - b*c)^4) - log((e*(a + b*x)^2)/(c
+ d*x)^2)^2*(B^2/(4*b^2*g^5*(4*a^3*x + a^4/b + b^3*x^4 + 6*a^2*b*x^2 + 4*a*
b^2*x^3)) - (B^2*d^4)/(4*b*g^5*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a
*b^3*c^3*d - 4*a^3*b*c*d^3))) - (log((e*(a + b*x)^2)/(c + d*x)^2)*((A*B)/(2
*b^2*d*g^5) + (B^2*d^4*(a*(a*((4*a^2*d^2 + b^2*c^2 - 5*a*b*c*d)/(6*b*d^3) +
 (a*(a*d - b*c))/(2*b*d^2)) + (6*a^3*d^3 - b^3*c^3 + 5*a*b^2*c^2*d - 10*a^2
*b*c*d^2)/(6*b*d^4)) + (4*a^4*d^4 + b^4*c^4 + 10*a^2*b^2*c^2*d^2 - 5*a*b^3*
c^3*d - 10*a^3*b*c*d^3)/(2*b*d^5)))/(2*b*g^5*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2
*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)) + (B^2*d^4*x^2*(b*(b*((4*a^2*d^2
 + b^2*c^2 - 5*a*b*c*d)/(6*b*d^3) + (a*(a*d - b*c))/(2*b*d^2)) + (4*a^2*d^2
 + b^2*c^2 - 5*a*b*c*d)/(3*d^3) + (a*(a*d - b*c))/d^2) - a*((b^2*c - a*b*d)
/(2*d^2) - (b*(a*d - b*c))/d^2) + (b^3*c^2 + 4*a^2*b*d^2 - 5*a*b^2*c*d)/(2*
d^3)))/(2*b*g^5*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*
a^3*b*c*d^3)) - (B^2*d^4*x^3*(b*((b^2*c - a*b*d)/(2*d^2) - (b*(a*d - b*c))/
d^2) + (b^3*c - a*b^2*d)/(2*d^2)))/(2*b*g^5*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*
c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)) + (B^2*d^4*x*(b*(a*((4*a^2*d^2 +
b^2*c^2 - 5*a*b*c*d)/(6*b*d^3) + (a*(a*d - b*c))/(2*b*d^2)) + (6*a^3*d^3 -
b^3*c^3 + 5*a*b^2*c^2*d - 10*a^2*b*c*d^2)/(6*b*d^4)) + a*(b*((4*a^2*d^2 + b
^2*c^2 - 5*a*b*c*d)/(6*b*d^3) + (a*(a*d - b*c))/(2*b*d^2)) + (4*a^2*d^2 + b
^2*c^2 - 5*a*b*c*d)/(3*d^3) + (a*(a*d - b*c))/d^2) + (6*a^3*d^3 - b^3*c^3 +
 5*a*b^2*c^2*d - 10*a^2*b*c*d^2)/(2*d^4)))/(2*b*g^5*(a^4*d^4 + b^4*c^4 + 6*
a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))))/((4*a^3*x)/d + a^4/(b*d
) + (b^3*x^4)/d + (6*a^2*b*x^2)/d + (4*a*b^2*x^3)/d) - ((18*A^2*a^3*d^3 - 1
```

8*A^2*b^3*c^3 + 415*B^2*a^3*d^3 - 9*B^2*b^3*c^3 + 150*A*B*a^3*d^3 - 18*A*B*
b^3*c^3 + 54*A^2*a*b^2*c^2*d - 54*A^2*a^2*b*c*d^2 + 55*B^2*a*b^2*c^2*d - 16
1*B^2*a^2*b*c*d^2 + 78*A*B*a*b^2*c^2*d - 138*A*B*a^2*b*c*d^2)/(12*(a*d - b*
c)) + (x^2*(163*B^2*a*b^2*d^3 - 13*B^2*b^3*c*d^2 + 42*A*B*a*b^2*d^3 - 6*A*B
*b^3*c*d^2))/(2*(a*d - b*c)) + (x*(271*B^2*a^2*b*d^3 + 7*B^2*b^3*c^2*d - 53
*B^2*a*b^2*c*d^2 + 78*A*B*a^2*b*d^3 + 6*A*B*b^3*c^2*d - 30*A*B*a*b^2*c*d^2)
)/(3*(a*d - b*c)) + (d*x^3*(25*B^2*b^3*d^2 + 6*A*B*b^3*d^2))/(a*d - b*c))/(
x*(24*a^3*b^4*c^2*g^5 + 24*a^5*b^2*d^2*g^5 - 48*a^4*b^3*c*d*g^5) + x^3*(24*
a*b^6*c^2*g^5 + 24*a^3*b^4*d^2*g^5 - 48*a^2*b^5*c*d*g^5) + x^4*(6*b^7*c^2*g
^5 + 6*a^2*b^5*d^2*g^5 - 12*a*b^6*c*d*g^5) + x^2*(36*a^2*b^5*c^2*g^5 + 36*a
^4*b^3*d^2*g^5 - 72*a^3*b^4*c*d*g^5) + 6*a^6*b*d^2*g^5 + 6*a^4*b^3*c^2*g^5
- 12*a^5*b^2*c*d*g^5)

3.137 $\int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx$

Optimal. Leaf size=37

$$\text{Int}\left(\frac{(ag+bgx)^2}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)},x\right)$$

```
[Out] Unintegrable((b*g*x+a*g)^2/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a*g + b*g*x)^2/(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]),x]
```

```
[Out] Defer[Int][(a*g + b*g*x)^2/(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]), x]
```

Rubi steps

$$\int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx = \int \left(\frac{a^2g^2}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}+\frac{2abg^2x}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}+\frac{b^2g^2x^2}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\right)\,dx$$
$$= \left(a^2g^2\right)\int \frac{1}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx+\left(2abg^2\right)\int \frac{x}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx+\left(b^2g\right.$$

Mathematica [A]
time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a*g + b*g*x)^2/(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]),x]
```

```
[Out] Integrate[(a*g + b*g*x)^2/(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]), x]
```

Maple [A]
time = 1.04, size = 0, normalized size = 0.00

$$\int \frac{(bgx + ag)^2}{A + B \ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*g*x+a*g)^2/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)
```

```
[Out] int((b*g*x+a*g)^2/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^2/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="maxi
ma")
```

```
[Out] integrate((b*g*x + a*g)^2/(B*log((b*x + a)^2*e/(d*x + c)^2) + A), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^2/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="fric
as")
```

```
[Out] integral((b^2*g^2*x^2 + 2*a*b*g^2*x + a^2*g^2)/(B*log((b^2*x^2 + 2*a*b*x +
a^2)*e/(d^2*x^2 + 2*c*d*x + c^2)) + A), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$g^2\left(\int \frac{a^2}{A + B\log\left(\frac{a^2e}{c^2+2cdx+d^2x^2} + \frac{2abex}{c^2+2cdx+d^2x^2} + \frac{b^2ex^2}{c^2+2cdx+d^2x^2}\right)}\, dx + \int \frac{b^2x^2}{A + B\log\left(\frac{a^2e}{c^2+2cdx+d^2x^2} + \frac{2abex}{c^2+2cdx+d^2x^2} + \frac{b^2ex^2}{c^2+2cdx+d^2x^2}\right)}\, dx + \int \frac{2abx}{A + B\log\left(\frac{a^2e}{c^2+2cdx+d^2x^2} + \frac{2abex}{c^2+2cdx+d^2x^2} + \frac{b^2ex^2}{c^2+2cdx+d^2x^2}\right)}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)**2/(A+B*ln(e*(b*x+a)**2/(d*x+c)**2)),x)
```

```
[Out] g**2*(Integral(a**2/(A + B*log(a**2*e/(c**2 + 2*c*d*x + d**2*x**2) + 2*a*b*
e*x/(c**2 + 2*c*d*x + d**2*x**2) + b**2*e*x**2/(c**2 + 2*c*d*x + d**2*x**2)
```

```
)), x) + Integral(b**2*x**2/(A + B*log(a**2*e/(c**2 + 2*c*d*x + d**2*x**2)
+ 2*a*b*e*x/(c**2 + 2*c*d*x + d**2*x**2) + b**2*e*x**2/(c**2 + 2*c*d*x + d*
*2*x**2))), x) + Integral(2*a*b*x/(A + B*log(a**2*e/(c**2 + 2*c*d*x + d**2*
x**2) + 2*a*b*e*x/(c**2 + 2*c*d*x + d**2*x**2) + b**2*e*x**2/(c**2 + 2*c*d*
x + d**2*x**2))), x))
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^2/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="giac
")
```

```
[Out] integrate((b*g*x + a*g)^2/(B*log((b*x + a)^2*e/(d*x + c)^2) + A), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{{\left(a\,g+b\,g\,x\right)}^2}{A+B\,\ln\left(\frac{e\,{\left(a+b\,x\right)}^2}{{\left(c+d\,x\right)}^2}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*g + b*g*x)^2/(A + B*log((e*(a + b*x)^2)/(c + d*x)^2)),x)
```

```
[Out] int((a*g + b*g*x)^2/(A + B*log((e*(a + b*x)^2)/(c + d*x)^2)), x)
```

3.138 $\int \frac{ag+bgx}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx$

Optimal. Leaf size=35

$$\text{Int}\left(\frac{ag+bgx}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)},x\right)$$

```
[Out] Unintegrable((b*g*x+a*g)/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{ag+bgx}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a*g + b*g*x)/(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]),x]
[Out] Defer[Int][(a*g + b*g*x)/(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]), x]
```

Rubi steps

$$\begin{aligned}\int \frac{ag+bgx}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx &= \int \left(\frac{ag}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}+\frac{bgx}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\right)dx\\ &= (ag)\int \frac{1}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx+(bg)\int \frac{x}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx\end{aligned}$$

Mathematica [A]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{ag+bgx}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a*g + b*g*x)/(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]),x]
[Out] Integrate[(a*g + b*g*x)/(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]), x]
```

Maple [A]
time = 1.05, size = 0, normalized size = 0.00

$$\int \frac{bgx + ag}{A + B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*g*x+a*g)/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)
```

```
[Out] int((b*g*x+a*g)/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="maxima
")
```

```
[Out] integrate((b*g*x + a*g)/(B*log((b*x + a)^2*e/(d*x + c)^2) + A), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="fricas
")
```

```
[Out] integral((b*g*x + a*g)/(B*log((b^2*x^2 + 2*a*b*x + a^2)*e/(d^2*x^2 + 2*c*d*
x + c^2)) + A), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$g\left(\int \frac{a}{A + B\log\left(\frac{a^2e}{c^2+2cdx+d^2x^2} + \frac{2abex}{c^2+2cdx+d^2x^2} + \frac{b^2ex^2}{c^2+2cdx+d^2x^2}\right)}dx + \int \frac{bx}{A + B\log\left(\frac{a^2e}{c^2+2cdx+d^2x^2} + \frac{2abex}{c^2+2cdx+d^2x^2} + \frac{b^2ex^2}{c^2+2cdx+d^2x^2}\right)}dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)/(A+B*ln(e*(b*x+a)**2/(d*x+c)**2)),x)
```

```
[Out] g*(Integral(a/(A + B*log(a**2*e/(c**2 + 2*c*d*x + d**2*x**2) + 2*a*b*e*x/(c
**2 + 2*c*d*x + d**2*x**2) + b**2*e*x**2/(c**2 + 2*c*d*x + d**2*x**2))), x)
 + Integral(b*x/(A + B*log(a**2*e/(c**2 + 2*c*d*x + d**2*x**2) + 2*a*b*e*x/
```

```
(c**2 + 2*c*d*x + d**2*x**2) + b**2*e*x**2/(c**2 + 2*c*d*x + d**2*x**2))),
x))
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="giac")
[Out] integrate((b*g*x + a*g)/(B*log((b*x + a)^2*e/(d*x + c)^2) + A), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{a\,g+b\,g\,x}{A+B\,\ln\left(\frac{e\,{\left(a+b\,x\right)}^2}{{\left(c+d\,x\right)}^2}\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*g + b*g*x)/(A + B*log((e*(a + b*x)^2)/(c + d*x)^2)),x)
[Out] int((a*g + b*g*x)/(A + B*log((e*(a + b*x)^2)/(c + d*x)^2)), x)
```

3.139 $\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx$

Optimal. Leaf size=37

$$\mathrm{Int}\left(\frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)},x\right)$$

```
[Out] Unintegrable(1/(b*g*x+a*g)/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((a*g + b*g*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])),x]
[Out] Defer[Int][1/((a*g + b*g*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])), x]
```

Rubi steps

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx = \int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx$$

Mathematica [A]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((a*g + b*g*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])),x]
[Out] Integrate[1/((a*g + b*g*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])), x]
```

Maple [A]
time = 1.03, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bgx+ag\right)\left(A+B\ln\left(\frac{e\left(bx+a\right)^{2}}{\left(dx+c\right)^{2}}\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*g*x+a*g)/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)
```

```
[Out] int(1/(b*g*x+a*g)/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="maxi
ma")
```

```
[Out] integrate(1/((b*g*x + a*g)*(B*log((b*x + a)^2*e/(d*x + c)^2) + A)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="fric
as")
```

```
[Out] integral(1/(A*b*g*x + A*a*g + (B*b*g*x + B*a*g)*log((b^2*x^2 + 2*a*b*x + a^
2)*e/(d^2*x^2 + 2*c*d*x + c^2))), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{Aa+Abx+Ba\log\left(\frac{a^{2}e}{c^{2}+2cdx+d^{2}x^{2}}+\frac{2abex}{c^{2}+2cdx+d^{2}x^{2}}+\frac{b^{2}ex^{2}}{c^{2}+2cdx+d^{2}x^{2}}\right)+Bbx\log\left(\frac{a^{2}e}{c^{2}+2cdx+d^{2}x^{2}}+\frac{2abex}{c^{2}+2cdx+d^{2}x^{2}}+\frac{b^{2}ex^{2}}{c^{2}+2cdx+d^{2}x^{2}}\right)}\,dx}{g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)/(A+B*ln(e*(b*x+a)**2/(d*x+c)**2)),x)
```

```
[Out] Integral(1/(A*a + A*b*x + B*a*log(a**2*e/(c**2 + 2*c*d*x + d**2*x**2) + 2*a
*b*e*x/(c**2 + 2*c*d*x + d**2*x**2) + b**2*e*x**2/(c**2 + 2*c*d*x + d**2*x*
```

```
*2)) + B*b*x*log(a**2*e/(c**2 + 2*c*d*x + d**2*x**2) + 2*a*b*e*x/(c**2 + 2*
c*d*x + d**2*x**2) + b**2*e*x**2/(c**2 + 2*c*d*x + d**2*x**2))), x)/g
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="giac
")
[Out] integrate(1/((b*g*x + a*g)*(B*log((b*x + a)^2*e/(d*x + c)^2) + A)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{(a\,g+b\,g\,x)\,\left(A+B\,\ln\left(\frac{e\,{(a+b\,x)}^2}{{(c+d\,x)}^2}\right)\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*g + b*g*x)*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))),x)
[Out] int(1/((a*g + b*g*x)*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))), x)
```

3.140 $\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx$

Optimal. Leaf size=94

$$\frac{e^{\frac{A}{2B}}\sqrt{\frac{e(a+bx)^2}{(c+dx)^2}}\,(c+dx)\text{Ei}\left(\frac{-A-B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{2B}\right)}{2B(bc-ad)g^2(a+bx)}$$

```
[Out] 1/2*exp(1/2*A/B)*(d*x+c)*Ei(1/2*(-A-B*ln(e*(b*x+a)^2/(d*x+c)^2))/B)*(e*(b*x
+a)^2/(d*x+c)^2)^(1/2)/B/(-a*d+b*c)/g^2/(b*x+a)
```

Rubi [A]
time = 0.08, antiderivative size = 91, normalized size of antiderivative = 0.97, number of steps used = 3, number of rules used = 3, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.088$, Rules used = {2550, 2347, 2209}

$$\frac{e^{\frac{A}{2B}}(c+dx)\sqrt{\frac{e(a+bx)^2}{(c+dx)^2}}\,\text{Ei}\left(-\frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{2B}\right)}{2Bg^2(a+bx)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])),x]
```

```
[Out] (E^(A/(2*B))*Sqrt[(e*(a + b*x)^2)/(c + d*x)^2]*(c + d*x)*ExpIntegralEi[-1/2
*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/B])/(2*B*(b*c - a*d)*g^2*(a + b*x
))
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2550

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(
m + 1)*(g/b)^m, Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x],
```

```
 x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] &&
 EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && Eq
Q[b*f - a*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Rubi steps

$$\int \frac{1}{(ag+bgx)^2 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx = \int \frac{1}{(ag+bgx)^2 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx$$

Mathematica [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{1}{(ag+bgx)^2 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])),x]
[Out] Integrate[1/((a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])), x]
```

Maple [F]
time = 0.63, size = 0, normalized size = 0.00

$$\int \frac{1}{(bgx+ag)^2 \left(A+B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*g*x+a*g)^2/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)
[Out] int(1/(b*g*x+a*g)^2/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="ma
xima")
[Out] integrate(1/((b*g*x + a*g)^2*(B*log((b*x + a)^2*e/(d*x + c)^2) + A)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="fr
icas")
```

```
[Out] integral(1/(A*b^2*g^2*x^2 + 2*A*a*b*g^2*x + A*a^2*g^2 + (B*b^2*g^2*x^2 + 2*
B*a*b*g^2*x + B*a^2*g^2)*log((b^2*x^2 + 2*a*b*x + a^2)*e/(d^2*x^2 + 2*c*d*x
 + c^2))), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{Aa^2+2Aabx+Ab^2x^2+Ba^2\log\left(\frac{a^2e}{c^2+2cdx+d^2x^2}+\frac{2abex}{c^2+2cdx+d^2x^2}+\frac{b^2ex^2}{c^2+2cdx+d^2x^2}\right)+2Babx\log\left(\frac{a^2e}{c^2+2cdx+d^2x^2}+\frac{2abex}{c^2+2cdx+d^2x^2}+\frac{b^2ex^2}{c^2+2cdx+d^2x^2}\right)+Bb^2x^2\log\left(\frac{a^2e}{c^2+2cdx+d^2x^2}+\frac{2abex}{c^2+2cdx+d^2x^2}+\frac{b^2ex^2}{c^2+2cdx+d^2x^2}\right)}\,dx}{g^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)**2/(A+B*ln(e*(b*x+a)**2/(d*x+c)**2)),x)
```

```
[Out] Integral(1/(A*a**2 + 2*A*a*b*x + A*b**2*x**2 + B*a**2*log(a**2*e/(c**2 + 2*
c*d*x + d**2*x**2) + 2*a*b*e*x/(c**2 + 2*c*d*x + d**2*x**2) + b**2*e*x**2/(
c**2 + 2*c*d*x + d**2*x**2)) + 2*B*a*b*x*log(a**2*e/(c**2 + 2*c*d*x + d**2*
x**2) + 2*a*b*e*x/(c**2 + 2*c*d*x + d**2*x**2) + b**2*e*x**2/(c**2 + 2*c*d*
x + d**2*x**2)) + B*b**2*x**2*log(a**2*e/(c**2 + 2*c*d*x + d**2*x**2) + 2*a
*b*e*x/(c**2 + 2*c*d*x + d**2*x**2) + b**2*e*x**2/(c**2 + 2*c*d*x + d**2*x*
*2))), x)/g**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="gi
ac")
```

```
[Out] integrate(1/((b*g*x + a*g)^2*(B*log((b*x + a)^2*e/(d*x + c)^2) + A)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a\,g+b\,g\,x\right)}^2\,\left(A+B\,\ln\left(\frac{e\,{\left(a+b\,x\right)}^2}{{\left(c+d\,x\right)}^2}\right)\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*g + b*g*x)^2*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))),x)
```

```
[Out] int(1/((a*g + b*g*x)^2*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))), x)
```

3.141 $\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx$

Optimal. Leaf size=152

$$-\frac{de^{\frac{A}{2B}}\sqrt{\frac{e(a+bx)^2}{(c+dx)^2}}\,(c+dx)\mathrm{Ei}\left(\frac{-A-B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{2B}\right)}{2B(bc-ad)^2g^3(a+bx)}+\frac{bee^{A/B}\mathrm{Ei}\left(-\frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{B}\right)}{2B(bc-ad)^2g^3}$$

```
[Out] 1/2*b*e*exp(A/B)*Ei((-A-B*ln(e*(b*x+a)^2/(d*x+c)^2))/B)/B/(-a*d+b*c)^2/g^3-
1/2*d*exp(1/2*A/B)*(d*x+c)*Ei(1/2*(-A-B*ln(e*(b*x+a)^2/(d*x+c)^2))/B)*(e*(b
*x+a)^2/(d*x+c)^2)^(1/2)/B/(-a*d+b*c)^2/g^3/(b*x+a)
```

Rubi [A]

time = 0.17, antiderivative size = 149, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2550, 2395, 2347, 2209}

$$\frac{bee^{A/B}\mathrm{Ei}\left(-\frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{B}\right)}{2Bg^3(bc-ad)^2}-\frac{de^{\frac{A}{2B}}(c+dx)\sqrt{\frac{e(a+bx)^2}{(c+dx)^2}}\,\mathrm{Ei}\left(-\frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{2B}\right)}{2Bg^3(a+bx)(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a*g + b*g*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])),x]
```

```
[Out] (b*e*E^(A/B)*ExpIntegralEi[-((A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/B)])/
(2*B*(b*c - a*d)^2*g^3) - (d*E^(A/(2*B))*Sqrt[(e*(a + b*x)^2)/(c + d*x)^2]*
(c + d*x)*ExpIntegralEi[-1/2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/B])/(
2*B*(b*c - a*d)^2*g^3*(a + b*x))
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2550

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(
m + 1)*(g/b)^m, Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x],
 x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] &&
 EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && Eq
Q[b*f - a*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Rubi steps

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx=\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx$$

Mathematica [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((a*g + b*g*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])),x]
[Out] Integrate[1/((a*g + b*g*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])), x]
```

Maple [F]
time = 1.18, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bgx+ag\right)^{3}\left(A+B\ln\left(\frac{e\left(bx+a\right)^{2}}{\left(dx+c\right)^{2}}\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*g*x+a*g)^3/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)
[Out] int(1/(b*g*x+a*g)^3/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)^3/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="ma
xima")
```

```
[Out] integrate(1/((b*g*x + a*g)^3*(B*log((b*x + a)^2*e/(d*x + c)^2) + A)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)^3/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="fr
icas")
```

```
[Out] integral(1/(A*b^3*g^3*x^3 + 3*A*a*b^2*g^3*x^2 + 3*A*a^2*b*g^3*x + A*a^3*g^3
 + (B*b^3*g^3*x^3 + 3*B*a*b^2*g^3*x^2 + 3*B*a^2*b*g^3*x + B*a^3*g^3)*log((b
^2*x^2 + 2*a*b*x + a^2)*e/(d^2*x^2 + 2*c*d*x + c^2))), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{Aa^3+3Aa^2bx+3Aab^2x^2+Ab^3x^3+Ba^3\log\left(\frac{a^2e}{c^2+2cdx+d^2x^2}+\frac{2abex}{c^2+2cdx+d^2x^2}+\frac{b^2ex^2}{c^2+2cdx+d^2x^2}\right)+3Ba^2bx\log\left(\frac{a^2e}{c^2+2cdx+d^2x^2}+\frac{2abex}{c^2+2cdx+d^2x^2}+\frac{b^2ex^2}{c^2+2cdx+d^2x^2}\right)+3Bab^2x^2\log\left(\frac{a^2e}{c^2+2cdx+d^2x^2}+\frac{2abex}{c^2+2cdx+d^2x^2}+\frac{b^2ex^2}{c^2+2cdx+d^2x^2}\right)+Bb^3x^3\log\left(\frac{a^2e}{c^2+2cdx+d^2x^2}+\frac{2abex}{c^2+2cdx+d^2x^2}+\frac{b^2ex^2}{c^2+2cdx+d^2x^2}\right)}\, dx}{g^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)**3/(A+B*ln(e*(b*x+a)**2/(d*x+c)**2)),x)
```

```
[Out] Integral(1/(A*a**3 + 3*A*a**2*b*x + 3*A*a*b**2*x**2 + A*b**3*x**3 + B*a**3*
log(a**2*e/(c**2 + 2*c*d*x + d**2*x**2) + 2*a*b*e*x/(c**2 + 2*c*d*x + d**2*
x**2) + b**2*e*x**2/(c**2 + 2*c*d*x + d**2*x**2)) + 3*B*a**2*b*x*log(a**2*e
/(c**2 + 2*c*d*x + d**2*x**2) + 2*a*b*e*x/(c**2 + 2*c*d*x + d**2*x**2) + b*
*2*e*x**2/(c**2 + 2*c*d*x + d**2*x**2)) + 3*B*a*b**2*x**2*log(a**2*e/(c**2
+ 2*c*d*x + d**2*x**2) + 2*a*b*e*x/(c**2 + 2*c*d*x + d**2*x**2) + b**2*e*x*
*2/(c**2 + 2*c*d*x + d**2*x**2)) + B*b**3*x**3*log(a**2*e/(c**2 + 2*c*d*x +
 d**2*x**2) + 2*a*b*e*x/(c**2 + 2*c*d*x + d**2*x**2) + b**2*e*x**2/(c**2 +
2*c*d*x + d**2*x**2))), x)/g**3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)^3/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="gi
ac")
```

```
[Out] integrate(1/((b*g*x + a*g)^3*(B*log((b*x + a)^2*e/(d*x + c)^2) + A)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a\,g+b\,g\,x\right)}^3\,\left(A+B\,\ln\left(\frac{e\,{\left(a+b\,x\right)}^2}{{\left(c+d\,x\right)}^2}\right)\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*g + b*g*x)^3*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))),x)
```

```
[Out] int(1/((a*g + b*g*x)^3*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))), x)
```

3.142 $\int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$

Optimal. Leaf size=37

$$\text{Int}\left(\frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2},x\right)$$

```
[Out] Unintegrable((b*g*x+a*g)^2/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a*g + b*g*x)^2/(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2,x]
[Out] Defer[Int][(a*g + b*g*x)^2/(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx &= \int \left(\frac{a^2g^2}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}+\frac{2abg^2x}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}+\frac{b^2g^2x}{\left(A+B\log\left(\frac{e}{}\right.\right.}\right.\\ &= \left(a^2g^2\right)\int \frac{1}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx+\left(2abg^2\right)\int \frac{x}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\end{aligned}$$

Mathematica [A]
time = 0.25, size = 0, normalized size = 0.00

$$\int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a*g + b*g*x)^2/(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2,x]
```

```
[Out] Integrate[(a*g + b*g*x)^2/(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2, x]
```

Maple [A]
time = 0.61, size = 0, normalized size = 0.00

$$\int \frac{(bgx+ag)^{2}}{\left(A+B\ln\left(\frac{e(bx+a)^{2}}{(dx+c)^{2}}\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*g*x+a*g)^2/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

```
[Out] int((b*g*x+a*g)^2/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^2/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="ma
xima")
```

```
[Out] -1/2*(b^3*d*g^2*x^4 + a^3*c*g^2 + (b^3*c*g^2 + 3*a*b^2*d*g^2)*x^3 + 3*(a*b^
2*c*g^2 + a^2*b*d*g^2)*x^2 + (3*a^2*b*c*g^2 + a^3*d*g^2)*x)/(2*(b*c - a*d)*
B^2*log(b*x + a) - 2*(b*c - a*d)*B^2*log(d*x + c) + (b*c - a*d)*A*B + (b*c
- a*d)*B^2) + integrate(1/2*(4*b^3*d*g^2*x^3 + 3*a^2*b*c*g^2 + a^3*d*g^2 +
3*(b^3*c*g^2 + 3*a*b^2*d*g^2)*x^2 + 6*(a*b^2*c*g^2 + a^2*b*d*g^2)*x)/(2*(b*
c - a*d)*B^2*log(b*x + a) - 2*(b*c - a*d)*B^2*log(d*x + c) + (b*c - a*d)*A*
B + (b*c - a*d)*B^2), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^2/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="fr
icas")
```

```
[Out] integral((b^2*g^2*x^2 + 2*a*b*g^2*x + a^2*g^2)/(B^2*log((b^2*x^2 + 2*a*b*x
+ a^2)*e/(d^2*x^2 + 2*c*d*x + c^2))^2 + 2*A*B*log((b^2*x^2 + 2*a*b*x + a^2)
*e/(d^2*x^2 + 2*c*d*x + c^2)) + A^2), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)**2/(A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^2/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="gi
ac")
```

```
[Out] integrate((b*g*x + a*g)^2/(B*log((b*x + a)^2*e/(d*x + c)^2) + A)^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{(a\,g+b\,g\,x)^2}{\left(A+B\,\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*g + b*g*x)^2/(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2,x)
```

```
[Out] int((a*g + b*g*x)^2/(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2, x)
```

3.143 $\int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$

Optimal. Leaf size=35

$$\text{Int}\left(\frac{ag+bgx}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2},x\right)$$

```
[Out] Unintegrable((b*g*x+a*g)/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a*g + b*g*x)/(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2,x]
[Out] Defer[Int][(a*g + b*g*x)/(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2, x]
```

Rubi steps

$$\begin{aligned}\int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx &= \int \left(\frac{ag}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}+\frac{bgx}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\right)dx\\ &= (ag)\int \frac{1}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx+(bg)\int \frac{x}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx\end{aligned}$$

Mathematica [A]
time = 0.20, size = 0, normalized size = 0.00

$$\int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a*g + b*g*x)/(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2,x]
```

[Out] Integrate[(a*g + b*g*x)/(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2, x]

Maple [A]
time = 0.52, size = 0, normalized size = 0.00

$$\int \frac{bgx+ag}{\left(A+B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*g*x+a*g)/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)

[Out] int((b*g*x+a*g)/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*g*x+a*g)/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="maxima")

[Out] -1/2*(b^2*d*g*x^3 + a^2*c*g + (b^2*c*g + 2*a*b*d*g)*x^2 + (2*a*b*c*g + a^2*d*g)*x)/(2*(b*c - a*d)*B^2*log(b*x + a) - 2*(b*c - a*d)*B^2*log(d*x + c) + (b*c - a*d)*A*B + (b*c - a*d)*B^2) + integrate(1/2*(3*b^2*d*g*x^2 + 2*a*b*c*g + a^2*d*g + 2*(b^2*c*g + 2*a*b*d*g)*x)/(2*(b*c - a*d)*B^2*log(b*x + a) - 2*(b*c - a*d)*B^2*log(d*x + c) + (b*c - a*d)*A*B + (b*c - a*d)*B^2), x)

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*g*x+a*g)/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="fricas")

[Out] integral((b*g*x + a*g)/(B^2*log((b^2*x^2 + 2*a*b*x + a^2)*e/(d^2*x^2 + 2*c*d*x + c^2))^2 + 2*A*B*log((b^2*x^2 + 2*a*b*x + a^2)*e/(d^2*x^2 + 2*c*d*x + c^2)) + A^2), x)

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^2cg+a^2dgx+2abcgx+2abdgx^2+b^2cgx^2+b^2dgx^3}{2ABad-2ABbc+(2B^2ad-2B^2bc)\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}-\frac{g\left(\int\frac{a^2d}{A+B\log\left(\frac{a^2e}{c^2+2cdx+d^2x^2}+\frac{2abex}{c^2+2cdx+d^2x^2}+\frac{b^2ex^2}{c^2+2cdx+d^2x^2}\right)}\,dx+\int\frac{2abc}{A+B\log\left(\frac{a^2e}{c^2+2cdx+d^2x^2}+\frac{2abex}{c^2+2cdx+d^2x^2}+\frac{b^2ex^2}{c^2+2cdx+d^2x^2}\right)}\,dx+\int\frac{2b^2cx}{A+B\log\left(\frac{a^2e}{c^2+2cdx+d^2x^2}+\frac{2abex}{c^2+2cdx+d^2x^2}+\frac{b^2ex^2}{c^2+2cdx+d^2x^2}\right)}\,dx+\int\frac{3b^2dx^2}{A+B\log\left(\frac{a^2e}{c^2+2cdx+d^2x^2}+\frac{2abex}{c^2+2cdx+d^2x^2}+\frac{b^2ex^2}{c^2+2cdx+d^2x^2}\right)}\,dx+\int\frac{4abdx}{A+B\log\left(\frac{a^2e}{c^2+2cdx+d^2x^2}+\frac{2abex}{c^2+2cdx+d^2x^2}+\frac{b^2ex^2}{c^2+2cdx+d^2x^2}\right)}\,dx\right)}{2B(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)/(A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2,x)
```

```
[Out] (a**2*c*g + a**2*d*g*x + 2*a*b*c*g*x + 2*a*b*d*g*x**2 + b**2*c*g*x**2 + b**
2*d*g*x**3)/(2*A*B*a*d - 2*A*B*b*c + (2*B**2*a*d - 2*B**2*b*c)*log(e*(a + b
*x)**2/(c + d*x)**2)) - g*(Integral(a**2*d/(A + B*log(a**2*e/(c**2 + 2*c*d*
x + d**2*x**2) + 2*a*b*e*x/(c**2 + 2*c*d*x + d**2*x**2) + b**2*e*x**2/(c**2
 + 2*c*d*x + d**2*x**2))), x) + Integral(2*a*b*c/(A + B*log(a**2*e/(c**2 +
2*c*d*x + d**2*x**2) + 2*a*b*e*x/(c**2 + 2*c*d*x + d**2*x**2) + b**2*e*x**2
/(c**2 + 2*c*d*x + d**2*x**2))), x) + Integral(2*b**2*c*x/(A + B*log(a**2*e
/(c**2 + 2*c*d*x + d**2*x**2) + 2*a*b*e*x/(c**2 + 2*c*d*x + d**2*x**2) + b*
*2*e*x**2/(c**2 + 2*c*d*x + d**2*x**2))), x) + Integral(3*b**2*d*x**2/(A +
B*log(a**2*e/(c**2 + 2*c*d*x + d**2*x**2) + 2*a*b*e*x/(c**2 + 2*c*d*x + d**
2*x**2) + b**2*e*x**2/(c**2 + 2*c*d*x + d**2*x**2))), x) + Integral(4*a*b*d
*x/(A + B*log(a**2*e/(c**2 + 2*c*d*x + d**2*x**2) + 2*a*b*e*x/(c**2 + 2*c*d
*x + d**2*x**2) + b**2*e*x**2/(c**2 + 2*c*d*x + d**2*x**2))), x))/(2*B*(a*d
 - b*c))
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="giac
")
```

```
[Out] integrate((b*g*x + a*g)/(B*log((b*x + a)^2*e/(d*x + c)^2) + A)^2, x)
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{a\,g+b\,g\,x}{{\left(A+B\,\ln\left(\frac{e\,{\left(a+b\,x\right)}^2}{{\left(c+d\,x\right)}^2}\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*g + b*g*x)/(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2,x)
```

```
[Out] int((a*g + b*g*x)/(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2, x)
```

3.144 $\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$

Optimal. Leaf size=37

$$\text{Int}\left(\frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2},x\right)$$

[Out] Unintegrable(1/(b*g*x+a*g)/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$$

Verification is not applicable to the result.

[In] Int[1/((a*g + b*g*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2),x]

[Out] Defer[Int][1/((a*g + b*g*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2), x]

Rubi steps

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx=\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$$

Mathematica [A]
time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$$

Verification is not applicable to the result.

[In] Integrate[1/((a*g + b*g*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2),x]

[Out] Integrate[1/((a*g + b*g*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2), x]

Maple [A]
time = 0.48, size = 0, normalized size = 0.00

$$\int \frac{1}{(bgx+ag)\left(A+B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*g*x+a*g)/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

```
[Out] int(1/(b*g*x+a*g)/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="ma
xima")
```

```
[Out] d*integrate(1/2/(2*(b*c*g - a*d*g)*B^2*log(b*x + a) - 2*(b*c*g - a*d*g)*B^2
*log(d*x + c) + (b*c*g - a*d*g)*A*B + (b*c*g - a*d*g)*B^2), x) - 1/2*(d*x +
 c)/(2*(b*c*g - a*d*g)*B^2*log(b*x + a) - 2*(b*c*g - a*d*g)*B^2*log(d*x + c
) + (b*c*g - a*d*g)*A*B + (b*c*g - a*d*g)*B^2)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="fr
icas")
```

```
[Out] integral(1/(A^2*b*g*x + A^2*a*g + (B^2*b*g*x + B^2*a*g)*log((b^2*x^2 + 2*a*
b*x + a^2)*e/(d^2*x^2 + 2*c*d*x + c^2))^2 + 2*(A*B*b*g*x + A*B*a*g)*log((b^
2*x^2 + 2*a*b*x + a^2)*e/(d^2*x^2 + 2*c*d*x + c^2))), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{c+dx}{2ABadg-2ABbcg+(2B^2adg-2B^2bcg)\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)} - \frac{d\int \frac{1}{A+B\log\left(\frac{a^2e}{c^2+2cdx+d^2x^2}+\frac{2abex}{c^2+2cdx+d^2x^2}+\frac{b^2ex^2}{c^2+2cdx+d^2x^2}\right)}\, dx}{2Bg\,(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)/(A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2,x)
```

```
[Out] (c + d*x)/(2*A*B*a*d*g - 2*A*B*b*c*g + (2*B**2*a*d*g - 2*B**2*b*c*g)*log(e*
(a + b*x)**2/(c + d*x)**2)) - d*Integral(1/(A + B*log(a**2*e/(c**2 + 2*c*d*
x + d**2*x**2) + 2*a*b*e*x/(c**2 + 2*c*d*x + d**2*x**2) + b**2*e*x**2/(c**2
 + 2*c*d*x + d**2*x**2))), x)/(2*B*g*(a*d - b*c))
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="gi
ac")
```

```
[Out] integrate(1/((b*g*x + a*g)*(B*log((b*x + a)^2*e/(d*x + c)^2) + A)^2), x)
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{(a\,g+b\,g\,x)\,\left(A+B\,\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*g + b*g*x)*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2),x)
```

```
[Out] int(1/((a*g + b*g*x)*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2), x)
```

3.145 $\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$

Optimal. Leaf size=150

$$-\frac{e^{\frac{A}{2B}}\sqrt{\frac{e(a+bx)^2}{(c+dx)^2}}\,(c+dx)\operatorname{Ei}\left(\frac{-A-B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{2B}\right)}{4B^2(bc-ad)g^2(a+bx)}-\frac{c+dx}{2B(bc-ad)g^2(a+bx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}$$

```
[Out] 1/2*(-d*x-c)/B/(-a*d+b*c)/g^2/(b*x+a)/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))-1/4*e
xp(1/2*A/B)*(d*x+c)*Ei(1/2*(-A-B*ln(e*(b*x+a)^2/(d*x+c)^2))/B)*(e*(b*x+a)^2
/(d*x+c)^2)^(1/2)/B^2/(-a*d+b*c)/g^2/(b*x+a)
```

Rubi [A]

time = 0.10, antiderivative size = 147, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2550, 2343, 2347, 2209}

$$-\frac{e^{\frac{A}{2B}}(c+dx)\sqrt{\frac{e(a+bx)^2}{(c+dx)^2}}\operatorname{Ei}\left(-\frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{2B}\right)}{4B^2g^2(a+bx)(bc-ad)}-\frac{c+dx}{2Bg^2(a+bx)(bc-ad)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2),x]
```

```
[Out] -1/4*(E^(A/(2*B))*Sqrt[(e*(a + b*x)^2)/(c + d*x)^2]*(c + d*x)*ExpIntegralEi
[-1/2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/B])/(B^2*(b*c - a*d)*g^2*(a
+ b*x)) - (c + d*x)/(2*B*(b*c - a*d)*g^2*(a + b*x)*(A + B*Log[(e*(a + b*x)^
2)/(c + d*x)^2]))
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2343

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^(p + 1)/(b*d*n*(p + 1))), x] -
Dist[(m + 1)/(b*n*(p + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p + 1), x], x]
/; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2550

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(
m + 1)*(g/b)^m, Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x],
 x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] &&
 EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && Eq
Q[b*f - a*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Rubi steps

$$\int \frac{1}{(ag+bgx)^2 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx=\int \frac{1}{(ag+bgx)^2 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$$

Mathematica [F]
time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{1}{(ag+bgx)^2 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2),x]
[Out] Integrate[1/((a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2), x
]
```

Maple [F]
time = 0.63, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bgx+ag\right)^{2} \left(A+B\ln\left(\frac{e\left(bx+a\right)^{2}}{\left(dx+c\right)^{2}}\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*g*x+a*g)^2/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

```
[Out] int(1/(b*g*x+a*g)^2/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="
maxima")
```

```
[Out] -1/2*(d*x + c)/((a*b*c*g^2 - a^2*d*g^2)*A*B + (a*b*c*g^2 - a^2*d*g^2)*B^2 +
 ((b^2*c*g^2 - a*b*d*g^2)*A*B + (b^2*c*g^2 - a*b*d*g^2)*B^2)*x + 2*((b^2*c*
g^2 - a*b*d*g^2)*B^2*x + (a*b*c*g^2 - a^2*d*g^2)*B^2)*log(b*x + a) - 2*((b^
2*c*g^2 - a*b*d*g^2)*B^2*x + (a*b*c*g^2 - a^2*d*g^2)*B^2)*log(d*x + c)) + i
ntegrate(-1/2/(A*B*a^2*g^2 + B^2*a^2*g^2 + (A*B*b^2*g^2 + B^2*b^2*g^2)*x^2
+ 2*(A*B*a*b*g^2 + B^2*a*b*g^2)*x + 2*(B^2*b^2*g^2*x^2 + 2*B^2*a*b*g^2*x +
B^2*a^2*g^2)*log(b*x + a) - 2*(B^2*b^2*g^2*x^2 + 2*B^2*a*b*g^2*x + B^2*a^2*
g^2)*log(d*x + c)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="
fricas")
```

```
[Out] integral(1/(A^2*b^2*g^2*x^2 + 2*A^2*a*b*g^2*x + A^2*a^2*g^2 + (B^2*b^2*g^2*
x^2 + 2*B^2*a*b*g^2*x + B^2*a^2*g^2)*log((b^2*x^2 + 2*a*b*x + a^2)*e/(d^2*x
^2 + 2*c*d*x + c^2))^2 + 2*(A*B*b^2*g^2*x^2 + 2*A*B*a*b*g^2*x + A*B*a^2*g^2
)*log((b^2*x^2 + 2*a*b*x + a^2)*e/(d^2*x^2 + 2*c*d*x + c^2))), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{c+dx}{2ABa^2dg^2-2ABabcg^2+2ABabdg^2x-2ABb^2cg^2x+(2B^2a^2dg^2-2B^2abcg^2+2B^2abdg^2x-2B^2b^2cg^2x)\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}-\frac{\int\frac{1}{Aa^2+2Aabx+Ab^2x^2+Ba^2\log\left(\frac{a^2e}{c^2+2cdx+d^2x^2}+\frac{2abex}{c^2+2cdx+d^2x^2}+\frac{b^2ex^2}{c^2+2cdx+d^2x^2}\right)+2Babx\log\left(\frac{a^2e}{c^2+2cdx+d^2x^2}+\frac{2abex}{c^2+2cdx+d^2x^2}+\frac{b^2ex^2}{c^2+2cdx+d^2x^2}\right)+Bb^2x^2\log\left(\frac{a^2e}{c^2+2cdx+d^2x^2}+\frac{2abex}{c^2+2cdx+d^2x^2}+\frac{b^2ex^2}{c^2+2cdx+d^2x^2}\right)}\,dx}{2Bg^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)**2/(A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2,x)
```

```
[Out] (c + d*x)/(2*A*B*a**2*d*g**2 - 2*A*B*a*b*c*g**2 + 2*A*B*a*b*d*g**2*x - 2*A*
B*b**2*c*g**2*x + (2*B**2*a**2*d*g**2 - 2*B**2*a*b*c*g**2 + 2*B**2*a*b*d*g*
*2*x - 2*B**2*b**2*c*g**2*x)*log(e*(a + b*x)**2/(c + d*x)**2)) - Integral(1
/(A*a**2 + 2*A*a*b*x + A*b**2*x**2 + B*a**2*log(a**2*e/(c**2 + 2*c*d*x + d*
```

```
*2*x**2) + 2*a*b*e*x/(c**2 + 2*c*d*x + d**2*x**2) + b**2*e*x**2/(c**2 + 2*c
*d*x + d**2*x**2)) + 2*B*a*b*x*log(a**2*e/(c**2 + 2*c*d*x + d**2*x**2) + 2*
a*b*e*x/(c**2 + 2*c*d*x + d**2*x**2) + b**2*e*x**2/(c**2 + 2*c*d*x + d**2*x
**2)) + B*b**2*x**2*log(a**2*e/(c**2 + 2*c*d*x + d**2*x**2) + 2*a*b*e*x/(c*
*2 + 2*c*d*x + d**2*x**2) + b**2*e*x**2/(c**2 + 2*c*d*x + d**2*x**2))), x)/
(2*B*g**2)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="
giac")
[Out] integrate(1/((b*g*x + a*g)^2*(B*log((b*x + a)^2*e/(d*x + c)^2) + A)^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a\,g+b\,g\,x\right)}^2\,{\left(A+B\,\ln\left(\frac{e\,{\left(a+b\,x\right)}^2}{{\left(c+d\,x\right)}^2}\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*g + b*g*x)^2*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2),x)
[Out] int(1/((a*g + b*g*x)^2*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2), x)
```

3.146 $\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$

Optimal. Leaf size=266

$$\frac{de^{\frac{A}{2B}}\sqrt{\frac{e(a+bx)^2}{(c+dx)^2}}\,(c+dx)\text{Ei}\left(\frac{-A-B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{2B}\right)}{4B^2(bc-ad)^2g^3(a+bx)}-\frac{bee^{A/B}\text{Ei}\left(-\frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{B}\right)}{2B^2(bc-ad)^2g^3}+\frac{d(}{2B(bc-ad)^2g^3(a+}$$

```
[Out] -1/2*b*e*exp(A/B)*Ei((-A-B*ln(e*(b*x+a)^2/(d*x+c)^2))/B)/B^2/(-a*d+b*c)^2/g
^3+1/2*d*(d*x+c)/B/(-a*d+b*c)^2/g^3/(b*x+a)/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))
-1/2*b*(d*x+c)^2/B/(-a*d+b*c)^2/g^3/(b*x+a)^2/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2
))+1/4*d*exp(1/2*A/B)*(d*x+c)*Ei(1/2*(-A-B*ln(e*(b*x+a)^2/(d*x+c)^2))/B)*(e
*(b*x+a)^2/(d*x+c)^2)^(1/2)/B^2/(-a*d+b*c)^2/g^3/(b*x+a)
```

Rubi [A]
time = 0.21, antiderivative size = 263, normalized size of antiderivative = 0.99, number of steps used = 9, number of rules used = 5, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.147$, Rules used = {2550, 2395, 2343, 2347, 2209}

$$\frac{de^{\frac{A}{2B}}(c+dx)\sqrt{\frac{e(a+bx)^2}{(c+dx)^2}}\,\text{Ei}\left(-\frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{2B}\right)}{4B^2g^3(a+bx)(bc-ad)^2}-\frac{bee^{A/B}\text{Ei}\left(-\frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{B}\right)}{2B^2g^3(bc-ad)^2}-\frac{b(c+dx)^2}{2Bg^3(a+bx)^2(bc-ad)^2\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}+\frac{d(c+dx)}{2Bg^3(a+bx)(bc-ad)^2\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a*g + b*g*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2),x]
```

```
[Out] -1/2*(b*e*E^(A/B)*ExpIntegralEi[-((A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/
B)])/(B^2*(b*c - a*d)^2*g^3) + (d*E^(A/(2*B))*Sqrt[(e*(a + b*x)^2)/(c + d*x
)^2]*(c + d*x)*ExpIntegralEi[-1/2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/
B])/(4*B^2*(b*c - a*d)^2*g^3*(a + b*x)) + (d*(c + d*x))/(2*B*(b*c - a*d)^2*
g^3*(a + b*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])) - (b*(c + d*x)^2)/(
2*B*(b*c - a*d)^2*g^3*(a + b*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]))
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2343

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^(p + 1)/(b*d*n*(p + 1))), x] -
Dist[(m + 1)/(b*n*(p + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p + 1), x], x]
```

```
/; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2550

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(
m + 1)*(g/b)^m, Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x],
 x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] &&
 EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && Eq
Q[b*f - a*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Rubi steps

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx=\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$$

Mathematica [F]

time = 0.22, size = 0, normalized size = 0.00

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((a*g + b*g*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2),x]
```

```
[Out] Integrate[1/((a*g + b*g*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2), x
]
```

Maple [F]
time = 1.14, size = 0, normalized size = 0.00

$$\int \frac{1}{(bgx+ag)^3 \left(A+B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*g*x+a*g)^3/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

```
[Out] int(1/(b*g*x+a*g)^3/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)^3/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="
maxima")
```

```
[Out] -1/2*(d*x + c)/((a^2*b*c*g^3 - a^3*d*g^3)*A*B + (a^2*b*c*g^3 - a^3*d*g^3)*B
^2 + ((b^3*c*g^3 - a*b^2*d*g^3)*A*B + (b^3*c*g^3 - a*b^2*d*g^3)*B^2)*x^2 +
2*((a*b^2*c*g^3 - a^2*b*d*g^3)*A*B + (a*b^2*c*g^3 - a^2*b*d*g^3)*B^2)*x + 2
*((b^3*c*g^3 - a*b^2*d*g^3)*B^2*x^2 + 2*(a*b^2*c*g^3 - a^2*b*d*g^3)*B^2*x +
 (a^2*b*c*g^3 - a^3*d*g^3)*B^2)*log(b*x + a) - 2*((b^3*c*g^3 - a*b^2*d*g^3)
*B^2*x^2 + 2*(a*b^2*c*g^3 - a^2*b*d*g^3)*B^2*x + (a^2*b*c*g^3 - a^3*d*g^3)*
B^2)*log(d*x + c)) - integrate(1/2*(b*d*x + 2*b*c - a*d)/(((b^4*c*g^3 - a*b
^3*d*g^3)*A*B + (b^4*c*g^3 - a*b^3*d*g^3)*B^2)*x^3 + (a^3*b*c*g^3 - a^4*d*g
^3)*A*B + (a^3*b*c*g^3 - a^4*d*g^3)*B^2 + 3*((a*b^3*c*g^3 - a^2*b^2*d*g^3)*
A*B + (a*b^3*c*g^3 - a^2*b^2*d*g^3)*B^2)*x^2 + 3*((a^2*b^2*c*g^3 - a^3*b*d*
g^3)*A*B + (a^2*b^2*c*g^3 - a^3*b*d*g^3)*B^2)*x + 2*((b^4*c*g^3 - a*b^3*d*g
^3)*B^2*x^3 + 3*(a*b^3*c*g^3 - a^2*b^2*d*g^3)*B^2*x^2 + 3*(a^2*b^2*c*g^3 -
a^3*b*d*g^3)*B^2*x + (a^3*b*c*g^3 - a^4*d*g^3)*B^2)*log(b*x + a) - 2*((b^4*
c*g^3 - a*b^3*d*g^3)*B^2*x^3 + 3*(a*b^3*c*g^3 - a^2*b^2*d*g^3)*B^2*x^2 + 3*
(a^2*b^2*c*g^3 - a^3*b*d*g^3)*B^2*x + (a^3*b*c*g^3 - a^4*d*g^3)*B^2)*log(d*
x + c)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)^3/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="
fricas")
```

```
[Out] integral(1/(A^2*b^3*g^3*x^3 + 3*A^2*a*b^2*g^3*x^2 + 3*A^2*a^2*b*g^3*x + A^2
*a^3*g^3 + (B^2*b^3*g^3*x^3 + 3*B^2*a*b^2*g^3*x^2 + 3*B^2*a^2*b*g^3*x + B^2
*a^3*g^3)*log((b^2*x^2 + 2*a*b*x + a^2)*e/(d^2*x^2 + 2*c*d*x + c^2))^2 + 2*
(A*B*b^3*g^3*x^3 + 3*A*B*a*b^2*g^3*x^2 + 3*A*B*a^2*b*g^3*x + A*B*a^3*g^3)*l
og((b^2*x^2 + 2*a*b*x + a^2)*e/(d^2*x^2 + 2*c*d*x + c^2))), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)**3/(A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)^3/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="
giac")
```

```
[Out] integrate(1/((b*g*x + a*g)^3*(B*log((b*x + a)^2*e/(d*x + c)^2) + A)^2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(a\,g+b\,g\,x\right)}^3\,{\left(A+B\,\ln\left(\frac{e\,{\left(a+b\,x\right)}^2}{{\left(c+d\,x\right)}^2}\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*g + b*g*x)^3*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2),x)
```

```
[Out] int(1/((a*g + b*g*x)^3*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2), x)
```

3.147 $\int (a+bx)^4 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right) dx$

Optimal. Leaf size=171

$$\frac{B(bc-ad)^4nx}{5d^4}-\frac{B(bc-ad)^3n(a+bx)^2}{10bd^3}+\frac{B(bc-ad)^2n(a+bx)^3}{15bd^2}-\frac{B(bc-ad)n(a+bx)^4}{20bd}-\frac{B(bc-ad)^5n\log}{5bd^5}$$

[Out] 1/5*B*(-a*d+b*c)^4*n*x/d^4-1/10*B*(-a*d+b*c)^3*n*(b*x+a)^2/b/d^3+1/15*B*(-a*d+b*c)^2*n*(b*x+a)^3/b/d^2-1/20*B*(-a*d+b*c)*n*(b*x+a)^4/b/d-1/5*B*(-a*d+b*c)^5*n*ln(d*x+c)/b/d^5+1/5*(b*x+a)^5*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/b

Rubi [A]
time = 0.06, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {2548, 45}

$$\frac{(a+bx)^5\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{5b}-\frac{Bn(bc-ad)^5\log(c+dx)}{5bd^5}+\frac{Bnx(bc-ad)^4}{5d^4}-\frac{Bn(a+bx)^2(bc-ad)^3}{10bd^3}+\frac{Bn(a+bx)^3(bc-ad)^2}{15bd^2}-\frac{Bn(a+bx)^4(bc-ad)}{20bd}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x)^4*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]),x]

[Out] (B*(b*c - a*d)^4*n*x)/(5*d^4) - (B*(b*c - a*d)^3*n*(a + b*x)^2)/(10*b*d^3) + (B*(b*c - a*d)^2*n*(a + b*x)^3)/(15*b*d^2) - (B*(b*c - a*d)*n*(a + b*x)^4)/(20*b*d) - (B*(b*c - a*d)^5*n*Log[c + d*x])/(5*b*d^5) + ((a + b*x)^5*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]))/(5*b)

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2548

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Dist[B*n*((b*c
- a*d)/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /;
 FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c -
 a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^4 \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right) \, dx &= \int \left(A(a+bx)^4+B(a+bx)^4\log \left(e(a+bx)^n(c+dx)^{-\right.\\ &= \frac{A(a+bx)^5}{5b}+B\int (a+bx)^4\log \left(e(a+bx)^n(c+dx)^{-\right.\\ &= \frac{A(a+bx)^5}{5b}+\frac{B(a+bx)^5\log \left(e(a+bx)^n(c+dx)^{-n}\right)}{5b}\\ &= \frac{A(a+bx)^5}{5b}+\frac{B(a+bx)^5\log \left(e(a+bx)^n(c+dx)^{-n}\right)}{5b}\\ &= \frac{B(bc-ad)^4nx}{5d^4}-\frac{B(bc-ad)^3n(a+bx)^2}{10bd^3}+\frac{B(bc-a}{1}\end{aligned}$$

Mathematica [A]
time = 0.25, size = 338, normalized size = 1.98

$$\frac{1}{60}\left(\frac{12a^5Bn\log(a+bx)}{b}-\frac{12Bc(b^4c^4-5ab^3c^3d+10a^2b^2c^2d^2-10a^3bcd^3+5a^4d^4)\,n\log(c+dx)}{d^5}+\frac{x(12a^4d^4(5A+4Bn)+12a^3bd^3(-10Bcn+10Adx+3Bdnx)+4a^2b^2d^2(30Ad^2x^2+Bn(30c^2-15cdx+4d^2x^2))+b^4(12Ad^4x^4+Bcn(12c^3-6c^2dx+4cd^2x^2-3d^3x^3))+ab^3d(60Ad^3x^3+Bn(-60c^3+30c^2dx-20cd^2x^2+3d^3x^3))+12Bd^4(5a^4+10a^3bx+10a^2b^2x^2+5ab^3x^3+b^4x^4)\log\left(e(a+bx)^n(c+dx)^{-n}\right))}{d^4}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^4*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]),x]
```

```
[Out] ((12*a^5*B*n*Log[a + b*x])/b - (12*B*c*(b^4*c^4 - 5*a*b^3*c^3*d + 10*a^2*b^
2*c^2*d^2 - 10*a^3*b*c*d^3 + 5*a^4*d^4)*n*Log[c + d*x])/d^5 + (x*(12*a^4*d^
4*(5*A + 4*B*n) + 12*a^3*b*d^3*(-10*B*c*n + 10*A*d*x + 3*B*d*n*x) + 4*a^2*b
^2*d^2*(30*A*d^2*x^2 + B*n*(30*c^2 - 15*c*d*x + 4*d^2*x^2)) + b^4*(12*A*d^4
*x^4 + B*c*n*(12*c^3 - 6*c^2*d*x + 4*c*d^2*x^2 - 3*d^3*x^3)) + a*b^3*d*(60*
A*d^3*x^3 + B*n*(-60*c^3 + 30*c^2*d*x - 20*c*d^2*x^2 + 3*d^3*x^3)) + 12*B*d
^4*(5*a^4 + 10*a^3*b*x + 10*a^2*b^2*x^2 + 5*a*b^3*x^3 + b^4*x^4)*Log[(e*(a
+ b*x)^n)/(c + d*x)^n]))/d^4)/60
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.46, size = 2372, normalized size = 13.87

method	result	size
risch	Expression too large to display	2372

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^4*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n))),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*A*b^4*x^5+x*A*a^4-1/d^3*b^3*B*a*c^3*n*x+2/d^2*b*B*ln(d*x+c)*a^3*c^2*n-2
/d^3*b^2*B*ln(d*x+c)*a^2*c^3*n+1/d^4*b^3*B*ln(d*x+c)*a*c^4*n+1/10*I*b^4*B*P
i*x^5*csgn(I*(b*x+a)^n)*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+1/10*I*b^4*B*Pi*x^5
```

*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+1/10*I*b^4*B*Pi*x^5*cs
gn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2+1/10*I*b^4*B*
Pi*x^5*csgn(I*e)*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2+4/15*b^2*B*a^2*n*x^3+1/1
5/d^2*b^4*B*c^2*n*x^3+3/5*b*B*a^3*n*x^2-1/10/d^3*b^4*B*c^3*n*x^2+4/5*B*a^4*
n*x+1/5/d^4*b^4*B*c^4*n*x-1/5/d^5*b^4*B*ln(d*x+c)*c^5*n-1/5*(b*x+a)^5*B/b*l
n((d*x+c)^n)-1/d*B*ln(d*x+c)*a^4*c*n-1/2*I*B*Pi*a^4*x*csgn(I*(b*x+a)^n/((d*
x+c)^n))^3-1/2*I*B*Pi*a^4*x*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^3-1/10*I*b^4*B*
Pi*x^5*csgn(I*(b*x+a)^n/((d*x+c)^n))^3-1/10*I*b^4*B*Pi*x^5*csgn(I*e/((d*x+c
)^n)*(b*x+a)^n)^3-1/3/d*b^3*B*a*c*n*x^3-1/d*b^2*B*a^2*c*n*x^2+1/2/d^2*b^3*B
*a*c^2*n*x^2-2/d*b*B*a^3*c*n*x+2/d^2*b^2*B*a^2*c^2*n*x+I*b^2*B*Pi*a^2*x^3*c
sgn(I*(b*x+a)^n)*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+I*b^2*B*Pi*a^2*x^3*csgn(I/
((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+I*b^2*B*Pi*a^2*x^3*csgn(I*(b*x
+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2+I*b^2*B*Pi*a^2*x^3*csg
n(I*e)*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2+I*b*B*Pi*a^3*x^2*csgn(I*(b*x+a)^n)
csgn(I(b*x+a)^n/((d*x+c)^n))^2+I*b*B*Pi*a^3*x^2*csgn(I/((d*x+c)^n))*csgn(
I*(b*x+a)^n/((d*x+c)^n))^2+I*b*B*Pi*a^3*x^2*csgn(I*(b*x+a)^n/((d*x+c)^n))*c
sgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2+I*b*B*Pi*a^3*x^2*csgn(I*e)*csgn(I*e/((d*x+
c)^n)*(b*x+a)^n)^2-1/2*I*B*Pi*a^4*x*csgn(I*e)*csgn(I*(b*x+a)^n/((d*x+c)^n))
*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)-1/2*I*B*Pi*a^4*x*csgn(I*(b*x+a)^n)*csgn(I/
((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))-1/10*I*b^4*B*Pi*x^5*csgn(I*e)*cs
gn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)-1/10*I*b^4*B*Pi
*x^5*csgn(I*(b*x+a)^n)*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))+1/
2*I*b^3*B*Pi*a*x^4*csgn(I*(b*x+a)^n)*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+1/2*I*
b^3*B*Pi*a*x^4*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+1/2*I*b^
3*B*Pi*a*x^4*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^
2+1/2*I*b^3*B*Pi*a*x^4*csgn(I*e)*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2+1/2*I*B*
Pi*a^4*x*csgn(I*(b*x+a)^n)*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+1/2*I*B*Pi*a^4*x
*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+b^3*A*a*x^4+2*b^2*A*a^
2*x^3+2*b*A*a^3*x^2+2*b^2*B*a^2*x^3*ln((b*x+a)^n)+2*b*B*ln(e)*a^3*x^2+2*b*B
*a^3*x^2*ln((b*x+a)^n)+1/5/b*B*ln(d*x+c)*a^5*n+b^3*B*ln(e)*a*x^4+b^3*B*a*x^
4*ln((b*x+a)^n)+2*b^2*B*ln(e)*a^2*x^3-1/2*I*b^3*B*Pi*a*x^4*csgn(I*e)*csgn(I
*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)-1/2*I*b^3*B*Pi*a*x^
4*csgn(I*(b*x+a)^n)*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))-I*b^2
*B*Pi*a^2*x^3*csgn(I*e)*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*
(b*x+a)^n)-I*b^2*B*Pi*a^2*x^3*csgn(I*(b*x+a)^n)*csgn(I/((d*x+c)^n))*csgn(I*
(b*x+a)^n/((d*x+c)^n))-I*b*B*Pi*a^3*x^2*csgn(I*e)*csgn(I*(b*x+a)^n/((d*x+c)
^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)-I*b*B*Pi*a^3*x^2*csgn(I*(b*x+a)^n)*csg
n(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))+B*ln(e)*a^4*x+B*a^4*x*ln((b*
x+a)^n)+1/5*b^4*B*ln(e)*x^5+1/5*b^4*B*x^5*ln((b*x+a)^n)+1/5/b*B*a^5*ln((b*x
+a)^n)+1/20*b^3*B*a*n*x^4-1/20/d*b^4*B*c*n*x^4-1/2*I*b^3*B*Pi*a*x^4*csgn(I*
(b*x+a)^n/((d*x+c)^n))^3-1/2*I*b^3*B*Pi*a*x^4*csgn(I*e/((d*x+c)^n)*(b*x+a)^
n)^3-I*b^2*B*Pi*a^2*x^3*csgn(I*(b*x+a)^n/((d*x+c)^n))^3-I*b^2*B*Pi*a^2*x^3*
csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^3-I*b*B*Pi*a^3*x^2*csgn(I*(b*x+a)^n/((d*x+c
)^n))^3-I*b*B*Pi*a^3*x^2*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^3+1/2*I*B*Pi*a^4*x
csgn(I(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2+1/2*I*B*Pi

```
*a^4*x*csgn(I*e)*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 681 vs. $2(160) = 320$.
time = 0.32, size = 681, normalized size = 3.98

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4*(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="maxima
")
```

```
[Out] 1/5*B*b^4*x^5*log((b*x + a)^n*e/(d*x + c)^n) + 1/5*A*b^4*x^5 + B*a*b^3*x^4*
log((b*x + a)^n*e/(d*x + c)^n) + A*a*b^3*x^4 + 2*B*a^2*b^2*x^3*log((b*x + a
)^n*e/(d*x + c)^n) + 2*A*a^2*b^2*x^3 + 2*B*a^3*b*x^2*log((b*x + a)^n*e/(d*x
 + c)^n) + 2*A*a^3*b*x^2 + (a*n*e*log(b*x + a)/b - c*n*e*log(d*x + c)/d)*B*
a^4*e^(-1) - 2*(a^2*n*e*log(b*x + a)/b^2 - c^2*n*e*log(d*x + c)/d^2 + (b*c*
n - a*d*n)*x*e/(b*d))*B*a^3*b*e^(-1) + (2*a^3*n*e*log(b*x + a)/b^3 - 2*c^3*
n*e*log(d*x + c)/d^3 - ((b^2*c*d*n - a*b*d^2*n)*x^2*e - 2*(b^2*c^2*n - a^2*
d^2*n)*x*e)/(b^2*d^2))*B*a^2*b^2*e^(-1) - 1/6*(6*a^4*n*e*log(b*x + a)/b^4 -
 6*c^4*n*e*log(d*x + c)/d^4 + (2*(b^3*c*d^2*n - a*b^2*d^3*n)*x^3*e - 3*(b^3
*c^2*d*n - a^2*b*d^3*n)*x^2*e + 6*(b^3*c^3*n - a^3*d^3*n)*x*e)/(b^3*d^3))*B
*a*b^3*e^(-1) + 1/60*(12*a^5*n*e*log(b*x + a)/b^5 - 12*c^5*n*e*log(d*x + c)
/d^5 - (3*(b^4*c*d^3*n - a*b^3*d^4*n)*x^4*e - 4*(b^4*c^2*d^2*n - a^2*b^2*d^
4*n)*x^3*e + 6*(b^4*c^3*d*n - a^3*b*d^4*n)*x^2*e - 12*(b^4*c^4*n - a^4*d^4*
n)*x*e)/(b^4*d^4))*B*b^4*e^(-1) + B*a^4*x*log((b*x + a)^n*e/(d*x + c)^n) +
A*a^4*x
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 503 vs. $2(160) = 320$.
time = 0.37, size = 503, normalized size = 2.94

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4*(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="fricas
")
```

```
[Out] 1/60*(12*(A + B)*b^5*d^5*x^5 + 3*(20*(A + B)*a*b^4*d^5 - (B*b^5*c*d^4 - B*a
*b^4*d^5)*n)*x^4 + 4*(30*(A + B)*a^2*b^3*d^5 + (B*b^5*c^2*d^3 - 5*B*a*b^4*c
*d^4 + 4*B*a^2*b^3*d^5)*n)*x^3 + 6*(20*(A + B)*a^3*b^2*d^5 - (B*b^5*c^3*d^2
 - 5*B*a*b^4*c^2*d^3 + 10*B*a^2*b^3*c*d^4 - 6*B*a^3*b^2*d^5)*n)*x^2 + 12*(5
*(A + B)*a^4*b*d^5 + (B*b^5*c^4*d - 5*B*a*b^4*c^3*d^2 + 10*B*a^2*b^3*c^2*d^
3 - 10*B*a^3*b^2*c*d^4 + 4*B*a^4*b*d^5)*n)*x + 12*(B*b^5*d^5*n*x^5 + 5*B*a*
b^4*d^5*n*x^4 + 10*B*a^2*b^3*d^5*n*x^3 + 10*B*a^3*b^2*d^5*n*x^2 + 5*B*a^4*b
*d^5*n*x + B*a^5*d^5*n)*log(b*x + a) - 12*(B*b^5*d^5*n*x^5 + 5*B*a*b^4*d^5*
```

```
n*x^4 + 10*B*a^2*b^3*d^5*n*x^3 + 10*B*a^3*b^2*d^5*n*x^2 + 5*B*a^4*b*d^5*n*x
 + (B*b^5*c^5 - 5*B*a*b^4*c^4*d + 10*B*a^2*b^3*c^3*d^2 - 10*B*a^3*b^2*c^2*d
^3 + 5*B*a^4*b*c*d^4)*n)*log(d*x + c))/(b*d^5)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**4*(A+B*ln(e*(b*x+a)**n/((d*x+c)**n))),x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 497 vs. 2(160) = 320.
time = 39.24, size = 497, normalized size = 2.91

$$\frac{Ba^5n\log(bx+a)}{5b} + \frac{1}{5}(Ab^4+Bb^4)x^5 - \frac{(Bb^4cn - Bab^3dn - 20Aab^3d - 20Bab^3d)x^4}{20d} + \frac{(Bb^4c^2n - 5Bab^3cdn + 4Ba^2b^2d^2n + 30Aa^2b^2d^2 + 30Ba^2b^2d^2)x^3}{15d^2} + \frac{1}{5}(Bb^4nx^5 + 5Bab^3nx^4 + 10Ba^2b^2nx^3 + 10Ba^3bnx^2 + 5Ba^4nx)\log(bx+a) - \frac{1}{5}(Bb^4nx^5 + 5Bab^3nx^4 + 10Ba^2b^2nx^3 + 10Ba^3bnx^2 + 5Ba^4nx)\log(dx+c) - \frac{(Bb^4c^3n - 5Bab^3c^2dn + 10Ba^2b^2cd^2n - 6Ba^3bd^3n - 20Aa^3bd^3 - 20Ba^3bd^3)x^2}{10d^3} + \frac{(Bb^4c^4n - 5Bab^3c^3dn + 10Ba^2b^2c^2d^2n - 10Ba^3bcd^3n + 4Ba^4d^4n + 5Aa^4d^4 + 5Ba^4d^4)x}{5d^4} - \frac{(Bb^4c^5n - 5Bab^3c^4dn + 10Ba^2b^2c^3d^2n - 10Ba^3bc^2d^3n + 5Ba^4cd^4n)\log(-dx-c)}{5d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4*(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="giac")
```

```
[Out] 1/5*B*a^5*n*log(b*x + a)/b + 1/5*(A*b^4 + B*b^4)*x^5 - 1/20*(B*b^4*c*n - B*
a*b^3*d*n - 20*A*a*b^3*d - 20*B*a*b^3*d)*x^4/d + 1/15*(B*b^4*c^2*n - 5*B*a*
b^3*c*d*n + 4*B*a^2*b^2*d^2*n + 30*A*a^2*b^2*d^2 + 30*B*a^2*b^2*d^2)*x^3/d^
2 + 1/5*(B*b^4*n*x^5 + 5*B*a*b^3*n*x^4 + 10*B*a^2*b^2*n*x^3 + 10*B*a^3*b*n*
x^2 + 5*B*a^4*n*x)*log(b*x + a) - 1/5*(B*b^4*n*x^5 + 5*B*a*b^3*n*x^4 + 10*B
*a^2*b^2*n*x^3 + 10*B*a^3*b*n*x^2 + 5*B*a^4*n*x)*log(d*x + c) - 1/10*(B*b^4
*c^3*n - 5*B*a*b^3*c^2*d*n + 10*B*a^2*b^2*c*d^2*n - 6*B*a^3*b*d^3*n - 20*A*
a^3*b*d^3 - 20*B*a^3*b*d^3)*x^2/d^3 + 1/5*(B*b^4*c^4*n - 5*B*a*b^3*c^3*d*n
+ 10*B*a^2*b^2*c^2*d^2*n - 10*B*a^3*b*c*d^3*n + 4*B*a^4*d^4*n + 5*A*a^4*d^4
 + 5*B*a^4*d^4)*x/d^4 - 1/5*(B*b^4*c^5*n - 5*B*a*b^3*c^4*d*n + 10*B*a^2*b^2
*c^3*d^2*n - 10*B*a^3*b*c^2*d^3*n + 5*B*a^4*c*d^4*n)*log(-d*x - c)/d^5
```

Mupad [**B**]
time = 4.56, size = 936, normalized size = 5.47

$$x^4\left(\frac{b^3(25Aad+5Abc+Badn-Bbcn)}{20d}-\frac{Ab^3(5ad+5bc)}{20d}\right)-x^3\left(\frac{(5ad+5bc)\left(\frac{b^3(25Aad+5Abc+Badn-Bbcn)}{5d}-\frac{Ab^3(5ad+5bc)}{5d}\right)}{15bd}-\frac{ab^2(10Aad+5Abc+Badn-Bbcn)}{3d}+\frac{Aab^3c}{3d}\right)+\ln\left(\frac{e(a+bx)^n}{(c+dx)^n}\right)\left(Ba^4x+2Ba^3bx^2+2Ba^2b^2x^3+Bab^3x^4+\frac{Bb^4x^5}{5}\right)+x\,[illegible]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))*(a + b*x)^4,x)
```

```
[Out] x^4*((b^3*(25*A*a*d + 5*A*b*c + B*a*d*n - B*b*c*n))/(20*d) - (A*b^3*(5*a*d
 + 5*b*c))/(20*d)) - x^3*(((5*a*d + 5*b*c)*((b^3*(25*A*a*d + 5*A*b*c + B*a*d
*n - B*b*c*n))/(5*d) - (A*b^3*(5*a*d + 5*b*c))/(5*d)))/(15*b*d) - (a*b^2*(1
```

```
0*A*a*d + 5*A*b*c + B*a*d*n - B*b*c*n))/(3*d) + (A*a*b^3*c)/(3*d)) + log((e
*(a + b*x)^n)/(c + d*x)^n)*((B*b^4*x^5)/5 + B*a^4*x + 2*B*a^3*b*x^2 + B*a*b
^3*x^4 + 2*B*a^2*b^2*x^3) + x*((a^3*(5*A*a*d + 10*A*b*c + 2*B*a*d*n - 2*B*b
*c*n))/d - ((5*a*d + 5*b*c)*((2*a^2*b*(5*A*a*d + 5*A*b*c + B*a*d*n - B*b*c*
n))/d + ((5*a*d + 5*b*c)*(((5*a*d + 5*b*c)*((b^3*(25*A*a*d + 5*A*b*c + B*a*
d*n - B*b*c*n))/(5*d) - (A*b^3*(5*a*d + 5*b*c))/(5*d)))/(5*b*d) - (a*b^2*(1
0*A*a*d + 5*A*b*c + B*a*d*n - B*b*c*n))/d + (A*a*b^3*c)/d))/(5*b*d) - (a*c*
((b^3*(25*A*a*d + 5*A*b*c + B*a*d*n - B*b*c*n))/(5*d) - (A*b^3*(5*a*d + 5*b
*c))/(5*d)))/(b*d)))/(5*b*d) + (a*c*(((5*a*d + 5*b*c)*((b^3*(25*A*a*d + 5*A
*b*c + B*a*d*n - B*b*c*n))/(5*d) - (A*b^3*(5*a*d + 5*b*c))/(5*d)))/(5*b*d)
- (a*b^2*(10*A*a*d + 5*A*b*c + B*a*d*n - B*b*c*n))/d + (A*a*b^3*c)/d))/(b*d
)) + x^2*((a^2*b*(5*A*a*d + 5*A*b*c + B*a*d*n - B*b*c*n))/d + ((5*a*d + 5*b
*c)*(((5*a*d + 5*b*c)*((b^3*(25*A*a*d + 5*A*b*c + B*a*d*n - B*b*c*n))/(5*d)
 - (A*b^3*(5*a*d + 5*b*c))/(5*d)))/(5*b*d) - (a*b^2*(10*A*a*d + 5*A*b*c + B
*a*d*n - B*b*c*n))/d + (A*a*b^3*c)/d))/(10*b*d) - (a*c*((b^3*(25*A*a*d + 5*
A*b*c + B*a*d*n - B*b*c*n))/(5*d) - (A*b^3*(5*a*d + 5*b*c))/(5*d)))/(2*b*d)
) + (A*b^4*x^5)/5 - (log(c + d*x)*(B*b^4*c^5*n + 5*B*a^4*c*d^4*n + 10*B*a^2
*b^2*c^3*d^2*n - 5*B*a*b^3*c^4*d*n - 10*B*a^3*b*c^2*d^3*n))/(5*d^5) + (B*a^
5*n*log(a + b*x))/(5*b)
```

3.148 $\int (a+bx)^3 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right) dx$

Optimal. Leaf size=142

$$-\frac{B(bc-ad)^3nx}{4d^3}+\frac{B(bc-ad)^2n(a+bx)^2}{8bd^2}-\frac{B(bc-ad)n(a+bx)^3}{12bd}+\frac{B(bc-ad)^4n\log(c+dx)}{4bd^4}+\frac{(a+bx)^4\,(A}{}$$

```
[Out] -1/4*B*(-a*d+b*c)^3*n*x/d^3+1/8*B*(-a*d+b*c)^2*n*(b*x+a)^2/b/d^2-1/12*B*(-a
*d+b*c)*n*(b*x+a)^3/b/d+1/4*B*(-a*d+b*c)^4*n*ln(d*x+c)/b/d^4+1/4*(b*x+a)^4*
(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/b
```

Rubi [A]

time = 0.05, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2548, 45}

$$\frac{(a+bx)^4\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{4b}+\frac{Bn(bc-ad)^4\log(c+dx)}{4bd^4}-\frac{Bnx(bc-ad)^3}{4d^3}+\frac{Bn(a+bx)^2(bc-ad)^2}{8bd^2}-\frac{Bn(a+bx)^3(bc-ad)}{12bd}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]),x]
```

```
[Out] -1/4*(B*(b*c - a*d)^3*n*x)/d^3 + (B*(b*c - a*d)^2*n*(a + b*x)^2)/(8*b*d^2)
- (B*(b*c - a*d)*n*(a + b*x)^3)/(12*b*d) + (B*(b*c - a*d)^4*n*Log[c + d*x])
/(4*b*d^4) + ((a + b*x)^4*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]))/(4*b)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2548

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Dist[B*n*((b*c
- a*d)/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /;
 FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c -
 a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Rubi steps

$$
\begin{aligned}
\int (a+bx)^3 \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right)\, dx &= \int \left(A(a+bx)^3+B(a+bx)^3\log \left(e(a+bx)^n(c+dx)^{-\right.\\
&= \frac{A(a+bx)^4}{4b}+B\int (a+bx)^3\log \left(e(a+bx)^n(c+dx)^{-\right.\\
&= \frac{A(a+bx)^4}{4b}+\frac{B(a+bx)^4\log \left(e(a+bx)^n(c+dx)^{-n}\right)}{4b}\\
&= \frac{A(a+bx)^4}{4b}+\frac{B(a+bx)^4\log \left(e(a+bx)^n(c+dx)^{-n}\right)}{4b}\\
&= -\frac{B(bc-ad)^3nx}{4d^3}+\frac{B(bc-ad)^2n(a+bx)^2}{8bd^2}-\frac{B(bc-}{}
\end{aligned}
$$

Mathematica [A]
time = 0.16, size = 250, normalized size = 1.76

$$\frac{6a^4Bd^4n\log(a+bx)+6bBc(b^3c^3-4ab^2c^2d+6a^2bcd^2-4a^3d^3)n\log(c+dx)+bdx(6a^3d^3(4A+3Bn)+9a^2bd^2(-4Bcn+4Adx+Bdnx)+b^3(6Ad^3x^3+Bcn(-6c^2+3cdx-2d^2x^2))+2ab^2d(12Ad^2x^2+Bn(12c^2-6cdx+d^2x^2))+6Bd^3(4a^3+6a^2bx+4ab^2x^2+b^3x^3)\log\left(e(a+bx)^n(c+dx)^{-n}\right))}{24bd^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]),x]
```

```
[Out] (6*a^4*B*d^4*n*Log[a + b*x] + 6*b*B*c*(b^3*c^3 - 4*a*b^2*c^2*d + 6*a^2*b*c*
d^2 - 4*a^3*d^3)*n*Log[c + d*x] + b*d*x*(6*a^3*d^3*(4*A + 3*B*n) + 9*a^2*b*
d^2*(-4*B*c*n + 4*A*d*x + B*d*n*x) + b^3*(6*A*d^3*x^3 + B*c*n*(-6*c^2 + 3*c
*d*x - 2*d^2*x^2)) + 2*a*b^2*d*(12*A*d^2*x^2 + B*n*(12*c^2 - 6*c*d*x + d^2*
x^2)) + 6*B*d^3*(4*a^3 + 6*a^2*b*x + 4*a*b^2*x^2 + b^3*x^3)*Log[(e*(a + b*x
)^n)/(c + d*x)^n]))/(24*b*d^4)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.38, size = 1838, normalized size = 12.94

method	result	size
risch	Expression too large to display	1838

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n))),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*A*b^3*x^4+x*A*a^3-1/4*(b*x+a)^4*B/b*ln((d*x+c)^n)-1/2/d*b^2*B*a*c*n*x^2
-3/2/d*b*B*a^2*c*n*x+1/d^2*b^2*B*a*c^2*n*x+3/2/d^2*b*B*ln(d*x+c)*a^2*c^2*n-
1/d^3*b^2*B*ln(d*x+c)*a*c^3*n+1/2*I*B*Pi*a^3*x*csgn(I*e)*csgn(I*e/((d*x+c)^
n)*(b*x+a)^n)^2+1/2*I*B*Pi*a^3*x*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d
*x+c)^n)*(b*x+a)^n)^2-1/2*I*b^2*B*Pi*a*x^3*csgn(I*(b*x+a)^n)*csgn(I/((d*x+c
```

```
)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))-3/4*I*b*B*Pi*a^2*x^2*csgn(I*e)*csgn(I*(
b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)-3/4*I*b*B*Pi*a^2*x^2*
csgn(I*(b*x+a)^n)*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))+1/4*b^3
*B*x^4*ln((b*x+a)^n)+1/4*b^3*B*ln(e)*x^4+1/4/b*B*a^4*ln((b*x+a)^n)+B*a^3*x*
ln((b*x+a)^n)+B*ln(e)*a^3*x+b^2*A*a*x^3+3/2*b*A*a^2*x^2+b^2*B*a*x^3*ln((b*x
+a)^n)+b^2*B*ln(e)*a*x^3+3/2*b*B*a^2*x^2*ln((b*x+a)^n)+3/2*b*B*ln(e)*a^2*x^
2+1/4/b*B*ln(d*x+c)*a^4*n+1/12*b^2*B*a*n*x^3-1/12/d*b^3*B*c*n*x^3+3/8*b*B*a
^2*n*x^2+1/8/d^2*b^3*B*c^2*n*x^2+3/4*B*a^3*n*x-1/4/d^3*b^3*B*c^3*n*x-1/d*B*
ln(d*x+c)*a^3*c*n+1/4/d^4*b^3*B*ln(d*x+c)*c^4*n-1/8*I*b^3*B*Pi*x^4*csgn(I*e
/((d*x+c)^n)*(b*x+a)^n)^3-1/8*I*b^3*B*Pi*x^4*csgn(I*(b*x+a)^n/((d*x+c)^n))^
3-1/2*I*B*Pi*a^3*x*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^3-1/2*I*B*Pi*a^3*x*csgn(
I*(b*x+a)^n/((d*x+c)^n))^3+1/2*I*B*Pi*a^3*x*csgn(I*(b*x+a)^n)*csgn(I*(b*x+a
)^n/((d*x+c)^n))^2+1/2*I*B*Pi*a^3*x*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((
d*x+c)^n))^2+1/8*I*b^3*B*Pi*x^4*csgn(I*e)*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2
+1/8*I*b^3*B*Pi*x^4*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x
+a)^n)^2+1/8*I*b^3*B*Pi*x^4*csgn(I*(b*x+a)^n)*csgn(I*(b*x+a)^n/((d*x+c)^n))
^2+1/8*I*b^3*B*Pi*x^4*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))^2-1
/2*I*b^2*B*Pi*a*x^3*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^3-1/2*I*b^2*B*Pi*a*x^3*
csgn(I*(b*x+a)^n/((d*x+c)^n))^3-3/4*I*b*B*Pi*a^2*x^2*csgn(I*e/((d*x+c)^n)*(
b*x+a)^n)^3-3/4*I*b*B*Pi*a^2*x^2*csgn(I*(b*x+a)^n/((d*x+c)^n))^3-1/2*I*b^2*
B*Pi*a*x^3*csgn(I*e)*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*
x+a)^n)-1/2*I*B*Pi*a^3*x*csgn(I*(b*x+a)^n)*csgn(I/((d*x+c)^n))*csgn(I*(b*x+
a)^n/((d*x+c)^n))-1/8*I*b^3*B*Pi*x^4*csgn(I*e)*csgn(I*(b*x+a)^n/((d*x+c)^n)
)*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)-1/8*I*b^3*B*Pi*x^4*csgn(I*(b*x+a)^n)*csgn
(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))+1/2*I*b^2*B*Pi*a*x^3*csgn(I*e
)*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2+1/2*I*b^2*B*Pi*a*x^3*csgn(I*(b*x+a)^n/(
(d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2+1/2*I*b^2*B*Pi*a*x^3*csgn(I*(
b*x+a)^n)*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+1/2*I*b^2*B*Pi*a*x^3*csgn(I/((d*x
+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+3/4*I*b*B*Pi*a^2*x^2*csgn(I*e)*csgn
(I*e/((d*x+c)^n)*(b*x+a)^n)^2+3/4*I*b*B*Pi*a^2*x^2*csgn(I*(b*x+a)^n/((d*x+c
)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2+3/4*I*b*B*Pi*a^2*x^2*csgn(I*(b*x+a)
^n)*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+3/4*I*b*B*Pi*a^2*x^2*csgn(I/((d*x+c)^n)
)*csgn(I*(b*x+a)^n/((d*x+c)^n))^2-1/2*I*B*Pi*a^3*x*csgn(I*e)*csgn(I*(b*x+a)
^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 475 vs. $2(133) = 266$.
time = 0.33, size = 475, normalized size = 3.35

$$\frac{1}{4} Bb^3x^4 \log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + \frac{1}{4} Ab^3x^4 + Bab^2x^3 \log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + Aab^2x^3 + \frac{3}{2} Ba^2bx^2 \log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + \frac{3}{2} Aa^2bx^2 + \left(\frac{ane \log(bx+a)}{b} - \frac{cne \log(dx+c)}{d}\right) Ba^3e^{(-1)} - \frac{3}{2} \left(\frac{a^2ne \log(bx+a)}{b^2} - \frac{c^2ne \log(dx+c)}{d^2} + \frac{(bcn-adn)xe}{bd}\right) Ba^2be^{(-1)} + \frac{1}{2} \left(\frac{2\,a^3ne \log(bx+a)}{b^3} - \frac{2\,c^3ne \log(dx+c)}{d^3} - \frac{(b^2cdn - abd^2n)x^2e - 2\,(b^2c^2n - a^2d^2n)xe}{b^2d^2}\right) Bab^2e^{(-1)} - \frac{1}{24} \left(\frac{6\,a^4ne \log(bx+a)}{b^4} - \frac{6\,c^4ne \log(dx+c)}{d^4} + \frac{2\,(b^3cd^2n - ab^2d^3n)x^3e - 3\,(b^3c^2dn - a^2bd^3n)x^2e + 6\,(b^3c^3n - a^3d^3n)xe}{b^3d^3}\right) Bb^3e^{(-1)} + Ba^3x \log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + Aa^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="maxima
")
```

```
[Out] 1/4*B*b^3*x^4*log((b*x + a)^n*e/(d*x + c)^n) + 1/4*A*b^3*x^4 + B*a*b^2*x^3*
log((b*x + a)^n*e/(d*x + c)^n) + A*a*b^2*x^3 + 3/2*B*a^2*b*x^2*log((b*x + a
)^n*e/(d*x + c)^n) + 3/2*A*a^2*b*x^2 + (a*n*e*log(b*x + a)/b - c*n*e*log(d*
x + c)/d)*B*a^3*e^(-1) - 3/2*(a^2*n*e*log(b*x + a)/b^2 - c^2*n*e*log(d*x +
c)/d^2 + (b*c*n - a*d*n)*x*e/(b*d))*B*a^2*b*e^(-1) + 1/2*(2*a^3*n*e*log(b*x
 + a)/b^3 - 2*c^3*n*e*log(d*x + c)/d^3 - ((b^2*c*d*n - a*b*d^2*n)*x^2*e - 2
*(b^2*c^2*n - a^2*d^2*n)*x*e)/(b^2*d^2))*B*a*b^2*e^(-1) - 1/24*(6*a^4*n*e*l
og(b*x + a)/b^4 - 6*c^4*n*e*log(d*x + c)/d^4 + (2*(b^3*c*d^2*n - a*b^2*d^3*
n)*x^3*e - 3*(b^3*c^2*d*n - a^2*b*d^3*n)*x^2*e + 6*(b^3*c^3*n - a^3*d^3*n)*
x*e)/(b^3*d^3))*B*b^3*e^(-1) + B*a^3*x*log((b*x + a)^n*e/(d*x + c)^n) + A*a
^3*x
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 370 vs. 2(133) = 266.
time = 0.35, size = 370, normalized size = 2.61

$$\frac{6\,(A+B)b^4d^4x^4+2\,(12\,(A+B)ab^3d^4-(Bb^4cd^3-Bab^3d^4)n)x^3+3\,(12\,(A+B)a^2b^2d^4+(Bb^4c^2d^2-4\,Bab^3cd^3+3\,Ba^2b^2d^4)n)x^2+6\,(4\,(A+B)a^3bd^4-(Bb^4c^3d-4\,Bab^3c^2d^2+6\,Ba^2b^2cd^3-3\,Ba^3bd^4)n)x+6\,(Bb^4d^4nx^4+4\,Bab^3d^4nx^3+6\,Ba^2b^2d^4nx^2+4\,Ba^3bd^4nx+Ba^4d^4n)\log(bx+a)-6\,(Bb^4d^4nx^4+4\,Bab^3d^4nx^3+6\,Ba^2b^2d^4nx^2+4\,Ba^3bd^4nx-(Bb^4c^4-4\,Bab^3c^3d+6\,Ba^2b^2c^2d^2-4\,Ba^3bcd^3)n)\log(dx+c)}{24\,bd^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="fricas
")
```

```
[Out] 1/24*(6*(A + B)*b^4*d^4*x^4 + 2*(12*(A + B)*a*b^3*d^4 - (B*b^4*c*d^3 - B*a*
b^3*d^4)*n)*x^3 + 3*(12*(A + B)*a^2*b^2*d^4 + (B*b^4*c^2*d^2 - 4*B*a*b^3*c*
d^3 + 3*B*a^2*b^2*d^4)*n)*x^2 + 6*(4*(A + B)*a^3*b*d^4 - (B*b^4*c^3*d - 4*B
*a*b^3*c^2*d^2 + 6*B*a^2*b^2*c*d^3 - 3*B*a^3*b*d^4)*n)*x + 6*(B*b^4*d^4*n*x
^4 + 4*B*a*b^3*d^4*n*x^3 + 6*B*a^2*b^2*d^4*n*x^2 + 4*B*a^3*b*d^4*n*x + B*a^
4*d^4*n)*log(b*x + a) - 6*(B*b^4*d^4*n*x^4 + 4*B*a*b^3*d^4*n*x^3 + 6*B*a^2*
b^2*d^4*n*x^2 + 4*B*a^3*b*d^4*n*x - (B*b^4*c^4 - 4*B*a*b^3*c^3*d + 6*B*a^2*
b^2*c^2*d^2 - 4*B*a^3*b*c*d^3)*n)*log(d*x + c))/(b*d^4)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3*(A+B*ln(e*(b*x+a)**n/((d*x+c)**n))),x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 355 vs. 2(133) = 266.
time = 8.90, size = 355, normalized size = 2.50

$$\frac{Ba^4n\log(bx+a)}{4\,b}+\frac{1}{4}\,(Ab^3+Bb^3)x^4-\frac{(Bb^3cn-Bab^2dn-12\,Aab^2d-12\,Bab^2d)x^3}{12\,d}+\frac{1}{4}\,(Bb^3nx^4+4\,Bab^2nx^3+6\,Ba^2bnx^2+4\,Ba^3nx)\log(bx+a)-\frac{1}{4}\,(Bb^3nx^4+4\,Bab^2nx^3+6\,Ba^2bnx^2+4\,Ba^3nx)\log(dx+c)+\frac{(Bb^3c^2n-4\,Bab^2cdn+3\,Ba^2bd^2n+12\,Aa^2bd^2+12\,Ba^2bd^2)x^2}{8\,d^2}-\frac{(Bb^3c^3n-4\,Bab^2c^2dn+6\,Ba^2bcd^2n-3\,Ba^3d^3n-4\,Aa^3d^3-4\,Ba^3d^3)x}{4\,d^3}+\frac{(Bb^3c^4n-4\,Bab^2c^3dn+6\,Ba^2bc^2d^2n-4\,Ba^3cd^3n)\log(dx+c)}{4\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="giac")
```

```
[Out] 1/4*B*a^4*n*log(b*x + a)/b + 1/4*(A*b^3 + B*b^3)*x^4 - 1/12*(B*b^3*c*n - B*
a*b^2*d*n - 12*A*a*b^2*d - 12*B*a*b^2*d)*x^3/d + 1/4*(B*b^3*n*x^4 + 4*B*a*b
^2*n*x^3 + 6*B*a^2*b*n*x^2 + 4*B*a^3*n*x)*log(b*x + a) - 1/4*(B*b^3*n*x^4 +
 4*B*a*b^2*n*x^3 + 6*B*a^2*b*n*x^2 + 4*B*a^3*n*x)*log(d*x + c) + 1/8*(B*b^3
*c^2*n - 4*B*a*b^2*c*d*n + 3*B*a^2*b*d^2*n + 12*A*a^2*b*d^2 + 12*B*a^2*b*d^
2)*x^2/d^2 - 1/4*(B*b^3*c^3*n - 4*B*a*b^2*c^2*d*n + 6*B*a^2*b*c*d^2*n - 3*B
*a^3*d^3*n - 4*A*a^3*d^3 - 4*B*a^3*d^3)*x/d^3 + 1/4*(B*b^3*c^4*n - 4*B*a*b^
2*c^3*d*n + 6*B*a^2*b*c^2*d^2*n - 4*B*a^3*c*d^3*n)*log(d*x + c)/d^4
```

Mupad [B]
time = 4.49, size = 520, normalized size = 3.66

$$x^3\left(\frac{b^2(16Aad+4Abc+Badn-Bbcn)}{12d}-\frac{Ab^2(4ad+4bc)}{12d}\right)+\ln\left(\frac{e(a+bx)^n}{(c+dx)^n}\right)\left(Ba^3x+\frac{3Ba^2bx^2}{2}+Bab^2x^3+\frac{Bb^3x^4}{4}\right)+x\left(\frac{a^2(8Aad+12Abc+3Badn-3Bbcn)}{2d}+\frac{(4ad+4bc)\left(\frac{(4ad+4bc)\left(\frac{b^2(16Aad+4Abc+Badn-Bbcn)}{4d}-\frac{Ab^2(4ad+4bc)}{4d}\right)}{4bd}-\frac{ab(6Aad+4Abc+Badn-Bbcn)}{d}+\frac{Aab^2c}{d}\right)}{4bd}-\frac{ac\left(\frac{b^2(16Aad+4Abc+Badn-Bbcn)}{4d}-\frac{Ab^2(4ad+4bc)}{4d}\right)}{bd}\right)-x^2\left(\frac{(4ad+4bc)\left(\frac{b^2(16Aad+4Abc+Badn-Bbcn)}{4d}-\frac{Ab^2(4ad+4bc)}{4d}\right)}{8bd}-\frac{ab(6Aad+4Abc+Badn-Bbcn)}{2d}+\frac{Aab^2c}{2d}\right)+\frac{Ab^3x^4}{4}+\frac{\ln(c+dx)\,(-4Bna^3cd^3+6Bna^2bc^2d^2-4Bnab^2c^3d+Bnb^3c^4)}{4d^4}+\frac{Ba^4n\ln(a+bx)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))*(a + b*x)^3,x)
```

```
[Out] x^3*((b^2*(16*A*a*d + 4*A*b*c + B*a*d*n - B*b*c*n))/(12*d) - (A*b^2*(4*a*d
+ 4*b*c))/(12*d)) + log((e*(a + b*x)^n)/(c + d*x)^n)*((B*b^3*x^4)/4 + B*a^3
*x + (3*B*a^2*b*x^2)/2 + B*a*b^2*x^3) + x*((a^2*(8*A*a*d + 12*A*b*c + 3*B*a
*d*n - 3*B*b*c*n))/(2*d) + ((4*a*d + 4*b*c)*(((4*a*d + 4*b*c)*((b^2*(16*A*a
*d + 4*A*b*c + B*a*d*n - B*b*c*n))/(4*d) - (A*b^2*(4*a*d + 4*b*c))/(4*d)))/
(4*b*d) - (a*b*(6*A*a*d + 4*A*b*c + B*a*d*n - B*b*c*n))/d + (A*a*b^2*c)/d))
/(4*b*d) - (a*c*((b^2*(16*A*a*d + 4*A*b*c + B*a*d*n - B*b*c*n))/(4*d) - (A*
b^2*(4*a*d + 4*b*c))/(4*d)))/(b*d)) - x^2*(((4*a*d + 4*b*c)*((b^2*(16*A*a*d
 + 4*A*b*c + B*a*d*n - B*b*c*n))/(4*d) - (A*b^2*(4*a*d + 4*b*c))/(4*d)))/(8
*b*d) - (a*b*(6*A*a*d + 4*A*b*c + B*a*d*n - B*b*c*n))/(2*d) + (A*a*b^2*c)/(
2*d)) + (A*b^3*x^4)/4 + (log(c + d*x)*(B*b^3*c^4*n - 4*B*a^3*c*d^3*n - 4*B*
a*b^2*c^3*d*n + 6*B*a^2*b*c^2*d^2*n))/(4*d^4) + (B*a^4*n*log(a + b*x))/(4*b
)
```

3.149 $\int (a+bx)^2 \left(A + B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right) dx$

Optimal. Leaf size=113

$$\frac{B(bc-ad)^2nx}{3d^2}-\frac{B(bc-ad)n(a+bx)^2}{6bd}-\frac{B(bc-ad)^3n\log(c+dx)}{3bd^3}+\frac{(a+bx)^3\left(A+B\log\left(e(a+bx)^n(c+dx\right.\right.}{3b}$$

```
[Out] 1/3*B*(-a*d+b*c)^2*n*x/d^2-1/6*B*(-a*d+b*c)*n*(b*x+a)^2/b/d-1/3*B*(-a*d+b*c
)^3*n*ln(d*x+c)/b/d^3+1/3*(b*x+a)^3*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/b
```

Rubi [A]
time = 0.04, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {2548, 45}

$$\frac{(a+bx)^3\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{3b}-\frac{Bn(bc-ad)^3\log(c+dx)}{3bd^3}+\frac{Bnx(bc-ad)^2}{3d^2}-\frac{Bn(a+bx)^2(bc-ad)}{6bd}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]),x]
```

```
[Out] (B*(b*c - a*d)^2*n*x)/(3*d^2) - (B*(b*c - a*d)*n*(a + b*x)^2)/(6*b*d) - (B*
(b*c - a*d)^3*n*Log[c + d*x])/(3*b*d^3) + ((a + b*x)^3*(A + B*Log[(e*(a + b
*x)^n)/(c + d*x)^n]))/(3*b)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2548

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Dist[B*n*((b*c
- a*d)/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /;
 FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c -
 a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^2 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right) dx &= \int \left(A(a+bx)^2+B(a+bx)^2\log\left(e(a+bx)^n(c+dx\right.\right.\\ &= \frac{A(a+bx)^3}{3b}+B\int (a+bx)^2\log\left(e(a+bx)^n(c+dx)\right.\\ &= \frac{A(a+bx)^3}{3b}+\frac{B(a+bx)^3\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{3b}\\ &= \frac{A(a+bx)^3}{3b}+\frac{B(a+bx)^3\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{3b}\\ &= \frac{B(bc-ad)^2nx}{3d^2}-\frac{B(bc-ad)n(a+bx)^2}{6bd}+\frac{A(a+bx}{3b}\end{aligned}$$

Mathematica [A]
time = 0.11, size = 171, normalized size = 1.51

$$\frac{2a^3Bd^3n\log(a+bx)-2bBc(b^2c^2-3abcd+3a^2d^2)\,n\log(c+dx)+bdx(2a^2d^2(3A+2Bn)+abd(-6Bcn+6Adx+Bdnx)+b^2(2Ad^2x^2+Bcn(2c-dx))+2Bd^2(3a^2+3abx+b^2x^2)\log\left(e(a+bx)^n(c+dx)^{-n}\right))}{6bd^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]),x]
```

```
[Out] (2*a^3*B*d^3*n*Log[a + b*x] - 2*b*B*c*(b^2*c^2 - 3*a*b*c*d + 3*a^2*d^2)*n*L
og[c + d*x] + b*d*x*(2*a^2*d^2*(3*A + 2*B*n) + a*b*d*(-6*B*c*n + 6*A*d*x +
B*d*n*x) + b^2*(2*A*d^2*x^2 + B*c*n*(2*c - d*x)) + 2*B*d^2*(3*a^2 + 3*a*b*x
 + b^2*x^2)*Log[(e*(a + b*x)^n)/(c + d*x)^n]))/(6*b*d^3)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.33, size = 1323, normalized size = 11.71

method	result	size
risch	Expression too large to display	1323

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n))),x,method=_RETURNVERBOSE)
```

```
[Out] x*A*a^2+B*a^2*x*ln((b*x+a)^n)+B*ln(e)*a^2*x+1/3*b^2*B*x^3*ln((b*x+a)^n)+1/3
*b^2*B*ln(e)*x^3+1/3/b*B*a^3*ln((b*x+a)^n)+1/3*A*b^2*x^3+1/6*b*B*a*n*x^2-1/
6/d*b^2*B*c*n*x^2+2/3*B*a^2*n*x+1/3/d^2*b^2*B*c^2*n*x-1/3/d^3*b^2*B*ln(d*x+
c)*c^3*n-1/d*B*ln(d*x+c)*a^2*c*n-1/2*I*B*Pi*a^2*x*csgn(I*(b*x+a)^n/((d*x+c)
^n))^3-1/3*(b*x+a)^3*B/b*ln((d*x+c)^n)+1/2*I*b*B*Pi*a*x^2*csgn(I/((d*x+c)^n
))*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+1/2*I*b*B*Pi*a*x^2*csgn(I*(b*x+a)^n/((d*
x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2-1/2*I*B*Pi*a^2*x*csgn(I*e)*csgn(
```

```
I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)-1/2*I*B*Pi*a^2*x*c
sgn(I*(b*x+a)^n)*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))+b*A*a*x^
2-1/d*b*B*a*c*n*x+1/d^2*b*B*ln(d*x+c)*a*c^2*n-1/2*I*B*Pi*a^2*x*csgn(I*e/((d
*x+c)^n)*(b*x+a)^n)^3-1/6*I*b^2*B*Pi*x^3*csgn(I*(b*x+a)^n/((d*x+c)^n))^3-1/
6*I*b^2*B*Pi*x^3*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^3-1/6*I*b^2*B*Pi*x^3*csgn(
I*e)*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)-1/6*I*b^
2*B*Pi*x^3*csgn(I*(b*x+a)^n)*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^
n))+1/2*I*b*B*Pi*a*x^2*csgn(I*e)*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2+1/2*I*b*
B*Pi*a*x^2*csgn(I*(b*x+a)^n)*csgn(I*(b*x+a)^n/((d*x+c)^n))^2-1/2*I*b*B*Pi*a
*x^2*csgn(I*e)*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n
)-1/2*I*b*B*Pi*a*x^2*csgn(I*(b*x+a)^n)*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n
/((d*x+c)^n))-1/2*I*b*B*Pi*a*x^2*csgn(I*(b*x+a)^n/((d*x+c)^n))^3-1/2*I*b*B*
Pi*a*x^2*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^3+1/2*I*B*Pi*a^2*x*csgn(I*e)*csgn(
I*e/((d*x+c)^n)*(b*x+a)^n)^2+1/2*I*B*Pi*a^2*x*csgn(I*(b*x+a)^n)*csgn(I*(b*x
+a)^n/((d*x+c)^n))^2+1/2*I*B*Pi*a^2*x*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/
((d*x+c)^n))^2+1/2*I*B*Pi*a^2*x*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*
x+c)^n)*(b*x+a)^n)^2+1/6*I*b^2*B*Pi*x^3*csgn(I*e)*csgn(I*e/((d*x+c)^n)*(b*x
+a)^n)^2+1/6*I*b^2*B*Pi*x^3*csgn(I*(b*x+a)^n)*csgn(I*(b*x+a)^n/((d*x+c)^n))
^2+1/6*I*b^2*B*Pi*x^3*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+1
/6*I*b^2*B*Pi*x^3*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a
)^n)^2+b*B*ln(e)*a*x^2+1/3/b*B*ln(d*x+c)*a^3*n+b*B*a*x^2*ln((b*x+a)^n)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 300 vs. $2(106) = 212$.
time = 0.33, size = 300, normalized size = 2.65

$$\frac{1}{3}\,Bb^{2}x^{3}\log\left(\frac{(bx+a)^{n}e}{(dx+c)^{n}}\right)+\frac{1}{3}\,Ab^{2}x^{3}+Babx^{2}\log\left(\frac{(bx+a)^{n}e}{(dx+c)^{n}}\right)+Aabx^{2}+\left(\frac{ane\log\left(bx+a\right)}{b}-\frac{cne\log\left(dx+c\right)}{d}\right)Ba^{2}e^{(-1)}-\left(\frac{a^{2}ne\log\left(bx+a\right)}{b^{2}}-\frac{c^{2}ne\log\left(dx+c\right)}{d^{2}}+\frac{(bcn-adn)xe}{bd}\right)Babe^{(-1)}+\frac{1}{6}\left(\frac{2\,a^{3}ne\log\left(bx+a\right)}{b^{3}}-\frac{2\,c^{3}ne\log\left(dx+c\right)}{d^{3}}-\frac{(b^{2}cdn-abd^{2}n)x^{2}e-2\,(b^{2}c^{2}n-a^{2}d^{2}n)xe}{b^{2}d^{2}}\right)Bb^{2}e^{(-1)}+Ba^{2}x\log\left(\frac{(bx+a)^{n}e}{(dx+c)^{n}}\right)+Aa^{2}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="maxima
")
```

```
[Out] 1/3*B*b^2*x^3*log((b*x + a)^n*e/(d*x + c)^n) + 1/3*A*b^2*x^3 + B*a*b*x^2*lo
g((b*x + a)^n*e/(d*x + c)^n) + A*a*b*x^2 + (a*n*e*log(b*x + a)/b - c*n*e*lo
g(d*x + c)/d)*B*a^2*e^(-1) - (a^2*n*e*log(b*x + a)/b^2 - c^2*n*e*log(d*x +
c)/d^2 + (b*c*n - a*d*n)*x*e/(b*d))*B*a*b*e^(-1) + 1/6*(2*a^3*n*e*log(b*x +
 a)/b^3 - 2*c^3*n*e*log(d*x + c)/d^3 - ((b^2*c*d*n - a*b*d^2*n)*x^2*e - 2*(
b^2*c^2*n - a^2*d^2*n)*x*e)/(b^2*d^2))*B*b^2*e^(-1) + B*a^2*x*log((b*x + a)
^n*e/(d*x + c)^n) + A*a^2*x
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 248 vs. $2(106) = 212$.
time = 0.35, size = 248, normalized size = 2.19

$$\frac{2\,(A+B)b^{3}d^{3}x^{3}+(6\,(A+B)ab^{2}d^{3}-(Bb^{3}cd^{2}-Bab^{2}d^{3})n)x^{2}+2\,(3\,(A+B)a^{2}bd^{3}+(Bb^{3}c^{2}d-3\,Bab^{2}cd^{2}+2\,Ba^{2}bd^{3})n)x+2\,(Bb^{3}d^{3}nx^{3}+3\,Bab^{2}d^{3}nx^{2}+3\,Ba^{2}bd^{3}nx+Ba^{3}d^{3}n)\log\left(bx+a\right)-2\,(Bb^{3}d^{3}nx^{3}+3\,Bab^{2}d^{3}nx^{2}+3\,Ba^{2}bd^{3}nx+(Bb^{3}c^{3}-3\,Bab^{2}c^{2}d+3\,Ba^{2}bcd^{2})n)\log\left(dx+c\right)}{6\,bd^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="fricas
")
```

```
[Out] 1/6*(2*(A + B)*b^3*d^3*x^3 + (6*(A + B)*a*b^2*d^3 - (B*b^3*c*d^2 - B*a*b^2*
d^3)*n)*x^2 + 2*(3*(A + B)*a^2*b*d^3 + (B*b^3*c^2*d - 3*B*a*b^2*c*d^2 + 2*B
*a^2*b*d^3)*n)*x + 2*(B*b^3*d^3*n*x^3 + 3*B*a*b^2*d^3*n*x^2 + 3*B*a^2*b*d^3
*n*x + B*a^3*d^3*n)*log(b*x + a) - 2*(B*b^3*d^3*n*x^3 + 3*B*a*b^2*d^3*n*x^2
 + 3*B*a^2*b*d^3*n*x + (B*b^3*c^3 - 3*B*a*b^2*c^2*d + 3*B*a^2*b*c*d^2)*n)*l
og(d*x + c))/(b*d^3)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*(A+B*ln(e*(b*x+a)**n/((d*x+c)**n))),x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 235 vs. 2(106) = 212.
time = 4.56, size = 235, normalized size = 2.08

$$\frac{Ba^3n\log(bx+a)}{3b}+\frac{1}{3}(Ab^2+Bb^2)x^3-\frac{(Bb^2cn-Babdn-6\,Aabd-6\,Babd)x^2}{6\,d}+\frac{1}{3}(Bb^2nx^3+3\,Babnx^2+3\,Ba^2nx)\log(bx+a)-\frac{1}{3}(Bb^2nx^3+3\,Babnx^2+3\,Ba^2nx)\log(dx+c)+\frac{(Bb^2c^2n-3\,Babcdn+2\,Ba^2d^2n+3\,Aa^2d^2+3\,Ba^2d^2)x}{3\,d^2}-\frac{(Bb^2c^3n-3\,Babc^2dn+3\,Ba^2cd^2n)\log(-dx-c)}{3\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="giac")
```

```
[Out] 1/3*B*a^3*n*log(b*x + a)/b + 1/3*(A*b^2 + B*b^2)*x^3 - 1/6*(B*b^2*c*n - B*a
*b*d*n - 6*A*a*b*d - 6*B*a*b*d)*x^2/d + 1/3*(B*b^2*n*x^3 + 3*B*a*b*n*x^2 +
3*B*a^2*n*x)*log(b*x + a) - 1/3*(B*b^2*n*x^3 + 3*B*a*b*n*x^2 + 3*B*a^2*n*x)
*log(d*x + c) + 1/3*(B*b^2*c^2*n - 3*B*a*b*c*d*n + 2*B*a^2*d^2*n + 3*A*a^2*
d^2 + 3*B*a^2*d^2)*x/d^2 - 1/3*(B*b^2*c^3*n - 3*B*a*b*c^2*d*n + 3*B*a^2*c*d
^2*n)*log(-d*x - c)/d^3
```

Mupad [**B**]
time = 4.24, size = 262, normalized size = 2.32

$$\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\,\left(B\,a^2\,x+B\,a\,b\,x^2+\frac{B\,b^2\,x^3}{3}\right)+x^2\,\left(\frac{b\,(9\,A\,a\,d+3\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{6\,d}-\frac{A\,b\,(3\,a\,d+3\,b\,c)}{6\,d}\right)-x\,\left(\frac{\left(\frac{b\,(9\,A\,a\,d+3\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{3\,d}-\frac{A\,b\,(3\,a\,d+3\,b\,c)}{3\,d}\right)\,(3\,a\,d+3\,b\,c)}{3\,b\,d}-\frac{a\,(3\,A\,a\,d+3\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{d}+\frac{A\,a\,b\,c}{d}\right)+\frac{A\,b^2\,x^3}{3}-\frac{\ln(c+d\,x)\,(3\,B\,n\,a^2\,c\,d^2-3\,B\,n\,a\,b\,c^2\,d+B\,n\,b^2\,c^3)}{3\,d^3}+\frac{B\,a^3\,n\,\ln(a+b\,x)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))*(a + b*x)^2,x)
```

```
[Out] log((e*(a + b*x)^n)/(c + d*x)^n)*((B*b^2*x^3)/3 + B*a^2*x + B*a*b*x^2) + x^
2*((b*(9*A*a*d + 3*A*b*c + B*a*d*n - B*b*c*n))/(6*d) - (A*b*(3*a*d + 3*b*c)
)/(6*d)) - x*((((b*(9*A*a*d + 3*A*b*c + B*a*d*n - B*b*c*n))/(3*d) - (A*b*(3
```

```
*a*d + 3*b*c))/(3*d))*(3*a*d + 3*b*c))/(3*b*d) - (a*(3*A*a*d + 3*A*b*c + B*
a*d*n - B*b*c*n))/d + (A*a*b*c)/d) + (A*b^2*x^3)/3 - (log(c + d*x)*(B*b^2*c
^3*n + 3*B*a^2*c*d^2*n - 3*B*a*b*c^2*d*n))/(3*d^3) + (B*a^3*n*log(a + b*x))
/(3*b)
```

3.150 $\int (a+bx) (A+B\log (e(a+bx)^n(c+dx)^{-n})) \, dx$

Optimal. Leaf size=84

$$-\frac{B(bc-ad)nx}{2d}+\frac{B(bc-ad)^2n\log(c+dx)}{2bd^2}+\frac{(a+bx)^2\,(A+B\log(e(a+bx)^n(c+dx)^{-n}))}{2b}$$

```
[Out] -1/2*B*(-a*d+b*c)*n*x/d+1/2*B*(-a*d+b*c)^2*n*ln(d*x+c)/b/d^2+1/2*(b*x+a)^2*
(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/b
```

Rubi [A]
time = 0.03, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {2548, 45}

$$\frac{(a+bx)^2\,(B\log(e(a+bx)^n(c+dx)^{-n})+A)}{2b}+\frac{Bn(bc-ad)^2\log(c+dx)}{2bd^2}-\frac{Bnx(bc-ad)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]),x]
```

```
[Out] -1/2*(B*(b*c - a*d)*n*x)/d + (B*(b*c - a*d)^2*n*Log[c + d*x])/(2*b*d^2) + (
(a + b*x)^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]))/(2*b)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2548

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Dist[B*n*((b*c
- a*d)/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /;
 FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c -
 a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Rubi steps

$$\int (a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\,dx = \int \left(A(a+bx)+B(a+bx)\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right.$$
$$= \frac{A(a+bx)^2}{2b} + B\int (a+bx)\log\left(e(a+bx)^n(c+dx)^{-n}\right)$$
$$= \frac{A(a+bx)^2}{2b} + \frac{B(a+bx)^2\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{2b} -$$
$$= \frac{A(a+bx)^2}{2b} + \frac{B(a+bx)^2\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{2b} -$$
$$= -\frac{B(bc-ad)nx}{2d} + \frac{A(a+bx)^2}{2b} + \frac{B(bc-ad)^2 n\log(c+}{2bd^2}$$

Mathematica [A]
time = 0.06, size = 100, normalized size = 1.19

$$\frac{a^2Bd^2n\log(a+bx)+bBc(bc-2ad)n\log(c+dx)+bdx(-bBcn+ad(2A+Bn)+Abdx+Bd(2a+bx)\log\left(e(a+bx)^n(c+dx)^{-n}\right))}{2bd^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]),x]
```

```
[Out] (a^2*B*d^2*n*Log[a + b*x] + b*B*c*(b*c - 2*a*d)*n*Log[c + d*x] + b*d*x*(-(b
*B*c*n) + a*d*(2*A + B*n) + A*b*d*x + B*d*(2*a + b*x)*Log[(e*(a + b*x)^n)/(
c + d*x)^n]))/(2*b*d^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.20, size = 817, normalized size = 9.73

method	result
risch	$B\ln(e)\,ax+\frac{B\ln(e)b\,x^2}{2}+\frac{bB\,x^2\ln((bx+a)^n)}{2}+\frac{Ab\,x^2}{2}+xAa-\frac{Bx(bx+2a)\ln((dx+c)^n)}{2}+\ln((bx+a)^n)\,xBa$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n))),x,method=_RETURNVERBOSE)
```

```
[Out] B*ln(e)*a*x+1/2*B*ln(e)*b*x^2+1/2*b*B*x^2*ln((b*x+a)^n)+1/2*A*b*x^2+x*A*a-1
/2*B*x*(b*x+2*a)*ln((d*x+c)^n)+ln((b*x+a)^n)*x*B*a+1/2*I*B*Pi*a*x*csgn(I/((
d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+1/2*I*B*Pi*a*x*csgn(I*(b*x+a)^n)
*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+1/2*I*B*Pi*a*x*csgn(I*(b*x+a)^n/((d*x+c)^n
))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2+1/2*I*B*Pi*a*x*csgn(I*e)*csgn(I*e/((d*
x+c)^n)*(b*x+a)^n)^2+1/4*I*b*B*Pi*x^2*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/
((d*x+c)^n))^2+1/4*I*b*B*Pi*x^2*csgn(I*(b*x+a)^n)*csgn(I*(b*x+a)^n/((d*x+c)
^n))^2+1/4*I*b*B*Pi*x^2*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*
```

```
(b*x+a)^n)^2+1/4*I*b*B*Pi*x^2*csgn(I*e)*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2+1
/2*B*n*a*x+1/2*B*a^2*n/b*ln(-b*x-a)-1/4*I*b*B*Pi*x^2*csgn(I*e)*csgn(I*(b*x+
a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)-1/2*I*B*Pi*a*x*csgn(I*(b*
x+a)^n)*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))-1/2*I*B*Pi*a*x*cs
gn(I*e)*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)-1/4*I
*b*B*Pi*x^2*csgn(I*(b*x+a)^n)*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)
^n))-1/2*b/d*B*c*n*x-1/d*B*ln(d*x+c)*a*c*n+1/2*b/d^2*B*ln(d*x+c)*c^2*n-1/2*
I*B*Pi*a*x*csgn(I*(b*x+a)^n/((d*x+c)^n))^3-1/2*I*B*Pi*a*x*csgn(I*e/((d*x+c)
^n)*(b*x+a)^n)^3-1/4*I*b*B*Pi*x^2*csgn(I*(b*x+a)^n/((d*x+c)^n))^3-1/4*I*b*B
*Pi*x^2*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^3
```

Maxima [A]
time = 0.29, size = 158, normalized size = 1.88

$$\frac{1}{2}Bbx^2\log\left(\frac{(bx+a)^ne}{(dx+c)^n}\right)+\frac{1}{2}Abx^2+\left(\frac{ane\log(bx+a)}{b}-\frac{cne\log(dx+c)}{d}\right)Bae^{(-1)}-\frac{1}{2}\left(\frac{a^2ne\log(bx+a)}{b^2}-\frac{c^2ne\log(dx+c)}{d^2}+\frac{(bcn-adn)xe}{bd}\right)Bbe^{(-1)}+Bax\log\left(\frac{(bx+a)^ne}{(dx+c)^n}\right)+Aax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="maxima")
```

```
[Out] 1/2*B*b*x^2*log((b*x + a)^n*e/(d*x + c)^n) + 1/2*A*b*x^2 + (a*n*e*log(b*x +
 a)/b - c*n*e*log(d*x + c)/d)*B*a*e^(-1) - 1/2*(a^2*n*e*log(b*x + a)/b^2 -
c^2*n*e*log(d*x + c)/d^2 + (b*c*n - a*d*n)*x*e/(b*d))*B*b*e^(-1) + B*a*x*lo
g((b*x + a)^n*e/(d*x + c)^n) + A*a*x
```

Fricas [A]
time = 0.35, size = 143, normalized size = 1.70

$$\frac{(A+B)b^2d^2x^2+(2(A+B)abd^2-(Bb^2cd-Babd^2)n)x+(Bb^2d^2nx^2+2Babd^2nx+Ba^2d^2n)\log(bx+a)-(Bb^2d^2nx^2+2Babd^2nx-(Bb^2c^2-2Babcd)n)\log(dx+c)}{2bd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="fricas")
```

```
[Out] 1/2*((A + B)*b^2*d^2*x^2 + (2*(A + B)*a*b*d^2 - (B*b^2*c*d - B*a*b*d^2)*n)*
x + (B*b^2*d^2*n*x^2 + 2*B*a*b*d^2*n*x + B*a^2*d^2*n)*log(b*x + a) - (B*b^2
*d^2*n*x^2 + 2*B*a*b*d^2*n*x - (B*b^2*c^2 - 2*B*a*b*c*d)*n)*log(d*x + c))/(
b*d^2)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(A+B*ln(e*(b*x+a)**n/((d*x+c)**n))),x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [A]
time = 3.98, size = 127, normalized size = 1.51

$$\frac{Ba^2n\log(bx+a)}{2b}+\frac{1}{2}(Ab+Bb)x^2+\frac{1}{2}(Bbnx^2+2Banx)\log(bx+a)-\frac{1}{2}(Bbnx^2+2Banx)\log(dx+c)-\frac{(Bbcn-Badn-2Aad-2Bad)x}{2d}+\frac{(Bbc^2n-2Bacdn)\log(dx+c)}{2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="giac")
```

```
[Out] 1/2*B*a^2*n*log(b*x + a)/b + 1/2*(A*b + B*b)*x^2 + 1/2*(B*b*n*x^2 + 2*B*a*n
*x)*log(b*x + a) - 1/2*(B*b*n*x^2 + 2*B*a*n*x)*log(d*x + c) - 1/2*(B*b*c*n
- B*a*d*n - 2*A*a*d - 2*B*a*d)*x/d + 1/2*(B*b*c^2*n - 2*B*a*c*d*n)*log(d*x
+ c)/d^2
```

Mupad [B]
time = 4.28, size = 127, normalized size = 1.51

$$\ln\left(\frac{e\,{\left(a+b\,x\right)}^n}{{\left(c+d\,x\right)}^n}\right)\,\left(\frac{B\,b\,x^2}{2}+B\,a\,x\right)+x\,\left(\frac{4\,A\,a\,d+2\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n}{2\,d}-\frac{A\,\left(2\,a\,d+2\,b\,c\right)}{2\,d}\right)+\frac{\ln\left(c+d\,x\right)\,\left(B\,b\,c^2\,n-2\,B\,a\,c\,d\,n\right)}{2\,d^2}+\frac{A\,b\,x^2}{2}+\frac{B\,a^2\,n\,\ln\left(a+b\,x\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))*(a + b*x),x)
```

```
[Out] log((e*(a + b*x)^n)/(c + d*x)^n)*(B*a*x + (B*b*x^2)/2) + x*((4*A*a*d + 2*A*
b*c + B*a*d*n - B*b*c*n)/(2*d) - (A*(2*a*d + 2*b*c))/(2*d)) + (log(c + d*x)
*(B*b*c^2*n - 2*B*a*c*d*n))/(2*d^2) + (A*b*x^2)/2 + (B*a^2*n*log(a + b*x))/
(2*b)
```

3.151 $\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{a+bx}\,dx$

Optimal. Leaf size=79

$$-\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{b}+\frac{Bn\text{Li}_2\left(1+\frac{bc-ad}{d(a+bx)}\right)}{b}$$

[Out] -ln((a*d-b*c)/d/(b*x+a))*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/b+B*n*polylog(2,1+(-a*d+b*c)/d/(b*x+a))/b

Rubi [A]
time = 0.14, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {2542, 2458, 2378, 2370, 2352}

$$\frac{Bn\text{PolyLog}\left(2,\frac{bc-ad}{d(a+bx)}+1\right)}{b}-\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{b}$$

Antiderivative was successfully verified.

[In] Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(a + b*x),x]

[Out] -((Log[-((b*c - a*d)/(d*(a + b*x)))]*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]))/b) + (B*n*PolyLog[2, 1 + (b*c - a*d)/(d*(a + b*x))])/b

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2370

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)/(x_))^(q_.)*(
x_)^(m_.), x_Symbol] :> Int[(e + d*x)^q*(a + b*Log[c*x^n])^p, x] /; FreeQ[{
a, b, c, d, e, m, n, p}, x] && EqQ[m, q] && IntegerQ[q]
```

Rule 2378

```
Int[((a_.) + Log[(c_.)*(x_)^(n_)]*(b_.))/((x_)*((d_) + (e_.)*(x_)^(r_.))),
x_Symbol] :> Dist[1/n, Subst[Int[(a + b*Log[c*x])/(x*(d + e*x^(r/n))), x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IntegerQ[r/n]
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
```

```
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2542

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))/((f_.) + (g_.)*(x_)), x_Symbol] :> Simp[(-Log[-(b*c - a*d)/(d*(a
+ b*x))])*((A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/g), x] + Dist[B*n*((b*c
 - a*d)/g), Int[Log[-(b*c - a*d)/(d*(a + b*x))]/((a + b*x)*(c + d*x)), x],
x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
 - a*d, 0] && EqQ[b*f - a*g, 0]
```

Rubi steps

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{a+bx}\,dx=\int\left(\frac{A}{a+bx}+\frac{B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{a+bx}\right)dx$$

$$=\frac{A\log(a+bx)}{b}+B\int\frac{\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{a+bx}\,dx$$

$$=\frac{A\log(a+bx)}{b}-\frac{B\log\left(-\frac{bc-ad}{d(a+bx)}\right)\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{b}+\frac{(}{}$$

$$=\frac{A\log(a+bx)}{b}-\frac{B\log\left(-\frac{bc-ad}{d(a+bx)}\right)\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{b}+\frac{(}{}$$

$$=\frac{A\log(a+bx)}{b}-\frac{B\log\left(-\frac{bc-ad}{d(a+bx)}\right)\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{b}-\frac{(}{}$$

$$=\frac{A\log(a+bx)}{b}-\frac{B\log\left(-\frac{bc-ad}{d(a+bx)}\right)\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{b}-\frac{(}{}$$

$$=\frac{A\log(a+bx)}{b}-\frac{B\log\left(-\frac{bc-ad}{d(a+bx)}\right)\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{b}+\frac{B}{}$$

Mathematica [A]
time = 0.06, size = 95, normalized size = 1.20

$$\frac{\log(a+bx)\left(-Bn\log(a+bx)+2\left(A+Bn\log\left(\frac{b(c+dx)}{bc-ad}\right)+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\right)+2Bn\,\mathrm{Li}_2\left(\frac{d(a+bx)}{-bc+ad}\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(a + b*x),x]
```

```
[Out] (Log[a + b*x]*(-(B*n*Log[a + b*x]) + 2*(A + B*n*Log[(b*(c + d*x))/(b*c - a*
d)] + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])) + 2*B*n*PolyLog[2, (d*(a + b*x))
/(-(b*c) + a*d)])/(2*b)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.50, size = 523, normalized size = 6.62

method	result
risch	$-\frac{B\ln(bx+a)\ln((dx+c)^{n})}{b}+\frac{Bn\,\mathrm{dilog}\left(\frac{-ad+cb+(bx+a)d}{-ad+cb}\right)}{b}+\frac{Bn\ln(bx+a)\ln\left(\frac{-ad+cb+(bx+a)d}{-ad+cb}\right)}{b}+\frac{iB\ln(bx+a)\pi\,\mathrm{csgn}(i(bx+a)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] -B/b*ln(b*x+a)*ln((d*x+c)^n)+1/b*B*n*dilog((-a*d+c*b+(b*x+a)*d)/(-a*d+b*c))
+1/b*B*n*ln(b*x+a)*ln((-a*d+c*b+(b*x+a)*d)/(-a*d+b*c))+1/2*I/b*B*ln(b*x+a)*
Pi*csgn(I*(b*x+a)^n)*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+1/2*I/b*B*ln(b*x+a)*Pi
*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+1/2*I/b*B*ln(b*x+a)*Pi
*csgn(I*e)*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2+1/2*I/b*B*ln(b*x+a)*Pi*csgn(I*
(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2+A*ln(b*x+a)/b+1/b*
B*ln(b*x+a)*ln(e)+1/2/b*B/n*ln((b*x+a)^n)^2-1/2*I/b*B*ln(b*x+a)*Pi*csgn(I*(
b*x+a)^n/((d*x+c)^n))^3-1/2*I/b*B*ln(b*x+a)*Pi*csgn(I*e/((d*x+c)^n)*(b*x+a)
^n)^3-1/2*I/b*B*ln(b*x+a)*Pi*csgn(I*(b*x+a)^n)*csgn(I/((d*x+c)^n))*csgn(I*(
b*x+a)^n/((d*x+c)^n))-1/2*I/b*B*ln(b*x+a)*Pi*csgn(I*e)*csgn(I*(b*x+a)^n/((d
*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(b*x+a),x, algorithm="maxima")
```

```
[Out] B*((log(b*x + a)*log((b*x + a)^n) - log(b*x + a)*log((d*x + c)^n))/b + inte
grate((b*d*x + b*c - (b*c*n - a*d*n)*log(b*x + a))/(b^2*d*x^2 + a*b*c + (b^
2*c + a*b*d)*x), x)) + A*log(b*x + a)/b
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(b*x+a),x, algorithm="fricas")
```

```
[Out] integral((B*log((b*x + a)^n*e/(d*x + c)^n) + A)/(b*x + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B\log\left(e(a + bx)^{n}\left(c + dx\right)^{-n}\right)}{a + bx}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))/(b*x+a),x)
[Out] Integral((A + B*log(e*(a + b*x)**n/(c + d*x)**n))/(a + b*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(b*x+a),x, algorithm="giac")
[Out] integrate((B*log((b*x + a)^n*e/(d*x + c)^n) + A)/(b*x + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A + B\,\ln\left(\frac{e\,{\left(a+b\,x\right)}^n}{{\left(c+d\,x\right)}^n}\right)}{a + b\,x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))/(a + b*x),x)
[Out] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))/(a + b*x), x)
```

3.152 $\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^2}\,dx$

Optimal. Leaf size=97

$$-\frac{Bn}{b(a+bx)}-\frac{Bdn\log(a+bx)}{b(bc-ad)}+\frac{Bdn\log(c+dx)}{b(bc-ad)}-\frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{b(a+bx)}$$

```
[Out] -B*n/b/(b*x+a)-B*d*n*ln(b*x+a)/b/(-a*d+b*c)+B*d*n*ln(d*x+c)/b/(-a*d+b*c)+(-
A-B*ln(e*(b*x+a)^n/((d*x+c)^n)))/b/(b*x+a)
```

Rubi [A]
time = 0.05, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2548, 46}

$$-\frac{B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A}{b(a+bx)}-\frac{Bdn\log(a+bx)}{b(bc-ad)}+\frac{Bdn\log(c+dx)}{b(bc-ad)}-\frac{Bn}{b(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(a + b*x)^2,x]
```

```
[Out] -((B*n)/(b*(a + b*x))) - (B*d*n*Log[a + b*x])/(b*(b*c - a*d)) + (B*d*n*Log[
c + d*x])/(b*(b*c - a*d)) - (A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(b*(a
+ b*x))
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2548

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Dist[B*n*((b*c
- a*d)/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /;
 FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c -
 a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Rubi steps

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^2}\,dx = \int \left(\frac{A}{(a+bx)^2}+\frac{B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^2}\right)dx$$
$$= -\frac{A}{b(a+bx)}+B\int \frac{\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^2}\,dx$$
$$= -\frac{A}{b(a+bx)}-\frac{B(c+dx)\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(bc-ad)(a+bx)}+(Bn)\int \frac{}{(a}$$
$$= -\frac{A}{b(a+bx)}-\frac{Bn}{b(a+bx)}-\frac{B(c+dx)\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(bc-ad)(a+bx)}$$

Mathematica [A]
time = 0.07, size = 89, normalized size = 0.92

$$\frac{-Bdn(a+bx)\log(a+bx)+Bdn(a+bx)\log(c+dx)-(bc-ad)\left(A+Bn+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{b(bc-ad)(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(a + b*x)^2,x]
```

```
[Out] (-(B*d*n*(a + b*x)*Log[a + b*x]) + B*d*n*(a + b*x)*Log[c + d*x] - (b*c - a*
d)*(A + B*n + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]))/(b*(b*c - a*d)*(a + b*x)
)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.25, size = 823, normalized size = 8.48

method	result
risch	$\frac{B\ln((dx+c)^n)}{b(bx+a)}-\frac{2Abc+iB\pi ad\,\mathrm{csgn}(ie)\mathrm{csgn}\left(i(bx+a)^n(dx+c)^{-n}\right)\mathrm{csgn}\left(ie(dx+c)^{-n}(bx+a)^n\right)-iB\pi bc\,\mathrm{csgn}(ie)\mathrm{csgn}\left(i(bx+a)^n(dx+c\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] B/b/(b*x+a)*ln((d*x+c)^n)-1/2*(2*A*b*c-2*B*a*d*n+2*B*b*c*n-I*B*Pi*b*c*csgn(
I*e/((d*x+c)^n)*(b*x+a)^n)^3+I*B*Pi*a*d*csgn(I*(b*x+a)^n)*csgn(I/((d*x+c)^n
))*csgn(I*(b*x+a)^n/((d*x+c)^n))-I*B*Pi*b*c*csgn(I*(b*x+a)^n)*csgn(I/((d*x+
c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))+I*B*Pi*a*d*csgn(I*e)*csgn(I*(b*x+a)^n/
((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)-I*B*Pi*b*c*csgn(I*e)*csgn(I*(b
*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)-2*B*a*d*ln((b*x+a)^n)+
2*B*ln(e)*b*c-2*B*ln(e)*a*d-2*B*ln(d*x+c)*a*d*n+2*B*ln(-b*x-a)*a*d*n-2*A*a*
d+2*B*ln((b*x+a)^n)*b*c+I*B*Pi*b*c*csgn(I*e)*csgn(I*e/((d*x+c)^n)*(b*x+a)^n
)^2+I*B*Pi*b*c*csgn(I*(b*x+a)^n)*csgn(I*(b*x+a)^n/((d*x+c)^n))^2-I*B*Pi*a*d
```

```
*csgn(I*(b*x+a)^n)*csgn(I*(b*x+a)^n/((d*x+c)^n))^2-2*B*ln(d*x+c)*b*d*n*x+2*
B*ln(-b*x-a)*b*d*n*x+I*B*Pi*a*d*csgn(I*(b*x+a)^n/((d*x+c)^n))^3+I*B*Pi*a*d*
csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^3-I*B*Pi*b*c*csgn(I*(b*x+a)^n/((d*x+c)^n))^
3-I*B*Pi*a*d*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^
2+I*B*Pi*b*c*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+I*B*Pi*b*c
*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2-I*B*Pi*a*d
*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))^2-I*B*Pi*a*d*csgn(I*e)*c
sgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2)/(b*x+a)/b/(-a*d+b*c)
```

Maxima [**A**]
time = 0.29, size = 119, normalized size = 1.23

$$-\left(\frac{dne\log\left(bx+a\right)}{b^2c-abd}-\frac{dne\log\left(dx+c\right)}{b^2c-abd}+\frac{ne}{b^2x+ab}\right)Be^{(-1)}-\frac{B\log\left(\frac{(bx+a)^ne}{(dx+c)^n}\right)}{b^2x+ab}-\frac{A}{b^2x+ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(b*x+a)^2,x, algorithm="maxima
")

[Out] -(d*n*e*log(b*x + a)/(b^2*c - a*b*d) - d*n*e*log(d*x + c)/(b^2*c - a*b*d) +
 n*e/(b^2*x + a*b))*B*e^(-1) - B*log((b*x + a)^n*e/(d*x + c)^n)/(b^2*x + a*
b) - A/(b^2*x + a*b)
```

Fricas [**A**]
time = 0.36, size = 98, normalized size = 1.01

$$-\frac{(A+B)bc-(A+B)ad+(Bbc-Bad)n+(Bbdnx+Bbcn)\log\left(bx+a\right)-(Bbdnx+Bbcn)\log\left(dx+c\right)}{ab^2c-a^2bd+(b^3c-ab^2d)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(b*x+a)^2,x, algorithm="fricas
")

[Out] -((A + B)*b*c - (A + B)*a*d + (B*b*c - B*a*d)*n + (B*b*d*n*x + B*b*c*n)*log
(b*x + a) - (B*b*d*n*x + B*b*c*n)*log(d*x + c))/(a*b^2*c - a^2*b*d + (b^3*c
 - a*b^2*d)*x)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))/(b*x+a)**2,x)

[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [A]
time = 3.64, size = 108, normalized size = 1.11

$$-\frac{Bdn\log\left(bx+a\right)}{b^2c-abd}+\frac{Bdn\log\left(dx+c\right)}{b^2c-abd}-\frac{Bn\log\left(bx+a\right)}{b^2x+ab}+\frac{Bn\log\left(dx+c\right)}{b^2x+ab}-\frac{Bn+A+B}{b^2x+ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -B*d*n*log(b*x + a)/(b^2*c - a*b*d) + B*d*n*log(d*x + c)/(b^2*c - a*b*d) -
B*n*log(b*x + a)/(b^2*x + a*b) + B*n*log(d*x + c)/(b^2*x + a*b) - (B*n + A
+ B)/(b^2*x + a*b)
```

Mupad [B]
time = 4.91, size = 97, normalized size = 1.00

$$-\frac{A+B\,n}{x\,b^2+a\,b}-\frac{B\,\ln\left(\frac{e\,{\left(a+b\,x\right)}^n}{{\left(c+d\,x\right)}^n}\right)}{b\,\left(a+b\,x\right)}-\frac{B\,d\,n\,\mathrm{atan}\left(\frac{b\,c\,2{}\mathrm{i}+b\,d\,x\,2{}\mathrm{i}}{a\,d-b\,c}+1{}\mathrm{i}\right)\,2{}\mathrm{i}}{b\,\left(a\,d-b\,c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))/(a + b*x)^2,x)
```

```
[Out] - (A + B*n)/(a*b + b^2*x) - (B*log((e*(a + b*x)^n)/(c + d*x)^n))/(b*(a + b*
x)) - (B*d*n*atan((b*c*2i + b*d*x*2i)/(a*d - b*c) + 1i)*2i)/(b*(a*d - b*c))
```

3.153 $\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^3}\,dx$

Optimal. Leaf size=137

$$-\frac{Bn}{4b(a+bx)^2}+\frac{Bdn}{2b(bc-ad)(a+bx)}+\frac{Bd^2n\log(a+bx)}{2b(bc-ad)^2}-\frac{Bd^2n\log(c+dx)}{2b(bc-ad)^2}-\frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{2b(a+bx)^2}$$

```
[Out] -1/4*B*n/b/(b*x+a)^2+1/2*B*d*n/b/(-a*d+b*c)/(b*x+a)+1/2*B*d^2*n*ln(b*x+a)/b
/(-a*d+b*c)^2-1/2*B*d^2*n*ln(d*x+c)/b/(-a*d+b*c)^2+1/2*(-A-B*ln(e*(b*x+a)^n
/((d*x+c)^n)))/b/(b*x+a)^2
```

Rubi [A]
time = 0.05, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {2548, 46}

$$-\frac{B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A}{2b(a+bx)^2}+\frac{Bd^2n\log(a+bx)}{2b(bc-ad)^2}-\frac{Bd^2n\log(c+dx)}{2b(bc-ad)^2}+\frac{Bdn}{2b(a+bx)(bc-ad)}-\frac{Bn}{4b(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(a + b*x)^3,x]
```

```
[Out] -1/4*(B*n)/(b*(a + b*x)^2) + (B*d*n)/(2*b*(b*c - a*d)*(a + b*x)) + (B*d^2*n
*Log[a + b*x])/(2*b*(b*c - a*d)^2) - (B*d^2*n*Log[c + d*x])/(2*b*(b*c - a*d
)^2) - (A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(2*b*(a + b*x)^2)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2548

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Dist[B*n*((b*c
- a*d)/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /;
 FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c -
 a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Rubi steps

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^3}\,dx = \int \left(\frac{A}{(a+bx)^3}+\frac{B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^3}\right)dx$$

$$= -\frac{A}{2b(a+bx)^2}+B\int \frac{\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^3}\,dx$$

$$= -\frac{A}{2b(a+bx)^2}-\frac{B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{2b(a+bx)^2}+\frac{(B(bc-ad)n)\int}{2b}$$

$$= -\frac{A}{2b(a+bx)^2}-\frac{B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{2b(a+bx)^2}+\frac{(B(bc-ad)n)\int}{}$$

$$= -\frac{A}{2b(a+bx)^2}-\frac{Bn}{4b(a+bx)^2}+\frac{Bdn}{2b(bc-ad)(a+bx)}+\frac{Bd^2n\log(a}{2b(bc-}$$

Mathematica [A]
time = 0.09, size = 132, normalized size = 0.96

$$-\frac{-2Bd^2n(a+bx)^2\log(a+bx)+2Bd^2n(a+bx)^2\log(c+dx)+(bc-ad)\left(2A(bc-ad)+Bn(bc-3ad-2bdx)+2B(bc-ad)\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{4b(bc-ad)^2(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(a + b*x)^3,x]
```

```
[Out] -1/4*(-2*B*d^2*n*(a + b*x)^2*Log[a + b*x] + 2*B*d^2*n*(a + b*x)^2*Log[c + d
*x] + (b*c - a*d)*(2*A*(b*c - a*d) + B*n*(b*c - 3*a*d - 2*b*d*x) + 2*B*(b*c
 - a*d)*Log[(e*(a + b*x)^n)/(c + d*x)^n]))/(b*(b*c - a*d)^2*(a + b*x)^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.32, size = 1379, normalized size = 10.07

method	result	size
risch	Expression too large to display	1379

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*B/b/(b*x+a)^2*ln((d*x+c)^n)-1/4*(-I*B*Pi*b^2*c^2*csgn(I*(b*x+a)^n)*csgn
(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))-I*B*Pi*a^2*d^2*csgn(I*e)*csgn
(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)-I*B*Pi*a^2*d^2*cs
gn(I*(b*x+a)^n)*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))+2*B*ln(e)
*a^2*d^2+2*B*ln(e)*b^2*c^2-4*B*ln(-b*x-a)*a*b*d^2*n*x+2*A*b^2*c^2+2*A*a^2*d
^2+I*B*Pi*b^2*c^2*csgn(I*e)*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2+3*d^2*B*a^2*n
+2*I*B*Pi*a*b*c*d*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^3-I*B*Pi*b^2*c^2*csgn(I*e
)*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)-4*A*a*b*c*d
```

```
+B*b^2*c^2*n+I*B*Pi*b^2*c^2*csgn(I*(b*x+a)^n)*csgn(I*(b*x+a)^n/((d*x+c)^n))
^2+I*B*Pi*b^2*c^2*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+I*B*P
i*b^2*c^2*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2+I
*B*Pi*a^2*d^2*csgn(I*e)*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2-4*B*ln(e)*a*b*c*d
+2*I*B*Pi*a*b*c*d*csgn(I*(b*x+a)^n/((d*x+c)^n))^3-2*I*B*Pi*a*b*c*d*csgn(I*e
)*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2-2*I*B*Pi*a*b*c*d*csgn(I*(b*x+a)^n)*csgn
(I*(b*x+a)^n/((d*x+c)^n))^2-2*B*a^2*n*ln(-b*x-a)*d^2-I*B*Pi*b^2*c^2*csgn(I*
e/((d*x+c)^n)*(b*x+a)^n)^3+2*B*ln(d*x+c)*b^2*d^2*n*x^2-2*B*ln(-b*x-a)*b^2*d
^2*n*x^2-4*d*b*B*a*c*n-I*B*Pi*b^2*c^2*csgn(I*(b*x+a)^n/((d*x+c)^n))^3+2*B*l
n(d*x+c)*a^2*d^2*n+2*B*a*b*d^2*n*x-2*B*b^2*c*d*n*x+2*I*B*Pi*a*b*c*d*csgn(I*
e)*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)+2*B*a^2*d^
2*ln((b*x+a)^n)-2*I*B*Pi*a*b*c*d*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x
+c)^n))^2-2*I*B*Pi*a*b*c*d*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^
n)*(b*x+a)^n)^2+2*I*B*Pi*a*b*c*d*csgn(I*(b*x+a)^n)*csgn(I/((d*x+c)^n))*csgn
(I*(b*x+a)^n/((d*x+c)^n))+4*B*ln(d*x+c)*a*b*d^2*n*x+I*B*Pi*a^2*d^2*csgn(I*(
b*x+a)^n)*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+I*B*Pi*a^2*d^2*csgn(I/((d*x+c)^n)
)*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+I*B*Pi*a^2*d^2*csgn(I*(b*x+a)^n/((d*x+c)^
n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2-I*B*Pi*a^2*d^2*csgn(I*(b*x+a)^n/((d*x
+c)^n))^3-I*B*Pi*a^2*d^2*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^3-4*B*a*b*c*d*ln((
b*x+a)^n)+2*B*b^2*c^2*ln((b*x+a)^n))/(b*x+a)^2/(-a*d+b*c)^2/b
```

Maxima [A]
time = 0.31, size = 235, normalized size = 1.72

$$\frac{1}{4}\left(\frac{2\,d^2ne\log\left(bx+a\right)}{b^3c^2-2\,ab^2cd+a^2bd^2}-\frac{2\,d^2ne\log\left(dx+c\right)}{b^3c^2-2\,ab^2cd+a^2bd^2}+\frac{2\,bdnxe-(bcn-3\,adn)e}{a^2b^2c-a^3bd+(b^4c-ab^3d)x^2+2\,(ab^3c-a^2b^2d)x}\right)Be^{(-1)}-\frac{B\log\left(\frac{(bx+a)^ne}{(dx+c)^n}\right)}{2\,(b^3x^2+2\,ab^2x+a^2b)}-\frac{A}{2\,(b^3x^2+2\,ab^2x+a^2b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(b*x+a)^3,x, algorithm="maxima
")
```

```
[Out] 1/4*(2*d^2*n*e*log(b*x + a)/(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2) - 2*d^2*n*e
*log(d*x + c)/(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2) + (2*b*d*n*x*e - (b*c*n -
 3*a*d*n)*e)/(a^2*b^2*c - a^3*b*d + (b^4*c - a*b^3*d)*x^2 + 2*(a*b^3*c - a^
2*b^2*d)*x))*B*e^(-1) - 1/2*B*log((b*x + a)^n*e/(d*x + c)^n)/(b^3*x^2 + 2*a
*b^2*x + a^2*b) - 1/2*A/(b^3*x^2 + 2*a*b^2*x + a^2*b)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 274 vs. $2(128) = 256$.
time = 0.35, size = 274, normalized size = 2.00

$$-\frac{2\,(A+B)b^2c^2-4\,(A+B)abcd+2\,(A+B)a^2d^2-2\,(Bb^2cd-Babd^2)nx+(Bb^2c^2-4\,Babcd+3\,Ba^2d^2)n-2\,(Bb^2d^2nx^2+2\,Babd^2nx-(Bb^2c^2-2\,Babcd)n)\log\left(bx+a\right)+2\,(Bb^2d^2nx^2+2\,Babd^2nx-(Bb^2c^2-2\,Babcd)n)\log\left(dx+c\right)}{4\,(a^2b^3c^2-2\,a^3b^2cd+a^4bd^2+(b^5c^2-2\,ab^4cd+a^2b^3d^2)x^2+2\,(ab^4c^2-2\,a^2b^3cd+a^3b^2d^2)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(b*x+a)^3,x, algorithm="fricas
")
```

```
[Out] -1/4*(2*(A + B)*b^2*c^2 - 4*(A + B)*a*b*c*d + 2*(A + B)*a^2*d^2 - 2*(B*b^2*
c*d - B*a*b*d^2)*n*x + (B*b^2*c^2 - 4*B*a*b*c*d + 3*B*a^2*d^2)*n - 2*(B*b^2
*d^2*n*x^2 + 2*B*a*b*d^2*n*x - (B*b^2*c^2 - 2*B*a*b*c*d)*n)*log(b*x + a) +
2*(B*b^2*d^2*n*x^2 + 2*B*a*b*d^2*n*x - (B*b^2*c^2 - 2*B*a*b*c*d)*n)*log(d*x
 + c))/(a^2*b^3*c^2 - 2*a^3*b^2*c*d + a^4*b*d^2 + (b^5*c^2 - 2*a*b^4*c*d +
a^2*b^3*d^2)*x^2 + 2*(a*b^4*c^2 - 2*a^2*b^3*c*d + a^3*b^2*d^2)*x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))/(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 5.14, size = 239, normalized size = 1.74

$$\frac{Bd^2n\log(bx+a)}{2(b^3c^2-2\,ab^2cd+a^2bd^2)}-\frac{Bd^2n\log(dx+c)}{2(b^3c^2-2\,ab^2cd+a^2bd^2)}-\frac{Bn\log(bx+a)}{2(b^3x^2+2\,ab^2x+a^2b)}+\frac{Bn\log(dx+c)}{2(b^3x^2+2\,ab^2x+a^2b)}+\frac{2\,Bbdnx-Bbcn+3\,Badn-2\,Abc-2\,Bbc+2\,Aad+2\,Bad}{4(b^4cx^2-ab^3dx^2+2\,ab^3cx-2\,a^2b^2dx+a^2b^2c-a^3bd)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/2*B*d^2*n*log(b*x + a)/(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2) - 1/2*B*d^2*n*
log(d*x + c)/(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2) - 1/2*B*n*log(b*x + a)/(b^
3*x^2 + 2*a*b^2*x + a^2*b) + 1/2*B*n*log(d*x + c)/(b^3*x^2 + 2*a*b^2*x + a^
2*b) + 1/4*(2*B*b*d*n*x - B*b*c*n + 3*B*a*d*n - 2*A*b*c - 2*B*b*c + 2*A*a*d
 + 2*B*a*d)/(b^4*c*x^2 - a*b^3*d*x^2 + 2*a*b^3*c*x - 2*a^2*b^2*d*x + a^2*b^
2*c - a^3*b*d)
```

Mupad [B]
time = 4.66, size = 192, normalized size = 1.40

$$-\frac{\frac{2\,A\,a\,d-2\,A\,b\,c+3\,B\,a\,d\,n-B\,b\,c\,n}{2\,(a\,d-b\,c)}+\frac{B\,b\,d\,n\,x}{a\,d-b\,c}}{2\,a^2\,b+4\,a\,b^2\,x+2\,b^3\,x^2}-\frac{B\,\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)}{2\,b\,\left(a^2+2\,a\,b\,x+b^2\,x^2\right)}-\frac{B\,d^2\,n\,\mathrm{atanh}\left(\frac{2\,b^3\,c^2-2\,a^2\,b\,d^2}{2\,b\,(a\,d-b\,c)^2}-\frac{2\,b\,d\,x}{a\,d-b\,c}\right)}{b\,(a\,d-b\,c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))/(a + b*x)^3,x)
```

```
[Out] - ((2*A*a*d - 2*A*b*c + 3*B*a*d*n - B*b*c*n)/(2*(a*d - b*c)) + (B*b*d*n*x)/
(a*d - b*c))/(2*a^2*b + 2*b^3*x^2 + 4*a*b^2*x) - (B*log((e*(a + b*x)^n)/(c
+ d*x)^n))/(2*b*(a^2 + b^2*x^2 + 2*a*b*x)) - (B*d^2*n*atanh((2*b^3*c^2 - 2*
a^2*b*d^2)/(2*b*(a*d - b*c)^2) - (2*b*d*x)/(a*d - b*c)))/(b*(a*d - b*c)^2)
```

3.154 $\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^4}\,dx$

Optimal. Leaf size=166

$$-\frac{Bn}{9b(a+bx)^3}+\frac{Bdn}{6b(bc-ad)(a+bx)^2}-\frac{Bd^2n}{3b(bc-ad)^2(a+bx)}-\frac{Bd^3n\log(a+bx)}{3b(bc-ad)^3}+\frac{Bd^3n\log(c+dx)}{3b(bc-ad)^3}-\frac{A+B}{}$$

```
[Out] -1/9*B*n/b/(b*x+a)^3+1/6*B*d*n/b/(-a*d+b*c)/(b*x+a)^2-1/3*B*d^2*n/b/(-a*d+b
*c)^2/(b*x+a)-1/3*B*d^3*n*ln(b*x+a)/b/(-a*d+b*c)^3+1/3*B*d^3*n*ln(d*x+c)/b/
(-a*d+b*c)^3+1/3*(-A-B*ln(e*(b*x+a)^n/((d*x+c)^n)))/b/(b*x+a)^3
```

Rubi [A]
time = 0.07, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {2548, 46}

$$-\frac{B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A}{3b(a+bx)^3}-\frac{Bd^3n\log(a+bx)}{3b(bc-ad)^3}+\frac{Bd^3n\log(c+dx)}{3b(bc-ad)^3}-\frac{Bd^2n}{3b(a+bx)(bc-ad)^2}+\frac{Bdn}{6b(a+bx)^2(bc-ad)}-\frac{Bn}{9b(a+bx)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(a + b*x)^4,x]
```

```
[Out] -1/9*(B*n)/(b*(a + b*x)^3) + (B*d*n)/(6*b*(b*c - a*d)*(a + b*x)^2) - (B*d^2
*n)/(3*b*(b*c - a*d)^2*(a + b*x)) - (B*d^3*n*Log[a + b*x])/(3*b*(b*c - a*d)
^3) + (B*d^3*n*Log[c + d*x])/(3*b*(b*c - a*d)^3) - (A + B*Log[(e*(a + b*x)^
n)/(c + d*x)^n])/(3*b*(a + b*x)^3)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2548

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Dist[B*n*((b*c
- a*d)/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /;
 FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c -
 a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Rubi steps

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^4}\,dx = \int \left(\frac{A}{(a+bx)^4}+\frac{B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^4}\right)dx$$
$$= -\frac{A}{3b(a+bx)^3}+B\int \frac{\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^4}\,dx$$
$$= -\frac{A}{3b(a+bx)^3}-\frac{B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{3b(a+bx)^3}+\frac{(B(bc-ad)n)\int}{3b}$$
$$= -\frac{A}{3b(a+bx)^3}-\frac{B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{3b(a+bx)^3}+\frac{(B(bc-ad)n)\int}{}$$
$$= -\frac{A}{3b(a+bx)^3}-\frac{Bn}{9b(a+bx)^3}+\frac{Bdn}{6b(bc-ad)(a+bx)^2}-\frac{B}{3b(bc-a}$$

Mathematica [A]
time = 0.14, size = 167, normalized size = 1.01

$$-\frac{6Bd^3n(a+bx)^3\log(a+bx)-6Bd^3n(a+bx)^3\log(c+dx)+(bc-ad)\left(6A(bc-ad)^2+Bn(11a^2d^2+abd(-7c+15dx)+b^2(2c^2-3cdx+6d^2x^2))+6B(bc-ad)^2\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{18b(bc-ad)^3(a+bx)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(a + b*x)^4,x]
```

```
[Out] -1/18*(6*B*d^3*n*(a + b*x)^3*Log[a + b*x] - 6*B*d^3*n*(a + b*x)^3*Log[c + d
*x] + (b*c - a*d)*(6*A*(b*c - a*d)^2 + B*n*(11*a^2*d^2 + a*b*d*(-7*c + 15*d
*x) + b^2*(2*c^2 - 3*c*d*x + 6*d^2*x^2)) + 6*B*(b*c - a*d)^2*Log[(e*(a + b*
x)^n)/(c + d*x)^n]))/(b*(b*c - a*d)^3*(a + b*x)^3)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.33, size = 1976, normalized size = 11.90

method	result	size
risch	Expression too large to display	1976

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/(b*x+a)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*B/b/(b*x+a)^3*ln((d*x+c)^n)-1/18*(-6*B*a^3*d^3*ln((b*x+a)^n)+6*B*b^3*c^
3*ln((b*x+a)^n)-6*B*ln(d*x+c)*a^3*d^3*n-6*A*a^3*d^3+6*A*b^3*c^3+18*B*a^2*b*
c*d^2*ln((b*x+a)^n)-3*I*B*Pi*a^3*d^3*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e
/((d*x+c)^n)*(b*x+a)^n)^2+9*I*B*Pi*a^2*b*c*d^2*csgn(I*(b*x+a)^n)*csgn(I*(b*
x+a)^n/((d*x+c)^n))^2+9*I*B*Pi*a^2*b*c*d^2*csgn(I/((d*x+c)^n))*csgn(I*(b*x+
a)^n/((d*x+c)^n))^2+9*I*B*Pi*a^2*b*c*d^2*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn
(I*e/((d*x+c)^n)*(b*x+a)^n)^2-18*A*a*b^2*c^2*d+3*I*B*Pi*a^3*d^3*csgn(I*e/((
```

```
d*x+c)^n)*(b*x+a)^n)^3+18*A*a^2*b*c*d^2+2*B*c^3*n*b^3-11*B*a^3*d^3*n+9*I*B*
Pi*a*b^2*c^2*d*csgn(I*(b*x+a)^n/((d*x+c)^n))^3-3*I*B*Pi*b^3*c^3*csgn(I*e)*c
sgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)-3*I*B*Pi*a^3*d
^3*csgn(I*(b*x+a)^n)*csgn(I*(b*x+a)^n/((d*x+c)^n))^2-3*I*B*Pi*a^3*d^3*csgn(
I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))^2-6*B*ln(e)*a^3*d^3+6*B*ln(e)*
b^3*c^3+3*I*B*Pi*a^3*d^3*csgn(I*(b*x+a)^n/((d*x+c)^n))^3+3*I*B*Pi*b^3*c^3*c
sgn(I*e)*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2+6*B*ln(-b*x-a)*a^3*d^3*n-9*I*B*P
i*a*b^2*c^2*d*csgn(I*e)*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2-9*I*B*Pi*a*b^2*c^
2*d*csgn(I*(b*x+a)^n)*csgn(I*(b*x+a)^n/((d*x+c)^n))^2-9*I*B*Pi*a^2*b*c*d^2*
csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^3-9*I*B*Pi*a*b^2*c^2*d*csgn(I/((d*x+c)^n))*
csgn(I*(b*x+a)^n/((d*x+c)^n))^2+9*I*B*Pi*a*b^2*c^2*d*csgn(I*(b*x+a)^n)*csgn
(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))+3*I*B*Pi*a^3*d^3*csgn(I*e)*cs
gn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)+18*B*a*b^2*c*d^
2*n*x-3*I*B*Pi*b^3*c^3*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^3-3*I*B*Pi*a^3*d^3*c
sgn(I*e)*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2+3*I*B*Pi*b^3*c^3*csgn(I*(b*x+a)^
n)*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+3*I*B*Pi*b^3*c^3*csgn(I/((d*x+c)^n))*csg
n(I*(b*x+a)^n/((d*x+c)^n))^2+3*I*B*Pi*b^3*c^3*csgn(I*(b*x+a)^n/((d*x+c)^n))
*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2-9*I*B*Pi*a^2*b*c*d^2*csgn(I*e)*csgn(I*(b
*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)-9*I*B*Pi*a^2*b*c*d^2*c
sgn(I*(b*x+a)^n)*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))+9*I*B*Pi
*a*b^2*c^2*d*csgn(I*e)*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(
b*x+a)^n)+3*I*B*Pi*a^3*d^3*csgn(I*(b*x+a)^n)*csgn(I/((d*x+c)^n))*csgn(I*(b*
x+a)^n/((d*x+c)^n))-9*B*a*c^2*d*n*b^2+18*B*a^2*c*d^2*n*b+18*B*ln(e)*a^2*b*c
*d^2-18*B*ln(e)*a*b^2*c^2*d-6*B*a*b^2*d^3*n*x^2+6*B*b^3*c*d^2*n*x^2-15*B*a^
2*b*d^3*n*x-3*B*b^3*c^2*d*n*x+9*I*B*Pi*a*b^2*c^2*d*csgn(I*e/((d*x+c)^n)*(b*
x+a)^n)^3-9*I*B*Pi*a^2*b*c*d^2*csgn(I*(b*x+a)^n/((d*x+c)^n))^3-9*I*B*Pi*a*b
^2*c^2*d*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2+9*
I*B*Pi*a^2*b*c*d^2*csgn(I*e)*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2-3*I*B*Pi*b^3
*c^3*csgn(I*(b*x+a)^n)*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))-18
*B*a*b^2*c^2*d*ln((b*x+a)^n)-3*I*B*Pi*b^3*c^3*csgn(I*(b*x+a)^n/((d*x+c)^n))
^3+18*B*ln(-b*x-a)*a*b^2*d^3*n*x^2-18*B*ln(d*x+c)*a^2*b*d^3*n*x+18*B*ln(-b*
x-a)*a^2*b*d^3*n*x-18*B*ln(d*x+c)*a*b^2*d^3*n*x^2-6*B*ln(d*x+c)*b^3*d^3*n*x
^3+6*B*ln(-b*x-a)*b^3*d^3*n*x^3)/(b*x+a)^3/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(-a*
d+b*c)/b
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 404 vs. $2(155) = 310$.
time = 0.31, size = 404, normalized size = 2.43

$$-\frac{1}{18}\left(\frac{6\,d^3ne\log\left(bx+a\right)}{b^4c^3-3\,ab^3c^2d+3\,a^2b^2cd^2-a^3bd^3}-\frac{6\,d^3ne\log\left(dx+c\right)}{b^4c^3-3\,ab^3c^2d+3\,a^2b^2cd^2-a^3bd^3}+\frac{6\,b^2d^2nx^2e-3\,(b^2cdn-5\,abd^2n)xe+(2\,b^2c^2n-7\,abcdn+11\,a^2d^2n)e}{a^3b^3c^2-2\,a^4b^2cd+a^5bd^2+(b^6c^2-2\,ab^5cd+a^2b^4d^2)x^3+3\,(ab^5c^2-2\,a^2b^4cd+a^3b^3d^2)x^2+3\,(a^2b^4c^2-2\,a^3b^3cd+a^4b^2d^2)x}\right)Be^{(-1)}-\frac{B\log\left(\frac{(bx+a)^ne}{(dx+c)^n}\right)}{3\,(b^4x^3+3\,ab^3x^2+3\,a^2b^2x+a^3b)}-\frac{A}{3\,(b^4x^3+3\,ab^3x^2+3\,a^2b^2x+a^3b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(b*x+a)^4,x, algorithm="maxima
")
```

```
[Out] -1/18*(6*d^3*n*e*log(b*x + a)/(b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 -
a^3*b*d^3) - 6*d^3*n*e*log(d*x + c)/(b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*
d^2 - a^3*b*d^3) + (6*b^2*d^2*n*x^2*e - 3*(b^2*c*d*n - 5*a*b*d^2*n)*x*e + (
2*b^2*c^2*n - 7*a*b*c*d*n + 11*a^2*d^2*n)*e)/(a^3*b^3*c^2 - 2*a^4*b^2*c*d +
 a^5*b*d^2 + (b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2)*x^3 + 3*(a*b^5*c^2 - 2*a
^2*b^4*c*d + a^3*b^3*d^2)*x^2 + 3*(a^2*b^4*c^2 - 2*a^3*b^3*c*d + a^4*b^2*d^
2)*x))*B*e^(-1) - 1/3*B*log((b*x + a)^n*e/(d*x + c)^n)/(b^4*x^3 + 3*a*b^3*x
^2 + 3*a^2*b^2*x + a^3*b) - 1/3*A/(b^4*x^3 + 3*a*b^3*x^2 + 3*a^2*b^2*x + a^
3*b)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 504 vs. $2(155) = 310$.
time = 0.37, size = 504, normalized size = 3.04

$$-\frac{6(A+B)b^3c^3 - 18(A+B)ab^2c^2d + 18(A+B)a^2bcd^2 - 6(A+B)a^3d^3 + 6(Bb^3cd^2 - Bab^2d^3)nx^2 - 3(Bb^3c^2d - 6Bab^2cd^2 + 5Ba^2bd^3)nx + (2Bb^3c^3 - 9Bab^2c^2d + 18Ba^2bcd^2 - 11Ba^3d^3)n + 6(Bb^3d^3nx^3 + 3Bab^2d^3nx^2 + 3Ba^2bd^3nx + (Bb^3c^3 - 3Bab^2c^2d + 3Ba^2bcd^2)n)\log(bx+a) - 6(Bb^3d^3nx^3 + 3Bab^2d^3nx^2 + 3Ba^2bd^3nx + (Bb^3c^3 - 3Bab^2c^2d + 3Ba^2bcd^2)n)\log(dx+c)}{18(a^3b^4c^3 - 3a^4b^3c^2d + 3a^5b^2cd^2 - a^6bd^3 + (b^7c^3 - 3ab^6c^2d + 3a^2b^5cd^2 - a^3b^4d^3)x^3 + 3(ab^6c^3 - 3a^2b^5c^2d + 3a^3b^4cd^2 - a^4b^3d^3)x^2 + 3(a^2b^5c^3 - 3a^3b^4c^2d + 3a^4b^3cd^2 - a^5b^2d^3)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(b*x+a)^4,x, algorithm="fricas
")
```

```
[Out] -1/18*(6*(A + B)*b^3*c^3 - 18*(A + B)*a*b^2*c^2*d + 18*(A + B)*a^2*b*c*d^2
- 6*(A + B)*a^3*d^3 + 6*(B*b^3*c*d^2 - B*a*b^2*d^3)*n*x^2 - 3*(B*b^3*c^2*d
- 6*B*a*b^2*c*d^2 + 5*B*a^2*b*d^3)*n*x + (2*B*b^3*c^3 - 9*B*a*b^2*c^2*d + 1
8*B*a^2*b*c*d^2 - 11*B*a^3*d^3)*n + 6*(B*b^3*d^3*n*x^3 + 3*B*a*b^2*d^3*n*x^
2 + 3*B*a^2*b*d^3*n*x + (B*b^3*c^3 - 3*B*a*b^2*c^2*d + 3*B*a^2*b*c*d^2)*n)*
log(b*x + a) - 6*(B*b^3*d^3*n*x^3 + 3*B*a*b^2*d^3*n*x^2 + 3*B*a^2*b*d^3*n*x
 + (B*b^3*c^3 - 3*B*a*b^2*c^2*d + 3*B*a^2*b*c*d^2)*n)*log(d*x + c))/(a^3*b^
4*c^3 - 3*a^4*b^3*c^2*d + 3*a^5*b^2*c*d^2 - a^6*b*d^3 + (b^7*c^3 - 3*a*b^6*
c^2*d + 3*a^2*b^5*c*d^2 - a^3*b^4*d^3)*x^3 + 3*(a*b^6*c^3 - 3*a^2*b^5*c^2*d
 + 3*a^3*b^4*c*d^2 - a^4*b^3*d^3)*x^2 + 3*(a^2*b^5*c^3 - 3*a^3*b^4*c^2*d +
3*a^4*b^3*c*d^2 - a^5*b^2*d^3)*x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))/(b*x+a)**4,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 448 vs. $2(155) = 310$.
time = 4.21, size = 448, normalized size = 2.70

$$-\frac{Bd^3n\log(bx+a)}{3(b^4c^3 - 3ab^3c^2d + 3a^2b^2cd^2 - a^3bd^3)} + \frac{Bd^3n\log(dx+c)}{3(b^4c^3 - 3ab^3c^2d + 3a^2b^2cd^2 - a^3bd^3)} - \frac{Bn\log(bx+a)}{3(b^4x^3 + 3ab^3x^2 + 3a^2b^2x + a^3b)} + \frac{Bn\log(dx+c)}{3(b^4x^3 + 3ab^3x^2 + 3a^2b^2x + a^3b)} - \frac{6Bb^2d^2nx^2 - 3Bb^2cdnx + 15Babd^2nx + 2Bb^2c^2n - 7Babcdn + 11Ba^2d^2n + 6Ab^2c^2 + 6Bb^2c^2 - 12Aabcd - 12Babcd + 6Aa^2d^2 + 6Ba^2d^2}{18(b^6c^2x^3 - 2ab^5cdx^3 + a^2b^4d^2x^3 + 3ab^5c^2x^2 - 6a^2b^4cdx^2 + 3a^3b^3d^2x^2 + 3a^2b^4c^2x - 6a^3b^3cdx + 3a^4b^2d^2x + a^3b^3c^2 - 2a^4b^2cd + a^5bd^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(b*x+a)^4,x, algorithm="giac")
```

```
[Out] -1/3*B*d^3*n*log(b*x + a)/(b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*
b*d^3) + 1/3*B*d^3*n*log(d*x + c)/(b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^
2 - a^3*b*d^3) - 1/3*B*n*log(b*x + a)/(b^4*x^3 + 3*a*b^3*x^2 + 3*a^2*b^2*x
+ a^3*b) + 1/3*B*n*log(d*x + c)/(b^4*x^3 + 3*a*b^3*x^2 + 3*a^2*b^2*x + a^3*
b) - 1/18*(6*B*b^2*d^2*n*x^2 - 3*B*b^2*c*d*n*x + 15*B*a*b*d^2*n*x + 2*B*b^2
*c^2*n - 7*B*a*b*c*d*n + 11*B*a^2*d^2*n + 6*A*b^2*c^2 + 6*B*b^2*c^2 - 12*A*
a*b*c*d - 12*B*a*b*c*d + 6*A*a^2*d^2 + 6*B*a^2*d^2)/(b^6*c^2*x^3 - 2*a*b^5*
c*d*x^3 + a^2*b^4*d^2*x^3 + 3*a*b^5*c^2*x^2 - 6*a^2*b^4*c*d*x^2 + 3*a^3*b^3
*d^2*x^2 + 3*a^2*b^4*c^2*x - 6*a^3*b^3*c*d*x + 3*a^4*b^2*d^2*x + a^3*b^3*c^
2 - 2*a^4*b^2*c*d + a^5*b*d^2)
```

Mupad [B]

time = 4.91, size = 317, normalized size = 1.91

$$\frac{2\,A\,a\,c\,d}{3\,{\left(a\,d-b\,c\right)}^2\,{\left(a+b\,x\right)}^3}-\frac{A\,b\,c^2}{3\,{\left(a\,d-b\,c\right)}^2\,{\left(a+b\,x\right)}^3}-\frac{B\,\ln\left(\frac{e\,{\left(a+b\,x\right)}^n}{{\left(c+d\,x\right)}^n}\right)}{3\,b\,{\left(a+b\,x\right)}^3}-\frac{A\,a^2\,d^2}{3\,b\,{\left(a\,d-b\,c\right)}^2\,{\left(a+b\,x\right)}^3}-\frac{B\,b\,c^2\,n}{9\,{\left(a\,d-b\,c\right)}^2\,{\left(a+b\,x\right)}^3}-\frac{5\,B\,a\,d^2\,n\,x}{6\,{\left(a\,d-b\,c\right)}^2\,{\left(a+b\,x\right)}^3}-\frac{B\,b\,d^2\,n\,x^2}{3\,{\left(a\,d-b\,c\right)}^2\,{\left(a+b\,x\right)}^3}+\frac{7\,B\,a\,c\,d\,n}{18\,{\left(a\,d-b\,c\right)}^2\,{\left(a+b\,x\right)}^3}-\frac{11\,B\,a^2\,d^2\,n}{18\,b\,{\left(a\,d-b\,c\right)}^2\,{\left(a+b\,x\right)}^3}+\frac{B\,b\,c\,d\,n\,x}{6\,{\left(a\,d-b\,c\right)}^2\,{\left(a+b\,x\right)}^3}-\frac{B\,d^3\,n\,\mathrm{atan}\left(\frac{a\,d\,1{}\mathrm{i}+b\,c\,1{}\mathrm{i}+b\,d\,x\,2{}\mathrm{i}}{a\,d-b\,c}\right)\,2{}\mathrm{i}}{3\,b\,{\left(a\,d-b\,c\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))/(a + b*x)^4,x)
```

```
[Out] (2*A*a*c*d)/(3*(a*d - b*c)^2*(a + b*x)^3) - (A*b*c^2)/(3*(a*d - b*c)^2*(a +
 b*x)^3) - (B*log((e*(a + b*x)^n)/(c + d*x)^n))/(3*b*(a + b*x)^3) - (A*a^2*
d^2)/(3*b*(a*d - b*c)^2*(a + b*x)^3) - (B*b*c^2*n)/(9*(a*d - b*c)^2*(a + b*
x)^3) - (B*d^3*n*atan((a*d*1i + b*c*1i + b*d*x*2i)/(a*d - b*c))*2i)/(3*b*(a
*d - b*c)^3) - (5*B*a*d^2*n*x)/(6*(a*d - b*c)^2*(a + b*x)^3) - (B*b*d^2*n*x
^2)/(3*(a*d - b*c)^2*(a + b*x)^3) + (7*B*a*c*d*n)/(18*(a*d - b*c)^2*(a + b*
x)^3) - (11*B*a^2*d^2*n)/(18*b*(a*d - b*c)^2*(a + b*x)^3) + (B*b*c*d*n*x)/(
6*(a*d - b*c)^2*(a + b*x)^3)
```

3.155 $\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^5}\,dx$

Optimal. Leaf size=195

$$-\frac{Bn}{16b(a+bx)^4}+\frac{Bdn}{12b(bc-ad)(a+bx)^3}-\frac{Bd^2n}{8b(bc-ad)^2(a+bx)^2}+\frac{Bd^3n}{4b(bc-ad)^3(a+bx)}+\frac{Bd^4n\log(a+bx)}{4b(bc-ad)^4}-$$

```
[Out] -1/16*B*n/b/(b*x+a)^4+1/12*B*d*n/b/(-a*d+b*c)/(b*x+a)^3-1/8*B*d^2*n/b/(-a*d
+b*c)^2/(b*x+a)^2+1/4*B*d^3*n/b/(-a*d+b*c)^3/(b*x+a)+1/4*B*d^4*n*ln(b*x+a)/
b/(-a*d+b*c)^4-1/4*B*d^4*n*ln(d*x+c)/b/(-a*d+b*c)^4+1/4*(-A-B*ln(e*(b*x+a)^
n/((d*x+c)^n)))/b/(b*x+a)^4
```

Rubi [A]
time = 0.08, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {2548, 46}

$$-\frac{B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A}{4b(a+bx)^4}+\frac{Bd^4n\log(a+bx)}{4b(bc-ad)^4}-\frac{Bd^4n\log(c+dx)}{4b(bc-ad)^4}+\frac{Bd^3n}{4b(a+bx)(bc-ad)^3}-\frac{Bd^2n}{8b(a+bx)^2(bc-ad)^2}+\frac{Bdn}{12b(a+bx)^3(bc-ad)}-\frac{Bn}{16b(a+bx)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(a + b*x)^5,x]
```

```
[Out] -1/16*(B*n)/(b*(a + b*x)^4) + (B*d*n)/(12*b*(b*c - a*d)*(a + b*x)^3) - (B*d
^2*n)/(8*b*(b*c - a*d)^2*(a + b*x)^2) + (B*d^3*n)/(4*b*(b*c - a*d)^3*(a + b
*x)) + (B*d^4*n*Log[a + b*x])/(4*b*(b*c - a*d)^4) - (B*d^4*n*Log[c + d*x])/
(4*b*(b*c - a*d)^4) - (A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(4*b*(a + b*
x)^4)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2548

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Dist[B*n*((b*c
- a*d)/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /;
 FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c -
 a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Rubi steps

$$\int \frac{A + B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^5}\,dx = \int \left(\frac{A}{(a+bx)^5} + \frac{B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^5}\right)dx$$

$$= -\frac{A}{4b(a+bx)^4} + B\int \frac{\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^5}\,dx$$

$$= -\frac{A}{4b(a+bx)^4} - \frac{B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{4b(a+bx)^4} + \frac{(B(bc-ad)n)}{4}$$

$$= -\frac{A}{4b(a+bx)^4} - \frac{B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{4b(a+bx)^4} + \frac{(B(bc-ad)n)}{}$$

$$= -\frac{A}{4b(a+bx)^4} - \frac{Bn}{16b(a+bx)^4} + \frac{Bdn}{12b(bc-ad)(a+bx)^3} - \frac{}{8b(bc}$$

Mathematica [A]
time = 0.19, size = 160, normalized size = 0.82

$$\frac{\frac{12Bd^4n\log(a+bx)}{(bc-ad)^4} - \frac{12Bd^4n\log(c+dx)}{(bc-ad)^4} + \frac{-12A-3Bn+\frac{4Bdn(a+bx)}{bc-ad}-\frac{6Bd^2n(a+bx)^2}{(bc-ad)^2}+\frac{12Bd^3n(a+bx)^3}{(bc-ad)^3}-12B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^4}}{48b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(a + b*x)^5,x]
```

```
[Out] ((12*B*d^4*n*Log[a + b*x])/(b*c - a*d)^4 - (12*B*d^4*n*Log[c + d*x])/(b*c -
 a*d)^4 + (-12*A - 3*B*n + (4*B*d*n*(a + b*x))/(b*c - a*d) - (6*B*d^2*n*(a
+ b*x)^2)/(b*c - a*d)^2 + (12*B*d^3*n*(a + b*x)^3)/(b*c - a*d)^3 - 12*B*Log
[(e*(a + b*x)^n)/(c + d*x)^n])/(a + b*x)^4)/(48*b)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.39, size = 2583, normalized size = 13.25

method	result	size
risch	Expression too large to display	2583

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/(b*x+a)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*B/b/(b*x+a)^4*ln((d*x+c)^n)+1/48*(-12*A*b^4*c^4+16*B*a*c^3*d*n*b^3-36*B
*a^2*c^2*d^2*n*b^2+48*B*a^3*c*d^3*n*b-6*I*B*Pi*b^4*c^4*csgn(I/((d*x+c)^n))*
csgn(I*(b*x+a)^n/((d*x+c)^n))^2-6*I*B*Pi*b^4*c^4*csgn(I*(b*x+a)^n/((d*x+c)^
n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2-24*I*B*Pi*a*b^3*c^3*d*csgn(I*(b*x+a)^
n)*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))-24*I*B*Pi*a^3*b*c*d^3*
```

```
csgn(I*e)*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)-12*
B*b^4*c^4*ln((b*x+a)^n)+6*I*B*Pi*b^4*c^4*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^3+
36*I*B*Pi*a^2*b^2*c^2*d^2*csgn(I*(b*x+a)^n/((d*x+c)^n))^3-3*B*c^4*n*b^4-25*
B*a^4*d^4*n+6*I*B*Pi*b^4*c^4*csgn(I*e)*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I
*e/((d*x+c)^n)*(b*x+a)^n)-24*I*B*Pi*a^3*b*c*d^3*csgn(I*(b*x+a)^n)*csgn(I/((
d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))-72*A*a^2*b^2*c^2*d^2-12*B*ln(d*x+c
)*b^4*d^4*n*x^4+12*B*ln(-b*x-a)*b^4*d^4*n*x^4-12*B*a*b^3*d^4*n*x^3+12*B*b^4
*c*d^3*n*x^3-42*B*a^2*b^2*d^4*n*x^2-6*B*b^4*c^2*d^2*n*x^2-52*B*a^3*b*d^4*n*
x+4*B*b^4*c^3*d*n*x+48*A*a^3*b*c*d^3+48*A*a*b^3*c^3*d-36*I*B*Pi*a^2*b^2*c^2
*d^2*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))^2-12*A*a^4*d^4+6*I*B
*Pi*a^4*d^4*csgn(I*(b*x+a)^n)*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)
^n))-24*I*B*Pi*a^3*b*c*d^3*csgn(I*(b*x+a)^n/((d*x+c)^n))^3-24*I*B*Pi*a^3*b*
c*d^3*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^3+48*B*a*b^3*c*d^3*n*x^2+72*B*a^2*b^2
*c*d^3*n*x-24*B*a*b^3*c^2*d^2*n*x+48*B*ln(e)*a^3*b*c*d^3-72*B*ln(e)*a^2*b^2
*c^2*d^2+12*B*ln(-b*x-a)*a^4*d^4*n-12*B*a^4*d^4*ln((b*x+a)^n)-24*I*B*Pi*a*b
^3*c^3*d*csgn(I*(b*x+a)^n/((d*x+c)^n))^3+24*I*B*Pi*a^3*b*c*d^3*csgn(I/((d*x
+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+24*I*B*Pi*a^3*b*c*d^3*csgn(I*(b*x+a
)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2-36*I*B*Pi*a^2*b^2*c^2*d^
2*csgn(I*e)*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2-36*I*B*Pi*a^2*b^2*c^2*d^2*csg
n(I*(b*x+a)^n)*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+36*I*B*Pi*a^2*b^2*c^2*d^2*cs
gn(I*e/((d*x+c)^n)*(b*x+a)^n)^3+24*I*B*Pi*a*b^3*c^3*d*csgn(I*(b*x+a)^n/((d*
x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2+48*B*ln(e)*a*b^3*c^3*d+6*I*B*Pi*
a^4*d^4*csgn(I*e)*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a
)^n)-6*I*B*Pi*a^4*d^4*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))^2-1
2*B*ln(e)*a^4*d^4-12*B*ln(e)*b^4*c^4+6*I*B*Pi*b^4*c^4*csgn(I*(b*x+a)^n/((d*
x+c)^n))^3-6*I*B*Pi*a^4*d^4*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)
^n)*(b*x+a)^n)^2+24*I*B*Pi*a*b^3*c^3*d*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n
/((d*x+c)^n))^2-24*I*B*Pi*a*b^3*c^3*d*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^3+6*I
*B*Pi*b^4*c^4*csgn(I*(b*x+a)^n)*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+
c)^n))+48*B*a^3*b*c*d^3*ln((b*x+a)^n)-72*B*a^2*b^2*c^2*d^2*ln((b*x+a)^n)+48
*B*a*b^3*c^3*d*ln((b*x+a)^n)+6*I*B*Pi*a^4*d^4*csgn(I*(b*x+a)^n/((d*x+c)^n))
^3+6*I*B*Pi*a^4*d^4*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^3-12*B*ln(d*x+c)*a^4*d^
4*n-36*I*B*Pi*a^2*b^2*c^2*d^2*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+
c)^n)*(b*x+a)^n)^2+24*I*B*Pi*a*b^3*c^3*d*csgn(I*e)*csgn(I*e/((d*x+c)^n)*(b*
x+a)^n)^2+24*I*B*Pi*a*b^3*c^3*d*csgn(I*(b*x+a)^n)*csgn(I*(b*x+a)^n/((d*x+c)
^n))^2-6*I*B*Pi*b^4*c^4*csgn(I*e)*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2-6*I*B*P
i*b^4*c^4*csgn(I*(b*x+a)^n)*csgn(I*(b*x+a)^n/((d*x+c)^n))^2-6*I*B*Pi*a^4*d^
4*csgn(I*(b*x+a)^n)*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+36*I*B*Pi*a^2*b^2*c^2*d
^2*csgn(I*e)*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)+
36*I*B*Pi*a^2*b^2*c^2*d^2*csgn(I*(b*x+a)^n)*csgn(I/((d*x+c)^n))*csgn(I*(b*x
+a)^n/((d*x+c)^n))-24*I*B*Pi*a*b^3*c^3*d*csgn(I*e)*csgn(I*(b*x+a)^n/((d*x+c
)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)-6*I*B*Pi*a^4*d^4*csgn(I*e)*csgn(I*e/(
(d*x+c)^n)*(b*x+a)^n)^2+24*I*B*Pi*a^3*b*c*d^3*csgn(I*e)*csgn(I*e/((d*x+c)^n
)*(b*x+a)^n)^2+24*I*B*Pi*a^3*b*c*d^3*csgn(I*(b*x+a)^n)*csgn(I*(b*x+a)^n/((d
*x+c)^n))^2-48*B*ln(d*x+c)*a*b^3*d^4*n*x^3+48*B*ln(-b*x-a)*a*b^3*d^4*n*x^3-
```

```
72*B*ln(d*x+c)*a^2*b^2*d^4*n*x^2+72*B*ln(-b*x-a)*a^2*b^2*d^4*n*x^2-48*B*ln(
d*x+c)*a^3*b*d^4*n*x+48*B*ln(-b*x-a)*a^3*b*d^4*n*x)/(b*x+a)^4/(-a^3*d^3+3*a
^2*b*c*d^2-3*a*b^2*c^2*d+b^3*c^3)/(-a*d+b*c)/b
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 621 vs. $2(182) = 364$.
time = 0.36, size = 621, normalized size = 3.18

$$\frac{1}{48}\left(\frac{12\,d^4ne\log\left(bx+a\right)}{b^5c^4-4\,ab^4c^3d+6\,a^2b^3c^2d^2-4\,a^3b^2cd^3+a^4bd^4}-\frac{12\,d^4ne\log\left(dx+c\right)}{b^5c^4-4\,ab^4c^3d+6\,a^2b^3c^2d^2-4\,a^3b^2cd^3+a^4bd^4}+\frac{12\,b^3d^3nx^3e-6\,(b^3cd^2n-7\,ab^2d^3n)x^2e+4\,(b^3c^2dn-5\,ab^2cd^2n+13\,a^2bd^3n)xe-(3\,b^3c^3n-13\,ab^2c^2dn+23\,a^2bcd^2n-25\,a^3d^3n)e}{a^4b^4c^3-3\,a^5b^3c^2d+3\,a^6b^2cd^2-a^7bd^3+(b^8c^3-3\,ab^7c^2d+3\,a^2b^6cd^2-a^3b^5d^3)x^4+4\,(ab^7c^3-3\,a^2b^6c^2d+3\,a^3b^5cd^2-a^4b^4d^3)x^3+6\,(a^2b^6c^3-3\,a^3b^5c^2d+3\,a^4b^4cd^2-a^5b^3d^3)x^2+4\,(a^3b^5c^3-3\,a^4b^4c^2d+3\,a^5b^3cd^2-a^6b^2d^3)x}\right)Be^{(-1)}-\frac{B\log\left(\frac{(bx+a)^ne}{(dx+c)^n}\right)}{4\,(b^5x^4+4\,ab^4x^3+6\,a^2b^3x^2+4\,a^3b^2x+a^4b)}-\frac{A}{4\,(b^5x^4+4\,ab^4x^3+6\,a^2b^3x^2+4\,a^3b^2x+a^4b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(b*x+a)^5,x, algorithm="maxima
")
```

```
[Out] 1/48*(12*d^4*n*e*log(b*x + a)/(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2
- 4*a^3*b^2*c*d^3 + a^4*b*d^4) - 12*d^4*n*e*log(d*x + c)/(b^5*c^4 - 4*a*b^4
*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4) + (12*b^3*d^3*n*x
^3*e - 6*(b^3*c*d^2*n - 7*a*b^2*d^3*n)*x^2*e + 4*(b^3*c^2*d*n - 5*a*b^2*c*d
^2*n + 13*a^2*b*d^3*n)*x*e - (3*b^3*c^3*n - 13*a*b^2*c^2*d*n + 23*a^2*b*c*d
^2*n - 25*a^3*d^3*n)*e)/(a^4*b^4*c^3 - 3*a^5*b^3*c^2*d + 3*a^6*b^2*c*d^2 -
a^7*b*d^3 + (b^8*c^3 - 3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 - a^3*b^5*d^3)*x^4 +
 4*(a*b^7*c^3 - 3*a^2*b^6*c^2*d + 3*a^3*b^5*c*d^2 - a^4*b^4*d^3)*x^3 + 6*(a
^2*b^6*c^3 - 3*a^3*b^5*c^2*d + 3*a^4*b^4*c*d^2 - a^5*b^3*d^3)*x^2 + 4*(a^3*
b^5*c^3 - 3*a^4*b^4*c^2*d + 3*a^5*b^3*c*d^2 - a^6*b^2*d^3)*x))*B*e^(-1) - 1
/4*B*log((b*x + a)^n*e/(d*x + c)^n)/(b^5*x^4 + 4*a*b^4*x^3 + 6*a^2*b^3*x^2
+ 4*a^3*b^2*x + a^4*b) - 1/4*A/(b^5*x^4 + 4*a*b^4*x^3 + 6*a^2*b^3*x^2 + 4*a
^3*b^2*x + a^4*b)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 772 vs. $2(182) = 364$.
time = 0.37, size = 772, normalized size = 3.96

$$-\frac{12\,(A+B)b^4c^4-48\,(A+B)ab^3c^3d+72\,(A+B)a^2b^2c^2d^2-48\,(A+B)a^3bcd^3+12\,(A+B)a^4d^4-12\,(Bb^4cd^3-Bab^3d^4)nx^3+6\,(Bb^4c^2d^2-8\,Bab^3cd^3+7\,Ba^2b^2d^4)nx^2-4\,(Bb^4c^3d-6\,Bab^3c^2d^2+18\,Ba^2b^2cd^3-13\,Ba^3bd^4)nx+(3\,Bb^4c^4-16\,Bab^3c^3d+36\,Ba^2b^2c^2d^2-48\,Ba^3bcd^3+25\,Ba^4d^4)n-12\,(Bb^4d^4nx^4+4\,Bab^3d^4nx^3+6\,Ba^2b^2d^4nx^2+4\,Ba^3bd^4nx-(Bb^4c^4-4\,Bab^3c^3d+6\,Ba^2b^2c^2d^2-4\,Ba^3bcd^3)n)\log\left(bx+a\right)+12\,(Bb^4d^4nx^4+4\,Bab^3d^4nx^3+6\,Ba^2b^2d^4nx^2+4\,Ba^3bd^4nx-(Bb^4c^4-4\,Bab^3c^3d+6\,Ba^2b^2c^2d^2-4\,Ba^3bcd^3)n)\log\left(dx+c\right)}{48\,(a^4b^5c^4-4\,a^5b^4c^3d+6\,a^6b^3c^2d^2-4\,a^7b^2cd^3+a^8bd^4+(b^9c^4-4\,ab^8c^3d+6\,a^2b^7c^2d^2-4\,a^3b^6cd^3+a^4b^5d^4)x^4+4\,(ab^8c^4-4\,a^2b^7c^3d+6\,a^3b^6c^2d^2-4\,a^4b^5cd^3+a^5b^4d^4)x^3+6\,(a^2b^7c^4-4\,a^3b^6c^3d+6\,a^4b^5c^2d^2-4\,a^5b^4cd^3+a^6b^3d^4)x^2+4\,(a^3b^6c^4-4\,a^4b^5c^3d+6\,a^5b^4c^2d^2-4\,a^6b^3cd^3+a^7b^2d^4)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(b*x+a)^5,x, algorithm="fricas
")
```

```
[Out] -1/48*(12*(A + B)*b^4*c^4 - 48*(A + B)*a*b^3*c^3*d + 72*(A + B)*a^2*b^2*c^2
*d^2 - 48*(A + B)*a^3*b*c*d^3 + 12*(A + B)*a^4*d^4 - 12*(B*b^4*c*d^3 - B*a*
b^3*d^4)*n*x^3 + 6*(B*b^4*c^2*d^2 - 8*B*a*b^3*c*d^3 + 7*B*a^2*b^2*d^4)*n*x^
2 - 4*(B*b^4*c^3*d - 6*B*a*b^3*c^2*d^2 + 18*B*a^2*b^2*c*d^3 - 13*B*a^3*b*d^
4)*n*x + (3*B*b^4*c^4 - 16*B*a*b^3*c^3*d + 36*B*a^2*b^2*c^2*d^2 - 48*B*a^3*
b*c*d^3 + 25*B*a^4*d^4)*n - 12*(B*b^4*d^4*n*x^4 + 4*B*a*b^3*d^4*n*x^3 + 6*B
*a^2*b^2*d^4*n*x^2 + 4*B*a^3*b*d^4*n*x - (B*b^4*c^4 - 4*B*a*b^3*c^3*d + 6*B
*a^2*b^2*c^2*d^2 - 4*B*a^3*b*c*d^3)*n)*log(b*x + a) + 12*(B*b^4*d^4*n*x^4 +
```

```
 4*B*a*b^3*d^4*n*x^3 + 6*B*a^2*b^2*d^4*n*x^2 + 4*B*a^3*b*d^4*n*x - (B*b^4*c
^4 - 4*B*a*b^3*c^3*d + 6*B*a^2*b^2*c^2*d^2 - 4*B*a^3*b*c*d^3)*n)*log(d*x +
c))/(a^4*b^5*c^4 - 4*a^5*b^4*c^3*d + 6*a^6*b^3*c^2*d^2 - 4*a^7*b^2*c*d^3 +
a^8*b*d^4 + (b^9*c^4 - 4*a*b^8*c^3*d + 6*a^2*b^7*c^2*d^2 - 4*a^3*b^6*c*d^3
+ a^4*b^5*d^4)*x^4 + 4*(a*b^8*c^4 - 4*a^2*b^7*c^3*d + 6*a^3*b^6*c^2*d^2 - 4
*a^4*b^5*c*d^3 + a^5*b^4*d^4)*x^3 + 6*(a^2*b^7*c^4 - 4*a^3*b^6*c^3*d + 6*a^
4*b^5*c^2*d^2 - 4*a^5*b^4*c*d^3 + a^6*b^3*d^4)*x^2 + 4*(a^3*b^6*c^4 - 4*a^4
*b^5*c^3*d + 6*a^5*b^4*c^2*d^2 - 4*a^6*b^3*c*d^3 + a^7*b^2*d^4)*x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))/(b*x+a)**5,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 710 vs. 2(182) = 364.
time = 4.65, size = 710, normalized size = 3.64

$$\frac{Bd^4n\log\left(bx+a\right)}{4\left(b^5c^4-4ab^4c^3d+6a^2b^3c^2d^2-4a^3b^2cd^3+a^4bd^4\right)}-\frac{Bd^4n\log\left(dx+c\right)}{4\left(b^5c^4-4ab^4c^3d+6a^2b^3c^2d^2-4a^3b^2cd^3+a^4bd^4\right)}-\frac{Bn\log\left(bx+a\right)}{4\left(b^5x^4+4ab^4x^3+6a^2b^3x^2+4a^3b^2x+a^4b\right)}+\frac{Bn\log\left(dx+c\right)}{4\left(b^5x^4+4ab^4x^3+6a^2b^3x^2+4a^3b^2x+a^4b\right)}+\frac{12Bb^3d^3nx^3-6Bb^3cd^2nx^2+42Bab^2d^3nx^2+4Bb^3c^2dnx-20Bab^2cd^2nx+52Ba^2bd^3nx-3Bb^3c^3n+13Bab^2c^2dn-23Ba^2bcd^2n+25Ba^3d^3n-12Ab^3c^3-12Bb^3c^3+36Aab^2c^2d+36Bab^2c^2d-36Aa^2bcd^2-36Ba^2bcd^2+12Aa^3d^3+12Ba^3d^3}{48\left(b^8c^3x^4-3ab^7c^2dx^4+3a^2b^6cd^2x^4-a^3b^5d^3x^4+4ab^7c^3x^3-12a^2b^6c^2dx^3+12a^3b^5cd^2x^3-4a^4b^4d^3x^3+6a^2b^6c^3x^2-18a^3b^5c^2dx^2+18a^4b^4cd^2x^2-6a^5b^3d^3x^2+4a^3b^5c^3x-12a^4b^4c^2dx+12a^5b^3cd^2x-4a^6b^2d^3x+a^4b^4c^3-3a^5b^3c^2d+3a^6b^2cd^2-a^7bd^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(b*x+a)^5,x, algorithm="giac")
```

```
[Out] 1/4*B*d^4*n*log(b*x + a)/(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a
^3*b^2*c*d^3 + a^4*b*d^4) - 1/4*B*d^4*n*log(d*x + c)/(b^5*c^4 - 4*a*b^4*c^3
*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4) - 1/4*B*n*log(b*x + a
)/(b^5*x^4 + 4*a*b^4*x^3 + 6*a^2*b^3*x^2 + 4*a^3*b^2*x + a^4*b) + 1/4*B*n*l
og(d*x + c)/(b^5*x^4 + 4*a*b^4*x^3 + 6*a^2*b^3*x^2 + 4*a^3*b^2*x + a^4*b) +
 1/48*(12*B*b^3*d^3*n*x^3 - 6*B*b^3*c*d^2*n*x^2 + 42*B*a*b^2*d^3*n*x^2 + 4*
B*b^3*c^2*d*n*x - 20*B*a*b^2*c*d^2*n*x + 52*B*a^2*b*d^3*n*x - 3*B*b^3*c^3*n
 + 13*B*a*b^2*c^2*d*n - 23*B*a^2*b*c*d^2*n + 25*B*a^3*d^3*n - 12*A*b^3*c^3
- 12*B*b^3*c^3 + 36*A*a*b^2*c^2*d + 36*B*a*b^2*c^2*d - 36*A*a^2*b*c*d^2 - 3
6*B*a^2*b*c*d^2 + 12*A*a^3*d^3 + 12*B*a^3*d^3)/(b^8*c^3*x^4 - 3*a*b^7*c^2*d
*x^4 + 3*a^2*b^6*c*d^2*x^4 - a^3*b^5*d^3*x^4 + 4*a*b^7*c^3*x^3 - 12*a^2*b^6
*c^2*d*x^3 + 12*a^3*b^5*c*d^2*x^3 - 4*a^4*b^4*d^3*x^3 + 6*a^2*b^6*c^3*x^2 -
 18*a^3*b^5*c^2*d*x^2 + 18*a^4*b^4*c*d^2*x^2 - 6*a^5*b^3*d^3*x^2 + 4*a^3*b^
5*c^3*x - 12*a^4*b^4*c^2*d*x + 12*a^5*b^3*c*d^2*x - 4*a^6*b^2*d^3*x + a^4*b
^4*c^3 - 3*a^5*b^3*c^2*d + 3*a^6*b^2*c*d^2 - a^7*b*d^3)
```

Mupad [**B**]
time = 5.33, size = 555, normalized size = 2.85

$$-\frac{\frac{12\,A\,a^3\,d^3-12\,A\,b^3\,c^3+25\,B\,a^3\,d^3\,n-3\,B\,b^3\,c^3\,n+36\,A\,a\,b^2\,c^2\,d-36\,A\,a^2\,b\,c\,d^2+13\,B\,a\,b^2\,c^2\,d\,n-23\,B\,a^2\,b\,c\,d^2\,n}{12\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}+\frac{d\,x\,\left(13\,B\,n\,a^2\,b\,d^2-5\,B\,n\,a\,b^2\,c\,d+B\,n\,b^3\,c^2\right)}{3\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}-\frac{d^2\,x^2\,\left(B\,b^3\,c\,n-7\,B\,a\,b^2\,d\,n\right)}{2\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}+\frac{B\,b^3\,d^3\,n\,x^3}{a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3}}{4\,a^4\,b+16\,a^3\,b^2\,x+24\,a^2\,b^3\,x^2+16\,a\,b^4\,x^3+4\,b^5\,x^4}-\frac{B\,\ln\left(\frac{e\,{\left(a+b\,x\right)}^n}{{\left(c+d\,x\right)}^n}\right)}{4\,b\,\left(a^4+4\,a^3\,b\,x+6\,a^2\,b^2\,x^2+4\,a\,b^3\,x^3+b^4\,x^4\right)}-\frac{B\,d^4\,n\,\mathrm{atanh}\left(\frac{-4\,a^4\,b\,d^4+8\,a^3\,b^2\,c\,d^3-8\,a\,b^4\,c^3\,d+4\,b^5\,c^4}{4\,b\,{\left(a\,d-b\,c\right)}^4}-\frac{2\,b\,d\,x\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}{{\left(a\,d-b\,c\right)}^4}\right)}{2\,b\,{\left(a\,d-b\,c\right)}^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))/(a + b*x)^5,x)
```

```
[Out] - ((12*A*a^3*d^3 - 12*A*b^3*c^3 + 25*B*a^3*d^3*n - 3*B*b^3*c^3*n + 36*A*a*b
^2*c^2*d - 36*A*a^2*b*c*d^2 + 13*B*a*b^2*c^2*d*n - 23*B*a^2*b*c*d^2*n)/(12*
(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) + (d*x*(B*b^3*c^2*n +
13*B*a^2*b*d^2*n - 5*B*a*b^2*c*d*n))/(3*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d
- 3*a^2*b*c*d^2)) - (d^2*x^2*(B*b^3*c*n - 7*B*a*b^2*d*n))/(2*(a^3*d^3 - b^3
*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) + (B*b^3*d^3*n*x^3)/(a^3*d^3 - b^3*c
^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))/(4*a^4*b + 4*b^5*x^4 + 16*a^3*b^2*x +
16*a*b^4*x^3 + 24*a^2*b^3*x^2) - (B*log((e*(a + b*x)^n)/(c + d*x)^n))/(4*b*
(a^4 + b^4*x^4 + 4*a*b^3*x^3 + 6*a^2*b^2*x^2 + 4*a^3*b*x)) - (B*d^4*n*atanh
((4*b^5*c^4 - 4*a^4*b*d^4 + 8*a^3*b^2*c*d^3 - 8*a*b^4*c^3*d)/(4*b*(a*d - b*
c)^4) - (2*b*d*x*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))/(a*d
- b*c)^4))/(2*b*(a*d - b*c)^4)
```

3.156 $\int (a+bx)^3 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2 \, dx$

Optimal. Leaf size=322

$$-\frac{B(bc-ad)n(a+bx)^3\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{6bd}+\frac{(a+bx)^4\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{4b}$$

```
[Out] -1/6*B*(-a*d+b*c)*n*(b*x+a)^3*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/b/d+1/4*(b*
x+a)^4*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/b+1/12*B*(-a*d+b*c)^2*n*(b*x+a)^
2*(3*A+B*n+3*B*ln(e*(b*x+a)^n/((d*x+c)^n)))/b/d^2-1/12*B*(-a*d+b*c)^3*n*(b*
x+a)*(6*A+5*B*n+6*B*ln(e*(b*x+a)^n/((d*x+c)^n)))/b/d^3-1/12*B*(-a*d+b*c)^4*
n*ln((-a*d+b*c)/b/(d*x+c))*(6*A+11*B*n+6*B*ln(e*(b*x+a)^n/((d*x+c)^n)))/b/d
^4-1/2*B^2*(-a*d+b*c)^4*n^2*polylog(2,d*(b*x+a)/b/(d*x+c))/b/d^4
```

Rubi [A]
time = 0.29, antiderivative size = 322, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2573, 2549, 2381, 2384, 2354, 2438}

$$-\frac{B^2n^2(bc-ad)^4\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{2bd^4}-\frac{Bn(bc-ad)^4\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(6B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+6A+11Bn\right)}{12bd^4}-\frac{Bn(a+bx)(bc-ad)^3\left(6B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+6A+5Bn\right)}{12bd^3}+\frac{Bn(a+bx)^2(bc-ad)^2\left(3B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+3A+Bn\right)}{12bd^2}-\frac{Bn(a+bx)^3(bc-ad)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{6bd}+\frac{(a+bx)^4\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^2}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2,x]
```

```
[Out] -1/6*(B*(b*c - a*d)*n*(a + b*x)^3*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]))
/(b*d) + ((a + b*x)^4*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2)/(4*b) + (
B*(b*c - a*d)^2*n*(a + b*x)^2*(3*A + B*n + 3*B*Log[(e*(a + b*x)^n)/(c + d*x
)^n]))/(12*b*d^2) - (B*(b*c - a*d)^3*n*(a + b*x)*(6*A + 5*B*n + 6*B*Log[(e*
(a + b*x)^n)/(c + d*x)^n]))/(12*b*d^3) - (B*(b*c - a*d)^4*n*Log[(b*c - a*d)
/(b*(c + d*x))]*(6*A + 11*B*n + 6*B*Log[(e*(a + b*x)^n)/(c + d*x)^n]))/(12*
b*d^4) - (B^2*(b*c - a*d)^4*n^2*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/(2
*b*d^4)
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2381

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + Dist[b*n*(p/(d*(q + 1))), Int[(f*x)^
m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, d
```

```
, e, f, m, n, q}, x] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 2384

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Dist[f/(e*(q + 1)), Int[(f*x)^(m - 1)*(d + e*x)^(q + 1
)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x
] && ILtQ[q, -1] && GtQ[m, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2549

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(m +
1)*(g/b)^m, Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x], x,
(a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ
[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[b*f - a*g, 0] && (GtQ[p, 0] || LtQ
[m, -1])
```

Rule 2573

```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

Rubi steps

$$\int (a+bx)^3 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2 dx = \int \left(A^2(a+bx)^3+2AB(a+bx)^3\log\left(e(a+bx)^n(c+\right.\right.$$

$$= \frac{A^2(a+bx)^4}{4b} + (2AB)\int (a+bx)^3\log\left(e(a+bx)^n(c+\right.$$

$$= \frac{A^2(a+bx)^4}{4b} + \frac{AB(a+bx)^4\log\left(e(a+bx)^n(c+dx)^{-}\right.}{2b}$$

$$= \frac{A^2(a+bx)^4}{4b} + \frac{AB(a+bx)^4\log\left(e(a+bx)^n(c+dx)^{-}\right.}{2b}$$

$$= -\frac{AB(bc-ad)^3nx}{2d^3} + \frac{AB(bc-ad)^2n(a+bx)^2}{4bd^2} - \frac{AB}{}$$

$$= -\frac{AB(bc-ad)^3nx}{2d^3} + \frac{AB(bc-ad)^2n(a+bx)^2}{4bd^2} - \frac{AB}{}$$

$$= -\frac{AB(bc-ad)^3nx}{2d^3} + \frac{AB(bc-ad)^2n(a+bx)^2}{4bd^2} - \frac{AB}{}$$

$$= -\frac{AB(bc-ad)^3nx}{2d^3} - \frac{5B^2(bc-ad)^3n^2x}{12d^3} + \frac{AB(bc-}{}$$

$$= -\frac{AB(bc-ad)^3nx}{2d^3} - \frac{5B^2(bc-ad)^3n^2x}{12d^3} + \frac{AB(bc-}{}$$

$$= -\frac{AB(bc-ad)^3nx}{2d^3} - \frac{5B^2(bc-ad)^3n^2x}{12d^3} + \frac{AB(bc-}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 1709 vs. $2(322) = 644$.
time = 0.70, size = 1709, normalized size = 5.31

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2,x]
```

```
[Out] (-24*a^4*A*B*d^4*n + 6*a*b^3*B^2*c^3*d*n^2 - 24*a^2*b^2*B^2*c^2*d^2*n^2 + 3
6*a^3*b*B^2*c*d^3*n^2 - 24*a^4*B^2*d^4*n^2 + 12*a^3*A^2*b*d^4*x - 6*A*b^4*B
*c^3*d*n*x + 24*a*A*b^3*B*c^2*d^2*n*x - 36*a^2*A*b^2*B*c*d^3*n*x + 18*a^3*A
*b*B*d^4*n*x - 5*b^4*B^2*c^3*d*n^2*x + 17*a*b^3*B^2*c^2*d^2*n^2*x - 19*a^2*
b^2*B^2*c*d^3*n^2*x + 7*a^3*b*B^2*d^4*n^2*x + 18*a^2*A^2*b^2*d^4*x^2 + 3*A*
```

```
b^4*B*c^2*d^2*n*x^2 - 12*a*A*b^3*B*c*d^3*n*x^2 + 9*a^2*A*b^2*B*d^4*n*x^2 +
b^4*B^2*c^2*d^2*n^2*x^2 - 2*a*b^3*B^2*c*d^3*n^2*x^2 + a^2*b^2*B^2*d^4*n^2*x
^2 + 12*a*A^2*b^3*d^4*x^3 - 2*A*b^4*B*c*d^3*n*x^3 + 2*a*A*b^3*B*d^4*n*x^3 +
 3*A^2*b^4*d^4*x^4 - 3*a^4*B^2*d^4*n^2*Log[a + b*x]^2 + 6*A*b^4*B*c^4*n*Log
[c + d*x] - 24*a*A*b^3*B*c^3*d*n*Log[c + d*x] + 36*a^2*A*b^2*B*c^2*d^2*n*Lo
g[c + d*x] - 24*a^3*A*b*B*c*d^3*n*Log[c + d*x] + 11*b^4*B^2*c^4*n^2*Log[c +
 d*x] - 38*a*b^3*B^2*c^3*d*n^2*Log[c + d*x] + 45*a^2*b^2*B^2*c^2*d^2*n^2*Lo
g[c + d*x] - 18*a^3*b*B^2*c*d^3*n^2*Log[c + d*x] - 24*a^4*B^2*d^4*n^2*Log[c
 + d*x] + 3*b^4*B^2*c^4*n^2*Log[c + d*x]^2 - 12*a*b^3*B^2*c^3*d*n^2*Log[c +
 d*x]^2 + 18*a^2*b^2*B^2*c^2*d^2*n^2*Log[c + d*x]^2 - 12*a^3*b*B^2*c*d^3*n^
2*Log[c + d*x]^2 - 24*a^4*B^2*d^4*n*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 24*a
^3*A*b*B*d^4*x*Log[(e*(a + b*x)^n)/(c + d*x)^n] - 6*b^4*B^2*c^3*d*n*x*Log[(
e*(a + b*x)^n)/(c + d*x)^n] + 24*a*b^3*B^2*c^2*d^2*n*x*Log[(e*(a + b*x)^n)/
(c + d*x)^n] - 36*a^2*b^2*B^2*c*d^3*n*x*Log[(e*(a + b*x)^n)/(c + d*x)^n] +
18*a^3*b*B^2*d^4*n*x*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 36*a^2*A*b^2*B*d^4*
x^2*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 3*b^4*B^2*c^2*d^2*n*x^2*Log[(e*(a +
b*x)^n)/(c + d*x)^n] - 12*a*b^3*B^2*c*d^3*n*x^2*Log[(e*(a + b*x)^n)/(c + d*
x)^n] + 9*a^2*b^2*B^2*d^4*n*x^2*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 24*a*A*b
^3*B*d^4*x^3*Log[(e*(a + b*x)^n)/(c + d*x)^n] - 2*b^4*B^2*c*d^3*n*x^3*Log[(
e*(a + b*x)^n)/(c + d*x)^n] + 2*a*b^3*B^2*d^4*n*x^3*Log[(e*(a + b*x)^n)/(c
+ d*x)^n] + 6*A*b^4*B*d^4*x^4*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 6*b^4*B^2*
c^4*n*Log[c + d*x]*Log[(e*(a + b*x)^n)/(c + d*x)^n] - 24*a*b^3*B^2*c^3*d*n*
Log[c + d*x]*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 36*a^2*b^2*B^2*c^2*d^2*n*Lo
g[c + d*x]*Log[(e*(a + b*x)^n)/(c + d*x)^n] - 24*a^3*b*B^2*c*d^3*n*Log[c +
d*x]*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 12*a^3*b*B^2*d^4*x*Log[(e*(a + b*x)
^n)/(c + d*x)^n]^2 + 18*a^2*b^2*B^2*d^4*x^2*Log[(e*(a + b*x)^n)/(c + d*x)^n
]^2 + 12*a*b^3*B^2*d^4*x^3*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2 + 3*b^4*B^2*d
^4*x^4*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2 + B*n*Log[a + b*x]*(-6*b*B*c*(b^3
*c^3 - 4*a*b^2*c^2*d + 6*a^2*b*c*d^2 - 4*a^3*d^3)*n*Log[c + d*x] + 6*B*(b*c
 - a*d)^4*n*Log[(b*(c + d*x))/(b*c - a*d)] + a*d*(-6*b^3*B*c^3*n + 21*a*b^2
*B*c^2*d*n - 26*a^2*b*B*c*d^2*n + a^3*d^3*(6*A + 35*B*n) + 6*a^3*B*d^3*Log[
(e*(a + b*x)^n)/(c + d*x)^n])) + 6*B^2*(b*c - a*d)^4*n^2*PolyLog[2, (d*(a +
 b*x))/(-(b*c) + a*d)])/(12*b*d^4)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 2.60, size = 26938, normalized size = 83.66

method	result	size
risch	Expression too large to display	26938

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1809 vs. $2(314) = 628$.
time = 0.82, size = 1809, normalized size = 5.62

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2,x, algorithm="maxi
ma")
```

```
[Out] 1/2*A*B*b^3*x^4*log((b*x + a)^n*e/(d*x + c)^n) + 1/4*A^2*b^3*x^4 + 2*A*B*a*
b^2*x^3*log((b*x + a)^n*e/(d*x + c)^n) + A^2*a*b^2*x^3 + 3*A*B*a^2*b*x^2*lo
g((b*x + a)^n*e/(d*x + c)^n) + 3/2*A^2*a^2*b*x^2 + 2*(a*n*e*log(b*x + a)/b
- c*n*e*log(d*x + c)/d)*A*B*a^3*e^(-1) - 3*(a^2*n*e*log(b*x + a)/b^2 - c^2*
n*e*log(d*x + c)/d^2 + (b*c*n - a*d*n)*x*e/(b*d))*A*B*a^2*b*e^(-1) + (2*a^3
*n*e*log(b*x + a)/b^3 - 2*c^3*n*e*log(d*x + c)/d^3 - ((b^2*c*d*n - a*b*d^2*
n)*x^2*e - 2*(b^2*c^2*n - a^2*d^2*n)*x*e)/(b^2*d^2))*A*B*a*b^2*e^(-1) - 1/1
2*(6*a^4*n*e*log(b*x + a)/b^4 - 6*c^4*n*e*log(d*x + c)/d^4 + (2*(b^3*c*d^2*
n - a*b^2*d^3*n)*x^3*e - 3*(b^3*c^2*d*n - a^2*b*d^3*n)*x^2*e + 6*(b^3*c^3*n
 - a^3*d^3*n)*x*e)/(b^3*d^3))*A*B*b^3*e^(-1) + 2*A*B*a^3*x*log((b*x + a)^n*
e/(d*x + c)^n) + A^2*a^3*x + 1/12*((11*n^2 + 6*n)*b^3*c^4 - 2*(19*n^2 + 12*
n)*a*b^2*c^3*d + 9*(5*n^2 + 4*n)*a^2*b*c^2*d^2 - 6*(3*n^2 + 4*n)*a^3*c*d^3)
*B^2*log(d*x + c)/d^4 + 1/2*(b^4*c^4*n^2 - 4*a*b^3*c^3*d*n^2 + 6*a^2*b^2*c^
2*d^2*n^2 - 4*a^3*b*c*d^3*n^2 + a^4*d^4*n^2)*(log(b*x + a)*log((b*d*x + a*d
)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B^2/(b*d^4) + 1/12*
(3*B^2*b^4*d^4*x^4 - 3*B^2*a^4*d^4*n^2*log(b*x + a)^2 + 2*(a*b^3*d^4*(n + 6
) - b^4*c*d^3*n)*B^2*x^3 + ((n^2 + 3*n)*b^4*c^2*d^2 - 2*(n^2 + 6*n)*a*b^3*c
*d^3 + (n^2 + 9*n + 18)*a^2*b^2*d^4)*B^2*x^2 - 6*(b^4*c^4*n^2 - 4*a*b^3*c^3
*d*n^2 + 6*a^2*b^2*c^2*d^2*n^2 - 4*a^3*b*c*d^3*n^2)*B^2*log(b*x + a)*log(d*
x + c) + 3*(b^4*c^4*n^2 - 4*a*b^3*c^3*d*n^2 + 6*a^2*b^2*c^2*d^2*n^2 - 4*a^3
*b*c*d^3*n^2)*B^2*log(d*x + c)^2 - ((5*n^2 + 6*n)*b^4*c^3*d - (17*n^2 + 24*
n)*a*b^3*c^2*d^2 + (19*n^2 + 36*n)*a^2*b^2*c*d^3 - (7*n^2 + 18*n + 12)*a^3*
b*d^4)*B^2*x - (6*a*b^3*c^3*d*n^2 - 21*a^2*b^2*c^2*d^2*n^2 + 26*a^3*b*c*d^3
*n^2 - (11*n^2 + 6*n)*a^4*d^4)*B^2*log(b*x + a) + 3*(B^2*b^4*d^4*x^4 + 4*B^
2*a*b^3*d^4*x^3 + 6*B^2*a^2*b^2*d^4*x^2 + 4*B^2*a^3*b*d^4*x)*log((b*x + a)^
n)^2 + 3*(B^2*b^4*d^4*x^4 + 4*B^2*a*b^3*d^4*x^3 + 6*B^2*a^2*b^2*d^4*x^2 + 4
*B^2*a^3*b*d^4*x)*log((d*x + c)^n)^2 + (6*B^2*b^4*d^4*x^4 + 6*B^2*a^4*d^4*n
*log(b*x + a) + 2*(a*b^3*d^4*(n + 12) - b^4*c*d^3*n)*B^2*x^3 + 3*(3*a^2*b^2
*d^4*(n + 4) + b^4*c^2*d^2*n - 4*a*b^3*c*d^3*n)*B^2*x^2 + 6*(a^3*b*d^4*(3*n
 + 4) - b^4*c^3*d*n + 4*a*b^3*c^2*d^2*n - 6*a^2*b^2*c*d^3*n)*B^2*x + 6*(b^4
*c^4*n - 4*a*b^3*c^3*d*n + 6*a^2*b^2*c^2*d^2*n - 4*a^3*b*c*d^3*n)*B^2*log(d
*x + c))*log((b*x + a)^n) - (6*B^2*b^4*d^4*x^4 + 6*B^2*a^4*d^4*n*log(b*x +
a) + 2*(a*b^3*d^4*(n + 12) - b^4*c*d^3*n)*B^2*x^3 + 3*(3*a^2*b^2*d^4*(n + 4
) + b^4*c^2*d^2*n - 4*a*b^3*c*d^3*n)*B^2*x^2 + 6*(a^3*b*d^4*(3*n + 4) - b^4
*c^3*d*n + 4*a*b^3*c^2*d^2*n - 6*a^2*b^2*c*d^3*n)*B^2*x + 6*(b^4*c^4*n - 4*
```

```
a*b^3*c^3*d*n + 6*a^2*b^2*c^2*d^2*n - 4*a^3*b*c*d^3*n)*B^2*log(d*x + c) + 6
*(B^2*b^4*d^4*x^4 + 4*B^2*a*b^3*d^4*x^3 + 6*B^2*a^2*b^2*d^4*x^2 + 4*B^2*a^3
*b*d^4*x)*log((b*x + a)^n))*log((d*x + c)^n))/(b*d^4)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2,x, algorithm="fric
as")
```

```
[Out] integral(A^2*b^3*x^3 + 3*A^2*a*b^2*x^2 + 3*A^2*a^2*b*x + A^2*a^3 + (B^2*b^3
*x^3 + 3*B^2*a*b^2*x^2 + 3*B^2*a^2*b*x + B^2*a^3)*log((b*x + a)^n*e/(d*x +
c)^n)^2 + 2*(A*B*b^3*x^3 + 3*A*B*a*b^2*x^2 + 3*A*B*a^2*b*x + A*B*a^3)*log((
b*x + a)^n*e/(d*x + c)^n), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3*(A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**2,x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2,x, algorithm="giac
")
```

```
[Out] integrate((b*x + a)^3*(B*log((b*x + a)^n*e/(d*x + c)^n) + A)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(A+B\,\ln\left(\frac{e\,{\left(a+b\,x\right)}^n}{{\left(c+d\,x\right)}^n}\right)\right)^2\,{\left(a+b\,x\right)}^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^2*(a + b*x)^3,x)
```

```
[Out] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^2*(a + b*x)^3, x)
```

3.157 $\int (a+bx)^2 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2 dx$

Optimal. Leaf size=263

$$-\frac{B(bc-ad)n(a+bx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{3bd}+\frac{(a+bx)^3\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{3b}$$

```
[Out] -1/3*B*(-a*d+b*c)*n*(b*x+a)^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/b/d+1/3*(b*
x+a)^3*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/b+1/3*B*(-a*d+b*c)^2*n*(b*x+a)*(
2*A+B*n+2*B*ln(e*(b*x+a)^n/((d*x+c)^n)))/b/d^2+1/3*B*(-a*d+b*c)^3*n*ln((-a*
d+b*c)/b/(d*x+c))*(2*A+3*B*n+2*B*ln(e*(b*x+a)^n/((d*x+c)^n)))/b/d^3+2/3*B^2
*(-a*d+b*c)^3*n^2*polylog(2,d*(b*x+a)/b/(d*x+c))/b/d^3
```

Rubi [A]
time = 0.23, antiderivative size = 263, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2573, 2549, 2381, 2384, 2354, 2438}

$$\frac{2B^2n^2(bc-ad)^3\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{3bd^3}+\frac{Bn(bc-ad)^3\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(2B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+2A+3Bn\right)}{3bd^3}+\frac{Bn(a+bx)(bc-ad)^2\left(2B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+2A+Bn\right)}{3bd^2}-\frac{Bn(a+bx)^2(bc-ad)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{3bd}+\frac{(a+bx)^3\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^2}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2,x]
```

```
[Out] -1/3*(B*(b*c - a*d)*n*(a + b*x)^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]))
/(b*d) + ((a + b*x)^3*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2)/(3*b) + (
B*(b*c - a*d)^2*n*(a + b*x)*(2*A + B*n + 2*B*Log[(e*(a + b*x)^n)/(c + d*x)^
n]))/(3*b*d^2) + (B*(b*c - a*d)^3*n*Log[(b*c - a*d)/(b*(c + d*x))]*(2*A + 3
*B*n + 2*B*Log[(e*(a + b*x)^n)/(c + d*x)^n]))/(3*b*d^3) + (2*B^2*(b*c - a*d
)^3*n^2*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/(3*b*d^3)
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2381

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + Dist[b*n*(p/(d*(q + 1))), Int[(f*x)^
m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, d
, e, f, m, n, q}, x] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 2384

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Dist[f/(e*(q + 1)), Int[(f*x)^(m - 1)*(d + e*x)^(q + 1
)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x
] && ILtQ[q, -1] && GtQ[m, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2549

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(m +
1)*(g/b)^m, Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x], x,
(a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ
[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[b*f - a*g, 0] && (GtQ[p, 0] || LtQ
[m, -1])
```

Rule 2573

```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

Rubi steps

$$\int (a+bx)^2 \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2 \, dx = \int \left(A^2(a+bx)^2+2AB(a+bx)^2\log \left(e(a+bx)^n(c+\right.\right.$$
$$= \frac{A^2(a+bx)^3}{3b}+(2AB)\int (a+bx)^2\log \left(e(a+bx)^n(c+\right.$$
$$= \frac{A^2(a+bx)^3}{3b}+\frac{2AB(a+bx)^3\log \left(e(a+bx)^n(c+dx)\right.}{3b}$$
$$= \frac{A^2(a+bx)^3}{3b}+\frac{2AB(a+bx)^3\log \left(e(a+bx)^n(c+dx)\right.}{3b}$$
$$= \frac{2AB(bc-ad)^2nx}{3d^2}-\frac{AB(bc-ad)n(a+bx)^2}{3bd}+\frac{A^2(a}{}$$
$$= \frac{2AB(bc-ad)^2nx}{3d^2}-\frac{AB(bc-ad)n(a+bx)^2}{3bd}+\frac{A^2(a}{}$$
$$= \frac{2AB(bc-ad)^2nx}{3d^2}-\frac{AB(bc-ad)n(a+bx)^2}{3bd}+\frac{A^2(a}{}$$
$$= \frac{2AB(bc-ad)^2nx}{3d^2}+\frac{B^2(bc-ad)^2n^2x}{3d^2}-\frac{AB(bc-ac}{3}$$
$$= \frac{2AB(bc-ad)^2nx}{3d^2}+\frac{B^2(bc-ad)^2n^2x}{3d^2}-\frac{AB(bc-ac}{3}$$
$$= \frac{2AB(bc-ad)^2nx}{3d^2}+\frac{B^2(bc-ad)^2n^2x}{3d^2}-\frac{AB(bc-ac}{3}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 1149 vs. $2(263) = 526$.
time = 0.48, size = 1149, normalized size = 4.37

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2,x]
```

```
[Out] (-6*a^3*A*B*d^3*n - 2*a*b^2*B^2*c^2*d*n^2 + 6*a^2*b*B^2*c*d^2*n^2 - 6*a^3*B
^2*d^3*n^2 + 3*a^2*A^2*b*d^3*x + 2*A*b^3*B*c^2*d*n*x - 6*a*A*b^2*B*c*d^2*n*
x + 4*a^2*A*b*B*d^3*n*x + b^3*B^2*c^2*d*n^2*x - 2*a*b^2*B^2*c*d^2*n^2*x + a
^2*b*B^2*d^3*n^2*x + 3*a*A^2*b^2*d^3*x^2 - A*b^3*B*c*d^2*n*x^2 + a*A*b^2*B*
d^3*n*x^2 + A^2*b^3*d^3*x^3 - a^3*B^2*d^3*n^2*Log[a + b*x]^2 - 2*A*b^3*B*c^
```

```
3*n*Log[c + d*x] + 6*a*A*b^2*B*c^2*d*n*Log[c + d*x] - 6*a^2*A*b*B*c*d^2*n*L
og[c + d*x] - 3*b^3*B^2*c^3*n^2*Log[c + d*x] + 7*a*b^2*B^2*c^2*d*n^2*Log[c
+ d*x] - 4*a^2*b*B^2*c*d^2*n^2*Log[c + d*x] - 6*a^3*B^2*d^3*n^2*Log[c + d*x
] - b^3*B^2*c^3*n^2*Log[c + d*x]^2 + 3*a*b^2*B^2*c^2*d*n^2*Log[c + d*x]^2 -
 3*a^2*b*B^2*c*d^2*n^2*Log[c + d*x]^2 - 6*a^3*B^2*d^3*n*Log[(e*(a + b*x)^n)
/(c + d*x)^n] + 6*a^2*A*b*B*d^3*x*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 2*b^3*
B^2*c^2*d*n*x*Log[(e*(a + b*x)^n)/(c + d*x)^n] - 6*a*b^2*B^2*c*d^2*n*x*Log[
(e*(a + b*x)^n)/(c + d*x)^n] + 4*a^2*b*B^2*d^3*n*x*Log[(e*(a + b*x)^n)/(c +
 d*x)^n] + 6*a*A*b^2*B*d^3*x^2*Log[(e*(a + b*x)^n)/(c + d*x)^n] - b^3*B^2*c
*d^2*n*x^2*Log[(e*(a + b*x)^n)/(c + d*x)^n] + a*b^2*B^2*d^3*n*x^2*Log[(e*(a
 + b*x)^n)/(c + d*x)^n] + 2*A*b^3*B*d^3*x^3*Log[(e*(a + b*x)^n)/(c + d*x)^n
] - 2*b^3*B^2*c^3*n*Log[c + d*x]*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 6*a*b^2
*B^2*c^2*d*n*Log[c + d*x]*Log[(e*(a + b*x)^n)/(c + d*x)^n] - 6*a^2*b*B^2*c*
d^2*n*Log[c + d*x]*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 3*a^2*b*B^2*d^3*x*Log
[(e*(a + b*x)^n)/(c + d*x)^n]^2 + 3*a*b^2*B^2*d^3*x^2*Log[(e*(a + b*x)^n)/(
c + d*x)^n]^2 + b^3*B^2*d^3*x^3*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2 + B*n*Lo
g[a + b*x]*(2*b*B*c*(b^2*c^2 - 3*a*b*c*d + 3*a^2*d^2)*n*Log[c + d*x] - 2*B*
(b*c - a*d)^3*n*Log[(b*(c + d*x))/(b*c - a*d)] + a*d*(2*b^2*B*c^2*n - 5*a*b
*B*c*d*n + a^2*d^2*(2*A + 9*B*n) + 2*a^2*B*d^2*Log[(e*(a + b*x)^n)/(c + d*x
)^n])) - 2*B^2*(b*c - a*d)^3*n^2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)])/
(3*b*d^3)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 2.09, size = 19970, normalized size = 75.93

method	result	size
risch	Expression too large to display	19970

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1244 vs. 2(256) = 512.
time = 0.88, size = 1244, normalized size = 4.73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2,x, algorithm="maxi
ma")
```

```
[Out] 2/3*A*B*b^2*x^3*log((b*x + a)^n*e/(d*x + c)^n) + 1/3*A^2*b^2*x^3 + 2*A*B*a*
b*x^2*log((b*x + a)^n*e/(d*x + c)^n) + A^2*a*b*x^2 + 2*(a*n*e*log(b*x + a)/
```

```
b - c*n*e*log(d*x + c)/d)*A*B*a^2*e^(-1) - 2*(a^2*n*e*log(b*x + a)/b^2 - c^
2*n*e*log(d*x + c)/d^2 + (b*c*n - a*d*n)*x*e/(b*d))*A*B*a*b*e^(-1) + 1/3*(2
*a^3*n*e*log(b*x + a)/b^3 - 2*c^3*n*e*log(d*x + c)/d^3 - ((b^2*c*d*n - a*b*
d^2*n)*x^2*e - 2*(b^2*c^2*n - a^2*d^2*n)*x*e)/(b^2*d^2))*A*B*b^2*e^(-1) + 2
*A*B*a^2*x*log((b*x + a)^n*e/(d*x + c)^n) + A^2*a^2*x - 1/3*((3*n^2 + 2*n)*
b^2*c^3 - (7*n^2 + 6*n)*a*b*c^2*d + 2*(2*n^2 + 3*n)*a^2*c*d^2)*B^2*log(d*x
+ c)/d^3 - 2/3*(b^3*c^3*n^2 - 3*a*b^2*c^2*d*n^2 + 3*a^2*b*c*d^2*n^2 - a^3*d
^3*n^2)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x +
a*d)/(b*c - a*d)))*B^2/(b*d^3) - 1/3*(B^2*a^3*d^3*n^2*log(b*x + a)^2 - B^2*
b^3*d^3*x^3 - (a*b^2*d^3*(n + 3) - b^3*c*d^2*n)*B^2*x^2 - 2*(b^3*c^3*n^2 -
3*a*b^2*c^2*d*n^2 + 3*a^2*b*c*d^2*n^2)*B^2*log(b*x + a)*log(d*x + c) + (b^3
*c^3*n^2 - 3*a*b^2*c^2*d*n^2 + 3*a^2*b*c*d^2*n^2)*B^2*log(d*x + c)^2 - ((n^
2 + 2*n)*b^3*c^2*d - 2*(n^2 + 3*n)*a*b^2*c*d^2 + (n^2 + 4*n + 3)*a^2*b*d^3)
*B^2*x - (2*a*b^2*c^2*d*n^2 - 5*a^2*b*c*d^2*n^2 + (3*n^2 + 2*n)*a^3*d^3)*B^
2*log(b*x + a) - (B^2*b^3*d^3*x^3 + 3*B^2*a*b^2*d^3*x^2 + 3*B^2*a^2*b*d^3*x
)*log((b*x + a)^n)^2 - (B^2*b^3*d^3*x^3 + 3*B^2*a*b^2*d^3*x^2 + 3*B^2*a^2*b
*d^3*x)*log((d*x + c)^n)^2 - (2*B^2*b^3*d^3*x^3 + 2*B^2*a^3*d^3*n*log(b*x +
 a) + (a*b^2*d^3*(n + 6) - b^3*c*d^2*n)*B^2*x^2 + 2*(a^2*b*d^3*(2*n + 3) +
b^3*c^2*d*n - 3*a*b^2*c*d^2*n)*B^2*x - 2*(b^3*c^3*n - 3*a*b^2*c^2*d*n + 3*a
^2*b*c*d^2*n)*B^2*log(d*x + c))*log((b*x + a)^n) + (2*B^2*b^3*d^3*x^3 + 2*B
^2*a^3*d^3*n*log(b*x + a) + (a*b^2*d^3*(n + 6) - b^3*c*d^2*n)*B^2*x^2 + 2*(
a^2*b*d^3*(2*n + 3) + b^3*c^2*d*n - 3*a*b^2*c*d^2*n)*B^2*x - 2*(b^3*c^3*n -
 3*a*b^2*c^2*d*n + 3*a^2*b*c*d^2*n)*B^2*log(d*x + c) + 2*(B^2*b^3*d^3*x^3 +
 3*B^2*a*b^2*d^3*x^2 + 3*B^2*a^2*b*d^3*x)*log((b*x + a)^n))*log((d*x + c)^n
))/(b*d^3)
```

Fricas [**F**]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2,x, algorithm="fric
as")
```

```
[Out] integral(A^2*b^2*x^2 + 2*A^2*a*b*x + A^2*a^2 + (B^2*b^2*x^2 + 2*B^2*a*b*x +
 B^2*a^2)*log((b*x + a)^n*e/(d*x + c)^n)^2 + 2*(A*B*b^2*x^2 + 2*A*B*a*b*x +
 A*B*a^2)*log((b*x + a)^n*e/(d*x + c)^n), x)
```

Sympy [**F(-2)**]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*(A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**2,x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2,x, algorithm="giac
")
```

```
[Out] integrate((b*x + a)^2*(B*log((b*x + a)^n*e/(d*x + c)^n) + A)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(A+B\,\ln\left(\frac{e\,{\left(a+b\,x\right)}^n}{{\left(c+d\,x\right)}^n}\right)\right)^2\,{\left(a+b\,x\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^2*(a + b*x)^2,x)
```

```
[Out] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^2*(a + b*x)^2, x)
```

3.158 $\int (a+bx) \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2 \, dx$

Optimal. Leaf size=195

$$-\frac{B(bc-ad)n(a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{bd}+\frac{(a+bx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{2b}-$$

```
[Out] -B*(-a*d+b*c)*n*(b*x+a)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/b/d+1/2*(b*x+a)^2
*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/b-B*(-a*d+b*c)^2*n*ln((-a*d+b*c)/b/(d*
x+c))*(A+B*n+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/b/d^2-B^2*(-a*d+b*c)^2*n^2*poly
log(2,d*(b*x+a)/b/(d*x+c))/b/d^2
```

Rubi [A]
time = 0.14, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {2573, 2549, 2381, 2384, 2354, 2438}

$$-\frac{B^2n^2(bc-ad)^2\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{bd^2}-\frac{Bn(bc-ad)^2\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A+Bn\right)}{bd^2}-\frac{Bn(a+bx)(bc-ad)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{bd}+\frac{(a+bx)^2\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^2}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2,x]
```

```
[Out] -((B*(b*c - a*d)*n*(a + b*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]))/(b*d
)) + ((a + b*x)^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2)/(2*b) - (B*(b
*c - a*d)^2*n*Log[(b*c - a*d)/(b*(c + d*x))]*(A + B*n + B*Log[(e*(a + b*x)^
n)/(c + d*x)^n]))/(b*d^2) - (B^2*(b*c - a*d)^2*n^2*PolyLog[2, (d*(a + b*x))
/(b*(c + d*x))])/(b*d^2)
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2381

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + Dist[b*n*(p/(d*(q + 1))), Int[(f*x)^
m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, d
, e, f, m, n, q}, x] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 2384

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
```

```
)/(e*(q + 1))), x] - Dist[f/(e*(q + 1)), Int[(f*x)^(m - 1)*(d + e*x)^(q + 1
)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x
] && ILtQ[q, -1] && GtQ[m, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2549

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(m +
1)*(g/b)^m, Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x], x,
(a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ
[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[b*f - a*g, 0] && (GtQ[p, 0] || LtQ
[m, -1])
```

Rule 2573

```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

Rubi steps

$$\int (a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2\,dx = \int \left(A^2(a+bx)+2AB(a+bx)\log\left(e(a+bx)^n(c+dx\right.\right.$$

$$= \frac{A^2(a+bx)^2}{2b}+(2AB)\int (a+bx)\log\left(e(a+bx)^n(c+\right.$$

$$= \frac{A^2(a+bx)^2}{2b}+\frac{AB(a+bx)^2\log\left(e(a+bx)^n(c+dx)^{-}\right.}{b}$$

$$= \frac{A^2(a+bx)^2}{2b}+\frac{AB(a+bx)^2\log\left(e(a+bx)^n(c+dx)^{-}\right.}{b}$$

$$= -\frac{AB(bc-ad)nx}{d}+\frac{A^2(a+bx)^2}{2b}+\frac{AB(bc-ad)^2n\,\mathrm{lo}}{bd^2}$$

$$= -\frac{AB(bc-ad)nx}{d}+\frac{A^2(a+bx)^2}{2b}+\frac{AB(bc-ad)^2n\,\mathrm{lo}}{bd^2}$$

$$= -\frac{AB(bc-ad)nx}{d}+\frac{A^2(a+bx)^2}{2b}+\frac{AB(bc-ad)^2n\,\mathrm{lo}}{bd^2}$$

$$= -\frac{AB(bc-ad)nx}{d}+\frac{A^2(a+bx)^2}{2b}+\frac{AB(bc-ad)^2n\,\mathrm{lo}}{bd^2}$$

$$= -\frac{AB(bc-ad)nx}{d}+\frac{A^2(a+bx)^2}{2b}+\frac{AB(bc-ad)^2n\,\mathrm{lo}}{bd^2}$$

$$= -\frac{AB(bc-ad)nx}{d}+\frac{A^2(a+bx)^2}{2b}+\frac{AB(bc-ad)^2n\,\mathrm{lo}}{bd^2}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 656 vs. 2(195) = 390.
time = 0.35, size = 656, normalized size = 3.36

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2,x]
```

```
[Out] (-2*a^2*A*B*n)/b - (2*a^2*B^2*n^2)/b + (a*B^2*c*n^2)/d + a*A^2*x + a*A*B*n*
x - (A*b*B*c*n*x)/d + (A^2*b*x^2)/2 - (a^2*B^2*n^2*Log[a + b*x]^2)/(2*b) +
(A*b*B*c^2*n*Log[c + d*x])/d^2 - (2*a*A*B*c*n*Log[c + d*x])/d - (2*a^2*B^2*
n^2*Log[c + d*x])/b + (b*B^2*c^2*n^2*Log[c + d*x])/d^2 - (a*B^2*c*n^2*Log[c
 + d*x])/d + (b*B^2*c^2*n^2*Log[c + d*x]^2)/(2*d^2) - (a*B^2*c*n^2*Log[c +
```

```
d*x]^2)/d - (2*a^2*B^2*n*Log[(e*(a + b*x)^n)/(c + d*x)^n])/b + 2*a*A*B*x*Lo
g[(e*(a + b*x)^n)/(c + d*x)^n] + a*B^2*n*x*Log[(e*(a + b*x)^n)/(c + d*x)^n]
 - (b*B^2*c*n*x*Log[(e*(a + b*x)^n)/(c + d*x)^n])/d + A*b*B*x^2*Log[(e*(a +
 b*x)^n)/(c + d*x)^n] + (b*B^2*c^2*n*Log[c + d*x]*Log[(e*(a + b*x)^n)/(c +
d*x)^n])/d^2 - (2*a*B^2*c*n*Log[c + d*x]*Log[(e*(a + b*x)^n)/(c + d*x)^n])/
d + a*B^2*x*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2 + (b*B^2*x^2*Log[(e*(a + b*x
)^n)/(c + d*x)^n]^2)/2 + (B*n*Log[a + b*x]*(b*B*c*(-(b*c) + 2*a*d)*n*Log[c
+ d*x] + B*(b*c - a*d)^2*n*Log[(b*(c + d*x))/(b*c - a*d)] + a*d*(-(b*B*c*n)
 + a*d*(A + 3*B*n) + a*B*d*Log[(e*(a + b*x)^n)/(c + d*x)^n])))/(b*d^2) + (B
^2*(b*c - a*d)^2*n^2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)])/(b*d^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.37, size = 10210, normalized size = 52.36

method	result	size
risch	Expression too large to display	10210

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 753 vs. 2(195) = 390.
time = 0.81, size = 753, normalized size = 3.86

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2,x, algorithm="maxima
")
```

```
[Out] A*B*b*x^2*log((b*x + a)^n*e/(d*x + c)^n) + 1/2*A^2*b*x^2 + 2*(a*n*e*log(b*x
 + a)/b - c*n*e*log(d*x + c)/d)*A*B*a*e^(-1) - (a^2*n*e*log(b*x + a)/b^2 -
c^2*n*e*log(d*x + c)/d^2 + (b*c*n - a*d*n)*x*e/(b*d))*A*B*b*e^(-1) + 2*A*B*
a*x*log((b*x + a)^n*e/(d*x + c)^n) + A^2*a*x + ((n^2 + n)*b*c^2 - (n^2 + 2*
n)*a*c*d)*B^2*log(d*x + c)/d^2 + (b^2*c^2*n^2 - 2*a*b*c*d*n^2 + a^2*d^2*n^2
)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(
b*c - a*d)))*B^2/(b*d^2) - 1/2*(B^2*a^2*d^2*n^2*log(b*x + a)^2 - B^2*b^2*d^
2*x^2 + 2*(b^2*c^2*n^2 - 2*a*b*c*d*n^2)*B^2*log(b*x + a)*log(d*x + c) - (b^
2*c^2*n^2 - 2*a*b*c*d*n^2)*B^2*log(d*x + c)^2 - 2*(a*b*d^2*(n + 1) - b^2*c*
d*n)*B^2*x + 2*(a*b*c*d*n^2 - (n^2 + n)*a^2*d^2)*B^2*log(b*x + a) - (B^2*b^
2*d^2*x^2 + 2*B^2*a*b*d^2*x)*log((b*x + a)^n)^2 - (B^2*b^2*d^2*x^2 + 2*B^2*
a*b*d^2*x)*log((d*x + c)^n)^2 - 2*(B^2*b^2*d^2*x^2 + B^2*a^2*d^2*n*log(b*x
+ a) + (a*b*d^2*(n + 2) - b^2*c*d*n)*B^2*x + (b^2*c^2*n - 2*a*b*c*d*n)*B^2*
```

```
log(d*x + c))*log((b*x + a)^n) + 2*(B^2*b^2*d^2*x^2 + B^2*a^2*d^2*n*log(b*x
 + a) + (a*b*d^2*(n + 2) - b^2*c*d*n)*B^2*x + (b^2*c^2*n - 2*a*b*c*d*n)*B^2
*log(d*x + c) + (B^2*b^2*d^2*x^2 + 2*B^2*a*b*d^2*x)*log((b*x + a)^n))*log((
d*x + c)^n))/(b*d^2)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2,x, algorithm="fricas
")
```

```
[Out] integral(A^2*b*x + A^2*a + (B^2*b*x + B^2*a)*log((b*x + a)^n*e/(d*x + c)^n)
^2 + 2*(A*B*b*x + A*B*a)*log((b*x + a)^n*e/(d*x + c)^n), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**2,x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2,x, algorithm="giac")
```

```
[Out] integrate((b*x + a)*(B*log((b*x + a)^n*e/(d*x + c)^n) + A)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(A+B\,\ln\left(\frac{e\,{\left(a+b\,x\right)}^n}{{\left(c+d\,x\right)}^n}\right)\right)}^2\,\left(a+b\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^2*(a + b*x),x)
```

```
[Out] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^2*(a + b*x), x)
```

3.159 $\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{a+bx}\,dx$

Optimal. Leaf size=131

$$-\frac{(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right))^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b}+\frac{2Bn(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right))\operatorname{Li}_2\left(\frac{b(c+dx)}{d(a+bx)}\right)}{b}$$

```
[Out] -(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2*ln(1-b*(d*x+c)/d/(b*x+a))/b+2*B*n*(A+B
*ln(e*(b*x+a)^n/((d*x+c)^n)))*polylog(2,b*(d*x+c)/d/(b*x+a))/b+2*B^2*n^2*po
lylog(3,b*(d*x+c)/d/(b*x+a))/b
```

Rubi [A]
time = 0.12, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {2573, 2549, 2379, 2421, 6724}

$$\frac{2Bn\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{b}+\frac{2B^2n^2\text{PolyLog}\left(3,\frac{b(c+dx)}{d(a+bx)}\right)}{b}-\frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^2}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2/(a + b*x),x]
```

```
[Out] -(((A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2*Log[1 - (b*(c + d*x))/(d*(a +
 b*x))])/b) + (2*B*n*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])*PolyLog[2, (b
*(c + d*x))/(d*(a + b*x))])/b + (2*B^2*n^2*PolyLog[3, (b*(c + d*x))/(d*(a +
 b*x))])/b
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2549

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(m +
```

```
1)*(g/b)^m, Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x], x,
(a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ
[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[b*f - a*g, 0] && (GtQ[p, 0] || LtQ
[m, -1])
```

Rule 2573

```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{a+bx}\,dx=\int\left(\frac{A^2}{a+bx}+\frac{2AB\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{a+bx}+\frac{B^2\log^2\left(e(a\right.}{a}\right.$$

$$=\frac{A^2\log(a+bx)}{b}+(2AB)\int\frac{\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{a+bx}\,dx+B^2$$

$$=\frac{A^2\log(a+bx)}{b}-\frac{2AB\log\left(-\frac{bc-ad}{d(a+bx)}\right)\log\left(e(a+bx)^n(c+dx)^{-}\right.}{b}$$

$$=\frac{A^2\log(a+bx)}{b}-\frac{2AB\log\left(-\frac{bc-ad}{d(a+bx)}\right)\log\left(e(a+bx)^n(c+dx)^{-}\right.}{b}$$

$$=\frac{A^2\log(a+bx)}{b}-\frac{2AB\log\left(-\frac{bc-ad}{d(a+bx)}\right)\log\left(e(a+bx)^n(c+dx)^{-}\right.}{b}$$

$$=\frac{A^2\log(a+bx)}{b}-\frac{2AB\log\left(-\frac{bc-ad}{d(a+bx)}\right)\log\left(e(a+bx)^n(c+dx)^{-}\right.}{b}$$

$$=\frac{A^2\log(a+bx)}{b}-\frac{2AB\log\left(-\frac{bc-ad}{d(a+bx)}\right)\log\left(e(a+bx)^n(c+dx)^{-}\right.}{b}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 443 vs. 2(131) = 262.

time = 0.13, size = 443, normalized size = 3.38

$$\frac{B^2n^2\log^3(a+bx)+3Bn\log^2(a+bx)\left(A+B(-n\log(a+bx)+n\log(c+dx)+\log\left(e(a+bx)^n(c+dx)^{-n}\right))\right)+3\log(a+bx)\left(A+B(-n\log(a+bx)+n\log(c+dx)+\log\left(e(a+bx)^n(c+dx)^{-n}\right))\right)^2-6ABn\left(\log\left(\frac{d(a+bx)}{-bc+ad}\right)\log(c+dx)+\mathrm{Li}_2\left(\frac{b(c+dx)}{bc-ad}\right)\right)-6B^2n(-n\log(a+bx)+n\log(c+dx)+\log\left(e(a+bx)^n(c+dx)^{-n}\right))\left(\log\left(\frac{d(a+bx)}{-bc+ad}\right)\log(c+dx)+\mathrm{Li}_2\left(\frac{b(c+dx)}{bc-ad}\right)\right)-6B^2n^2\left(\frac{1}{2}\log^2(a+bx)\left(\log(c+dx)-\log\left(\frac{b(c+dx)}{bc-ad}\right)\right)-\log(a+bx)\mathrm{Li}_2\left(\frac{d(a+bx)}{-bc+ad}\right)+\mathrm{Li}_3\left(\frac{d(a+bx)}{-bc+ad}\right)\right)+3B^2n^2\left(\log\left(\frac{d(a+bx)}{-bc+ad}\right)\log^2(c+dx)+2\log(c+dx)\mathrm{Li}_2\left(\frac{b(c+dx)}{bc-ad}\right)-2\mathrm{Li}_3\left(\frac{b(c+dx)}{bc-ad}\right)\right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2/(a + b*x),x]
```

```
[Out] (B^2*n^2*Log[a + b*x]^3 + 3*B*n*Log[a + b*x]^2*(A + B*(-(n*Log[a + b*x]) +
n*Log[c + d*x] + Log[(e*(a + b*x)^n)/(c + d*x)^n])) + 3*Log[a + b*x]*(A + B
*(-(n*Log[a + b*x]) + n*Log[c + d*x] + Log[(e*(a + b*x)^n)/(c + d*x)^n]))^2
 - 6*A*B*n*(Log[(d*(a + b*x))/(-(b*c) + a*d)]*Log[c + d*x] + PolyLog[2, (b*
(c + d*x))/(b*c - a*d)]) - 6*B^2*n*(-(n*Log[a + b*x]) + n*Log[c + d*x] + Lo
g[(e*(a + b*x)^n)/(c + d*x)^n])*(Log[(d*(a + b*x))/(-(b*c) + a*d)]*Log[c +
d*x] + PolyLog[2, (b*(c + d*x))/(b*c - a*d)]) - 6*B^2*n^2*((Log[a + b*x]^2*
(Log[c + d*x] - Log[(b*(c + d*x))/(b*c - a*d)]))/2 - Log[a + b*x]*PolyLog[2
, (d*(a + b*x))/(-(b*c) + a*d)] + PolyLog[3, (d*(a + b*x))/(-(b*c) + a*d)])
 + 3*B^2*n^2*(Log[(d*(a + b*x))/(-(b*c) + a*d)]*Log[c + d*x]^2 + 2*Log[c +
d*x]*PolyLog[2, (b*(c + d*x))/(b*c - a*d)] - 2*PolyLog[3, (b*(c + d*x))/(b*
c - a*d)]))/(3*b)
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{\left(A+B\ln\left(e(bx+a)^{n}\left(dx+c\right)^{-n}\right)\right)^{2}}{bx+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(b*x+a),x)
```

```
[Out] int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(b*x+a),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(b*x+a),x, algorithm="maxima
")
```

```
[Out] B^2*log(b*x + a)*log((d*x + c)^n)^2/b + A^2*log(b*x + a)/b - integrate(-(2*
A*B*b*c + B^2*b*c + (B^2*b*d*x + B^2*b*c)*log((b*x + a)^n)^2 + (2*A*B*b*d +
 B^2*b*d)*x + 2*(A*B*b*c + B^2*b*c + (A*B*b*d + B^2*b*d)*x)*log((b*x + a)^n
) - 2*(A*B*b*c + B^2*b*c + (A*B*b*d + B^2*b*d)*x + (B^2*b*d*n*x + B^2*a*d*n
)*log(b*x + a) + (B^2*b*d*x + B^2*b*c)*log((b*x + a)^n))*log((d*x + c)^n))/
(b^2*d*x^2 + a*b*c + (b^2*c + a*b*d)*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(b*x+a),x, algorithm="fricas
")
[Out] integral((B^2*log((b*x + a)^n*e/(d*x + c)^n)^2 + 2*A*B*log((b*x + a)^n*e/(d
*x + c)^n) + A^2)/(b*x + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(A + B \log \left(e(a + bx)^{n} \left(c + dx\right)^{-n}\right)\right)^{2}}{a + bx}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**2/(b*x+a),x)
[Out] Integral((A + B*log(e*(a + b*x)**n/(c + d*x)**n))**2/(a + b*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(b*x+a),x, algorithm="giac")
[Out] integrate((B*log((b*x + a)^n*e/(d*x + c)^n) + A)^2/(b*x + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(A+B\,\ln\left(\frac{e\,{\left(a+b\,x\right)}^n}{{\left(c+d\,x\right)}^n}\right)\right)}^2}{a+b\,x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^2/(a + b*x),x)
[Out] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^2/(a + b*x), x)
```

3.160 $\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{(a+bx)^2}\,dx$

Optimal. Leaf size=129

$$-\frac{2B^2n^2(c+dx)}{(bc-ad)(a+bx)}-\frac{2Bn(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{(bc-ad)(a+bx)}-\frac{(c+dx)\left(A+B\log\left(e(a+bx)^n(c+\right.\right.}{(bc-ad)(a+bx)}$$

```
[Out] -2*B^2*n^2*(d*x+c)/(-a*d+b*c)/(b*x+a)-2*B*n*(d*x+c)*(A+B*ln(e*(b*x+a)^n/((d
*x+c)^n)))/(-a*d+b*c)/(b*x+a)-(d*x+c)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(
-a*d+b*c)/(b*x+a)
```

Rubi [A]
time = 0.08, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {2573, 2549, 2342, 2341}

$$-\frac{2Bn(c+dx)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{(a+bx)(bc-ad)}-\frac{(c+dx)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^2}{(a+bx)(bc-ad)}-\frac{2B^2n^2(c+dx)}{(a+bx)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2/(a + b*x)^2,x]
```

```
[Out] (-2*B^2*n^2*(c + d*x))/((b*c - a*d)*(a + b*x)) - (2*B*n*(c + d*x)*(A + B*Lo
g[(e*(a + b*x)^n)/(c + d*x)^n]))/((b*c - a*d)*(a + b*x)) - ((c + d*x)*(A +
B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2)/((b*c - a*d)*(a + b*x))
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2549

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(m +
1)*(g/b)^m, Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x], x,
(a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ
[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[b*f - a*g, 0] && (GtQ[p, 0] || LtQ
```

```
[m, -1])
```

Rule 2573

```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

Rubi steps

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{(a+bx)^2}\,dx = \int \left(\frac{A^2}{(a+bx)^2} + \frac{2AB\log(e(a+bx)^n(c+dx)^{-n})}{(a+bx)^2} + \frac{B^2\log^2(e($$
$$= -\frac{A^2}{b(a+bx)} + (2AB)\int \frac{\log(e(a+bx)^n(c+dx)^{-n})}{(a+bx)^2}\,dx + B^2\int$$
$$= -\frac{A^2}{b(a+bx)} - \frac{2AB(c+dx)\log(e(a+bx)^n(c+dx)^{-n})}{(bc-ad)(a+bx)} - \frac{B^2(c$$
$$= -\frac{A^2}{b(a+bx)} - \frac{2ABn}{b(a+bx)} - \frac{2AB(c+dx)\log(e(a+bx)^n(c+dx}{(bc-ad)(a+bx)}$$
$$= -\frac{A^2}{b(a+bx)} - \frac{2ABn}{b(a+bx)} - \frac{2B^2n^2}{b(a+bx)} - \frac{2AB(c+dx)\log(e(a+}{(bc-ad)(a}$$

Mathematica [A]
time = 0.20, size = 236, normalized size = 1.83

$$\frac{B^2dn^2(a+bx)\log^2(a+bx)+B^2dn^2(a+bx)\log^2(c+dx)+2Bdn(a+bx)\log(c+dx)\left(A+Bn+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)-2Bdn(a+bx)\log(a+bx)\left(A+Bn+Bn\log(c+dx)+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)-(bc-ad)\left(A^2+2ABn+2B^2n^2+2B(A+Bn)\log\left(e(a+bx)^n(c+dx)^{-n}\right)+B^2\log^2\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{b(bc-ad)(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2/(a + b*x)^2,x]
```

```
[Out] (B^2*d*n^2*(a + b*x)*Log[a + b*x]^2 + B^2*d*n^2*(a + b*x)*Log[c + d*x]^2 +
2*B*d*n*(a + b*x)*Log[c + d*x]*(A + B*n + B*Log[(e*(a + b*x)^n)/(c + d*x)^n
]) - 2*B*d*n*(a + b*x)*Log[a + b*x]*(A + B*n + B*n*Log[c + d*x] + B*Log[(e*
(a + b*x)^n)/(c + d*x)^n]) - (b*c - a*d)*(A^2 + 2*A*B*n + 2*B^2*n^2 + 2*B*(
A + B*n)*Log[(e*(a + b*x)^n)/(c + d*x)^n] + B^2*Log[(e*(a + b*x)^n)/(c + d*
x)^n]^2))/(b*(b*c - a*d)*(a + b*x))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 1.33, size = 10098, normalized size = 78.28

---

method	result	size
risch	Expression too large to display	10098

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 442 vs. $2(131) = 262$.
time = 0.31, size = 442, normalized size = 3.43

$$-2\left(\frac{dne\log(bx+a)}{b^2c-abd}-\frac{dne\log(dx+c)}{b^2c-abd}+\frac{ne}{b^2x+ab}\right)ABe^{(-1)}-\left(2\left(\frac{dne\log(bx+a)}{b^2c-abd}-\frac{dne\log(dx+c)}{b^2c-abd}+\frac{ne}{b^2x+ab}\right)e^{(-1)}\log\left(\frac{(bx+a)^ne}{(dx+c)^n}\right)-\frac{\left((bdn^2xe^2+adn^2e^2)\log(bx+a)^2+(bdn^2xe^2+adn^2e^2)\log(dx+c)^2-2(bcn^2-adn^2)e^2-2(bdn^2xe^2+adn^2e^2)\log(bx+a)+2(bdn^2xe^2+adn^2e^2-(bdn^2xe^2+adn^2e^2)\log(bx+a))\log(dx+c)\right)e^{(-2)}}{ab^2c-a^2bd+(b^3c-ab^2d)x}\right)B^2-\frac{B^2\log\left(\frac{(bx+a)^ne}{(dx+c)^n}\right)^2}{b^2x+ab}-\frac{2AB\log\left(\frac{(bx+a)^ne}{(dx+c)^n}\right)}{b^2x+ab}-\frac{A^2}{b^2x+ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(b*x+a)^2,x, algorithm="maxi
ma")
```

```
[Out] -2*(d*n*e*log(b*x + a)/(b^2*c - a*b*d) - d*n*e*log(d*x + c)/(b^2*c - a*b*d)
 + n*e/(b^2*x + a*b))*A*B*e^(-1) - (2*(d*n*e*log(b*x + a)/(b^2*c - a*b*d) -
 d*n*e*log(d*x + c)/(b^2*c - a*b*d) + n*e/(b^2*x + a*b))*e^(-1)*log((b*x +
a)^n*e/(d*x + c)^n) - ((b*d*n^2*x*e^2 + a*d*n^2*e^2)*log(b*x + a)^2 + (b*d*
n^2*x*e^2 + a*d*n^2*e^2)*log(d*x + c)^2 - 2*(b*c*n^2 - a*d*n^2)*e^2 - 2*(b*
d*n^2*x*e^2 + a*d*n^2*e^2)*log(b*x + a) + 2*(b*d*n^2*x*e^2 + a*d*n^2*e^2 -
(b*d*n^2*x*e^2 + a*d*n^2*e^2)*log(b*x + a))*log(d*x + c))*e^(-2)/(a*b^2*c -
 a^2*b*d + (b^3*c - a*b^2*d)*x))*B^2 - B^2*log((b*x + a)^n*e/(d*x + c)^n)^2
/(b^2*x + a*b) - 2*A*B*log((b*x + a)^n*e/(d*x + c)^n)/(b^2*x + a*b) - A^2/(
b^2*x + a*b)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 296 vs. $2(131) = 262$.
time = 0.38, size = 296, normalized size = 2.29

$$-\frac{(A^2+2AB+B^2)bc-(A^2+2AB+B^2)ad+2(B^2bc-B^2ad)n^2+(B^2bdn^2x+B^2bcn^2)\log(bx+a)^2+(B^2bdn^2x+B^2bcn^2)\log(dx+c)^2+2((AB+B^2)bc-(AB+B^2)ad)n+2(B^2bcn^2+(AB+B^2)bcn+(B^2bdn^2+(AB+B^2)bdn)x)\log(bx+a)-2(B^2bcn^2+(AB+B^2)bcn+(B^2bdn^2+(AB+B^2)bdn)x+(B^2bdn^2x+B^2bcn^2)\log(bx+a))\log(dx+c)}{ab^2c-a^2bd+(b^3c-ab^2d)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(b*x+a)^2,x, algorithm="fric
as")
```

```
[Out] -((A^2 + 2*A*B + B^2)*b*c - (A^2 + 2*A*B + B^2)*a*d + 2*(B^2*b*c - B^2*a*d)
*n^2 + (B^2*b*d*n^2*x + B^2*b*c*n^2)*log(b*x + a)^2 + (B^2*b*d*n^2*x + B^2*
b*c*n^2)*log(d*x + c)^2 + 2*((A*B + B^2)*b*c - (A*B + B^2)*a*d)*n + 2*(B^2*
b*c*n^2 + (A*B + B^2)*b*c*n + (B^2*b*d*n^2 + (A*B + B^2)*b*d*n)*x)*log(b*x
+ a) - 2*(B^2*b*c*n^2 + (A*B + B^2)*b*c*n + (B^2*b*d*n^2 + (A*B + B^2)*b*d*
```

```
n)*x + (B^2*b*d*n^2*x + B^2*b*c*n^2)*log(b*x + a))*log(d*x + c))/(a*b^2*c -
 a^2*b*d + (b^3*c - a*b^2*d)*x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**2/(b*x+a)**2,x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(b*x+a)^2,x, algorithm="giac
")
```

```
[Out] integrate((B*log((b*x + a)^n*e/(d*x + c)^n) + A)^2/(b*x + a)^2, x)
```

Mupad [B]
time = 5.27, size = 200, normalized size = 1.55

$$-\ln\left(\frac{e\,{\left(a+b\,x\right)}^n}{{\left(c+d\,x\right)}^n}\right)\,\left(\frac{2\,A\,B}{x\,b^2+a\,b}+\frac{2\,B^2\,n}{x\,b^2+a\,b}\right)-{\ln\left(\frac{e\,{\left(a+b\,x\right)}^n}{{\left(c+d\,x\right)}^n}\right)}^2\,\left(\frac{B^2}{b\,\left(a+b\,x\right)}-\frac{B^2\,d}{b\,\left(a\,d-b\,c\right)}\right)-\frac{A^2+2\,A\,B\,n+2\,B^2\,n^2}{x\,b^2+a\,b}-\frac{B\,d\,n\,\mathrm{atan}\left(\frac{\left(\frac{c\,b^2+a\,d\,b}{b}+2\,b\,d\,x\right)\,1{}\mathrm{i}}{a\,d-b\,c}\right)\,\left(A+B\,n\right)\,4{}\mathrm{i}}{b\,\left(a\,d-b\,c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^2/(a + b*x)^2,x)
```

```
[Out] - log((e*(a + b*x)^n)/(c + d*x)^n)*((2*A*B)/(a*b + b^2*x) + (2*B^2*n)/(a*b
+ b^2*x)) - log((e*(a + b*x)^n)/(c + d*x)^n)^2*(B^2/(b*(a + b*x)) - (B^2*d)
/(b*(a*d - b*c))) - (A^2 + 2*B^2*n^2 + 2*A*B*n)/(a*b + b^2*x) - (B*d*n*atan
((((b^2*c + a*b*d)/b + 2*b*d*x)*1i)/(a*d - b*c))*(A + B*n)*4i)/(b*(a*d - b*
c))
```

3.161 $\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{(a+bx)^3}\,dx$

Optimal. Leaf size=274

$$\frac{2B^2dn^2(c+dx)}{(bc-ad)^2(a+bx)}-\frac{bB^2n^2(c+dx)^2}{4(bc-ad)^2(a+bx)^2}+\frac{2Bdn(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{(bc-ad)^2(a+bx)}-\frac{bBn(c+dx)}{}$$

```
[Out] 2*B^2*d*n^2*(d*x+c)/(-a*d+b*c)^2/(b*x+a)-1/4*b*B^2*n^2*(d*x+c)^2/(-a*d+b*c)
^2/(b*x+a)^2+2*B*d*n*(d*x+c)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/(-a*d+b*c)^2
/(b*x+a)-1/2*b*B*n*(d*x+c)^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/(-a*d+b*c)^2
/(b*x+a)^2+d*(d*x+c)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(-a*d+b*c)^2/(b*x+
a)-1/2*b*(d*x+c)^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(-a*d+b*c)^2/(b*x+a)
^2
```

Rubi [A]
time = 0.17, antiderivative size = 274, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {2573, 2549, 2395, 2342, 2341}

$$-\frac{bBn(c+dx)^2\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{2(a+bx)^2(bc-ad)^2}+\frac{2Bdn(c+dx)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{(a+bx)(bc-ad)^2}-\frac{b(c+dx)^2\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^2}{2(a+bx)^2(bc-ad)^2}+\frac{d(c+dx)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^2}{(a+bx)(bc-ad)^2}-\frac{bB^2n^2(c+dx)^2}{4(a+bx)^2(bc-ad)^2}+\frac{2B^2dn^2(c+dx)}{(a+bx)(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2/(a + b*x)^3,x]
```

```
[Out] (2*B^2*d*n^2*(c + d*x))/((b*c - a*d)^2*(a + b*x)) - (b*B^2*n^2*(c + d*x)^2)
/(4*(b*c - a*d)^2*(a + b*x)^2) + (2*B*d*n*(c + d*x)*(A + B*Log[(e*(a + b*x)
^n)/(c + d*x)^n]))/((b*c - a*d)^2*(a + b*x)) - (b*B*n*(c + d*x)^2*(A + B*Lo
g[(e*(a + b*x)^n)/(c + d*x)^n]))/(2*(b*c - a*d)^2*(a + b*x)^2) + (d*(c + d*
x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2)/((b*c - a*d)^2*(a + b*x)) -
(b*(c + d*x)^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2)/(2*(b*c - a*d)^2
*(a + b*x)^2)
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2549

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(m +
1)*(g/b)^m, Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x], x,
(a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ
[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[b*f - a*g, 0] && (GtQ[p, 0] || LtQ
[m, -1])
```

Rule 2573

```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

Rubi steps

$$\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{(a+bx)^3}\,dx = \int \left(\frac{A^2}{(a+bx)^3}+\frac{2AB\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^3}+\frac{B^2\log^2\left(e(\right.}{}\right.$$

$$= -\frac{A^2}{2b(a+bx)^2}+(2AB)\int \frac{\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^3}\,dx+B^2$$

$$= -\frac{A^2}{2b(a+bx)^2}-\frac{AB\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{b(a+bx)^2}+\frac{(bB^2)\int \frac{(c+d}{}}{}$$

$$= -\frac{A^2}{2b(a+bx)^2}-\frac{AB\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{b(a+bx)^2}+\frac{B^2d(c+dx}{(}$$

$$= -\frac{A^2}{2b(a+bx)^2}-\frac{ABn}{2b(a+bx)^2}+\frac{ABdn}{b(bc-ad)(a+bx)}+\frac{ABd^2n\log}{b(bc-}$$

$$= -\frac{A^2}{2b(a+bx)^2}-\frac{ABn}{2b(a+bx)^2}+\frac{ABdn}{b(bc-ad)(a+bx)}+\frac{2B^2}{b(bc-ad}$$

Mathematica [A]

time = 0.29, size = 332, normalized size = 1.21

$$-\frac{2B^2d^2n^2(a+bx)^2\log^2(a+bx)+2B^2d^2n^2(a+bx)^2\log^2(c+dx)+2Bd^2n(a+bx)^2\log(c+dx)\left(2A+3Bn+2B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)-2Bd^2n(a+bx)^2\log(a+bx)\left(2A+3Bn+2Bn\log(c+dx)+2B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)+(bc-ad)\left(2A^2(bc-ad)+B^2n^2(bc-7ad-6bdx)+2ABn(bc-3ad-2bdx)+2B(2A(bc-ad)+Bn(bc-3ad-2bdx))\log\left(e(a+bx)^n(c+dx)^{-n}\right)+2B^2(bc-ad)\log^2\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{4b(bc-ad)^2(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2/(a + b*x)^3,x]
```

```
[Out] -1/4*(2*B^2*d^2*n^2*(a + b*x)^2*Log[a + b*x]^2 + 2*B^2*d^2*n^2*(a + b*x)^2*
Log[c + d*x]^2 + 2*B*d^2*n*(a + b*x)^2*Log[c + d*x]*(2*A + 3*B*n + 2*B*Log[
(e*(a + b*x)^n)/(c + d*x)^n]) - 2*B*d^2*n*(a + b*x)^2*Log[a + b*x]*(2*A + 3
*B*n + 2*B*n*Log[c + d*x] + 2*B*Log[(e*(a + b*x)^n)/(c + d*x)^n]) + (b*c -
a*d)*(2*A^2*(b*c - a*d) + B^2*n^2*(b*c - 7*a*d - 6*b*d*x) + 2*A*B*n*(b*c -
3*a*d - 2*b*d*x) + 2*B*(2*A*(b*c - a*d) + B*n*(b*c - 3*a*d - 2*b*d*x))*Log[
(e*(a + b*x)^n)/(c + d*x)^n] + 2*B^2*(b*c - a*d)*Log[(e*(a + b*x)^n)/(c + d
*x)^n]^2))/(b*(b*c - a*d)^2*(a + b*x)^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 2.18, size = 17300, normalized size = 63.14

method	result	size
risch	Expression too large to display	17300

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 885 vs. $2(272) = 544$.
time = 0.34, size = 885, normalized size = 3.23

$$\frac{1}{2}\left(\frac{2\,d^2ne\log(bx+a)}{b^3c^2-2\,ab^2cd+a^2bd^2}-\frac{2\,d^2ne\log(dx+c)}{b^3c^2-2\,ab^2cd+a^2bd^2}+\frac{2\,bdnxe-(bcn-3\,adn)e}{a^2b^2c-a^3bd+(b^4c-ab^3d)x^2+2\,(ab^3c-a^2b^2d)x}\right)ABe^{(-1)}+\frac{1}{4}\left(2\left(\frac{2\,d^2ne\log(bx+a)}{b^3c^2-2\,ab^2cd+a^2bd^2}-\frac{2\,d^2ne\log(dx+c)}{b^3c^2-2\,ab^2cd+a^2bd^2}+\frac{2\,bdnxe-(bcn-3\,adn)e}{a^2b^2c-a^3bd+(b^4c-ab^3d)x^2+2\,(ab^3c-a^2b^2d)x}\right)e^{(-1)}\log\left(\frac{(bx+a)^ne}{(dx+c)^n}\right)+\frac{\left(6\,(b^2cdn^2-abd^2n^2)xe^2-2\,(b^2d^2n^2x^2e^2+2\,abd^2n^2xe^2+a^2d^2n^2e^2)\log(bx+a)^2-2\,(b^2d^2n^2x^2e^2+2\,abd^2n^2xe^2+a^2d^2n^2e^2)\log(dx+c)^2-(b^2c^2n^2-8\,abcdn^2+7\,a^2d^2n^2)e^2+6\,(b^2d^2n^2x^2e^2+2\,abd^2n^2xe^2+a^2d^2n^2e^2)\log(bx+a)-2\,(3\,b^2d^2n^2x^2e^2+6\,abd^2n^2xe^2+3\,a^2d^2n^2e^2-2\,(b^2d^2n^2x^2e^2+2\,abd^2n^2xe^2+a^2d^2n^2e^2)\log(bx+a))\log(dx+c)\right)e^{(-2)}}{a^2b^3c^2-2\,a^3b^2cd+a^4bd^2+(b^5c^2-2\,ab^4cd+a^2b^3d^2)x^2+2\,(ab^4c^2-2\,a^2b^3cd+a^3b^2d^2)x}\right)B^2-\frac{B^2\log\left(\frac{(bx+a)^ne}{(dx+c)^n}\right)^2}{2\,(b^3x^2+2\,ab^2x+a^2b)}-\frac{AB\log\left(\frac{(bx+a)^ne}{(dx+c)^n}\right)}{b^3x^2+2\,ab^2x+a^2b}-\frac{A^2}{2\,(b^3x^2+2\,ab^2x+a^2b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(b*x+a)^3,x, algorithm="maxi
ma")
```

```
[Out] 1/2*(2*d^2*n*e*log(b*x + a)/(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2) - 2*d^2*n*e
*log(d*x + c)/(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2) + (2*b*d*n*x*e - (b*c*n -
 3*a*d*n)*e)/(a^2*b^2*c - a^3*b*d + (b^4*c - a*b^3*d)*x^2 + 2*(a*b^3*c - a^
2*b^2*d)*x))*A*B*e^(-1) + 1/4*(2*(2*d^2*n*e*log(b*x + a)/(b^3*c^2 - 2*a*b^2
*c*d + a^2*b*d^2) - 2*d^2*n*e*log(d*x + c)/(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d
^2) + (2*b*d*n*x*e - (b*c*n - 3*a*d*n)*e)/(a^2*b^2*c - a^3*b*d + (b^4*c - a
*b^3*d)*x^2 + 2*(a*b^3*c - a^2*b^2*d)*x))*e^(-1)*log((b*x + a)^n*e/(d*x + c
)^n) + (6*(b^2*c*d*n^2 - a*b*d^2*n^2)*x*e^2 - 2*(b^2*d^2*n^2*x^2*e^2 + 2*a*
```

```
b*d^2*n^2*x*e^2 + a^2*d^2*n^2*e^2)*log(b*x + a)^2 - 2*(b^2*d^2*n^2*x^2*e^2
+ 2*a*b*d^2*n^2*x*e^2 + a^2*d^2*n^2*e^2)*log(d*x + c)^2 - (b^2*c^2*n^2 - 8*
a*b*c*d*n^2 + 7*a^2*d^2*n^2)*e^2 + 6*(b^2*d^2*n^2*x^2*e^2 + 2*a*b*d^2*n^2*x
*e^2 + a^2*d^2*n^2*e^2)*log(b*x + a) - 2*(3*b^2*d^2*n^2*x^2*e^2 + 6*a*b*d^2
*n^2*x*e^2 + 3*a^2*d^2*n^2*e^2 - 2*(b^2*d^2*n^2*x^2*e^2 + 2*a*b*d^2*n^2*x*e
^2 + a^2*d^2*n^2*e^2)*log(b*x + a))*log(d*x + c))*e^(-2)/(a^2*b^3*c^2 - 2*a
^3*b^2*c*d + a^4*b*d^2 + (b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*x^2 + 2*(a*b
^4*c^2 - 2*a^2*b^3*c*d + a^3*b^2*d^2)*x))*B^2 - 1/2*B^2*log((b*x + a)^n*e/(
d*x + c)^n)^2/(b^3*x^2 + 2*a*b^2*x + a^2*b) - A*B*log((b*x + a)^n*e/(d*x +
c)^n)/(b^3*x^2 + 2*a*b^2*x + a^2*b) - 1/2*A^2/(b^3*x^2 + 2*a*b^2*x + a^2*b)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 774 vs. $2(272) = 544$.
time = 0.37, size = 774, normalized size = 2.82

$$-\frac{2(A^2+2AB+B^2)b^2c^2-4(A^2+2AB+B^2)abcd+2(A^2+2AB+B^2)a^2d^2+(B^2b^2c^2-8B^2abcd+7B^2a^2d^2)n^2-2(B^2b^2d^2n^2x^2+2B^2abd^2n^2x-(B^2b^2c^2-2B^2abcd)n^2)\log(bx+a)^2-2(B^2b^2d^2n^2x^2+2B^2abd^2n^2x-(B^2b^2c^2-2B^2abcd)n^2)\log(dx+c)^2+2((AB+B^2)b^2c^2-4(AB+B^2)abcd+3(AB+B^2)a^2d^2)n-2(3(B^2b^2cd-B^2abd^2)n^2+2((AB+B^2)b^2cd-(AB+B^2)abd^2)n)x+2((B^2b^2c^2-4B^2abcd)n^2-(3B^2b^2d^2n^2+2(AB+B^2)b^2d^2n)x^2+2((AB+B^2)b^2c^2-2(AB+B^2)abcd)n-2(2(AB+B^2)abd^2n+(B^2b^2cd+2B^2abd^2)n^2)x)\log(bx+a)-2((B^2b^2c^2-4B^2abcd)n^2-(3B^2b^2d^2n^2+2(AB+B^2)b^2d^2n)x^2+2((AB+B^2)b^2c^2-2(AB+B^2)abcd)n-2(2(AB+B^2)abd^2n+(B^2b^2cd+2B^2abd^2)n^2)x-2(B^2b^2d^2n^2x^2+2B^2abd^2n^2x-(B^2b^2c^2-2B^2abcd)n^2)\log(bx+a))\log(dx+c)}{4(a^2b^3c^2-2a^3b^2cd+a^4bd^2+(b^5c^2-2ab^4cd+a^2b^3d^2)x^2+2(ab^4c^2-2a^2b^3cd+a^3b^2d^2)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(b*x+a)^3,x, algorithm="fric
as")
```

```
[Out] -1/4*(2*(A^2 + 2*A*B + B^2)*b^2*c^2 - 4*(A^2 + 2*A*B + B^2)*a*b*c*d + 2*(A^
2 + 2*A*B + B^2)*a^2*d^2 + (B^2*b^2*c^2 - 8*B^2*a*b*c*d + 7*B^2*a^2*d^2)*n^
2 - 2*(B^2*b^2*d^2*n^2*x^2 + 2*B^2*a*b*d^2*n^2*x - (B^2*b^2*c^2 - 2*B^2*a*b
*c*d)*n^2)*log(b*x + a)^2 - 2*(B^2*b^2*d^2*n^2*x^2 + 2*B^2*a*b*d^2*n^2*x -
(B^2*b^2*c^2 - 2*B^2*a*b*c*d)*n^2)*log(d*x + c)^2 + 2*((A*B + B^2)*b^2*c^2
- 4*(A*B + B^2)*a*b*c*d + 3*(A*B + B^2)*a^2*d^2)*n - 2*(3*(B^2*b^2*c*d - B^
2*a*b*d^2)*n^2 + 2*((A*B + B^2)*b^2*c*d - (A*B + B^2)*a*b*d^2)*n)*x + 2*((B
^2*b^2*c^2 - 4*B^2*a*b*c*d)*n^2 - (3*B^2*b^2*d^2*n^2 + 2*(A*B + B^2)*b^2*d^
2*n)*x^2 + 2*((A*B + B^2)*b^2*c^2 - 2*(A*B + B^2)*a*b*c*d)*n - 2*(2*(A*B +
B^2)*a*b*d^2*n + (B^2*b^2*c*d + 2*B^2*a*b*d^2)*n^2)*x)*log(b*x + a) - 2*((B
^2*b^2*c^2 - 4*B^2*a*b*c*d)*n^2 - (3*B^2*b^2*d^2*n^2 + 2*(A*B + B^2)*b^2*d^
2*n)*x^2 + 2*((A*B + B^2)*b^2*c^2 - 2*(A*B + B^2)*a*b*c*d)*n - 2*(2*(A*B +
B^2)*a*b*d^2*n + (B^2*b^2*c*d + 2*B^2*a*b*d^2)*n^2)*x - 2*(B^2*b^2*d^2*n^2*
x^2 + 2*B^2*a*b*d^2*n^2*x - (B^2*b^2*c^2 - 2*B^2*a*b*c*d)*n^2)*log(b*x + a)
)*log(d*x + c))/(a^2*b^3*c^2 - 2*a^3*b^2*c*d + a^4*b*d^2 + (b^5*c^2 - 2*a*b
^4*c*d + a^2*b^3*d^2)*x^2 + 2*(a*b^4*c^2 - 2*a^2*b^3*c*d + a^3*b^2*d^2)*x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**2/(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(b*x+a)^3,x, algorithm="giac
")
```

```
[Out] integrate((B*log((b*x + a)^n*e/(d*x + c)^n) + A)^2/(b*x + a)^3, x)
```

Mupad [B]
time = 5.32, size = 444, normalized size = 1.62

$$-\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)^2\left(\frac{B^2}{2\,b\,(a^2+2\,a\,b\,x+b^2\,x^2)}-\frac{B^2\,d^2}{2\,b\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}\right)-\frac{\frac{2\,A^2\,a\,d-2\,A^2\,b\,c+7\,B^2\,a\,d\,n^2-B^2\,b\,c\,n^2+6\,A\,B\,a\,d\,n-2\,A\,B\,b\,c\,n}{2\,(a\,d-b\,c)}+\frac{d\,x\,(3\,b\,B^2\,n^2+2\,A\,b\,B\,n)}{a\,d-b\,c}}{2\,a^2\,b+4\,a\,b^2\,x+2\,b^3\,x^2}-\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\left(\frac{A\,B}{a^2\,b+2\,a\,b^2\,x+b^3\,x^2}+\frac{B^2\,d^2\,\left(\frac{b\,n\,(a\,d-b\,c)\,(2\,a\,d-b\,c)}{2\,d^2}+\frac{b^2\,n\,x\,(a\,d-b\,c)}{d}+\frac{a\,b\,n\,(a\,d-b\,c)}{2\,d}\right)}{b\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)\,(a^2\,b+2\,a\,b^2\,x+b^3\,x^2)}\right)-\frac{B\,d^2\,n\,\mathrm{atan}\left(\frac{\left(2\,b\,d\,x-\frac{2\,b^3\,c^2-2\,a^2\,b\,d^2}{2\,b\,(a\,d-b\,c)}\right)\,1{}\mathrm{i}}{a\,d-b\,c}\right)\,(2\,A+3\,B\,n)\,1{}\mathrm{i}}{b\,(a\,d-b\,c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^2/(a + b*x)^3,x)
```

```
[Out] - log((e*(a + b*x)^n)/(c + d*x)^n)^2*(B^2/(2*b*(a^2 + b^2*x^2 + 2*a*b*x)) -
 (B^2*d^2)/(2*b*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))) - ((2*A^2*a*d - 2*A^2*b*c
 + 7*B^2*a*d*n^2 - B^2*b*c*n^2 + 6*A*B*a*d*n - 2*A*B*b*c*n)/(2*(a*d - b*c))
 + (d*x*(3*B^2*b*n^2 + 2*A*B*b*n))/(a*d - b*c))/(2*a^2*b + 2*b^3*x^2 + 4*a*
b^2*x) - log((e*(a + b*x)^n)/(c + d*x)^n)*((A*B)/(a^2*b + b^3*x^2 + 2*a*b^2
*x) + (B^2*d^2*((b*n*(a*d - b*c)*(2*a*d - b*c))/(2*d^2) + (b^2*n*x*(a*d - b
*c))/d + (a*b*n*(a*d - b*c))/(2*d)))/(b*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)*(a^
2*b + b^3*x^2 + 2*a*b^2*x))) - (B*d^2*n*atan(((2*b*d*x - (2*b^3*c^2 - 2*a^2
*b*d^2)/(2*b*(a*d - b*c)))*1i)/(a*d - b*c))*(2*A + 3*B*n)*1i)/(b*(a*d - b*c
)^2)
```

3.162 $\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{(a+bx)^4}\,dx$

Optimal. Leaf size=427

$$-\frac{2B^2d^2n^2(c+dx)}{(bc-ad)^3(a+bx)}+\frac{bB^2dn^2(c+dx)^2}{2(bc-ad)^3(a+bx)^2}-\frac{2b^2B^2n^2(c+dx)^3}{27(bc-ad)^3(a+bx)^3}-\frac{2Bd^2n(c+dx)\left(A+B\log\left(e(a+bx)^n\right.\right.}{(bc-ad)^3(a+bx)}$$

```
[Out] -2*B^2*d^2*n^2*(d*x+c)/(-a*d+b*c)^3/(b*x+a)+1/2*b*B^2*d*n^2*(d*x+c)^2/(-a*d
+b*c)^3/(b*x+a)^2-2/27*b^2*B^2*n^2*(d*x+c)^3/(-a*d+b*c)^3/(b*x+a)^3-2*B*d^2
*n*(d*x+c)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/(-a*d+b*c)^3/(b*x+a)+b*B*d*n*(
d*x+c)^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/(-a*d+b*c)^3/(b*x+a)^2-2/9*b^2*B
*n*(d*x+c)^3*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/(-a*d+b*c)^3/(b*x+a)^3-d^2*(
d*x+c)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(-a*d+b*c)^3/(b*x+a)+b*d*(d*x+c)
^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(-a*d+b*c)^3/(b*x+a)^2-1/3*b^2*(d*x+
c)^3*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(-a*d+b*c)^3/(b*x+a)^3
```

Rubi [A]
time = 0.24, antiderivative size = 427, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {2573, 2549, 2395, 2342, 2341}

$$-\frac{b^2(c+dx)^3\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^2}{3(a+bx)^3(bc-ad)^3}-\frac{2b^2Bn(c+dx)^3\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{9(a+bx)^3(bc-ad)^3}-\frac{d^2(c+dx)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^2}{(a+bx)(bc-ad)^3}-\frac{2Bd^2n(c+dx)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{(a+bx)(bc-ad)^3}+\frac{bd(c+dx)^2\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^2}{(a+bx)^2(bc-ad)^3}+\frac{bBdn(c+dx)^2\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{(a+bx)^2(bc-ad)^3}-\frac{2b^2B^2n^2(c+dx)^3}{27(a+bx)^3(bc-ad)^3}-\frac{2B^2d^2n^2(c+dx)}{(a+bx)(bc-ad)^3}+\frac{bB^2dn^2(c+dx)^2}{2(a+bx)^2(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2/(a + b*x)^4,x]
```

```
[Out] (-2*B^2*d^2*n^2*(c + d*x))/((b*c - a*d)^3*(a + b*x)) + (b*B^2*d*n^2*(c + d*
x)^2)/(2*(b*c - a*d)^3*(a + b*x)^2) - (2*b^2*B^2*n^2*(c + d*x)^3)/(27*(b*c
- a*d)^3*(a + b*x)^3) - (2*B*d^2*n*(c + d*x)*(A + B*Log[(e*(a + b*x)^n)/(c
+ d*x)^n]))/((b*c - a*d)^3*(a + b*x)) + (b*B*d*n*(c + d*x)^2*(A + B*Log[(e*
(a + b*x)^n)/(c + d*x)^n]))/((b*c - a*d)^3*(a + b*x)^2) - (2*b^2*B*n*(c + d
*x)^3*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]))/(9*(b*c - a*d)^3*(a + b*x)^
3) - (d^2*(c + d*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2)/((b*c - a*d
)^3*(a + b*x)) + (b*d*(c + d*x)^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^
2)/((b*c - a*d)^3*(a + b*x)^2) - (b^2*(c + d*x)^3*(A + B*Log[(e*(a + b*x)^n
)/(c + d*x)^n])^2)/(3*(b*c - a*d)^3*(a + b*x)^3)
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2549

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(m +
1)*(g/b)^m, Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x], x,
(a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ
[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[b*f - a*g, 0] && (GtQ[p, 0] || LtQ
[m, -1])
```

Rule 2573

```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

Rubi steps

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{(a+bx)^4}\,dx = \int \left(\frac{A^2}{(a+bx)^4} + \frac{2AB\log(e(a+bx)^n(c+dx)^{-n})}{(a+bx)^4} + \frac{B^2\log^2(e(\ldots}{\ldots}\right.$$

$$= -\frac{A^2}{3b(a+bx)^3} + (2AB)\int \frac{\log(e(a+bx)^n(c+dx)^{-n})}{(a+bx)^4}\,dx + B^2 \ldots$$

$$= -\frac{A^2}{3b(a+bx)^3} - \frac{2AB\log(e(a+bx)^n(c+dx)^{-n})}{3b(a+bx)^3} - \frac{B^2\log^2(e(\ldots}{3\ldots}$$

$$= -\frac{A^2}{3b(a+bx)^3} - \frac{2AB\log(e(a+bx)^n(c+dx)^{-n})}{3b(a+bx)^3} - \frac{B^2\log^2(e(\ldots}{3\ldots}$$

$$= -\frac{A^2}{3b(a+bx)^3} - \frac{2ABn}{9b(a+bx)^3} + \frac{ABdn}{3b(bc-ad)(a+bx)^2} - \frac{2\ldots}{3b(bc-\ldots}$$

$$= -\frac{A^2}{3b(a+bx)^3} - \frac{2ABn}{9b(a+bx)^3} + \frac{ABdn}{3b(bc-ad)(a+bx)^2} - \frac{2\ldots}{3b(bc-\ldots}$$

$$= -\frac{A^2}{3b(a+bx)^3} - \frac{2ABn}{9b(a+bx)^3} + \frac{ABdn}{3b(bc-ad)(a+bx)^2} - \frac{2\ldots}{3b(bc-\ldots}$$

$$= -\frac{A^2}{3b(a+bx)^3} - \frac{2ABn}{9b(a+bx)^3} - \frac{2B^2n^2}{27b(a+bx)^3} + \frac{ABdn}{3b(bc-ad)(a+\ldots}$$

$$= -\frac{A^2}{3b(a+bx)^3} - \frac{2ABn}{9b(a+bx)^3} - \frac{2B^2n^2}{27b(a+bx)^3} + \frac{ABdn}{3b(bc-ad)(a+\ldots}$$

$$= -\frac{A^2}{3b(a+bx)^3} - \frac{2ABn}{9b(a+bx)^3} - \frac{2B^2n^2}{27b(a+bx)^3} + \frac{ABdn}{3b(bc-ad)(a+\ldots}$$

Mathematica [A]
time = 0.40, size = 432, normalized size = 1.01

$$\frac{18B^2d^3n^2(a+bx)^3\log^2(a+bx)+18B^2d^3n^2(a+bx)^3\log^2(c+dx)+6Bd^3n(a+bx)^3\log(c+dx)\left(6A+11Bn+6B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)-6Bd^3n(a+bx)^3\log(a+bx)\left(6A+11Bn+6Bn\log(c+dx)+6B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)-(bc-ad)\left(18A^2(bc-ad)^2+6ABn(11a^2d^2+abd(-7c+15dx)+b^2(2c^2-3cdx+6d^2x^2))+B^2n^2(85a^2d^2+abd(-23c+147dx)+b^2(4c^2-15cdx+66d^2x^2))+6B(6A(bc-ad)^2+Bn(11a^2d^2+abd(-7c+15dx)+b^2(2c^2-3cdx+6d^2x^2)))\log\left(e(a+bx)^n(c+dx)^{-n}\right)+18B^2(bc-ad)^2\log^2\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{54b(bc-ad)^3(a+bx)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2/(a + b*x)^4,x]
```

```
[Out] (18*B^2*d^3*n^2*(a + b*x)^3*Log[a + b*x]^2 + 18*B^2*d^3*n^2*(a + b*x)^3*Log
[c + d*x]^2 + 6*B*d^3*n*(a + b*x)^3*Log[c + d*x]*(6*A + 11*B*n + 6*B*Log[(e
*(a + b*x)^n)/(c + d*x)^n]) - 6*B*d^3*n*(a + b*x)^3*Log[a + b*x]*(6*A + 11*
B*n + 6*B*n*Log[c + d*x] + 6*B*Log[(e*(a + b*x)^n)/(c + d*x)^n]) - (b*c - a
```

```
*d)*(18*A^2*(b*c - a*d)^2 + 6*A*B*n*(11*a^2*d^2 + a*b*d*(-7*c + 15*d*x) + b
^2*(2*c^2 - 3*c*d*x + 6*d^2*x^2)) + B^2*n^2*(85*a^2*d^2 + a*b*d*(-23*c + 14
7*d*x) + b^2*(4*c^2 - 15*c*d*x + 66*d^2*x^2)) + 6*B*(6*A*(b*c - a*d)^2 + B*
n*(11*a^2*d^2 + a*b*d*(-7*c + 15*d*x) + b^2*(2*c^2 - 3*c*d*x + 6*d^2*x^2)))
*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 18*B^2*(b*c - a*d)^2*Log[(e*(a + b*x)^n
)/(c + d*x)^n]^2))/(54*b*(b*c - a*d)^3*(a + b*x)^3)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 3.63, size = 25057, normalized size = 58.68

method	result	size
risch	Expression too large to display	25057

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(b*x+a)^4,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1469 vs. $2(425) = 850$.
time = 0.42, size = 1469, normalized size = 3.44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(b*x+a)^4,x, algorithm="maxi
ma")
```

```
[Out] -1/9*(6*d^3*n*e*log(b*x + a)/(b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a
^3*b*d^3) - 6*d^3*n*e*log(d*x + c)/(b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d
^2 - a^3*b*d^3) + (6*b^2*d^2*n*x^2*e - 3*(b^2*c*d*n - 5*a*b*d^2*n)*x*e + (2
*b^2*c^2*n - 7*a*b*c*d*n + 11*a^2*d^2*n)*e)/(a^3*b^3*c^2 - 2*a^4*b^2*c*d +
a^5*b*d^2 + (b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2)*x^3 + 3*(a*b^5*c^2 - 2*a^
2*b^4*c*d + a^3*b^3*d^2)*x^2 + 3*(a^2*b^4*c^2 - 2*a^3*b^3*c*d + a^4*b^2*d^2
)*x))*A*B*e^(-1) - 1/54*(6*(6*d^3*n*e*log(b*x + a)/(b^4*c^3 - 3*a*b^3*c^2*d
 + 3*a^2*b^2*c*d^2 - a^3*b*d^3) - 6*d^3*n*e*log(d*x + c)/(b^4*c^3 - 3*a*b^3
*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3) + (6*b^2*d^2*n*x^2*e - 3*(b^2*c*d*n -
 5*a*b*d^2*n)*x*e + (2*b^2*c^2*n - 7*a*b*c*d*n + 11*a^2*d^2*n)*e)/(a^3*b^3*
c^2 - 2*a^4*b^2*c*d + a^5*b*d^2 + (b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2)*x^3
 + 3*(a*b^5*c^2 - 2*a^2*b^4*c*d + a^3*b^3*d^2)*x^2 + 3*(a^2*b^4*c^2 - 2*a^3
*b^3*c*d + a^4*b^2*d^2)*x))*e^(-1)*log((b*x + a)^n*e/(d*x + c)^n) + (66*(b^
3*c*d^2*n^2 - a*b^2*d^3*n^2)*x^2*e^2 - 3*(5*b^3*c^2*d*n^2 - 54*a*b^2*c*d^2*
n^2 + 49*a^2*b*d^3*n^2)*x*e^2 - 18*(b^3*d^3*n^2*x^3*e^2 + 3*a*b^2*d^3*n^2*x
^2*e^2 + 3*a^2*b*d^3*n^2*x*e^2 + a^3*d^3*n^2*e^2)*log(b*x + a)^2 - 18*(b^3*
d^3*n^2*x^3*e^2 + 3*a*b^2*d^3*n^2*x^2*e^2 + 3*a^2*b*d^3*n^2*x*e^2 + a^3*d^3
```

```
*n^2*e^2)*log(d*x + c)^2 + (4*b^3*c^3*n^2 - 27*a*b^2*c^2*d*n^2 + 108*a^2*b*
c*d^2*n^2 - 85*a^3*d^3*n^2)*e^2 + 66*(b^3*d^3*n^2*x^3*e^2 + 3*a*b^2*d^3*n^2
*x^2*e^2 + 3*a^2*b*d^3*n^2*x*e^2 + a^3*d^3*n^2*e^2)*log(b*x + a) - 6*(11*b^
3*d^3*n^2*x^3*e^2 + 33*a*b^2*d^3*n^2*x^2*e^2 + 33*a^2*b*d^3*n^2*x*e^2 + 11*
a^3*d^3*n^2*e^2 - 6*(b^3*d^3*n^2*x^3*e^2 + 3*a*b^2*d^3*n^2*x^2*e^2 + 3*a^2*
b*d^3*n^2*x*e^2 + a^3*d^3*n^2*e^2)*log(b*x + a))*log(d*x + c))*e^(-2)/(a^3*
b^4*c^3 - 3*a^4*b^3*c^2*d + 3*a^5*b^2*c*d^2 - a^6*b*d^3 + (b^7*c^3 - 3*a*b^
6*c^2*d + 3*a^2*b^5*c*d^2 - a^3*b^4*d^3)*x^3 + 3*(a*b^6*c^3 - 3*a^2*b^5*c^2
*d + 3*a^3*b^4*c*d^2 - a^4*b^3*d^3)*x^2 + 3*(a^2*b^5*c^3 - 3*a^3*b^4*c^2*d
+ 3*a^4*b^3*c*d^2 - a^5*b^2*d^3)*x))*B^2 - 1/3*B^2*log((b*x + a)^n*e/(d*x +
 c)^n)^2/(b^4*x^3 + 3*a*b^3*x^2 + 3*a^2*b^2*x + a^3*b) - 2/3*A*B*log((b*x +
 a)^n*e/(d*x + c)^n)/(b^4*x^3 + 3*a*b^3*x^2 + 3*a^2*b^2*x + a^3*b) - 1/3*A^
2/(b^4*x^3 + 3*a*b^3*x^2 + 3*a^2*b^2*x + a^3*b)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1371 vs. $2(425) = 850$.
time = 0.41, size = 1371, normalized size = 3.21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(b*x+a)^4,x, algorithm="fric
as")
```

```
[Out] -1/54*(18*(A^2 + 2*A*B + B^2)*b^3*c^3 - 54*(A^2 + 2*A*B + B^2)*a*b^2*c^2*d
+ 54*(A^2 + 2*A*B + B^2)*a^2*b*c*d^2 - 18*(A^2 + 2*A*B + B^2)*a^3*d^3 + (4*
B^2*b^3*c^3 - 27*B^2*a*b^2*c^2*d + 108*B^2*a^2*b*c*d^2 - 85*B^2*a^3*d^3)*n^
2 + 6*(11*(B^2*b^3*c*d^2 - B^2*a*b^2*d^3)*n^2 + 6*((A*B + B^2)*b^3*c*d^2 -
(A*B + B^2)*a*b^2*d^3)*n)*x^2 + 18*(B^2*b^3*d^3*n^2*x^3 + 3*B^2*a*b^2*d^3*n
^2*x^2 + 3*B^2*a^2*b*d^3*n^2*x + (B^2*b^3*c^3 - 3*B^2*a*b^2*c^2*d + 3*B^2*a
^2*b*c*d^2)*n^2)*log(b*x + a)^2 + 18*(B^2*b^3*d^3*n^2*x^3 + 3*B^2*a*b^2*d^3
*n^2*x^2 + 3*B^2*a^2*b*d^3*n^2*x + (B^2*b^3*c^3 - 3*B^2*a*b^2*c^2*d + 3*B^2
*a^2*b*c*d^2)*n^2)*log(d*x + c)^2 + 6*(2*(A*B + B^2)*b^3*c^3 - 9*(A*B + B^2
)*a*b^2*c^2*d + 18*(A*B + B^2)*a^2*b*c*d^2 - 11*(A*B + B^2)*a^3*d^3)*n - 3*
((5*B^2*b^3*c^2*d - 54*B^2*a*b^2*c*d^2 + 49*B^2*a^2*b*d^3)*n^2 + 6*((A*B +
B^2)*b^3*c^2*d - 6*(A*B + B^2)*a*b^2*c*d^2 + 5*(A*B + B^2)*a^2*b*d^3)*n)*x
+ 6*((11*B^2*b^3*d^3*n^2 + 6*(A*B + B^2)*b^3*d^3*n)*x^3 + (2*B^2*b^3*c^3 -
9*B^2*a*b^2*c^2*d + 18*B^2*a^2*b*c*d^2)*n^2 + 3*(6*(A*B + B^2)*a*b^2*d^3*n
+ (2*B^2*b^3*c*d^2 + 9*B^2*a*b^2*d^3)*n^2)*x^2 + 6*((A*B + B^2)*b^3*c^3 - 3
*(A*B + B^2)*a*b^2*c^2*d + 3*(A*B + B^2)*a^2*b*c*d^2)*n + 3*(6*(A*B + B^2)*
a^2*b*d^3*n - (B^2*b^3*c^2*d - 6*B^2*a*b^2*c*d^2 - 6*B^2*a^2*b*d^3)*n^2)*x)
*log(b*x + a) - 6*((11*B^2*b^3*d^3*n^2 + 6*(A*B + B^2)*b^3*d^3*n)*x^3 + (2*
B^2*b^3*c^3 - 9*B^2*a*b^2*c^2*d + 18*B^2*a^2*b*c*d^2)*n^2 + 3*(6*(A*B + B^2
)*a*b^2*d^3*n + (2*B^2*b^3*c*d^2 + 9*B^2*a*b^2*d^3)*n^2)*x^2 + 6*((A*B + B^
2)*b^3*c^3 - 3*(A*B + B^2)*a*b^2*c^2*d + 3*(A*B + B^2)*a^2*b*c*d^2)*n + 3*(
```

```
6*(A*B + B^2)*a^2*b*d^3*n - (B^2*b^3*c^2*d - 6*B^2*a*b^2*c*d^2 - 6*B^2*a^2*
b*d^3)*n^2)*x + 6*(B^2*b^3*d^3*n^2*x^3 + 3*B^2*a*b^2*d^3*n^2*x^2 + 3*B^2*a^
2*b*d^3*n^2*x + (B^2*b^3*c^3 - 3*B^2*a*b^2*c^2*d + 3*B^2*a^2*b*c*d^2)*n^2)*
log(b*x + a))*log(d*x + c))/(a^3*b^4*c^3 - 3*a^4*b^3*c^2*d + 3*a^5*b^2*c*d^
2 - a^6*b*d^3 + (b^7*c^3 - 3*a*b^6*c^2*d + 3*a^2*b^5*c*d^2 - a^3*b^4*d^3)*x
^3 + 3*(a*b^6*c^3 - 3*a^2*b^5*c^2*d + 3*a^3*b^4*c*d^2 - a^4*b^3*d^3)*x^2 +
3*(a^2*b^5*c^3 - 3*a^3*b^4*c^2*d + 3*a^4*b^3*c*d^2 - a^5*b^2*d^3)*x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**2/(b*x+a)**4,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(b*x+a)^4,x, algorithm="giac
")
```

```
[Out] integrate((B*log((b*x + a)^n*e/(d*x + c)^n) + A)^2/(b*x + a)^4, x)
```

Mupad [**B**]
time = 6.84, size = 911, normalized size = 2.13

$$\frac{\frac{18A^2a^2d^2-36A^2abcd+18A^2b^2c^2+66ABa^2d^2n-42ABabcdn+12ABb^2c^2n+85B^2a^2d^2n^2-23B^2abcdn^2+4B^2b^2c^2n^2}{6(ad-bc)}+\frac{x(-5cB^2b^2dn^2+49aB^2bd^2n^2-6AcBb^2dn+30AaBbd^2n)}{2(ad-bc)}+\frac{dx^2(11dB^2b^2n^2+6AdBb^2n)}{ad-bc}}{x^3(9b^5c-9ab^4d)+x(27a^2b^3c-27a^3b^2d)-x^2(27a^2b^3d-27ab^4c)+9a^3b^2c-9a^4bd}-\ln\left(\frac{e(a+bx)^n}{(c+dx)^n}\right)^2\left(\frac{B^2}{3b(a^3+3a^2bx+3ab^2x^2+b^3x^3)}-\frac{B^2d^3}{3b(a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3)}\right)-\ln\left(\frac{e(a+bx)^n}{(c+dx)^n}\right)\left(\frac{2AB}{3(a^3b+3a^2b^2x+3ab^3x^2+b^4x^3)}+\frac{2B^2d^3\left(a\left(\frac{bn(ad-bc)(3ad-bc)}{2d^2}+\frac{abn(ad-bc)}{d}\right)+x\left(b\left(\frac{bn(ad-bc)(3ad-bc)}{2d^2}+\frac{abn(ad-bc)}{d}\right)+\frac{2ab^2n(ad-bc)}{d}+\frac{b^2n(ad-bc)(3ad-bc)}{d^2}\right)+\frac{bn(ad-bc)(3a^2d^2-3abcd+b^2c^2)}{d^3}+\frac{3b^3nx^2(ad-bc)}{d}\right)}{9b(a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3)(a^3b+3a^2b^2x+3ab^3x^2+b^4x^3)}\right)-\frac{Bd^3n\,\mathrm{atan}\left(\frac{Bd^3n(6A+11Bn)\left(\frac{a^3bd^3-a^2b^2cd^2-ab^3c^2d+b^4c^3}{a^2bd^2-2ab^2cd+b^3c^2}+2bdx\right)(a^2bd^2-2ab^2cd+b^3c^2)\,1i}{b(11B^2d^3n^2+6ABd^3n)(ad-bc)^3}\right)(6A+11Bn)\,2i}{9b(ad-bc)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^2/(a + b*x)^4,x)
```

```
[Out] ((18*A^2*a^2*d^2 + 18*A^2*b^2*c^2 + 85*B^2*a^2*d^2*n^2 + 4*B^2*b^2*c^2*n^2
- 36*A^2*a*b*c*d + 66*A*B*a^2*d^2*n + 12*A*B*b^2*c^2*n - 23*B^2*a*b*c*d*n^2
 - 42*A*B*a*b*c*d*n)/(6*(a*d - b*c)) + (x*(49*B^2*a*b*d^2*n^2 - 5*B^2*b^2*c
*d*n^2 + 30*A*B*a*b*d^2*n - 6*A*B*b^2*c*d*n))/(2*(a*d - b*c)) + (d*x^2*(11*
B^2*b^2*d*n^2 + 6*A*B*b^2*d*n))/(a*d - b*c))/(x^3*(9*b^5*c - 9*a*b^4*d) + x
*(27*a^2*b^3*c - 27*a^3*b^2*d) - x^2*(27*a^2*b^3*d - 27*a*b^4*c) + 9*a^3*b^
2*c - 9*a^4*b*d) - log((e*(a + b*x)^n)/(c + d*x)^n)^2*(B^2/(3*b*(a^3 + b^3*
x^3 + 3*a*b^2*x^2 + 3*a^2*b*x)) - (B^2*d^3)/(3*b*(a^3*d^3 - b^3*c^3 + 3*a*b
^2*c^2*d - 3*a^2*b*c*d^2))) - log((e*(a + b*x)^n)/(c + d*x)^n)*((2*A*B)/(3*
```

```
(a^3*b + b^4*x^3 + 3*a^2*b^2*x + 3*a*b^3*x^2)) + (2*B^2*d^3*(a*((b*n*(a*d -
 b*c)*(3*a*d - b*c))/(2*d^2) + (a*b*n*(a*d - b*c))/d) + x*(b*((b*n*(a*d - b
*c)*(3*a*d - b*c))/(2*d^2) + (a*b*n*(a*d - b*c))/d) + (2*a*b^2*n*(a*d - b*c
))/d + (b^2*n*(a*d - b*c)*(3*a*d - b*c))/d^2) + (b*n*(a*d - b*c)*(3*a^2*d^2
 + b^2*c^2 - 3*a*b*c*d))/d^3 + (3*b^3*n*x^2*(a*d - b*c))/d))/(9*b*(a^3*d^3
- b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)*(a^3*b + b^4*x^3 + 3*a^2*b^2*x +
 3*a*b^3*x^2))) - (B*d^3*n*atan((B*d^3*n*(6*A + 11*B*n)*((b^4*c^3 + a^3*b*d
^3 - a^2*b^2*c*d^2 - a*b^3*c^2*d)/(b^3*c^2 + a^2*b*d^2 - 2*a*b^2*c*d) + 2*b
*d*x)*(b^3*c^2 + a^2*b*d^2 - 2*a*b^2*c*d)*1i)/(b*(11*B^2*d^3*n^2 + 6*A*B*d^
3*n)*(a*d - b*c)^3))*(6*A + 11*B*n)*2i)/(9*b*(a*d - b*c)^3)
```

3.163 $\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{(a+bx)^5}\,dx$

Optimal. Leaf size=587

$$\frac{2B^2d^3n^2(c+dx)}{(bc-ad)^4(a+bx)}-\frac{3bB^2d^2n^2(c+dx)^2}{4(bc-ad)^4(a+bx)^2}+\frac{2b^2B^2dn^2(c+dx)^3}{9(bc-ad)^4(a+bx)^3}-\frac{b^3B^2n^2(c+dx)^4}{32(bc-ad)^4(a+bx)^4}+\frac{2Bd^3n(c+dx)\,(}{}$$

```
[Out] 2*B^2*d^3*n^2*(d*x+c)/(-a*d+b*c)^4/(b*x+a)-3/4*b*B^2*d^2*n^2*(d*x+c)^2/(-a*
d+b*c)^4/(b*x+a)^2+2/9*b^2*B^2*d*n^2*(d*x+c)^3/(-a*d+b*c)^4/(b*x+a)^3-1/32*
b^3*B^2*n^2*(d*x+c)^4/(-a*d+b*c)^4/(b*x+a)^4+2*B*d^3*n*(d*x+c)*(A+B*ln(e*(b
*x+a)^n/((d*x+c)^n)))/(-a*d+b*c)^4/(b*x+a)-3/2*b*B*d^2*n*(d*x+c)^2*(A+B*ln(
e*(b*x+a)^n/((d*x+c)^n)))/(-a*d+b*c)^4/(b*x+a)^2+2/3*b^2*B*d*n*(d*x+c)^3*(A
+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/(-a*d+b*c)^4/(b*x+a)^3-1/8*b^3*B*n*(d*x+c)^
4*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/(-a*d+b*c)^4/(b*x+a)^4+d^3*(d*x+c)*(A+B
*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(-a*d+b*c)^4/(b*x+a)-3/2*b*d^2*(d*x+c)^2*(A
+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(-a*d+b*c)^4/(b*x+a)^2+b^2*d*(d*x+c)^3*(A
+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(-a*d+b*c)^4/(b*x+a)^3-1/4*b^3*(d*x+c)^4*
(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(-a*d+b*c)^4/(b*x+a)^4
```

Rubi [A]
time = 0.31, antiderivative size = 587, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {2573, 2549, 2395, 2342, 2341}

$$-\frac{b^3(c+dx)^4\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^2}{4(a+bx)^4(bc-ad)^4}-\frac{b^3Bn(c+dx)^4\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{8(a+bx)^4(bc-ad)^4}+\frac{b^2d(c+dx)^3\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^2}{(a+bx)^3(bc-ad)^4}+\frac{2b^2Bdn(c+dx)^3\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{3(a+bx)^3(bc-ad)^4}+\frac{d^3(c+dx)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^2}{(a+bx)(bc-ad)^4}+\frac{2Bd^3n(c+dx)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{(a+bx)(bc-ad)^4}-\frac{3bd^2(c+dx)^2\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^2}{2(a+bx)^2(bc-ad)^4}-\frac{3bBd^2n(c+dx)^2\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{2(a+bx)^2(bc-ad)^4}-\frac{b^3B^2n^2(c+dx)^4}{32(a+bx)^4(bc-ad)^4}+\frac{2b^2B^2dn^2(c+dx)^3}{9(a+bx)^3(bc-ad)^4}+\frac{2B^2d^3n^2(c+dx)}{(a+bx)(bc-ad)^4}-\frac{3bB^2d^2n^2(c+dx)^2}{4(a+bx)^2(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2/(a + b*x)^5,x]
```

```
[Out] (2*B^2*d^3*n^2*(c + d*x))/((b*c - a*d)^4*(a + b*x)) - (3*b*B^2*d^2*n^2*(c +
 d*x)^2)/(4*(b*c - a*d)^4*(a + b*x)^2) + (2*b^2*B^2*d*n^2*(c + d*x)^3)/(9*(
b*c - a*d)^4*(a + b*x)^3) - (b^3*B^2*n^2*(c + d*x)^4)/(32*(b*c - a*d)^4*(a
+ b*x)^4) + (2*B*d^3*n*(c + d*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]))/
((b*c - a*d)^4*(a + b*x)) - (3*b*B*d^2*n*(c + d*x)^2*(A + B*Log[(e*(a + b*x
)^n)/(c + d*x)^n]))/(2*(b*c - a*d)^4*(a + b*x)^2) + (2*b^2*B*d*n*(c + d*x)^
3*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]))/(3*(b*c - a*d)^4*(a + b*x)^3) -
 (b^3*B*n*(c + d*x)^4*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]))/(8*(b*c - a
*d)^4*(a + b*x)^4) + (d^3*(c + d*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]
)^2)/((b*c - a*d)^4*(a + b*x)) - (3*b*d^2*(c + d*x)^2*(A + B*Log[(e*(a + b*
x)^n)/(c + d*x)^n])^2)/(2*(b*c - a*d)^4*(a + b*x)^2) + (b^2*d*(c + d*x)^3*(
A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2)/((b*c - a*d)^4*(a + b*x)^3) - (b
^3*(c + d*x)^4*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2)/(4*(b*c - a*d)^4
*(a + b*x)^4)
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2549

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(m +
1)*(g/b)^m, Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x], x,
(a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ
[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[b*f - a*g, 0] && (GtQ[p, 0] || LtQ
[m, -1])
```

Rule 2573

```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

Rubi steps

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{(a+bx)^5}\,dx = \int \left(\frac{A^2}{(a+bx)^5} + \frac{2AB\log(e(a+bx)^n(c+dx)^{-n})}{(a+bx)^5} + \frac{B^2\log^2(e\right.$$

$$= -\frac{A^2}{4b(a+bx)^4} + (2AB)\int \frac{\log(e(a+bx)^n(c+dx)^{-n})}{(a+bx)^5}\,dx + B$$

$$= -\frac{A^2}{4b(a+bx)^4} - \frac{AB\log(e(a+bx)^n(c+dx)^{-n})}{2b(a+bx)^4} - \frac{B^2\log^2(e(}{4}$$

$$= -\frac{A^2}{4b(a+bx)^4} - \frac{AB\log(e(a+bx)^n(c+dx)^{-n})}{2b(a+bx)^4} - \frac{B^2\log^2(e(}{4}$$

$$= -\frac{A^2}{4b(a+bx)^4} - \frac{ABn}{8b(a+bx)^4} + \frac{ABdn}{6b(bc-ad)(a+bx)^3} - \frac{}{4b(bc}$$

$$= -\frac{A^2}{4b(a+bx)^4} - \frac{ABn}{8b(a+bx)^4} + \frac{ABdn}{6b(bc-ad)(a+bx)^3} - \frac{}{4b(bc}$$

$$= -\frac{A^2}{4b(a+bx)^4} - \frac{ABn}{8b(a+bx)^4} + \frac{ABdn}{6b(bc-ad)(a+bx)^3} - \frac{}{4b(bc}$$

$$= -\frac{A^2}{4b(a+bx)^4} - \frac{ABn}{8b(a+bx)^4} - \frac{B^2n^2}{32b(a+bx)^4} + \frac{ABdn}{6b(bc-ad)(a}$$

$$= -\frac{A^2}{4b(a+bx)^4} - \frac{ABn}{8b(a+bx)^4} - \frac{B^2n^2}{32b(a+bx)^4} + \frac{ABdn}{6b(bc-ad)(a}$$

$$= -\frac{A^2}{4b(a+bx)^4} - \frac{ABn}{8b(a+bx)^4} - \frac{B^2n^2}{32b(a+bx)^4} + \frac{ABdn}{6b(bc-ad)(a}$$

Mathematica [A]
time = 0.53, size = 1011, normalized size = 1.72

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2/(a + b*x)^5,x]
```

```
[Out] -1/288*(72*b*B^2*n^2*(-4*a^3*d^3*(c + d*x) + 6*a^2*b*d^2*(c^2 - d^2*x^2) -
4*a*b^2*d*(c^3 + d^3*x^3) + b^3*(c^4 - d^4*x^4))*Log[a + b*x]^2 + 72*b*B^2*
n^2*(-4*a^3*d^3*(c + d*x) + 6*a^2*b*d^2*(c^2 - d^2*x^2) - 4*a*b^2*d*(c^3 +
d^3*x^3) + b^3*(c^4 - d^4*x^4))*Log[c + d*x]^2 - 4*B*d*(b*c - a*d)^3*n*(a +
```

```
b*x)*(12*A + 7*B*n + 12*B*(-(n*Log[a + b*x]) + n*Log[c + d*x] + Log[(e*(a
+ b*x)^n)/(c + d*x)^n])) + 6*B*d^2*(b*c - a*d)^2*n*(a + b*x)^2*(12*A + 13*B
*n + 12*B*(-(n*Log[a + b*x]) + n*Log[c + d*x] + Log[(e*(a + b*x)^n)/(c + d*
x)^n])) - 12*B*d^3*(b*c - a*d)*n*(a + b*x)^3*(12*A + 25*B*n + 12*B*(-(n*Log
[a + b*x]) + n*Log[c + d*x] + Log[(e*(a + b*x)^n)/(c + d*x)^n])) - 12*B*d^4
*n*(a + b*x)^4*Log[a + b*x]*(12*A + 25*B*n + 12*B*(-(n*Log[a + b*x]) + n*Lo
g[c + d*x] + Log[(e*(a + b*x)^n)/(c + d*x)^n])) + 12*B*d^4*n*(a + b*x)^4*Lo
g[c + d*x]*(12*A + 25*B*n + 12*B*(-(n*Log[a + b*x]) + n*Log[c + d*x] + Log[
(e*(a + b*x)^n)/(c + d*x)^n])) + 9*(b*c - a*d)^4*(8*A^2 + 4*A*B*n + B^2*n^2
 + 16*A*B*(-(n*Log[a + b*x]) + n*Log[c + d*x] + Log[(e*(a + b*x)^n)/(c + d*
x)^n]) + 4*B^2*n*(-(n*Log[a + b*x]) + n*Log[c + d*x] + Log[(e*(a + b*x)^n)/
(c + d*x)^n]) + 8*B^2*(-(n*Log[a + b*x]) + n*Log[c + d*x] + Log[(e*(a + b*x
)^n)/(c + d*x)^n])^2) - 12*B*(b*c - a*d)*n*Log[a + b*x]*(4*B*d*(b*c - a*d)^
2*n*(a + b*x) + 6*B*d^2*(-(b*c) + a*d)*n*(a + b*x)^2 + 12*B*d^3*n*(a + b*x)
^3 - 3*(b*c - a*d)^3*(4*A + B*n + 4*B*(-(n*Log[a + b*x]) + n*Log[c + d*x] +
 Log[(e*(a + b*x)^n)/(c + d*x)^n]))) + 12*B*n*Log[c + d*x]*(4*B*d*(b*c - a*
d)^3*n*(a + b*x) - 6*B*d^2*(b*c - a*d)^2*n*(a + b*x)^2 + 12*B*d^3*(b*c - a*
d)*n*(a + b*x)^3 - 12*B*(b*c - a*d)^4*n*Log[a + b*x] + 12*B*d^4*n*(a + b*x)
^4*Log[a + b*x] - 3*(b*c - a*d)^4*(4*A + B*n + 4*B*(-(n*Log[a + b*x]) + n*L
og[c + d*x] + Log[(e*(a + b*x)^n)/(c + d*x)^n]))))/(b*(b*c - a*d)^4*(a + b*
x)^4)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 4.32, size = 33370, normalized size = 56.85

method	result	size
risch	Expression too large to display	33370

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(b*x+a)^5,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 2187 vs. 2(579) = 1158.
time = 0.52, size = 2187, normalized size = 3.73

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(b*x+a)^5,x, algorithm="maxi
ma")
```

```
[Out] 1/24*(12*d^4*n*e*log(b*x + a)/(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2
- 4*a^3*b^2*c*d^3 + a^4*b*d^4) - 12*d^4*n*e*log(d*x + c)/(b^5*c^4 - 4*a*b^4
```

```
*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4) + (12*b^3*d^3*n*x
^3*e - 6*(b^3*c*d^2*n - 7*a*b^2*d^3*n)*x^2*e + 4*(b^3*c^2*d*n - 5*a*b^2*c*d
^2*n + 13*a^2*b*d^3*n)*x*e - (3*b^3*c^3*n - 13*a*b^2*c^2*d*n + 23*a^2*b*c*d
^2*n - 25*a^3*d^3*n)*e)/(a^4*b^4*c^3 - 3*a^5*b^3*c^2*d + 3*a^6*b^2*c*d^2 -
a^7*b*d^3 + (b^8*c^3 - 3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 - a^3*b^5*d^3)*x^4 +
 4*(a*b^7*c^3 - 3*a^2*b^6*c^2*d + 3*a^3*b^5*c*d^2 - a^4*b^4*d^3)*x^3 + 6*(a
^2*b^6*c^3 - 3*a^3*b^5*c^2*d + 3*a^4*b^4*c*d^2 - a^5*b^3*d^3)*x^2 + 4*(a^3*
b^5*c^3 - 3*a^4*b^4*c^2*d + 3*a^5*b^3*c*d^2 - a^6*b^2*d^3)*x))*A*B*e^(-1) +
 1/288*(12*(12*d^4*n*e*log(b*x + a)/(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^
2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4) - 12*d^4*n*e*log(d*x + c)/(b^5*c^4 - 4
*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4) + (12*b^3*d
^3*n*x^3*e - 6*(b^3*c*d^2*n - 7*a*b^2*d^3*n)*x^2*e + 4*(b^3*c^2*d*n - 5*a*b
^2*c*d^2*n + 13*a^2*b*d^3*n)*x*e - (3*b^3*c^3*n - 13*a*b^2*c^2*d*n + 23*a^2
*b*c*d^2*n - 25*a^3*d^3*n)*e)/(a^4*b^4*c^3 - 3*a^5*b^3*c^2*d + 3*a^6*b^2*c*
d^2 - a^7*b*d^3 + (b^8*c^3 - 3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 - a^3*b^5*d^3)
*x^4 + 4*(a*b^7*c^3 - 3*a^2*b^6*c^2*d + 3*a^3*b^5*c*d^2 - a^4*b^4*d^3)*x^3
+ 6*(a^2*b^6*c^3 - 3*a^3*b^5*c^2*d + 3*a^4*b^4*c*d^2 - a^5*b^3*d^3)*x^2 + 4
*(a^3*b^5*c^3 - 3*a^4*b^4*c^2*d + 3*a^5*b^3*c*d^2 - a^6*b^2*d^3)*x))*e^(-1)
*log((b*x + a)^n*e/(d*x + c)^n) + (300*(b^4*c*d^3*n^2 - a*b^3*d^4*n^2)*x^3*
e^2 - 6*(13*b^4*c^2*d^2*n^2 - 176*a*b^3*c*d^3*n^2 + 163*a^2*b^2*d^4*n^2)*x^
2*e^2 + 4*(7*b^4*c^3*d*n^2 - 60*a*b^3*c^2*d^2*n^2 + 324*a^2*b^2*c*d^3*n^2 -
 271*a^3*b*d^4*n^2)*x*e^2 - 72*(b^4*d^4*n^2*x^4*e^2 + 4*a*b^3*d^4*n^2*x^3*e
^2 + 6*a^2*b^2*d^4*n^2*x^2*e^2 + 4*a^3*b*d^4*n^2*x*e^2 + a^4*d^4*n^2*e^2)*l
og(b*x + a)^2 - 72*(b^4*d^4*n^2*x^4*e^2 + 4*a*b^3*d^4*n^2*x^3*e^2 + 6*a^2*b
^2*d^4*n^2*x^2*e^2 + 4*a^3*b*d^4*n^2*x*e^2 + a^4*d^4*n^2*e^2)*log(d*x + c)^
2 - (9*b^4*c^4*n^2 - 64*a*b^3*c^3*d*n^2 + 216*a^2*b^2*c^2*d^2*n^2 - 576*a^3
*b*c*d^3*n^2 + 415*a^4*d^4*n^2)*e^2 + 300*(b^4*d^4*n^2*x^4*e^2 + 4*a*b^3*d^
4*n^2*x^3*e^2 + 6*a^2*b^2*d^4*n^2*x^2*e^2 + 4*a^3*b*d^4*n^2*x*e^2 + a^4*d^4
*n^2*e^2)*log(b*x + a) - 12*(25*b^4*d^4*n^2*x^4*e^2 + 100*a*b^3*d^4*n^2*x^3
*e^2 + 150*a^2*b^2*d^4*n^2*x^2*e^2 + 100*a^3*b*d^4*n^2*x*e^2 + 25*a^4*d^4*n
^2*e^2 - 12*(b^4*d^4*n^2*x^4*e^2 + 4*a*b^3*d^4*n^2*x^3*e^2 + 6*a^2*b^2*d^4*
n^2*x^2*e^2 + 4*a^3*b*d^4*n^2*x*e^2 + a^4*d^4*n^2*e^2)*log(b*x + a))*log(d*
x + c))*e^(-2)/(a^4*b^5*c^4 - 4*a^5*b^4*c^3*d + 6*a^6*b^3*c^2*d^2 - 4*a^7*b
^2*c*d^3 + a^8*b*d^4 + (b^9*c^4 - 4*a*b^8*c^3*d + 6*a^2*b^7*c^2*d^2 - 4*a^3
*b^6*c*d^3 + a^4*b^5*d^4)*x^4 + 4*(a*b^8*c^4 - 4*a^2*b^7*c^3*d + 6*a^3*b^6*
c^2*d^2 - 4*a^4*b^5*c*d^3 + a^5*b^4*d^4)*x^3 + 6*(a^2*b^7*c^4 - 4*a^3*b^6*c
^3*d + 6*a^4*b^5*c^2*d^2 - 4*a^5*b^4*c*d^3 + a^6*b^3*d^4)*x^2 + 4*(a^3*b^6*
c^4 - 4*a^4*b^5*c^3*d + 6*a^5*b^4*c^2*d^2 - 4*a^6*b^3*c*d^3 + a^7*b^2*d^4)*
x))*B^2 - 1/4*B^2*log((b*x + a)^n*e/(d*x + c)^n)^2/(b^5*x^4 + 4*a*b^4*x^3 +
 6*a^2*b^3*x^2 + 4*a^3*b^2*x + a^4*b) - 1/2*A*B*log((b*x + a)^n*e/(d*x + c)
^n)/(b^5*x^4 + 4*a*b^4*x^3 + 6*a^2*b^3*x^2 + 4*a^3*b^2*x + a^4*b) - 1/4*A^2
/(b^5*x^4 + 4*a*b^4*x^3 + 6*a^2*b^3*x^2 + 4*a^3*b^2*x + a^4*b)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2062 vs. $2(579) = 1158$.

time = 0.40, size = 2062, normalized size = 3.51

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(b*x+a)^5,x, algorithm="fric
as")
```

```
[Out] -1/288*(72*(A^2 + 2*A*B + B^2)*b^4*c^4 - 288*(A^2 + 2*A*B + B^2)*a*b^3*c^3*
d + 432*(A^2 + 2*A*B + B^2)*a^2*b^2*c^2*d^2 - 288*(A^2 + 2*A*B + B^2)*a^3*b
*c*d^3 + 72*(A^2 + 2*A*B + B^2)*a^4*d^4 - 12*(25*(B^2*b^4*c*d^3 - B^2*a*b^3
*d^4)*n^2 + 12*((A*B + B^2)*b^4*c*d^3 - (A*B + B^2)*a*b^3*d^4)*n)*x^3 + (9*
B^2*b^4*c^4 - 64*B^2*a*b^3*c^3*d + 216*B^2*a^2*b^2*c^2*d^2 - 576*B^2*a^3*b*
c*d^3 + 415*B^2*a^4*d^4)*n^2 + 6*((13*B^2*b^4*c^2*d^2 - 176*B^2*a*b^3*c*d^3
 + 163*B^2*a^2*b^2*d^4)*n^2 + 12*((A*B + B^2)*b^4*c^2*d^2 - 8*(A*B + B^2)*a
*b^3*c*d^3 + 7*(A*B + B^2)*a^2*b^2*d^4)*n)*x^2 - 72*(B^2*b^4*d^4*n^2*x^4 +
4*B^2*a*b^3*d^4*n^2*x^3 + 6*B^2*a^2*b^2*d^4*n^2*x^2 + 4*B^2*a^3*b*d^4*n^2*x
 - (B^2*b^4*c^4 - 4*B^2*a*b^3*c^3*d + 6*B^2*a^2*b^2*c^2*d^2 - 4*B^2*a^3*b*c
*d^3)*n^2)*log(b*x + a)^2 - 72*(B^2*b^4*d^4*n^2*x^4 + 4*B^2*a*b^3*d^4*n^2*x
^3 + 6*B^2*a^2*b^2*d^4*n^2*x^2 + 4*B^2*a^3*b*d^4*n^2*x - (B^2*b^4*c^4 - 4*B
^2*a*b^3*c^3*d + 6*B^2*a^2*b^2*c^2*d^2 - 4*B^2*a^3*b*c*d^3)*n^2)*log(d*x +
c)^2 + 12*(3*(A*B + B^2)*b^4*c^4 - 16*(A*B + B^2)*a*b^3*c^3*d + 36*(A*B + B
^2)*a^2*b^2*c^2*d^2 - 48*(A*B + B^2)*a^3*b*c*d^3 + 25*(A*B + B^2)*a^4*d^4)*
n - 4*((7*B^2*b^4*c^3*d - 60*B^2*a*b^3*c^2*d^2 + 324*B^2*a^2*b^2*c*d^3 - 27
1*B^2*a^3*b*d^4)*n^2 + 12*((A*B + B^2)*b^4*c^3*d - 6*(A*B + B^2)*a*b^3*c^2*
d^2 + 18*(A*B + B^2)*a^2*b^2*c*d^3 - 13*(A*B + B^2)*a^3*b*d^4)*n)*x - 12*((
25*B^2*b^4*d^4*n^2 + 12*(A*B + B^2)*b^4*d^4*n)*x^4 + 4*(12*(A*B + B^2)*a*b^
3*d^4*n + (3*B^2*b^4*c*d^3 + 22*B^2*a*b^3*d^4)*n^2)*x^3 - (3*B^2*b^4*c^4 -
16*B^2*a*b^3*c^3*d + 36*B^2*a^2*b^2*c^2*d^2 - 48*B^2*a^3*b*c*d^3)*n^2 + 6*(
12*(A*B + B^2)*a^2*b^2*d^4*n - (B^2*b^4*c^2*d^2 - 8*B^2*a*b^3*c*d^3 - 18*B^
2*a^2*b^2*d^4)*n^2)*x^2 - 12*((A*B + B^2)*b^4*c^4 - 4*(A*B + B^2)*a*b^3*c^3
*d + 6*(A*B + B^2)*a^2*b^2*c^2*d^2 - 4*(A*B + B^2)*a^3*b*c*d^3)*n + 4*(12*(
A*B + B^2)*a^3*b*d^4*n + (B^2*b^4*c^3*d - 6*B^2*a*b^3*c^2*d^2 + 18*B^2*a^2*
b^2*c*d^3 + 12*B^2*a^3*b*d^4)*n^2)*x)*log(b*x + a) + 12*((25*B^2*b^4*d^4*n^
2 + 12*(A*B + B^2)*b^4*d^4*n)*x^4 + 4*(12*(A*B + B^2)*a*b^3*d^4*n + (3*B^2*
b^4*c*d^3 + 22*B^2*a*b^3*d^4)*n^2)*x^3 - (3*B^2*b^4*c^4 - 16*B^2*a*b^3*c^3*
d + 36*B^2*a^2*b^2*c^2*d^2 - 48*B^2*a^3*b*c*d^3)*n^2 + 6*(12*(A*B + B^2)*a^
2*b^2*d^4*n - (B^2*b^4*c^2*d^2 - 8*B^2*a*b^3*c*d^3 - 18*B^2*a^2*b^2*d^4)*n^
2)*x^2 - 12*((A*B + B^2)*b^4*c^4 - 4*(A*B + B^2)*a*b^3*c^3*d + 6*(A*B + B^2
)*a^2*b^2*c^2*d^2 - 4*(A*B + B^2)*a^3*b*c*d^3)*n + 4*(12*(A*B + B^2)*a^3*b*
d^4*n + (B^2*b^4*c^3*d - 6*B^2*a*b^3*c^2*d^2 + 18*B^2*a^2*b^2*c*d^3 + 12*B^
2*a^3*b*d^4)*n^2)*x + 12*(B^2*b^4*d^4*n^2*x^4 + 4*B^2*a*b^3*d^4*n^2*x^3 + 6
*B^2*a^2*b^2*d^4*n^2*x^2 + 4*B^2*a^3*b*d^4*n^2*x - (B^2*b^4*c^4 - 4*B^2*a*b
^3*c^3*d + 6*B^2*a^2*b^2*c^2*d^2 - 4*B^2*a^3*b*c*d^3)*n^2)*log(b*x + a))*lo
g(d*x + c))/(a^4*b^5*c^4 - 4*a^5*b^4*c^3*d + 6*a^6*b^3*c^2*d^2 - 4*a^7*b^2*
```

```
c*d^3 + a^8*b*d^4 + (b^9*c^4 - 4*a*b^8*c^3*d + 6*a^2*b^7*c^2*d^2 - 4*a^3*b^
6*c*d^3 + a^4*b^5*d^4)*x^4 + 4*(a*b^8*c^4 - 4*a^2*b^7*c^3*d + 6*a^3*b^6*c^2
*d^2 - 4*a^4*b^5*c*d^3 + a^5*b^4*d^4)*x^3 + 6*(a^2*b^7*c^4 - 4*a^3*b^6*c^3*
d + 6*a^4*b^5*c^2*d^2 - 4*a^5*b^4*c*d^3 + a^6*b^3*d^4)*x^2 + 4*(a^3*b^6*c^4
 - 4*a^4*b^5*c^3*d + 6*a^5*b^4*c^2*d^2 - 4*a^6*b^3*c*d^3 + a^7*b^2*d^4)*x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**2/(b*x+a)**5,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(b*x+a)^5,x, algorithm="giac
")
```

```
[Out] integrate((B*log((b*x + a)^n*e/(d*x + c)^n) + A)^2/(b*x + a)^5, x)
```

Mupad [**B**]
time = 9.61, size = 1579, normalized size = 2.69

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^2/(a + b*x)^5,x)
```

```
[Out] (B*d^4*n*atan((B*d^4*n*(12*A + 25*B*n)*((b^5*c^4 - a^4*b*d^4 + 2*a^3*b^2*c*
d^3 - 2*a*b^4*c^3*d)/(b^4*c^3 - a^3*b*d^3 + 3*a^2*b^2*c*d^2 - 3*a*b^3*c^2*d
) + 2*b*d*x)*(b^4*c^3 - a^3*b*d^3 + 3*a^2*b^2*c*d^2 - 3*a*b^3*c^2*d)*1i)/(b
*(25*B^2*d^4*n^2 + 12*A*B*d^4*n)*(a*d - b*c)^4))*(12*A + 25*B*n)*1i)/(12*b*
(a*d - b*c)^4) - log((e*(a + b*x)^n)/(c + d*x)^n)^2*(B^2/(4*b*(a^4 + b^4*x^
4 + 4*a*b^3*x^3 + 6*a^2*b^2*x^2 + 4*a^3*b*x)) - (B^2*d^4)/(4*b*(a^4*d^4 + b
^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))) - ((72*A^2*a^
3*d^3 - 72*A^2*b^3*c^3 + 415*B^2*a^3*d^3*n^2 - 9*B^2*b^3*c^3*n^2 + 216*A^2*
a*b^2*c^2*d - 216*A^2*a^2*b*c*d^2 + 300*A*B*a^3*d^3*n - 36*A*B*b^3*c^3*n +
55*B^2*a*b^2*c^2*d*n^2 - 161*B^2*a^2*b*c*d^2*n^2 + 156*A*B*a*b^2*c^2*d*n -
276*A*B*a^2*b*c*d^2*n)/(12*(a*d - b*c)) + (x^2*(163*B^2*a*b^2*d^3*n^2 - 13*
```

```
B^2*b^3*c*d^2*n^2 + 84*A*B*a*b^2*d^3*n - 12*A*B*b^3*c*d^2*n))/(2*(a*d - b*c
)) + (x*(271*B^2*a^2*b*d^3*n^2 + 7*B^2*b^3*c^2*d*n^2 - 53*B^2*a*b^2*c*d^2*n
^2 + 156*A*B*a^2*b*d^3*n + 12*A*B*b^3*c^2*d*n - 60*A*B*a*b^2*c*d^2*n))/(3*(
a*d - b*c)) + (d*x^3*(25*B^2*b^3*d^2*n^2 + 12*A*B*b^3*d^2*n))/(a*d - b*c))/
(x*(96*a^3*b^4*c^2 + 96*a^5*b^2*d^2 - 192*a^4*b^3*c*d) + x^3*(96*a*b^6*c^2
+ 96*a^3*b^4*d^2 - 192*a^2*b^5*c*d) + x^4*(24*b^7*c^2 + 24*a^2*b^5*d^2 - 48
*a*b^6*c*d) + x^2*(144*a^2*b^5*c^2 + 144*a^4*b^3*d^2 - 288*a^3*b^4*c*d) + 2
4*a^6*b*d^2 + 24*a^4*b^3*c^2 - 48*a^5*b^2*c*d) - log((e*(a + b*x)^n)/(c + d
*x)^n)*((A*B)/(2*(a^4*b + b^5*x^4 + 4*a^3*b^2*x + 4*a*b^4*x^3 + 6*a^2*b^3*x
^2)) + (B^2*d^4*(x^2*(b*(b*((b*n*(a*d - b*c)*(4*a*d - b*c))/(6*d^2) + (a*b*
n*(a*d - b*c))/(2*d)) + (a*b^2*n*(a*d - b*c))/d + (b^2*n*(a*d - b*c)*(4*a*d
 - b*c))/(3*d^2)) + (3*a*b^3*n*(a*d - b*c))/(2*d) + (b^3*n*(a*d - b*c)*(4*a
*d - b*c))/(2*d^2)) + a*(a*((b*n*(a*d - b*c)*(4*a*d - b*c))/(6*d^2) + (a*b*
n*(a*d - b*c))/(2*d)) + (b*n*(a*d - b*c)*(6*a^2*d^2 + b^2*c^2 - 4*a*b*c*d))
/(6*d^3)) + x*(b*(a*((b*n*(a*d - b*c)*(4*a*d - b*c))/(6*d^2) + (a*b*n*(a*d
- b*c))/(2*d)) + (b*n*(a*d - b*c)*(6*a^2*d^2 + b^2*c^2 - 4*a*b*c*d))/(6*d^3
)) + a*(b*((b*n*(a*d - b*c)*(4*a*d - b*c))/(6*d^2) + (a*b*n*(a*d - b*c))/(2
*d)) + (a*b^2*n*(a*d - b*c))/d + (b^2*n*(a*d - b*c)*(4*a*d - b*c))/(3*d^2))
 + (b^2*n*(a*d - b*c)*(6*a^2*d^2 + b^2*c^2 - 4*a*b*c*d))/(2*d^3)) + (b*n*(a
*d - b*c)*(4*a^3*d^3 - b^3*c^3 + 4*a*b^2*c^2*d - 6*a^2*b*c*d^2))/(2*d^4) +
(2*b^4*n*x^3*(a*d - b*c))/d))/(4*b*(a^4*b + b^5*x^4 + 4*a^3*b^2*x + 4*a*b^4
*x^3 + 6*a^2*b^3*x^2)*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*
d - 4*a^3*b*c*d^3)))
```

3.164 $\int (a+bx)^3 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3 dx$

Optimal. Leaf size=809

$$-\frac{B^3(bc-ad)^3n^3x}{4d^3}-\frac{B^3(bc-ad)^4n^3\log\left(\frac{a+bx}{c+dx}\right)}{4bd^4}+\frac{3B^3(bc-ad)^4n^3\log(c+dx)}{2bd^4}-\frac{7B^2(bc-ad)^3n^2(a+bx)\,(A}{}$$

```
[Out] -1/4*B^3*(-a*d+b*c)^3*n^3*x/d^3-1/4*B^3*(-a*d+b*c)^4*n^3*ln((b*x+a)/(d*x+c)
)/b/d^4+3/2*B^3*(-a*d+b*c)^4*n^3*ln(d*x+c)/b/d^4-7/4*B^2*(-a*d+b*c)^3*n^2*(
b*x+a)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/b/d^3+1/4*b*B^2*(-a*d+b*c)^2*n^2*(
d*x+c)^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/d^4-9/2*B^2*(-a*d+b*c)^4*n^2*ln(
(-a*d+b*c)/b/(d*x+c))*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/b/d^4-9/4*B*(-a*d+b
*c)^3*n*(b*x+a)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/b/d^3+9/8*b*B*(-a*d+b*c
)^2*n*(d*x+c)^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/d^4-1/4*b^2*B*(-a*d+b*c
)*n*(d*x+c)^3*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/d^4-3/4*B*(-a*d+b*c)^4*n*
ln((-a*d+b*c)/b/(d*x+c))*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/b/d^4+1/4*(b*x
+a)^4*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/b+7/4*B^2*(-a*d+b*c)^4*n^2*(A+B*l
n(e*(b*x+a)^n/((d*x+c)^n)))*ln(1-b*(d*x+c)/d/(b*x+a))/b/d^4-9/2*B^3*(-a*d+b
*c)^4*n^3*polylog(2,d*(b*x+a)/b/(d*x+c))/b/d^4-3/2*B^2*(-a*d+b*c)^4*n^2*(A+
B*ln(e*(b*x+a)^n/((d*x+c)^n)))*polylog(2,d*(b*x+a)/b/(d*x+c))/b/d^4-7/4*B^3
*(-a*d+b*c)^4*n^3*polylog(2,b*(d*x+c)/d/(b*x+a))/b/d^4+3/2*B^3*(-a*d+b*c)^4
*n^3*polylog(3,d*(b*x+a)/b/(d*x+c))/b/d^4
```

Rubi [A]
time = 0.74, antiderivative size = 809, normalized size of antiderivative = 1.00, number of steps used = 27, number of rules used = 15, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.454$, Rules used = {2573, 2549, 2381, 2395, 2356, 2389, 2379, 2438, 2351, 31, 46, 2355, 2354, 2421, 6724}

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3,x]
```

```
[Out] -1/4*(B^3*(b*c - a*d)^3*n^3*x)/d^3 - (B^3*(b*c - a*d)^4*n^3*Log[(a + b*x)/(
c + d*x)])/(4*b*d^4) + (3*B^3*(b*c - a*d)^4*n^3*Log[c + d*x])/(2*b*d^4) - (
7*B^2*(b*c - a*d)^3*n^2*(a + b*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]))
/(4*b*d^3) + (b*B^2*(b*c - a*d)^2*n^2*(c + d*x)^2*(A + B*Log[(e*(a + b*x)^n
)/(c + d*x)^n]))/(4*d^4) - (9*B^2*(b*c - a*d)^4*n^2*Log[(b*c - a*d)/(b*(c +
 d*x))]*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]))/(2*b*d^4) - (9*B*(b*c - a
*d)^3*n*(a + b*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2)/(4*b*d^3) + (
9*b*B*(b*c - a*d)^2*n*(c + d*x)^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^
2)/(8*d^4) - (b^2*B*(b*c - a*d)*n*(c + d*x)^3*(A + B*Log[(e*(a + b*x)^n)/(c
 + d*x)^n])^2)/(4*d^4) - (3*B*(b*c - a*d)^4*n*Log[(b*c - a*d)/(b*(c + d*x))
]*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2)/(4*b*d^4) + ((a + b*x)^4*(A +
```

```
B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3)/(4*b) + (7*B^2*(b*c - a*d)^4*n^2*(A
 + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])*Log[1 - (b*(c + d*x))/(d*(a + b*x))]
)/(4*b*d^4) - (9*B^3*(b*c - a*d)^4*n^3*PolyLog[2, (d*(a + b*x))/(b*(c + d*x
))])/(2*b*d^4) - (3*B^2*(b*c - a*d)^4*n^2*(A + B*Log[(e*(a + b*x)^n)/(c + d
*x)^n])*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/(2*b*d^4) - (7*B^3*(b*c -
a*d)^4*n^3*PolyLog[2, (b*(c + d*x))/(d*(a + b*x))])/(4*b*d^4) + (3*B^3*(b*c
 - a*d)^4*n^3*PolyLog[3, (d*(a + b*x))/(b*(c + d*x))])/(2*b*d^4)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2355

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[x*((a + b*Log[c*x^n])^p/(d*(d + e*x))), x] - Dist[b*n*(p/d),
Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n
, p}, x] && GtQ[p, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
```

```
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2381

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + Dist[b*n*(p/(d*(q + 1))), Int[(f*x)^
m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, d
, e, f, m, n, q}, x] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2549

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(m +
1)*(g/b)^m, Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x], x,
(a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ
[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[b*f - a*g, 0] && (GtQ[p, 0] || LtQ
[m, -1])
```

Rule 2573

```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (a+bx)^3 \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3 \, dx = \int \left(A^3(a+bx)^3+3A^2B(a+bx)^3\log \left(e(a+bx)^n(c\right.\right.$$

$$= \frac{A^3(a+bx)^4}{4b} + \left(3A^2B\right) \int (a+bx)^3 \log \left(e(a+bx)^n(\right.$$

$$= \frac{A^3(a+bx)^4}{4b} + \frac{3A^2B(a+bx)^4 \log \left(e(a+bx)^n(c+\right.}{4b}$$

$$= \frac{A^3(a+bx)^4}{4b} + \frac{3A^2B(a+bx)^4 \log \left(e(a+bx)^n(c+\right.}{4b}$$

$$= -\frac{3A^2B(bc-ad)^3nx}{4d^3} + \frac{3A^2B(bc-ad)^2n(a+bx)^2}{8bd^2}$$

$$= -\frac{3A^2B(bc-ad)^3nx}{4d^3} + \frac{3A^2B(bc-ad)^2n(a+bx)^2}{8bd^2}$$

$$= -\frac{3A^2B(bc-ad)^3nx}{4d^3} + \frac{3A^2B(bc-ad)^2n(a+bx)^2}{8bd^2}$$

$$= -\frac{3A^2B(bc-ad)^3nx}{4d^3} - \frac{5AB^2(bc-ad)^3n^2x}{4d^3} + \frac{3A^2}{}$$

$$= -\frac{3A^2B(bc-ad)^3nx}{4d^3} - \frac{5AB^2(bc-ad)^3n^2x}{4d^3} + \frac{3A^2}{}$$

$$= -\frac{3A^2B(bc-ad)^3nx}{4d^3} - \frac{5AB^2(bc-ad)^3n^2x}{4d^3} + \frac{3A^2}{}$$

$$= -\frac{3A^2B(bc-ad)^3nx}{4d^3} - \frac{5AB^2(bc-ad)^3n^2x}{4d^3} - \frac{B^3(}{}$$

$$= -\frac{3A^2B(bc-ad)^3nx}{4d^3} - \frac{5AB^2(bc-ad)^3n^2x}{4d^3} - \frac{B^3(}{}$$

$$= -\frac{3A^2B(bc-ad)^3nx}{4d^3} - \frac{5AB^2(bc-ad)^3n^2x}{4d^3} - \frac{B^3(}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 9054 vs. $2(809) = 1618$.
time = 5.48, size = 9054, normalized size = 11.19

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3,x]
```

```
[Out] Result too large to show
```

Maple [**F**]
time = 0.13, size = 0, normalized size = 0.00

$$\int (bx+a)^3 \left(A+B\ln\left(e(bx+a)^n (dx+c)^{-n}\right)\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3,x)
```

```
[Out] int((b*x+a)^3*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3,x, algorithm="maxi
ma")
```

```
[Out] 3/4*A^2*B*b^3*x^4*log((b*x + a)^n*e/(d*x + c)^n) + 1/4*A^3*b^3*x^4 + 3*A^2*
B*a*b^2*x^3*log((b*x + a)^n*e/(d*x + c)^n) + A^3*a*b^2*x^3 + 9/2*A^2*B*a^2*
b*x^2*log((b*x + a)^n*e/(d*x + c)^n) + 3/2*A^3*a^2*b*x^2 + 3*(a*n*e*log(b*x
 + a)/b - c*n*e*log(d*x + c)/d)*A^2*B*a^3*e^(-1) - 9/2*(a^2*n*e*log(b*x + a
)/b^2 - c^2*n*e*log(d*x + c)/d^2 + (b*c*n - a*d*n)*x*e/(b*d))*A^2*B*a^2*b*e
^(-1) + 3/2*(2*a^3*n*e*log(b*x + a)/b^3 - 2*c^3*n*e*log(d*x + c)/d^3 - ((b^
2*c*d*n - a*b*d^2*n)*x^2*e - 2*(b^2*c^2*n - a^2*d^2*n)*x*e)/(b^2*d^2))*A^2*
B*a*b^2*e^(-1) - 1/8*(6*a^4*n*e*log(b*x + a)/b^4 - 6*c^4*n*e*log(d*x + c)/d
^4 + (2*(b^3*c*d^2*n - a*b^2*d^3*n)*x^3*e - 3*(b^3*c^2*d*n - a^2*b*d^3*n)*x
^2*e + 6*(b^3*c^3*n - a^3*d^3*n)*x*e)/(b^3*d^3))*A^2*B*b^3*e^(-1) + 3*A^2*B
*a^3*x*log((b*x + a)^n*e/(d*x + c)^n) + A^3*a^3*x - 1/8*(2*(B^3*b^4*d^4*x^4
 + 4*B^3*a*b^3*d^4*x^3 + 6*B^3*a^2*b^2*d^4*x^2 + 4*B^3*a^3*b*d^4*x)*log((d*
x + c)^n)^3 - (6*B^3*a^4*d^4*n*log(b*x + a) + 6*(A*B^2*b^4*d^4 + B^3*b^4*d^
4)*x^4 + 6*(b^4*c^4*n - 4*a*b^3*c^3*d*n + 6*a^2*b^2*c^2*d^2*n - 4*a^3*b*c*d
^3*n)*B^3*log(d*x + c) + 2*(12*A*B^2*a*b^3*d^4 + (a*b^3*d^4*(n + 12) - b^4*
c*d^3*n)*B^3)*x^3 + 3*(12*A*B^2*a^2*b^2*d^4 + (3*a^2*b^2*d^4*(n + 4) + b^4*
c^2*d^2*n - 4*a*b^3*c*d^3*n)*B^3)*x^2 + 6*(4*A*B^2*a^3*b*d^4 + (a^3*b*d^4*(
3*n + 4) - b^4*c^3*d*n + 4*a*b^3*c^2*d^2*n - 6*a^2*b^2*c*d^3*n)*B^3)*x + 6*
(B^3*b^4*d^4*x^4 + 4*B^3*a*b^3*d^4*x^3 + 6*B^3*a^2*b^2*d^4*x^2 + 4*B^3*a^3*
b*d^4*x)*log((b*x + a)^n))*log((d*x + c)^n)^2)/(b*d^4) - integrate(-1/4*(12
*A*B^2*a^3*b*c*d^3 + 4*B^3*a^3*b*c*d^3 + 4*(3*A*B^2*b^4*d^4 + B^3*b^4*d^4)*
```

```
x^4 + 4*(3*(b^4*c*d^3 + 3*a*b^3*d^4)*A*B^2 + (b^4*c*d^3 + 3*a*b^3*d^4)*B^3)
*x^3 + 4*(B^3*b^4*d^4*x^4 + B^3*a^3*b*c*d^3 + (b^4*c*d^3 + 3*a*b^3*d^4)*B^3
*x^3 + 3*(a*b^3*c*d^3 + a^2*b^2*d^4)*B^3*x^2 + (3*a^2*b^2*c*d^3 + a^3*b*d^4
)*B^3*x)*log((b*x + a)^n)^3 + 12*(3*(a*b^3*c*d^3 + a^2*b^2*d^4)*A*B^2 + (a*
b^3*c*d^3 + a^2*b^2*d^4)*B^3)*x^2 + 12*(A*B^2*a^3*b*c*d^3 + B^3*a^3*b*c*d^3
 + (A*B^2*b^4*d^4 + B^3*b^4*d^4)*x^4 + ((b^4*c*d^3 + 3*a*b^3*d^4)*A*B^2 + (
b^4*c*d^3 + 3*a*b^3*d^4)*B^3)*x^3 + 3*((a*b^3*c*d^3 + a^2*b^2*d^4)*A*B^2 +
(a*b^3*c*d^3 + a^2*b^2*d^4)*B^3)*x^2 + ((3*a^2*b^2*c*d^3 + a^3*b*d^4)*A*B^2
 + (3*a^2*b^2*c*d^3 + a^3*b*d^4)*B^3)*x)*log((b*x + a)^n)^2 + 4*(3*(3*a^2*b
^2*c*d^3 + a^3*b*d^4)*A*B^2 + (3*a^2*b^2*c*d^3 + a^3*b*d^4)*B^3)*x + 12*(2*
A*B^2*a^3*b*c*d^3 + B^3*a^3*b*c*d^3 + (2*A*B^2*b^4*d^4 + B^3*b^4*d^4)*x^4 +
 (2*(b^4*c*d^3 + 3*a*b^3*d^4)*A*B^2 + (b^4*c*d^3 + 3*a*b^3*d^4)*B^3)*x^3 +
3*(2*(a*b^3*c*d^3 + a^2*b^2*d^4)*A*B^2 + (a*b^3*c*d^3 + a^2*b^2*d^4)*B^3)*x
^2 + (2*(3*a^2*b^2*c*d^3 + a^3*b*d^4)*A*B^2 + (3*a^2*b^2*c*d^3 + a^3*b*d^4)
*B^3)*x)*log((b*x + a)^n) - (6*B^3*a^4*d^4*n^2*log(b*x + a) + 24*A*B^2*a^3*
b*c*d^3 + 12*B^3*a^3*b*c*d^3 + 6*(A*B^2*b^4*d^4*(n + 4) + B^3*b^4*d^4*(n +
2))*x^4 + 6*(b^4*c^4*n^2 - 4*a*b^3*c^3*d*n^2 + 6*a^2*b^2*c^2*d^2*n^2 - 4*a^
3*b*c*d^3*n^2)*B^3*log(d*x + c) + 2*(12*(a*b^3*d^4*(n + 3) + b^4*c*d^3)*A*B
^2 - ((n^2 - 6)*b^4*c*d^3 - (n^2 + 12*n + 18)*a*b^3*d^4)*B^3)*x^3 + 3*(12*(
a^2*b^2*d^4*(n + 2) + 2*a*b^3*c*d^3)*A*B^2 + (b^4*c^2*d^2*n^2 - 4*(n^2 - 3)
*a*b^3*c*d^3 + 3*(n^2 + 4*n + 4)*a^2*b^2*d^4)*B^3)*x^2 + 12*(B^3*b^4*d^4*x^
4 + B^3*a^3*b*c*d^3 + (b^4*c*d^3 + 3*a*b^3*d^4)*B^3*x^3 + 3*(a*b^3*c*d^3 +
a^2*b^2*d^4)*B^3*x^2 + (3*a^2*b^2*c*d^3 + a^3*b*d^4)*B^3*x)*log((b*x + a)^n
)^2 + 6*(4*(a^3*b*d^4*(n + 1) + 3*a^2*b^2*c*d^3)*A*B^2 - (b^4*c^3*d*n^2 - 4
*a*b^3*c^2*d^2*n^2 + 6*(n^2 - 1)*a^2*b^2*c*d^3 - (3*n^2 + 4*n + 2)*a^3*b*d^
4)*B^3)*x + 6*(4*A*B^2*a^3*b*c*d^3 + 4*B^3*a^3*b*c*d^3 + (B^3*b^4*d^4*(n +
4) + 4*A*B^2*b^4*d^4)*x^4 + 4*((b^4*c*d^3 + 3*a*b^3*d^4)*A*B^2 + (a*b^3*d^4
*(n + 3) + b^4*c*d^3)*B^3)*x^3 + 6*(2*(a*b^3*c*d^3 + a^2*b^2*d^4)*A*B^2 + (
a^2*b^2*d^4*(n + 2) + 2*a*b^3*c*d^3)*B^3)*x^2 + 4*((3*a^2*b^2*c*d^3 + a^3*b
*d^4)*A*B^2 + (a^3*b*d^4*(n + 1) + 3*a^2*b^2*c*d^3)*B^3)*x)*log((b*x + a)^n
))*log((d*x + c)^n))/(b*d^4*x + b*c*d^3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3,x, algorithm="fric
as")
```

```
[Out] integral(A^3*b^3*x^3 + 3*A^3*a*b^2*x^2 + 3*A^3*a^2*b*x + A^3*a^3 + (B^3*b^3
*x^3 + 3*B^3*a*b^2*x^2 + 3*B^3*a^2*b*x + B^3*a^3)*log((b*x + a)^n*e/(d*x +
c)^n)^3 + 3*(A*B^2*b^3*x^3 + 3*A*B^2*a*b^2*x^2 + 3*A*B^2*a^2*b*x + A*B^2*a^
3)*log((b*x + a)^n*e/(d*x + c)^n)^2 + 3*(A^2*B*b^3*x^3 + 3*A^2*B*a*b^2*x^2
+ 3*A^2*B*a^2*b*x + A^2*B*a^3)*log((b*x + a)^n*e/(d*x + c)^n), x)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3*(A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3006 deep
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3,x, algorithm="giac
")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(A+B\,\ln\left(\frac{e\,{\left(a+b\,x\right)}^n}{{\left(c+d\,x\right)}^n}\right)\right)^3\,{\left(a+b\,x\right)}^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^3*(a + b*x)^3,x)
```

```
[Out] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^3*(a + b*x)^3, x)
```

3.165 $\int (a+bx)^2 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3\,dx$

Optimal. Leaf size=614

$$-\frac{B^3(bc-ad)^3n^3\log(c+dx)}{bd^3}+\frac{B^2(bc-ad)^2n^2(a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{bd^2}+\frac{4B^2(bc-ad)^3}{}$$

```
[Out] -B^3*(-a*d+b*c)^3*n^3*ln(d*x+c)/b/d^3+B^2*(-a*d+b*c)^2*n^2*(b*x+a)*(A+B*ln(
e*(b*x+a)^n/((d*x+c)^n)))/b/d^2+4*B^2*(-a*d+b*c)^3*n^2*ln((-a*d+b*c)/b/(d*x
+c))*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/b/d^3+2*B*(-a*d+b*c)^2*n*(b*x+a)*(A+
B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/b/d^2-1/2*b*B*(-a*d+b*c)*n*(d*x+c)^2*(A+B*
ln(e*(b*x+a)^n/((d*x+c)^n)))^2/d^3+B*(-a*d+b*c)^3*n*ln((-a*d+b*c)/b/(d*x+c)
)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/b/d^3+1/3*(b*x+a)^3*(A+B*ln(e*(b*x+a)
^n/((d*x+c)^n)))^3/b-B^2*(-a*d+b*c)^3*n^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))
*ln(1-b*(d*x+c)/d/(b*x+a))/b/d^3+4*B^3*(-a*d+b*c)^3*n^3*polylog(2,d*(b*x+a)
/b/(d*x+c))/b/d^3+2*B^2*(-a*d+b*c)^3*n^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))*
polylog(2,d*(b*x+a)/b/(d*x+c))/b/d^3+B^3*(-a*d+b*c)^3*n^3*polylog(2,b*(d*x+
c)/d/(b*x+a))/b/d^3-2*B^3*(-a*d+b*c)^3*n^3*polylog(3,d*(b*x+a)/b/(d*x+c))/b
/d^3
```

Rubi [A]
time = 0.50, antiderivative size = 614, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 14, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.424$, Rules used = {2573, 2549, 2381, 2395, 2356, 2389, 2379, 2438, 2351, 31, 2355, 2354, 2421, 6724}

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3,x]
```

```
[Out] -((B^3*(b*c - a*d)^3*n^3*Log[c + d*x])/(b*d^3)) + (B^2*(b*c - a*d)^2*n^2*(a
 + b*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]))/(b*d^2) + (4*B^2*(b*c - a
*d)^3*n^2*Log[(b*c - a*d)/(b*(c + d*x))]*(A + B*Log[(e*(a + b*x)^n)/(c + d*
x)^n]))/(b*d^3) + (2*B*(b*c - a*d)^2*n*(a + b*x)*(A + B*Log[(e*(a + b*x)^n)
/(c + d*x)^n])^2)/(b*d^2) - (b*B*(b*c - a*d)*n*(c + d*x)^2*(A + B*Log[(e*(a
 + b*x)^n)/(c + d*x)^n])^2)/(2*d^3) + (B*(b*c - a*d)^3*n*Log[(b*c - a*d)/(b
*(c + d*x))]*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2)/(b*d^3) + ((a + b*
x)^3*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3)/(3*b) - (B^2*(b*c - a*d)^3
*n^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])*Log[1 - (b*(c + d*x))/(d*(a +
 b*x))])/(b*d^3) + (4*B^3*(b*c - a*d)^3*n^3*PolyLog[2, (d*(a + b*x))/(b*(c
+ d*x))])/(b*d^3) + (2*B^2*(b*c - a*d)^3*n^2*(A + B*Log[(e*(a + b*x)^n)/(c
+ d*x)^n])*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/(b*d^3) + (B^3*(b*c - a
*d)^3*n^3*PolyLog[2, (b*(c + d*x))/(d*(a + b*x))])/(b*d^3) - (2*B^3*(b*c -
a*d)^3*n^3*PolyLog[3, (d*(a + b*x))/(b*(c + d*x))])/(b*d^3)
```

<u>Rule 31</u>

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

<u>Rule 2351</u>

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

<u>Rule 2354</u>

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

<u>Rule 2355</u>

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[x*((a + b*Log[c*x^n])^p/(d*(d + e*x))), x] - Dist[b*n*(p/d),
Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n
, p}, x] && GtQ[p, 0]
```

<u>Rule 2356</u>

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

<u>Rule 2379</u>

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

<u>Rule 2381</u>

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + Dist[b*n*(p/(d*(q + 1))), Int[(f*x)^
```

```
m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, d
, e, f, m, n, q}, x] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2549

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(m +
1)*(g/b)^m, Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x], x,
(a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ
[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[b*f - a*g, 0] && (GtQ[p, 0] || LtQ
[m, -1])
```

Rule 2573

```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (a+bx)^2 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3 dx = \int \left(A^3(a+bx)^2+3A^2B(a+bx)^2\log\left(e(a+bx)^n(c\right.\right.$$

$$= \frac{A^3(a+bx)^3}{3b} + \left(3A^2B\right)\int (a+bx)^2\log\left(e(a+bx)^n(\right.$$

$$= \frac{A^3(a+bx)^3}{3b} + \frac{A^2B(a+bx)^3\log\left(e(a+bx)^n(c+dx\right.}{b}$$

$$= \frac{A^3(a+bx)^3}{3b} + \frac{A^2B(a+bx)^3\log\left(e(a+bx)^n(c+dx\right.}{b}$$

$$= \frac{A^2B(bc-ad)^2nx}{d^2} - \frac{A^2B(bc-ad)n(a+bx)^2}{2bd} + \frac{A^3}{}$$

$$= \frac{A^2B(bc-ad)^2nx}{d^2} - \frac{A^2B(bc-ad)n(a+bx)^2}{2bd} + \frac{A^3}{}$$

$$= \frac{A^2B(bc-ad)^2nx}{d^2} - \frac{A^2B(bc-ad)n(a+bx)^2}{2bd} + \frac{A^3}{}$$

$$= \frac{A^2B(bc-ad)^2nx}{d^2} + \frac{AB^2(bc-ad)^2n^2x}{d^2} - \frac{A^2B(bc}{}$$

$$= \frac{A^2B(bc-ad)^2nx}{d^2} + \frac{AB^2(bc-ad)^2n^2x}{d^2} - \frac{A^2B(bc}{}$$

$$= \frac{A^2B(bc-ad)^2nx}{d^2} + \frac{AB^2(bc-ad)^2n^2x}{d^2} - \frac{A^2B(bc}{}$$

$$= \frac{A^2B(bc-ad)^2nx}{d^2} + \frac{AB^2(bc-ad)^2n^2x}{d^2} - \frac{A^2B(bc}{}$$

$$= \frac{A^2B(bc-ad)^2nx}{d^2} + \frac{AB^2(bc-ad)^2n^2x}{d^2} - \frac{A^2B(bc}{}$$

$$= \frac{A^2B(bc-ad)^2nx}{d^2} + \frac{AB^2(bc-ad)^2n^2x}{d^2} - \frac{A^2B(bc}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 5668 vs. $2(614) = 1228$.

time = 1.93, size = 5668, normalized size = 9.23

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3,x]
```

```
[Out] Result too large to show
```

Maple [F]
time = 0.11, size = 0, normalized size = 0.00

$$\int (bx+a)^2 \left(A+B\ln\left(e(bx+a)^n (dx+c)^{-n}\right)\right)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3,x)
```

```
[Out] int((b*x+a)^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3,x, algorithm="maxi
ma")
```

```
[Out] A^2*B*b^2*x^3*log((b*x + a)^n*e/(d*x + c)^n) + 1/3*A^3*b^2*x^3 + 3*A^2*B*a*
b*x^2*log((b*x + a)^n*e/(d*x + c)^n) + A^3*a*b*x^2 + 3*(a*n*e*log(b*x + a)/
b - c*n*e*log(d*x + c)/d)*A^2*B*a^2*e^(-1) - 3*(a^2*n*e*log(b*x + a)/b^2 -
c^2*n*e*log(d*x + c)/d^2 + (b*c*n - a*d*n)*x*e/(b*d))*A^2*B*a*b*e^(-1) + 1/
2*(2*a^3*n*e*log(b*x + a)/b^3 - 2*c^3*n*e*log(d*x + c)/d^3 - ((b^2*c*d*n -
a*b*d^2*n)*x^2*e - 2*(b^2*c^2*n - a^2*d^2*n)*x*e)/(b^2*d^2))*A^2*B*b^2*e^(-
1) + 3*A^2*B*a^2*x*log((b*x + a)^n*e/(d*x + c)^n) + A^3*a^2*x - 1/6*(2*(B^3
*b^3*d^3*x^3 + 3*B^3*a*b^2*d^3*x^2 + 3*B^3*a^2*b*d^3*x)*log((d*x + c)^n)^3
- 3*(2*B^3*a^3*d^3*n*log(b*x + a) - 2*(b^3*c^3*n - 3*a*b^2*c^2*d*n + 3*a^2*
b*c*d^2*n)*B^3*log(d*x + c) + 2*(A*B^2*b^3*d^3 + B^3*b^3*d^3)*x^3 + (6*A*B^
2*a*b^2*d^3 + (a*b^2*d^3*(n + 6) - b^3*c*d^2*n)*B^3)*x^2 + 2*(3*A*B^2*a^2*b
*d^3 + (a^2*b*d^3*(2*n + 3) + b^3*c^2*d*n - 3*a*b^2*c*d^2*n)*B^3)*x + 2*(B^
3*b^3*d^3*x^3 + 3*B^3*a*b^2*d^3*x^2 + 3*B^3*a^2*b*d^3*x)*log((b*x + a)^n))*
log((d*x + c)^n)^2)/(b*d^3) - integrate(-(3*A*B^2*a^2*b*c*d^2 + B^3*a^2*b*c
*d^2 + (3*A*B^2*b^3*d^3 + B^3*b^3*d^3)*x^3 + (B^3*b^3*d^3*x^3 + B^3*a^2*b*c
*d^2 + (b^3*c*d^2 + 2*a*b^2*d^3)*B^3*x^2 + (2*a*b^2*c*d^2 + a^2*b*d^3)*B^3*
x)*log((b*x + a)^n)^3 + (3*(b^3*c*d^2 + 2*a*b^2*d^3)*A*B^2 + (b^3*c*d^2 + 2
*a*b^2*d^3)*B^3)*x^2 + 3*(A*B^2*a^2*b*c*d^2 + B^3*a^2*b*c*d^2 + (A*B^2*b^3*
d^3 + B^3*b^3*d^3)*x^3 + ((b^3*c*d^2 + 2*a*b^2*d^3)*A*B^2 + (b^3*c*d^2 + 2*
a*b^2*d^3)*B^3)*x^2 + ((2*a*b^2*c*d^2 + a^2*b*d^3)*A*B^2 + (2*a*b^2*c*d^2 +
 a^2*b*d^3)*B^3)*x)*log((b*x + a)^n)^2 + (3*(2*a*b^2*c*d^2 + a^2*b*d^3)*A*B
```

```
^2 + (2*a*b^2*c*d^2 + a^2*b*d^3)*B^3)*x + 3*(2*A*B^2*a^2*b*c*d^2 + B^3*a^2*
b*c*d^2 + (2*A*B^2*b^3*d^3 + B^3*b^3*d^3)*x^3 + (2*(b^3*c*d^2 + 2*a*b^2*d^3
)*A*B^2 + (b^3*c*d^2 + 2*a*b^2*d^3)*B^3)*x^2 + (2*(2*a*b^2*c*d^2 + a^2*b*d^
3)*A*B^2 + (2*a*b^2*c*d^2 + a^2*b*d^3)*B^3)*x)*log((b*x + a)^n) - (2*B^3*a^
3*d^3*n^2*log(b*x + a) + 6*A*B^2*a^2*b*c*d^2 + 3*B^3*a^2*b*c*d^2 - 2*(b^3*c
^3*n^2 - 3*a*b^2*c^2*d*n^2 + 3*a^2*b*c*d^2*n^2)*B^3*log(d*x + c) + (B^3*b^3
*d^3*(2*n + 3) + 2*A*B^2*b^3*d^3*(n + 3))*x^3 + (6*(a*b^2*d^3*(n + 2) + b^3
*c*d^2)*A*B^2 - ((n^2 - 3)*b^3*c*d^2 - (n^2 + 6*n + 6)*a*b^2*d^3)*B^3)*x^2
+ 3*(B^3*b^3*d^3*x^3 + B^3*a^2*b*c*d^2 + (b^3*c*d^2 + 2*a*b^2*d^3)*B^3*x^2
+ (2*a*b^2*c*d^2 + a^2*b*d^3)*B^3*x)*log((b*x + a)^n)^2 + (6*(a^2*b*d^3*(n
+ 1) + 2*a*b^2*c*d^2)*A*B^2 + (2*b^3*c^2*d*n^2 - 6*(n^2 - 1)*a*b^2*c*d^2 +
(4*n^2 + 6*n + 3)*a^2*b*d^3)*B^3)*x + 2*(3*A*B^2*a^2*b*c*d^2 + 3*B^3*a^2*b*
c*d^2 + (B^3*b^3*d^3*(n + 3) + 3*A*B^2*b^3*d^3)*x^3 + 3*((b^3*c*d^2 + 2*a*b
^2*d^3)*A*B^2 + (a*b^2*d^3*(n + 2) + b^3*c*d^2)*B^3)*x^2 + 3*((2*a*b^2*c*d^
2 + a^2*b*d^3)*A*B^2 + (a^2*b*d^3*(n + 1) + 2*a*b^2*c*d^2)*B^3)*x)*log((b*x
 + a)^n))*log((d*x + c)^n))/(b*d^3*x + b*c*d^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3,x, algorithm="fric
as")
```

```
[Out] integral(A^3*b^2*x^2 + 2*A^3*a*b*x + A^3*a^2 + (B^3*b^2*x^2 + 2*B^3*a*b*x +
 B^3*a^2)*log((b*x + a)^n*e/(d*x + c)^n)^3 + 3*(A*B^2*b^2*x^2 + 2*A*B^2*a*b
*x + A*B^2*a^2)*log((b*x + a)^n*e/(d*x + c)^n)^2 + 3*(A^2*B*b^2*x^2 + 2*A^2
*B*a*b*x + A^2*B*a^2)*log((b*x + a)^n*e/(d*x + c)^n), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*(A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**3,x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3,x, algorithm="giac
")
```

```
[Out] integrate((b*x + a)^2*(B*log((b*x + a)^n*e/(d*x + c)^n) + A)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(A+B\,\ln\left(\frac{e\,{\left(a+b\,x\right)}^n}{{\left(c+d\,x\right)}^n}\right)\right)^3\,{\left(a+b\,x\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^3*(a + b*x)^2,x)
```

```
[Out] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^3*(a + b*x)^2, x)
```

3.166 $\int (a+bx) \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3 \, dx$

Optimal. Leaf size=376

$$-\frac{3B^2(bc-ad)^2n^2\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{bd^2}-\frac{3B(bc-ad)n(a+bx)\left(A+B\log\left(e(a\right.\right.}{2bd}$$

```
[Out] -3*B^2*(-a*d+b*c)^2*n^2*ln((-a*d+b*c)/b/(d*x+c))*(A+B*ln(e*(b*x+a)^n/((d*x+
c)^n)))/b/d^2-3/2*B*(-a*d+b*c)*n*(b*x+a)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^
2/b/d-3/2*B*(-a*d+b*c)^2*n*ln((-a*d+b*c)/b/(d*x+c))*(A+B*ln(e*(b*x+a)^n/((d
*x+c)^n)))^2/b/d^2+1/2*(b*x+a)^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/b-3*B^
3*(-a*d+b*c)^2*n^3*polylog(2,d*(b*x+a)/b/(d*x+c))/b/d^2-3*B^2*(-a*d+b*c)^2*
n^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))*polylog(2,d*(b*x+a)/b/(d*x+c))/b/d^2+
3*B^3*(-a*d+b*c)^2*n^3*polylog(3,d*(b*x+a)/b/(d*x+c))/b/d^2
```

Rubi [A]
time = 0.26, antiderivative size = 376, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.290$, Rules used = {2573, 2549, 2381, 2395, 2355, 2354, 2438, 2421, 6724}

$$-\frac{3B^2n^2(bc-ad)^2\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{bd^2}-\frac{3B^3n^3(bc-ad)^2\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{bd^2}+\frac{3B^3n^3(bc-ad)^2\text{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)}{bd^2}-\frac{3B^2n^2(bc-ad)^2\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{bd^2}-\frac{3Bn(bc-ad)^2\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^2}{2bd^2}-\frac{3Bn(a+bx)(bc-ad)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^2}{2bd}+\frac{(a+bx)^2\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^3}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3,x]
```

```
[Out] (-3*B^2*(b*c - a*d)^2*n^2*Log[(b*c - a*d)/(b*(c + d*x))]*(A + B*Log[(e*(a +
 b*x)^n)/(c + d*x)^n]))/(b*d^2) - (3*B*(b*c - a*d)*n*(a + b*x)*(A + B*Log[(
e*(a + b*x)^n)/(c + d*x)^n])^2)/(2*b*d) - (3*B*(b*c - a*d)^2*n*Log[(b*c - a
*d)/(b*(c + d*x))]*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2)/(2*b*d^2) +
((a + b*x)^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3)/(2*b) - (3*B^3*(b*
c - a*d)^2*n^3*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/(b*d^2) - (3*B^2*(b
*c - a*d)^2*n^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])*PolyLog[2, (d*(a +
 b*x))/(b*(c + d*x))])/(b*d^2) + (3*B^3*(b*c - a*d)^2*n^3*PolyLog[3, (d*(a
+ b*x))/(b*(c + d*x))])/(b*d^2)
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2355

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[x*((a + b*Log[c*x^n])^p/(d*(d + e*x))), x] - Dist[b*n*(p/d),
```

```
Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n
, p}, x] && GtQ[p, 0]
```

Rule 2381

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + Dist[b*n*(p/(d*(q + 1))), Int[(f*x)^
m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, d
, e, f, m, n, q}, x] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2549

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(m +
1)*(g/b)^m, Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x], x,
(a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ
[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[b*f - a*g, 0] && (GtQ[p, 0] || LtQ
[m, -1])
```

Rule 2573

```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
```

```
rQ[n]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3\,dx=\int \left(A^3(a+bx)+3A^2B(a+bx)\log\left(e(a+bx)^n(c+\right.\right.$$

$$=\frac{A^3(a+bx)^2}{2b}+\left(3A^2B\right)\int (a+bx)\log\left(e(a+bx)^n(c\right.$$

$$=\frac{A^3(a+bx)^2}{2b}+\frac{3A^2B(a+bx)^2\log\left(e(a+bx)^n(c+dx\right.}{2b}$$

$$=\frac{A^3(a+bx)^2}{2b}+\frac{3A^2B(a+bx)^2\log\left(e(a+bx)^n(c+dx\right.}{2b}$$

$$=-\frac{3A^2B(bc-ad)nx}{2d}+\frac{A^3(a+bx)^2}{2b}+\frac{3A^2B(bc-ad}{2d}$$

$$=-\frac{3A^2B(bc-ad)nx}{2d}+\frac{A^3(a+bx)^2}{2b}+\frac{3A^2B(bc-ad}{2d}$$

$$=-\frac{3A^2B(bc-ad)nx}{2d}+\frac{A^3(a+bx)^2}{2b}+\frac{3A^2B(bc-ad}{2d}$$

$$=-\frac{3A^2B(bc-ad)nx}{2d}+\frac{A^3(a+bx)^2}{2b}+\frac{3A^2B(bc-ad}{2d}$$

$$=-\frac{3A^2B(bc-ad)nx}{2d}+\frac{A^3(a+bx)^2}{2b}+\frac{3A^2B(bc-ad}{2d}$$

$$=-\frac{3A^2B(bc-ad)nx}{2d}+\frac{A^3(a+bx)^2}{2b}+\frac{3A^2B(bc-ad}{2d}$$

$$=-\frac{3A^2B(bc-ad)nx}{2d}+\frac{A^3(a+bx)^2}{2b}+\frac{3A^2B(bc-ad}{2d}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 3813 vs. $2(376) = 752$.

time = 1.39, size = 3813, normalized size = 10.14

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3,x]
[Out] (4*a^2*B^3*d^2*n^3*Log[a + b*x]^3 - 6*a^2*B^2*d^2*n^2*Log[a + b*x]^2*(2*A -
 B*n + 2*B*n*Log[c + d*x] + 2*B*Log[(e*(a + b*x)^n)/(c + d*x)^n]) + 6*a^2*B
*d^2*n*Log[a + b*x]*(2*A^2 - 2*A*B*n + B^2*n^2 + 2*B^2*n^2*Log[c + d*x]^2 -
 2*B*(-2*A + B*n)*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 2*B^2*Log[(e*(a + b*x)
^n)/(c + d*x)^n]^2 + 2*B*n*Log[c + d*x]*(2*A - B*n + 2*B*Log[(e*(a + b*x)^n
)/(c + d*x)^n])) + b*(4*B^3*c*(b*c - 2*a*d)*n^3*Log[c + d*x]^3 + 6*B^2*d^2*
n^2*x*(2*a + b*x)*Log[c + d*x]^2*(2*A - B*n + 2*B*Log[(e*(a + b*x)^n)/(c +
d*x)^n]) + 6*B*d^2*n*x*(2*a + b*x)*Log[c + d*x]*(2*A^2 - 2*A*B*n + B^2*n^2
- 2*B*(-2*A + B*n)*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 2*B^2*Log[(e*(a + b*x
)^n)/(c + d*x)^n]^2) + d^2*x*(2*a + b*x)*(4*A^3 - 6*A^2*B*n + 6*A*B^2*n^2 -
 3*B^3*n^3 + 6*B*(2*A^2 - 2*A*B*n + B^2*n^2)*Log[(e*(a + b*x)^n)/(c + d*x)^
n] - 6*B^2*(-2*A + B*n)*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2 + 4*B^3*Log[(e*(
a + b*x)^n)/(c + d*x)^n]^3)))/(8*b*d^2) - (3*B*n*(-8*a*A*b*B*c*d*n + 16*a^2
*A*B*d^2*n - 8*b^2*B^2*c^2*n^2 + 16*a*b*B^2*c*d*n^2 - 8*a^2*B^2*d^2*n^2 + 4
*A^2*b^2*c*d*x - 8*a*A^2*b*d^2*x + 4*a*A*b*B*d^2*n*x - 2*a*b*B^2*d^2*n^2*x
- 2*A^2*b^2*d^2*x^2 + 2*A*b^2*B*d^2*n*x^2 - b^2*B^2*d^2*n^2*x^2 + 8*a*A*b*B
*c*d*n*Log[a + b*x] - 12*a^2*A*B*d^2*n*Log[a + b*x] + 8*a*b*B^2*c*d*n^2*Log
[a + b*x] - 14*a^2*B^2*d^2*n^2*Log[a + b*x] - 4*a*b*B^2*c*d*n^2*Log[a + b*x
]^2 + 6*a^2*B^2*d^2*n^2*Log[a + b*x]^2 - 4*A^2*b^2*c^2*Log[c + d*x] + 8*a*A
^2*b*c*d*Log[c + d*x] - 8*A*b^2*B*c^2*n*Log[c + d*x] + 8*a*A*b*B*c*d*n*Log[
c + d*x] - 8*a*b*B^2*c*d*n^2*Log[c + d*x] + 16*a^2*B^2*d^2*n^2*Log[c + d*x]
 + 8*a*A^2*b*d^2*x*Log[c + d*x] - 8*a*A*b*B*d^2*n*x*Log[c + d*x] + 4*a*b*B^
2*d^2*n^2*x*Log[c + d*x] + 4*A^2*b^2*d^2*x^2*Log[c + d*x] - 4*A*b^2*B*d^2*n
*x^2*Log[c + d*x] + 2*b^2*B^2*d^2*n^2*x^2*Log[c + d*x] + 8*A*b^2*B*c^2*n*Lo
g[a + b*x]*Log[c + d*x] - 16*a*A*b*B*c*d*n*Log[a + b*x]*Log[c + d*x] + 8*a^
2*A*B*d^2*n*Log[a + b*x]*Log[c + d*x] - 4*b^2*B^2*c^2*n^2*Log[a + b*x]*Log[
c + d*x] + 16*a*b*B^2*c*d*n^2*Log[a + b*x]*Log[c + d*x] - 16*a^2*B^2*d^2*n^
2*Log[a + b*x]*Log[c + d*x] - 4*b^2*B^2*c^2*n^2*Log[a + b*x]^2*Log[c + d*x]
 + 8*a*b*B^2*c*d*n^2*Log[a + b*x]^2*Log[c + d*x] - 4*a^2*B^2*d^2*n^2*Log[a
+ b*x]^2*Log[c + d*x] + 12*b^2*B^2*c^2*n^2*Log[(d*(a + b*x))/(-(b*c) + a*d)
]*Log[c + d*x] - 24*a*b*B^2*c*d*n^2*Log[(d*(a + b*x))/(-(b*c) + a*d)]*Log[c
 + d*x] + 12*a^2*B^2*d^2*n^2*Log[(d*(a + b*x))/(-(b*c) + a*d)]*Log[c + d*x]
 - 4*A*b^2*B*c^2*n*Log[c + d*x]^2 + 8*a*A*b*B*c*d*n*Log[c + d*x]^2 - 4*b^2*
B^2*c^2*n^2*Log[c + d*x]^2 + 4*a*b*B^2*c*d*n^2*Log[c + d*x]^2 + 8*a*A*b*B*d
^2*n*x*Log[c + d*x]^2 - 4*a*b*B^2*d^2*n^2*x*Log[c + d*x]^2 + 4*A*b^2*B*d^2*
n*x^2*Log[c + d*x]^2 - 2*b^2*B^2*d^2*n^2*x^2*Log[c + d*x]^2 + 8*b^2*B^2*c^2
*n^2*Log[a + b*x]*Log[c + d*x]^2 - 16*a*b*B^2*c*d*n^2*Log[a + b*x]*Log[c +
d*x]^2 + 8*a^2*B^2*d^2*n^2*Log[a + b*x]*Log[c + d*x]^2 - 4*b^2*B^2*c^2*n^2*
```

```
Log[(d*(a + b*x))/(-(b*c) + a*d)]*Log[c + d*x]^2 + 8*a*b*B^2*c*d*n^2*Log[(d
*(a + b*x))/(-(b*c) + a*d)]*Log[c + d*x]^2 - 4*a^2*B^2*d^2*n^2*Log[(d*(a +
b*x))/(-(b*c) + a*d)]*Log[c + d*x]^2 - 8*A*b^2*B*c^2*n*Log[a + b*x]*Log[(b*
(c + d*x))/(b*c - a*d)] + 16*a*A*b*B*c*d*n*Log[a + b*x]*Log[(b*(c + d*x))/(
b*c - a*d)] - 8*a^2*A*B*d^2*n*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)] +
 4*b^2*B^2*c^2*n^2*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)] - 8*a*b*B^2*
c*d*n^2*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)] + 4*a^2*B^2*d^2*n^2*Log
[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)] + 4*b^2*B^2*c^2*n^2*Log[a + b*x]^2
*Log[(b*(c + d*x))/(b*c - a*d)] - 8*a*b*B^2*c*d*n^2*Log[a + b*x]^2*Log[(b*(
c + d*x))/(b*c - a*d)] + 4*a^2*B^2*d^2*n^2*Log[a + b*x]^2*Log[(b*(c + d*x))
/(b*c - a*d)] - 8*b^2*B^2*c^2*n^2*Log[a + b*x]*Log[c + d*x]*Log[(b*(c + d*x
))/(b*c - a*d)] + 16*a*b*B^2*c*d*n^2*Log[a + b*x]*Log[c + d*x]*Log[(b*(c +
d*x))/(b*c - a*d)] - 8*a^2*B^2*d^2*n^2*Log[a + b*x]*Log[c + d*x]*Log[(b*(c
+ d*x))/(b*c - a*d)] - 8*a*b*B^2*c*d*n*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 1
6*a^2*B^2*d^2*n*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 8*A*b^2*B*c*d*x*Log[(e*(
a + b*x)^n)/(c + d*x)^n] - 16*a*A*b*B*d^2*x*Log[(e*(a + b*x)^n)/(c + d*x)^n
] + 4*a*b*B^2*d^2*n*x*Log[(e*(a + b*x)^n)/(c + d*x)^n] - 4*A*b^2*B*d^2*x^2*
Log[(e*(a + b*x)^n)/(c + d*x)^n] + 2*b^2*B^2*d^2*n*x^2*Log[(e*(a + b*x)^n)/
(c + d*x)^n] + 8*a*b*B^2*c*d*n*Log[a + b*x]*Log[(e*(a + b*x)^n)/(c + d*x)^n
] - 12*a^2*B^2*d^2*n*Log[a + b*x]*Log[(e*(a + b*x)^n)/(c + d*x)^n] - 8*A*b^
2*B*c^2*Log[c + d*x]*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 16*a*A*b*B*c*d*Log[
c + d*x]*Log[(e*(a + b*x)^n)/(c + d*x)^n] - 8*b^2*B^2*c^2*n*Log[c + d*x]*Lo
g[(e*(a + b*x)^n)/(c + d*x)^n] + 8*a*b*B^2*c*d*n*Log[c + d*x]*Log[(e*(a + b
*x)^n)/(c + d*x)^n] + 16*a*A*b*B*d^2*x*Log[c + d*x]*Log[(e*(a + b*x)^n)/(c
+ d*x)^n] - 8*a*b*B^2*d^2*n*x*Log[c + d*x]*Log[(e*(a + b*x)^n)/(c + d*x)^n]
 + 8*A*b^2*B*d^2*x^2*Log[c + d*x]*Log[(e*(a + b*x)^n)/(c + d*x)^n] - 4*b^2*
B^2*d^2*n*x^2*Log[c + d*x]*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 8*b^2*B^2*c^2
*n*Log[a + b*x]*Log[c + d*x]*Log[(e*(a + b*x)^n)/(c + d*x)^n] - 16*a*b*B^2*
c*d*n*Log[a + b*x]*Log[c + d*x]*Log[(e*(a + b*x...
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int (bx+a)\left(A+B\ln\left(e(bx+a)^{n}(dx+c)^{-n}\right)\right)^{3}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3,x)
```

```
[Out] int((b*x+a)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3,x, algorithm="maxima
")
```

```
[Out] 3/2*A^2*B*b*x^2*log((b*x + a)^n*e/(d*x + c)^n) + 1/2*A^3*b*x^2 + 3*(a*n*e*l
og(b*x + a)/b - c*n*e*log(d*x + c)/d)*A^2*B*a*e^(-1) - 3/2*(a^2*n*e*log(b*x
 + a)/b^2 - c^2*n*e*log(d*x + c)/d^2 + (b*c*n - a*d*n)*x*e/(b*d))*A^2*B*b*e
^(-1) + 3*A^2*B*a*x*log((b*x + a)^n*e/(d*x + c)^n) + A^3*a*x - 1/2*((B^3*b^
2*d^2*x^2 + 2*B^3*a*b*d^2*x)*log((d*x + c)^n)^3 - 3*(B^3*a^2*d^2*n*log(b*x
+ a) + (b^2*c^2*n - 2*a*b*c*d*n)*B^3*log(d*x + c) + (A*B^2*b^2*d^2 + B^3*b^
2*d^2)*x^2 + (2*A*B^2*a*b*d^2 + (a*b*d^2*(n + 2) - b^2*c*d*n)*B^3)*x + (B^3
*b^2*d^2*x^2 + 2*B^3*a*b*d^2*x)*log((b*x + a)^n))*log((d*x + c)^n)^2)/(b*d^
2) - integrate(-(3*A*B^2*a*b*c*d + B^3*a*b*c*d + (B^3*b^2*d^2*x^2 + B^3*a*b
*c*d + (b^2*c*d + a*b*d^2)*B^3*x)*log((b*x + a)^n)^3 + (3*A*B^2*b^2*d^2 + B
^3*b^2*d^2)*x^2 + 3*(A*B^2*a*b*c*d + B^3*a*b*c*d + (A*B^2*b^2*d^2 + B^3*b^2
*d^2)*x^2 + ((b^2*c*d + a*b*d^2)*A*B^2 + (b^2*c*d + a*b*d^2)*B^3)*x)*log((b
*x + a)^n)^2 + (3*(b^2*c*d + a*b*d^2)*A*B^2 + (b^2*c*d + a*b*d^2)*B^3)*x +
3*(2*A*B^2*a*b*c*d + B^3*a*b*c*d + (2*A*B^2*b^2*d^2 + B^3*b^2*d^2)*x^2 + (2
*(b^2*c*d + a*b*d^2)*A*B^2 + (b^2*c*d + a*b*d^2)*B^3)*x)*log((b*x + a)^n) -
 3*(B^3*a^2*d^2*n^2*log(b*x + a) + 2*A*B^2*a*b*c*d + B^3*a*b*c*d + (b^2*c^2
*n^2 - 2*a*b*c*d*n^2)*B^3*log(d*x + c) + (A*B^2*b^2*d^2*(n + 2) + B^3*b^2*d
^2*(n + 1))*x^2 + (B^3*b^2*d^2*x^2 + B^3*a*b*c*d + (b^2*c*d + a*b*d^2)*B^3*
x)*log((b*x + a)^n)^2 + (2*(a*b*d^2*(n + 1) + b^2*c*d)*A*B^2 - ((n^2 - 1)*b
^2*c*d - (n^2 + 2*n + 1)*a*b*d^2)*B^3)*x + (2*A*B^2*a*b*c*d + 2*B^3*a*b*c*d
 + (B^3*b^2*d^2*(n + 2) + 2*A*B^2*b^2*d^2)*x^2 + 2*((b^2*c*d + a*b*d^2)*A*B
^2 + (a*b*d^2*(n + 1) + b^2*c*d)*B^3)*x)*log((b*x + a)^n))*log((d*x + c)^n)
)/(b*d^2*x + b*c*d), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3,x, algorithm="fricas
")
```

```
[Out] integral(A^3*b*x + A^3*a + (B^3*b*x + B^3*a)*log((b*x + a)^n*e/(d*x + c)^n)
^3 + 3*(A*B^2*b*x + A*B^2*a)*log((b*x + a)^n*e/(d*x + c)^n)^2 + 3*(A^2*B*b*
x + A^2*B*a)*log((b*x + a)^n*e/(d*x + c)^n), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**3,x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3,x, algorithm="giac")
```

```
[Out] integrate((b*x + a)*(B*log((b*x + a)^n*e/(d*x + c)^n) + A)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(A+B\,\ln\left(\frac{e\,{\left(a+b\,x\right)}^n}{{\left(c+d\,x\right)}^n}\right)\right)^3\,\left(a+b\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^3*(a + b*x),x)
```

```
[Out] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^3*(a + b*x), x)
```

3.167 $\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{a+bx}\,dx$

Optimal. Leaf size=186

$$-\frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b}+\frac{3Bn\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2\text{Li}_2\left(\frac{b(c+d}{d(a+b}\right.}{b}$$

```
[Out] -(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3*ln(1-b*(d*x+c)/d/(b*x+a))/b+3*B*n*(A+B
*ln(e*(b*x+a)^n/((d*x+c)^n)))^2*polylog(2,b*(d*x+c)/d/(b*x+a))/b+6*B^2*n^2*
(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))*polylog(3,b*(d*x+c)/d/(b*x+a))/b+6*B^3*n^
3*polylog(4,b*(d*x+c)/d/(b*x+a))/b
```

Rubi [A]
time = 0.16, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2573, 2549, 2379, 2421, 2430, 6724}

$$\frac{6B^2n^2\text{PolyLog}\left(3,\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{b}+\frac{3Bn\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^2}{b}+\frac{6B^3n^3\text{PolyLog}\left(4,\frac{b(c+dx)}{d(a+bx)}\right)}{b}-\frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^3}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3/(a + b*x),x]
```

```
[Out] -(((A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3*Log[1 - (b*(c + d*x))/(d*(a +
 b*x))])/b) + (3*B*n*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2*PolyLog[2,
(b*(c + d*x))/(d*(a + b*x))])/b + (6*B^2*n^2*(A + B*Log[(e*(a + b*x)^n)/(c
+ d*x)^n])*PolyLog[3, (b*(c + d*x))/(d*(a + b*x))])/b + (6*B^3*n^3*PolyLog[
4, (b*(c + d*x))/(d*(a + b*x))])/b
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2430

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Dist[b*n*(p/q), Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p - 1
)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 2549

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(m +
1)*(g/b)^m, Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x], x,
(a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ
[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[b*f - a*g, 0] && (GtQ[p, 0] || LtQ
[m, -1])
```

Rule 2573

```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{a+bx}\,dx = \int \left(\frac{A^3}{a+bx} + \frac{3A^2B\log(e(a+bx)^n(c+dx)^{-n})}{a+bx} + \frac{3AB^2\log^2(e\right.$$

$$= \frac{A^3\log(a+bx)}{b} + (3A^2B)\int \frac{\log(e(a+bx)^n(c+dx)^{-n})}{a+bx}\,dx + ($$

$$= \frac{A^3\log(a+bx)}{b} - \frac{3A^2B\log\left(-\frac{bc-ad}{d(a+bx)}\right)\log(e(a+bx)^n(c+dx)}{b}$$

$$= \frac{A^3\log(a+bx)}{b} - \frac{3A^2B\log\left(-\frac{bc-ad}{d(a+bx)}\right)\log(e(a+bx)^n(c+dx)}{b}$$

$$= \frac{A^3\log(a+bx)}{b} - \frac{3A^2B\log\left(-\frac{bc-ad}{d(a+bx)}\right)\log(e(a+bx)^n(c+dx)}{b}$$

$$= \frac{A^3\log(a+bx)}{b} - \frac{3A^2B\log\left(-\frac{bc-ad}{d(a+bx)}\right)\log(e(a+bx)^n(c+dx)}{b}$$

$$= \frac{A^3\log(a+bx)}{b} - \frac{3A^2B\log\left(-\frac{bc-ad}{d(a+bx)}\right)\log(e(a+bx)^n(c+dx)}{b}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 2513 vs. $2(186) = 372$.
time = 0.44, size = 2513, normalized size = 13.51

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3/(a + b*x),x]
```

```
[Out] (4*A^3*Log[a + b*x] - 6*A^2*B*n*Log[a + b*x]^2 + 4*A*B^2*n^2*Log[a + b*x]^3
 - B^3*n^3*Log[a + b*x]^4 + B^3*n^3*Log[(d*(a + b*x))/(-(b*c) + a*d)]^4 - 4
*B^3*n^3*Log[(d*(a + b*x))/(-(b*c) + a*d)]^3*Log[-((d*(a + b*x))/(b*(c + d*
x)))] + 6*B^3*n^3*Log[(d*(a + b*x))/(-(b*c) + a*d)]^2*Log[-((d*(a + b*x))/(
b*(c + d*x)))]^2 - 4*B^3*n^3*Log[(d*(a + b*x))/(-(b*c) + a*d)]*Log[-((d*(a
+ b*x))/(b*(c + d*x)))]^3 + B^3*n^3*Log[-((d*(a + b*x))/(b*(c + d*x)))]^4 -
 12*A*B^2*n^2*Log[a + b*x]*Log[c + d*x]^2 + 12*B^3*n^3*Log[a + b*x]^2*Log[c
 + d*x]^2 + 12*A*B^2*n^2*Log[(d*(a + b*x))/(-(b*c) + a*d)]*Log[c + d*x]^2 -
 12*B^3*n^3*Log[a + b*x]*Log[(d*(a + b*x))/(-(b*c) + a*d)]*Log[c + d*x]^2 -
 8*B^3*n^3*Log[a + b*x]*Log[c + d*x]^3 + 8*B^3*n^3*Log[(d*(a + b*x))/(-(b*c
) + a*d)]*Log[c + d*x]^3 + 12*A^2*B*n*Log[a + b*x]*Log[(b*(c + d*x))/(b*c -
 a*d)] - 12*A*B^2*n^2*Log[a + b*x]^2*Log[(b*(c + d*x))/(b*c - a*d)] + 4*B^3
```

*n^3*Log[a + b*x]^3*Log[(b*(c + d*x))/(b*c - a*d)] + 8*B^3*n^3*Log[(d*(a +
b*x))/(-(b*c) + a*d)]^3*Log[(b*(c + d*x))/(b*c - a*d)] - 12*B^3*n^3*Log[(d*
(a + b*x))/(-(b*c) + a*d)]^2*Log[-((d*(a + b*x))/(b*(c + d*x)))]*Log[(b*(c
+ d*x))/(b*c - a*d)] + 24*A*B^2*n^2*Log[a + b*x]*Log[c + d*x]*Log[(b*(c + d
*x))/(b*c - a*d)] - 24*B^3*n^3*Log[a + b*x]^2*Log[c + d*x]*Log[(b*(c + d*x)
)/(b*c - a*d)] + 12*B^3*n^3*Log[a + b*x]*Log[c + d*x]^2*Log[(b*(c + d*x))/(
b*c - a*d)] + 6*B^3*n^3*Log[a + b*x]^2*Log[(b*(c + d*x))/(b*c - a*d)]^2 + 1
2*B^3*n^3*Log[a + b*x]*Log[(d*(a + b*x))/(-(b*c) + a*d)]*Log[(b*(c + d*x))/
(b*c - a*d)]^2 - 18*B^3*n^3*Log[(d*(a + b*x))/(-(b*c) + a*d)]^2*Log[(b*(c +
 d*x))/(b*c - a*d)]^2 + 12*A^2*B*Log[a + b*x]*Log[(e*(a + b*x)^n)/(c + d*x)
^n] - 12*A*B^2*n*Log[a + b*x]^2*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 4*B^3*n^
2*Log[a + b*x]^3*Log[(e*(a + b*x)^n)/(c + d*x)^n] - 12*B^3*n^2*Log[a + b*x]
*Log[c + d*x]^2*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 12*B^3*n^2*Log[(d*(a + b
*x))/(-(b*c) + a*d)]*Log[c + d*x]^2*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 24*A
*B^2*n*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)]*Log[(e*(a + b*x)^n)/(c +
 d*x)^n] - 12*B^3*n^2*Log[a + b*x]^2*Log[(b*(c + d*x))/(b*c - a*d)]*Log[(e*
(a + b*x)^n)/(c + d*x)^n] + 24*B^3*n^2*Log[a + b*x]*Log[c + d*x]*Log[(b*(c
+ d*x))/(b*c - a*d)]*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 12*A*B^2*Log[a + b*
x]*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2 - 6*B^3*n*Log[a + b*x]^2*Log[(e*(a +
b*x)^n)/(c + d*x)^n]^2 + 12*B^3*n*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d
)]*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2 + 4*B^3*Log[a + b*x]*Log[(e*(a + b*x)
^n)/(c + d*x)^n]^3 - 4*B^3*n^3*Log[-((d*(a + b*x))/(b*(c + d*x)))]^3*Log[(b
*c - a*d)/(b*c + b*d*x)] + 12*B*n*(A^2 + B^2*n^2*Log[(d*(a + b*x))/(-(b*c)
+ a*d)]^2 + B^2*n^2*Log[c + d*x]^2 + 2*B^2*n^2*Log[a + b*x]*Log[(b*(c + d*x
))/(b*c - a*d)] - 2*B^2*n^2*Log[(d*(a + b*x))/(-(b*c) + a*d)]*(Log[-((d*(a
+ b*x))/(b*(c + d*x)))] + Log[(b*(c + d*x))/(b*c - a*d)]) + 2*A*B*Log[(e*(a
 + b*x)^n)/(c + d*x)^n] + B^2*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2 + 2*B*n*Lo
g[c + d*x]*(A - B*n*Log[a + b*x] + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]))*Pol
yLog[2, (d*(a + b*x))/(-(b*c) + a*d)] - 12*B^3*n^3*Log[-((d*(a + b*x))/(b*(
c + d*x)))]^2*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))] + 12*B^3*n^3*Log[(d*(
a + b*x))/(-(b*c) + a*d)]^2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)] - 24*B^3*
n^3*Log[(d*(a + b*x))/(-(b*c) + a*d)]*Log[-((d*(a + b*x))/(b*(c + d*x)))]*P
olyLog[2, (b*(c + d*x))/(b*c - a*d)] + 12*B^3*n^3*Log[-((d*(a + b*x))/(b*(c
 + d*x)))]^2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)] + 24*A*B^2*n^2*Log[c + d
*x]*PolyLog[2, (b*(c + d*x))/(b*c - a*d)] - 24*B^3*n^3*Log[a + b*x]*Log[c +
 d*x]*PolyLog[2, (b*(c + d*x))/(b*c - a*d)] + 12*B^3*n^3*Log[c + d*x]^2*Pol
yLog[2, (b*(c + d*x))/(b*c - a*d)] + 24*B^3*n^3*Log[a + b*x]*Log[(b*(c + d*
x))/(b*c - a*d)]*PolyLog[2, (b*(c + d*x))/(b*c - a*d)] - 24*B^3*n^3*Log[(d*
(a + b*x))/(-(b*c) + a*d)]*Log[(b*(c + d*x))/(b*c - a*d)]*PolyLog[2, (b*(c
+ d*x))/(b*c - a*d)] + 24*B^3*n^2*Log[c + d*x]*Log[(e*(a + b*x)^n)/(c + d*x
)^n]*PolyLog[2, (b*(c + d*x))/(b*c - a*d)] - 24*A*B^2*n^2*PolyLog[3, (d*(a
+ b*x))/(-(b*c) + a*d)] + 24*B^3*n^3*Log[-((d*(a + b*x))/(b*(c + d*x)))]*Po
lyLog[3, (d*(a + b*x))/(-(b*c) + a*d)] - 24*B^3*n^2*Log[(e*(a + b*x)^n)/(c
+ d*x)^n]*PolyLog[3, (d*(a + b*x))/(-(b*c) + a*d)] + 24*B^3*n^3*Log[-((d*(a
 + b*x))/(b*(c + d*x)))]*PolyLog[3, (d*(a + b*x))/(b*(c + d*x))] - 24*A*B^2

```
*n^2*PolyLog[3, (b*(c + d*x))/(b*c - a*d)] + 24*B^3*n^3*Log[-((d*(a + b*x))
/(b*(c + d*x)))]*PolyLog[3, (b*(c + d*x))/(b*c - a*d)] - 24*B^3*n^2*Log[(e*
(a + b*x)^n)/(c + d*x)^n]*PolyLog[3, (b*(c + d*x))/(b*c - a*d)] - 24*B^3*n^
3*PolyLog[4, (d*(a + b*x))/(b*(c + d*x))])/(4*b)
```

Maple [**F**]
time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{\left(A+B\ln \left(e(bx+a)^{n}\left(dx+c\right)^{-n}\right)\right)^{3}}{bx+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/(b*x+a),x)
```

```
[Out] int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/(b*x+a),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(b*x+a),x, algorithm="maxima
")
```

```
[Out] -B^3*log(b*x + a)*log((d*x + c)^n)^3/b + A^3*log(b*x + a)/b + integrate((3*
A^2*B*b*c + 3*A*B^2*b*c + B^3*b*c + (B^3*b*d*x + B^3*b*c)*log((b*x + a)^n)^
3 + 3*(A*B^2*b*c + B^3*b*c + (A*B^2*b*d + B^3*b*d)*x)*log((b*x + a)^n)^2 +
3*(A*B^2*b*c + B^3*b*c + (A*B^2*b*d + B^3*b*d)*x + (B^3*b*d*n*x + B^3*a*d*n
)*log(b*x + a) + (B^3*b*d*x + B^3*b*c)*log((b*x + a)^n))*log((d*x + c)^n)^2
 + (3*A^2*B*b*d + 3*A*B^2*b*d + B^3*b*d)*x + 3*(A^2*B*b*c + 2*A*B^2*b*c + B
^3*b*c + (A^2*B*b*d + 2*A*B^2*b*d + B^3*b*d)*x)*log((b*x + a)^n) - 3*(A^2*B
*b*c + 2*A*B^2*b*c + B^3*b*c + (B^3*b*d*x + B^3*b*c)*log((b*x + a)^n)^2 + (
A^2*B*b*d + 2*A*B^2*b*d + B^3*b*d)*x + 2*(A*B^2*b*c + B^3*b*c + (A*B^2*b*d
+ B^3*b*d)*x)*log((b*x + a)^n))*log((d*x + c)^n))/(b^2*d*x^2 + a*b*c + (b^2
*c + a*b*d)*x), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(b*x+a),x, algorithm="fricas
")
```

```
[Out] integral((B^3*log((b*x + a)^n*e/(d*x + c)^n)^3 + 3*A*B^2*log((b*x + a)^n*e/
(d*x + c)^n)^2 + 3*A^2*B*log((b*x + a)^n*e/(d*x + c)^n) + A^3)/(b*x + a), x
)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**3/(b*x+a),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((B*log((b*x + a)^n*e/(d*x + c)^n) + A)^3/(b*x + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A+B\,\ln\left(\frac{e\,{\left(a+b\,x\right)}^n}{{\left(c+d\,x\right)}^n}\right)\right)^3}{a+b\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^3/(a + b*x),x)
```

```
[Out] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^3/(a + b*x), x)
```

3.168 $\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{(a+bx)^2}\,dx$

Optimal. Leaf size=184

$$-\frac{6B^3n^3(c+dx)}{(bc-ad)(a+bx)}-\frac{6B^2n^2(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{(bc-ad)(a+bx)}-\frac{3Bn(c+dx)\left(A+B\log\left(e(a+bx\right.\right.}{(bc-ad)(a+bx)}$$

```
[Out] -6*B^3*n^3*(d*x+c)/(-a*d+b*c)/(b*x+a)-6*B^2*n^2*(d*x+c)*(A+B*ln(e*(b*x+a)^n
/((d*x+c)^n)))/(-a*d+b*c)/(b*x+a)-3*B*n*(d*x+c)*(A+B*ln(e*(b*x+a)^n/((d*x+c
)^n)))^2/(-a*d+b*c)/(b*x+a)-(d*x+c)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/(-a
*d+b*c)/(b*x+a)
```

Rubi [A]

time = 0.10, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {2573, 2549, 2342, 2341}

$$-\frac{6B^2n^2(c+dx)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{(a+bx)(bc-ad)}-\frac{3Bn(c+dx)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^2}{(a+bx)(bc-ad)}-\frac{(c+dx)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^3}{(a+bx)(bc-ad)}-\frac{6B^3n^3(c+dx)}{(a+bx)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3/(a + b*x)^2,x]
```

```
[Out] (-6*B^3*n^3*(c + d*x))/((b*c - a*d)*(a + b*x)) - (6*B^2*n^2*(c + d*x)*(A +
B*Log[(e*(a + b*x)^n)/(c + d*x)^n]))/((b*c - a*d)*(a + b*x)) - (3*B*n*(c +
d*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2)/((b*c - a*d)*(a + b*x)) -
((c + d*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3)/((b*c - a*d)*(a + b*
x))
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2549

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(m +
```

```
1)*(g/b)^m, Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x], x,
(a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ
[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[b*f - a*g, 0] && (GtQ[p, 0] || LtQ
[m, -1])
```

Rule 2573

```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

Rubi steps

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{(a+bx)^2}\,dx = \int \left(\frac{A^3}{(a+bx)^2} + \frac{3A^2B\log(e(a+bx)^n(c+dx)^{-n})}{(a+bx)^2} + \frac{3AB^2\,\mathrm{lo}\ldots}{}\right.$$
$$= -\frac{A^3}{b(a+bx)} + (3A^2B)\int \frac{\log(e(a+bx)^n(c+dx)^{-n})}{(a+bx)^2}\,dx + (3\ldots$$
$$= -\frac{A^3}{b(a+bx)} - \frac{3A^2B(c+dx)\log(e(a+bx)^n(c+dx)^{-n})}{(bc-ad)(a+bx)} - \frac{3\ldots}{}$$
$$= -\frac{A^3}{b(a+bx)} - \frac{3A^2Bn}{b(a+bx)} - \frac{3A^2B(c+dx)\log(e(a+bx)^n(c+\ldots}{(bc-ad)(a+bx)}$$
$$= -\frac{A^3}{b(a+bx)} - \frac{3A^2Bn}{b(a+bx)} - \frac{6AB^2n^2}{b(a+bx)} - \frac{3A^2B(c+dx)\log(e(\ldots}{(bc-ad)\ldots}$$
$$= -\frac{A^3}{b(a+bx)} - \frac{3A^2Bn}{b(a+bx)} - \frac{6AB^2n^2}{b(a+bx)} - \frac{6B^3n^3}{b(a+bx)} - \frac{3A^2B(c\ldots}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 524 vs. 2(184) = 368.
time = 0.47, size = 524, normalized size = 2.85

$$\frac{-B^3dn^3(a+bx)\log^3(a+bx)+B^3dn^3(a+bx)\log^3(c+dx)+3B^2dn^2(a+bx)\log^2(c+dx)\left(A+Bn+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)+3B^2dn^2(a+bx)\log^2(a+bx)\left(A+Bn+Bn\log(c+dx)+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)+3Bdn(a+bx)\log(c+dx)\left(A^2+2ABn+2B^2n^2+2B(A+Bn)\log\left(e(a+bx)^n(c+dx)^{-n}\right)+B^2\log^2\left(e(a+bx)^n(c+dx)^{-n}\right)\right)-(bc-ad)\left(A^3+3A^2Bn+6AB^2n^2+6B^3n^3+3B(A^2+2ABn+2B^2n^2)\log\left(e(a+bx)^n(c+dx)^{-n}\right)+3B^2(A+Bn)\log^2\left(e(a+bx)^n(c+dx)^{-n}\right)+B^3\log^3\left(e(a+bx)^n(c+dx)^{-n}\right)\right)-3Bdn(a+bx)\log(a+bx)\left(A^2+2ABn+2B^2n^2+B^2n^2\log^2(c+dx)+2B(A+Bn)\log\left(e(a+bx)^n(c+dx)^{-n}\right)+B^2\log^2\left(e(a+bx)^n(c+dx)^{-n}\right)+2Bn\log(c+dx)\left(A+Bn+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\right)}{b(bc-ad)(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3/(a + b*x)^2,x]
```

```
[Out] (-(B^3*d*n^3*(a + b*x)*Log[a + b*x]^3) + B^3*d*n^3*(a + b*x)*Log[c + d*x]^3
 + 3*B^2*d*n^2*(a + b*x)*Log[c + d*x]^2*(A + B*n + B*Log[(e*(a + b*x)^n)/(c
 + d*x)^n]) + 3*B^2*d*n^2*(a + b*x)*Log[a + b*x]^2*(A + B*n + B*n*Log[c + d
*x] + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]) + 3*B*d*n*(a + b*x)*Log[c + d*x]*
```

```
(A^2 + 2*A*B*n + 2*B^2*n^2 + 2*B*(A + B*n)*Log[(e*(a + b*x)^n)/(c + d*x)^n]
 + B^2*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2) - (b*c - a*d)*(A^3 + 3*A^2*B*n +
 6*A*B^2*n^2 + 6*B^3*n^3 + 3*B*(A^2 + 2*A*B*n + 2*B^2*n^2)*Log[(e*(a + b*x)
^n)/(c + d*x)^n] + 3*B^2*(A + B*n)*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2 + B^3
*Log[(e*(a + b*x)^n)/(c + d*x)^n]^3) - 3*B*d*n*(a + b*x)*Log[a + b*x]*(A^2
+ 2*A*B*n + 2*B^2*n^2 + B^2*n^2*Log[c + d*x]^2 + 2*B*(A + B*n)*Log[(e*(a +
b*x)^n)/(c + d*x)^n] + B^2*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2 + 2*B*n*Log[c
 + d*x]*(A + B*n + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])))/(b*(b*c - a*d)*(a
+ b*x))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 14.08, size = 69354, normalized size = 376.92

method	result	size
risch	Expression too large to display	69354

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1092 vs. 2(187) = 374.
time = 0.36, size = 1092, normalized size = 5.93

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(b*x+a)^2,x, algorithm="maxi
ma")
```

```
[Out] -3*(d*n*e*log(b*x + a)/(b^2*c - a*b*d) - d*n*e*log(d*x + c)/(b^2*c - a*b*d)
 + n*e/(b^2*x + a*b))*A^2*B*e^(-1) - B^3*log((b*x + a)^n*e/(d*x + c)^n)^3/(
b^2*x + a*b) - 3*(2*(d*n*e*log(b*x + a)/(b^2*c - a*b*d) - d*n*e*log(d*x + c
)/(b^2*c - a*b*d) + n*e/(b^2*x + a*b))*e^(-1)*log((b*x + a)^n*e/(d*x + c)^n
) - ((b*d*n^2*x*e^2 + a*d*n^2*e^2)*log(b*x + a)^2 + (b*d*n^2*x*e^2 + a*d*n^
2*e^2)*log(d*x + c)^2 - 2*(b*c*n^2 - a*d*n^2)*e^2 - 2*(b*d*n^2*x*e^2 + a*d*
n^2*e^2)*log(b*x + a) + 2*(b*d*n^2*x*e^2 + a*d*n^2*e^2 - (b*d*n^2*x*e^2 + a
*d*n^2*e^2)*log(b*x + a))*log(d*x + c))*e^(-2)/(a*b^2*c - a^2*b*d + (b^3*c
- a*b^2*d)*x))*A*B^2 - (3*(d*n*e*log(b*x + a)/(b^2*c - a*b*d) - d*n*e*log(d
*x + c)/(b^2*c - a*b*d) + n*e/(b^2*x + a*b))*e^(-1)*log((b*x + a)^n*e/(d*x
+ c)^n)^2 - (3*((b*d*n^2*x*e^2 + a*d*n^2*e^2)*log(b*x + a)^2 + (b*d*n^2*x*e
^2 + a*d*n^2*e^2)*log(d*x + c)^2 - 2*(b*c*n^2 - a*d*n^2)*e^2 - 2*(b*d*n^2*x
*e^2 + a*d*n^2*e^2)*log(b*x + a) + 2*(b*d*n^2*x*e^2 + a*d*n^2*e^2 - (b*d*n^
2*x*e^2 + a*d*n^2*e^2)*log(b*x + a))*log(d*x + c))*e^(-1)*log((b*x + a)^n*e
```

```
/(d*x + c)^n)/(a*b^2*c - a^2*b*d + (b^3*c - a*b^2*d)*x) - ((b*d*n^3*x*e^3 +
 a*d*n^3*e^3)*log(b*x + a)^3 - (b*d*n^3*x*e^3 + a*d*n^3*e^3)*log(d*x + c)^3
 - 3*(b*d*n^3*x*e^3 + a*d*n^3*e^3)*log(b*x + a)^2 - 3*(b*d*n^3*x*e^3 + a*d*
n^3*e^3 - (b*d*n^3*x*e^3 + a*d*n^3*e^3)*log(b*x + a))*log(d*x + c)^2 + 6*(b
*c*n^3 - a*d*n^3)*e^3 + 6*(b*d*n^3*x*e^3 + a*d*n^3*e^3)*log(b*x + a) - 3*(2
*b*d*n^3*x*e^3 + 2*a*d*n^3*e^3 + (b*d*n^3*x*e^3 + a*d*n^3*e^3)*log(b*x + a)
^2 - 2*(b*d*n^3*x*e^3 + a*d*n^3*e^3)*log(b*x + a))*log(d*x + c))*e^(-2)/(a*
b^2*c - a^2*b*d + (b^3*c - a*b^2*d)*x))*e^(-1))*B^3 - 3*A*B^2*log((b*x + a)
^n*e/(d*x + c)^n)^2/(b^2*x + a*b) - 3*A^2*B*log((b*x + a)^n*e/(d*x + c)^n)/
(b^2*x + a*b) - A^3/(b^2*x + a*b)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 673 vs. $2(187) = 374$.
time = 0.42, size = 673, normalized size = 3.66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(b*x+a)^2,x, algorithm="fric
as")
```

```
[Out] -(6*(B^3*b*c - B^3*a*d)*n^3 + (B^3*b*d*n^3*x + B^3*b*c*n^3)*log(b*x + a)^3
- (B^3*b*d*n^3*x + B^3*b*c*n^3)*log(d*x + c)^3 + (A^3 + 3*A^2*B + 3*A*B^2 +
 B^3)*b*c - (A^3 + 3*A^2*B + 3*A*B^2 + B^3)*a*d + 6*((A*B^2 + B^3)*b*c - (A
*B^2 + B^3)*a*d)*n^2 + 3*(B^3*b*c*n^3 + (A*B^2 + B^3)*b*c*n^2 + (B^3*b*d*n^
3 + (A*B^2 + B^3)*b*d*n^2)*x)*log(b*x + a)^2 + 3*(B^3*b*c*n^3 + (A*B^2 + B^
3)*b*c*n^2 + (B^3*b*d*n^3 + (A*B^2 + B^3)*b*d*n^2)*x + (B^3*b*d*n^3*x + B^3
*b*c*n^3)*log(b*x + a))*log(d*x + c)^2 + 3*((A^2*B + 2*A*B^2 + B^3)*b*c - (
A^2*B + 2*A*B^2 + B^3)*a*d)*n + 3*(2*B^3*b*c*n^3 + 2*(A*B^2 + B^3)*b*c*n^2
+ (A^2*B + 2*A*B^2 + B^3)*b*c*n + (2*B^3*b*d*n^3 + 2*(A*B^2 + B^3)*b*d*n^2
+ (A^2*B + 2*A*B^2 + B^3)*b*d*n)*x)*log(b*x + a) - 3*(2*B^3*b*c*n^3 + 2*(A*
B^2 + B^3)*b*c*n^2 + (A^2*B + 2*A*B^2 + B^3)*b*c*n + (B^3*b*d*n^3*x + B^3*b
*c*n^3)*log(b*x + a)^2 + (2*B^3*b*d*n^3 + 2*(A*B^2 + B^3)*b*d*n^2 + (A^2*B
+ 2*A*B^2 + B^3)*b*d*n)*x + 2*(B^3*b*c*n^3 + (A*B^2 + B^3)*b*c*n^2 + (B^3*b
*d*n^3 + (A*B^2 + B^3)*b*d*n^2)*x)*log(b*x + a))*log(d*x + c))/(a*b^2*c - a
^2*b*d + (b^3*c - a*b^2*d)*x)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**3/(b*x+a)**2,x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(b*x+a)^2,x, algorithm="giac
")
```

```
[Out] integrate((B*log((b*x + a)^n*e/(d*x + c)^n) + A)^3/(b*x + a)^2, x)
```

Mupad [B]
time = 6.06, size = 474, normalized size = 2.58

$$-\ln\left(\frac{e\,{\left(a+b\,x\right)}^n}{{\left(c+d\,x\right)}^n}\right)\,\left(\frac{3\,B\,b\,d\,A^2\,x^2+3\,B\,\left(a\,d+b\,c\right)\,A^2\,x+3\,B\,a\,c\,A^2}{b\,{\left(a+b\,x\right)}^2\,\left(c+d\,x\right)}+\frac{6\,d\,\left(n\,B^3+A\,B^2\right)\,\left(b^2\,n\,x^2\,\left(a\,d-b\,c\right)+\frac{a\,b\,c\,n\,\left(a\,d-b\,c\right)}{d}+\frac{b\,n\,x\,\left(a\,d+b\,c\right)\,\left(a\,d-b\,c\right)}{d}\right)}{b^2\,\left(a\,d-b\,c\right)\,{\left(a+b\,x\right)}^2\,\left(c+d\,x\right)}\right)-\frac{A^3+3\,A^2\,B\,n+6\,A\,B^2\,n^2+6\,B^3\,n^3}{x\,b^2+a\,b}-\ln\left(\frac{e\,{\left(a+b\,x\right)}^n}{{\left(c+d\,x\right)}^n}\right)^2\,\left(\frac{3\,A\,B^2}{x\,b^2+a\,b}+\frac{3\,B^3\,n}{x\,b^2+a\,b}-\frac{3\,d\,\left(n\,B^3+A\,B^2\right)}{b\,\left(a\,d-b\,c\right)}\right)-\ln\left(\frac{e\,{\left(a+b\,x\right)}^n}{{\left(c+d\,x\right)}^n}\right)^3\,\left(\frac{B^3}{b\,\left(a+b\,x\right)}-\frac{B^3\,d}{b\,\left(a\,d-b\,c\right)}\right)-\frac{B\,d\,n\,\mathrm{atan}\left(\frac{B\,d\,n\,\left(\frac{c\,b^2+a\,d\,b}{b}+2\,b\,d\,x\right)\,\left(A^2+2\,A\,B\,n+2\,B^2\,n^2\right)\,3{}\mathrm{i}}{\left(a\,d-b\,c\right)\,\left(3\,d\,A^2\,B\,n+6\,d\,A\,B^2\,n^2+6\,d\,B^3\,n^3\right)}\right)\,\left(A^2+2\,A\,B\,n+2\,B^2\,n^2\right)\,6{}\mathrm{i}}{b\,\left(a\,d-b\,c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^3/(a + b*x)^2,x)
```

```
[Out] - log((e*(a + b*x)^n)/(c + d*x)^n)*((3*A^2*B*a*c + 3*A^2*B*x*(a*d + b*c) +
3*A^2*B*b*d*x^2)/(b*(a + b*x)^2*(c + d*x)) + (6*d*(A*B^2 + B^3*n)*(b^2*n*x^
2*(a*d - b*c) + (a*b*c*n*(a*d - b*c))/d + (b*n*x*(a*d + b*c)*(a*d - b*c))/d
))/(b^2*(a*d - b*c)*(a + b*x)^2*(c + d*x))) - (A^3 + 6*B^3*n^3 + 6*A*B^2*n^
2 + 3*A^2*B*n)/(a*b + b^2*x) - log((e*(a + b*x)^n)/(c + d*x)^n)^2*((3*A*B^2
)/(a*b + b^2*x) + (3*B^3*n)/(a*b + b^2*x) - (3*d*(A*B^2 + B^3*n))/(b*(a*d -
 b*c))) - log((e*(a + b*x)^n)/(c + d*x)^n)^3*(B^3/(b*(a + b*x)) - (B^3*d)/(
b*(a*d - b*c))) - (B*d*n*atan((B*d*n*((b^2*c + a*b*d)/b + 2*b*d*x)*(A^2 + 2
*B^2*n^2 + 2*A*B*n)*3i)/((a*d - b*c)*(6*B^3*d*n^3 + 3*A^2*B*d*n + 6*A*B^2*d
*n^2)))*(A^2 + 2*B^2*n^2 + 2*A*B*n)*6i)/(b*(a*d - b*c))
```

3.169 $\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{(a+bx)^3}\,dx$

Optimal. Leaf size=390

$$\frac{6B^3dn^3(c+dx)}{(bc-ad)^2(a+bx)}-\frac{3bB^3n^3(c+dx)^2}{8(bc-ad)^2(a+bx)^2}+\frac{6B^2dn^2(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{(bc-ad)^2(a+bx)}-\frac{3bB^2n^2(c$$

```
[Out] 6*B^3*d*n^3*(d*x+c)/(-a*d+b*c)^2/(b*x+a)-3/8*b*B^3*n^3*(d*x+c)^2/(-a*d+b*c)
^2/(b*x+a)^2+6*B^2*d*n^2*(d*x+c)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/(-a*d+b*
c)^2/(b*x+a)-3/4*b*B^2*n^2*(d*x+c)^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/(-a*
d+b*c)^2/(b*x+a)^2+3*B*d*n*(d*x+c)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(-a*
d+b*c)^2/(b*x+a)-3/4*b*B*n*(d*x+c)^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(-
a*d+b*c)^2/(b*x+a)^2+d*(d*x+c)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/(-a*d+b*
c)^2/(b*x+a)-1/2*b*(d*x+c)^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/(-a*d+b*c)
^2/(b*x+a)^2
```

Rubi [A]
time = 0.22, antiderivative size = 390, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {2573, 2549, 2395, 2342, 2341}

$$-\frac{3bB^2n^2(c+dx)^2\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{4(a+bx)^2(bc-ad)^2}+\frac{6B^2dn^2(c+dx)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{(a+bx)(bc-ad)^2}-\frac{3bBn(c+dx)^2\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^2}{4(a+bx)^2(bc-ad)^2}+\frac{3Bdn(c+dx)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^2}{(a+bx)(bc-ad)^2}-\frac{b(c+dx)^2\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^3}{2(a+bx)^2(bc-ad)^2}+\frac{d(c+dx)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^3}{(a+bx)(bc-ad)^2}-\frac{3bB^3n^3(c+dx)^2}{8(a+bx)^2(bc-ad)^2}+\frac{6B^3dn^3(c+dx)}{(a+bx)(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3/(a + b*x)^3,x]
```

```
[Out] (6*B^3*d*n^3*(c + d*x))/((b*c - a*d)^2*(a + b*x)) - (3*b*B^3*n^3*(c + d*x)^
2)/(8*(b*c - a*d)^2*(a + b*x)^2) + (6*B^2*d*n^2*(c + d*x)*(A + B*Log[(e*(a
+ b*x)^n)/(c + d*x)^n]))/((b*c - a*d)^2*(a + b*x)) - (3*b*B^2*n^2*(c + d*x)
^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]))/(4*(b*c - a*d)^2*(a + b*x)^2)
+ (3*B*d*n*(c + d*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2)/((b*c - a*
d)^2*(a + b*x)) - (3*b*B*n*(c + d*x)^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)
^n])^2)/(4*(b*c - a*d)^2*(a + b*x)^2) + (d*(c + d*x)*(A + B*Log[(e*(a + b*x
)^n)/(c + d*x)^n])^3)/((b*c - a*d)^2*(a + b*x)) - (b*(c + d*x)^2*(A + B*Log
[(e*(a + b*x)^n)/(c + d*x)^n])^3)/(2*(b*c - a*d)^2*(a + b*x)^2)
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
```

```
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2549

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(m +
1)*(g/b)^m, Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x], x,
(a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ
[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[b*f - a*g, 0] && (GtQ[p, 0] || LtQ
[m, -1])
```

Rule 2573

```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

Rubi steps

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{(a+bx)^3}\,dx = \int \left(\frac{A^3}{(a+bx)^3} + \frac{3A^2B\log(e(a+bx)^n(c+dx)^{-n})}{(a+bx)^3} + \frac{3AB^2\,\mathrm{lo}}{}\right.$$

$$= -\frac{A^3}{2b(a+bx)^2} + (3A^2B)\int \frac{\log(e(a+bx)^n(c+dx)^{-n})}{(a+bx)^3}\,dx + ($$

$$= -\frac{A^3}{2b(a+bx)^2} - \frac{3A^2B\log(e(a+bx)^n(c+dx)^{-n})}{2b(a+bx)^2} + \frac{(3AbB^2)}{}$$

$$= -\frac{A^3}{2b(a+bx)^2} - \frac{3A^2B\log(e(a+bx)^n(c+dx)^{-n})}{2b(a+bx)^2} + \frac{3AB^2d(}{}$$

$$= -\frac{A^3}{2b(a+bx)^2} - \frac{3A^2Bn}{4b(a+bx)^2} + \frac{3A^2Bdn}{2b(bc-ad)(a+bx)} + \frac{3A^2Bd}{2b}$$

$$= -\frac{A^3}{2b(a+bx)^2} - \frac{3A^2Bn}{4b(a+bx)^2} + \frac{3A^2Bdn}{2b(bc-ad)(a+bx)} + \frac{6A}{b(bc-}$$

$$= -\frac{A^3}{2b(a+bx)^2} - \frac{3A^2Bn}{4b(a+bx)^2} + \frac{3A^2Bdn}{2b(bc-ad)(a+bx)} + \frac{6A}{b(bc-}$$

<u>Mathematica</u> [A]
time = 0.69, size = 693, normalized size = 1.78

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3/(a + b*x)^3,x]
```

```
[Out] -1/8*(-4*B^3*d^2*n^3*(a + b*x)^2*Log[a + b*x]^3 + 4*B^3*d^2*n^3*(a + b*x)^2
*Log[c + d*x]^3 + 6*B^2*d^2*n^2*(a + b*x)^2*Log[c + d*x]^2*(2*A + 3*B*n + 2
*B*Log[(e*(a + b*x)^n)/(c + d*x)^n]) + 6*B^2*d^2*n^2*(a + b*x)^2*Log[a + b*
x]^2*(2*A + 3*B*n + 2*B*n*Log[c + d*x] + 2*B*Log[(e*(a + b*x)^n)/(c + d*x)^
n]) + 6*B*d^2*n*(a + b*x)^2*Log[c + d*x]*(2*A^2 + 6*A*B*n + 7*B^2*n^2 + 2*B
*(2*A + 3*B*n)*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 2*B^2*Log[(e*(a + b*x)^n)
/(c + d*x)^n]^2) + (b*c - a*d)*(4*A^3*(b*c - a*d) + 3*B^3*n^3*(-15*a*d + b*
(c - 14*d*x)) + 6*A*B^2*n^2*(-7*a*d + b*(c - 6*d*x)) + 6*A^2*B*n*(-3*a*d +
b*(c - 2*d*x)) + 6*B*(2*A^2*(b*c - a*d) + B^2*n^2*(-7*a*d + b*(c - 6*d*x))
+ 2*A*B*n*(-3*a*d + b*(c - 2*d*x)))*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 6*B^
2*(2*A*(b*c - a*d) + B*n*(-3*a*d + b*(c - 2*d*x)))*Log[(e*(a + b*x)^n)/(c +
 d*x)^n]^2 + 4*B^3*(b*c - a*d)*Log[(e*(a + b*x)^n)/(c + d*x)^n]^3) - 6*B*d^
2*n*(a + b*x)^2*Log[a + b*x]*(2*A^2 + 6*A*B*n + 7*B^2*n^2 + 2*B^2*n^2*Log[c
 + d*x]^2 + 2*B*(2*A + 3*B*n)*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 2*B^2*Log[
(e*(a + b*x)^n)/(c + d*x)^n]^2 + 2*B*n*Log[c + d*x]*(2*A + 3*B*n + 2*B*Log[
(e*(a + b*x)^n)/(c + d*x)^n])))/(b*(b*c - a*d)^2*(a + b*x)^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 21.28, size = 120138, normalized size = 308.05

method	result	size
risch	Expression too large to display	120138

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 2176 vs. 2(388) = 776.
time = 0.46, size = 2176, normalized size = 5.58

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(b*x+a)^3,x, algorithm="maxi
ma")
```

```
[Out] 3/4*(2*d^2*n*e*log(b*x + a)/(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2) - 2*d^2*n*e
*log(d*x + c)/(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2) + (2*b*d*n*x*e - (b*c*n -
 3*a*d*n)*e)/(a^2*b^2*c - a^3*b*d + (b^4*c - a*b^3*d)*x^2 + 2*(a*b^3*c - a^
2*b^2*d)*x))*A^2*B*e^(-1) - 1/2*B^3*log((b*x + a)^n*e/(d*x + c)^n)^3/(b^3*x
^2 + 2*a*b^2*x + a^2*b) + 3/4*(2*(2*d^2*n*e*log(b*x + a)/(b^3*c^2 - 2*a*b^2
*c*d + a^2*b*d^2) - 2*d^2*n*e*log(d*x + c)/(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d
^2) + (2*b*d*n*x*e - (b*c*n - 3*a*d*n)*e)/(a^2*b^2*c - a^3*b*d + (b^4*c - a
*b^3*d)*x^2 + 2*(a*b^3*c - a^2*b^2*d)*x))*e^(-1)*log((b*x + a)^n*e/(d*x + c
)^n) + (6*(b^2*c*d*n^2 - a*b*d^2*n^2)*x*e^2 - 2*(b^2*d^2*n^2*x^2*e^2 + 2*a*
b*d^2*n^2*x*e^2 + a^2*d^2*n^2*e^2)*log(b*x + a)^2 - 2*(b^2*d^2*n^2*x^2*e^2
+ 2*a*b*d^2*n^2*x*e^2 + a^2*d^2*n^2*e^2)*log(d*x + c)^2 - (b^2*c^2*n^2 - 8*
a*b*c*d*n^2 + 7*a^2*d^2*n^2)*e^2 + 6*(b^2*d^2*n^2*x^2*e^2 + 2*a*b*d^2*n^2*x
*e^2 + a^2*d^2*n^2*e^2)*log(b*x + a) - 2*(3*b^2*d^2*n^2*x^2*e^2 + 6*a*b*d^2
*n^2*x*e^2 + 3*a^2*d^2*n^2*e^2 - 2*(b^2*d^2*n^2*x^2*e^2 + 2*a*b*d^2*n^2*x*e
^2 + a^2*d^2*n^2*e^2)*log(b*x + a))*log(d*x + c))*e^(-2)/(a^2*b^3*c^2 - 2*a
^3*b^2*c*d + a^4*b*d^2 + (b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*x^2 + 2*(a*b
^4*c^2 - 2*a^2*b^3*c*d + a^3*b^2*d^2)*x))*A*B^2 + 1/8*(6*(2*d^2*n*e*log(b*x
 + a)/(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2) - 2*d^2*n*e*log(d*x + c)/(b^3*c^2
 - 2*a*b^2*c*d + a^2*b*d^2) + (2*b*d*n*x*e - (b*c*n - 3*a*d*n)*e)/(a^2*b^2*
c - a^3*b*d + (b^4*c - a*b^3*d)*x^2 + 2*(a*b^3*c - a^2*b^2*d)*x))*e^(-1)*lo
g((b*x + a)^n*e/(d*x + c)^n)^2 + (6*(6*(b^2*c*d*n^2 - a*b*d^2*n^2)*x*e^2 -
2*(b^2*d^2*n^2*x^2*e^2 + 2*a*b*d^2*n^2*x*e^2 + a^2*d^2*n^2*e^2)*log(b*x + a
)^2 - 2*(b^2*d^2*n^2*x^2*e^2 + 2*a*b*d^2*n^2*x*e^2 + a^2*d^2*n^2*e^2)*log(d
*x + c)^2 - (b^2*c^2*n^2 - 8*a*b*c*d*n^2 + 7*a^2*d^2*n^2)*e^2 + 6*(b^2*d^2*
```

```
n^2*x^2*e^2 + 2*a*b*d^2*n^2*x*e^2 + a^2*d^2*n^2*e^2)*log(b*x + a) - 2*(3*b^
2*d^2*n^2*x^2*e^2 + 6*a*b*d^2*n^2*x*e^2 + 3*a^2*d^2*n^2*e^2 - 2*(b^2*d^2*n^
2*x^2*e^2 + 2*a*b*d^2*n^2*x*e^2 + a^2*d^2*n^2*e^2)*log(b*x + a))*log(d*x +
c))*e^(-1)*log((b*x + a)^n*e/(d*x + c)^n)/(a^2*b^3*c^2 - 2*a^3*b^2*c*d + a^
4*b*d^2 + (b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*x^2 + 2*(a*b^4*c^2 - 2*a^2*
b^3*c*d + a^3*b^2*d^2)*x) + (4*(b^2*d^2*n^3*x^2*e^3 + 2*a*b*d^2*n^3*x*e^3 +
 a^2*d^2*n^3*e^3)*log(b*x + a)^3 - 4*(b^2*d^2*n^3*x^2*e^3 + 2*a*b*d^2*n^3*x
*e^3 + a^2*d^2*n^3*e^3)*log(d*x + c)^3 + 42*(b^2*c*d*n^3 - a*b*d^2*n^3)*x*e
^3 - 18*(b^2*d^2*n^3*x^2*e^3 + 2*a*b*d^2*n^3*x*e^3 + a^2*d^2*n^3*e^3)*log(b
*x + a)^2 - 6*(3*b^2*d^2*n^3*x^2*e^3 + 6*a*b*d^2*n^3*x*e^3 + 3*a^2*d^2*n^3*
e^3 - 2*(b^2*d^2*n^3*x^2*e^3 + 2*a*b*d^2*n^3*x*e^3 + a^2*d^2*n^3*e^3)*log(b
*x + a))*log(d*x + c)^2 - 3*(b^2*c^2*n^3 - 16*a*b*c*d*n^3 + 15*a^2*d^2*n^3)
*e^3 + 42*(b^2*d^2*n^3*x^2*e^3 + 2*a*b*d^2*n^3*x*e^3 + a^2*d^2*n^3*e^3)*log
(b*x + a) - 6*(7*b^2*d^2*n^3*x^2*e^3 + 14*a*b*d^2*n^3*x*e^3 + 7*a^2*d^2*n^3
*e^3 + 2*(b^2*d^2*n^3*x^2*e^3 + 2*a*b*d^2*n^3*x*e^3 + a^2*d^2*n^3*e^3)*log(
b*x + a)^2 - 6*(b^2*d^2*n^3*x^2*e^3 + 2*a*b*d^2*n^3*x*e^3 + a^2*d^2*n^3*e^3
)*log(b*x + a))*log(d*x + c))*e^(-2)/(a^2*b^3*c^2 - 2*a^3*b^2*c*d + a^4*b*d
^2 + (b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*x^2 + 2*(a*b^4*c^2 - 2*a^2*b^3*c
*d + a^3*b^2*d^2)*x))*e^(-1))*B^3 - 3/2*A*B^2*log((b*x + a)^n*e/(d*x + c)^n
)^2/(b^3*x^2 + 2*a*b^2*x + a^2*b) - 3/2*A^2*B*log((b*x + a)^n*e/(d*x + c)^n
)/(b^3*x^2 + 2*a*b^2*x + a^2*b) - 1/2*A^3/(b^3*x^2 + 2*a*b^2*x + a^2*b)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1725 vs. $2(388) = 776$.
time = 0.43, size = 1725, normalized size = 4.42

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(b*x+a)^3,x, algorithm="fric
as")
```

```
[Out] -1/8*(4*(A^3 + 3*A^2*B + 3*A*B^2 + B^3)*b^2*c^2 - 8*(A^3 + 3*A^2*B + 3*A*B^
2 + B^3)*a*b*c*d + 4*(A^3 + 3*A^2*B + 3*A*B^2 + B^3)*a^2*d^2 + 3*(B^3*b^2*c
^2 - 16*B^3*a*b*c*d + 15*B^3*a^2*d^2)*n^3 - 4*(B^3*b^2*d^2*n^3*x^2 + 2*B^3*
a*b*d^2*n^3*x - (B^3*b^2*c^2 - 2*B^3*a*b*c*d)*n^3)*log(b*x + a)^3 + 4*(B^3*
b^2*d^2*n^3*x^2 + 2*B^3*a*b*d^2*n^3*x - (B^3*b^2*c^2 - 2*B^3*a*b*c*d)*n^3)*
log(d*x + c)^3 + 6*((A*B^2 + B^3)*b^2*c^2 - 8*(A*B^2 + B^3)*a*b*c*d + 7*(A*
B^2 + B^3)*a^2*d^2)*n^2 + 6*((B^3*b^2*c^2 - 4*B^3*a*b*c*d)*n^3 + 2*((A*B^2
+ B^3)*b^2*c^2 - 2*(A*B^2 + B^3)*a*b*c*d)*n^2 - (3*B^3*b^2*d^2*n^3 + 2*(A*B
^2 + B^3)*b^2*d^2*n^2)*x^2 - 2*(2*(A*B^2 + B^3)*a*b*d^2*n^2 + (B^3*b^2*c*d
+ 2*B^3*a*b*d^2)*n^3)*x)*log(b*x + a)^2 + 6*((B^3*b^2*c^2 - 4*B^3*a*b*c*d)*
n^3 + 2*((A*B^2 + B^3)*b^2*c^2 - 2*(A*B^2 + B^3)*a*b*c*d)*n^2 - (3*B^3*b^2*
d^2*n^3 + 2*(A*B^2 + B^3)*b^2*d^2*n^2)*x^2 - 2*(2*(A*B^2 + B^3)*a*b*d^2*n^2
 + (B^3*b^2*c*d + 2*B^3*a*b*d^2)*n^3)*x - 2*(B^3*b^2*d^2*n^3*x^2 + 2*B^3*a*
```

```
b*d^2*n^3*x - (B^3*b^2*c^2 - 2*B^3*a*b*c*d)*n^3)*log(b*x + a))*log(d*x + c)
^2 + 6*((A^2*B + 2*A*B^2 + B^3)*b^2*c^2 - 4*(A^2*B + 2*A*B^2 + B^3)*a*b*c*d
 + 3*(A^2*B + 2*A*B^2 + B^3)*a^2*d^2)*n - 6*(7*(B^3*b^2*c*d - B^3*a*b*d^2)*
n^3 + 6*((A*B^2 + B^3)*b^2*c*d - (A*B^2 + B^3)*a*b*d^2)*n^2 + 2*((A^2*B + 2
*A*B^2 + B^3)*b^2*c*d - (A^2*B + 2*A*B^2 + B^3)*a*b*d^2)*n)*x + 6*((B^3*b^2
*c^2 - 8*B^3*a*b*c*d)*n^3 + 2*((A*B^2 + B^3)*b^2*c^2 - 4*(A*B^2 + B^3)*a*b*
c*d)*n^2 - (7*B^3*b^2*d^2*n^3 + 6*(A*B^2 + B^3)*b^2*d^2*n^2 + 2*(A^2*B + 2*
A*B^2 + B^3)*b^2*d^2*n)*x^2 + 2*((A^2*B + 2*A*B^2 + B^3)*b^2*c^2 - 2*(A^2*B
 + 2*A*B^2 + B^3)*a*b*c*d)*n - 2*(2*(A^2*B + 2*A*B^2 + B^3)*a*b*d^2*n + (3*
B^3*b^2*c*d + 4*B^3*a*b*d^2)*n^3 + 2*((A*B^2 + B^3)*b^2*c*d + 2*(A*B^2 + B^
3)*a*b*d^2)*n^2)*x)*log(b*x + a) - 6*((B^3*b^2*c^2 - 8*B^3*a*b*c*d)*n^3 + 2
*((A*B^2 + B^3)*b^2*c^2 - 4*(A*B^2 + B^3)*a*b*c*d)*n^2 - (7*B^3*b^2*d^2*n^3
 + 6*(A*B^2 + B^3)*b^2*d^2*n^2 + 2*(A^2*B + 2*A*B^2 + B^3)*b^2*d^2*n)*x^2 -
 2*(B^3*b^2*d^2*n^3*x^2 + 2*B^3*a*b*d^2*n^3*x - (B^3*b^2*c^2 - 2*B^3*a*b*c*
d)*n^3)*log(b*x + a)^2 + 2*((A^2*B + 2*A*B^2 + B^3)*b^2*c^2 - 2*(A^2*B + 2*
A*B^2 + B^3)*a*b*c*d)*n - 2*(2*(A^2*B + 2*A*B^2 + B^3)*a*b*d^2*n + (3*B^3*b
^2*c*d + 4*B^3*a*b*d^2)*n^3 + 2*((A*B^2 + B^3)*b^2*c*d + 2*(A*B^2 + B^3)*a*
b*d^2)*n^2)*x + 2*((B^3*b^2*c^2 - 4*B^3*a*b*c*d)*n^3 + 2*((A*B^2 + B^3)*b^2
*c^2 - 2*(A*B^2 + B^3)*a*b*c*d)*n^2 - (3*B^3*b^2*d^2*n^3 + 2*(A*B^2 + B^3)*
b^2*d^2*n^2)*x^2 - 2*(2*(A*B^2 + B^3)*a*b*d^2*n^2 + (B^3*b^2*c*d + 2*B^3*a*
b*d^2)*n^3)*x)*log(b*x + a))*log(d*x + c))/(a^2*b^3*c^2 - 2*a^3*b^2*c*d + a
^4*b*d^2 + (b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*x^2 + 2*(a*b^4*c^2 - 2*a^2
*b^3*c*d + a^3*b^2*d^2)*x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**3/(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(b*x+a)^3,x, algorithm="giac
")
```

```
[Out] integrate((B*log((b*x + a)^n*e/(d*x + c)^n) + A)^3/(b*x + a)^3, x)
```

Mupad [B]
time = 8.99, size = 966, normalized size = 2.48

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^3/(a + b*x)^3,x)
```

```
[Out] - log((e*(a + b*x)^n)/(c + d*x)^n)^3*(B^3/(2*b*(a^2 + b^2*x^2 + 2*a*b*x)) -
 (B^3*d^2)/(2*b*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))) - ((4*A^3*a*d - 4*A^3*b*c
 + 45*B^3*a*d*n^3 - 3*B^3*b*c*n^3 + 18*A^2*B*a*d*n - 6*A^2*B*b*c*n + 42*A*B
^2*a*d*n^2 - 6*A*B^2*b*c*n^2)/(2*(a*d - b*c)) + (3*x*(7*B^3*b*d*n^3 + 2*A^2
*B*b*d*n + 6*A*B^2*b*d*n^2))/(a*d - b*c))/(4*a^2*b + 4*b^3*x^2 + 8*a*b^2*x)
 - log((e*(a + b*x)^n)/(c + d*x)^n)^2*((3*A*B^2)/(2*(a^2*b + b^3*x^2 + 2*a*
b^2*x)) - (3*d^2*(2*A*B^2 + 3*B^3*n))/(4*b*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))
 + (3*B^3*d^2*((b*n*(a*d - b*c)*(2*a*d - b*c))/d^2 + (2*b^2*n*x*(a*d - b*c)
)/d + (a*b*n*(a*d - b*c))/d))/(4*b*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)*(a^2*b +
 b^3*x^2 + 2*a*b^2*x))) - log((e*(a + b*x)^n)/(c + d*x)^n)*((3*B*a*c*(A^2 -
 B^2*n^2) + 3*B*x*(a*d + b*c)*(A^2 - B^2*n^2) + 3*B*b*d*x^2*(A^2 - B^2*n^2)
)/(2*b*(a + b*x)^3*(c + d*x)) + (3*d^2*(2*A*B^2 + 3*B^3*n)*(x*(((b*n*(a*d -
 b*c)*(2*a*d - b*c))/d^2 + (a*b*n*(a*d - b*c))/d)*(a*d + b*c) + (2*a*b^2*c*
n*(a*d - b*c))/d) + x^2*(b*d*((b*n*(a*d - b*c)*(2*a*d - b*c))/d^2 + (a*b*n*
(a*d - b*c))/d) + (2*b^2*n*(a*d + b*c)*(a*d - b*c))/d) + a*c*((b*n*(a*d - b
*c)*(2*a*d - b*c))/d^2 + (a*b*n*(a*d - b*c))/d) + 2*b^3*n*x^3*(a*d - b*c)))
/(4*b^2*(a + b*x)^3*(c + d*x)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))) - (B*d^2*n*
atan((B*d^2*n*(2*b*d*x - (b^3*c^2 - a^2*b*d^2)/(b*(a*d - b*c)))*(2*A^2 + 7*
B^2*n^2 + 6*A*B*n)*3i)/((a*d - b*c)*(21*B^3*d^2*n^3 + 6*A^2*B*d^2*n + 18*A*
B^2*d^2*n^2)))*(2*A^2 + 7*B^2*n^2 + 6*A*B*n)*3i)/(2*b*(a*d - b*c)^2)
```

3.170 $\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{(a+bx)^4}\,dx$

Optimal. Leaf size=611

$$-\frac{6B^3d^2n^3(c+dx)}{(bc-ad)^3(a+bx)}+\frac{3bB^3dn^3(c+dx)^2}{4(bc-ad)^3(a+bx)^2}-\frac{2b^2B^3n^3(c+dx)^3}{27(bc-ad)^3(a+bx)^3}-\frac{6B^2d^2n^2(c+dx)\left(A+B\log\left(e(a+bx\right.\right.}{(bc-ad)^3(a+bx)}$$

```
[Out] -6*B^3*d^2*n^3*(d*x+c)/(-a*d+b*c)^3/(b*x+a)+3/4*b*B^3*d*n^3*(d*x+c)^2/(-a*d
+b*c)^3/(b*x+a)^2-2/27*b^2*B^3*n^3*(d*x+c)^3/(-a*d+b*c)^3/(b*x+a)^3-6*B^2*d
^2*n^2*(d*x+c)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/(-a*d+b*c)^3/(b*x+a)+3/2*b
*B^2*d*n^2*(d*x+c)^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/(-a*d+b*c)^3/(b*x+a)
^2-2/9*b^2*B^2*n^2*(d*x+c)^3*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/(-a*d+b*c)^3
/(b*x+a)^3-3*B*d^2*n*(d*x+c)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(-a*d+b*c)
^3/(b*x+a)+3/2*b*B*d*n*(d*x+c)^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(-a*d+
b*c)^3/(b*x+a)^2-1/3*b^2*B*n*(d*x+c)^3*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/
(-a*d+b*c)^3/(b*x+a)^3-d^2*(d*x+c)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/(-a*
d+b*c)^3/(b*x+a)+b*d*(d*x+c)^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/(-a*d+b*
c)^3/(b*x+a)^2-1/3*b^2*(d*x+c)^3*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/(-a*d+
b*c)^3/(b*x+a)^3
```

Rubi [A]

time = 0.32, antiderivative size = 611, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {2573, 2549, 2395, 2342, 2341}

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3/(a + b*x)^4,x]
```

```
[Out] (-6*B^3*d^2*n^3*(c + d*x))/((b*c - a*d)^3*(a + b*x)) + (3*b*B^3*d*n^3*(c +
d*x)^2)/(4*(b*c - a*d)^3*(a + b*x)^2) - (2*b^2*B^3*n^3*(c + d*x)^3)/(27*(b*
c - a*d)^3*(a + b*x)^3) - (6*B^2*d^2*n^2*(c + d*x)*(A + B*Log[(e*(a + b*x)^
n)/(c + d*x)^n]))/((b*c - a*d)^3*(a + b*x)) + (3*b*B^2*d*n^2*(c + d*x)^2*(A
 + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]))/(2*(b*c - a*d)^3*(a + b*x)^2) - (2*
b^2*B^2*n^2*(c + d*x)^3*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]))/(9*(b*c -
 a*d)^3*(a + b*x)^3) - (3*B*d^2*n*(c + d*x)*(A + B*Log[(e*(a + b*x)^n)/(c +
 d*x)^n])^2)/((b*c - a*d)^3*(a + b*x)) + (3*b*B*d*n*(c + d*x)^2*(A + B*Log[
(e*(a + b*x)^n)/(c + d*x)^n])^2)/(2*(b*c - a*d)^3*(a + b*x)^2) - (b^2*B*n*(
c + d*x)^3*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2)/(3*(b*c - a*d)^3*(a
+ b*x)^3) - (d^2*(c + d*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3)/((b*
c - a*d)^3*(a + b*x)) + (b*d*(c + d*x)^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*
x)^n])^3)/((b*c - a*d)^3*(a + b*x)^2) - (b^2*(c + d*x)^3*(A + B*Log[(e*(a +
 b*x)^n)/(c + d*x)^n])^3)/(3*(b*c - a*d)^3*(a + b*x)^3)
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2549

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(m +
1)*(g/b)^m, Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x], x,
(a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ
[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[b*f - a*g, 0] && (GtQ[p, 0] || LtQ
[m, -1])
```

Rule 2573

```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

Rubi steps

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{(a+bx)^4}\,dx = \int \left(\frac{A^3}{(a+bx)^4} + \frac{3A^2B\log(e(a+bx)^n(c+dx)^{-n})}{(a+bx)^4} + \frac{3AB^2\log$$

$$= -\frac{A^3}{3b(a+bx)^3} + (3A^2B)\int \frac{\log(e(a+bx)^n(c+dx)^{-n})}{(a+bx)^4}\,dx + (3$$

$$= -\frac{A^3}{3b(a+bx)^3} - \frac{A^2B\log(e(a+bx)^n(c+dx)^{-n})}{b(a+bx)^3} - \frac{AB^2\log^2(e$$

$$= -\frac{A^3}{3b(a+bx)^3} - \frac{A^2B\log(e(a+bx)^n(c+dx)^{-n})}{b(a+bx)^3} - \frac{AB^2\log^2(e$$

$$= -\frac{A^3}{3b(a+bx)^3} - \frac{A^2Bn}{3b(a+bx)^3} + \frac{A^2Bdn}{2b(bc-ad)(a+bx)^2} - \frac{A}{b(bc-}$$

$$= -\frac{A^3}{3b(a+bx)^3} - \frac{A^2Bn}{3b(a+bx)^3} + \frac{A^2Bdn}{2b(bc-ad)(a+bx)^2} - \frac{A}{b(bc-}$$

$$= -\frac{A^3}{3b(a+bx)^3} - \frac{A^2Bn}{3b(a+bx)^3} + \frac{A^2Bdn}{2b(bc-ad)(a+bx)^2} - \frac{A}{b(bc-}$$

$$= -\frac{A^3}{3b(a+bx)^3} - \frac{A^2Bn}{3b(a+bx)^3} - \frac{2AB^2n^2}{9b(a+bx)^3} + \frac{A^2Bdn}{2b(bc-ad)(a+}$$

$$= -\frac{A^3}{3b(a+bx)^3} - \frac{A^2Bn}{3b(a+bx)^3} - \frac{2AB^2n^2}{9b(a+bx)^3} + \frac{A^2Bdn}{2b(bc-ad)(a+}$$

$$= -\frac{A^3}{3b(a+bx)^3} - \frac{A^2Bn}{3b(a+bx)^3} - \frac{2AB^2n^2}{9b(a+bx)^3} + \frac{A^2Bdn}{2b(bc-ad)(a+}$$

$$= -\frac{A^3}{3b(a+bx)^3} - \frac{A^2Bn}{3b(a+bx)^3} - \frac{2AB^2n^2}{9b(a+bx)^3} - \frac{2B^3n^3}{27b(a+bx)^3} +$$

$$= -\frac{A^3}{3b(a+bx)^3} - \frac{A^2Bn}{3b(a+bx)^3} - \frac{2AB^2n^2}{9b(a+bx)^3} - \frac{2B^3n^3}{27b(a+bx)^3} +$$

$$= -\frac{A^3}{3b(a+bx)^3} - \frac{A^2Bn}{3b(a+bx)^3} - \frac{2AB^2n^2}{9b(a+bx)^3} - \frac{2B^3n^3}{27b(a+bx)^3} +$$

Mathematica [A]

time = 0.89, size = 1003, normalized size = 1.64

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3/(a + b*x)^4,x]
```

```
[Out] (-36*B^3*d^3*n^3*(a + b*x)^3*Log[a + b*x]^3 + 36*B^3*d^3*n^3*(a + b*x)^3*Lo
g[c + d*x]^3 + 18*B^2*d^3*n^2*(a + b*x)^3*Log[c + d*x]^2*(6*A + 11*B*n + 6*
B*Log[(e*(a + b*x)^n)/(c + d*x)^n]) + 18*B^2*d^3*n^2*(a + b*x)^3*Log[a + b*
x]^2*(6*A + 11*B*n + 6*B*n*Log[c + d*x] + 6*B*Log[(e*(a + b*x)^n)/(c + d*x)
^n]) + 6*B*d^3*n*(a + b*x)^3*Log[c + d*x]*(18*A^2 + 66*A*B*n + 85*B^2*n^2 +
 6*B*(6*A + 11*B*n)*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 18*B^2*Log[(e*(a + b
*x)^n)/(c + d*x)^n]^2) - (b*c - a*d)*(36*A^3*b^2*c^2 - 72*a*A^3*b*c*d + 36*
a^2*A^3*d^2 + 36*A^2*b^2*B*c^2*n - 126*a*A^2*b*B*c*d*n + 198*a^2*A^2*B*d^2*
n + 24*A*b^2*B^2*c^2*n^2 - 138*a*A*b*B^2*c*d*n^2 + 510*a^2*A*B^2*d^2*n^2 +
8*b^2*B^3*c^2*n^3 - 73*a*b*B^3*c*d*n^3 + 575*a^2*B^3*d^2*n^3 - 54*A^2*b^2*B
*c*d*n*x + 270*a*A^2*b*B*d^2*n*x - 90*A*b^2*B^2*c*d*n^2*x + 882*a*A*b*B^2*d
^2*n^2*x - 57*b^2*B^3*c*d*n^3*x + 1077*a*b*B^3*d^2*n^3*x + 108*A^2*b^2*B*d^
2*n*x^2 + 396*A*b^2*B^2*d^2*n^2*x^2 + 510*b^2*B^3*d^2*n^3*x^2 + 6*B*(18*A^2
*(b*c - a*d)^2 + 6*A*B*n*(11*a^2*d^2 + a*b*d*(-7*c + 15*d*x) + b^2*(2*c^2 -
 3*c*d*x + 6*d^2*x^2)) + B^2*n^2*(85*a^2*d^2 + a*b*d*(-23*c + 147*d*x) + b^
2*(4*c^2 - 15*c*d*x + 66*d^2*x^2)))*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 18*B
^2*(6*A*(b*c - a*d)^2 + B*n*(11*a^2*d^2 + a*b*d*(-7*c + 15*d*x) + b^2*(2*c^
2 - 3*c*d*x + 6*d^2*x^2)))*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2 + 36*B^3*(b*c
 - a*d)^2*Log[(e*(a + b*x)^n)/(c + d*x)^n]^3) - 6*B*d^3*n*(a + b*x)^3*Log[a
 + b*x]*(18*A^2 + 66*A*B*n + 85*B^2*n^2 + 18*B^2*n^2*Log[c + d*x]^2 + 6*B*(
6*A + 11*B*n)*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 18*B^2*Log[(e*(a + b*x)^n)
/(c + d*x)^n]^2 + 6*B*n*Log[c + d*x]*(6*A + 11*B*n + 6*B*Log[(e*(a + b*x)^n
)/(c + d*x)^n])))/(108*b*(b*c - a*d)^3*(a + b*x)^3)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 30.04, size = 175812, normalized size = 287.74

method	result	size
risch	Expression too large to display	175812

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/(b*x+a)^4,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 3508 vs. $2(606) = 1212$.
time = 0.63, size = 3508, normalized size = 5.74

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(b*x+a)^4,x, algorithm="maxi
ma")
```

```
[Out] -1/6*(6*d^3*n*e*log(b*x + a)/(b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a
^3*b*d^3) - 6*d^3*n*e*log(d*x + c)/(b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d
^2 - a^3*b*d^3) + (6*b^2*d^2*n*x^2*e - 3*(b^2*c*d*n - 5*a*b*d^2*n)*x*e + (2
*b^2*c^2*n - 7*a*b*c*d*n + 11*a^2*d^2*n)*e)/(a^3*b^3*c^2 - 2*a^4*b^2*c*d +
a^5*b*d^2 + (b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2)*x^3 + 3*(a*b^5*c^2 - 2*a^
2*b^4*c*d + a^3*b^3*d^2)*x^2 + 3*(a^2*b^4*c^2 - 2*a^3*b^3*c*d + a^4*b^2*d^2
)*x))*A^2*B*e^(-1) - 1/3*B^3*log((b*x + a)^n*e/(d*x + c)^n)^3/(b^4*x^3 + 3*
a*b^3*x^2 + 3*a^2*b^2*x + a^3*b) - 1/18*(6*(6*d^3*n*e*log(b*x + a)/(b^4*c^3
 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3) - 6*d^3*n*e*log(d*x + c)/(b
^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3) + (6*b^2*d^2*n*x^2*e
- 3*(b^2*c*d*n - 5*a*b*d^2*n)*x*e + (2*b^2*c^2*n - 7*a*b*c*d*n + 11*a^2*d^2
*n)*e)/(a^3*b^3*c^2 - 2*a^4*b^2*c*d + a^5*b*d^2 + (b^6*c^2 - 2*a*b^5*c*d +
a^2*b^4*d^2)*x^3 + 3*(a*b^5*c^2 - 2*a^2*b^4*c*d + a^3*b^3*d^2)*x^2 + 3*(a^2
*b^4*c^2 - 2*a^3*b^3*c*d + a^4*b^2*d^2)*x))*e^(-1)*log((b*x + a)^n*e/(d*x +
 c)^n) + (66*(b^3*c*d^2*n^2 - a*b^2*d^3*n^2)*x^2*e^2 - 3*(5*b^3*c^2*d*n^2 -
 54*a*b^2*c*d^2*n^2 + 49*a^2*b*d^3*n^2)*x*e^2 - 18*(b^3*d^3*n^2*x^3*e^2 + 3
*a*b^2*d^3*n^2*x^2*e^2 + 3*a^2*b*d^3*n^2*x*e^2 + a^3*d^3*n^2*e^2)*log(b*x +
 a)^2 - 18*(b^3*d^3*n^2*x^3*e^2 + 3*a*b^2*d^3*n^2*x^2*e^2 + 3*a^2*b*d^3*n^2
*x*e^2 + a^3*d^3*n^2*e^2)*log(d*x + c)^2 + (4*b^3*c^3*n^2 - 27*a*b^2*c^2*d*
n^2 + 108*a^2*b*c*d^2*n^2 - 85*a^3*d^3*n^2)*e^2 + 66*(b^3*d^3*n^2*x^3*e^2 +
 3*a*b^2*d^3*n^2*x^2*e^2 + 3*a^2*b*d^3*n^2*x*e^2 + a^3*d^3*n^2*e^2)*log(b*x
 + a) - 6*(11*b^3*d^3*n^2*x^3*e^2 + 33*a*b^2*d^3*n^2*x^2*e^2 + 33*a^2*b*d^3
*n^2*x*e^2 + 11*a^3*d^3*n^2*e^2 - 6*(b^3*d^3*n^2*x^3*e^2 + 3*a*b^2*d^3*n^2*
x^2*e^2 + 3*a^2*b*d^3*n^2*x*e^2 + a^3*d^3*n^2*e^2)*log(b*x + a))*log(d*x +
c))*e^(-2)/(a^3*b^4*c^3 - 3*a^4*b^3*c^2*d + 3*a^5*b^2*c*d^2 - a^6*b*d^3 + (
b^7*c^3 - 3*a*b^6*c^2*d + 3*a^2*b^5*c*d^2 - a^3*b^4*d^3)*x^3 + 3*(a*b^6*c^3
 - 3*a^2*b^5*c^2*d + 3*a^3*b^4*c*d^2 - a^4*b^3*d^3)*x^2 + 3*(a^2*b^5*c^3 -
3*a^3*b^4*c^2*d + 3*a^4*b^3*c*d^2 - a^5*b^2*d^3)*x))*A*B^2 - 1/108*(18*(6*d
^3*n*e*log(b*x + a)/(b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)
 - 6*d^3*n*e*log(d*x + c)/(b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*
b*d^3) + (6*b^2*d^2*n*x^2*e - 3*(b^2*c*d*n - 5*a*b*d^2*n)*x*e + (2*b^2*c^2*
n - 7*a*b*c*d*n + 11*a^2*d^2*n)*e)/(a^3*b^3*c^2 - 2*a^4*b^2*c*d + a^5*b*d^2
 + (b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2)*x^3 + 3*(a*b^5*c^2 - 2*a^2*b^4*c*d
 + a^3*b^3*d^2)*x^2 + 3*(a^2*b^4*c^2 - 2*a^3*b^3*c*d + a^4*b^2*d^2)*x))*e^(
-1)*log((b*x + a)^n*e/(d*x + c)^n)^2 + (6*(66*(b^3*c*d^2*n^2 - a*b^2*d^3*n^
2)*x^2*e^2 - 3*(5*b^3*c^2*d*n^2 - 54*a*b^2*c*d^2*n^2 + 49*a^2*b*d^3*n^2)*x*
e^2 - 18*(b^3*d^3*n^2*x^3*e^2 + 3*a*b^2*d^3*n^2*x^2*e^2 + 3*a^2*b*d^3*n^2*x
*e^2 + a^3*d^3*n^2*e^2)*log(b*x + a)^2 - 18*(b^3*d^3*n^2*x^3*e^2 + 3*a*b^2*
d^3*n^2*x^2*e^2 + 3*a^2*b*d^3*n^2*x*e^2 + a^3*d^3*n^2*e^2)*log(d*x + c)^2 +
 (4*b^3*c^3*n^2 - 27*a*b^2*c^2*d*n^2 + 108*a^2*b*c*d^2*n^2 - 85*a^3*d^3*n^2
```

```
)*e^2 + 66*(b^3*d^3*n^2*x^3*e^2 + 3*a*b^2*d^3*n^2*x^2*e^2 + 3*a^2*b*d^3*n^2
*x*e^2 + a^3*d^3*n^2*e^2)*log(b*x + a) - 6*(11*b^3*d^3*n^2*x^3*e^2 + 33*a*b
^2*d^3*n^2*x^2*e^2 + 33*a^2*b*d^3*n^2*x*e^2 + 11*a^3*d^3*n^2*e^2 - 6*(b^3*d
^3*n^2*x^3*e^2 + 3*a*b^2*d^3*n^2*x^2*e^2 + 3*a^2*b*d^3*n^2*x*e^2 + a^3*d^3*
n^2*e^2)*log(b*x + a))*log(d*x + c))*e^(-1)*log((b*x + a)^n*e/(d*x + c)^n)/
(a^3*b^4*c^3 - 3*a^4*b^3*c^2*d + 3*a^5*b^2*c*d^2 - a^6*b*d^3 + (b^7*c^3 - 3
*a*b^6*c^2*d + 3*a^2*b^5*c*d^2 - a^3*b^4*d^3)*x^3 + 3*(a*b^6*c^3 - 3*a^2*b^
5*c^2*d + 3*a^3*b^4*c*d^2 - a^4*b^3*d^3)*x^2 + 3*(a^2*b^5*c^3 - 3*a^3*b^4*c
^2*d + 3*a^4*b^3*c*d^2 - a^5*b^2*d^3)*x) + (510*(b^3*c*d^2*n^3 - a*b^2*d^3*
n^3)*x^2*e^3 + 36*(b^3*d^3*n^3*x^3*e^3 + 3*a*b^2*d^3*n^3*x^2*e^3 + 3*a^2*b*
d^3*n^3*x*e^3 + a^3*d^3*n^3*e^3)*log(b*x + a)^3 - 36*(b^3*d^3*n^3*x^3*e^3 +
 3*a*b^2*d^3*n^3*x^2*e^3 + 3*a^2*b*d^3*n^3*x*e^3 + a^3*d^3*n^3*e^3)*log(d*x
 + c)^3 - 3*(19*b^3*c^2*d*n^3 - 378*a*b^2*c*d^2*n^3 + 359*a^2*b*d^3*n^3)*x*
e^3 - 198*(b^3*d^3*n^3*x^3*e^3 + 3*a*b^2*d^3*n^3*x^2*e^3 + 3*a^2*b*d^3*n^3*
x*e^3 + a^3*d^3*n^3*e^3)*log(b*x + a)^2 - 18*(11*b^3*d^3*n^3*x^3*e^3 + 33*a
*b^2*d^3*n^3*x^2*e^3 + 33*a^2*b*d^3*n^3*x*e^3 + 11*a^3*d^3*n^3*e^3 - 6*(b^3
*d^3*n^3*x^3*e^3 + 3*a*b^2*d^3*n^3*x^2*e^3 + 3*a^2*b*d^3*n^3*x*e^3 + a^3*d^
3*n^3*e^3)*log(b*x + a))*log(d*x + c)^2 + (8*b^3*c^3*n^3 - 81*a*b^2*c^2*d*n
^3 + 648*a^2*b*c*d^2*n^3 - 575*a^3*d^3*n^3)*e^3 + 510*(b^3*d^3*n^3*x^3*e^3
+ 3*a*b^2*d^3*n^3*x^2*e^3 + 3*a^2*b*d^3*n^3*x*e^3 + a^3*d^3*n^3*e^3)*log(b*
x + a) - 6*(85*b^3*d^3*n^3*x^3*e^3 + 255*a*b^2*d^3*n^3*x^2*e^3 + 255*a^2*b*
d^3*n^3*x*e^3 + 85*a^3*d^3*n^3*e^3 + 18*(b^3*d^3*n^3*x^3*e^3 + 3*a*b^2*d^3*
n^3*x^2*e^3 + 3*a^2*b*d^3*n^3*x*e^3 + a^3*d^3*n^3*e^3)*log(b*x + a)^2 - 66*
(b^3*d^3*n^3*x^3*e^3 + 3*a*b^2*d^3*n^3*x^2*e^3 + 3*a^2*b*d^3*n^3*x*e^3 + a^
3*d^3*n^3*e^3)*log(b*x + a))*log(d*x + c))*e^(-2)/(a^3*b^4*c^3 - 3*a^4*b^3*
c^2*d + 3*a^5*b^2*c*d^2 - a^6*b*d^3 + (b^7*c^3 - 3*a*b^6*c^2*d + 3*a^2*b^5*
c*d^2 - a^3*b^4*d^3)*x^3 + 3*(a*b^6*c^3 - 3*a^2...
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3044 vs. $2(606) = 1212$.
time = 0.52, size = 3044, normalized size = 4.98

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(b*x+a)^4,x, algorithm="fric
as")
```

```
[Out] -1/108*(36*(A^3 + 3*A^2*B + 3*A*B^2 + B^3)*b^3*c^3 - 108*(A^3 + 3*A^2*B + 3
*A*B^2 + B^3)*a*b^2*c^2*d + 108*(A^3 + 3*A^2*B + 3*A*B^2 + B^3)*a^2*b*c*d^2
 - 36*(A^3 + 3*A^2*B + 3*A*B^2 + B^3)*a^3*d^3 + (8*B^3*b^3*c^3 - 81*B^3*a*b
^2*c^2*d + 648*B^3*a^2*b*c*d^2 - 575*B^3*a^3*d^3)*n^3 + 36*(B^3*b^3*d^3*n^3
*x^3 + 3*B^3*a*b^2*d^3*n^3*x^2 + 3*B^3*a^2*b*d^3*n^3*x + (B^3*b^3*c^3 - 3*B
^3*a*b^2*c^2*d + 3*B^3*a^2*b*c*d^2)*n^3)*log(b*x + a)^3 - 36*(B^3*b^3*d^3*n
^3*x^3 + 3*B^3*a*b^2*d^3*n^3*x^2 + 3*B^3*a^2*b*d^3*n^3*x + (B^3*b^3*c^3 - 3
```

*B^3*a*b^2*c^2*d + 3*B^3*a^2*b*c*d^2)*n^3)*log(d*x + c)^3 + 6*(4*(A*B^2 + B
^3)*b^3*c^3 - 27*(A*B^2 + B^3)*a*b^2*c^2*d + 108*(A*B^2 + B^3)*a^2*b*c*d^2
- 85*(A*B^2 + B^3)*a^3*d^3)*n^2 + 6*(85*(B^3*b^3*c*d^2 - B^3*a*b^2*d^3)*n^3
+ 66*((A*B^2 + B^3)*b^3*c*d^2 - (A*B^2 + B^3)*a*b^2*d^3)*n^2 + 18*((A^2*B
+ 2*A*B^2 + B^3)*b^3*c*d^2 - (A^2*B + 2*A*B^2 + B^3)*a*b^2*d^3)*n)*x^2 + 18
*((2*B^3*b^3*c^3 - 9*B^3*a*b^2*c^2*d + 18*B^3*a^2*b*c*d^2)*n^3 + (11*B^3*b^
3*d^3*n^3 + 6*(A*B^2 + B^3)*b^3*d^3*n^2)*x^3 + 6*((A*B^2 + B^3)*b^3*c^3 - 3
*(A*B^2 + B^3)*a*b^2*c^2*d + 3*(A*B^2 + B^3)*a^2*b*c*d^2)*n^2 + 3*(6*(A*B^2
+ B^3)*a*b^2*d^3*n^2 + (2*B^3*b^3*c*d^2 + 9*B^3*a*b^2*d^3)*n^3)*x^2 + 3*(6
*(A*B^2 + B^3)*a^2*b*d^3*n^2 - (B^3*b^3*c^2*d - 6*B^3*a*b^2*c*d^2 - 6*B^3*a
^2*b*d^3)*n^3)*x)*log(b*x + a)^2 + 18*((2*B^3*b^3*c^3 - 9*B^3*a*b^2*c^2*d +
18*B^3*a^2*b*c*d^2)*n^3 + (11*B^3*b^3*d^3*n^3 + 6*(A*B^2 + B^3)*b^3*d^3*n^
2)*x^3 + 6*((A*B^2 + B^3)*b^3*c^3 - 3*(A*B^2 + B^3)*a*b^2*c^2*d + 3*(A*B^2
+ B^3)*a^2*b*c*d^2)*n^2 + 3*(6*(A*B^2 + B^3)*a*b^2*d^3*n^2 + (2*B^3*b^3*c*d
^2 + 9*B^3*a*b^2*d^3)*n^3)*x^2 + 3*(6*(A*B^2 + B^3)*a^2*b*d^3*n^2 - (B^3*b^
3*c^2*d - 6*B^3*a*b^2*c*d^2 - 6*B^3*a^2*b*d^3)*n^3)*x + 6*(B^3*b^3*d^3*n^3*
x^3 + 3*B^3*a*b^2*d^3*n^3*x^2 + 3*B^3*a^2*b*d^3*n^3*x + (B^3*b^3*c^3 - 3*B^
3*a*b^2*c^2*d + 3*B^3*a^2*b*c*d^2)*n^3)*log(b*x + a))*log(d*x + c)^2 + 18*(
2*(A^2*B + 2*A*B^2 + B^3)*b^3*c^3 - 9*(A^2*B + 2*A*B^2 + B^3)*a*b^2*c^2*d +
18*(A^2*B + 2*A*B^2 + B^3)*a^2*b*c*d^2 - 11*(A^2*B + 2*A*B^2 + B^3)*a^3*d^
3)*n - 3*((19*B^3*b^3*c^2*d - 378*B^3*a*b^2*c*d^2 + 359*B^3*a^2*b*d^3)*n^3
+ 6*(5*(A*B^2 + B^3)*b^3*c^2*d - 54*(A*B^2 + B^3)*a*b^2*c*d^2 + 49*(A*B^2 +
B^3)*a^2*b*d^3)*n^2 + 18*((A^2*B + 2*A*B^2 + B^3)*b^3*c^2*d - 6*(A^2*B + 2
*A*B^2 + B^3)*a*b^2*c*d^2 + 5*(A^2*B + 2*A*B^2 + B^3)*a^2*b*d^3)*n)*x + 6*(
(4*B^3*b^3*c^3 - 27*B^3*a*b^2*c^2*d + 108*B^3*a^2*b*c*d^2)*n^3 + (85*B^3*b^
3*d^3*n^3 + 66*(A*B^2 + B^3)*b^3*d^3*n^2 + 18*(A^2*B + 2*A*B^2 + B^3)*b^3*d
^3*n)*x^3 + 6*(2*(A*B^2 + B^3)*b^3*c^3 - 9*(A*B^2 + B^3)*a*b^2*c^2*d + 18*(
A*B^2 + B^3)*a^2*b*c*d^2)*n^2 + 3*(18*(A^2*B + 2*A*B^2 + B^3)*a*b^2*d^3*n +
(22*B^3*b^3*c*d^2 + 63*B^3*a*b^2*d^3)*n^3 + 6*(2*(A*B^2 + B^3)*b^3*c*d^2 +
9*(A*B^2 + B^3)*a*b^2*d^3)*n^2)*x^2 + 18*((A^2*B + 2*A*B^2 + B^3)*b^3*c^3
- 3*(A^2*B + 2*A*B^2 + B^3)*a*b^2*c^2*d + 3*(A^2*B + 2*A*B^2 + B^3)*a^2*b*c
*d^2)*n + 3*(18*(A^2*B + 2*A*B^2 + B^3)*a^2*b*d^3*n - (5*B^3*b^3*c^2*d - 54
*B^3*a*b^2*c*d^2 - 36*B^3*a^2*b*d^3)*n^3 - 6*((A*B^2 + B^3)*b^3*c^2*d - 6*(
A*B^2 + B^3)*a*b^2*c*d^2 - 6*(A*B^2 + B^3)*a^2*b*d^3)*n^2)*x)*log(b*x + a)
- 6*((4*B^3*b^3*c^3 - 27*B^3*a*b^2*c^2*d + 108*B^3*a^2*b*c*d^2)*n^3 + (85*B
^3*b^3*d^3*n^3 + 66*(A*B^2 + B^3)*b^3*d^3*n^2 + 18*(A^2*B + 2*A*B^2 + B^3)*
b^3*d^3*n)*x^3 + 6*(2*(A*B^2 + B^3)*b^3*c^3 - 9*(A*B^2 + B^3)*a*b^2*c^2*d +
18*(A*B^2 + B^3)*a^2*b*c*d^2)*n^2 + 3*(18*(A^2*B + 2*A*B^2 + B^3)*a*b^2*d^
3*n + (22*B^3*b^3*c*d^2 + 63*B^3*a*b^2*d^3)*n^3 + 6*(2*(A*B^2 + B^3)*b^3*c*
d^2 + 9*(A*B^2 + B^3)*a*b^2*d^3)*n^2)*x^2 + 18*(B^3*b^3*d^3*n^3*x^3 + 3*B^3
*a*b^2*d^3*n^3*x^2 + 3*B^3*a^2*b*d^3*n^3*x + (B^3*b^3*c^3 - 3*B^3*a*b^2*c^2
*d + 3*B^3*a^2*b*c*d^2)*n^3)*log(b*x + a)^2 + 18*((A^2*B + 2*A*B^2 + B^3)*b
^3*c^3 - 3*(A^2*B + 2*A*B^2 + B^3)*a*b^2*c^2*d + 3*(A^2*B + 2*A*B^2 + B^3)*
a^2*b*c*d^2)*n + 3*(18*(A^2*B + 2*A*B^2 + B^3)*a^2*b*d^3*n - (5*B^3*b^3*c^2
*d - 54*B^3*a*b^2*c*d^2 - 36*B^3*a^2*b*d^3)*n^3 - 6*((A*B^2 + B^3)*b^3*c^2*

```
d - 6*(A*B^2 + B^3)*a*b^2*c*d^2 - 6*(A*B^2 + B^3)*a^2*b*d^3)*n^2)*x + 6*((2
*B^3*b^3*c^3 - 9*B^3*a*b^2*c^2*d + 18*B^3*a^2*b*c*d^2)*n^3 + (11*B^3*b^3*d^
3*n^3 + 6*(A*B^2 + B^3)*b^3*d^3*n^2)*x^3 + 6*((A*B^2 + B^3)*b^3*c^3 - 3*(A*
B^2 + B^3)*a*b^2*c^2*d + 3*(A*B^2 + B^3)*a^2*b*c*d^2)*n^2 + 3*(6*(A*B^2 + B
^3)*a*b^2*d^3*n^2 + (2*B^3*b^3*c*d^2 + 9*B^3*a*b^2*d^3)*n^3)*x^2 + 3*(6*(A*
B^2 + B^3)*a^2*b*d^3*n^2 - (B^3*b^3*c^2*d - 6*B^3*a*b^2*c*d^2 - 6*B^3*a^2*b
*d^3)*n^3)*x)*log(b*x + a))*log(d*x + c))/(a^3*b^4*c^3 - 3*a^4*b^3*c^2*d +
3*a^5*b^2*c*d^2 - a^6*b*d^3 + (b^7*c^3 - 3*a*b^6*c^2*d + 3*a^2*b^5*c*d^2 -
a^3*b^4*d^3)*x^3 + 3*(a*b^6*c^3 - 3*a^2*b^5*c^2*d + 3*a^3*b^4*c*d^2 - a^4*b
^3*d^3)*x^2 + 3*(a^2*b^5*c^3 - 3*a^3*b^4*c^2*d + 3*a^4*b^3*c*d^2 - a^5*b^2*
d^3)*x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**3/(b*x+a)**4,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(b*x+a)^4,x, algorithm="giac
")
```

```
[Out] integrate((B*log((b*x + a)^n*e/(d*x + c)^n) + A)^3/(b*x + a)^4, x)
```

Mupad [**B**]
time = 10.73, size = 2069, normalized size = 3.39

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^3/(a + b*x)^4,x)
```

```
[Out] ((36*A^3*a^2*d^2 + 36*A^3*b^2*c^2 + 575*B^3*a^2*d^2*n^3 + 8*B^3*b^2*c^2*n^3
 + 198*A^2*B*a^2*d^2*n + 36*A^2*B*b^2*c^2*n - 72*A^3*a*b*c*d + 510*A*B^2*a^
2*d^2*n^2 + 24*A*B^2*b^2*c^2*n^2 - 73*B^3*a*b*c*d*n^3 - 126*A^2*B*a*b*c*d*n
 - 138*A*B^2*a*b*c*d*n^2)/(6*(a*d - b*c)) + (x*(359*B^3*a*b*d^2*n^3 - 19*B^
3*b^2*c*d*n^3 + 90*A^2*B*a*b*d^2*n - 18*A^2*B*b^2*c*d*n + 294*A*B^2*a*b*d^2
```

```
*n^2 - 30*A*B^2*b^2*c*d*n^2))/(2*(a*d - b*c)) + (x^2*(85*B^3*b^2*d^2*n^3 +
18*A^2*B*b^2*d^2*n + 66*A*B^2*b^2*d^2*n^2))/(a*d - b*c))/(x^3*(18*b^5*c - 1
8*a*b^4*d) + x*(54*a^2*b^3*c - 54*a^3*b^2*d) - x^2*(54*a^2*b^3*d - 54*a*b^4
*c) + 18*a^3*b^2*c - 18*a^4*b*d) - log((e*(a + b*x)^n)/(c + d*x)^n)^3*(B^3/
(3*b*(a^3 + b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x)) - (B^3*d^3)/(3*b*(a^3*d^3 -
 b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))) - log((e*(a + b*x)^n)/(c + d*x)
^n)^2*((A*B^2)/(a^3*b + b^4*x^3 + 3*a^2*b^2*x + 3*a*b^3*x^2) - (d^3*(6*A*B^
2 + 11*B^3*n))/(6*b*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) +
(B^3*d^3*(a*((b*n*(a*d - b*c)*(3*a*d - b*c))/(6*d^2) + (a*b*n*(a*d - b*c))/
(3*d)) + x*(b*((b*n*(a*d - b*c)*(3*a*d - b*c))/(6*d^2) + (a*b*n*(a*d - b*c)
)/(3*d)) + (2*a*b^2*n*(a*d - b*c))/(3*d) + (b^2*n*(a*d - b*c)*(3*a*d - b*c)
)/(3*d^2)) + (b*n*(a*d - b*c)*(3*a^2*d^2 + b^2*c^2 - 3*a*b*c*d))/(3*d^3) +
(b^3*n*x^2*(a*d - b*c))/d))/(b*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b
*c*d^2)*(a^3*b + b^4*x^3 + 3*a^2*b^2*x + 3*a*b^3*x^2))) - log((e*(a + b*x)^
n)/(c + d*x)^n)*((x*((a*d + b*c)*(3*A^2*B*a*d - 3*A^2*B*b*c - 6*B^3*a*d*n^2
 + 3*B^3*b*c*n^2) - 3*B^3*a*b*c*d*n^2) + x^2*(b*d*(3*A^2*B*a*d - 3*A^2*B*b*
c - 6*B^3*a*d*n^2 + 3*B^3*b*c*n^2) - 3*B^3*b*d*n^2*(a*d + b*c)) + a*c*(3*A^
2*B*a*d - 3*A^2*B*b*c - 6*B^3*a*d*n^2 + 3*B^3*b*c*n^2) - 3*B^3*b^2*d^2*n^2*
x^3)/(3*b*(a*d - b*c)*(a + b*x)^4*(c + d*x)) + (d^3*(6*A*B^2 + 11*B^3*n)*(x
*((a*((a*b*n*(a*d - b*c)^2)/d + (b*n*(a*d - b*c)^2*(3*a*d - b*c))/(2*d^2))
+ (b*n*(a*d - b*c)^2*(3*a^2*d^2 + b^2*c^2 - 3*a*b*c*d))/d^3)*(a*d + b*c) +
a*c*(b*((a*b*n*(a*d - b*c)^2)/d + (b*n*(a*d - b*c)^2*(3*a*d - b*c))/(2*d^2)
) + (b^2*n*(a*d - b*c)^2*(3*a*d - b*c))/d^2 + (2*a*b^2*n*(a*d - b*c)^2)/d))
 + x^2*((a*d + b*c)*(b*((a*b*n*(a*d - b*c)^2)/d + (b*n*(a*d - b*c)^2*(3*a*d
 - b*c))/(2*d^2)) + (b^2*n*(a*d - b*c)^2*(3*a*d - b*c))/d^2 + (2*a*b^2*n*(a
*d - b*c)^2)/d) + b*d*(a*((a*b*n*(a*d - b*c)^2)/d + (b*n*(a*d - b*c)^2*(3*a
*d - b*c))/(2*d^2)) + (b*n*(a*d - b*c)^2*(3*a^2*d^2 + b^2*c^2 - 3*a*b*c*d))
/d^3) + (3*a*b^3*c*n*(a*d - b*c)^2)/d) + x^3*(b*d*(b*((a*b*n*(a*d - b*c)^2)
/d + (b*n*(a*d - b*c)^2*(3*a*d - b*c))/(2*d^2)) + (b^2*n*(a*d - b*c)^2*(3*a
*d - b*c))/d^2 + (2*a*b^2*n*(a*d - b*c)^2)/d) + (3*b^3*n*(a*d + b*c)*(a*d -
 b*c)^2)/d) + a*c*(a*((a*b*n*(a*d - b*c)^2)/d + (b*n*(a*d - b*c)^2*(3*a*d -
 b*c))/(2*d^2)) + (b*n*(a*d - b*c)^2*(3*a^2*d^2 + b^2*c^2 - 3*a*b*c*d))/d^3
) + 3*b^4*n*x^4*(a*d - b*c)^2))/(9*b^2*(a*d - b*c)*(a + b*x)^4*(c + d*x)*(a
^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))) - (B*d^3*n*atan((B*d^3*
n*((b^4*c^3 + a^3*b*d^3 - a^2*b^2*c*d^2 - a*b^3*c^2*d)/(b^3*c^2 + a^2*b*d^2
 - 2*a*b^2*c*d) + 2*b*d*x)*(18*A^2 + 85*B^2*n^2 + 66*A*B*n)*(b^3*c^2 + a^2*
b*d^2 - 2*a*b^2*c*d)*1i)/(b*(a*d - b*c)^3*(85*B^3*d^3*n^3 + 18*A^2*B*d^3*n
+ 66*A*B^2*d^3*n^2)))*(18*A^2 + 85*B^2*n^2 + 66*A*B*n)*1i)/(9*b*(a*d - b*c)
^3)
```

3.171 $\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{(a+bx)^5}\,dx$

Optimal. Leaf size=830

$$\frac{6B^3d^3n^3(c+dx)}{(bc-ad)^4(a+bx)}-\frac{9bB^3d^2n^3(c+dx)^2}{8(bc-ad)^4(a+bx)^2}+\frac{2b^2B^3dn^3(c+dx)^3}{9(bc-ad)^4(a+bx)^3}-\frac{3b^3B^3n^3(c+dx)^4}{128(bc-ad)^4(a+bx)^4}+\frac{6B^2d^3n^2(c+d}{}$$

```
[Out] 6*B^3*d^3*n^3*(d*x+c)/(-a*d+b*c)^4/(b*x+a)-9/8*b*B^3*d^2*n^3*(d*x+c)^2/(-a*
d+b*c)^4/(b*x+a)^2+2/9*b^2*B^3*d*n^3*(d*x+c)^3/(-a*d+b*c)^4/(b*x+a)^3-3/128
*b^3*B^3*n^3*(d*x+c)^4/(-a*d+b*c)^4/(b*x+a)^4+6*B^2*d^3*n^2*(d*x+c)*(A+B*ln
(e*(b*x+a)^n/((d*x+c)^n)))/(-a*d+b*c)^4/(b*x+a)-9/4*b*B^2*d^2*n^2*(d*x+c)^2
*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/(-a*d+b*c)^4/(b*x+a)^2+2/3*b^2*B^2*d*n^2
*(d*x+c)^3*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/(-a*d+b*c)^4/(b*x+a)^3-3/32*b^
3*B^2*n^2*(d*x+c)^4*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/(-a*d+b*c)^4/(b*x+a)^
4+3*B*d^3*n*(d*x+c)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(-a*d+b*c)^4/(b*x+a
)-9/4*b*B*d^2*n*(d*x+c)^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(-a*d+b*c)^4/
(b*x+a)^2+b^2*B*d*n*(d*x+c)^3*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(-a*d+b*c
)^4/(b*x+a)^3-3/16*b^3*B*n*(d*x+c)^4*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(-
a*d+b*c)^4/(b*x+a)^4+d^3*(d*x+c)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/(-a*d+
b*c)^4/(b*x+a)-3/2*b*d^2*(d*x+c)^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/(-a*
d+b*c)^4/(b*x+a)^2+b^2*d*(d*x+c)^3*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/(-a*
d+b*c)^4/(b*x+a)^3-1/4*b^3*(d*x+c)^4*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/(-
a*d+b*c)^4/(b*x+a)^4
```

Rubi [A]
time = 0.39, antiderivative size = 830, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {2573, 2549, 2395, 2342, 2341}

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3/(a + b*x)^5,x]
```

```
[Out] (6*B^3*d^3*n^3*(c + d*x))/((b*c - a*d)^4*(a + b*x)) - (9*b*B^3*d^2*n^3*(c +
 d*x)^2)/(8*(b*c - a*d)^4*(a + b*x)^2) + (2*b^2*B^3*d*n^3*(c + d*x)^3)/(9*(
b*c - a*d)^4*(a + b*x)^3) - (3*b^3*B^3*n^3*(c + d*x)^4)/(128*(b*c - a*d)^4*
(a + b*x)^4) + (6*B^2*d^3*n^2*(c + d*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x
)^n]))/((b*c - a*d)^4*(a + b*x)) - (9*b*B^2*d^2*n^2*(c + d*x)^2*(A + B*Log[
(e*(a + b*x)^n)/(c + d*x)^n]))/(4*(b*c - a*d)^4*(a + b*x)^2) + (2*b^2*B^2*d
*n^2*(c + d*x)^3*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]))/(3*(b*c - a*d)^4
*(a + b*x)^3) - (3*b^3*B^2*n^2*(c + d*x)^4*(A + B*Log[(e*(a + b*x)^n)/(c +
d*x)^n]))/(32*(b*c - a*d)^4*(a + b*x)^4) + (3*B*d^3*n*(c + d*x)*(A + B*Log[
(e*(a + b*x)^n)/(c + d*x)^n])^2)/((b*c - a*d)^4*(a + b*x)) - (9*b*B*d^2*n*(
```

```
c + d*x)^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2)/(4*(b*c - a*d)^4*(a
+ b*x)^2) + (b^2*B*d*n*(c + d*x)^3*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])
^2)/((b*c - a*d)^4*(a + b*x)^3) - (3*b^3*B*n*(c + d*x)^4*(A + B*Log[(e*(a +
 b*x)^n)/(c + d*x)^n])^2)/(16*(b*c - a*d)^4*(a + b*x)^4) + (d^3*(c + d*x)*(
A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3)/((b*c - a*d)^4*(a + b*x)) - (3*b
*d^2*(c + d*x)^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3)/(2*(b*c - a*d)
^4*(a + b*x)^2) + (b^2*d*(c + d*x)^3*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n
])^3)/((b*c - a*d)^4*(a + b*x)^3) - (b^3*(c + d*x)^4*(A + B*Log[(e*(a + b*x
)^n)/(c + d*x)^n])^3)/(4*(b*c - a*d)^4*(a + b*x)^4)
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2549

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(m +
1)*(g/b)^m, Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x], x,
(a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ
[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[b*f - a*g, 0] && (GtQ[p, 0] || LtQ
[m, -1])
```

Rule 2573

```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

Rubi steps

$$\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{(a+bx)^5}\,dx = \int \left(\frac{A^3}{(a+bx)^5}+\frac{3A^2B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^5}+\frac{3AB^2\text{lo}}{}\right.$$

$$= -\frac{A^3}{4b(a+bx)^4}+\left(3A^2B\right)\int \frac{\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)^5}\,dx+($$

$$= -\frac{A^3}{4b(a+bx)^4}-\frac{3A^2B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{4b(a+bx)^4}-\frac{3AB^2\text{log}}{}$$

$$= -\frac{A^3}{4b(a+bx)^4}-\frac{3A^2B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{4b(a+bx)^4}-\frac{3AB^2\text{log}}{}$$

$$= -\frac{A^3}{4b(a+bx)^4}-\frac{3A^2Bn}{16b(a+bx)^4}+\frac{A^2Bdn}{4b(bc-ad)(a+bx)^3}-\frac{}{8b(bc}$$

$$= -\frac{A^3}{4b(a+bx)^4}-\frac{3A^2Bn}{16b(a+bx)^4}+\frac{A^2Bdn}{4b(bc-ad)(a+bx)^3}-\frac{}{8b(bc}$$

$$= -\frac{A^3}{4b(a+bx)^4}-\frac{3A^2Bn}{16b(a+bx)^4}+\frac{A^2Bdn}{4b(bc-ad)(a+bx)^3}-\frac{}{8b(bc}$$

$$= -\frac{A^3}{4b(a+bx)^4}-\frac{3A^2Bn}{16b(a+bx)^4}-\frac{3AB^2n^2}{32b(a+bx)^4}+\frac{A^2Bd}{4b(bc-ad)(}$$

$$= -\frac{A^3}{4b(a+bx)^4}-\frac{3A^2Bn}{16b(a+bx)^4}-\frac{3AB^2n^2}{32b(a+bx)^4}+\frac{A^2Bd}{4b(bc-ad)(}$$

$$= -\frac{A^3}{4b(a+bx)^4}-\frac{3A^2Bn}{16b(a+bx)^4}-\frac{3AB^2n^2}{32b(a+bx)^4}+\frac{A^2Bd}{4b(bc-ad)(}$$

$$= -\frac{A^3}{4b(a+bx)^4}-\frac{3A^2Bn}{16b(a+bx)^4}-\frac{3AB^2n^2}{32b(a+bx)^4}-\frac{3B^3n^3}{128b(a+bx}$$

$$= -\frac{A^3}{4b(a+bx)^4}-\frac{3A^2Bn}{16b(a+bx)^4}-\frac{3AB^2n^2}{32b(a+bx)^4}-\frac{3B^3n^3}{128b(a+bx}$$

$$= -\frac{A^3}{4b(a+bx)^4}-\frac{3A^2Bn}{16b(a+bx)^4}-\frac{3AB^2n^2}{32b(a+bx)^4}-\frac{3B^3n^3}{128b(a+bx}$$

Mathematica [A]

time = 1.31, size = 1370, normalized size = 1.65

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3/(a + b*x)^5,x]
```

```
[Out] -1/1152*(-288*B^3*d^4*n^3*(a + b*x)^4*Log[a + b*x]^3 + 288*B^3*d^4*n^3*(a +
 b*x)^4*Log[c + d*x]^3 + 72*B^2*d^4*n^2*(a + b*x)^4*Log[c + d*x]^2*(12*A +
25*B*n + 12*B*Log[(e*(a + b*x)^n)/(c + d*x)^n]) + 72*B^2*d^4*n^2*(a + b*x)^
4*Log[a + b*x]^2*(12*A + 25*B*n + 12*B*n*Log[c + d*x] + 12*B*Log[(e*(a + b*
x)^n)/(c + d*x)^n]) + 12*B*d^4*n*(a + b*x)^4*Log[c + d*x]*(72*A^2 + 300*A*B
*n + 415*B^2*n^2 + 12*B*(12*A + 25*B*n)*Log[(e*(a + b*x)^n)/(c + d*x)^n] +
72*B^2*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2) + (b*c - a*d)*(288*A^3*b^3*c^3 -
 864*a*A^3*b^2*c^2*d + 864*a^2*A^3*b*c*d^2 - 288*a^3*A^3*d^3 + 216*A^2*b^3*
B*c^3*n - 936*a*A^2*b^2*B*c^2*d*n + 1656*a^2*A^2*b*B*c*d^2*n - 1800*a^3*A^2
*B*d^3*n + 108*A*b^3*B^2*c^3*n^2 - 660*a*A*b^2*B^2*c^2*d*n^2 + 1932*a^2*A*b
*B^2*c*d^2*n^2 - 4980*a^3*A*B^2*d^3*n^2 + 27*b^3*B^3*c^3*n^3 - 229*a*b^2*B^
3*c^2*d*n^3 + 1067*a^2*b*B^3*c*d^2*n^3 - 5845*a^3*B^3*d^3*n^3 - 288*A^2*b^3
*B*c^2*d*n*x + 1440*a*A^2*b^2*B*c*d^2*n*x - 3744*a^2*A^2*b*B*d^3*n*x - 336*
A*b^3*B^2*c^2*d*n^2*x + 2544*a*A*b^2*B^2*c*d^2*n^2*x - 13008*a^2*A*b*B^2*d^
3*n^2*x - 148*b^3*B^3*c^2*d*n^3*x + 1676*a*b^2*B^3*c*d^2*n^3*x - 16468*a^2*
b*B^3*d^3*n^3*x + 432*A^2*b^3*B*c*d^2*n*x^2 - 3024*a*A^2*b^2*B*d^3*n*x^2 +
936*A*b^3*B^2*c*d^2*n^2*x^2 - 11736*a*A*b^2*B^2*d^3*n^2*x^2 + 690*b^3*B^3*c
*d^2*n^3*x^2 - 15630*a*b^2*B^3*d^3*n^3*x^2 - 864*A^2*b^3*B*d^3*n*x^3 - 3600
*A*b^3*B^2*d^3*n^2*x^3 - 4980*b^3*B^3*d^3*n^3*x^3 + 12*B*(72*A^2*(b*c - a*d
)^3 + B^2*n^2*(-415*a^3*d^3 + a^2*b*d^2*(161*c - 1084*d*x) + a*b^2*d*(-55*c
^2 + 212*c*d*x - 978*d^2*x^2) + b^3*(9*c^3 - 28*c^2*d*x + 78*c*d^2*x^2 - 30
0*d^3*x^3)) + 12*A*B*n*(-25*a^3*d^3 + a^2*b*d^2*(23*c - 52*d*x) + a*b^2*d*(
-13*c^2 + 20*c*d*x - 42*d^2*x^2) + b^3*(3*c^3 - 4*c^2*d*x + 6*c*d^2*x^2 - 1
2*d^3*x^3)))*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 72*B^2*(12*A*(b*c - a*d)^3
+ B*n*(-25*a^3*d^3 + a^2*b*d^2*(23*c - 52*d*x) + a*b^2*d*(-13*c^2 + 20*c*d*
x - 42*d^2*x^2) + b^3*(3*c^3 - 4*c^2*d*x + 6*c*d^2*x^2 - 12*d^3*x^3)))*Log[
(e*(a + b*x)^n)/(c + d*x)^n]^2 + 288*B^3*(b*c - a*d)^3*Log[(e*(a + b*x)^n)/
(c + d*x)^n]^3) - 12*B*d^4*n*(a + b*x)^4*Log[a + b*x]*(72*A^2 + 300*A*B*n +
 415*B^2*n^2 + 72*B^2*n^2*Log[c + d*x]^2 + 12*B*(12*A + 25*B*n)*Log[(e*(a +
 b*x)^n)/(c + d*x)^n] + 72*B^2*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2 + 12*B*n*
Log[c + d*x]*(12*A + 25*B*n + 12*B*Log[(e*(a + b*x)^n)/(c + d*x)^n])))/(b*(
b*c - a*d)^4*(a + b*x)^4)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 38.20, size = 236754, normalized size = 285.25

method	result	size

risch	Expression too large to display	236754

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/(b*x+a)^5,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 5098 vs. $2(822) = 1644$.
time = 0.85, size = 5098, normalized size = 6.14

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(b*x+a)^5,x, algorithm="maxi
ma")
```

```
[Out] 1/16*(12*d^4*n*e*log(b*x + a)/(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2
- 4*a^3*b^2*c*d^3 + a^4*b*d^4) - 12*d^4*n*e*log(d*x + c)/(b^5*c^4 - 4*a*b^4
*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4) + (12*b^3*d^3*n*x
^3*e - 6*(b^3*c*d^2*n - 7*a*b^2*d^3*n)*x^2*e + 4*(b^3*c^2*d*n - 5*a*b^2*c*d
^2*n + 13*a^2*b*d^3*n)*x*e - (3*b^3*c^3*n - 13*a*b^2*c^2*d*n + 23*a^2*b*c*d
^2*n - 25*a^3*d^3*n)*e)/(a^4*b^4*c^3 - 3*a^5*b^3*c^2*d + 3*a^6*b^2*c*d^2 -
a^7*b*d^3 + (b^8*c^3 - 3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 - a^3*b^5*d^3)*x^4 +
 4*(a*b^7*c^3 - 3*a^2*b^6*c^2*d + 3*a^3*b^5*c*d^2 - a^4*b^4*d^3)*x^3 + 6*(a
^2*b^6*c^3 - 3*a^3*b^5*c^2*d + 3*a^4*b^4*c*d^2 - a^5*b^3*d^3)*x^2 + 4*(a^3*
b^5*c^3 - 3*a^4*b^4*c^2*d + 3*a^5*b^3*c*d^2 - a^6*b^2*d^3)*x))*A^2*B*e^(-1)
 - 1/4*B^3*log((b*x + a)^n*e/(d*x + c)^n)^3/(b^5*x^4 + 4*a*b^4*x^3 + 6*a^2*
b^3*x^2 + 4*a^3*b^2*x + a^4*b) + 1/96*(12*(12*d^4*n*e*log(b*x + a)/(b^5*c^4
 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4) - 12*d^
4*n*e*log(d*x + c)/(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2
*c*d^3 + a^4*b*d^4) + (12*b^3*d^3*n*x^3*e - 6*(b^3*c*d^2*n - 7*a*b^2*d^3*n)
*x^2*e + 4*(b^3*c^2*d*n - 5*a*b^2*c*d^2*n + 13*a^2*b*d^3*n)*x*e - (3*b^3*c^
3*n - 13*a*b^2*c^2*d*n + 23*a^2*b*c*d^2*n - 25*a^3*d^3*n)*e)/(a^4*b^4*c^3 -
 3*a^5*b^3*c^2*d + 3*a^6*b^2*c*d^2 - a^7*b*d^3 + (b^8*c^3 - 3*a*b^7*c^2*d +
 3*a^2*b^6*c*d^2 - a^3*b^5*d^3)*x^4 + 4*(a*b^7*c^3 - 3*a^2*b^6*c^2*d + 3*a^
3*b^5*c*d^2 - a^4*b^4*d^3)*x^3 + 6*(a^2*b^6*c^3 - 3*a^3*b^5*c^2*d + 3*a^4*b
^4*c*d^2 - a^5*b^3*d^3)*x^2 + 4*(a^3*b^5*c^3 - 3*a^4*b^4*c^2*d + 3*a^5*b^3*
c*d^2 - a^6*b^2*d^3)*x))*e^(-1)*log((b*x + a)^n*e/(d*x + c)^n) + (300*(b^4*
c*d^3*n^2 - a*b^3*d^4*n^2)*x^3*e^2 - 6*(13*b^4*c^2*d^2*n^2 - 176*a*b^3*c*d^
3*n^2 + 163*a^2*b^2*d^4*n^2)*x^2*e^2 + 4*(7*b^4*c^3*d*n^2 - 60*a*b^3*c^2*d^
2*n^2 + 324*a^2*b^2*c*d^3*n^2 - 271*a^3*b*d^4*n^2)*x*e^2 - 72*(b^4*d^4*n^2*
x^4*e^2 + 4*a*b^3*d^4*n^2*x^3*e^2 + 6*a^2*b^2*d^4*n^2*x^2*e^2 + 4*a^3*b*d^4
*n^2*x*e^2 + a^4*d^4*n^2*e^2)*log(b*x + a)^2 - 72*(b^4*d^4*n^2*x^4*e^2 + 4*
```

```
a*b^3*d^4*n^2*x^3*e^2 + 6*a^2*b^2*d^4*n^2*x^2*e^2 + 4*a^3*b*d^4*n^2*x*e^2 +
 a^4*d^4*n^2*e^2)*log(d*x + c)^2 - (9*b^4*c^4*n^2 - 64*a*b^3*c^3*d*n^2 + 21
6*a^2*b^2*c^2*d^2*n^2 - 576*a^3*b*c*d^3*n^2 + 415*a^4*d^4*n^2)*e^2 + 300*(b
^4*d^4*n^2*x^4*e^2 + 4*a*b^3*d^4*n^2*x^3*e^2 + 6*a^2*b^2*d^4*n^2*x^2*e^2 +
4*a^3*b*d^4*n^2*x*e^2 + a^4*d^4*n^2*e^2)*log(b*x + a) - 12*(25*b^4*d^4*n^2*
x^4*e^2 + 100*a*b^3*d^4*n^2*x^3*e^2 + 150*a^2*b^2*d^4*n^2*x^2*e^2 + 100*a^3
*b*d^4*n^2*x*e^2 + 25*a^4*d^4*n^2*e^2 - 12*(b^4*d^4*n^2*x^4*e^2 + 4*a*b^3*d
^4*n^2*x^3*e^2 + 6*a^2*b^2*d^4*n^2*x^2*e^2 + 4*a^3*b*d^4*n^2*x*e^2 + a^4*d^
4*n^2*e^2)*log(b*x + a))*log(d*x + c))*e^(-2)/(a^4*b^5*c^4 - 4*a^5*b^4*c^3*
d + 6*a^6*b^3*c^2*d^2 - 4*a^7*b^2*c*d^3 + a^8*b*d^4 + (b^9*c^4 - 4*a*b^8*c^
3*d + 6*a^2*b^7*c^2*d^2 - 4*a^3*b^6*c*d^3 + a^4*b^5*d^4)*x^4 + 4*(a*b^8*c^4
 - 4*a^2*b^7*c^3*d + 6*a^3*b^6*c^2*d^2 - 4*a^4*b^5*c*d^3 + a^5*b^4*d^4)*x^3
 + 6*(a^2*b^7*c^4 - 4*a^3*b^6*c^3*d + 6*a^4*b^5*c^2*d^2 - 4*a^5*b^4*c*d^3 +
 a^6*b^3*d^4)*x^2 + 4*(a^3*b^6*c^4 - 4*a^4*b^5*c^3*d + 6*a^5*b^4*c^2*d^2 -
4*a^6*b^3*c*d^3 + a^7*b^2*d^4)*x))*A*B^2 + 1/1152*(72*(12*d^4*n*e*log(b*x +
 a)/(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*
d^4) - 12*d^4*n*e*log(d*x + c)/(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2
 - 4*a^3*b^2*c*d^3 + a^4*b*d^4) + (12*b^3*d^3*n*x^3*e - 6*(b^3*c*d^2*n - 7*
a*b^2*d^3*n)*x^2*e + 4*(b^3*c^2*d*n - 5*a*b^2*c*d^2*n + 13*a^2*b*d^3*n)*x*e
 - (3*b^3*c^3*n - 13*a*b^2*c^2*d*n + 23*a^2*b*c*d^2*n - 25*a^3*d^3*n)*e)/(a
^4*b^4*c^3 - 3*a^5*b^3*c^2*d + 3*a^6*b^2*c*d^2 - a^7*b*d^3 + (b^8*c^3 - 3*a
*b^7*c^2*d + 3*a^2*b^6*c*d^2 - a^3*b^5*d^3)*x^4 + 4*(a*b^7*c^3 - 3*a^2*b^6*
c^2*d + 3*a^3*b^5*c*d^2 - a^4*b^4*d^3)*x^3 + 6*(a^2*b^6*c^3 - 3*a^3*b^5*c^2
*d + 3*a^4*b^4*c*d^2 - a^5*b^3*d^3)*x^2 + 4*(a^3*b^5*c^3 - 3*a^4*b^4*c^2*d
+ 3*a^5*b^3*c*d^2 - a^6*b^2*d^3)*x))*e^(-1)*log((b*x + a)^n*e/(d*x + c)^n)^
2 + (12*(300*(b^4*c*d^3*n^2 - a*b^3*d^4*n^2)*x^3*e^2 - 6*(13*b^4*c^2*d^2*n^
2 - 176*a*b^3*c*d^3*n^2 + 163*a^2*b^2*d^4*n^2)*x^2*e^2 + 4*(7*b^4*c^3*d*n^2
 - 60*a*b^3*c^2*d^2*n^2 + 324*a^2*b^2*c*d^3*n^2 - 271*a^3*b*d^4*n^2)*x*e^2
- 72*(b^4*d^4*n^2*x^4*e^2 + 4*a*b^3*d^4*n^2*x^3*e^2 + 6*a^2*b^2*d^4*n^2*x^2
*e^2 + 4*a^3*b*d^4*n^2*x*e^2 + a^4*d^4*n^2*e^2)*log(b*x + a)^2 - 72*(b^4*d^
4*n^2*x^4*e^2 + 4*a*b^3*d^4*n^2*x^3*e^2 + 6*a^2*b^2*d^4*n^2*x^2*e^2 + 4*a^3
*b*d^4*n^2*x*e^2 + a^4*d^4*n^2*e^2)*log(d*x + c)^2 - (9*b^4*c^4*n^2 - 64*a*
b^3*c^3*d*n^2 + 216*a^2*b^2*c^2*d^2*n^2 - 576*a^3*b*c*d^3*n^2 + 415*a^4*d^4
*n^2)*e^2 + 300*(b^4*d^4*n^2*x^4*e^2 + 4*a*b^3*d^4*n^2*x^3*e^2 + 6*a^2*b^2*
d^4*n^2*x^2*e^2 + 4*a^3*b*d^4*n^2*x*e^2 + a^4*d^4*n^2*e^2)*log(b*x + a) - 1
2*(25*b^4*d^4*n^2*x^4*e^2 + 100*a*b^3*d^4*n^2*x^3*e^2 + 150*a^2*b^2*d^4*n^2
*x^2*e^2 + 100*a^3*b*d^4*n^2*x*e^2 + 25*a^4*d^4*n^2*e^2 - 12*(b^4*d^4*n^2*x
^4*e^2 + 4*a*b^3*d^4*n^2*x^3*e^2 + 6*a^2*b^2*d^4*n^2*x^2*e^2 + 4*a^3*b*d^4*
n^2*x*e^2 + a^4*d^4*n^2*e^2)*log(b*x + a))*log(d*x + c))*e^(-1)*log((b*x +
a)^n*e/(d*x + c)^n)/(a^4*b^5*c^4 - 4*a^5*b^4*c^...
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 4585 vs. $2(822) = 1644$.

time = 0.78, size = 4585, normalized size = 5.52

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(b*x+a)^5,x, algorithm="fric
as")
```

```
[Out] -1/1152*(288*(A^3 + 3*A^2*B + 3*A*B^2 + B^3)*b^4*c^4 - 1152*(A^3 + 3*A^2*B
+ 3*A*B^2 + B^3)*a*b^3*c^3*d + 1728*(A^3 + 3*A^2*B + 3*A*B^2 + B^3)*a^2*b^2
*c^2*d^2 - 1152*(A^3 + 3*A^2*B + 3*A*B^2 + B^3)*a^3*b*c*d^3 + 288*(A^3 + 3*
A^2*B + 3*A*B^2 + B^3)*a^4*d^4 + (27*B^3*b^4*c^4 - 256*B^3*a*b^3*c^3*d + 12
96*B^3*a^2*b^2*c^2*d^2 - 6912*B^3*a^3*b*c*d^3 + 5845*B^3*a^4*d^4)*n^3 - 12*
(415*(B^3*b^4*c*d^3 - B^3*a*b^3*d^4)*n^3 + 300*((A*B^2 + B^3)*b^4*c*d^3 - (
A*B^2 + B^3)*a*b^3*d^4)*n^2 + 72*((A^2*B + 2*A*B^2 + B^3)*b^4*c*d^3 - (A^2*
B + 2*A*B^2 + B^3)*a*b^3*d^4)*n)*x^3 - 288*(B^3*b^4*d^4*n^3*x^4 + 4*B^3*a*b
^3*d^4*n^3*x^3 + 6*B^3*a^2*b^2*d^4*n^3*x^2 + 4*B^3*a^3*b*d^4*n^3*x - (B^3*b
^4*c^4 - 4*B^3*a*b^3*c^3*d + 6*B^3*a^2*b^2*c^2*d^2 - 4*B^3*a^3*b*c*d^3)*n^3
)*log(b*x + a)^3 + 288*(B^3*b^4*d^4*n^3*x^4 + 4*B^3*a*b^3*d^4*n^3*x^3 + 6*B
^3*a^2*b^2*d^4*n^3*x^2 + 4*B^3*a^3*b*d^4*n^3*x - (B^3*b^4*c^4 - 4*B^3*a*b^3
*c^3*d + 6*B^3*a^2*b^2*c^2*d^2 - 4*B^3*a^3*b*c*d^3)*n^3)*log(d*x + c)^3 + 1
2*(9*(A*B^2 + B^3)*b^4*c^4 - 64*(A*B^2 + B^3)*a*b^3*c^3*d + 216*(A*B^2 + B^
3)*a^2*b^2*c^2*d^2 - 576*(A*B^2 + B^3)*a^3*b*c*d^3 + 415*(A*B^2 + B^3)*a^4*
d^4)*n^2 + 6*(5*(23*B^3*b^4*c^2*d^2 - 544*B^3*a*b^3*c*d^3 + 521*B^3*a^2*b^2
*d^4)*n^3 + 12*(13*(A*B^2 + B^3)*b^4*c^2*d^2 - 176*(A*B^2 + B^3)*a*b^3*c*d^
3 + 163*(A*B^2 + B^3)*a^2*b^2*d^4)*n^2 + 72*((A^2*B + 2*A*B^2 + B^3)*b^4*c^
2*d^2 - 8*(A^2*B + 2*A*B^2 + B^3)*a*b^3*c*d^3 + 7*(A^2*B + 2*A*B^2 + B^3)*a
^2*b^2*d^4)*n)*x^2 - 72*((25*B^3*b^4*d^4*n^3 + 12*(A*B^2 + B^3)*b^4*d^4*n^2
)*x^4 - (3*B^3*b^4*c^4 - 16*B^3*a*b^3*c^3*d + 36*B^3*a^2*b^2*c^2*d^2 - 48*B
^3*a^3*b*c*d^3)*n^3 + 4*(12*(A*B^2 + B^3)*a*b^3*d^4*n^2 + (3*B^3*b^4*c*d^3
+ 22*B^3*a*b^3*d^4)*n^3)*x^3 - 12*((A*B^2 + B^3)*b^4*c^4 - 4*(A*B^2 + B^3)*
a*b^3*c^3*d + 6*(A*B^2 + B^3)*a^2*b^2*c^2*d^2 - 4*(A*B^2 + B^3)*a^3*b*c*d^3
)*n^2 + 6*(12*(A*B^2 + B^3)*a^2*b^2*d^4*n^2 - (B^3*b^4*c^2*d^2 - 8*B^3*a*b^
3*c*d^3 - 18*B^3*a^2*b^2*d^4)*n^3)*x^2 + 4*(12*(A*B^2 + B^3)*a^3*b*d^4*n^2
+ (B^3*b^4*c^3*d - 6*B^3*a*b^3*c^2*d^2 + 18*B^3*a^2*b^2*c*d^3 + 12*B^3*a^3*
b*d^4)*n^3)*x)*log(b*x + a)^2 - 72*((25*B^3*b^4*d^4*n^3 + 12*(A*B^2 + B^3)*
b^4*d^4*n^2)*x^4 - (3*B^3*b^4*c^4 - 16*B^3*a*b^3*c^3*d + 36*B^3*a^2*b^2*c^2
*d^2 - 48*B^3*a^3*b*c*d^3)*n^3 + 4*(12*(A*B^2 + B^3)*a*b^3*d^4*n^2 + (3*B^3
*b^4*c*d^3 + 22*B^3*a*b^3*d^4)*n^3)*x^3 - 12*((A*B^2 + B^3)*b^4*c^4 - 4*(A*
B^2 + B^3)*a*b^3*c^3*d + 6*(A*B^2 + B^3)*a^2*b^2*c^2*d^2 - 4*(A*B^2 + B^3)*
a^3*b*c*d^3)*n^2 + 6*(12*(A*B^2 + B^3)*a^2*b^2*d^4*n^2 - (B^3*b^4*c^2*d^2 -
 8*B^3*a*b^3*c*d^3 - 18*B^3*a^2*b^2*d^4)*n^3)*x^2 + 4*(12*(A*B^2 + B^3)*a^3
*b*d^4*n^2 + (B^3*b^4*c^3*d - 6*B^3*a*b^3*c^2*d^2 + 18*B^3*a^2*b^2*c*d^3 +
12*B^3*a^3*b*d^4)*n^3)*x + 12*(B^3*b^4*d^4*n^3*x^4 + 4*B^3*a*b^3*d^4*n^3*x^
3 + 6*B^3*a^2*b^2*d^4*n^3*x^2 + 4*B^3*a^3*b*d^4*n^3*x - (B^3*b^4*c^4 - 4*B^
3*a*b^3*c^3*d + 6*B^3*a^2*b^2*c^2*d^2 - 4*B^3*a^3*b*c*d^3)*n^3)*log(b*x + a
))*log(d*x + c)^2 + 72*(3*(A^2*B + 2*A*B^2 + B^3)*b^4*c^4 - 16*(A^2*B + 2*A
*B^2 + B^3)*a*b^3*c^3*d + 36*(A^2*B + 2*A*B^2 + B^3)*a^2*b^2*c^2*d^2 - 48*(
```

```
A^2*B + 2*A*B^2 + B^3)*a^3*b*c*d^3 + 25*(A^2*B + 2*A*B^2 + B^3)*a^4*d^4)*n
- 4*((37*B^3*b^4*c^3*d - 456*B^3*a*b^3*c^2*d^2 + 4536*B^3*a^2*b^2*c*d^3 - 4
117*B^3*a^3*b*d^4)*n^3 + 12*(7*(A*B^2 + B^3)*b^4*c^3*d - 60*(A*B^2 + B^3)*a
*b^3*c^2*d^2 + 324*(A*B^2 + B^3)*a^2*b^2*c*d^3 - 271*(A*B^2 + B^3)*a^3*b*d^
4)*n^2 + 72*((A^2*B + 2*A*B^2 + B^3)*b^4*c^3*d - 6*(A^2*B + 2*A*B^2 + B^3)*
a*b^3*c^2*d^2 + 18*(A^2*B + 2*A*B^2 + B^3)*a^2*b^2*c*d^3 - 13*(A^2*B + 2*A*
B^2 + B^3)*a^3*b*d^4)*n)*x - 12*((415*B^3*b^4*d^4*n^3 + 300*(A*B^2 + B^3)*b
^4*d^4*n^2 + 72*(A^2*B + 2*A*B^2 + B^3)*b^4*d^4*n)*x^4 - (9*B^3*b^4*c^4 - 6
4*B^3*a*b^3*c^3*d + 216*B^3*a^2*b^2*c^2*d^2 - 576*B^3*a^3*b*c*d^3)*n^3 + 4*
(72*(A^2*B + 2*A*B^2 + B^3)*a*b^3*d^4*n + 5*(15*B^3*b^4*c*d^3 + 68*B^3*a*b^
3*d^4)*n^3 + 12*(3*(A*B^2 + B^3)*b^4*c*d^3 + 22*(A*B^2 + B^3)*a*b^3*d^4)*n^
2)*x^3 - 12*(3*(A*B^2 + B^3)*b^4*c^4 - 16*(A*B^2 + B^3)*a*b^3*c^3*d + 36*(A
*B^2 + B^3)*a^2*b^2*c^2*d^2 - 48*(A*B^2 + B^3)*a^3*b*c*d^3)*n^2 + 6*(72*(A^
2*B + 2*A*B^2 + B^3)*a^2*b^2*d^4*n - (13*B^3*b^4*c^2*d^2 - 176*B^3*a*b^3*c*
d^3 - 252*B^3*a^2*b^2*d^4)*n^3 - 12*((A*B^2 + B^3)*b^4*c^2*d^2 - 8*(A*B^2 +
 B^3)*a*b^3*c*d^3 - 18*(A*B^2 + B^3)*a^2*b^2*d^4)*n^2)*x^2 - 72*((A^2*B + 2
*A*B^2 + B^3)*b^4*c^4 - 4*(A^2*B + 2*A*B^2 + B^3)*a*b^3*c^3*d + 6*(A^2*B +
2*A*B^2 + B^3)*a^2*b^2*c^2*d^2 - 4*(A^2*B + 2*A*B^2 + B^3)*a^3*b*c*d^3)*n +
 4*(72*(A^2*B + 2*A*B^2 + B^3)*a^3*b*d^4*n + (7*B^3*b^4*c^3*d - 60*B^3*a*b^
3*c^2*d^2 + 324*B^3*a^2*b^2*c*d^3 + 144*B^3*a^3*b*d^4)*n^3 + 12*((A*B^2 + B
^3)*b^4*c^3*d - 6*(A*B^2 + B^3)*a*b^3*c^2*d^2 + 18*(A*B^2 + B^3)*a^2*b^2*c*
d^3 + 12*(A*B^2 + B^3)*a^3*b*d^4)*n^2)*x)*log(b*x + a) + 12*((415*B^3*b^4*d
^4*n^3 + 300*(A*B^2 + B^3)*b^4*d^4*n^2 + 72*(A^2*B + 2*A*B^2 + B^3)*b^4*d^4
*n)*x^4 - (9*B^3*b^4*c^4 - 64*B^3*a*b^3*c^3*d + 216*B^3*a^2*b^2*c^2*d^2 - 5
76*B^3*a^3*b*c*d^3)*n^3 + 4*(72*(A^2*B + 2*A*B^2 + B^3)*a*b^3*d^4*n + 5*(15
*B^3*b^4*c*d^3 + 68*B^3*a*b^3*d^4)*n^3 + 12*(3*(A*B^2 + B^3)*b^4*c*d^3 + 22
*(A*B^2 + B^3)*a*b^3*d^4)*n^2)*x^3 - 12*(3*(A*B...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**3/(b*x+a)**5,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(b*x+a)^5,x, algorithm="giac
")
```

```
[Out] integrate((B*log((b*x + a)^n*e/(d*x + c)^n) + A)^3/(b*x + a)^5, x)
```

Mupad [B]
time = 11.29, size = 2500, normalized size = 3.01

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^3/(a + b*x)^5,x)

[Out] log((e*(a + b*x)^n)/(c + d*x)^n)*((x*((a*d + b*c)*(a*((9*B^3*a*d^2*n^2)/2 -
 (3*B^3*b*c*d*n^2)/2) + 13*B^3*a^2*d^2*n^2 + (11*B^3*b^2*c^2*n^2)/2 - 6*A^2
*B*a^2*d^2 - 6*A^2*B*b^2*c^2 - (31*B^3*a*b*c*d*n^2)/2 + 12*A^2*B*a*b*c*d) +
 a*c*(b*((9*B^3*a*d^2*n^2)/2 - (3*B^3*b*c*d*n^2)/2) + (27*B^3*a*b*d^2*n^2)/
2 - (9*B^3*b^2*c*d*n^2)/2)) + x^2*((a*d + b*c)*(b*((9*B^3*a*d^2*n^2)/2 - (3
*B^3*b*c*d*n^2)/2) + (27*B^3*a*b*d^2*n^2)/2 - (9*B^3*b^2*c*d*n^2)/2) + b*d*
(a*((9*B^3*a*d^2*n^2)/2 - (3*B^3*b*c*d*n^2)/2) + 13*B^3*a^2*d^2*n^2 + (11*B
^3*b^2*c^2*n^2)/2 - 6*A^2*B*a^2*d^2 - 6*A^2*B*b^2*c^2 - (31*B^3*a*b*c*d*n^2
)/2 + 12*A^2*B*a*b*c*d) + 6*B^3*a*b^2*c*d^2*n^2) + x^3*(b*d*(b*((9*B^3*a*d^
2*n^2)/2 - (3*B^3*b*c*d*n^2)/2) + (27*B^3*a*b*d^2*n^2)/2 - (9*B^3*b^2*c*d*n
^2)/2) + 6*B^3*b^2*d^2*n^2*(a*d + b*c)) + a*c*(a*((9*B^3*a*d^2*n^2)/2 - (3*
B^3*b*c*d*n^2)/2) + 13*B^3*a^2*d^2*n^2 + (11*B^3*b^2*c^2*n^2)/2 - 6*A^2*B*a
^2*d^2 - 6*A^2*B*b^2*c^2 - (31*B^3*a*b*c*d*n^2)/2 + 12*A^2*B*a*b*c*d) + 6*B
^3*b^3*d^3*n^2*x^4)/(8*b*(a*d - b*c)^2*(a + b*x)^5*(c + d*x)) - (d^4*(12*A*
B^2 + 25*B^3*n)*(x^3*((a*d + b*c)*(b*(b*((2*a*b*n*(a*d - b*c)^3)/d + (2*b*n
*(a*d - b*c)^3*(4*a*d - b*c))/(3*d^2)) + (4*b^2*n*(a*d - b*c)^3*(4*a*d - b*
c))/(3*d^2) + (4*a*b^2*n*(a*d - b*c)^3)/d) + (2*b^3*n*(a*d - b*c)^3*(4*a*d
- b*c))/d^2 + (6*a*b^3*n*(a*d - b*c)^3)/d) + b*d*(b*(a*((2*a*b*n*(a*d - b*c
)^3)/d + (2*b*n*(a*d - b*c)^3*(4*a*d - b*c))/(3*d^2)) + (2*b*n*(a*d - b*c)^
3*(6*a^2*d^2 + b^2*c^2 - 4*a*b*c*d))/(3*d^3)) + a*(b*((2*a*b*n*(a*d - b*c)^
3)/d + (2*b*n*(a*d - b*c)^3*(4*a*d - b*c))/(3*d^2)) + (4*b^2*n*(a*d - b*c)^
3*(4*a*d - b*c))/(3*d^2) + (4*a*b^2*n*(a*d - b*c)^3)/d) + (2*b^2*n*(a*d - b
*c)^3*(6*a^2*d^2 + b^2*c^2 - 4*a*b*c*d))/d^3) + (8*a*b^4*c*n*(a*d - b*c)^3)
/d) + x^2*((a*d + b*c)*(b*(a*((2*a*b*n*(a*d - b*c)^3)/d + (2*b*n*(a*d - b*c
)^3*(4*a*d - b*c))/(3*d^2)) + (2*b*n*(a*d - b*c)^3*(6*a^2*d^2 + b^2*c^2 - 4
*a*b*c*d))/(3*d^3)) + a*(b*((2*a*b*n*(a*d - b*c)^3)/d + (2*b*n*(a*d - b*c)^
3*(4*a*d - b*c))/(3*d^2)) + (4*b^2*n*(a*d - b*c)^3*(4*a*d - b*c))/(3*d^2) +
 (4*a*b^2*n*(a*d - b*c)^3)/d) + (2*b^2*n*(a*d - b*c)^3*(6*a^2*d^2 + b^2*c^2
 - 4*a*b*c*d))/d^3) + a*c*(b*(b*((2*a*b*n*(a*d - b*c)^3)/d + (2*b*n*(a*d -
b*c)^3*(4*a*d - b*c))/(3*d^2)) + (4*b^2*n*(a*d - b*c)^3*(4*a*d - b*c))/(3*d
^2) + (4*a*b^2*n*(a*d - b*c)^3)/d) + (2*b^3*n*(a*d - b*c)^3*(4*a*d - b*c))/
d^2 + (6*a*b^3*n*(a*d - b*c)^3)/d) + b*d*(a*(a*((2*a*b*n*(a*d - b*c)^3)/d +
 (2*b*n*(a*d - b*c)^3*(4*a*d - b*c))/(3*d^2)) + (2*b*n*(a*d - b*c)^3*(6*a^2
*d^2 + b^2*c^2 - 4*a*b*c*d))/(3*d^3)) + (2*b*n*(a*d - b*c)^3*(4*a^3*d^3 - b
^3*c^3 + 4*a*b^2*c^2*d - 6*a^2*b*c*d^2))/d^4)) + x*((a*(a*((2*a*b*n*(a*d -
```

```
b*c)^3)/d + (2*b*n*(a*d - b*c)^3*(4*a*d - b*c))/(3*d^2)) + (2*b*n*(a*d - b*
c)^3*(6*a^2*d^2 + b^2*c^2 - 4*a*b*c*d))/(3*d^3)) + (2*b*n*(a*d - b*c)^3*(4*
a^3*d^3 - b^3*c^3 + 4*a*b^2*c^2*d - 6*a^2*b*c*d^2))/d^4)*(a*d + b*c) + a*c*
(b*(a*((2*a*b*n*(a*d - b*c)^3)/d + (2*b*n*(a*d - b*c)^3*(4*a*d - b*c))/(3*d
^2)) + (2*b*n*(a*d - b*c)^3*(6*a^2*d^2 + b^2*c^2 - 4*a*b*c*d))/(3*d^3)) + a
*(b*((2*a*b*n*(a*d - b*c)^3)/d + (2*b*n*(a*d - b*c)^3*(4*a*d - b*c))/(3*d^2
)) + (4*b^2*n*(a*d - b*c)^3*(4*a*d - b*c))/(3*d^2) + (4*a*b^2*n*(a*d - b*c)
^3)/d) + (2*b^2*n*(a*d - b*c)^3*(6*a^2*d^2 + b^2*c^2 - 4*a*b*c*d))/d^3)) +
x^4*(b*d*(b*(b*((2*a*b*n*(a*d - b*c)^3)/d + (2*b*n*(a*d - b*c)^3*(4*a*d - b
*c))/(3*d^2)) + (4*b^2*n*(a*d - b*c)^3*(4*a*d - b*c))/(3*d^2) + (4*a*b^2*n*
(a*d - b*c)^3)/d) + (2*b^3*n*(a*d - b*c)^3*(4*a*d - b*c))/d^2 + (6*a*b^3*n*
(a*d - b*c)^3)/d) + (8*b^4*n*(a*d + b*c)*(a*d - b*c)^3)/d) + a*c*(a*(a*((2*
a*b*n*(a*d - b*c)^3)/d + (2*b*n*(a*d - b*c)^3*(4*a*d - b*c))/(3*d^2)) + (2*
b*n*(a*d - b*c)^3*(6*a^2*d^2 + b^2*c^2 - 4*a*b*c*d))/(3*d^3)) + (2*b*n*(a*d
 - b*c)^3*(4*a^3*d^3 - b^3*c^3 + 4*a*b^2*c^2*d - 6*a^2*b*c*d^2))/d^4) + 8*b
^5*n*x^5*(a*d - b*c)^3))/(64*b^2*(a*d - b*c)^2*(a + b*x)^5*(c + d*x)*(a^4*d
^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))) - log((
e*(a + b*x)^n)/(c + d*x)^n)^3*(B^3/(4*b*(a^4 + b^4*x^4 + 4*a*b^3*x^3 + 6*a^
2*b^2*x^2 + 4*a^3*b*x)) - (B^3*d^4)/(4*b*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2
*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))) - log((e*(a + b*x)^n)/(c + d*x)^n)^
2*((3*A*B^2)/(4*(a^4*b + b^5*x^4 + 4*a^3*b^2*x + 4*a*b^4*x^3 + 6*a^2*b^3*x^
2)) - (d^4*(12*A*B^2 + 25*B^3*n))/(16*b*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*
d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)) + (3*B^3*d^4*(x^2*(b*(b*((b*n*(a*d -
b*c)*(4*a*d - b*c))/(3*d^2) + (a*b*n*(a*d - b*c))/d) + (2*a*b^2*n*(a*d - b*
c))/d + (2*b^2*n*(a*d - b*c)*(4*a*d - b*c))/(3*d^2)) + (3*a*b^3*n*(a*d - b*
c))/d + (b^3*n*(a*d - b*c)*(4*a*d - b*c))/d^2) + a*(a*((b*n*(a*d - b*c)*(4*
a*d - b*c))/(3*d^2) + (a*b*n*(a*d - b*c))/d) + (b*n*(a*d - b*c)*(6*a^2*d^2
+ b^2*c^2 - 4*a*b*c*d))/(3*d^3)) + x*(b*(a*((b*n*(a*d - b*c)*(4*a*d - b*c))
/(3*d^2) + (a*b*n*(a*d - b*c))/d) + (b*n*(a*d - b*c)*(6*a^2*d^2 + b^2*c^2 -
 4*a*b*c*d))/(3*d^3)) + a*(b*((b*n*(a*d - b*c)*(4*a*d - b*c))/(3*d^2) + (a*
b*n*(a*d - b*c))/d) + (2*a*b^2*n*(a*d - b*c))/d + (2*b^2*n*(a*d - b*c)*(4*a
*d - b*c))/(3*d^2)) + (b^2*n*(a*d - b*c)*(6*a^2...
```

3.172 $\int \frac{1}{(ag+bgx)^2(A+B\log(e(a+bx)^n(c+dx)^{-n}))}\,dx$

Optimal. Leaf size=96

$$\frac{e^{\frac{A}{Bn}}(c+dx)\left(e(a+bx)^n(c+dx)^{-n}\right)^{\frac{1}{n}}\mathrm{Ei}\left(-\frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{Bn}\right)}{B(bc-ad)g^2n(a+bx)}$$

```
[Out] exp(A/B/n)*(d*x+c)*(e*(b*x+a)^n/((d*x+c)^n))^(1/n)*Ei((-A-B*ln(e*(b*x+a)^n/
((d*x+c)^n)))/B/n)/B/(-a*d+b*c)/g^2/n/(b*x+a)
```

Rubi [A]
time = 0.10, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2573, 2549, 2347, 2209}

$$\frac{e^{\frac{A}{Bn}}(c+dx)\left(e(a+bx)^n(c+dx)^{-n}\right)^{\frac{1}{n}}\mathrm{Ei}\left(-\frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{Bn}\right)}{Bg^2n(a+bx)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])),x]
```

```
[Out] (E^(A/(B*n))*(c + d*x)*((e*(a + b*x)^n)/(c + d*x)^n)^n^(-1)*ExpIntegralEi[-
((A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(B*n))])/(B*(b*c - a*d)*g^2*n*(a
+ b*x))
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2549

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(m +
1)*(g/b)^m, Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x], x,
(a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ
[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[b*f - a*g, 0] && (GtQ[p, 0] || LtQ
[m, -1])
```

Rule 2573

```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] && !Intege
rQ[n]
```

Rubi steps

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}dx=\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}d$$

Mathematica [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])),x]
[Out] Integrate[1/((a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])), x]
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bgx+ag\right)^{2}\left(A+B\ln\left(e\left(bx+a\right)^{n}\left(dx+c\right)^{-n}\right)\right)}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*g*x+a*g)^2/(A+B*ln(e*(b*x+a)^n/((d*x+c)^n))),x)
[Out] int(1/(b*g*x+a*g)^2/(A+B*ln(e*(b*x+a)^n/((d*x+c)^n))),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="
maxima")
[Out] integrate(1/((b*g*x + a*g)^2*(B*log((b*x + a)^n*e/(d*x + c)^n) + A)), x)
```

Fricas [A]
time = 0.38, size = 56, normalized size = 0.58

$$\frac{e^{\left(\frac{A+B}{Bn}\right)}\,\mathrm{log\_integral}\left(\frac{(dx+c)e^{\left(-\frac{A+B}{Bn}\right)}}{bx+a}\right)}{(Bbc-Bad)g^2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="
fricas")
```

```
[Out] e^((A + B)/(B*n))*log_integral((d*x + c)*e^(-(A + B)/(B*n))/(b*x + a))/((B*
b*c - B*a*d)*g^2*n)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)**2/(A+B*ln(e*(b*x+a)**n/((d*x+c)**n))),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="
giac")
```

```
[Out] integrate(1/((b*g*x + a*g)^2*(B*log((b*x + a)^n*e/(d*x + c)^n) + A)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a\,g+b\,g\,x\right)}^2\,\left(A+B\,\ln\left(\frac{e\,{\left(a+b\,x\right)}^n}{{\left(c+d\,x\right)}^n}\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*g + b*g*x)^2*(A + B*log((e*(a + b*x)^n)/(c + d*x)^n))),x)
```

```
[Out] int(1/((a*g + b*g*x)^2*(A + B*log((e*(a + b*x)^n)/(c + d*x)^n))), x)
```

3.173 $\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right) dx$

Optimal. Leaf size=180

$$-\frac{B(bc-ad)^4g^4x}{5d^4}+\frac{B(bc-ad)^3g^4(a+bx)^2}{10bd^3}-\frac{B(bc-ad)^2g^4(a+bx)^3}{15bd^2}+\frac{B(bc-ad)g^4(a+bx)^4}{20bd}+\frac{B(bc-ad)^5}{5}$$

```
[Out] -1/5*B*(-a*d+b*c)^4*g^4*x/d^4+1/10*B*(-a*d+b*c)^3*g^4*(b*x+a)^2/b/d^3-1/15*
B*(-a*d+b*c)^2*g^4*(b*x+a)^3/b/d^2+1/20*B*(-a*d+b*c)*g^4*(b*x+a)^4/b/d+1/5*
B*(-a*d+b*c)^5*g^4*ln(d*x+c)/b/d^5+1/5*g^4*(b*x+a)^5*(A+B*ln(e*(d*x+c)/(b*x
+a)))/b
```

Rubi [A]
time = 0.09, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2548, 21, 45}

$$\frac{g^4(a+bx)^5\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{5b}+\frac{Bg^4(bc-ad)^5\log(c+dx)}{5bd^5}-\frac{Bg^4x(bc-ad)^4}{5d^4}+\frac{Bg^4(a+bx)^2(bc-ad)^3}{10bd^3}-\frac{Bg^4(a+bx)^3(bc-ad)^2}{15bd^2}+\frac{Bg^4(a+bx)^4(bc-ad)}{20bd}$$

Antiderivative was successfully verified.

```
[In] Int[(a*g + b*g*x)^4*(A + B*Log[(e*(c + d*x))/(a + b*x)]),x]
```

```
[Out] -1/5*(B*(b*c - a*d)^4*g^4*x)/d^4 + (B*(b*c - a*d)^3*g^4*(a + b*x)^2)/(10*b*
d^3) - (B*(b*c - a*d)^2*g^4*(a + b*x)^3)/(15*b*d^2) + (B*(b*c - a*d)*g^4*(a
 + b*x)^4)/(20*b*d) + (B*(b*c - a*d)^5*g^4*Log[c + d*x])/(5*b*d^5) + (g^4*(
a + b*x)^5*(A + B*Log[(e*(c + d*x))/(a + b*x)]))/(5*b)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2548

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
```

```
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Dist[B*n*((b*c
- a*d)/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /;
 FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c -
 a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Rubi steps

$$\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right) dx = \frac{g^4(a+bx)^5\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{5b} - \frac{B\int \frac{(-bc+ad)g^5(a+bx)^4}{c+dx}}{5bg}$$

$$= \frac{g^4(a+bx)^5\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{5b} + \frac{(B(bc-ad)g^4)\int \frac{(a+}{c+}}{5b}$$

$$= \frac{g^4(a+bx)^5\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{5b} + \frac{(B(bc-ad)g^4)\int \Big(-}{}$$

$$= -\frac{B(bc-ad)^4g^4x}{5d^4} + \frac{B(bc-ad)^3g^4(a+bx)^2}{10bd^3} - \frac{B(bc-ad)}{15}$$

Mathematica [A]
time = 0.07, size = 142, normalized size = 0.79

$$\frac{g^4\left(-\frac{B(-bc+ad)\left(-12bd(bc-ad)^3x+6d^2(bc-ad)^2(a+bx)^2+4d^3(-bc+ad)(a+bx)^3+3d^4(a+bx)^4+12(bc-ad)^4\log(c+dx)\right)}{12d^5}+(a+bx)^5\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)\right)}{5b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*g + b*g*x)^4*(A + B*Log[(e*(c + d*x))/(a + b*x)]),x]
```

```
[Out] (g^4*(-1/12*(B*(-(b*c) + a*d)*(-12*b*d*(b*c - a*d)^3*x + 6*d^2*(b*c - a*d)^
2*(a + b*x)^2 + 4*d^3*(-(b*c) + a*d)*(a + b*x)^3 + 3*d^4*(a + b*x)^4 + 12*(
b*c - a*d)^4*Log[c + d*x]))/d^5 + (a + b*x)^5*(A + B*Log[(e*(c + d*x))/(a +
 b*x)])))/(5*b)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 4272 vs. $2(168) = 336$.
time = 0.42, size = 4273, normalized size = 23.74

method	result
risch	$\frac{g^4b^4Ax^5}{5} - \frac{g^4B\ln(dx+c)a^5}{5b} + \frac{(bx+a)^5g^4B\ln\left(\frac{e(dx+c)}{bx+a}\right)}{5b} - \frac{2g^4bB\ln(dx+c)a^3c^2}{d^2} + \frac{2g^4b^2B\ln(dx+c)a^2c^3}{d^3} - g^4$
derivativedivides	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*g*x+a*g)^4*(A+B*ln(e*(d*x+c)/(b*x+a))),x,method=_RETURNVERBOSE)

[Out] 1/b^2*e*(a*d-b*c)*(2/5*B*b^2*e*g^4/((d*e/b-e*(a*d-b*c)/b/(b*x+a))*b-e*d)^2*a^3*c-4/15*B*b^4*e^2*g^4/d/((d*e/b-e*(a*d-b*c)/b/(b*x+a))*b-e*d)^3*a*c^3+1/5*B*b^4*e^3*g^4/((d*e/b-e*(a*d-b*c)/b/(b*x+a))*b-e*d)^4*a*c^3-4/5*B*b^4*g^4/d^3/((d*e/b-e*(a*d-b*c)/b/(b*x+a))*b-e*d)*a*c^3+2/5*B*b^3*e^2*g^4/((d*e/b-e*(a*d-b*c)/b/(b*x+a))*b-e*d)^3*a^2*c^2-4/5*B*b^2*g^4/d/((d*e/b-e*(a*d-b*c)/b/(b*x+a))*b-e*d)*a^3*c-1/10*B*b*e*g^4*d/((d*e/b-e*(a*d-b*c)/b/(b*x+a))*b-e*d)^2*a^4-4/5*B*b^4/e*g^4/d^4*ln((d*e/b-e*(a*d-b*c)/b/(b*x+a))*b-e*d)*a*c^3+B*b^9*g^4*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))*(d*e/b-e*(a*d-b*c)/b/(b*x+a))^4/d^4/((d*e/b-e*(a*d-b*c)/b/(b*x+a))*b-e*d)^5*c^4-4/15*B*b^2*e^2*g^4*d/((d*e/b-e*(a*d-b*c)/b/(b*x+a))*b-e*d)^3*a^3*c-3/10*B*b^3*e^3*g^4*d/((d*e/b-e*(a*d-b*c)/b/(b*x+a))*b-e*d)^4*a^2*c^2+6/5*B*b^3/e*g^4/d^3*ln((d*e/b-e*(a*d-b*c)/b/(b*x+a))*b-e*d)*a^2*c^2-4/5*B*b^2/e*g^4/d^2*ln((d*e/b-e*(a*d-b*c)/b/(b*x+a))*b-e*d)*a^3*c+2/5*B*b^4*e*g^4/d^2/((d*e/b-e*(a*d-b*c)/b/(b*x+a))*b-e*d)^2*a*c^3+1/5*B*b^2*e^3*g^4*d^2/((d*e/b-e*(a*d-b*c)/b/(b*x+a))*b-e*d)^4*a^3*c-3/5*B*b^3*e*g^4/d/((d*e/b-e*(a*d-b*c)/b/(b*x+a))*b-e*d)^2*a^2*c^2+2*B*b^7*e^2*g^4*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))*(d*e/b-e*(a*d-b*c)/b/(b*x+a))^2/d^2/((d*e/b-e*(a*d-b*c)/b/(b*x+a))*b-e*d)^5*c^4-1/10*B*b^5*e*g^4/d^3/((d*e/b-e*(a*d-b*c)/b/(b*x+a))*b-e*d)^2*c^4+B*b^5*g^4*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))*(d*e/b-e*(a*d-b*c)/b/(b*x+a))^4/((d*e/b-e*(a*d-b*c)/b/(b*x+a))*b-e*d)^5*a^4+1/5*B*b/e*g^4/d*ln((d*e/b-e*(a*d-b*c)/b/(b*x+a))*b-e*d)*a^4-1/20*B*b^5*e^3*g^4/d/((d*e/b-e*(a*d-b*c)/b/(b*x+a))*b-e*d)^4*c^4+1/5*B*b^5/e*g^4/d^5*ln((d*e/b-e*(a*d-b*c)/b/(b*x+a))*b-e*d)*c^4-4*B*b^6*g^4*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))*(d*e/b-e*(a*d-b*c)/b/(b*x+a))^4/d/((d*e/b-e*(a*d-b*c)/b/(b*x+a))*b-e*d)^5*a^3*c-4*B*b^8*g^4*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))*(d*e/b-e*(a*d-b*c)/b/(b*x+a))^4/d^3/((d*e/b-e*(a*d-b*c)/b/(b*x+a))*b-e*d)^5*a*c^3-2*B*b^4*e*g^4*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))*(d*e/b-e*(a*d-b*c)/b/(b*x+a))^3*d/((d*e/b-e*(a*d-b*c)/b/(b*x+a))*b-e*d)^5*a^4+4*B*b^5*e^3*g^4*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))*(d*e/b-e*(a*d-b*c)/b/(b*x+a))/((d*e/b-e*(a*d-b*c)/b/(b*x+a))*b-e*d)^5*a*c^3-1/20*B*b*e^3*g^4*d^3/((d*e/b-e*(a*d-b*c)/b/(b*x+a))*b-e*d)^4*a^4+1/15*B*b*e^2*g^4*d^2/((d*e/b-e*(a*d-b*c)/b/(b*x+a))*b-e*d)^3*a^4+1/15*B*b^5*e^2*g^4/d^2/((d*e/b-e*(a*d-b*c)/b/(b*x+a))*b-e*d)^3*c^4+6/5*B*b^3*g^4/d^2/((d*e/b-e*(a*d-b*c)/b/(b*x+a))*b-e*d)*a^2*c^2+1/5*B*b*g^4/((d*e/b-e*(a*d-b*c)/b/(b*x+a))*b-e*d)*a^4-2*B*b^8*e*g^4*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))*(d*e/b-e*(a*d-b*c)/b/(b*x+a))^3/d^3/((d*e/b-e*(a*d-b*c)/b/(b*x+a))*b-e*d)^5*c^4-6/5*B*b^8/e*g^4*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))*(d*e/b-e*(a*d-b*c)/b/(b*x+a))^5/d^3/((d*e/b-e*(a*d-b*c)/b/(b*x+a))*b-e*d)^5*a^2*c^2+2*B*b^3*e^2*g^4*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))*(d*e/b-e*(a*d-b*c)/b/(b*x+a))^2*d^2/((d*e/b-e*(a*d-b*c)/b/(b*x+a))*b-e*d)^5*a^4+6*B*b^7*g^4*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))*(d*e/b-e*(a*d-b*c)/b/(b*x+a))^4/d^2/((d*e/b-e*(a*d-b*c)/b/(b*x+a))*b-e*d)^5*a^2*c^2-B*b^2*e^3*g^4*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))*(d*e/b-e

```
*(a*d-b*c)/b/(b*x+a))*d^3/((d*e/b-e*(a*d-b*c)/b/(b*x+a))*b-e*d)^5*a^4+4/5*B
*b^7/e*g^4*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))*(d*e/b-e*(a*d-b*c)/b/(b*x+a))^5/
d^2/((d*e/b-e*(a*d-b*c)/b/(b*x+a))*b-e*d)^5*a^3*c-6*B*b^4*e^3*g^4*ln(d*e/b-
e*(a*d-b*c)/b/(b*x+a))*(d*e/b-e*(a*d-b*c)/b/(b*x+a))*d/((d*e/b-e*(a*d-b*c)/
b/(b*x+a))*b-e*d)^5*a^2*c^2-8*B*b^6*e^2*g^4*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))
*(d*e/b-e*(a*d-b*c)/b/(b*x+a))^2/d/((d*e/b-e*(a*d-b*c)/b/(b*x+a))*b-e*d)^5*
a*c^3+4*B*b^3*e^3*g^4*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))*(d*e/b-e*(a*d-b*c)/b/
(b*x+a))*d^2/((d*e/b-e*(a*d-b*c)/b/(b*x+a))*b-e*d)^5*a^3*c-12*B*b^6*e*g^4*l
n(d*e/b-e*(a*d-b*c)/b/(b*x+a))*(d*e/b-e*(a*d-b*c)/b/(b*x+a))^3/d/((d*e/b-e*
(a*d-b*c)/b/(b*x+a))*b-e*d)^5*a^2*c^2+8*B*b^7*e*g^4*ln(d*e/b-e*(a*d-b*c)/b/
(b*x+a))*(d*e/b-e*(a*d-b*c)/b/(b*x+a))^3/d^2/((d*e/b-e*(a*d-b*c)/b/(b*x+a))
*b-e*d)^5*a*c^3-8*B*b^4*e^2*g^4*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))*(d*e/b-e*(a
*d-b*c)/b/(b*x+a))^2*d/((d*e/b-e*(a*d-b*c)/b/(b*x+a))*b-e*d)^5*a^3*c+4/5*B*
b^9/e*g^4*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))*(d*e/b-e*(a*d-b*c)/b/(b*x+a))^5/d
^4/((d*e/b-e*(a*d-b*c)/b/(b*x+a))*b-e*d)^5*a*c^3+12*B*b^5*e^2*g^4*ln(d*e/b-
e*(a*d-b*c)/b/(b*x+a))*(d*e/b-e*(a*d-b*c)/b/(b*x+a))^2/((d*e/b-e*(a*d-b*c)/
b/(b*x+a))*b-e*d)^5*a^2*c^2+8*B*b^5*e*g^4*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))*(
d*e/b-e*(a*d-b*c)/b/(b*x+a))^3/((d*e/b-e*(a*d-b*c)/b/(b*x+a))*b-e*d)^5*a^3*
c-B*b^6*e^3*g^4*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))*(d*e/b-e*(a*d-b*c)/b/(b*x+a
))/d/((d*e/b-e*(a*d-b*c)/b/(b*x+a))*b-e*d)^5*c^4-1/5*B*b^6/e*g^4*ln(d*e/b-e
*(a*d-b*c)/b/(b*x+a))*(d*e/b-e*(a*d-b*c)/b/(b*x+a))^5/d/((d*e/b-e*(a*d-b*c)
/b/(b*x+a))*b-e*d)^5*a^4-1/5*B*b^10/e*g^4*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))*(
d*e/b-e*(a*d-b*c)/b/(b*x+a))^5/d^5/((d*e/b-e*(a*d-b*c)/b/(b*x+a))*b-e*d)^5*
c^4-1/5*A*b*e^4*g^4*(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+
b^4*c^4)/((d*e/b-e*(a*d-b*c)/b/(b*x+a))*b-e*d)^5+1/5*B*b^5*g^4/d^4/((d*e/b-
e*(a*d-b*c)/b/(b*x+a))*b-e*d)*c^4)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 629 vs. $2(169) = 338$.
time = 0.29, size = 629, normalized size = 3.49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^4*(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="maxima")
```

```
[Out] 1/5*A*b^4*g^4*x^5 + A*a*b^3*g^4*x^4 + 2*A*a^2*b^2*g^4*x^3 + 2*A*a^3*b*g^4*x
^2 + (x*log(d*x*e/(b*x + a) + c*e/(b*x + a)) - a*log(b*x + a)/b + c*log(d*x
 + c)/d)*B*a^4*g^4 + 2*(x^2*log(d*x*e/(b*x + a) + c*e/(b*x + a)) + a^2*log(
b*x + a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d))*B*a^3*b*g^4 + (2
*x^3*log(d*x*e/(b*x + a) + c*e/(b*x + a)) - 2*a^3*log(b*x + a)/b^3 + 2*c^3*
log(d*x + c)/d^3 + ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2
*d^2))*B*a^2*b^2*g^4 + 1/6*(6*x^4*log(d*x*e/(b*x + a) + c*e/(b*x + a)) + 6*
a^4*log(b*x + a)/b^4 - 6*c^4*log(d*x + c)/d^4 + (2*(b^3*c*d^2 - a*b^2*d^3)*
x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3))*B
```

```
*a*b^3*g^4 + 1/60*(12*x^5*log(d*x*e/(b*x + a) + c*e/(b*x + a)) - 12*a^5*log
(b*x + a)/b^5 + 12*c^5*log(d*x + c)/d^5 + (3*(b^4*c*d^3 - a*b^3*d^4)*x^4 -
4*(b^4*c^2*d^2 - a^2*b^2*d^4)*x^3 + 6*(b^4*c^3*d - a^3*b*d^4)*x^2 - 12*(b^4
*c^4 - a^4*d^4)*x)/(b^4*d^4))*B*b^4*g^4 + A*a^4*g^4*x
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 432 vs. $2(169) = 338$.
time = 0.43, size = 432, normalized size = 2.40

$$\frac{12 Ab^5d^5g^4x^5 - 12 Ba^5d^5g^4\log(bx+a) + 3\left(Bb^5cd^4 + (20A-B)ab^4d^5\right)g^4x^4 - 4\left(Bb^5c^2d^3 - 5Bab^4cd^4 - 2(15A-2B)a^2b^3d^5\right)g^4x^3 + 6\left(Bb^5c^3d^2 - 5Bab^4c^2d^3 + 10Ba^2b^3cd^4 + 2(10A-3B)a^3b^2d^5\right)g^4x^2 - 12\left(Bb^5c^4d - 5Bab^4c^3d^2 + 10Ba^2b^3c^2d^3 - 10Ba^3b^2cd^4 - (5A-4B)a^4bd^5\right)g^4x + 12\left(Bb^5c^5 - 5Bab^4c^4d + 10Ba^2b^3c^3d^2 - 10Ba^3b^2c^2d^3 + 5Ba^4bcd^4\right)g^4\log(dx+c) + 12\left(Bb^5d^5g^4x^5 + 5Bab^4d^5g^4x^4 + 10Ba^2b^3d^5g^4x^3 + 10Ba^3b^2d^5g^4x^2 + 5Ba^4bd^5g^4x\right)\log\left(\frac{(dx+c)e}{bx+a}\right)}{60\,bd^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^4*(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="fricas")
```

```
[Out] 1/60*(12*A*b^5*d^5*g^4*x^5 - 12*B*a^5*d^5*g^4*log(b*x + a) + 3*(B*b^5*c*d^4
 + (20*A - B)*a*b^4*d^5)*g^4*x^4 - 4*(B*b^5*c^2*d^3 - 5*B*a*b^4*c*d^4 - 2*(
15*A - 2*B)*a^2*b^3*d^5)*g^4*x^3 + 6*(B*b^5*c^3*d^2 - 5*B*a*b^4*c^2*d^3 + 1
0*B*a^2*b^3*c*d^4 + 2*(10*A - 3*B)*a^3*b^2*d^5)*g^4*x^2 - 12*(B*b^5*c^4*d -
 5*B*a*b^4*c^3*d^2 + 10*B*a^2*b^3*c^2*d^3 - 10*B*a^3*b^2*c*d^4 - (5*A - 4*B
)*a^4*b*d^5)*g^4*x + 12*(B*b^5*c^5 - 5*B*a*b^4*c^4*d + 10*B*a^2*b^3*c^3*d^2
 - 10*B*a^3*b^2*c^2*d^3 + 5*B*a^4*b*c*d^4)*g^4*log(d*x + c) + 12*(B*b^5*d^5
*g^4*x^5 + 5*B*a*b^4*d^5*g^4*x^4 + 10*B*a^2*b^3*d^5*g^4*x^3 + 10*B*a^3*b^2*
d^5*g^4*x^2 + 5*B*a^4*b*d^5*g^4*x)*log((d*x + c)*e/(b*x + a)))/(b*d^5)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 969 vs. $2(155) = 310$.
time = 4.38, size = 969, normalized size = 5.38

$$\frac{Ab^4g^4x^5}{5} - \frac{Ba^5g^4\log\left(x + \frac{\frac{Ba^6d^5g^4}{b} + 5Ba^5cd^4g^4 - 10Ba^4bc^2d^3g^4 + 10Ba^3b^2c^3d^2g^4 - 5Ba^2b^3c^4dg^4 + Bab^4c^5g^4}{Ba^5d^5g^4 + 5Ba^4bcd^4g^4 - 10Ba^3b^2c^2d^3g^4 + 10Ba^2b^3c^3d^2g^4 - 5Bab^4c^4dg^4 + Bb^5c^5g^4}\right)}{5b} + \frac{Bcg^4\cdot\left(5a^4d^4 - 10a^3bcd^3 + 10a^2b^2c^2d^2 - 5ab^3c^3d + b^4c^4\right)\log\left(x + \frac{6Ba^5cd^4g^4 - 10Ba^4bc^2d^3g^4 + 10Ba^3b^2c^3d^2g^4 - 5Ba^2b^3c^4dg^4 + Bab^4c^5g^4 - Bacg^4\cdot\left(5a^4d^4 - 10a^3bcd^3 + 10a^2b^2c^2d^2 - 5ab^3c^3d + b^4c^4\right) + \frac{Bbc^2g^4\cdot\left(5a^4d^4 - 10a^3bcd^3 + 10a^2b^2c^2d^2 - 5ab^3c^3d + b^4c^4\right)}{d}}{Ba^5d^5g^4 + 5Ba^4bcd^4g^4 - 10Ba^3b^2c^2d^3g^4 + 10Ba^2b^3c^3d^2g^4 - 5Bab^4c^4dg^4 + Bb^5c^5g^4}\right)}{5d^5} + x^4\left(Aab^3g^4 - \frac{Bab^3g^4}{20} + \frac{Bb^4cg^4}{20d}\right) + x^3\cdot\left(2Aa^2b^2g^4 - \frac{4Ba^2b^2g^4}{15} + \frac{Bab^3cg^4}{3d} - \frac{Bb^4c^2g^4}{15d^2}\right) + x^2\cdot\left(2Aa^3bg^4 - \frac{3Ba^3bg^4}{5} + \frac{Ba^2b^2cg^4}{d} - \frac{Bab^3c^2g^4}{2d^2} + \frac{Bb^4c^3g^4}{10d^3}\right) + x\left(Aa^4g^4 - \frac{4Ba^4g^4}{5} + \frac{2Ba^3bcg^4}{d} - \frac{2Ba^2b^2c^2g^4}{d^2} + \frac{Bab^3c^3g^4}{d^3} - \frac{Bb^4c^4g^4}{5d^4}\right) + \left(Ba^4g^4x + 2Ba^3bg^4x^2 + 2Ba^2b^2g^4x^3 + Bab^3g^4x^4 + \frac{Bb^4g^4x^5}{5}\right)\log\left(\frac{e(c+dx)}{a+bx}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)**4*(A+B*ln(e*(d*x+c)/(b*x+a))),x)
```

```
[Out] A*b**4*g**4*x**5/5 - B*a**5*g**4*log(x + (B*a**6*d**5*g**4/b + 5*B*a**5*c*d
**4*g**4 - 10*B*a**4*b*c**2*d**3*g**4 + 10*B*a**3*b**2*c**3*d**2*g**4 - 5*B
*a**2*b**3*c**4*d*g**4 + B*a*b**4*c**5*g**4)/(B*a**5*d**5*g**4 + 5*B*a**4*b
*c*d**4*g**4 - 10*B*a**3*b**2*c**2*d**3*g**4 + 10*B*a**2*b**3*c**3*d**2*g**
4 - 5*B*a*b**4*c**4*d*g**4 + B*b**5*c**5*g**4))/(5*b) + B*c*g**4*(5*a**4*d*
*4 - 10*a**3*b*c*d**3 + 10*a**2*b**2*c**2*d**2 - 5*a*b**3*c**3*d + b**4*c**
4)*log(x + (6*B*a**5*c*d**4*g**4 - 10*B*a**4*b*c**2*d**3*g**4 + 10*B*a**3*b
**2*c**3*d**2*g**4 - 5*B*a**2*b**3*c**4*d*g**4 + B*a*b**4*c**5*g**4 - B*a*c
*g**4*(5*a**4*d**4 - 10*a**3*b*c*d**3 + 10*a**2*b**2*c**2*d**2 - 5*a*b**3*c
**3*d + b**4*c**4) + B*b*c**2*g**4*(5*a**4*d**4 - 10*a**3*b*c*d**3 + 10*a**
2*b**2*c**2*d**2 - 5*a*b**3*c**3*d + b**4*c**4)/d)/(B*a**5*d**5*g**4 + 5*B*
a**4*b*c*d**4*g**4 - 10*B*a**3*b**2*c**2*d**3*g**4 + 10*B*a**2*b**3*c**3*d*
*2*g**4 - 5*B*a*b**4*c**4*d*g**4 + B*b**5*c**5*g**4))/(5*d**5) + x**4*(A*a*
```

```
b**3*g**4 - B*a*b**3*g**4/20 + B*b**4*c*g**4/(20*d)) + x**3*(2*A*a**2*b**2*
g**4 - 4*B*a**2*b**2*g**4/15 + B*a*b**3*c*g**4/(3*d) - B*b**4*c**2*g**4/(15
*d**2)) + x**2*(2*A*a**3*b*g**4 - 3*B*a**3*b*g**4/5 + B*a**2*b**2*c*g**4/d
- B*a*b**3*c**2*g**4/(2*d**2) + B*b**4*c**3*g**4/(10*d**3)) + x*(A*a**4*g**
4 - 4*B*a**4*g**4/5 + 2*B*a**3*b*c*g**4/d - 2*B*a**2*b**2*c**2*g**4/d**2 +
B*a*b**3*c**3*g**4/d**3 - B*b**4*c**4*g**4/(5*d**4)) + (B*a**4*g**4*x + 2*B
*a**3*b*g**4*x**2 + 2*B*a**2*b**2*g**4*x**3 + B*a*b**3*g**4*x**4 + B*b**4*g
**4*x**5/5)*log(e*(c + d*x)/(a + b*x))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 5960 vs. $2(169) = 338$.
time = 3.41, size = 5960, normalized size = 33.11

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^4*(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="giac")
```

```
[Out] -1/60*(12*B*b^6*c^6*d^5*g^4*e^6*log(-d*e + (d*x*e + c*e)*b/(b*x + a)) - 72*
B*a*b^5*c^5*d^6*g^4*e^6*log(-d*e + (d*x*e + c*e)*b/(b*x + a)) + 180*B*a^2*b
^4*c^4*d^7*g^4*e^6*log(-d*e + (d*x*e + c*e)*b/(b*x + a)) - 240*B*a^3*b^3*c^
3*d^8*g^4*e^6*log(-d*e + (d*x*e + c*e)*b/(b*x + a)) + 180*B*a^4*b^2*c^2*d^9
*g^4*e^6*log(-d*e + (d*x*e + c*e)*b/(b*x + a)) - 72*B*a^5*b*c*d^10*g^4*e^6*
log(-d*e + (d*x*e + c*e)*b/(b*x + a)) + 12*B*a^6*d^11*g^4*e^6*log(-d*e + (d
*x*e + c*e)*b/(b*x + a)) - 60*(d*x*e + c*e)*B*b^7*c^6*d^4*g^4*e^5*log(-d*e
+ (d*x*e + c*e)*b/(b*x + a))/(b*x + a) + 360*(d*x*e + c*e)*B*a*b^6*c^5*d^5*
g^4*e^5*log(-d*e + (d*x*e + c*e)*b/(b*x + a))/(b*x + a) - 900*(d*x*e + c*e)
*B*a^2*b^5*c^4*d^6*g^4*e^5*log(-d*e + (d*x*e + c*e)*b/(b*x + a))/(b*x + a)
+ 1200*(d*x*e + c*e)*B*a^3*b^4*c^3*d^7*g^4*e^5*log(-d*e + (d*x*e + c*e)*b/(
b*x + a))/(b*x + a) - 900*(d*x*e + c*e)*B*a^4*b^3*c^2*d^8*g^4*e^5*log(-d*e
+ (d*x*e + c*e)*b/(b*x + a))/(b*x + a) + 360*(d*x*e + c*e)*B*a^5*b^2*c*d^9*
g^4*e^5*log(-d*e + (d*x*e + c*e)*b/(b*x + a))/(b*x + a) - 60*(d*x*e + c*e)*
B*a^6*b*d^10*g^4*e^5*log(-d*e + (d*x*e + c*e)*b/(b*x + a))/(b*x + a) + 120*
(d*x*e + c*e)^2*B*b^8*c^6*d^3*g^4*e^4*log(-d*e + (d*x*e + c*e)*b/(b*x + a))
/(b*x + a)^2 - 720*(d*x*e + c*e)^2*B*a*b^7*c^5*d^4*g^4*e^4*log(-d*e + (d*x*
e + c*e)*b/(b*x + a))/(b*x + a)^2 + 1800*(d*x*e + c*e)^2*B*a^2*b^6*c^4*d^5*
g^4*e^4*log(-d*e + (d*x*e + c*e)*b/(b*x + a))/(b*x + a)^2 - 2400*(d*x*e + c
*e)^2*B*a^3*b^5*c^3*d^6*g^4*e^4*log(-d*e + (d*x*e + c*e)*b/(b*x + a))/(b*x
+ a)^2 + 1800*(d*x*e + c*e)^2*B*a^4*b^4*c^2*d^7*g^4*e^4*log(-d*e + (d*x*e +
 c*e)*b/(b*x + a))/(b*x + a)^2 - 720*(d*x*e + c*e)^2*B*a^5*b^3*c*d^8*g^4*e^
4*log(-d*e + (d*x*e + c*e)*b/(b*x + a))/(b*x + a)^2 + 120*(d*x*e + c*e)^2*B
*a^6*b^2*d^9*g^4*e^4*log(-d*e + (d*x*e + c*e)*b/(b*x + a))/(b*x + a)^2 - 12
0*(d*x*e + c*e)^3*B*b^9*c^6*d^2*g^4*e^3*log(-d*e + (d*x*e + c*e)*b/(b*x + a
))/(b*x + a)^3 + 720*(d*x*e + c*e)^3*B*a*b^8*c^5*d^3*g^4*e^3*log(-d*e + (d*
x*e + c*e)*b/(b*x + a))/(b*x + a)^3 - 1800*(d*x*e + c*e)^3*B*a^2*b^7*c^4*d^
```

```
4*g^4*e^3*log(-d*e + (d*x*e + c*e)*b/(b*x + a))/(b*x + a)^3 + 2400*(d*x*e +
 c*e)^3*B*a^3*b^6*c^3*d^5*g^4*e^3*log(-d*e + (d*x*e + c*e)*b/(b*x + a))/(b*
x + a)^3 - 1800*(d*x*e + c*e)^3*B*a^4*b^5*c^2*d^6*g^4*e^3*log(-d*e + (d*x*e
 + c*e)*b/(b*x + a))/(b*x + a)^3 + 720*(d*x*e + c*e)^3*B*a^5*b^4*c*d^7*g^4*
e^3*log(-d*e + (d*x*e + c*e)*b/(b*x + a))/(b*x + a)^3 - 120*(d*x*e + c*e)^3
*B*a^6*b^3*d^8*g^4*e^3*log(-d*e + (d*x*e + c*e)*b/(b*x + a))/(b*x + a)^3 +
60*(d*x*e + c*e)^4*B*b^10*c^6*d*g^4*e^2*log(-d*e + (d*x*e + c*e)*b/(b*x + a
))/(b*x + a)^4 - 360*(d*x*e + c*e)^4*B*a*b^9*c^5*d^2*g^4*e^2*log(-d*e + (d*
x*e + c*e)*b/(b*x + a))/(b*x + a)^4 + 900*(d*x*e + c*e)^4*B*a^2*b^8*c^4*d^3
*g^4*e^2*log(-d*e + (d*x*e + c*e)*b/(b*x + a))/(b*x + a)^4 - 1200*(d*x*e +
c*e)^4*B*a^3*b^7*c^3*d^4*g^4*e^2*log(-d*e + (d*x*e + c*e)*b/(b*x + a))/(b*x
 + a)^4 + 900*(d*x*e + c*e)^4*B*a^4*b^6*c^2*d^5*g^4*e^2*log(-d*e + (d*x*e +
 c*e)*b/(b*x + a))/(b*x + a)^4 - 360*(d*x*e + c*e)^4*B*a^5*b^5*c*d^6*g^4*e^
2*log(-d*e + (d*x*e + c*e)*b/(b*x + a))/(b*x + a)^4 + 60*(d*x*e + c*e)^4*B*
a^6*b^4*d^7*g^4*e^2*log(-d*e + (d*x*e + c*e)*b/(b*x + a))/(b*x + a)^4 - 12*
(d*x*e + c*e)^5*B*b^11*c^6*g^4*e*log(-d*e + (d*x*e + c*e)*b/(b*x + a))/(b*x
 + a)^5 + 72*(d*x*e + c*e)^5*B*a*b^10*c^5*d*g^4*e*log(-d*e + (d*x*e + c*e)*
b/(b*x + a))/(b*x + a)^5 - 180*(d*x*e + c*e)^5*B*a^2*b^9*c^4*d^2*g^4*e*log(
-d*e + (d*x*e + c*e)*b/(b*x + a))/(b*x + a)^5 + 240*(d*x*e + c*e)^5*B*a^3*b
^8*c^3*d^3*g^4*e*log(-d*e + (d*x*e + c*e)*b/(b*x + a))/(b*x + a)^5 - 180*(d
*x*e + c*e)^5*B*a^4*b^7*c^2*d^4*g^4*e*log(-d*e + (d*x*e + c*e)*b/(b*x + a))
/(b*x + a)^5 + 72*(d*x*e + c*e)^5*B*a^5*b^6*c*d^5*g^4*e*log(-d*e + (d*x*e +
 c*e)*b/(b*x + a))/(b*x + a)^5 - 12*(d*x*e + c*e)^5*B*a^6*b^5*d^6*g^4*e*log
(-d*e + (d*x*e + c*e)*b/(b*x + a))/(b*x + a)^5 + 60*(d*x*e + c*e)*B*b^7*c^6
*d^4*g^4*e^5*log((d*x*e + c*e)/(b*x + a))/(b*x + a) - 360*(d*x*e + c*e)*B*a
*b^6*c^5*d^5*g^4*e^5*log((d*x*e + c*e)/(b*x + a))/(b*x + a) + 900*(d*x*e +
c*e)*B*a^2*b^5*c^4*d^6*g^4*e^5*log((d*x*e + c*e)/(b*x + a))/(b*x + a) - 120
0*(d*x*e + c*e)*B*a^3*b^4*c^3*d^7*g^4*e^5*log((d*x*e + c*e)/(b*x + a))/(b*x
 + a) + 900*(d*x*e + c*e)*B*a^4*b^3*c^2*d^8*g^4*e^5*log((d*x*e + c*e)/(b*x
+ a))/(b*x + a) - 360*(d*x*e + c*e)*B*a^5*b^2*c*d^9*g^4*e^5*log((d*x*e + c*
e)/(b*x + a))/(b*x + a) + 60*(d*x*e + c*e)*B*a^6*b*d^10*g^4*e^5*log((d*x*e
+ c*e)/(b*x + a))/(b*x + a) - 120*(d*x*e + c*e)^2*B*b^8*c^6*d^3*g^4*e^4*log
((d*x*e + c*e)/(b*x + a))/(b*x + a)^2 + 720*(d*x*e + c*e)^2*B*a*b^7*c^5*d^4
*g^4*e^4*log((d*x*e + c*e)/(b*x + a))/(b*x + a)^2 - 1800*(d*x*e + c*e)^2*B*
a^2*b^6*c^4*d^5*g^4*e^4*log((d*x*e + c*e)/(b*x + a))/(b*x + a)^2 + 2400*(d*
x*e + c*e)^2*B*a^3*b^5*c^3*d^6*g^4*e^4*log((d*x*e + c*e)/(b*x + a))/(b*x +
a)^2 - 1800*(d*x*e + c*e)^2*B*a^4*b^4*c^2*d^7*g^4*e^4*log((d*x*e + c*e)/(b*
x + a))/(b*x + a)^2 + 720*(d*x*e + c*e)^2*B*a^5*b^3*c*d^8*g^4*e^4*log((d*x*
e + c*e)/(b*x + a))/(b*x + a)^2 - 120*(d*x*e + c*e)^2*B*a^6*b^2*d^9*g^4*e^4
*log((d*x*e + c*e)/(b*x + a))/(b*x + a)^2 + 120...
```

Mupad [B]
time = 4.86, size = 1008, normalized size = 5.60

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*g + b*g*x)^4*(A + B*log((e*(c + d*x))/(a + b*x))),x)
```

```
[Out] log((e*(c + d*x))/(a + b*x))*((B*b^4*g^4*x^5)/5 + B*a^4*g^4*x + 2*B*a^3*b*g
^4*x^2 + B*a*b^3*g^4*x^4 + 2*B*a^2*b^2*g^4*x^3) - x^3*((((b^3*g^4*(25*A*a*d
 + 5*A*b*c - B*a*d + B*b*c))/(5*d) - (A*b^3*g^4*(5*a*d + 5*b*c))/(5*d))*(5*
a*d + 5*b*c))/(15*b*d) - (a*b^2*g^4*(10*A*a*d + 5*A*b*c - B*a*d + B*b*c))/(
3*d) + (A*a*b^3*c*g^4)/(3*d)) + x^2*(((5*a*d + 5*b*c)*((((b^3*g^4*(25*A*a*d
 + 5*A*b*c - B*a*d + B*b*c))/(5*d) - (A*b^3*g^4*(5*a*d + 5*b*c))/(5*d))*(5*
a*d + 5*b*c))/(5*b*d) - (a*b^2*g^4*(10*A*a*d + 5*A*b*c - B*a*d + B*b*c))/d
+ (A*a*b^3*c*g^4)/d))/(10*b*d) + (a^2*b*g^4*(5*A*a*d + 5*A*b*c - B*a*d + B*
b*c))/d - (a*c*((b^3*g^4*(25*A*a*d + 5*A*b*c - B*a*d + B*b*c))/(5*d) - (A*b
^3*g^4*(5*a*d + 5*b*c))/(5*d)))/(2*b*d)) + x*((a^3*g^4*(5*A*a*d + 10*A*b*c
- 2*B*a*d + 2*B*b*c))/d - ((5*a*d + 5*b*c)*(((5*a*d + 5*b*c)*((((b^3*g^4*(2
5*A*a*d + 5*A*b*c - B*a*d + B*b*c))/(5*d) - (A*b^3*g^4*(5*a*d + 5*b*c))/(5*
d))*(5*a*d + 5*b*c))/(5*b*d) - (a*b^2*g^4*(10*A*a*d + 5*A*b*c - B*a*d + B*b
*c))/d + (A*a*b^3*c*g^4)/d))/(5*b*d) + (2*a^2*b*g^4*(5*A*a*d + 5*A*b*c - B*
a*d + B*b*c))/d - (a*c*((b^3*g^4*(25*A*a*d + 5*A*b*c - B*a*d + B*b*c))/(5*d
) - (A*b^3*g^4*(5*a*d + 5*b*c))/(5*d)))/(b*d)))/(5*b*d) + (a*c*((((b^3*g^4*
(25*A*a*d + 5*A*b*c - B*a*d + B*b*c))/(5*d) - (A*b^3*g^4*(5*a*d + 5*b*c))/(
5*d))*(5*a*d + 5*b*c))/(5*b*d) - (a*b^2*g^4*(10*A*a*d + 5*A*b*c - B*a*d + B
*b*c))/d + (A*a*b^3*c*g^4)/d))/(b*d)) + x^4*((b^3*g^4*(25*A*a*d + 5*A*b*c -
 B*a*d + B*b*c))/(20*d) - (A*b^3*g^4*(5*a*d + 5*b*c))/(20*d)) + (log(c + d*
x)*((B*b^4*c^5*g^4)/5 + B*a^4*c*d^4*g^4 - 2*B*a^3*b*c^2*d^3*g^4 + 2*B*a^2*b
^2*c^3*d^2*g^4 - B*a*b^3*c^4*d*g^4))/d^5 + (A*b^4*g^4*x^5)/5 - (B*a^5*g^4*l
og(a + b*x))/(5*b)
```

3.174 $\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right) dx$

Optimal. Leaf size=149

$$\frac{B(bc-ad)^3g^3x}{4d^3}-\frac{B(bc-ad)^2g^3(a+bx)^2}{8bd^2}+\frac{B(bc-ad)g^3(a+bx)^3}{12bd}-\frac{B(bc-ad)^4g^3\log(c+dx)}{4bd^4}+\frac{g^3(a+bx)^4}{}$$

```
[Out] 1/4*B*(-a*d+b*c)^3*g^3*x/d^3-1/8*B*(-a*d+b*c)^2*g^3*(b*x+a)^2/b/d^2+1/12*B*
(-a*d+b*c)*g^3*(b*x+a)^3/b/d-1/4*B*(-a*d+b*c)^4*g^3*ln(d*x+c)/b/d^4+1/4*g^3
*(b*x+a)^4*(A+B*ln(e*(d*x+c)/(b*x+a)))/b
```

Rubi [A]

time = 0.07, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2548, 21, 45}

$$\frac{g^3(a+bx)^4\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{4b}-\frac{Bg^3(bc-ad)^4\log(c+dx)}{4bd^4}+\frac{Bg^3x(bc-ad)^3}{4d^3}-\frac{Bg^3(a+bx)^2(bc-ad)^2}{8bd^2}+\frac{Bg^3(a+bx)^3(bc-ad)}{12bd}$$

Antiderivative was successfully verified.

```
[In] Int[(a*g + b*g*x)^3*(A + B*Log[(e*(c + d*x))/(a + b*x)]),x]
```

```
[Out] (B*(b*c - a*d)^3*g^3*x)/(4*d^3) - (B*(b*c - a*d)^2*g^3*(a + b*x)^2)/(8*b*d^
2) + (B*(b*c - a*d)*g^3*(a + b*x)^3)/(12*b*d) - (B*(b*c - a*d)^4*g^3*Log[c
+ d*x])/(4*b*d^4) + (g^3*(a + b*x)^4*(A + B*Log[(e*(c + d*x))/(a + b*x)]))/
(4*b)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2548

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Dist[B*n*((b*c
```

```
- a*d)/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /;
 FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c -
 a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Rubi steps

$$\int (ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)dx = \frac{g^3(a+bx)^4\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{4b} - \frac{B\int\frac{(-bc+ad)g^4(a+bx)^3}{c+dx}}{4bg}$$

$$= \frac{g^3(a+bx)^4\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{4b} + \frac{(B(bc-ad)g^3)\int\frac{(a+}{c+}}{4b}$$

$$= \frac{g^3(a+bx)^4\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{4b} + \frac{(B(bc-ad)g^3)\int\left(\frac{b(}{}\right.}{}$$

$$= \frac{B(bc-ad)^3g^3x}{4d^3} - \frac{B(bc-ad)^2g^3(a+bx)^2}{8bd^2} + \frac{B(bc-ad)g}{12b}$$

Mathematica [A]
time = 0.06, size = 120, normalized size = 0.81

$$\frac{g^3\left(\frac{B(bc-ad)\left(6bd(bc-ad)^2x+3d^2(-bc+ad)(a+bx)^2+2d^3(a+bx)^3-6(bc-ad)^3\log(c+dx)\right)}{6d^4}+(a+bx)^4\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*g + b*g*x)^3*(A + B*Log[(e*(c + d*x))/(a + b*x)]),x]
[Out] (g^3*((B*(b*c - a*d)*(6*b*d*(b*c - a*d)^2*x + 3*d^2*(-(b*c) + a*d)*(a + b*x
)^2 + 2*d^3*(a + b*x)^3 - 6*(b*c - a*d)^3*Log[c + d*x]))/(6*d^4) + (a + b*x
)^4*(A + B*Log[(e*(c + d*x))/(a + b*x)])))/(4*b)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2729 vs. 2(139) = 278.
time = 0.44, size = 2730, normalized size = 18.32

method	result
risch	$\frac{g^3(bx+a)^4B\ln\left(\frac{e(dx+c)}{bx+a}\right)}{4b}+\frac{g^3b^3Ax^4}{4}+g^3b^2Aa\,x^3-\frac{g^3b^2Ba\,x^3}{12}+\frac{g^3b^3Bc\,x^3}{12d}+\frac{3g^3bA\,a^2x^2}{2}-\frac{3g^3bB\,a^2x}{8}$
derivativedivides	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*g*x+a*g)^3*(A+B*ln(e*(d*x+c)/(b*x+a))),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^2*e*(a*d-b*c)*(-3/8*B*b^3*e*g^3/d/((d*e/b-e*(a*d-b*c)/b/(b*x+a))*b-e*d)
^2*a*c^2+1/4*B*b*g^3/((d*e/b-e*(a*d-b*c)/b/(b*x+a))*b-e*d)*a^3+1/12*B*b*e^2
*g^3*d^2/((d*e/b-e*(a*d-b*c)/b/(b*x+a))*b-e*d)^3*a^3-1/12*B*b^4*e^2*g^3/d/(
(d*e/b-e*(a*d-b*c)/b/(b*x+a))*b-e*d)^3*c^3-1/8*B*b*e*g^3*d/((d*e/b-e*(a*d-b
*c)/b/(b*x+a))*b-e*d)^2*a^3+1/8*B*b^4*e*g^3/d^2/((d*e/b-e*(a*d-b*c)/b/(b*x+
a))*b-e*d)^2*c^3+3/4*B*b^3*g^3/d^2/((d*e/b-e*(a*d-b*c)/b/(b*x+a))*b-e*d)*a*
c^2-3/4*B*b^2*g^3/d/((d*e/b-e*(a*d-b*c)/b/(b*x+a))*b-e*d)*a^2*c+3/8*B*b^2*e
*g^3/((d*e/b-e*(a*d-b*c)/b/(b*x+a))*b-e*d)^2*a^2*c+1/4*A*b*e^3*g^3*(a^3*d^3
-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/((d*e/b-e*(a*d-b*c)/b/(b*x+a))*b-e*d)
^4-1/4*B*b^4*g^3/d^3/((d*e/b-e*(a*d-b*c)/b/(b*x+a))*b-e*d)*c^3-3/4*B*b^7/e*
g^3*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))*(d*e/b-e*(a*d-b*c)/b/(b*x+a))^4/d^3/((d
*e/b-e*(a*d-b*c)/b/(b*x+a))*b-e*d)^4*a*c^2+3/4*B*b^6/e*g^3*ln(d*e/b-e*(a*d-
b*c)/b/(b*x+a))*(d*e/b-e*(a*d-b*c)/b/(b*x+a))^4/d^2/((d*e/b-e*(a*d-b*c)/b/(
b*x+a))*b-e*d)^4*a^2*c-3*B*b^3*e^2*g^3*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))*(d*e
/b-e*(a*d-b*c)/b/(b*x+a))*d/((d*e/b-e*(a*d-b*c)/b/(b*x+a))*b-e*d)^4*a^2*c-9
/2*B*b^5*e*g^3*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))*(d*e/b-e*(a*d-b*c)/b/(b*x+a)
)^2/d/((d*e/b-e*(a*d-b*c)/b/(b*x+a))*b-e*d)^4*a*c^2+1/4*B*b^3*e^2*g^3/((d*e
/b-e*(a*d-b*c)/b/(b*x+a))*b-e*d)^3*a*c^2-B*b^7*g^3*ln(d*e/b-e*(a*d-b*c)/b/(
b*x+a))*(d*e/b-e*(a*d-b*c)/b/(b*x+a))^3/d^3/((d*e/b-e*(a*d-b*c)/b/(b*x+a))*
b-e*d)^4*c^3-3/4*B*b^2/e*g^3/d^2*ln((d*e/b-e*(a*d-b*c)/b/(b*x+a))*b-e*d)*a^
2*c+3/4*B*b^3/e*g^3/d^3*ln((d*e/b-e*(a*d-b*c)/b/(b*x+a))*b-e*d)*a*c^2-1/4*B
*b^2*e^2*g^3*d/((d*e/b-e*(a*d-b*c)/b/(b*x+a))*b-e*d)^3*a^2*c+B*b^4*g^3*ln(d
*e/b-e*(a*d-b*c)/b/(b*x+a))*(d*e/b-e*(a*d-b*c)/b/(b*x+a))^3/((d*e/b-e*(a*d-
b*c)/b/(b*x+a))*b-e*d)^4*a^3+1/4*B*b/e*g^3/d*ln((d*e/b-e*(a*d-b*c)/b/(b*x+a
))*b-e*d)*a^3-1/4*B*b^4/e*g^3/d^4*ln((d*e/b-e*(a*d-b*c)/b/(b*x+a))*b-e*d)*c
^3+1/4*B*b^8/e*g^3*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))*(d*e/b-e*(a*d-b*c)/b/(b*
x+a))^4/d^4/((d*e/b-e*(a*d-b*c)/b/(b*x+a))*b-e*d)^4*c^3-B*b^5*e^2*g^3*ln(d*
e/b-e*(a*d-b*c)/b/(b*x+a))*(d*e/b-e*(a*d-b*c)/b/(b*x+a))/d/((d*e/b-e*(a*d-b
*c)/b/(b*x+a))*b-e*d)^4*c^3+3*B*b^4*e^2*g^3*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))
*(d*e/b-e*(a*d-b*c)/b/(b*x+a))/((d*e/b-e*(a*d-b*c)/b/(b*x+a))*b-e*d)^4*a*c^
2+B*b^2*e^2*g^3*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))*(d*e/b-e*(a*d-b*c)/b/(b*x+a
))*d^2/((d*e/b-e*(a*d-b*c)/b/(b*x+a))*b-e*d)^4*a^3+9/2*B*b^4*e*g^3*ln(d*e/b
-e*(a*d-b*c)/b/(b*x+a))*(d*e/b-e*(a*d-b*c)/b/(b*x+a))^2/((d*e/b-e*(a*d-b*c)
/b/(b*x+a))*b-e*d)^4*a^2*c-3/2*B*b^3*e*g^3*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))*
(d*e/b-e*(a*d-b*c)/b/(b*x+a))^2*d/((d*e/b-e*(a*d-b*c)/b/(b*x+a))*b-e*d)^4*a
^3+3/2*B*b^6*e*g^3*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))*(d*e/b-e*(a*d-b*c)/b/(b*
x+a))^2/d^2/((d*e/b-e*(a*d-b*c)/b/(b*x+a))*b-e*d)^4*c^3-1/4*B*b^5/e*g^3*ln(
d*e/b-e*(a*d-b*c)/b/(b*x+a))*(d*e/b-e*(a*d-b*c)/b/(b*x+a))^4/d/((d*e/b-e*(a
*d-b*c)/b/(b*x+a))*b-e*d)^4*a^3-3*B*b^5*g^3*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))
*(d*e/b-e*(a*d-b*c)/b/(b*x+a))^3/d/((d*e/b-e*(a*d-b*c)/b/(b*x+a))*b-e*d)^4*
a^2*c+3*B*b^6*g^3*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))*(d*e/b-e*(a*d-b*c)/b/(b*x
+a))^3/d^2/((d*e/b-e*(a*d-b*c)/b/(b*x+a))*b-e*d)^4*a*c^2)
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 444 vs.

$2(140) = 280$.
time = 0.29, size = 444, normalized size = 2.98

$$\frac{1}{4}Ab^3g^3x^4 + Aab^2g^3x^3 + \frac{3}{2}Aa^2bg^3x^2 + \left(x\log\left(\frac{dxe}{bx+a} + \frac{ce}{bx+a}\right) - \frac{a\log(bx+a)}{b} + \frac{c\log(dx+c)}{d}\right)Ba^3g^3 + \frac{3}{2}\left(x^2\log\left(\frac{dxe}{bx+a} + \frac{ce}{bx+a}\right) + \frac{a^2\log(bx+a)}{b^2} - \frac{c^2\log(dx+c)}{d^2} + \frac{(bc-ad)x}{bd}\right)Ba^2bg^3 + \frac{1}{2}\left(2x^3\log\left(\frac{dxe}{bx+a} + \frac{ce}{bx+a}\right) - \frac{2a^3\log(bx+a)}{b^3} + \frac{2c^3\log(dx+c)}{d^3} + \frac{(b^2cd-abd^2)x^2 - 2(b^2c^2-a^2d^2)x}{b^2d^2}\right)Bab^2g^3 + \frac{1}{24}\left(6x^4\log\left(\frac{dxe}{bx+a} + \frac{ce}{bx+a}\right) + \frac{6a^4\log(bx+a)}{b^4} - \frac{6c^4\log(dx+c)}{d^4} + \frac{2(b^3cd^2-ab^2d^3)x^3 - 3(b^3c^2d-a^2bd^3)x^2 + 6(b^3c^3-a^3d^3)x}{b^3d^3}\right)Bb^3g^3 + Aa^3g^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^3*(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="maxima")
```

```
[Out] 1/4*A*b^3*g^3*x^4 + A*a*b^2*g^3*x^3 + 3/2*A*a^2*b*g^3*x^2 + (x*log(d*x*e/(b
*x + a) + c*e/(b*x + a)) - a*log(b*x + a)/b + c*log(d*x + c)/d)*B*a^3*g^3 +
 3/2*(x^2*log(d*x*e/(b*x + a) + c*e/(b*x + a)) + a^2*log(b*x + a)/b^2 - c^2
*log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d))*B*a^2*b*g^3 + 1/2*(2*x^3*log(d*x*e
/(b*x + a) + c*e/(b*x + a)) - 2*a^3*log(b*x + a)/b^3 + 2*c^3*log(d*x + c)/d
^3 + ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*B*a*b^2
*g^3 + 1/24*(6*x^4*log(d*x*e/(b*x + a) + c*e/(b*x + a)) + 6*a^4*log(b*x + a
)/b^4 - 6*c^4*log(d*x + c)/d^4 + (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^
2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3))*B*b^3*g^3 + A*a^
3*g^3*x
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 319 vs. $2(140) = 280$.
time = 0.41, size = 319, normalized size = 2.14

$$\frac{6Ab^4d^4g^3x^4 - 6Ba^4d^4g^3\log(bx+a) + 2(Bb^4cd^3 + (12A-B)ab^3d^4)g^3x^3 - 3(Bb^4c^2d^2 - 4Bab^3cd^3 - 3(4A-B)a^2b^2d^4)g^3x^2 + 6(Bb^4c^3d - 4Bab^3c^2d^2 + 6Ba^2b^2cd^3 + (4A-3B)a^3bd^4)g^3x - 6(Bb^4c^4 - 4Bab^3c^3d + 6Ba^2b^2c^2d^2 - 4Ba^3bcd^3)g^3\log(dx+c) + 6(Bb^4d^4g^3x^4 + 4Bab^3d^4g^3x^3 + 6Ba^2b^2d^4g^3x^2 + 4Ba^3bd^4g^3x)\log\left(\frac{(dx+c)e}{bx+a}\right)}{24bd^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^3*(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="fricas")
```

```
[Out] 1/24*(6*A*b^4*d^4*g^3*x^4 - 6*B*a^4*d^4*g^3*log(b*x + a) + 2*(B*b^4*c*d^3 +
 (12*A - B)*a*b^3*d^4)*g^3*x^3 - 3*(B*b^4*c^2*d^2 - 4*B*a*b^3*c*d^3 - 3*(4*
A - B)*a^2*b^2*d^4)*g^3*x^2 + 6*(B*b^4*c^3*d - 4*B*a*b^3*c^2*d^2 + 6*B*a^2*
b^2*c*d^3 + (4*A - 3*B)*a^3*b*d^4)*g^3*x - 6*(B*b^4*c^4 - 4*B*a*b^3*c^3*d +
 6*B*a^2*b^2*c^2*d^2 - 4*B*a^3*b*c*d^3)*g^3*log(d*x + c) + 6*(B*b^4*d^4*g^3
*x^4 + 4*B*a*b^3*d^4*g^3*x^3 + 6*B*a^2*b^2*d^4*g^3*x^2 + 4*B*a^3*b*d^4*g^3*
x)*log((d*x + c)*e/(b*x + a)))/(b*d^4)
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 706 vs. $2(128) = 256$.
time = 2.53, size = 706, normalized size = 4.74

$$\frac{Ab^3g^3x^4}{4} - \frac{Ba^4g^3\log\left(x + \frac{\frac{Ba^5d^4g^3}{b} + 4Ba^4cd^3g^3 - 6Ba^3bc^2d^2g^3 + 4Ba^2b^2c^3dg^3 - Bab^3c^4g^3}{Ba^4d^4g^3 + 4Ba^3bcd^3g^3 - 6Ba^2b^2c^2d^2g^3 + 4Bab^3c^3dg^3 - Bb^4c^4g^3}\right)}{4b} + \frac{Bcg^3\cdot(2ad-bc)(2a^2d^2-2abcd+b^2c^2)\log\left(x + \frac{5Ba^4cd^3g^3 - 6Ba^3bc^2d^2g^3 + 4Ba^2b^2c^3dg^3 - Bab^3c^4g^3 - Bacg^3\cdot(2ad-bc)(2a^2d^2-2abcd+b^2c^2) + \frac{Bbc^2g^3\cdot(2ad-bc)\left(2a^2d^2-2abcd+b^2c^2\right)}{d}}{Ba^4d^4g^3 + 4Ba^3bcd^3g^3 - 6Ba^2b^2c^2d^2g^3 + 4Bab^3c^3dg^3 - Bb^4c^4g^3}\right)}{4d^4} + x^3\left(Aab^2g^3 - \frac{Bab^2g^3}{12} + \frac{Bb^3cg^3}{12d}\right) + x^2\cdot\left(\frac{3Aa^2bg^3}{2} - \frac{3Ba^2bg^3}{8} + \frac{Bab^2cg^3}{2d} - \frac{Bb^3c^2g^3}{8d^2}\right) + x\left(Aa^3g^3 - \frac{3Ba^3g^3}{4} + \frac{3Ba^2bcg^3}{2d} - \frac{Bab^2c^2g^3}{d^2} + \frac{Bb^3c^3g^3}{4d^3}\right) + \left(Ba^3g^3x + \frac{3Ba^2bg^3x^2}{2} + Bab^2g^3x^3 + \frac{Bb^3g^3x^4}{4}\right)\log\left(\frac{e(c+dx)}{a+bx}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)**3*(A+B*ln(e*(d*x+c)/(b*x+a))),x)
```

```
[Out] A*b**3*g**3*x**4/4 - B*a**4*g**3*log(x + (B*a**5*d**4*g**3/b + 4*B*a**4*c*d
**3*g**3 - 6*B*a**3*b*c**2*d**2*g**3 + 4*B*a**2*b**2*c**3*d*g**3 - B*a*b**3
*c**4*g**3)/(B*a**4*d**4*g**3 + 4*B*a**3*b*c*d**3*g**3 - 6*B*a**2*b**2*c**2
*d**2*g**3 + 4*B*a*b**3*c**3*d*g**3 - B*b**4*c**4*g**3))/(4*b) + B*c*g**3*(
2*a*d - b*c)*(2*a**2*d**2 - 2*a*b*c*d + b**2*c**2)*log(x + (5*B*a**4*c*d**3
*g**3 - 6*B*a**3*b*c**2*d**2*g**3 + 4*B*a**2*b**2*c**3*d*g**3 - B*a*b**3*c*
*4*g**3 - B*a*c*g**3*(2*a*d - b*c)*(2*a**2*d**2 - 2*a*b*c*d + b**2*c**2) +
B*b*c**2*g**3*(2*a*d - b*c)*(2*a**2*d**2 - 2*a*b*c*d + b**2*c**2)/d)/(B*a**
4*d**4*g**3 + 4*B*a**3*b*c*d**3*g**3 - 6*B*a**2*b**2*c**2*d**2*g**3 + 4*B*a
*b**3*c**3*d*g**3 - B*b**4*c**4*g**3))/(4*d**4) + x**3*(A*a*b**2*g**3 - B*a
*b**2*g**3/12 + B*b**3*c*g**3/(12*d)) + x**2*(3*A*a**2*b*g**3/2 - 3*B*a**2*
b*g**3/8 + B*a*b**2*c*g**3/(2*d) - B*b**3*c**2*g**3/(8*d**2)) + x*(A*a**3*g
**3 - 3*B*a**3*g**3/4 + 3*B*a**2*b*c*g**3/(2*d) - B*a*b**2*c**2*g**3/d**2 +
 B*b**3*c**3*g**3/(4*d**3)) + (B*a**3*g**3*x + 3*B*a**2*b*g**3*x**2/2 + B*a
*b**2*g**3*x**3 + B*b**3*g**3*x**4/4)*log(e*(c + d*x)/(a + b*x))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 4137 vs. $2(140) = 280$.
time = 4.57, size = 4137, normalized size = 27.77

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^3*(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="giac")
```

```
[Out] 1/24*(6*B*b^5*c^5*d^4*g^3*e^5*log(-d*e + (d*x*e + c*e)*b/(b*x + a)) - 30*B*
a*b^4*c^4*d^5*g^3*e^5*log(-d*e + (d*x*e + c*e)*b/(b*x + a)) + 60*B*a^2*b^3*
c^3*d^6*g^3*e^5*log(-d*e + (d*x*e + c*e)*b/(b*x + a)) - 60*B*a^3*b^2*c^2*d^
7*g^3*e^5*log(-d*e + (d*x*e + c*e)*b/(b*x + a)) + 30*B*a^4*b*c*d^8*g^3*e^5*
log(-d*e + (d*x*e + c*e)*b/(b*x + a)) - 6*B*a^5*d^9*g^3*e^5*log(-d*e + (d*x
*e + c*e)*b/(b*x + a)) - 24*(d*x*e + c*e)*B*b^6*c^5*d^3*g^3*e^4*log(-d*e +
(d*x*e + c*e)*b/(b*x + a))/(b*x + a) + 120*(d*x*e + c*e)*B*a*b^5*c^4*d^4*g^
3*e^4*log(-d*e + (d*x*e + c*e)*b/(b*x + a))/(b*x + a) - 240*(d*x*e + c*e)*B
*a^2*b^4*c^3*d^5*g^3*e^4*log(-d*e + (d*x*e + c*e)*b/(b*x + a))/(b*x + a) +
240*(d*x*e + c*e)*B*a^3*b^3*c^2*d^6*g^3*e^4*log(-d*e + (d*x*e + c*e)*b/(b*x
 + a))/(b*x + a) - 120*(d*x*e + c*e)*B*a^4*b^2*c*d^7*g^3*e^4*log(-d*e + (d*
x*e + c*e)*b/(b*x + a))/(b*x + a) + 24*(d*x*e + c*e)*B*a^5*b*d^8*g^3*e^4*lo
g(-d*e + (d*x*e + c*e)*b/(b*x + a))/(b*x + a) + 36*(d*x*e + c*e)^2*B*b^7*c^
5*d^2*g^3*e^3*log(-d*e + (d*x*e + c*e)*b/(b*x + a))/(b*x + a)^2 - 180*(d*x*
e + c*e)^2*B*a*b^6*c^4*d^3*g^3*e^3*log(-d*e + (d*x*e + c*e)*b/(b*x + a))/(b
*x + a)^2 + 360*(d*x*e + c*e)^2*B*a^2*b^5*c^3*d^4*g^3*e^3*log(-d*e + (d*x*e
 + c*e)*b/(b*x + a))/(b*x + a)^2 - 360*(d*x*e + c*e)^2*B*a^3*b^4*c^2*d^5*g^
3*e^3*log(-d*e + (d*x*e + c*e)*b/(b*x + a))/(b*x + a)^2 + 180*(d*x*e + c*e)
^2*B*a^4*b^3*c*d^6*g^3*e^3*log(-d*e + (d*x*e + c*e)*b/(b*x + a))/(b*x + a)^
2 - 36*(d*x*e + c*e)^2*B*a^5*b^2*d^7*g^3*e^3*log(-d*e + (d*x*e + c*e)*b/(b*
```

x + a))/(b*x + a)^2 - 24*(d*x*e + c*e)^3*B*b^8*c^5*d*g^3*e^2*log(-d*e + (d*x*e + c*e)*b/(b*x + a))/(b*x + a)^3 + 120*(d*x*e + c*e)^3*B*a*b^7*c^4*d^2*g^3*e^2*log(-d*e + (d*x*e + c*e)*b/(b*x + a))/(b*x + a)^3 - 240*(d*x*e + c*e)^3*B*a^2*b^6*c^3*d^3*g^3*e^2*log(-d*e + (d*x*e + c*e)*b/(b*x + a))/(b*x + a)^3 + 240*(d*x*e + c*e)^3*B*a^3*b^5*c^2*d^4*g^3*e^2*log(-d*e + (d*x*e + c*e)*b/(b*x + a))/(b*x + a)^3 - 120*(d*x*e + c*e)^3*B*a^4*b^4*c*d^5*g^3*e^2*log(-d*e + (d*x*e + c*e)*b/(b*x + a))/(b*x + a)^3 + 24*(d*x*e + c*e)^3*B*a^5*b^3*d^6*g^3*e^2*log(-d*e + (d*x*e + c*e)*b/(b*x + a))/(b*x + a)^3 + 6*(d*x*e + c*e)^4*B*b^9*c^5*g^3*e*log(-d*e + (d*x*e + c*e)*b/(b*x + a))/(b*x + a)^4 - 30*(d*x*e + c*e)^4*B*a*b^8*c^4*d*g^3*e*log(-d*e + (d*x*e + c*e)*b/(b*x + a))/(b*x + a)^4 + 60*(d*x*e + c*e)^4*B*a^2*b^7*c^3*d^2*g^3*e*log(-d*e + (d*x*e + c*e)*b/(b*x + a))/(b*x + a)^4 - 60*(d*x*e + c*e)^4*B*a^3*b^6*c^2*d^3*g^3*e*log(-d*e + (d*x*e + c*e)*b/(b*x + a))/(b*x + a)^4 + 30*(d*x*e + c*e)^4*B*a^4*b^5*c*d^4*g^3*e*log(-d*e + (d*x*e + c*e)*b/(b*x + a))/(b*x + a)^4 - 6*(d*x*e + c*e)^4*B*a^5*b^4*d^5*g^3*e*log(-d*e + (d*x*e + c*e)*b/(b*x + a))/(b*x + a)^4 + 24*(d*x*e + c*e)*B*b^6*c^5*d^3*g^3*e^4*log((d*x*e + c*e)/(b*x + a))/(b*x + a) - 120*(d*x*e + c*e)*B*a*b^5*c^4*d^4*g^3*e^4*log((d*x*e + c*e)/(b*x + a))/(b*x + a) + 240*(d*x*e + c*e)*B*a^2*b^4*c^3*d^5*g^3*e^4*log((d*x*e + c*e)/(b*x + a))/(b*x + a) - 240*(d*x*e + c*e)*B*a^3*b^3*c^2*d^6*g^3*e^4*log((d*x*e + c*e)/(b*x + a))/(b*x + a) + 120*(d*x*e + c*e)*B*a^4*b^2*c*d^7*g^3*e^4*log((d*x*e + c*e)/(b*x + a))/(b*x + a) - 24*(d*x*e + c*e)*B*a^5*b*d^8*g^3*e^4*log((d*x*e + c*e)/(b*x + a))/(b*x + a) - 36*(d*x*e + c*e)^2*B*b^7*c^5*d^2*g^3*e^3*log((d*x*e + c*e)/(b*x + a))/(b*x + a)^2 + 180*(d*x*e + c*e)^2*B*a*b^6*c^4*d^3*g^3*e^3*log((d*x*e + c*e)/(b*x + a))/(b*x + a)^2 - 360*(d*x*e + c*e)^2*B*a^2*b^5*c^3*d^4*g^3*e^3*log((d*x*e + c*e)/(b*x + a))/(b*x + a)^2 + 360*(d*x*e + c*e)^2*B*a^3*b^4*c^2*d^5*g^3*e^3*log((d*x*e + c*e)/(b*x + a))/(b*x + a)^2 - 180*(d*x*e + c*e)^2*B*a^4*b^3*c*d^6*g^3*e^3*log((d*x*e + c*e)/(b*x + a))/(b*x + a)^2 + 36*(d*x*e + c*e)^2*B*a^5*b^2*d^7*g^3*e^3*log((d*x*e + c*e)/(b*x + a))/(b*x + a)^2 + 24*(d*x*e + c*e)^3*B*b^8*c^5*d*g^3*e^2*log((d*x*e + c*e)/(b*x + a))/(b*x + a)^3 - 120*(d*x*e + c*e)^3*B*a*b^7*c^4*d^2*g^3*e^2*log((d*x*e + c*e)/(b*x + a))/(b*x + a)^3 + 240*(d*x*e + c*e)^3*B*a^2*b^6*c^3*d^3*g^3*e^2*log((d*x*e + c*e)/(b*x + a))/(b*x + a)^3 - 240*(d*x*e + c*e)^3*B*a^3*b^5*c^2*d^4*g^3*e^2*log((d*x*e + c*e)/(b*x + a))/(b*x + a)^3 + 120*(d*x*e + c*e)^3*B*a^4*b^4*c*d^5*g^3*e^2*log((d*x*e + c*e)/(b*x + a))/(b*x + a)^3 - 24*(d*x*e + c*e)^3*B*a^5*b^3*d^6*g^3*e^2*log((d*x*e + c*e)/(b*x + a))/(b*x + a)^3 - 6*(d*x*e + c*e)^4*B*b^9*c^5*g^3*e*log((d*x*e + c*e)/(b*x + a))/(b*x + a)^4 + 30*(d*x*e + c*e)^4*B*a*b^8*c^4*d*g^3*e*log((d*x*e + c*e)/(b*x + a))/(b*x + a)^4 - 60*(d*x*e + c*e)^4*B*a^2*b^7*c^3*d^2*g^3*e*log((d*x*e + c*e)/(b*x + a))/(b*x + a)^4 + 60*(d*x*e + c*e)^4*B*a^3*b^6*c^2*d^3*g^3*e*log((d*x*e + c*e)/(b*x + a))/(b*x + a)^4 - 30*(d*x*e + c*e)^4*B*a^4*b^5*c*d^4*g^3*e*log((d*x*e + c*e)/(b*x + a))/(b*x + a)^4 + 6*(d*x*e + c*e)^4*B*a^5*b^4*d^5*g^3*e*log((d*x*e + c*e)/(b*x + a))/(b*x + a)^4 + 6*A*b^5*c^5*d^4*g^3*e^5 - 11*B*b^5*c^5*d^4*g^3*e^5 - 30*A*a*b^4*c^4*d^5*g^3*e^5 + 55*B*a*b^4*c^4*d^5*g^3*e^5 + 60*A*a^2*b^3*c^3*d^6*g^3*e^5 - 110*B*a^2*b^3*c^3*d^6*g^3*e^5 - 60*A*a^3*b^2*c^2*d^7*g^3*e^5

```
+ 110*B*a^3*b^2*c^2*d^7*g^3*e^5 + 30*A*a^4*b*c*d^8*g^3*e^5 - 55*B*a^4*b*c*d
^8*g^3*e^5 - 6*A*a^5*d^9*g^3*e^5 + 11*B*a^5*d^9...
```

Mupad [B]
time = 4.69, size = 566, normalized size = 3.80

$$x\left(\frac{(4ad+4bc)\left(\frac{\left(\frac{b^2g^3(16Aad+4Abc-Bad+Bbc)}{4d}-\frac{Ab^2g^3(4ad+4bc)}{4d}\right)(4ad+4bc)}{4bd}-\frac{abg^3(6Aad+4Abc-Bad+Bbc)}{d}+\frac{Aab^2cg^3}{d}\right)}{4bd}+\frac{a^2g^3(8Aad+12Abc-3Bad+3Bbc)}{2d}-\frac{ac\left(\frac{b^2g^3(16Aad+4Abc-Bad+Bbc)}{4d}-\frac{Ab^2g^3(4ad+4bc)}{4d}\right)}{bd}\right)-x^2\left(\frac{\left(\frac{b^2g^3(16Aad+4Abc-Bad+Bbc)}{4d}-\frac{Ab^2g^3(4ad+4bc)}{4d}\right)(4ad+4bc)}{8bd}-\frac{abg^3(6Aad+4Abc-Bad+Bbc)}{2d}+\frac{Aab^2cg^3}{2d}\right)+\ln\left(\frac{e(c+dx)}{a+bx}\right)\left(Ba^3g^3x+\frac{3Ba^2bg^3x^2}{2}+Bab^2g^3x^3+\frac{Bb^3g^3x^4}{4}\right)+x^3\left(\frac{b^2g^3(16Aad+4Abc-Bad+Bbc)}{12d}-\frac{Ab^2g^3(4ad+4bc)}{12d}\right)-\frac{\ln(c+dx)\left(-4Ba^3cd^3g^3+6Ba^2bc^2d^2g^3-4Bab^2c^3dg^3+Bb^3c^4g^3\right)}{4d^4}+\frac{Ab^3g^3x^4}{4}-\frac{Ba^4g^3\ln(a+bx)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*g + b*g*x)^3*(A + B*log((e*(c + d*x))/(a + b*x))),x)
```

```
[Out] x*(((4*a*d + 4*b*c)*((((b^2*g^3*(16*A*a*d + 4*A*b*c - B*a*d + B*b*c))/(4*d)
 - (A*b^2*g^3*(4*a*d + 4*b*c))/(4*d))*(4*a*d + 4*b*c))/(4*b*d) - (a*b*g^3*(
6*A*a*d + 4*A*b*c - B*a*d + B*b*c))/d + (A*a*b^2*c*g^3)/d))/(4*b*d) + (a^2*
g^3*(8*A*a*d + 12*A*b*c - 3*B*a*d + 3*B*b*c))/(2*d) - (a*c*((b^2*g^3*(16*A*
a*d + 4*A*b*c - B*a*d + B*b*c))/(4*d) - (A*b^2*g^3*(4*a*d + 4*b*c))/(4*d)))
/(b*d)) - x^2*((((b^2*g^3*(16*A*a*d + 4*A*b*c - B*a*d + B*b*c))/(4*d) - (A*
b^2*g^3*(4*a*d + 4*b*c))/(4*d))*(4*a*d + 4*b*c))/(8*b*d) - (a*b*g^3*(6*A*a*
d + 4*A*b*c - B*a*d + B*b*c))/(2*d) + (A*a*b^2*c*g^3)/(2*d)) + log((e*(c +
d*x))/(a + b*x))*((B*b^3*g^3*x^4)/4 + B*a^3*g^3*x + (3*B*a^2*b*g^3*x^2)/2 +
 B*a*b^2*g^3*x^3) + x^3*((b^2*g^3*(16*A*a*d + 4*A*b*c - B*a*d + B*b*c))/(12
*d) - (A*b^2*g^3*(4*a*d + 4*b*c))/(12*d)) - (log(c + d*x)*(B*b^3*c^4*g^3 -
4*B*a^3*c*d^3*g^3 + 6*B*a^2*b*c^2*d^2*g^3 - 4*B*a*b^2*c^3*d*g^3))/(4*d^4) +
 (A*b^3*g^3*x^4)/4 - (B*a^4*g^3*log(a + b*x))/(4*b)
```

3.175 $\int (ag+bgx)^2 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right) dx$

Optimal. Leaf size=118

$$-\frac{B(bc-ad)^2g^2x}{3d^2}+\frac{B(bc-ad)g^2(a+bx)^2}{6bd}+\frac{B(bc-ad)^3g^2\log(c+dx)}{3bd^3}+\frac{g^2(a+bx)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{3b}$$

```
[Out] -1/3*B*(-a*d+b*c)^2*g^2*x/d^2+1/6*B*(-a*d+b*c)*g^2*(b*x+a)^2/b/d+1/3*B*(-a*
d+b*c)^3*g^2*ln(d*x+c)/b/d^3+1/3*g^2*(b*x+a)^3*(A+B*ln(e*(d*x+c)/(b*x+a)))/
b
```

Rubi [A]
time = 0.05, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2548, 21, 45}

$$\frac{g^2(a+bx)^3\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{3b}+\frac{Bg^2(bc-ad)^3\log(c+dx)}{3bd^3}-\frac{Bg^2x(bc-ad)^2}{3d^2}+\frac{Bg^2(a+bx)^2(bc-ad)}{6bd}$$

Antiderivative was successfully verified.

```
[In] Int[(a*g + b*g*x)^2*(A + B*Log[(e*(c + d*x))/(a + b*x)]),x]
```

```
[Out] -1/3*(B*(b*c - a*d)^2*g^2*x)/d^2 + (B*(b*c - a*d)*g^2*(a + b*x)^2)/(6*b*d)
+ (B*(b*c - a*d)^3*g^2*Log[c + d*x])/(3*b*d^3) + (g^2*(a + b*x)^3*(A + B*Lo
g[(e*(c + d*x))/(a + b*x)]))/(3*b)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2548

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Dist[B*n*((b*c
```

```
- a*d)/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /;
 FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c -
 a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Rubi steps

$$\int (ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)dx=\frac{g^2(a+bx)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{3b}-\frac{B\int\frac{(-bc+ad)g^3(a+bx)^2}{c+dx}dx}{3bg}$$
$$=\frac{g^2(a+bx)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{3b}+\frac{(B(bc-ad)g^2)\int\frac{(a+bx)^2}{c+dx}}{3b}$$
$$=\frac{g^2(a+bx)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{3b}+\frac{(B(bc-ad)g^2)\int\left(-\frac{b(}{}\right.}{}$$
$$=-\frac{B(bc-ad)^2g^2x}{3d^2}+\frac{B(bc-ad)g^2(a+bx)^2}{6bd}+\frac{B(bc-ad)^3g}{3b}$$

Mathematica [A]
time = 0.04, size = 99, normalized size = 0.84

$$\frac{g^2\left(\frac{B(bc-ad)\left(d\left(a^2d+4abdx+b^2x(-2c+dx)\right)+2(bc-ad)^2\log(c+dx)\right)}{2d^3}+(a+bx)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)\right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*g + b*g*x)^2*(A + B*Log[(e*(c + d*x))/(a + b*x)]),x]
[Out] (g^2*((B*(b*c - a*d)*(d*(a^2*d + 4*a*b*d*x + b^2*x*(-2*c + d*x)) + 2*(b*c -
 a*d)^2*Log[c + d*x]))/(2*d^3) + (a + b*x)^3*(A + B*Log[(e*(c + d*x))/(a +
b*x)])))/(3*b)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1534 vs. 2(110) = 220.
time = 0.40, size = 1535, normalized size = 13.01

method	result
risch	$\frac{g^2(bx+a)^3B\ln\left(\frac{e(dx+c)}{bx+a}\right)}{3b}+\frac{g^2b^2A\,x^3}{3}+g^2bAa\,x^2-\frac{g^2bBa\,x^2}{6}+\frac{g^2b^2Bc\,x^2}{6d}+g^2A\,a^2x-\frac{g^2B\ln(dx+c)a^3}{3b}$
derivativedivides	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*g*x+a*g)^2*(A+B*ln(e*(d*x+c)/(b*x+a))),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^2*e*(a*d-b*c)*(-1/3*A*b*e^2*g^2*(a^2*d^2-2*a*b*c*d+b^2*c^2)/((d*e/b-e*(
a*d-b*c)/b/(b*x+a))*b-e*d)^3-1/6*B*b*e*g^2*d/((d*e/b-e*(a*d-b*c)/b/(b*x+a))
*b-e*d)^2*a^2+1/3*B*b^2*e*g^2/((d*e/b-e*(a*d-b*c)/b/(b*x+a))*b-e*d)^2*a*c-1
/6*B*b^3*e*g^2/d/((d*e/b-e*(a*d-b*c)/b/(b*x+a))*b-e*d)^2*c^2+1/3*B*b/e*g^2/
d*ln((d*e/b-e*(a*d-b*c)/b/(b*x+a))*b-e*d)*a^2-2/3*B*b^2/e*g^2/d^2*ln((d*e/b
-e*(a*d-b*c)/b/(b*x+a))*b-e*d)*a*c+1/3*B*b^3/e*g^2/d^3*ln((d*e/b-e*(a*d-b*c
)/b/(b*x+a))*b-e*d)*c^2+1/3*B*b*g^2/((d*e/b-e*(a*d-b*c)/b/(b*x+a))*b-e*d)*a
^2-2/3*B*b^2*g^2/d/((d*e/b-e*(a*d-b*c)/b/(b*x+a))*b-e*d)*a*c+1/3*B*b^3*g^2/
d^2/((d*e/b-e*(a*d-b*c)/b/(b*x+a))*b-e*d)*c^2-1/3*B*b^4/e*g^2*ln(d*e/b-e*(a
*d-b*c)/b/(b*x+a))*(d*e/b-e*(a*d-b*c)/b/(b*x+a))^3/d/((d*e/b-e*(a*d-b*c)/b/
(b*x+a))*b-e*d)^3*a^2+2/3*B*b^5/e*g^2*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))*(d*e/
b-e*(a*d-b*c)/b/(b*x+a))^3/d^2/((d*e/b-e*(a*d-b*c)/b/(b*x+a))*b-e*d)^3*a*c-
1/3*B*b^6/e*g^2*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))*(d*e/b-e*(a*d-b*c)/b/(b*x+a
))^3/d^3/((d*e/b-e*(a*d-b*c)/b/(b*x+a))*b-e*d)^3*c^2+B*b^3*g^2*ln(d*e/b-e*(
a*d-b*c)/b/(b*x+a))*(d*e/b-e*(a*d-b*c)/b/(b*x+a))^2/((d*e/b-e*(a*d-b*c)/b/(
b*x+a))*b-e*d)^3*a^2-2*B*b^4*g^2*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))*(d*e/b-e*(
a*d-b*c)/b/(b*x+a))^2/d/((d*e/b-e*(a*d-b*c)/b/(b*x+a))*b-e*d)^3*a*c+B*b^5*g
^2*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))*(d*e/b-e*(a*d-b*c)/b/(b*x+a))^2/d^2/((d*
e/b-e*(a*d-b*c)/b/(b*x+a))*b-e*d)^3*c^2-B*b^2*e*g^2*ln(d*e/b-e*(a*d-b*c)/b/
(b*x+a))*(d*e/b-e*(a*d-b*c)/b/(b*x+a))*d/((d*e/b-e*(a*d-b*c)/b/(b*x+a))*b-e
*d)^3*a^2+2*B*b^3*e*g^2*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))*(d*e/b-e*(a*d-b*c)/
b/(b*x+a))/((d*e/b-e*(a*d-b*c)/b/(b*x+a))*b-e*d)^3*a*c-B*b^4*e*g^2*ln(d*e/b
-e*(a*d-b*c)/b/(b*x+a))*(d*e/b-e*(a*d-b*c)/b/(b*x+a))/d/((d*e/b-e*(a*d-b*c)
/b/(b*x+a))*b-e*d)^3*c^2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 284 vs. $2(111) = 222$.

time = 0.29, size = 284, normalized size = 2.41

$$\frac{1}{3}Ab^2g^2x^3 + Aabg^2x^2 + \left(x\log\left(\frac{dxe}{bx+a} + \frac{ce}{bx+a}\right) - \frac{a\log(bx+a)}{b} + \frac{c\log(dx+c)}{d}\right)Ba^2g^2 + \left(x^2\log\left(\frac{dxe}{bx+a} + \frac{ce}{bx+a}\right) + \frac{a^2\log(bx+a)}{b^2} - \frac{c^2\log(dx+c)}{d^2} + \frac{(bc-ad)x}{bd}\right)Babg^2 + \frac{1}{6}\left(2x^3\log\left(\frac{dxe}{bx+a} + \frac{ce}{bx+a}\right) - \frac{2a^3\log(bx+a)}{b^3} + \frac{2c^3\log(dx+c)}{d^3} + \frac{(b^2cd-abd^2)x^2 - 2(b^2c^2-a^2d^2)x}{b^2d^2}\right)Bb^2g^2 + Aa^2g^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^2*(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="maxima")
```

```
[Out] 1/3*A*b^2*g^2*x^3 + A*a*b*g^2*x^2 + (x*log(d*x*e/(b*x + a) + c*e/(b*x + a))
 - a*log(b*x + a)/b + c*log(d*x + c)/d)*B*a^2*g^2 + (x^2*log(d*x*e/(b*x + a
) + c*e/(b*x + a)) + a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c - a
*d)*x/(b*d))*B*a*b*g^2 + 1/6*(2*x^3*log(d*x*e/(b*x + a) + c*e/(b*x + a)) -
2*a^3*log(b*x + a)/b^3 + 2*c^3*log(d*x + c)/d^3 + ((b^2*c*d - a*b*d^2)*x^2
- 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*B*b^2*g^2 + A*a^2*g^2*x
```

Fricas [A]

time = 0.36, size = 222, normalized size = 1.88

$$\frac{2Ab^3d^3g^2x^3 - 2Ba^3d^3g^2\log(bx+a) + (Bb^3cd^2 + (6A-B)ab^2d^3)g^2x^2 - 2(Bb^3c^2d - 3Bab^2cd^2 - (3A-2B)a^2bd^3)g^2x + 2(Bb^3c^3 - 3Bab^2c^2d + 3Ba^2bcd^2)g^2\log(dx+c) + 2(Bb^3d^3g^2x^3 + 3Bab^2d^3g^2x^2 + 3Ba^2bd^3g^2x)\log\left(\frac{(dx+c)e}{bx+a}\right)}{6bd^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^2*(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="fricas")
```

```
[Out] 1/6*(2*A*b^3*d^3*g^2*x^3 - 2*B*a^3*d^3*g^2*log(b*x + a) + (B*b^3*c*d^2 + (6
*A - B)*a*b^2*d^3)*g^2*x^2 - 2*(B*b^3*c^2*d - 3*B*a*b^2*c*d^2 - (3*A - 2*B)
*a^2*b*d^3)*g^2*x + 2*(B*b^3*c^3 - 3*B*a*b^2*c^2*d + 3*B*a^2*b*c*d^2)*g^2*l
og(d*x + c) + 2*(B*b^3*d^3*g^2*x^3 + 3*B*a*b^2*d^3*g^2*x^2 + 3*B*a^2*b*d^3*
g^2*x)*log((d*x + c)*e/(b*x + a)))/(b*d^3)
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 491 vs. $2(100) = 200$.
time = 1.67, size = 491, normalized size = 4.16

$$\frac{Ab^2g^2x^3}{3} - \frac{Ba^3g^2\log\left(x + \frac{\frac{Ba^4d^3g^2}{b}+3Ba^3cd^2g^2-3Ba^2bc^2dg^2+Bab^2c^3g^2}{Ba^3d^3g^2+3Ba^2bcd^2g^2-3Bab^2c^2dg^2+Bb^3c^3g^2}\right)}{3b} + \frac{Bcg^2\cdot(3a^2d^2-3abcd+b^2c^2)\log\left(x + \frac{4Ba^3cd^2g^2-3Ba^2bc^2dg^2+Bab^2c^3g^2-Bacg^2\cdot(3a^2d^2-3abcd+b^2c^2)+\frac{Bbc^2g^2\cdot(3a^2d^2-3abcd+b^2c^2)}{d}}{Ba^3d^3g^2+3Ba^2bcd^2g^2-3Bab^2c^2dg^2+Bb^3c^3g^2}\right)}{3d^3} + x^2\left(Aabg^2 - \frac{Babg^2}{6} + \frac{Bb^2cg^2}{6d}\right) + x\left(Aa^2g^2 - \frac{2Ba^2g^2}{3} + \frac{Babcg^2}{d} - \frac{Bb^2c^2g^2}{3d^2}\right) + \left(Ba^2g^2x + Babg^2x^2 + \frac{Bb^2g^2x^3}{3}\right)\log\left(\frac{e(c+dx)}{a+bx}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)**2*(A+B*ln(e*(d*x+c)/(b*x+a))),x)
```

```
[Out] A*b**2*g**2*x**3/3 - B*a**3*g**2*log(x + (B*a**4*d**3*g**2/b + 3*B*a**3*c*d
**2*g**2 - 3*B*a**2*b*c**2*d*g**2 + B*a*b**2*c**3*g**2)/(B*a**3*d**3*g**2 +
 3*B*a**2*b*c*d**2*g**2 - 3*B*a*b**2*c**2*d*g**2 + B*b**3*c**3*g**2))/(3*b)
 + B*c*g**2*(3*a**2*d**2 - 3*a*b*c*d + b**2*c**2)*log(x + (4*B*a**3*c*d**2*
g**2 - 3*B*a**2*b*c**2*d*g**2 + B*a*b**2*c**3*g**2 - B*a*c*g**2*(3*a**2*d**
2 - 3*a*b*c*d + b**2*c**2) + B*b*c**2*g**2*(3*a**2*d**2 - 3*a*b*c*d + b**2*
c**2)/d)/(B*a**3*d**3*g**2 + 3*B*a**2*b*c*d**2*g**2 - 3*B*a*b**2*c**2*d*g**
2 + B*b**3*c**3*g**2))/(3*d**3) + x**2*(A*a*b*g**2 - B*a*b*g**2/6 + B*b**2*
c*g**2/(6*d)) + x*(A*a**2*g**2 - 2*B*a**2*g**2/3 + B*a*b*c*g**2/d - B*b**2*
c**2*g**2/(3*d**2)) + (B*a**2*g**2*x + B*a*b*g**2*x**2 + B*b**2*g**2*x**3/3
)*log(e*(c + d*x)/(a + b*x))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2640 vs. $2(111) = 222$.
time = 4.41, size = 2640, normalized size = 22.37

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^2*(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="giac")
```

```
[Out] -1/6*(2*B*b^4*c^4*d^3*g^2*e^4*log(-d*e + (d*x*e + c*e)*b/(b*x + a)) - 8*B*a
*b^3*c^3*d^4*g^2*e^4*log(-d*e + (d*x*e + c*e)*b/(b*x + a)) + 12*B*a^2*b^2*c
^2*d^5*g^2*e^4*log(-d*e + (d*x*e + c*e)*b/(b*x + a)) - 8*B*a^3*b*c*d^6*g^2*
e^4*log(-d*e + (d*x*e + c*e)*b/(b*x + a)) + 2*B*a^4*d^7*g^2*e^4*log(-d*e +
(d*x*e + c*e)*b/(b*x + a)) - 6*(d*x*e + c*e)*B*b^5*c^4*d^2*g^2*e^3*log(-d*e
 + (d*x*e + c*e)*b/(b*x + a))/(b*x + a) + 24*(d*x*e + c*e)*B*a*b^4*c^3*d^3*
```

g^2*e^3*log(-d*e + (d*x*e + c*e)*b/(b*x + a))/(b*x + a) - 36*(d*x*e + c*e)*
B*a^2*b^3*c^2*d^4*g^2*e^3*log(-d*e + (d*x*e + c*e)*b/(b*x + a))/(b*x + a) +
 24*(d*x*e + c*e)*B*a^3*b^2*c*d^5*g^2*e^3*log(-d*e + (d*x*e + c*e)*b/(b*x +
 a))/(b*x + a) - 6*(d*x*e + c*e)*B*a^4*b*d^6*g^2*e^3*log(-d*e + (d*x*e + c*
e)*b/(b*x + a))/(b*x + a) + 6*(d*x*e + c*e)^2*B*b^6*c^4*d*g^2*e^2*log(-d*e
+ (d*x*e + c*e)*b/(b*x + a))/(b*x + a)^2 - 24*(d*x*e + c*e)^2*B*a*b^5*c^3*d
^2*g^2*e^2*log(-d*e + (d*x*e + c*e)*b/(b*x + a))/(b*x + a)^2 + 36*(d*x*e +
c*e)^2*B*a^2*b^4*c^2*d^3*g^2*e^2*log(-d*e + (d*x*e + c*e)*b/(b*x + a))/(b*x
 + a)^2 - 24*(d*x*e + c*e)^2*B*a^3*b^3*c*d^4*g^2*e^2*log(-d*e + (d*x*e + c*
e)*b/(b*x + a))/(b*x + a)^2 + 6*(d*x*e + c*e)^2*B*a^4*b^2*d^5*g^2*e^2*log(-
d*e + (d*x*e + c*e)*b/(b*x + a))/(b*x + a)^2 - 2*(d*x*e + c*e)^3*B*b^7*c^4*
g^2*e*log(-d*e + (d*x*e + c*e)*b/(b*x + a))/(b*x + a)^3 + 8*(d*x*e + c*e)^3
*B*a*b^6*c^3*d*g^2*e*log(-d*e + (d*x*e + c*e)*b/(b*x + a))/(b*x + a)^3 - 12
*(d*x*e + c*e)^3*B*a^2*b^5*c^2*d^2*g^2*e*log(-d*e + (d*x*e + c*e)*b/(b*x +
a))/(b*x + a)^3 + 8*(d*x*e + c*e)^3*B*a^3*b^4*c*d^3*g^2*e*log(-d*e + (d*x*e
 + c*e)*b/(b*x + a))/(b*x + a)^3 - 2*(d*x*e + c*e)^3*B*a^4*b^3*d^4*g^2*e*lo
g(-d*e + (d*x*e + c*e)*b/(b*x + a))/(b*x + a)^3 + 6*(d*x*e + c*e)*B*b^5*c^4
*d^2*g^2*e^3*log((d*x*e + c*e)/(b*x + a))/(b*x + a) - 24*(d*x*e + c*e)*B*a*
b^4*c^3*d^3*g^2*e^3*log((d*x*e + c*e)/(b*x + a))/(b*x + a) + 36*(d*x*e + c*
e)*B*a^2*b^3*c^2*d^4*g^2*e^3*log((d*x*e + c*e)/(b*x + a))/(b*x + a) - 24*(d
*x*e + c*e)*B*a^3*b^2*c*d^5*g^2*e^3*log((d*x*e + c*e)/(b*x + a))/(b*x + a)
+ 6*(d*x*e + c*e)*B*a^4*b*d^6*g^2*e^3*log((d*x*e + c*e)/(b*x + a))/(b*x + a
) - 6*(d*x*e + c*e)^2*B*b^6*c^4*d*g^2*e^2*log((d*x*e + c*e)/(b*x + a))/(b*x
 + a)^2 + 24*(d*x*e + c*e)^2*B*a*b^5*c^3*d^2*g^2*e^2*log((d*x*e + c*e)/(b*x
 + a))/(b*x + a)^2 - 36*(d*x*e + c*e)^2*B*a^2*b^4*c^2*d^3*g^2*e^2*log((d*x*
e + c*e)/(b*x + a))/(b*x + a)^2 + 24*(d*x*e + c*e)^2*B*a^3*b^3*c*d^4*g^2*e^
2*log((d*x*e + c*e)/(b*x + a))/(b*x + a)^2 - 6*(d*x*e + c*e)^2*B*a^4*b^2*d^
5*g^2*e^2*log((d*x*e + c*e)/(b*x + a))/(b*x + a)^2 + 2*(d*x*e + c*e)^3*B*b^
7*c^4*g^2*e*log((d*x*e + c*e)/(b*x + a))/(b*x + a)^3 - 8*(d*x*e + c*e)^3*B*
a*b^6*c^3*d*g^2*e*log((d*x*e + c*e)/(b*x + a))/(b*x + a)^3 + 12*(d*x*e + c*
e)^3*B*a^2*b^5*c^2*d^2*g^2*e*log((d*x*e + c*e)/(b*x + a))/(b*x + a)^3 - 8*(
d*x*e + c*e)^3*B*a^3*b^4*c*d^3*g^2*e*log((d*x*e + c*e)/(b*x + a))/(b*x + a)
^3 + 2*(d*x*e + c*e)^3*B*a^4*b^3*d^4*g^2*e*log((d*x*e + c*e)/(b*x + a))/(b*
x + a)^3 + 2*A*b^4*c^4*d^3*g^2*e^4 - 3*B*b^4*c^4*d^3*g^2*e^4 - 8*A*a*b^3*c^
3*d^4*g^2*e^4 + 12*B*a*b^3*c^3*d^4*g^2*e^4 + 12*A*a^2*b^2*c^2*d^5*g^2*e^4 -
 18*B*a^2*b^2*c^2*d^5*g^2*e^4 - 8*A*a^3*b*c*d^6*g^2*e^4 + 12*B*a^3*b*c*d^6*
g^2*e^4 + 2*A*a^4*d^7*g^2*e^4 - 3*B*a^4*d^7*g^2*e^4 + 5*(d*x*e + c*e)*B*b^5
*c^4*d^2*g^2*e^3/(b*x + a) - 20*(d*x*e + c*e)*B*a*b^4*c^3*d^3*g^2*e^3/(b*x
+ a) + 30*(d*x*e + c*e)*B*a^2*b^3*c^2*d^4*g^2*e^3/(b*x + a) - 20*(d*x*e + c
*e)*B*a^3*b^2*c*d^5*g^2*e^3/(b*x + a) + 5*(d*x*e + c*e)*B*a^4*b*d^6*g^2*e^3
/(b*x + a) - 2*(d*x*e + c*e)^2*B*b^6*c^4*d*g^2*e^2/(b*x + a)^2 + 8*(d*x*e +
 c*e)^2*B*a*b^5*c^3*d^2*g^2*e^2/(b*x + a)^2 - 12*(d*x*e + c*e)^2*B*a^2*b^4*
c^2*d^3*g^2*e^2/(b*x + a)^2 + 8*(d*x*e + c*e)^2*B*a^3*b^3*c*d^4*g^2*e^2/(b*
x + a)^2 - 2*(d*x*e + c*e)^2*B*a^4*b^2*d^5*g^2*e^2/(b*x + a)^2)*(b*c/((b*c*
e - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))/(b*d^6*e^3 - 3

```
*(d*x*e + c*e)*b^2*d^5*e^2/(b*x + a) + 3*(d*x*e + c*e)^2*b^3*d^4*e/(b*x + a
)^2 - (d*x*e + c*e)^3*b^4*d^3/(b*x + a)^3)
```

Mupad [B]
time = 4.57, size = 290, normalized size = 2.46

$$x^2\left(\frac{b\,g^2\,\left(9\,A\,a\,d+3\,A\,b\,c-B\,a\,d+B\,b\,c\right)}{6\,d}-\frac{A\,b\,g^2\,\left(3\,a\,d+3\,b\,c\right)}{6\,d}\right)-x\left(\frac{\left(3\,a\,d+3\,b\,c\right)\,\left(\frac{b\,g^2\,\left(9\,A\,a\,d+3\,A\,b\,c-B\,a\,d+B\,b\,c\right)}{3\,d}-\frac{A\,b\,g^2\,\left(3\,a\,d+3\,b\,c\right)}{3\,d}\right)}{3\,b\,d}-\frac{a\,g^2\,\left(3\,A\,a\,d+3\,A\,b\,c-B\,a\,d+B\,b\,c\right)}{d}+\frac{A\,a\,b\,c\,g^2}{d}\right)+\ln\left(\frac{e\,\left(c+d\,x\right)}{a+b\,x}\right)\,\left(B\,a^2\,g^2\,x+B\,a\,b\,g^2\,x^2+\frac{B\,b^2\,g^2\,x^3}{3}\right)+\frac{\ln\left(c+d\,x\right)\,\left(3\,B\,a^2\,c\,d^2\,g^2-3\,B\,a\,b\,c^2\,d\,g^2+B\,b^2\,c^3\,g^2\right)}{3\,d^3}+\frac{A\,b^2\,g^2\,x^3}{3}-\frac{B\,a^3\,g^2\,\ln\left(a+b\,x\right)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*g + b*g*x)^2*(A + B*log((e*(c + d*x))/(a + b*x))),x)
```

```
[Out] x^2*((b*g^2*(9*A*a*d + 3*A*b*c - B*a*d + B*b*c))/(6*d) - (A*b*g^2*(3*a*d +
3*b*c))/(6*d)) - x*(((3*a*d + 3*b*c)*((b*g^2*(9*A*a*d + 3*A*b*c - B*a*d + B
*b*c))/(3*d) - (A*b*g^2*(3*a*d + 3*b*c))/(3*d)))/(3*b*d) - (a*g^2*(3*A*a*d
+ 3*A*b*c - B*a*d + B*b*c))/d + (A*a*b*c*g^2)/d) + log((e*(c + d*x))/(a + b
*x))*((B*b^2*g^2*x^3)/3 + B*a^2*g^2*x + B*a*b*g^2*x^2) + (log(c + d*x)*(B*b
^2*c^3*g^2 + 3*B*a^2*c*d^2*g^2 - 3*B*a*b*c^2*d*g^2))/(3*d^3) + (A*b^2*g^2*x
^3)/3 - (B*a^3*g^2*log(a + b*x))/(3*b)
```

3.176 $\int (ag+bgx) \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)\,dx$

Optimal. Leaf size=81

$$\frac{B(bc-ad)gx}{2d}-\frac{B(bc-ad)^2g\log(c+dx)}{2bd^2}+\frac{g(a+bx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{2b}$$

[Out] 1/2*B*(-a*d+b*c)*g*x/d-1/2*B*(-a*d+b*c)^2*g*ln(d*x+c)/b/d^2+1/2*g*(b*x+a)^2*(A+B*ln(e*(d*x+c)/(b*x+a)))/b

Rubi [A]
time = 0.03, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {2548, 21, 45}

$$\frac{g(a+bx)^2\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{2b}-\frac{Bg(bc-ad)^2\log(c+dx)}{2bd^2}+\frac{Bgx(bc-ad)}{2d}$$

Antiderivative was successfully verified.

[In] Int[(a*g + b*g*x)*(A + B*Log[(e*(c + d*x))/(a + b*x)]),x]

[Out] (B*(b*c - a*d)*g*x)/(2*d) - (B*(b*c - a*d)^2*g*Log[c + d*x])/(2*b*d^2) + (g*(a + b*x)^2*(A + B*Log[(e*(c + d*x))/(a + b*x)]))/(2*b)

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2548

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Dist[B*n*((b*c
- a*d)/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /;
 FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c -
```

```
a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Rubi steps

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)dx = \frac{g(a+bx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{2b} - \frac{B\int \frac{(bc-ad)g^2(-a-bx)}{c+dx}\,dx}{2bg}$$

$$= \frac{g(a+bx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{2b} - \frac{(B(bc-ad)g)\int \frac{-a-bx}{c+dx}\,d}{2b}$$

$$= \frac{g(a+bx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{2b} - \frac{(B(bc-ad)g)\int\left(-\frac{b}{d}+\right.}{2b}$$

$$= \frac{B(bc-ad)gx}{2d} - \frac{B(bc-ad)^2g\log(c+dx)}{2bd^2} + \frac{g(a+bx)^2\left(A-\right.}{}$$

Mathematica [A]
time = 0.03, size = 69, normalized size = 0.85

$$\frac{g\left(\frac{B(bc-ad)(bdx+(-bc+ad)\log(c+dx))}{d^2}+(a+bx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*g + b*g*x)*(A + B*Log[(e*(c + d*x))/(a + b*x)]),x]
[Out] (g*((B*(b*c - a*d)*(b*d*x + (-(b*c) + a*d)*Log[c + d*x]))/d^2 + (a + b*x)^2
*(A + B*Log[(e*(c + d*x))/(a + b*x)])))/(2*b)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 680 vs. 2(75) = 150.
time = 0.36, size = 681, normalized size = 8.41

method	result
risch	$\frac{gBx(bx+2a)\ln\left(\frac{e(dx+c)}{bx+a}\right)}{2}+\frac{gbA\,x^2}{2}+gAax+\frac{gB\ln(-dx-c)ac}{d}-\frac{gbB\ln(-dx-c)c^2}{2d^2}-\frac{B\,a^2g\ln(bx+a)}{2b}-\frac{gB}{2}$
derivativedivides	$\frac{e(ad-cb)\left(\frac{Abeg(ad-cb)}{2\left(\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)b-ed\right)^2}+\frac{Bbg\ln\left(\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)b-ed\right)a}{2ed}-\frac{B\,b^2g\ln\left(\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)b-ed\right)c}{2e\,d^2}+\frac{Bbga}{2\left(\frac{de}{b}-\frac{e(ad-cb}{b(bx+a)}\right.}\right.}{}$
default	$\frac{e(ad-cb)\left(\frac{Abeg(ad-cb)}{2\left(\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)b-ed\right)^2}+\frac{Bbg\ln\left(\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)b-ed\right)a}{2ed}-\frac{B\,b^2g\ln\left(\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)b-ed\right)c}{2e\,d^2}+\frac{Bbga}{2\left(\frac{de}{b}-\frac{e(ad-cb}{b(bx+a)}\right.}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*g*x+a*g)*(A+B*ln(e*(d*x+c)/(b*x+a))),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^2*e*(a*d-b*c)*(1/2*A*b*e*g*(a*d-b*c)/((d*e/b-e*(a*d-b*c)/b/(b*x+a))*b-e
*d)^2+1/2*B*b/e*g/d*ln((d*e/b-e*(a*d-b*c)/b/(b*x+a))*b-e*d)*a-1/2*B*b^2/e*g
/d^2*ln((d*e/b-e*(a*d-b*c)/b/(b*x+a))*b-e*d)*c+1/2*B*b*g/((d*e/b-e*(a*d-b*c
)/b/(b*x+a))*b-e*d)*a-1/2*B*b^2*g/d/((d*e/b-e*(a*d-b*c)/b/(b*x+a))*b-e*d)*c
-1/2*B*b^3/e*g*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))*(d*e/b-e*(a*d-b*c)/b/(b*x+a)
)^2/d/((d*e/b-e*(a*d-b*c)/b/(b*x+a))*b-e*d)^2*a+1/2*B*b^4/e*g*ln(d*e/b-e*(a
*d-b*c)/b/(b*x+a))*(d*e/b-e*(a*d-b*c)/b/(b*x+a))^2/d^2/((d*e/b-e*(a*d-b*c)/
b/(b*x+a))*b-e*d)^2*c+B*b^2*g*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))*(d*e/b-e*(a*d
-b*c)/b/(b*x+a))/((d*e/b-e*(a*d-b*c)/b/(b*x+a))*b-e*d)^2*a-B*b^3*g*ln(d*e/b
-e*(a*d-b*c)/b/(b*x+a))*(d*e/b-e*(a*d-b*c)/b/(b*x+a))/d/((d*e/b-e*(a*d-b*c)
/b/(b*x+a))*b-e*d)^2*c)
```

Maxima [A]
time = 0.28, size = 147, normalized size = 1.81

$$\frac{1}{2}\,Abgx^2+\left(x\log\left(\frac{dxe}{bx+a}+\frac{ce}{bx+a}\right)-\frac{a\log\left(bx+a\right)}{b}+\frac{c\log\left(dx+c\right)}{d}\right)Bag+\frac{1}{2}\left(x^2\log\left(\frac{dxe}{bx+a}+\frac{ce}{bx+a}\right)+\frac{a^2\log\left(bx+a\right)}{b^2}-\frac{c^2\log\left(dx+c\right)}{d^2}+\frac{(bc-ad)x}{bd}\right)Bbg+Aagx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)*(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="maxima")
```

```
[Out] 1/2*A*b*g*x^2 + (x*log(d*x*e/(b*x + a) + c*e/(b*x + a)) - a*log(b*x + a)/b
+ c*log(d*x + c)/d)*B*a*g + 1/2*(x^2*log(d*x*e/(b*x + a) + c*e/(b*x + a)) +
 a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d))*B*b*g +
 A*a*g*x
```

Fricas [A]
time = 0.39, size = 126, normalized size = 1.56

$$\frac{Ab^2d^2gx^2-Ba^2d^2g\log\left(bx+a\right)+\left(Bb^2cd+(2\,A-B)abd^2\right)gx-\left(Bb^2c^2-2\,Babcd\right)g\log\left(dx+c\right)+\left(Bb^2d^2gx^2+2\,Babd^2gx\right)\log\left(\frac{(dx+c)e}{bx+a}\right)}{2\,bd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)*(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="fricas")
```

```
[Out] 1/2*(A*b^2*d^2*g*x^2 - B*a^2*d^2*g*log(b*x + a) + (B*b^2*c*d + (2*A - B)*a*
b*d^2)*g*x - (B*b^2*c^2 - 2*B*a*b*c*d)*g*log(d*x + c) + (B*b^2*d^2*g*x^2 +
2*B*a*b*d^2*g*x)*log((d*x + c)*e/(b*x + a)))/(b*d^2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 253 vs. $2(68) = 136$.
time = 1.07, size = 253, normalized size = 3.12

$$\frac{Abgx^2}{2}-\frac{Ba^2g\log\left(x+\frac{\frac{Ba^3d^2g}{b}+2Ba^2cdg-Babc^2g}{Ba^2d^2g+2Babcdg-Bb^2c^2g}\right)}{2b}+\frac{Bcg(2ad-bc)\log\left(x+\frac{3Ba^2cdg-Babc^2g-Bacg(2ad-bc)+\frac{Bbc^2g(2ad-bc)}{d}}{Ba^2d^2g+2Babcdg-Bb^2c^2g}\right)}{2d^2}+x\left(Aag-\frac{Bag}{2}+\frac{Bbcg}{2d}\right)+\left(Bagx+\frac{Bbgx^2}{2}\right)\log\left(\frac{e(c+dx)}{a+bx}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)*(A+B*ln(e*(d*x+c)/(b*x+a))),x)
```

```
[Out] A*b*g*x**2/2 - B*a**2*g*log(x + (B*a**3*d**2*g/b + 2*B*a**2*c*d*g - B*a*b*c
**2*g)/(B*a**2*d**2*g + 2*B*a*b*c*d*g - B*b**2*c**2*g))/(2*b) + B*c*g*(2*a*
d - b*c)*log(x + (3*B*a**2*c*d*g - B*a*b*c**2*g - B*a*c*g*(2*a*d - b*c) + B
*b*c**2*g*(2*a*d - b*c)/d)/(B*a**2*d**2*g + 2*B*a*b*c*d*g - B*b**2*c**2*g))
/(2*d**2) + x*(A*a*g - B*a*g/2 + B*b*c*g/(2*d)) + (B*a*g*x + B*b*g*x**2/2)*
log(e*(c + d*x)/(a + b*x))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1395 vs. $2(76) = 152$.
time = 4.28, size = 1395, normalized size = 17.22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)*(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="giac")
```

```
[Out] 1/2*(B*b^3*c^3*d^2*g*e^3*log(-d*e + (d*x*e + c*e)*b/(b*x + a)) - 3*B*a*b^2*
c^2*d^3*g*e^3*log(-d*e + (d*x*e + c*e)*b/(b*x + a)) + 3*B*a^2*b*c*d^4*g*e^3
*log(-d*e + (d*x*e + c*e)*b/(b*x + a)) - B*a^3*d^5*g*e^3*log(-d*e + (d*x*e
+ c*e)*b/(b*x + a)) - 2*(d*x*e + c*e)*B*b^4*c^3*d*g*e^2*log(-d*e + (d*x*e +
 c*e)*b/(b*x + a))/(b*x + a) + 6*(d*x*e + c*e)*B*a*b^3*c^2*d^2*g*e^2*log(-d
*e + (d*x*e + c*e)*b/(b*x + a))/(b*x + a) - 6*(d*x*e + c*e)*B*a^2*b^2*c*d^3
*g*e^2*log(-d*e + (d*x*e + c*e)*b/(b*x + a))/(b*x + a) + 2*(d*x*e + c*e)*B*
a^3*b*d^4*g*e^2*log(-d*e + (d*x*e + c*e)*b/(b*x + a))/(b*x + a) + (d*x*e +
c*e)^2*B*b^5*c^3*g*e*log(-d*e + (d*x*e + c*e)*b/(b*x + a))/(b*x + a)^2 - 3*
(d*x*e + c*e)^2*B*a*b^4*c^2*d*g*e*log(-d*e + (d*x*e + c*e)*b/(b*x + a))/(b*
x + a)^2 + 3*(d*x*e + c*e)^2*B*a^2*b^3*c*d^2*g*e*log(-d*e + (d*x*e + c*e)*b
/(b*x + a))/(b*x + a)^2 - (d*x*e + c*e)^2*B*a^3*b^2*d^3*g*e*log(-d*e + (d*x
*e + c*e)*b/(b*x + a))/(b*x + a)^2 + 2*(d*x*e + c*e)*B*b^4*c^3*d*g*e^2*log(
(d*x*e + c*e)/(b*x + a))/(b*x + a) - 6*(d*x*e + c*e)*B*a*b^3*c^2*d^2*g*e^2*
log((d*x*e + c*e)/(b*x + a))/(b*x + a) + 6*(d*x*e + c*e)*B*a^2*b^2*c*d^3*g*
e^2*log((d*x*e + c*e)/(b*x + a))/(b*x + a) - 2*(d*x*e + c*e)*B*a^3*b*d^4*g*
e^2*log((d*x*e + c*e)/(b*x + a))/(b*x + a) - (d*x*e + c*e)^2*B*b^5*c^3*g*e*
log((d*x*e + c*e)/(b*x + a))/(b*x + a)^2 + 3*(d*x*e + c*e)^2*B*a*b^4*c^2*d*
g*e*log((d*x*e + c*e)/(b*x + a))/(b*x + a)^2 - 3*(d*x*e + c*e)^2*B*a^2*b^3*
c*d^2*g*e*log((d*x*e + c*e)/(b*x + a))/(b*x + a)^2 + (d*x*e + c*e)^2*B*a^3*
b^2*d^3*g*e*log((d*x*e + c*e)/(b*x + a))/(b*x + a)^2 + A*b^3*c^3*d^2*g*e^3
- B*b^3*c^3*d^2*g*e^3 - 3*A*a*b^2*c^2*d^3*g*e^3 + 3*B*a*b^2*c^2*d^3*g*e^3 +
 3*A*a^2*b*c*d^4*g*e^3 - 3*B*a^2*b*c*d^4*g*e^3 - A*a^3*d^5*g*e^3 + B*a^3*d^
5*g*e^3 + (d*x*e + c*e)*B*b^4*c^3*d*g*e^2/(b*x + a) - 3*(d*x*e + c*e)*B*a*b
^3*c^2*d^2*g*e^2/(b*x + a) + 3*(d*x*e + c*e)*B*a^2*b^2*c*d^3*g*e^2/(b*x + a
) - (d*x*e + c*e)*B*a^3*b*d^4*g*e^2/(b*x + a))*(b*c/((b*c*e - a*d*e)*(b*c -
```

```
 a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))/(b*d^4*e^2 - 2*(d*x*e + c*e)*b^
2*d^3*e/(b*x + a) + (d*x*e + c*e)^2*b^3*d^2/(b*x + a)^2)
```

Mupad [B]
time = 4.31, size = 126, normalized size = 1.56

$$x\left(\frac{g\,(4\,A\,a\,d+2\,A\,b\,c-B\,a\,d+B\,b\,c)}{2\,d}-\frac{A\,g\,(2\,a\,d+2\,b\,c)}{2\,d}\right)+\ln\left(\frac{e\,(c+d\,x)}{a+b\,x}\right)\,\left(\frac{B\,b\,g\,x^2}{2}+B\,a\,g\,x\right)-\frac{\ln(c+d\,x)\,\left(B\,b\,c^2\,g-2\,B\,a\,c\,d\,g\right)}{2\,d^2}+\frac{A\,b\,g\,x^2}{2}-\frac{B\,a^2\,g\,\ln(a+b\,x)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*g + b*g*x)*(A + B*log((e*(c + d*x))/(a + b*x))),x)
```

```
[Out] x*((g*(4*A*a*d + 2*A*b*c - B*a*d + B*b*c))/(2*d) - (A*g*(2*a*d + 2*b*c))/(2
*d)) + log((e*(c + d*x))/(a + b*x))*((B*b*g*x^2)/2 + B*a*g*x) - (log(c + d*
x)*(B*b*c^2*g - 2*B*a*c*d*g))/(2*d^2) + (A*b*g*x^2)/2 - (B*a^2*g*log(a + b*
x))/(2*b)
```

3.177 $\int \frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{ag+bgx}\,dx$

Optimal. Leaf size=81

$$-\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{bg}-\frac{B\text{Li}_2\left(1+\frac{bc-ad}{d(a+bx)}\right)}{bg}$$

```
[Out] -ln((a*d-b*c)/d/(b*x+a))*(A+B*ln(e*(d*x+c)/(b*x+a)))/b/g-B*polylog(2,1+(-a*
d+b*c)/d/(b*x+a))/b/g
```

Rubi [A]

time = 0.14, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2544, 2458, 2378, 2370, 2352}

$$-\frac{B\text{PolyLog}\left(2,\frac{bc-ad}{d(a+bx)}+1\right)}{bg}-\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{bg}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(c + d*x))/(a + b*x)])/(a*g + b*g*x),x]
```

```
[Out] -((Log[-((b*c - a*d)/(d*(a + b*x)))]*(A + B*Log[(e*(c + d*x))/(a + b*x)]))/
(b*g)) - (B*PolyLog[2, 1 + (b*c - a*d)/(d*(a + b*x))])/(b*g)
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2370

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)/(x_))^(q_.)*(
x_)^(m_.), x_Symbol] :> Int[(e + d*x)^q*(a + b*Log[c*x^n])^p, x] /; FreeQ[{
a, b, c, d, e, m, n, p}, x] && EqQ[m, q] && IntegerQ[q]
```

Rule 2378

```
Int[((a_.) + Log[(c_.)*(x_)^(n_)]*(b_.))/((x_)*((d_) + (e_.)*(x_)^(r_.))),
x_Symbol] :> Dist[1/n, Subst[Int[(a + b*Log[c*x])/(x*(d + e*x^(r/n))), x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IntegerQ[r/n]
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
```

```
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2544

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))/((f_.) + (g_.)*(x_)), x_Symbol] :> Simp[(-Log[(b*c - a*d)/(b*(c +
 d*x))])*((A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/g), x] + Dist[B*n*((b*c
- a*d)/g), Int[Log[(b*c - a*d)/(b*(c + d*x))]/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[n + mn, 0] && NeQ[b*c -
 a*d, 0] && EqQ[d*f - c*g, 0]
```

Rubi steps

$$\int \frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{ag+bgx}\,dx = \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)\log(ag+bgx)}{bg} - \frac{B\int \frac{(a+bx)\left(\frac{de}{a+bx}-\frac{be(c+dx)}{(a+bx)^2}\right)\log(ag+bgx)}{e(c+dx)}\,dx}{bg}$$

$$= \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)\log(ag+bgx)}{bg} - \frac{B\int \frac{(a+bx)\left(\frac{de}{a+bx}-\frac{be(c+dx)}{(a+bx)^2}\right)\log(ag+bgx)}{c+dx}\,dx}{beg}$$

$$= \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)\log(ag+bgx)}{bg} - \frac{B\int\left(-\frac{be\log(ag+bgx)}{a+bx}+\frac{de\log(ag+bgx)}{c+dx}\right)dx}{beg}$$

$$= \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)\log(ag+bgx)}{bg} + \frac{B\int\frac{\log(ag+bgx)}{a+bx}\,dx}{g} - \frac{(Bd)\int\frac{\log(ag+bgx)}{c+dx}}{bg}$$

$$= -\frac{B\log\left(\frac{b(c+dx)}{bc-ad}\right)\log(ag+bgx)}{bg} + \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)\log(ag+bgx)}{bg} + B\int$$

$$= -\frac{B\log\left(\frac{b(c+dx)}{bc-ad}\right)\log(ag+bgx)}{bg} + \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)\log(ag+bgx)}{bg} + \frac{BS}{}$$

$$= \frac{B\log^2(g(a+bx))}{2bg} - \frac{B\log\left(\frac{b(c+dx)}{bc-ad}\right)\log(ag+bgx)}{bg} + \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)\log}{bg}$$

Mathematica [A]
time = 0.03, size = 95, normalized size = 1.17

$$\frac{\log(g(a+bx))\left(B\log(g(a+bx))+2\left(A-B\log\left(\frac{b(c+dx)}{bc-ad}\right)+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)\right)-2B\text{Li}_2\left(\frac{d(a+bx)}{-bc+ad}\right)}{2bg}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(c + d*x))/(a + b*x)])/(a*g + b*g*x),x]
```

```
[Out] (Log[g*(a + b*x)]*(B*Log[g*(a + b*x)] + 2*(A - B*Log[(b*(c + d*x))/(b*c - a
*d)] + B*Log[(e*(c + d*x))/(a + b*x)])) - 2*B*PolyLog[2, (d*(a + b*x))/(-(b
*c) + a*d)])/(2*b*g)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 227 vs. $2(80) = 160$.
time = 0.86, size = 228, normalized size = 2.81

method	result
risch	$\frac{A\ln(bx+a)}{gb}-\frac{B\,\mathrm{dilog}\left(-\frac{\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)b-ed}{ed}\right)}{gb}-\frac{B\ln\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)\ln\left(-\frac{\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)b-ed}{ed}\right)}{gb}$
derivativedivides	$\frac{e(ad-cb)\left(-\frac{bA\ln\left(\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)b-ed\right)}{ge(ad-cb)}-\frac{bB\,\mathrm{dilog}\left(-\frac{\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)b-ed}{ed}\right)}{ge(ad-cb)}-\frac{bB\ln\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)\ln\left(-\frac{\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)b-ed}{ed}\right)}{ge(ad-cb)}\right)}{b^2}$
default	$\frac{e(ad-cb)\left(-\frac{bA\ln\left(\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)b-ed\right)}{ge(ad-cb)}-\frac{bB\,\mathrm{dilog}\left(-\frac{\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)b-ed}{ed}\right)}{ge(ad-cb)}-\frac{bB\ln\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)\ln\left(-\frac{\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)b-ed}{ed}\right)}{ge(ad-cb)}\right)}{b^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*(d*x+c)/(b*x+a)))/(b*g*x+a*g),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^2*e*(a*d-b*c)*(-b/g/e/(a*d-b*c)*A*ln((d*e/b-e*(a*d-b*c)/b/(b*x+a))*b-e*
d)-b/g/e/(a*d-b*c)*B*dilog(-((d*e/b-e*(a*d-b*c)/b/(b*x+a))*b-e*d)/e/d)-b/g/
e/(a*d-b*c)*B*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))*ln(-((d*e/b-e*(a*d-b*c)/b/(b*
x+a))*b-e*d)/e/d))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(d*x+c)/(b*x+a)))/(b*g*x+a*g),x, algorithm="maxima")
```

```
[Out] B*(log(b*x + a)*log(d*x + c)/(b*g) - integrate(-(b*d*x + b*c - (2*b*d*x + b
*c + a*d)*log(b*x + a))/(b^2*d*g*x^2 + a*b*c*g + (b^2*c*g + a*b*d*g)*x), x)
) + A*log(b*g*x + a*g)/(b*g)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(d*x+c)/(b*x+a)))/(b*g*x+a*g),x, algorithm="fricas")
[Out] integral((B*log((d*x + c)*e/(b*x + a)) + A)/(b*g*x + a*g), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A}{a+bx}\,dx + \int \frac{B\log\left(\frac{ce}{a+bx}+\frac{dex}{a+bx}\right)}{a+bx}\,dx}{g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(d*x+c)/(b*x+a)))/(b*g*x+a*g),x)
[Out] (Integral(A/(a + b*x), x) + Integral(B*log(c*e/(a + b*x) + d*e*x/(a + b*x))
/(a + b*x), x))/g
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1368 vs. 2(81) = 162.
time = 43.16, size = 1368, normalized size = 16.89

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(d*x+c)/(b*x+a)))/(b*g*x+a*g),x, algorithm="giac")
[Out] -1/2*(B*b^3*c^3*d^2*e^3*log(-d*e + (d*x*e + c*e)*b/(b*x + a)) - 3*B*a*b^2*c
^2*d^3*e^3*log(-d*e + (d*x*e + c*e)*b/(b*x + a)) + 3*B*a^2*b*c*d^4*e^3*log(
-d*e + (d*x*e + c*e)*b/(b*x + a)) - B*a^3*d^5*e^3*log(-d*e + (d*x*e + c*e)*
b/(b*x + a)) - 2*(d*x*e + c*e)*B*b^4*c^3*d*e^2*log(-d*e + (d*x*e + c*e)*b/(
b*x + a))/(b*x + a) + 6*(d*x*e + c*e)*B*a*b^3*c^2*d^2*e^2*log(-d*e + (d*x*e
 + c*e)*b/(b*x + a))/(b*x + a) - 6*(d*x*e + c*e)*B*a^2*b^2*c*d^3*e^2*log(-d
*e + (d*x*e + c*e)*b/(b*x + a))/(b*x + a) + 2*(d*x*e + c*e)*B*a^3*b*d^4*e^2
*log(-d*e + (d*x*e + c*e)*b/(b*x + a))/(b*x + a) + (d*x*e + c*e)^2*B*b^5*c^
3*e*log(-d*e + (d*x*e + c*e)*b/(b*x + a))/(b*x + a)^2 - 3*(d*x*e + c*e)^2*B
*a*b^4*c^2*d*e*log(-d*e + (d*x*e + c*e)*b/(b*x + a))/(b*x + a)^2 + 3*(d*x*e
 + c*e)^2*B*a^2*b^3*c*d^2*e*log(-d*e + (d*x*e + c*e)*b/(b*x + a))/(b*x + a)
^2 - (d*x*e + c*e)^2*B*a^3*b^2*d^3*e*log(-d*e + (d*x*e + c*e)*b/(b*x + a))/
(b*x + a)^2 + 2*(d*x*e + c*e)*B*b^4*c^3*d*e^2*log((d*x*e + c*e)/(b*x + a))/
(b*x + a) - 6*(d*x*e + c*e)*B*a*b^3*c^2*d^2*e^2*log((d*x*e + c*e)/(b*x + a)
```

```
)/(b*x + a) + 6*(d*x*e + c*e)*B*a^2*b^2*c*d^3*e^2*log((d*x*e + c*e)/(b*x +
a))/(b*x + a) - 2*(d*x*e + c*e)*B*a^3*b*d^4*e^2*log((d*x*e + c*e)/(b*x + a)
)/(b*x + a) - (d*x*e + c*e)^2*B*b^5*c^3*e*log((d*x*e + c*e)/(b*x + a))/(b*x
 + a)^2 + 3*(d*x*e + c*e)^2*B*a*b^4*c^2*d*e*log((d*x*e + c*e)/(b*x + a))/(b
*x + a)^2 - 3*(d*x*e + c*e)^2*B*a^2*b^3*c*d^2*e*log((d*x*e + c*e)/(b*x + a)
)/(b*x + a)^2 + (d*x*e + c*e)^2*B*a^3*b^2*d^3*e*log((d*x*e + c*e)/(b*x + a)
)/(b*x + a)^2 + A*b^3*c^3*d^2*e^3 - B*b^3*c^3*d^2*e^3 - 3*A*a*b^2*c^2*d^3*e
^3 + 3*B*a*b^2*c^2*d^3*e^3 + 3*A*a^2*b*c*d^4*e^3 - 3*B*a^2*b*c*d^4*e^3 - A*
a^3*d^5*e^3 + B*a^3*d^5*e^3 + (d*x*e + c*e)*B*b^4*c^3*d*e^2/(b*x + a) - 3*(
d*x*e + c*e)*B*a*b^3*c^2*d^2*e^2/(b*x + a) + 3*(d*x*e + c*e)*B*a^2*b^2*c*d^
3*e^2/(b*x + a) - (d*x*e + c*e)*B*a^3*b*d^4*e^2/(b*x + a))*(b*c/((b*c*e - a
*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))^2/(b*d^4*g*e^2 - 2*
(d*x*e + c*e)*b^2*d^3*g*e/(b*x + a) + (d*x*e + c*e)^2*b^3*d^2*g/(b*x + a)^2
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\,\ln\left(\frac{e\,\left(c+d\,x\right)}{a+b\,x}\right)}{a\,g+b\,g\,x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(c + d*x))/(a + b*x)))/(a*g + b*g*x),x)
[Out] int((A + B*log((e*(c + d*x))/(a + b*x)))/(a*g + b*g*x), x)
```

3.178 $\int \frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{(ag+bgx)^2}\,dx$

Optimal. Leaf size=64

$$-\frac{A-B}{bg^2(a+bx)}-\frac{B(c+dx)\log\left(\frac{e(c+dx)}{a+bx}\right)}{(bc-ad)g^2(a+bx)}$$

```
[Out] (-A+B)/b/g^2/(b*x+a)-B*(d*x+c)*ln(e*(d*x+c)/(b*x+a))/(-a*d+b*c)/g^2/(b*x+a)
```

Rubi [A]
time = 0.03, antiderivative size = 99, normalized size of antiderivative = 1.55, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2552, 2332}

$$-\frac{A(c+dx)}{g^2(a+bx)(bc-ad)}-\frac{B(c+dx)\log\left(\frac{e(c+dx)}{a+bx}\right)}{g^2(a+bx)(bc-ad)}+\frac{B(c+dx)}{g^2(a+bx)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(c + d*x))/(a + b*x)])/(a*g + b*g*x)^2,x]
```

```
[Out] -((A*(c + d*x))/((b*c - a*d)*g^2*(a + b*x))) + (B*(c + d*x))/((b*c - a*d)*g
^2*(a + b*x)) - (B*(c + d*x)*Log[(e*(c + d*x))/(a + b*x)])/((b*c - a*d)*g^2
*(a + b*x))
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2552

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(
m + 1)*(g/d)^m, Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a
 + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[n
 + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[d*f
- c*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Rubi steps

$$\int \frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{(ag+bgx)^2}\,dx = -\frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{bg^2(a+bx)} + \frac{B\int\frac{-bc+ad}{g(a+bx)^2(c+dx)}\,dx}{bg}$$

$$= -\frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{bg^2(a+bx)} - \frac{(B(bc-ad))\int\frac{1}{(a+bx)^2(c+dx)}\,dx}{bg^2}$$

$$= -\frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{bg^2(a+bx)} - \frac{(B(bc-ad))\int\left(\frac{b}{(bc-ad)(a+bx)^2} - \frac{bd}{(bc-ad)^2(a+bx)} + \frac{d^2}{(bc-ad)^2}\right.}{bg^2}$$

$$= \frac{B}{bg^2(a+bx)} + \frac{Bd\log(a+bx)}{b(bc-ad)g^2} - \frac{Bd\log(c+dx)}{b(bc-ad)g^2} - \frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{bg^2(a+bx)}$$

Mathematica [A]
time = 0.04, size = 86, normalized size = 1.34

$$\frac{Bd(a+bx)\log(a+bx) - Bd(a+bx)\log(c+dx) - (bc-ad)\left(A-B+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{b(bc-ad)g^2(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(c + d*x))/(a + b*x)])/(a*g + b*g*x)^2,x]
```

```
[Out] (B*d*(a + b*x)*Log[a + b*x] - B*d*(a + b*x)*Log[c + d*x] - (b*c - a*d)*(A -
 B + B*Log[(e*(c + d*x))/(a + b*x)]))/(b*(b*c - a*d)*g^2*(a + b*x))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 170 vs. 2(63) = 126.
time = 0.36, size = 171, normalized size = 2.67

method	result	size
norman	$\frac{\frac{(A-B)x}{ga}+\frac{Bc\ln\left(\frac{e(dx+c)}{bx+a}\right)}{g(ad-cb)}+\frac{dBx\ln\left(\frac{e(dx+c)}{bx+a}\right)}{g(ad-cb)}}{g(bx+a)}$	89
risch	$-\frac{B\ln\left(\frac{e(dx+c)}{bx+a}\right)}{b\,g^2(bx+a)}-\frac{B\ln(bx+a)bdx-B\ln(-dx-c)bdx+B\ln(bx+a)ad-B\ln(-dx-c)ad+Aad-Abc-Bad+Bbc}{g^2(bx+a)b(ad-cb)}$	127
derivativedivides	$\frac{e(ad-cb)\left(\frac{b^2A\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)}{(ad-cb)^2e^2g^2}+\frac{b^2B\left(\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)\ln\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)+\frac{e(ad-cb)}{b(bx+a)}-\frac{de}{b}\right)}{(ad-cb)^2e^2g^2}\right)}{b^2}$	171
default	$\frac{e(ad-cb)\left(\frac{b^2A\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)}{(ad-cb)^2e^2g^2}+\frac{b^2B\left(\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)\ln\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)+\frac{e(ad-cb)}{b(bx+a)}-\frac{de}{b}\right)}{(ad-cb)^2e^2g^2}\right)}{b^2}$	171

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*(d*x+c)/(b*x+a)))/(b*g*x+a*g)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^2*e*(a*d-b*c)*(b^2/(a*d-b*c)^2/e^2/g^2*A*(d*e/b-e*(a*d-b*c)/b/(b*x+a))+
b^2/(a*d-b*c)^2/e^2/g^2*B*((d*e/b-e*(a*d-b*c)/b/(b*x+a))*ln(d*e/b-e*(a*d-b*
c)/b/(b*x+a))+e*(a*d-b*c)/b/(b*x+a)-d*e/b))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 136 vs. $2(65) = 130$.
time = 0.29, size = 136, normalized size = 2.12

$$-B\left(\frac{\log\left(\frac{dxe}{bx+a}+\frac{ce}{bx+a}\right)}{b^2g^2x+abg^2}-\frac{1}{b^2g^2x+abg^2}-\frac{d\log\left(bx+a\right)}{(b^2c-abd)g^2}+\frac{d\log\left(dx+c\right)}{(b^2c-abd)g^2}\right)-\frac{A}{b^2g^2x+abg^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(d*x+c)/(b*x+a)))/(b*g*x+a*g)^2,x, algorithm="maxima")
```

```
[Out] -B*(log(d*x*e/(b*x + a) + c*e/(b*x + a))/(b^2*g^2*x + a*b*g^2) - 1/(b^2*g^2
*x + a*b*g^2) - d*log(b*x + a)/((b^2*c - a*b*d)*g^2) + d*log(d*x + c)/((b^2
*c - a*b*d)*g^2)) - A/(b^2*g^2*x + a*b*g^2)
```

Fricas [A]
time = 0.34, size = 86, normalized size = 1.34

$$-\frac{(A-B)bc-(A-B)ad+(Bbdx+Bbc)\log\left(\frac{(dx+c)e}{bx+a}\right)}{(b^3c-ab^2d)g^2x+(ab^2c-a^2bd)g^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(d*x+c)/(b*x+a)))/(b*g*x+a*g)^2,x, algorithm="fricas")
```

```
[Out] -((A - B)*b*c - (A - B)*a*d + (B*b*d*x + B*b*c)*log((d*x + c)*e/(b*x + a)))
/((b^3*c - a*b^2*d)*g^2*x + (a*b^2*c - a^2*b*d)*g^2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 231 vs. $2(48) = 96$.
time = 1.02, size = 231, normalized size = 3.61

$$-\frac{B\log\left(\frac{e(c+dx)}{a+bx}\right)}{abg^2+b^2g^2x}+\frac{Bd\log\left(x+\frac{-\frac{Ba^2d^3}{ad-bc}+\frac{2Babcd^2}{ad-bc}+Bad^2-\frac{Bb^2c^2d}{ad-bc}+Bbcd}{2Bbd^2}\right)}{bg^2\left(ad-bc\right)}-\frac{Bd\log\left(x+\frac{\frac{Ba^2d^3}{ad-bc}-\frac{2Babcd^2}{ad-bc}+Bad^2+\frac{Bb^2c^2d}{ad-bc}+Bbcd}{2Bbd^2}\right)}{bg^2\left(ad-bc\right)}+\frac{-A+B}{abg^2+b^2g^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(d*x+c)/(b*x+a)))/(b*g*x+a*g)**2,x)
```

```
[Out] -B*log(e*(c + d*x)/(a + b*x))/(a*b*g**2 + b**2*g**2*x) + B*d*log(x + (-B*a*
*2*d**3/(a*d - b*c) + 2*B*a*b*c*d**2/(a*d - b*c) + B*a*d**2 - B*b**2*c**2*d
/(a*d - b*c) + B*b*c*d)/(2*B*b*d**2))/(b*g**2*(a*d - b*c)) - B*d*log(x + (B
*a**2*d**3/(a*d - b*c) - 2*B*a*b*c*d**2/(a*d - b*c) + B*a*d**2 + B*b**2*c**
```

```
2*d/(a*d - b*c) + B*b*c*d)/(2*B*b*d**2))/(b*g**2*(a*d - b*c)) + (-A + B)/(a
*b*g**2 + b**2*g**2*x)
```

Giac [A]
time = 3.09, size = 126, normalized size = 1.97

$$-\left(\frac{bc}{(bce-ade)(bc-ad)}-\frac{ad}{(bce-ade)(bc-ad)}\right)\left(\frac{(dxe+ce)B\log\left(\frac{dxe+ce}{bx+a}\right)}{(bx+a)g^2}+\frac{(dxe+ce)(A-B)}{(bx+a)g^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(d*x+c)/(b*x+a)))/(b*g*x+a*g)^2,x, algorithm="giac")
```

```
[Out] -(b*c/((b*c*e - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))*((
d*x*e + c*e)*B*log((d*x*e + c*e)/(b*x + a))/((b*x + a)*g^2) + (d*x*e + c*e)
*(A - B)/((b*x + a)*g^2))
```

Mupad [B]
time = 5.01, size = 106, normalized size = 1.66

$$-\frac{A-B}{x\,b^2\,g^2+a\,b\,g^2}-\frac{B\,\ln\left(\frac{e\,(c+d\,x)}{a+b\,x}\right)}{b^2\,g^2\,\left(x+\frac{a}{b}\right)}+\frac{B\,d\,\mathrm{atan}\left(\frac{b\,c\,2\mathrm{i}+b\,d\,x\,2\mathrm{i}}{a\,d-b\,c}+1\mathrm{i}\right)\,2\mathrm{i}}{b\,g^2\,\left(a\,d-b\,c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(c + d*x))/(a + b*x)))/(a*g + b*g*x)^2,x)
```

```
[Out] (B*d*atan((b*c*2i + b*d*x*2i)/(a*d - b*c) + 1i)*2i)/(b*g^2*(a*d - b*c)) - (
B*log((e*(c + d*x))/(a + b*x)))/(b^2*g^2*(x + a/b)) - (A - B)/(b^2*g^2*x +
a*b*g^2)
```

3.179 $\int \frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{(ag+bgx)^3}\,dx$

Optimal. Leaf size=144

$$\frac{B}{4bg^3(a+bx)^2}-\frac{Bd}{2b(bc-ad)g^3(a+bx)}-\frac{Bd^2\log(a+bx)}{2b(bc-ad)^2g^3}+\frac{Bd^2\log(c+dx)}{2b(bc-ad)^2g^3}-\frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{2bg^3(a+bx)^2}$$

```
[Out] 1/4*B/b/g^3/(b*x+a)^2-1/2*B*d/b/(-a*d+b*c)/g^3/(b*x+a)-1/2*B*d^2*ln(b*x+a)/
b/(-a*d+b*c)^2/g^3+1/2*B*d^2*ln(d*x+c)/b/(-a*d+b*c)^2/g^3+1/2*(-A-B*ln(e*(d
*x+c)/(b*x+a)))/b/g^3/(b*x+a)^2
```

Rubi [A]
time = 0.08, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2548, 21, 46}

$$-\frac{B\log\left(\frac{e(c+dx)}{a+bx}\right)+A}{2bg^3(a+bx)^2}-\frac{Bd^2\log(a+bx)}{2bg^3(bc-ad)^2}+\frac{Bd^2\log(c+dx)}{2bg^3(bc-ad)^2}-\frac{Bd}{2bg^3(a+bx)(bc-ad)}+\frac{B}{4bg^3(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(c + d*x))/(a + b*x)])/(a*g + b*g*x)^3,x]
```

```
[Out] B/(4*b*g^3*(a + b*x)^2) - (B*d)/(2*b*(b*c - a*d)*g^3*(a + b*x)) - (B*d^2*Lo
g[a + b*x])/(2*b*(b*c - a*d)^2*g^3) + (B*d^2*Log[c + d*x])/(2*b*(b*c - a*d)
^2*g^3) - (A + B*Log[(e*(c + d*x))/(a + b*x)])/(2*b*g^3*(a + b*x)^2)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2548

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
```

```
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Dist[B*n*((b*c
- a*d)/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /;
 FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c -
 a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{(ag+bgx)^3}\,dx &= -\frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{2bg^3(a+bx)^2}+\frac{B\int \frac{-bc+ad}{g^2(a+bx)^3(c+dx)}\,dx}{2bg}\\ &= -\frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{2bg^3(a+bx)^2}-\frac{(B(bc-ad))\int \frac{1}{(a+bx)^3(c+dx)}\,dx}{2bg^3}\\ &= -\frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{2bg^3(a+bx)^2}-\frac{(B(bc-ad))\int \left(\frac{b}{(bc-ad)(a+bx)^3}-\frac{bd}{(bc-ad)^2(a+bx)^2}+\frac{b}{(bc-ad)}\right.}{2bg^3}\\ &= \frac{B}{4bg^3(a+bx)^2}-\frac{Bd}{2b(bc-ad)g^3(a+bx)}-\frac{Bd^2\log(a+bx)}{2b(bc-ad)^2g^3}+\frac{Bd^2\log(c+dx)}{2b(bc-ad)^2g^3}-\end{aligned}$$

Mathematica [A]
time = 0.07, size = 128, normalized size = 0.89

$$-\frac{2Bd^2(a+bx)^2\log(a+bx)-2Bd^2(a+bx)^2\log(c+dx)+(bc-ad)\left(2Abc-bBc-2aAd+3aBd+2bBdx+2B(bc-ad)\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{4b(bc-ad)^2g^3(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(c + d*x))/(a + b*x)])/(a*g + b*g*x)^3,x]
[Out] -1/4*(2*B*d^2*(a + b*x)^2*Log[a + b*x] - 2*B*d^2*(a + b*x)^2*Log[c + d*x] +
 (b*c - a*d)*(2*A*b*c - b*B*c - 2*a*A*d + 3*a*B*d + 2*b*B*d*x + 2*B*(b*c -
a*d)*Log[(e*(c + d*x))/(a + b*x)]))/(b*(b*c - a*d)^2*g^3*(a + b*x)^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 340 vs. $2(137) = 274$.
time = 0.39, size = 341, normalized size = 2.37

method	result
norman	$\frac{\frac{B a d^2 x \ln\left(\frac{e(dx+c)}{bx+a}\right)}{\left(a^2d^2-2abcd+b^2c^2\right)g}-\frac{2Aabd-2Ab^2c-3Babd+Bb^2c}{4g\,b^2(ad-cb)}+\frac{Bdx}{2g(ad-cb)}+\frac{Bc(2ad-cb)\ln\left(\frac{e(dx+c)}{bx+a}\right)}{2g\left(a^2d^2-2abcd+b^2c^2\right)}+\frac{d^2Bbx^2\ln\left(\frac{e(dx+c)}{bx+a}\right)}{2\left(a^2d^2-2abcd+b^2c^2\right)g}}{g^2(bx+a)^2}$
risch	$-\frac{B\ln\left(\frac{e(dx+c)}{bx+a}\right)}{2b\,g^3(bx+a)^2}-\frac{-2B\ln(-dx-c)b^2d^2x^2+2B\ln(bx+a)b^2d^2x^2-4B\ln(-dx-c)ab\,d^2x+4B\ln(bx+a)ab\,d^2x-2B\ln(-}{4(a^2d^2-2a}$

derivativedivides	$\frac{e(ad-cb)\left(-\frac{b^3A\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)^2}{2(ad-cb)^3e^3g^3}+\frac{b^2Ad\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)}{(ad-cb)^3e^2g^3}-\frac{b^3B\left(\frac{\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)^2\ln\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)}{2}-\frac{\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)}{4}\right.}{(ad-cb)^3e^3g^3}\right.}{b^2}$
default	$\frac{e(ad-cb)\left(-\frac{b^3A\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)^2}{2(ad-cb)^3e^3g^3}+\frac{b^2Ad\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)}{(ad-cb)^3e^2g^3}-\frac{b^3B\left(\frac{\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)^2\ln\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)}{2}-\frac{\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)}{4}\right.}{(ad-cb)^3e^3g^3}\right.}{b^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*(d*x+c)/(b*x+a)))/(b*g*x+a*g)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^2*e*(a*d-b*c)*(-1/2*b^3/(a*d-b*c)^3/e^3/g^3*A*(d*e/b-e*(a*d-b*c)/b/(b*x
+a))^2+b^2/(a*d-b*c)^3/e^2/g^3*A*d*(d*e/b-e*(a*d-b*c)/b/(b*x+a))-b^3/(a*d-b
*c)^3/e^3/g^3*B*(1/2*(d*e/b-e*(a*d-b*c)/b/(b*x+a))^2*ln(d*e/b-e*(a*d-b*c)/b
/(b*x+a))-1/4*(d*e/b-e*(a*d-b*c)/b/(b*x+a))^2)+b^2/(a*d-b*c)^3/e^2/g^3*B*d*
((d*e/b-e*(a*d-b*c)/b/(b*x+a))*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))+e*(a*d-b*c)/
b/(b*x+a)-d*e/b))
```

Maxima [A]
time = 0.28, size = 257, normalized size = 1.78

$$-\frac{1}{4}B\left(\frac{2\,bdx-bc+3\,ad}{(b^4c-ab^3d)g^3x^2+2\,(ab^3c-a^2b^2d)g^3x+(a^2b^2c-a^3bd)g^3}+\frac{2\,\log\left(\frac{dxe}{bx+a}+\frac{ce}{bx+a}\right)}{b^3g^3x^2+2\,ab^2g^3x+a^2bg^3}+\frac{2\,d^2\log\left(bx+a\right)}{(b^3c^2-2\,ab^2cd+a^2bd^2)g^3}-\frac{2\,d^2\log\left(dx+c\right)}{(b^3c^2-2\,ab^2cd+a^2bd^2)g^3}\right)-\frac{A}{2\,(b^3g^3x^2+2\,ab^2g^3x+a^2bg^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(d*x+c)/(b*x+a)))/(b*g*x+a*g)^3,x, algorithm="maxima")
```

```
[Out] -1/4*B*((2*b*d*x - b*c + 3*a*d)/((b^4*c - a*b^3*d)*g^3*x^2 + 2*(a*b^3*c - a
^2*b^2*d)*g^3*x + (a^2*b^2*c - a^3*b*d)*g^3) + 2*log(d*x*e/(b*x + a) + c*e/
(b*x + a))/(b^3*g^3*x^2 + 2*a*b^2*g^3*x + a^2*b*g^3) + 2*d^2*log(b*x + a)/(
(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*g^3) - 2*d^2*log(d*x + c)/((b^3*c^2 - 2
*a*b^2*c*d + a^2*b*d^2)*g^3)) - 1/2*A/(b^3*g^3*x^2 + 2*a*b^2*g^3*x + a^2*b*
g^3)
```

Fricas [A]
time = 0.38, size = 220, normalized size = 1.53

$$-\frac{(2\,A-B)b^2c^2-4\,(A-B)abcd+(2\,A-3\,B)a^2d^2+2\,(Bb^2cd-Babd^2)x-2\,(Bb^2d^2x^2+2\,Babd^2x-Bb^2c^2+2\,Babcd)\log\left(\frac{(dx+c)e}{bx+a}\right)}{4\,((b^5c^2-2\,ab^4cd+a^2b^3d^2)g^3x^2+2\,(ab^4c^2-2\,a^2b^3cd+a^3b^2d^2)g^3x+(a^2b^3c^2-2\,a^3b^2cd+a^4bd^2)g^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(d*x+c)/(b*x+a)))/(b*g*x+a*g)^3,x, algorithm="fricas")
```

```
[Out] -1/4*((2*A - B)*b^2*c^2 - 4*(A - B)*a*b*c*d + (2*A - 3*B)*a^2*d^2 + 2*(B*b^
2*c*d - B*a*b*d^2)*x - 2*(B*b^2*d^2*x^2 + 2*B*a*b*d^2*x - B*b^2*c^2 + 2*B*a
*b*c*d)*log((d*x + c)*e/(b*x + a)))/((b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*
g^3*x^2 + 2*(a*b^4*c^2 - 2*a^2*b^3*c*d + a^3*b^2*d^2)*g^3*x + (a^2*b^3*c^2
- 2*a^3*b^2*c*d + a^4*b*d^2)*g^3)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 422 vs. $2(124) = 248$.
time = 1.58, size = 422, normalized size = 2.93

$$-\frac{B\log\left(\frac{e(c+dx)}{a+bx}\right)}{2a^2bg^3+4ab^2g^3x+2b^3g^3x^2}+\frac{Bd^2\log\left(x+\frac{-\frac{Ba^3d^5}{(ad-bc)^2}+\frac{3Ba^2bcd^4}{(ad-bc)^2}-\frac{3Bab^2c^2d^3}{(ad-bc)^2}+Bad^3+\frac{Bb^3c^3d^2}{(ad-bc)^2}+Bbcd^2}{2Bbd^3}\right)}{2bg^3(ad-bc)^2}-\frac{Bd^2\log\left(x+\frac{\frac{Ba^3d^5}{(ad-bc)^2}-\frac{3Ba^2bcd^4}{(ad-bc)^2}+\frac{3Bab^2c^2d^3}{(ad-bc)^2}+Bad^3-\frac{Bb^3c^3d^2}{(ad-bc)^2}+Bbcd^2}{2Bbd^3}\right)}{2bg^3(ad-bc)^2}+\frac{-2Aad+2Abc+3Bad-Bbc+2Bbdx}{4a^3bdg^3-4a^2b^2cg^3+x^2\cdot(4ab^3dg^3-4b^4cg^3)+x(8a^2b^2dg^3-8ab^3cg^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(d*x+c)/(b*x+a)))/(b*g*x+a*g)**3,x)
```

```
[Out] -B*log(e*(c + d*x)/(a + b*x))/(2*a**2*b*g**3 + 4*a*b**2*g**3*x + 2*b**3*g**
3*x**2) + B*d**2*log(x + (-B*a**3*d**5/(a*d - b*c)**2 + 3*B*a**2*b*c*d**4/(
a*d - b*c)**2 - 3*B*a*b**2*c**2*d**3/(a*d - b*c)**2 + B*a*d**3 + B*b**3*c**
3*d**2/(a*d - b*c)**2 + B*b*c*d**2)/(2*B*b*d**3))/(2*b*g**3*(a*d - b*c)**2)
 - B*d**2*log(x + (B*a**3*d**5/(a*d - b*c)**2 - 3*B*a**2*b*c*d**4/(a*d - b*
c)**2 + 3*B*a*b**2*c**2*d**3/(a*d - b*c)**2 + B*a*d**3 - B*b**3*c**3*d**2/(
a*d - b*c)**2 + B*b*c*d**2)/(2*B*b*d**3))/(2*b*g**3*(a*d - b*c)**2) + (-2*A
*a*d + 2*A*b*c + 3*B*a*d - B*b*c + 2*B*b*d*x)/(4*a**3*b*d*g**3 - 4*a**2*b**
2*c*g**3 + x**2*(4*a*b**3*d*g**3 - 4*b**4*c*g**3) + x*(8*a**2*b**2*d*g**3 -
 8*a*b**3*c*g**3))
```

Giac [A]
time = 2.54, size = 254, normalized size = 1.76

$$\frac{\left(\frac{4(dxe+ce)Bde\log\left(\frac{dxe+ce}{bx+a}\right)}{bx+a}+\frac{4(dxe+ce)Ade}{bx+a}-\frac{4(dxe+ce)Bde}{bx+a}-\frac{2(dxe+ce)^2Bb\log\left(\frac{dxe+ce}{bx+a}\right)}{(bx+a)^2}-\frac{2(dxe+ce)^2Ab}{(bx+a)^2}+\frac{(dxe+ce)^2Bb}{(bx+a)^2}\right)\left(\frac{bc}{(bce-ade)(bc-ad)}-\frac{ad}{(bce-ade)(bc-ad)}\right)}{4(bcg^3e-adg^3e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(d*x+c)/(b*x+a)))/(b*g*x+a*g)^3,x, algorithm="giac")
```

```
[Out] 1/4*(4*(d*x*e + c*e)*B*d*e*log((d*x*e + c*e)/(b*x + a))/(b*x + a) + 4*(d*x*
e + c*e)*A*d*e/(b*x + a) - 4*(d*x*e + c*e)*B*d*e/(b*x + a) - 2*(d*x*e + c*e
)^2*B*b*log((d*x*e + c*e)/(b*x + a))/(b*x + a)^2 - 2*(d*x*e + c*e)^2*A*b/(b
*x + a)^2 + (d*x*e + c*e)^2*B*b/(b*x + a)^2)*(b*c/((b*c*e - a*d*e)*(b*c - a
*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))/(b*c*g^3*e - a*d*g^3*e)
```

Mupad [B]
time = 5.19, size = 208, normalized size = 1.44

$$\frac{B\,d^2\,\mathrm{atanh}\left(\frac{2\,b^3\,c^2\,g^3-2\,a^2\,b\,d^2\,g^3}{2\,b\,g^3\,{\left(a\,d-b\,c\right)}^2}-\frac{2\,b\,d\,x}{a\,d-b\,c}\right)}{b\,g^3\,{\left(a\,d-b\,c\right)}^2}-\frac{B\,\ln\left(\frac{e\,\left(c+d\,x\right)}{a+b\,x}\right)}{2\,b^2\,g^3\,\left(2\,a\,x+b\,x^2+\frac{a^2}{b}\right)}-\frac{\frac{2\,A\,a\,d-2\,A\,b\,c-3\,B\,a\,d+B\,b\,c}{2\,\left(a\,d-b\,c\right)}-\frac{B\,b\,d\,x}{a\,d-b\,c}}{2\,a^2\,b\,g^3+4\,a\,b^2\,g^3\,x+2\,b^3\,g^3\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(c + d*x))/(a + b*x)))/(a*g + b*g*x)^3,x)
```

```
[Out] (B*d^2*atanh((2*b^3*c^2*g^3 - 2*a^2*b*d^2*g^3)/(2*b*g^3*(a*d - b*c)^2) - (2
*b*d*x)/(a*d - b*c)))/(b*g^3*(a*d - b*c)^2) - (B*log((e*(c + d*x))/(a + b*x
)))/(2*b^2*g^3*(2*a*x + b*x^2 + a^2/b)) - ((2*A*a*d - 2*A*b*c - 3*B*a*d + B
*b*c)/(2*(a*d - b*c)) - (B*b*d*x)/(a*d - b*c))/(2*a^2*b*g^3 + 2*b^3*g^3*x^2
 + 4*a*b^2*g^3*x)
```

3.180 $\int \frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{(ag+bgx)^4}\,dx$

Optimal. Leaf size=175

$$\frac{B}{9bg^4(a+bx)^3}-\frac{Bd}{6b(bc-ad)g^4(a+bx)^2}+\frac{Bd^2}{3b(bc-ad)^2g^4(a+bx)}+\frac{Bd^3\log(a+bx)}{3b(bc-ad)^3g^4}-\frac{Bd^3\log(c+dx)}{3b(bc-ad)^3g^4}-\frac{A+B}{3bg}$$

```
[Out] 1/9*B/b/g^4/(b*x+a)^3-1/6*B*d/b/(-a*d+b*c)/g^4/(b*x+a)^2+1/3*B*d^2/b/(-a*d+
b*c)^2/g^4/(b*x+a)+1/3*B*d^3*ln(b*x+a)/b/(-a*d+b*c)^3/g^4-1/3*B*d^3*ln(d*x+
c)/b/(-a*d+b*c)^3/g^4+1/3*(-A-B*ln(e*(d*x+c)/(b*x+a)))/b/g^4/(b*x+a)^3
```

Rubi [A]

time = 0.09, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2548, 21, 46}

$$-\frac{B\log\left(\frac{e(c+dx)}{a+bx}\right)+A}{3bg^4(a+bx)^3}+\frac{Bd^3\log(a+bx)}{3bg^4(bc-ad)^3}-\frac{Bd^3\log(c+dx)}{3bg^4(bc-ad)^3}+\frac{Bd^2}{3bg^4(a+bx)(bc-ad)^2}-\frac{Bd}{6bg^4(a+bx)^2(bc-ad)}+\frac{B}{9bg^4(a+bx)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(c + d*x))/(a + b*x)])/(a*g + b*g*x)^4,x]
```

```
[Out] B/(9*b*g^4*(a + b*x)^3) - (B*d)/(6*b*(b*c - a*d)*g^4*(a + b*x)^2) + (B*d^2)
/(3*b*(b*c - a*d)^2*g^4*(a + b*x)) + (B*d^3*Log[a + b*x])/(3*b*(b*c - a*d)^
3*g^4) - (B*d^3*Log[c + d*x])/(3*b*(b*c - a*d)^3*g^4) - (A + B*Log[(e*(c +
d*x))/(a + b*x)])/(3*b*g^4*(a + b*x)^3)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2548

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
```

```
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Dist[B*n*((b*c
- a*d)/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /;
 FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c -
 a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Rubi steps

$$\int \frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{(ag+bgx)^4}\,dx = -\frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{3bg^4(a+bx)^3}+\frac{B\int\frac{-bc+ad}{g^3(a+bx)^4(c+dx)}\,dx}{3bg}$$

$$= -\frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{3bg^4(a+bx)^3}-\frac{(B(bc-ad))\int\frac{1}{(a+bx)^4(c+dx)}\,dx}{3bg^4}$$

$$= -\frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{3bg^4(a+bx)^3}-\frac{(B(bc-ad))\int\left(\frac{b}{(bc-ad)(a+bx)^4}-\frac{bd}{(bc-ad)^2(a+bx)^3}+\frac{}{(bc-a}\right.}{3bg^4}$$

$$= \frac{B}{9bg^4(a+bx)^3}-\frac{Bd}{6b(bc-ad)g^4(a+bx)^2}+\frac{Bd^2}{3b(bc-ad)^2g^4(a+bx)}+\frac{Bd^3\log(}{3b(bc-}$$

Mathematica [A]
time = 0.10, size = 141, normalized size = 0.81

$$\frac{\frac{B\left((bc-ad)\left(11a^2d^2+abd(-7c+15dx)+b^2\left(2c^2-3cdx+6d^2x^2\right)\right)+6d^3(a+bx)^3\log(a+bx)-6d^3(a+bx)^3\log(c+dx)\right)}{(bc-ad)^3}-6\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{18bg^4(a+bx)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(c + d*x))/(a + b*x)])/(a*g + b*g*x)^4,x]
```

```
[Out] ((B*((b*c - a*d)*(11*a^2*d^2 + a*b*d*(-7*c + 15*d*x) + b^2*(2*c^2 - 3*c*d*x
 + 6*d^2*x^2)) + 6*d^3*(a + b*x)^3*Log[a + b*x] - 6*d^3*(a + b*x)^3*Log[c +
 d*x]))/(b*c - a*d)^3 - 6*(A + B*Log[(e*(c + d*x))/(a + b*x)]))/(18*b*g^4*(
a + b*x)^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 513 vs. $2(166) = 332$.
time = 0.41, size = 514, normalized size = 2.94 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*(d*x+c)/(b*x+a)))/(b*g*x+a*g)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^2*e*(a*d-b*c)*(1/3*b^4/(a*d-b*c)^4/e^4/g^4*A*(d*e/b-e*(a*d-b*c)/b/(b*x+
a))^3-b^3/(a*d-b*c)^4/e^3/g^4*A*d*(d*e/b-e*(a*d-b*c)/b/(b*x+a))^2+b^2/(a*d-
```

```
b*c)^4/e^2/g^4*A*d^2*(d*e/b-e*(a*d-b*c)/b/(b*x+a))+b^4/(a*d-b*c)^4/e^4/g^4*
B*(1/3*(d*e/b-e*(a*d-b*c)/b/(b*x+a))^3*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))-1/9*
(d*e/b-e*(a*d-b*c)/b/(b*x+a))^3)-2*b^3/(a*d-b*c)^4/e^3/g^4*B*d*(1/2*(d*e/b-
e*(a*d-b*c)/b/(b*x+a))^2*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))-1/4*(d*e/b-e*(a*d-
b*c)/b/(b*x+a))^2)+b^2/(a*d-b*c)^4/e^2/g^4*B*d^2*((d*e/b-e*(a*d-b*c)/b/(b*x
+a))*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))+e*(a*d-b*c)/b/(b*x+a)-d*e/b))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 430 vs. $2(164) = 328$.
time = 0.30, size = 430, normalized size = 2.46

$$\frac{1}{18}\,B\left(\frac{6\,b^2d^2x^2+2\,b^2c^2-7\,abcd+11\,a^2d^2-3\,(b^2cd-5\,abd^2)x}{(b^6c^2-2\,ab^5cd+a^2b^4d^2)g^4x^3+3\,(ab^5c^2-2\,a^2b^4cd+a^3b^3d^2)g^4x^2+3\,(a^2b^4c^2-2\,a^3b^3cd+a^4b^2d^2)g^4x+(a^3b^3c^2-2\,a^4b^2cd+a^5bd^2)g^4}-\frac{6\,\log\left(\frac{dxe}{bx+a}+\frac{ce}{bx+a}\right)}{b^4g^4x^3+3\,ab^3g^4x^2+3\,a^2b^2g^4x+a^3bg^4}+\frac{6\,d^3\log\left(bx+a\right)}{(b^4c^3-3\,ab^3c^2d+3\,a^2b^2cd^2-a^3bd^3)g^4}-\frac{6\,d^3\log\left(dx+c\right)}{(b^4c^3-3\,ab^3c^2d+3\,a^2b^2cd^2-a^3bd^3)g^4}\right)-\frac{A}{3\,(b^4g^4x^3+3\,ab^3g^4x^2+3\,a^2b^2g^4x+a^3bg^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(d*x+c)/(b*x+a)))/(b*g*x+a*g)^4,x, algorithm="maxima")
```

```
[Out] 1/18*B*((6*b^2*d^2*x^2 + 2*b^2*c^2 - 7*a*b*c*d + 11*a^2*d^2 - 3*(b^2*c*d -
5*a*b*d^2)*x)/((b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2)*g^4*x^3 + 3*(a*b^5*c^2
 - 2*a^2*b^4*c*d + a^3*b^3*d^2)*g^4*x^2 + 3*(a^2*b^4*c^2 - 2*a^3*b^3*c*d +
a^4*b^2*d^2)*g^4*x + (a^3*b^3*c^2 - 2*a^4*b^2*c*d + a^5*b*d^2)*g^4) - 6*log
(d*x*e/(b*x + a) + c*e/(b*x + a))/(b^4*g^4*x^3 + 3*a*b^3*g^4*x^2 + 3*a^2*b^
2*g^4*x + a^3*b*g^4) + 6*d^3*log(b*x + a)/((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2
*b^2*c*d^2 - a^3*b*d^3)*g^4) - 6*d^3*log(d*x + c)/((b^4*c^3 - 3*a*b^3*c^2*d
 + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*g^4)) - 1/3*A/(b^4*g^4*x^3 + 3*a*b^3*g^4*x^
2 + 3*a^2*b^2*g^4*x + a^3*b*g^4)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 411 vs. $2(164) = 328$.
time = 0.34, size = 411, normalized size = 2.35

$$-\frac{2\,(3\,A-B)b^3c^3-9\,(2\,A-B)ab^2c^2d+18\,(A-B)a^2bcd^2-(6\,A-11\,B)a^3d^3-6\,(Bb^3cd^2-Bab^2d^3)x^2+3\,(Bb^3c^2d-6\,Bab^2cd^2+5\,Ba^2bd^3)x+6\,(Bb^3d^3x^3+3\,Bab^2d^3x^2+3\,Ba^2bd^3x+Bb^3c^3-3\,Bab^2c^2d+3\,Ba^2bcd^2)\log\left(\frac{(dx+c)e}{bx+a}\right)}{18\,((b^7c^3-3\,ab^6c^2d+3\,a^2b^5cd^2-a^3b^4d^3)g^4x^3+3\,(ab^6c^3-3\,a^2b^5c^2d+3\,a^3b^4cd^2-a^4b^3d^3)g^4x^2+3\,(a^2b^5c^3-3\,a^3b^4c^2d+3\,a^4b^3cd^2-a^5b^2d^3)g^4x+(a^3b^4c^3-3\,a^4b^3c^2d+3\,a^5b^2cd^2-a^6bd^3)g^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(d*x+c)/(b*x+a)))/(b*g*x+a*g)^4,x, algorithm="fricas")
```

```
[Out] -1/18*(2*(3*A - B)*b^3*c^3 - 9*(2*A - B)*a*b^2*c^2*d + 18*(A - B)*a^2*b*c*d
^2 - (6*A - 11*B)*a^3*d^3 - 6*(B*b^3*c*d^2 - B*a*b^2*d^3)*x^2 + 3*(B*b^3*c^
2*d - 6*B*a*b^2*c*d^2 + 5*B*a^2*b*d^3)*x + 6*(B*b^3*d^3*x^3 + 3*B*a*b^2*d^3
*x^2 + 3*B*a^2*b*d^3*x + B*b^3*c^3 - 3*B*a*b^2*c^2*d + 3*B*a^2*b*c*d^2)*log
((d*x + c)*e/(b*x + a)))/((b^7*c^3 - 3*a*b^6*c^2*d + 3*a^2*b^5*c*d^2 - a^3*
b^4*d^3)*g^4*x^3 + 3*(a*b^6*c^3 - 3*a^2*b^5*c^2*d + 3*a^3*b^4*c*d^2 - a^4*b
^3*d^3)*g^4*x^2 + 3*(a^2*b^5*c^3 - 3*a^3*b^4*c^2*d + 3*a^4*b^3*c*d^2 - a^5*
b^2*d^3)*g^4*x + (a^3*b^4*c^3 - 3*a^4*b^3*c^2*d + 3*a^5*b^2*c*d^2 - a^6*b*d
^3)*g^4)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 656 vs. $2(151) = 302$.

time = 2.01, size = 656, normalized size = 3.75

$$-\frac{B\log\left(\frac{e(c+dx)}{a+bx}\right)}{3a^3bg^4+9a^2b^2g^4x+9ab^3g^4x^2+3b^4g^4x^3}+\frac{Bd^3\log\left(x+\frac{-\frac{Ba^4d^7}{(ad-bc)^3}+\frac{4Ba^3bcd^6}{(ad-bc)^3}-\frac{6Ba^2b^2c^2d^5}{(ad-bc)^3}+\frac{4Bab^3c^3d^4}{(ad-bc)^3}+Bad^4-\frac{Bb^4c^4d^3}{(ad-bc)^3}+Bbcd^3}{2Bbd^4}\right)}{3bg^4(ad-bc)^3}-\frac{Bd^3\log\left(x+\frac{\frac{Ba^4d^7}{(ad-bc)^3}-\frac{4Ba^3bcd^6}{(ad-bc)^3}+\frac{6Ba^2b^2c^2d^5}{(ad-bc)^3}-\frac{4Bab^3c^3d^4}{(ad-bc)^3}+Bad^4+\frac{Bb^4c^4d^3}{(ad-bc)^3}+Bbcd^3}{2Bbd^4}\right)}{3bg^4(ad-bc)^3}+\frac{-6Aa^2d^2+12Aabcd-6Ab^2c^2+11Ba^2d^2-7Babcd+2Bb^2c^2+6Bb^2d^2x^2+x(15Babd^2-3Bb^2cd)}{18a^5bd^2g^4-36a^4b^2cdg^4+18a^3b^3c^2g^4+x^3\cdot(18a^2b^4d^2g^4-36ab^5cdg^4+18b^6c^2g^4)+x^2\cdot(54a^3b^3d^2g^4-108a^2b^4cdg^4+54ab^5c^2g^4)+x(54a^4b^2d^2g^4-108a^3b^3cdg^4+54a^2b^4c^2g^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(d*x+c)/(b*x+a)))/(b*g*x+a*g)**4,x)
```

```
[Out] -B*log(e*(c + d*x)/(a + b*x))/(3*a**3*b*g**4 + 9*a**2*b**2*g**4*x + 9*a*b**
3*g**4*x**2 + 3*b**4*g**4*x**3) + B*d**3*log(x + (-B*a**4*d**7/(a*d - b*c)*
*3 + 4*B*a**3*b*c*d**6/(a*d - b*c)**3 - 6*B*a**2*b**2*c**2*d**5/(a*d - b*c)
**3 + 4*B*a*b**3*c**3*d**4/(a*d - b*c)**3 + B*a*d**4 - B*b**4*c**4*d**3/(a*
d - b*c)**3 + B*b*c*d**3)/(2*B*b*d**4))/(3*b*g**4*(a*d - b*c)**3) - B*d**3*
log(x + (B*a**4*d**7/(a*d - b*c)**3 - 4*B*a**3*b*c*d**6/(a*d - b*c)**3 + 6*
B*a**2*b**2*c**2*d**5/(a*d - b*c)**3 - 4*B*a*b**3*c**3*d**4/(a*d - b*c)**3
+ B*a*d**4 + B*b**4*c**4*d**3/(a*d - b*c)**3 + B*b*c*d**3)/(2*B*b*d**4))/(3
*b*g**4*(a*d - b*c)**3) + (-6*A*a**2*d**2 + 12*A*a*b*c*d - 6*A*b**2*c**2 +
11*B*a**2*d**2 - 7*B*a*b*c*d + 2*B*b**2*c**2 + 6*B*b**2*d**2*x**2 + x*(15*B
*a*b*d**2 - 3*B*b**2*c*d))/(18*a**5*b*d**2*g**4 - 36*a**4*b**2*c*d*g**4 + 1
8*a**3*b**3*c**2*g**4 + x**3*(18*a**2*b**4*d**2*g**4 - 36*a*b**5*c*d*g**4 +
 18*b**6*c**2*g**4) + x**2*(54*a**3*b**3*d**2*g**4 - 108*a**2*b**4*c*d*g**4
 + 54*a*b**5*c**2*g**4) + x*(54*a**4*b**2*d**2*g**4 - 108*a**3*b**3*c*d*g**
4 + 54*a**2*b**4*c**2*g**4))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 382 vs. $2(164) = 328$.

time = 2.93, size = 382, normalized size = 2.18

$$-\frac{\left(\frac{18(dxe+ce)Bd^2e^2\log\left(\frac{dxe+ce}{bx+a}\right)}{bx+a}-\frac{18(dxe+ce)^2Bbde\log\left(\frac{dxe+ce}{bx+a}\right)}{(bx+a)^2}+\frac{18(dxe+ce)Ad^2e^2}{bx+a}-\frac{18(dxe+ce)Bd^2e^2}{bx+a}-\frac{18(dxe+ce)^2Abde}{(bx+a)^2}+\frac{9(dxe+ce)^2Bbde}{(bx+a)^2}+\frac{6(dxe+ce)^3Bb^2\log\left(\frac{dxe+ce}{bx+a}\right)}{(bx+a)^3}+\frac{6(dxe+ce)^3Ab^2}{(bx+a)^3}-\frac{2(dxe+ce)^3Bb^2}{(bx+a)^3}\right)\left(\frac{bc}{(bce-ade)(bc-ad)}-\frac{ad}{(bce-ade)(bc-ad)}\right)}{18(b^2c^2g^4e^2-2abcdg^4e^2+a^2d^2g^4e^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(d*x+c)/(b*x+a)))/(b*g*x+a*g)^4,x, algorithm="giac")
```

```
[Out] -1/18*(18*(d*x*e + c*e)*B*d^2*e^2*log((d*x*e + c*e)/(b*x + a))/(b*x + a) -
18*(d*x*e + c*e)^2*B*b*d*e*log((d*x*e + c*e)/(b*x + a))/(b*x + a)^2 + 18*(d
*x*e + c*e)*A*d^2*e^2/(b*x + a) - 18*(d*x*e + c*e)*B*d^2*e^2/(b*x + a) - 18
*(d*x*e + c*e)^2*A*b*d*e/(b*x + a)^2 + 9*(d*x*e + c*e)^2*B*b*d*e/(b*x + a)^
2 + 6*(d*x*e + c*e)^3*B*b^2*log((d*x*e + c*e)/(b*x + a))/(b*x + a)^3 + 6*(d
*x*e + c*e)^3*A*b^2/(b*x + a)^3 - 2*(d*x*e + c*e)^3*B*b^2/(b*x + a)^3)*(b*c
/((b*c*e - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))/(b^2*c^
2*g^4*e^2 - 2*a*b*c*d*g^4*e^2 + a^2*d^2*g^4*e^2)
```

Mupad [B]

time = 5.88, size = 339, normalized size = 1.94

$$\frac{Bbc^2}{9g^4(ad-bc)^2(a+bx)^3}-\frac{Abc^2}{3g^4(ad-bc)^2(a+bx)^3}-\frac{B\ln\left(\frac{e(c+dx)}{a+bx}\right)}{3bg^4(a+bx)^3}-\frac{Aa^2d^2}{3bg^4(ad-bc)^2(a+bx)^3}+\frac{11Ba^2d^2}{18bg^4(ad-bc)^2(a+bx)^3}+\frac{5Bad^2x}{6g^4(ad-bc)^2(a+bx)^3}+\frac{Bbd^2x^2}{3g^4(ad-bc)^2(a+bx)^3}+\frac{2Aacd}{3g^4(ad-bc)^2(a+bx)^3}-\frac{7Bacd}{18g^4(ad-bc)^2(a+bx)^3}-\frac{Bbcdx}{6g^4(ad-bc)^2(a+bx)^3}+\frac{Bd^3\,\mathrm{atan}\left(\frac{ad\,1\mathrm{i}+bc\,1\mathrm{i}+bdx\,2\mathrm{i}}{ad-bc}\right)2\mathrm{i}}{3bg^4(ad-bc)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*log((e*(c + d*x))/(a + b*x)))/(a*g + b*g*x)^4,x)

[Out] (B*d^3*atan((a*d*1i + b*c*1i + b*d*x*2i)/(a*d - b*c))*2i)/(3*b*g^4*(a*d - b*c)^3) - (B*log((e*(c + d*x))/(a + b*x)))/(3*b*g^4*(a + b*x)^3) - (A*b*c^2)/(3*g^4*(a*d - b*c)^2*(a + b*x)^3) + (B*b*c^2)/(9*g^4*(a*d - b*c)^2*(a + b*x)^3) - (A*a^2*d^2)/(3*b*g^4*(a*d - b*c)^2*(a + b*x)^3) + (11*B*a^2*d^2)/(18*b*g^4*(a*d - b*c)^2*(a + b*x)^3) + (5*B*a*d^2*x)/(6*g^4*(a*d - b*c)^2*(a + b*x)^3) + (B*b*d^2*x^2)/(3*g^4*(a*d - b*c)^2*(a + b*x)^3) + (2*A*a*c*d)/(3*g^4*(a*d - b*c)^2*(a + b*x)^3) - (7*B*a*c*d)/(18*g^4*(a*d - b*c)^2*(a + b*x)^3) - (B*b*c*d*x)/(6*g^4*(a*d - b*c)^2*(a + b*x)^3)

3.181 $\int \frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{(ag+bgx)^5}\,dx$

Optimal. Leaf size=206

$$\frac{B}{16bg^5(a+bx)^4}-\frac{Bd}{12b(bc-ad)g^5(a+bx)^3}+\frac{Bd^2}{8b(bc-ad)^2g^5(a+bx)^2}-\frac{Bd^3}{4b(bc-ad)^3g^5(a+bx)}-\frac{Bd^4\log(a+}{4b(bc-ad)}$$

```
[Out] 1/16*B/b/g^5/(b*x+a)^4-1/12*B*d/b/(-a*d+b*c)/g^5/(b*x+a)^3+1/8*B*d^2/b/(-a*
d+b*c)^2/g^5/(b*x+a)^2-1/4*B*d^3/b/(-a*d+b*c)^3/g^5/(b*x+a)-1/4*B*d^4*ln(b*
x+a)/b/(-a*d+b*c)^4/g^5+1/4*B*d^4*ln(d*x+c)/b/(-a*d+b*c)^4/g^5+1/4*(-A-B*ln
(e*(d*x+c)/(b*x+a)))/b/g^5/(b*x+a)^4
```

Rubi [A]
time = 0.12, antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2548, 21, 46}

$$-\frac{B\log\left(\frac{e(c+dx)}{a+bx}\right)+A}{4bg^5(a+bx)^4}-\frac{Bd^4\log(a+bx)}{4bg^5(bc-ad)^4}+\frac{Bd^4\log(c+dx)}{4bg^5(bc-ad)^4}-\frac{Bd^3}{4bg^5(a+bx)(bc-ad)^3}+\frac{Bd^2}{8bg^5(a+bx)^2(bc-ad)^2}-\frac{Bd}{12bg^5(a+bx)^3(bc-ad)}+\frac{B}{16bg^5(a+bx)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(c + d*x))/(a + b*x)])/(a*g + b*g*x)^5,x]
```

```
[Out] B/(16*b*g^5*(a + b*x)^4) - (B*d)/(12*b*(b*c - a*d)*g^5*(a + b*x)^3) + (B*d^
2)/(8*b*(b*c - a*d)^2*g^5*(a + b*x)^2) - (B*d^3)/(4*b*(b*c - a*d)^3*g^5*(a
+ b*x)) - (B*d^4*Log[a + b*x])/(4*b*(b*c - a*d)^4*g^5) + (B*d^4*Log[c + d*x
])/(4*b*(b*c - a*d)^4*g^5) - (A + B*Log[(e*(c + d*x))/(a + b*x)])/(4*b*g^5*
(a + b*x)^4)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2548

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Dist[B*n*((b*c
- a*d)/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /;
 FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c -
 a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{(ag+bgx)^5}\,dx &= -\frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{4bg^5(a+bx)^4}+\frac{B\int \frac{-bc+ad}{g^4(a+bx)^5(c+dx)}\,dx}{4bg}\\ &= -\frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{4bg^5(a+bx)^4}-\frac{(B(bc-ad))\int \frac{1}{(a+bx)^5(c+dx)}\,dx}{4bg^5}\\ &= -\frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{4bg^5(a+bx)^4}-\frac{(B(bc-ad))\int \left(\frac{b}{(bc-ad)(a+bx)^5}-\frac{bd}{(bc-ad)^2(a+bx)^4}+\frac{}{(bc-ad)}\right.}{}\\ &= \frac{B}{16bg^5(a+bx)^4}-\frac{Bd}{12b(bc-ad)g^5(a+bx)^3}+\frac{Bd^2}{8b(bc-ad)^2g^5(a+bx)^2}-\frac{}{4b(bc-}\end{aligned}$$

Mathematica [A]
time = 0.14, size = 166, normalized size = 0.81

$$\frac{\frac{B(-bc+ad)\left(-\frac{3(bc-ad)^4}{(a+bx)^4}+\frac{4d(bc-ad)^3}{(a+bx)^3}-\frac{6d^2(bc-ad)^2}{(a+bx)^2}+\frac{12d^3(bc-ad)}{a+bx}+12d^4\log(a+bx)-12d^4\log(c+dx)\right)}{12(bc-ad)^5}-\frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{(a+bx)^4}}{4bg^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(c + d*x))/(a + b*x)])/(a*g + b*g*x)^5,x]
```

```
[Out] ((B*(-(b*c) + a*d)*((-3*(b*c - a*d)^4)/(a + b*x)^4 + (4*d*(b*c - a*d)^3)/(a
 + b*x)^3 - (6*d^2*(b*c - a*d)^2)/(a + b*x)^2 + (12*d^3*(b*c - a*d))/(a + b
*x) + 12*d^4*Log[a + b*x] - 12*d^4*Log[c + d*x]))/(12*(b*c - a*d)^5) - (A +
 B*Log[(e*(c + d*x))/(a + b*x)])/(a + b*x)^4)/(4*b*g^5)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 687 vs. 2(195) = 390.
time = 0.53, size = 688, normalized size = 3.34 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*(d*x+c)/(b*x+a)))/(b*g*x+a*g)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^2*e*(a*d-b*c)*(-1/4*b^5/(a*d-b*c)^5/e^5/g^5*A*(d*e/b-e*(a*d-b*c)/b/(b*x
+a))^4+b^4/(a*d-b*c)^5/e^4/g^5*A*d*(d*e/b-e*(a*d-b*c)/b/(b*x+a))^3-3/2*b^3/
(a*d-b*c)^5/e^3/g^5*A*d^2*(d*e/b-e*(a*d-b*c)/b/(b*x+a))^2+b^2/(a*d-b*c)^5/e
^2/g^5*A*d^3*(d*e/b-e*(a*d-b*c)/b/(b*x+a))-b^5/(a*d-b*c)^5/e^5/g^5*B*(1/4*(
d*e/b-e*(a*d-b*c)/b/(b*x+a))^4*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))-1/16*(d*e/b-
e*(a*d-b*c)/b/(b*x+a))^4)+3*b^4/(a*d-b*c)^5/e^4/g^5*B*d*(1/3*(d*e/b-e*(a*d-
b*c)/b/(b*x+a))^3*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))-1/9*(d*e/b-e*(a*d-b*c)/b/
(b*x+a))^3)-3*b^3/(a*d-b*c)^5/e^3/g^5*B*d^2*(1/2*(d*e/b-e*(a*d-b*c)/b/(b*x+
a))^2*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))-1/4*(d*e/b-e*(a*d-b*c)/b/(b*x+a))^2)+
b^2/(a*d-b*c)^5/e^2/g^5*B*d^3*((d*e/b-e*(a*d-b*c)/b/(b*x+a))*ln(d*e/b-e*(a*
d-b*c)/b/(b*x+a))+e*(a*d-b*c)/b/(b*x+a)-d*e/b))
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 649 vs. $2(193) = 386$.
time = 0.33, size = 649, normalized size = 3.15

$$-\frac{1}{48}B\left(\frac{12b^3d^3x^3 - 3b^3c^3 + 13ab^2c^2d - 23a^2bcd^2 + 25a^3d^3 - 6(b^3cd^2 - 7ab^2d^3)x^2 + 4(b^3c^2d - 5ab^2cd^2 + 13a^2bd^3)x}{(b^8c^3 - 3ab^7c^2d + 3a^2b^6cd^2 - a^3b^5d^3)g^5x^4 + 4(ab^7c^3 - 3a^2b^6c^2d + 3a^3b^5cd^2 - a^4b^4d^3)g^5x^3 + 6(a^2b^6c^3 - 3a^3b^5c^2d + 3a^4b^4cd^2 - a^5b^3d^3)g^5x^2 + 4(a^3b^5c^3 - 3a^4b^4c^2d + 3a^5b^3cd^2 - a^6b^2d^3)g^5x + (a^4b^4c^3 - 3a^5b^3c^2d + 3a^6b^2cd^2 - a^7bd^3)g^5} + \frac{12\log\left(\frac{dex}{bx+a}+\frac{ce}{bx+a}\right)}{b^5g^5x^4 + 4ab^4g^5x^3 + 6a^2b^3g^5x^2 + 4a^3b^2g^5x + a^4bg^5} + \frac{12d^4\log(bx+a)}{(b^5c^4 - 4ab^4c^3d + 6a^2b^3c^2d^2 - 4a^3b^2cd^3 + a^4bd^4)g^5} - \frac{12d^4\log(dx+c)}{(b^5c^4 - 4ab^4c^3d + 6a^2b^3c^2d^2 - 4a^3b^2cd^3 + a^4bd^4)g^5}\right) - \frac{A}{4(b^5g^5x^4 + 4ab^4g^5x^3 + 6a^2b^3g^5x^2 + 4a^3b^2g^5x + a^4bg^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(d*x+c)/(b*x+a)))/(b*g*x+a*g)^5,x, algorithm="maxima")
```

```
[Out] -1/48*B*((12*b^3*d^3*x^3 - 3*b^3*c^3 + 13*a*b^2*c^2*d - 23*a^2*b*c*d^2 + 25
*a^3*d^3 - 6*(b^3*c*d^2 - 7*a*b^2*d^3)*x^2 + 4*(b^3*c^2*d - 5*a*b^2*c*d^2 +
 13*a^2*b*d^3)*x)/((b^8*c^3 - 3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 - a^3*b^5*d^3
)*g^5*x^4 + 4*(a*b^7*c^3 - 3*a^2*b^6*c^2*d + 3*a^3*b^5*c*d^2 - a^4*b^4*d^3)
*g^5*x^3 + 6*(a^2*b^6*c^3 - 3*a^3*b^5*c^2*d + 3*a^4*b^4*c*d^2 - a^5*b^3*d^3
)*g^5*x^2 + 4*(a^3*b^5*c^3 - 3*a^4*b^4*c^2*d + 3*a^5*b^3*c*d^2 - a^6*b^2*d^
3)*g^5*x + (a^4*b^4*c^3 - 3*a^5*b^3*c^2*d + 3*a^6*b^2*c*d^2 - a^7*b*d^3)*g^
5) + 12*log(d*x*e/(b*x + a) + c*e/(b*x + a))/(b^5*g^5*x^4 + 4*a*b^4*g^5*x^3
 + 6*a^2*b^3*g^5*x^2 + 4*a^3*b^2*g^5*x + a^4*b*g^5) + 12*d^4*log(b*x + a)/(
(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)
*g^5) - 12*d^4*log(d*x + c)/((b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 -
 4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5)) - 1/4*A/(b^5*g^5*x^4 + 4*a*b^4*g^5*x^3
+ 6*a^2*b^3*g^5*x^2 + 4*a^3*b^2*g^5*x + a^4*b*g^5)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 636 vs. $2(193) = 386$.
time = 0.44, size = 636, normalized size = 3.09

$$-\frac{3(4A-B)b^4c^4 - 16(3A-B)ab^3c^3d + 36(2A-B)a^2b^2c^2d^2 - 48(A-B)a^3bcd^3 + (12A-25B)a^4d^4 + 12(Bb^4cd^3 - Bab^3d^4)x^3 - 6(Bb^4c^2d^2 - 8Bab^3cd^3 + 7Ba^2b^2d^4)x^2 + 4(Bb^4c^3d - 6Bab^3c^2d^2 + 18Ba^2b^2cd^3 - 13Ba^3bd^4)x - 12(Bb^4d^4x^4 + 4Bab^3d^4x^3 + 6Ba^2b^2d^4x^2 + 4Ba^3bd^4x - Bb^4c^4 + 4Bab^3c^3d - 6Ba^2b^2c^2d^2 + 4Ba^3bcd^3)\log\left(\frac{(dx+c)e}{bx+a}\right)}{48((b^9c^4 - 4ab^8c^3d + 6a^2b^7c^2d^2 - 4a^3b^6cd^3 + a^4b^5d^4)g^5x^4 + 4(ab^8c^4 - 4a^2b^7c^3d + 6a^3b^6c^2d^2 - 4a^4b^5cd^3 + a^5b^4d^4)g^5x^3 + 6(a^2b^7c^4 - 4a^3b^6c^3d + 6a^4b^5c^2d^2 - 4a^5b^4cd^3 + a^6b^3d^4)g^5x^2 + 4(a^3b^6c^4 - 4a^4b^5c^3d + 6a^5b^4c^2d^2 - 4a^6b^3cd^3 + a^7b^2d^4)g^5x + (a^4b^5c^4 - 4a^5b^4c^3d + 6a^6b^3c^2d^2 - 4a^7b^2cd^3 + a^8bd^4)g^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(d*x+c)/(b*x+a)))/(b*g*x+a*g)^5,x, algorithm="fricas")
```

```
[Out] -1/48*(3*(4*A - B)*b^4*c^4 - 16*(3*A - B)*a*b^3*c^3*d + 36*(2*A - B)*a^2*b^
2*c^2*d^2 - 48*(A - B)*a^3*b*c*d^3 + (12*A - 25*B)*a^4*d^4 + 12*(B*b^4*c*d^
```

3 - B*a*b^3*d^4)*x^3 - 6*(B*b^4*c^2*d^2 - 8*B*a*b^3*c*d^3 + 7*B*a^2*b^2*d^4
)*x^2 + 4*(B*b^4*c^3*d - 6*B*a*b^3*c^2*d^2 + 18*B*a^2*b^2*c*d^3 - 13*B*a^3*
b*d^4)*x - 12*(B*b^4*d^4*x^4 + 4*B*a*b^3*d^4*x^3 + 6*B*a^2*b^2*d^4*x^2 + 4*
B*a^3*b*d^4*x - B*b^4*c^4 + 4*B*a*b^3*c^3*d - 6*B*a^2*b^2*c^2*d^2 + 4*B*a^3
*b*c*d^3)*log((d*x + c)*e/(b*x + a)))/((b^9*c^4 - 4*a*b^8*c^3*d + 6*a^2*b^7
*c^2*d^2 - 4*a^3*b^6*c*d^3 + a^4*b^5*d^4)*g^5*x^4 + 4*(a*b^8*c^4 - 4*a^2*b^
7*c^3*d + 6*a^3*b^6*c^2*d^2 - 4*a^4*b^5*c*d^3 + a^5*b^4*d^4)*g^5*x^3 + 6*(a
^2*b^7*c^4 - 4*a^3*b^6*c^3*d + 6*a^4*b^5*c^2*d^2 - 4*a^5*b^4*c*d^3 + a^6*b^
3*d^4)*g^5*x^2 + 4*(a^3*b^6*c^4 - 4*a^4*b^5*c^3*d + 6*a^5*b^4*c^2*d^2 - 4*a
^6*b^3*c*d^3 + a^7*b^2*d^4)*g^5*x + (a^4*b^5*c^4 - 4*a^5*b^4*c^3*d + 6*a^6*
b^3*c^2*d^2 - 4*a^7*b^2*c*d^3 + a^8*b*d^4)*g^5)

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 944 vs. $2(178) = 356$.
time = 3.05, size = 944, normalized size = 4.58

$$-\frac{B\log\left(\frac{e(c+dx)}{a+bx}\right)}{4a^4bg^5+16a^3b^2g^5x+24a^2b^3g^5x^2+16ab^4g^5x^3+4b^5g^5x^4}+\frac{Bd^4\log\left(x+\frac{-\frac{Ba^5d^9}{(ad-bc)^4}+\frac{5Ba^4bcd^8}{(ad-bc)^4}-\frac{10Ba^3b^2c^2d^7}{(ad-bc)^4}+\frac{10Ba^2b^3c^3d^6}{(ad-bc)^4}-\frac{5Bab^4c^4d^5}{(ad-bc)^4}+Bad^5+\frac{Bb^5c^5d^4}{(ad-bc)^4}+Bbcd^4}{2Bbd^5}\right)}{4bg^5(ad-bc)^4}-\frac{Bd^4\log\left(x+\frac{\frac{Ba^5d^9}{(ad-bc)^4}-\frac{5Ba^4bcd^8}{(ad-bc)^4}+\frac{10Ba^3b^2c^2d^7}{(ad-bc)^4}-\frac{10Ba^2b^3c^3d^6}{(ad-bc)^4}+\frac{5Bab^4c^4d^5}{(ad-bc)^4}+Bad^5-\frac{Bb^5c^5d^4}{(ad-bc)^4}+Bbcd^4}{2Bbd^5}\right)}{4bg^5(ad-bc)^4}+\frac{-12Aa^3d^3+36Aa^2bcd^2-36Aab^2c^2d+12Ab^3c^3+25Ba^3d^3-23Ba^2bcd^2+13Bab^2c^2d-3Bb^3c^3+12Bb^3d^3x^3+x^2\cdot(42Bab^2d^3-6Bb^3cd^2)+x(52Ba^2bd^3-20Bab^2cd^2+4Bb^3c^2d)}{48a^7bd^3g^5-144a^6b^2cd^2g^5+144a^5b^3c^2dg^5-48a^4b^4c^3g^5+x^4\cdot(48a^3b^5d^3g^5-144a^2b^6cd^2g^5+144ab^7c^2dg^5-48b^8c^3g^5)+x^3\cdot(192a^4b^4d^3g^5-576a^3b^5cd^2g^5+576a^2b^6c^2dg^5-192ab^7c^3g^5)+x^2\cdot(288a^5b^3d^3g^5-864a^4b^4cd^2g^5+864a^3b^5c^2dg^5-288a^2b^6c^3g^5)+x(192a^6b^2d^3g^5-576a^5b^3cd^2g^5+576a^4b^4c^2dg^5-192a^3b^5c^3g^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*ln(e*(d*x+c)/(b*x+a)))/(b*g*x+a*g)**5,x)

[Out] -B*log(e*(c + d*x)/(a + b*x))/(4*a**4*b*g**5 + 16*a**3*b**2*g**5*x + 24*a**
2*b**3*g**5*x**2 + 16*a*b**4*g**5*x**3 + 4*b**5*g**5*x**4) + B*d**4*log(x +
(-B*a**5*d**9/(a*d - b*c)**4 + 5*B*a**4*b*c*d**8/(a*d - b*c)**4 - 10*B*a**
3*b**2*c**2*d**7/(a*d - b*c)**4 + 10*B*a**2*b**3*c**3*d**6/(a*d - b*c)**4 -
5*B*a*b**4*c**4*d**5/(a*d - b*c)**4 + B*a*d**5 + B*b**5*c**5*d**4/(a*d - b
*c)**4 + B*b*c*d**4)/(2*B*b*d**5))/(4*b*g**5*(a*d - b*c)**4) - B*d**4*log(x
+ (B*a**5*d**9/(a*d - b*c)**4 - 5*B*a**4*b*c*d**8/(a*d - b*c)**4 + 10*B*a*
*3*b**2*c**2*d**7/(a*d - b*c)**4 - 10*B*a**2*b**3*c**3*d**6/(a*d - b*c)**4
+ 5*B*a*b**4*c**4*d**5/(a*d - b*c)**4 + B*a*d**5 - B*b**5*c**5*d**4/(a*d -
b*c)**4 + B*b*c*d**4)/(2*B*b*d**5))/(4*b*g**5*(a*d - b*c)**4) + (-12*A*a**3
*d**3 + 36*A*a**2*b*c*d**2 - 36*A*a*b**2*c**2*d + 12*A*b**3*c**3 + 25*B*a**
3*d**3 - 23*B*a**2*b*c*d**2 + 13*B*a*b**2*c**2*d - 3*B*b**3*c**3 + 12*B*b**
3*d**3*x**3 + x**2*(42*B*a*b**2*d**3 - 6*B*b**3*c*d**2) + x*(52*B*a**2*b*d*
*3 - 20*B*a*b**2*c*d**2 + 4*B*b**3*c**2*d))/(48*a**7*b*d**3*g**5 - 144*a**6
*b**2*c*d**2*g**5 + 144*a**5*b**3*c**2*d*g**5 - 48*a**4*b**4*c**3*g**5 + x*
4(48*a**3*b**5*d**3*g**5 - 144*a**2*b**6*c*d**2*g**5 + 144*a*b**7*c**2*d*
g**5 - 48*b**8*c**3*g**5) + x**3*(192*a**4*b**4*d**3*g**5 - 576*a**3*b**5*c
*d**2*g**5 + 576*a**2*b**6*c**2*d*g**5 - 192*a*b**7*c**3*g**5) + x**2*(288*
a**5*b**3*d**3*g**5 - 864*a**4*b**4*c*d**2*g**5 + 864*a**3*b**5*c**2*d*g**5
- 288*a**2*b**6*c**3*g**5) + x*(192*a**6*b**2*d**3*g**5 - 576*a**5*b**3*c*
d**2*g**5 + 576*a**4*b**4*c**2*d*g**5 - 192*a**3*b**5*c**3*g**5))

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 511 vs. $2(193) = 386$.

time = 3.68, size = 511, normalized size = 2.48

$$\frac{\left(\frac{48\,(dxe+ce)Bd^3e^3\log\left(\frac{dxe+ce}{bx+a}\right)}{bx+a}-\frac{72\,(dxe+ce)^2Bbd^2e^2\log\left(\frac{dxe+ce}{bx+a}\right)}{(bx+a)^2}+\frac{48\,(dxe+ce)^3Bb^2de\log\left(\frac{dxe+ce}{bx+a}\right)}{(bx+a)^3}+\frac{48\,(dxe+ce)Ad^3e^3}{bx+a}-\frac{48\,(dxe+ce)Bd^3e^3}{bx+a}-\frac{72\,(dxe+ce)^2Abd^2e^2}{(bx+a)^2}+\frac{36\,(dxe+ce)^2Bbd^2e^2}{(bx+a)^2}+\frac{48\,(dxe+ce)^3Ab^2de}{(bx+a)^3}-\frac{16\,(dxe+ce)^3Bb^2de}{(bx+a)^3}-\frac{12\,(dxe+ce)^4Bb^3\log\left(\frac{dxe+ce}{bx+a}\right)}{(bx+a)^4}-\frac{12\,(dxe+ce)^4Ab^3}{(bx+a)^4}+\frac{3\,(dxe+ce)^4Bb^3}{(bx+a)^4}\right)\left(\frac{bc}{(bce-ade)(bc-ad)}-\frac{ad}{(bce-ade)(bc-ad)}\right)}{48\,(b^3c^3g^5e^3-3\,ab^2c^2dg^5e^3+3\,a^2bcd^2g^5e^3-a^3d^3g^5e^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(d*x+c)/(b*x+a)))/(b*g*x+a*g)^5,x, algorithm="giac")
```

```
[Out] 1/48*(48*(d*x*e + c*e)*B*d^3*e^3*log((d*x*e + c*e)/(b*x + a))/(b*x + a) - 7
2*(d*x*e + c*e)^2*B*b*d^2*e^2*log((d*x*e + c*e)/(b*x + a))/(b*x + a)^2 + 48
*(d*x*e + c*e)^3*B*b^2*d*e*log((d*x*e + c*e)/(b*x + a))/(b*x + a)^3 + 48*(d
*x*e + c*e)*A*d^3*e^3/(b*x + a) - 48*(d*x*e + c*e)*B*d^3*e^3/(b*x + a) - 72
*(d*x*e + c*e)^2*A*b*d^2*e^2/(b*x + a)^2 + 36*(d*x*e + c*e)^2*B*b*d^2*e^2/(
b*x + a)^2 + 48*(d*x*e + c*e)^3*A*b^2*d*e/(b*x + a)^3 - 16*(d*x*e + c*e)^3*
B*b^2*d*e/(b*x + a)^3 - 12*(d*x*e + c*e)^4*B*b^3*log((d*x*e + c*e)/(b*x + a
))/(b*x + a)^4 - 12*(d*x*e + c*e)^4*A*b^3/(b*x + a)^4 + 3*(d*x*e + c*e)^4*B
*b^3/(b*x + a)^4)*(b*c/((b*c*e - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)
*(b*c - a*d)))/(b^3*c^3*g^5*e^3 - 3*a*b^2*c^2*d*g^5*e^3 + 3*a^2*b*c*d^2*g^5
*e^3 - a^3*d^3*g^5*e^3)
```

Mupad [B]
time = 6.62, size = 578, normalized size = 2.81

$$\frac{B\,d^4\,\mathrm{atanh}\left(\frac{-4\,a^4\,b\,d^4\,g^5+8\,a^3\,b^2\,c\,d^3\,g^5-8\,a\,b^4\,c^3\,d\,g^5+4\,b^5\,c^4\,g^5}{4\,b\,g^5\,(a\,d-b\,c)^4}-\frac{2\,b\,d\,x\,(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3)}{(a\,d-b\,c)^4}\right)}{2\,b\,g^5\,(a\,d-b\,c)^4}-\frac{B\,\ln\left(\frac{e\,(c+d\,x)}{a+b\,x}\right)}{4\,b^2\,g^5\,\left(4\,a^3\,x+\frac{a^4}{b}+b^3\,x^4+6\,a^2\,b\,x^2+4\,a\,b^2\,x^3\right)}-\frac{\frac{12\,A\,a^3\,d^3-12\,A\,b^3\,c^3-25\,B\,a^3\,d^3+3\,B\,b^3\,c^3+36\,A\,a\,b^2\,c^2\,d-36\,A\,a^2\,b\,c\,d^2-13\,B\,a\,b^2\,c^2\,d+23\,B\,a^2\,b\,c\,d^2}{12\,(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3)}+\frac{d^2\,x^2\,(B\,b^3\,c-7\,B\,a\,b^2\,d)}{2\,(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3)}-\frac{d\,x\,(13\,B\,a^2\,b\,d^2-5\,B\,a\,b^2\,c\,d+B\,b^3\,c^2)}{3\,(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3)}-\frac{B\,b^3\,d^3\,x^3}{a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3}}{4\,a^4\,b\,g^5+16\,a^3\,b^2\,g^5\,x+24\,a^2\,b^3\,g^5\,x^2+16\,a\,b^4\,g^5\,x^3+4\,b^5\,g^5\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(c + d*x))/(a + b*x)))/(a*g + b*g*x)^5,x)
```

```
[Out] (B*d^4*atanh((4*b^5*c^4*g^5 - 4*a^4*b*d^4*g^5 - 8*a*b^4*c^3*d*g^5 + 8*a^3*b
^2*c*d^3*g^5)/(4*b*g^5*(a*d - b*c)^4) - (2*b*d*x*(a^3*d^3 - b^3*c^3 + 3*a*b
^2*c^2*d - 3*a^2*b*c*d^2))/(a*d - b*c)^4))/(2*b*g^5*(a*d - b*c)^4) - (B*log
((e*(c + d*x))/(a + b*x)))/(4*b^2*g^5*(4*a^3*x + a^4/b + b^3*x^4 + 6*a^2*b*
x^2 + 4*a*b^2*x^3)) - ((12*A*a^3*d^3 - 12*A*b^3*c^3 - 25*B*a^3*d^3 + 3*B*b^
3*c^3 + 36*A*a*b^2*c^2*d - 36*A*a^2*b*c*d^2 - 13*B*a*b^2*c^2*d + 23*B*a^2*b
*c*d^2)/(12*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) + (d^2*x^2
*(B*b^3*c - 7*B*a*b^2*d))/(2*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c
*d^2)) - (d*x*(B*b^3*c^2 + 13*B*a^2*b*d^2 - 5*B*a*b^2*c*d))/(3*(a^3*d^3 - b
^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) - (B*b^3*d^3*x^3)/(a^3*d^3 - b^3*c
^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))/(4*a^4*b*g^5 + 4*b^5*g^5*x^4 + 16*a^3*
b^2*g^5*x + 16*a*b^4*g^5*x^3 + 24*a^2*b^3*g^5*x^2)
```

3.182 $\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2 dx$

Optimal. Leaf size=503

$$\frac{13B^2(bc-ad)^4g^4x}{30d^4}-\frac{7B^2(bc-ad)^3g^4(a+bx)^2}{60bd^3}+\frac{B^2(bc-ad)^2g^4(a+bx)^3}{30bd^2}-\frac{5B^2(bc-ad)^5g^4\log(a+bx)}{6bd^5}-13$$

```
[Out] 13/30*B^2*(-a*d+b*c)^4*g^4*x/d^4-7/60*B^2*(-a*d+b*c)^3*g^4*(b*x+a)^2/b/d^3+
1/30*B^2*(-a*d+b*c)^2*g^4*(b*x+a)^3/b/d^2-5/6*B^2*(-a*d+b*c)^5*g^4*ln(b*x+a
)/b/d^5-13/30*B^2*(-a*d+b*c)^5*g^4*ln((d*x+c)/(b*x+a))/b/d^5+1/5*B*(-a*d+b*
c)^3*g^4*(b*x+a)^2*(A+B*ln(e*(d*x+c)/(b*x+a)))/b/d^3-2/15*B*(-a*d+b*c)^2*g^
4*(b*x+a)^3*(A+B*ln(e*(d*x+c)/(b*x+a)))/b/d^2+1/10*B*(-a*d+b*c)*g^4*(b*x+a)
^4*(A+B*ln(e*(d*x+c)/(b*x+a)))/b/d-2/5*B*(-a*d+b*c)^4*g^4*(d*x+c)*(A+B*ln(e
*(d*x+c)/(b*x+a)))/d^5+1/5*g^4*(b*x+a)^5*(A+B*ln(e*(d*x+c)/(b*x+a)))^2/b-2/
5*B*(-a*d+b*c)^5*g^4*(A+B*ln(e*(d*x+c)/(b*x+a)))*ln(1-d*(b*x+a)/b/(d*x+c))/
b/d^5+2/5*B^2*(-a*d+b*c)^5*g^4*polylog(2,d*(b*x+a)/b/(d*x+c))/b/d^5
```

Rubi [A]
time = 0.46, antiderivative size = 503, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 8, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2552, 2356, 2389, 2379, 2438, 2351, 31, 46}

$$\frac{2B^2g^4(bc-ad)^5\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{5bd^5}-\frac{2Bg^4(bc-ad)^5\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{5bd^5}-\frac{2Bg^4(c+dx)(bc-ad)^4\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{5d^5}+\frac{Bg^4(a+bx)^2(bc-ad)^3\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{5bd^3}-\frac{2Bg^4(a+bx)^3(bc-ad)^2\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{15bd^2}+\frac{Bg^4(a+bx)^4(bc-ad)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{10bd}+\frac{g^4(a+bx)^5\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}{5b}-\frac{5B^2g^4(bc-ad)^5\log(a+bx)}{6bd^5}-\frac{13B^2g^4(bc-ad)^5\log\left(\frac{c+dx}{a+bx}\right)}{30bd^5}+\frac{13B^2g^4x(bc-ad)^4}{30d^4}-\frac{7B^2g^4(a+bx)^2(bc-ad)^3}{60bd^3}+\frac{B^2g^4(a+bx)^3(bc-ad)^2}{30bd^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*g + b*g*x)^4*(A + B*Log[(e*(c + d*x))/(a + b*x)])^2,x]
```

```
[Out] (13*B^2*(b*c - a*d)^4*g^4*x)/(30*d^4) - (7*B^2*(b*c - a*d)^3*g^4*(a + b*x)^
2)/(60*b*d^3) + (B^2*(b*c - a*d)^2*g^4*(a + b*x)^3)/(30*b*d^2) - (5*B^2*(b*
c - a*d)^5*g^4*Log[a + b*x])/(6*b*d^5) - (13*B^2*(b*c - a*d)^5*g^4*Log[(c +
 d*x)/(a + b*x)])/(30*b*d^5) + (B*(b*c - a*d)^3*g^4*(a + b*x)^2*(A + B*Log[
(e*(c + d*x))/(a + b*x)]))/(5*b*d^3) - (2*B*(b*c - a*d)^2*g^4*(a + b*x)^3*(
A + B*Log[(e*(c + d*x))/(a + b*x)]))/(15*b*d^2) + (B*(b*c - a*d)*g^4*(a + b
*x)^4*(A + B*Log[(e*(c + d*x))/(a + b*x)]))/(10*b*d) - (2*B*(b*c - a*d)^4*g
^4*(c + d*x)*(A + B*Log[(e*(c + d*x))/(a + b*x)]))/(5*d^5) + (g^4*(a + b*x)
^5*(A + B*Log[(e*(c + d*x))/(a + b*x)])^2)/(5*b) - (2*B*(b*c - a*d)^5*g^4*(
A + B*Log[(e*(c + d*x))/(a + b*x)])*Log[1 - (d*(a + b*x))/(b*(c + d*x))])/(
5*b*d^5) + (2*B^2*(b*c - a*d)^5*g^4*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))]
)/(5*b*d^5)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2552

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(
m + 1)*(g/d)^m, Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a
```

```
 + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[n
 + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[d*f
- c*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Rubi steps

$$\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2 dx = \frac{g^4(a+bx)^5\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{5b} - \frac{(2B)\int \frac{(bc-ad)g^5(a+bx)}{}}{5}$$

$$= \frac{g^4(a+bx)^5\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{5b} - \frac{(2B(bc-ad)g^4)\int \frac{(a}{}}{}$$

$$= \frac{g^4(a+bx)^5\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{5b} - \frac{(2B(bc-ad)g^4)\int \Big(}{}$$

$$= \frac{g^4(a+bx)^5\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{5b} - \frac{(2B(bc-ad)g^4)\int (a}{}$$

$$= -\frac{2AB(bc-ad)^4g^4x}{5d^4} + \frac{B(bc-ad)^3g^4(a+bx)^2\Big(A+B\log}{5bd^3}$$

$$= -\frac{2AB(bc-ad)^4g^4x}{5d^4} - \frac{2B^2(bc-ad)^4g^4(a+bx)\log\Big(\frac{e(c+dx}{a+bx}}{5bd^4}$$

$$= -\frac{2AB(bc-ad)^4g^4x}{5d^4} - \frac{2B^2(bc-ad)^5g^4\log(c+dx)}{5bd^5} - \frac{2B^2}{}$$

$$= -\frac{2AB(bc-ad)^4g^4x}{5d^4} + \frac{13B^2(bc-ad)^4g^4x}{30d^4} - \frac{7B^2(bc-ad)}{60b}$$

$$= -\frac{2AB(bc-ad)^4g^4x}{5d^4} + \frac{13B^2(bc-ad)^4g^4x}{30d^4} - \frac{7B^2(bc-ad)}{60b}$$

$$= -\frac{2AB(bc-ad)^4g^4x}{5d^4} + \frac{13B^2(bc-ad)^4g^4x}{30d^4} - \frac{7B^2(bc-ad)}{60b}$$

$$= -\frac{2AB(bc-ad)^4g^4x}{5d^4} + \frac{13B^2(bc-ad)^4g^4x}{30d^4} - \frac{7B^2(bc-ad)}{60b}$$

Mathematica [A]
time = 0.33, size = 512, normalized size = 1.02

$$\frac{g^4\left((a+bx)^5\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2-\frac{B(bc-ad)\left(24Abd(bc-ad)^3x+24B(bc-ad)^4\log(a+bx)-4B(bc-ad)^2\left(2bd(bc-ad)x-d^2(a+bx)^2-2(bc-ad)^2\log(c+dx)\right)-B(bc-ad)\left(6bd(bc-ad)^2x+3d^2(-bc+ad)(a+bx)^2+2d^3(a+bx)^3-6(bc-ad)^3\log(c+dx)\right)-12B(bc-ad)^3(bdx+(-bc+ad)\log(c+dx))+24bB(bc-ad)^3(c+dx)\log\left(\frac{e(c+dx)}{a+bx}\right)-12d^2(bc-ad)^2(a+bx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)+8d^3(bc-ad)(a+bx)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)-6d^4(a+bx)^4\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)-24(bc-ad)^4\log(c+dx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)-12B(bc-ad)^4\left(\left(2\log\left(\frac{d(a+bx)}{-bc+ad}\right)-\log(c+dx)\right)\log(c+dx)+2\text{Li}_2\left(\frac{b(c+dx)}{bc-ad}\right)\right)\right)}{12d^5}\right)}{5b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*g + b*g*x)^4*(A + B*Log[(e*(c + d*x))/(a + b*x)])^2,x]
```

```
[Out] (g^4*((a + b*x)^5*(A + B*Log[(e*(c + d*x))/(a + b*x)])^2 - (B*(b*c - a*d)*(
24*A*b*d*(b*c - a*d)^3*x + 24*B*(b*c - a*d)^4*Log[a + b*x] - 4*B*(b*c - a*d
)^2*(2*b*d*(b*c - a*d)*x - d^2*(a + b*x)^2 - 2*(b*c - a*d)^2*Log[c + d*x])
- B*(b*c - a*d)*(6*b*d*(b*c - a*d)^2*x + 3*d^2*(-(b*c) + a*d)*(a + b*x)^2 +
 2*d^3*(a + b*x)^3 - 6*(b*c - a*d)^3*Log[c + d*x]) - 12*B*(b*c - a*d)^3*(b*
d*x + (-(b*c) + a*d)*Log[c + d*x]) + 24*b*B*(b*c - a*d)^3*(c + d*x)*Log[(e*
(c + d*x))/(a + b*x)] - 12*d^2*(b*c - a*d)^2*(a + b*x)^2*(A + B*Log[(e*(c +
 d*x))/(a + b*x)]) + 8*d^3*(b*c - a*d)*(a + b*x)^3*(A + B*Log[(e*(c + d*x))
/(a + b*x)]) - 6*d^4*(a + b*x)^4*(A + B*Log[(e*(c + d*x))/(a + b*x)]) - 24*
(b*c - a*d)^4*Log[c + d*x]*(A + B*Log[(e*(c + d*x))/(a + b*x)]) - 12*B*(b*c
 - a*d)^4*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x
] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(12*d^5)))/(5*b)
```

Maple [F]
time = 0.24, size = 0, normalized size = 0.00

$$\int (bgx+ag)^4 \left(A+B\ln\left(\frac{e(dx+c)}{bx+a}\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*g*x+a*g)^4*(A+B*ln(e*(d*x+c)/(b*x+a)))^2,x)
```

```
[Out] int((b*g*x+a*g)^4*(A+B*ln(e*(d*x+c)/(b*x+a)))^2,x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 2101 vs. 2(484) = 968.
time = 0.40, size = 2101, normalized size = 4.18

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^4*(A+B*log(e*(d*x+c)/(b*x+a)))^2,x, algorithm="maxima
")
```

```
[Out] 1/5*A^2*b^4*g^4*x^5 + A^2*a*b^3*g^4*x^4 + 2*A^2*a^2*b^2*g^4*x^3 + 2*A^2*a^3
*b*g^4*x^2 + 2*(x*log(d*x*e/(b*x + a) + c*e/(b*x + a)) - a*log(b*x + a)/b +
 c*log(d*x + c)/d)*A*B*a^4*g^4 + 4*(x^2*log(d*x*e/(b*x + a) + c*e/(b*x + a)
) + a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d))*A*B*
a^3*b*g^4 + 2*(2*x^3*log(d*x*e/(b*x + a) + c*e/(b*x + a)) - 2*a^3*log(b*x +
 a)/b^3 + 2*c^3*log(d*x + c)/d^3 + ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 -
a^2*d^2)*x)/(b^2*d^2))*A*B*a^2*b^2*g^4 + 1/3*(6*x^4*log(d*x*e/(b*x + a) + c
*e/(b*x + a)) + 6*a^4*log(b*x + a)/b^4 - 6*c^4*log(d*x + c)/d^4 + (2*(b^3*c
```

```
*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^
3)*x)/(b^3*d^3))*A*B*a*b^3*g^4 + 1/30*(12*x^5*log(d*x*e/(b*x + a) + c*e/(b*
x + a)) - 12*a^5*log(b*x + a)/b^5 + 12*c^5*log(d*x + c)/d^5 + (3*(b^4*c*d^3
 - a*b^3*d^4)*x^4 - 4*(b^4*c^2*d^2 - a^2*b^2*d^4)*x^3 + 6*(b^4*c^3*d - a^3*
b*d^4)*x^2 - 12*(b^4*c^4 - a^4*d^4)*x)/(b^4*d^4))*A*B*b^4*g^4 + A^2*a^4*g^4
*x - 1/30*(13*b^4*c^5*g^4 - 53*a*b^3*c^4*d*g^4 + 76*a^2*b^2*c^3*d^2*g^4 - 3
6*a^3*b*c^2*d^3*g^4 - 12*a^4*c*d^4*g^4)*B^2*log(d*x + c)/d^5 - 2/5*(b^5*c^5
*g^4 - 5*a*b^4*c^4*d*g^4 + 10*a^2*b^3*c^3*d^2*g^4 - 10*a^3*b^2*c^2*d^3*g^4
+ 5*a^4*b*c*d^4*g^4 - a^5*d^5*g^4)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a
*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B^2/(b*d^5) + 1/60*(12*B^2*b^
5*d^5*g^4*x^5 + 6*(b^5*c*d^4*g^4 + 9*a*b^4*d^5*g^4)*B^2*x^4 - 6*(b^5*c^2*d^
3*g^4 - 6*a*b^4*c*d^4*g^4 - 15*a^2*b^3*d^5*g^4)*B^2*x^3 + (5*b^5*c^3*d^2*g^
4 - 33*a*b^4*c^2*d^3*g^4 + 87*a^2*b^3*c*d^4*g^4 + 61*a^3*b^2*d^5*g^4)*B^2*x
^2 + 2*(b^5*c^4*d*g^4 + a*b^4*c^3*d^2*g^4 - 18*a^2*b^3*c^2*d^3*g^4 + 41*a^3
*b^2*c*d^4*g^4 + 5*a^4*b*d^5*g^4)*B^2*x + 12*(B^2*b^5*d^5*g^4*x^5 + 5*B^2*a
*b^4*d^5*g^4*x^4 + 10*B^2*a^2*b^3*d^5*g^4*x^3 + 10*B^2*a^3*b^2*d^5*g^4*x^2
+ 5*B^2*a^4*b*d^5*g^4*x + B^2*a^5*d^5*g^4)*log(b*x + a)^2 + 12*(B^2*b^5*d^5
*g^4*x^5 + 5*B^2*a*b^4*d^5*g^4*x^4 + 10*B^2*a^2*b^3*d^5*g^4*x^3 + 10*B^2*a^
3*b^2*d^5*g^4*x^2 + 5*B^2*a^4*b*d^5*g^4*x + (b^5*c^5*g^4 - 5*a*b^4*c^4*d*g^
4 + 10*a^2*b^3*c^3*d^2*g^4 - 10*a^3*b^2*c^2*d^3*g^4 + 5*a^4*b*c*d^4*g^4)*B^
2)*log(d*x + c)^2 - 2*(12*B^2*b^5*d^5*g^4*x^5 + 3*(b^5*c*d^4*g^4 + 19*a*b^4
*d^5*g^4)*B^2*x^4 - 4*(b^5*c^2*d^3*g^4 - 5*a*b^4*c*d^4*g^4 - 26*a^2*b^3*d^5
*g^4)*B^2*x^3 + 6*(b^5*c^3*d^2*g^4 - 5*a*b^4*c^2*d^3*g^4 + 10*a^2*b^3*c*d^4
*g^4 + 14*a^3*b^2*d^5*g^4)*B^2*x^2 - 12*(b^5*c^4*d*g^4 - 5*a*b^4*c^3*d^2*g^
4 + 10*a^2*b^3*c^2*d^3*g^4 - 10*a^3*b^2*c*d^4*g^4 - a^4*b*d^5*g^4)*B^2*x -
(12*a*b^4*c^4*d*g^4 - 54*a^2*b^3*c^3*d^2*g^4 + 94*a^3*b^2*c^2*d^3*g^4 - 77*
a^4*b*c*d^4*g^4 + 13*a^5*d^5*g^4)*B^2)*log(b*x + a) + 2*(12*B^2*b^5*d^5*g^4
*x^5 + 3*(b^5*c*d^4*g^4 + 19*a*b^4*d^5*g^4)*B^2*x^4 - 4*(b^5*c^2*d^3*g^4 -
5*a*b^4*c*d^4*g^4 - 26*a^2*b^3*d^5*g^4)*B^2*x^3 + 6*(b^5*c^3*d^2*g^4 - 5*a*
b^4*c^2*d^3*g^4 + 10*a^2*b^3*c*d^4*g^4 + 14*a^3*b^2*d^5*g^4)*B^2*x^2 - 12*(
b^5*c^4*d*g^4 - 5*a*b^4*c^3*d^2*g^4 + 10*a^2*b^3*c^2*d^3*g^4 - 10*a^3*b^2*c
*d^4*g^4 - a^4*b*d^5*g^4)*B^2*x - 12*(B^2*b^5*d^5*g^4*x^5 + 5*B^2*a*b^4*d^5
*g^4*x^4 + 10*B^2*a^2*b^3*d^5*g^4*x^3 + 10*B^2*a^3*b^2*d^5*g^4*x^2 + 5*B^2*
a^4*b*d^5*g^4*x + B^2*a^5*d^5*g^4)*log(b*x + a))*log(d*x + c))/(b*d^5)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^4*(A+B*log(e*(d*x+c)/(b*x+a)))^2,x, algorithm="fricas
")
```

```
[Out] integral(A^2*b^4*g^4*x^4 + 4*A^2*a*b^3*g^4*x^3 + 6*A^2*a^2*b^2*g^4*x^2 + 4*
A^2*a^3*b*g^4*x + A^2*a^4*g^4 + (B^2*b^4*g^4*x^4 + 4*B^2*a*b^3*g^4*x^3 + 6*
```

```
B^2*a^2*b^2*g^4*x^2 + 4*B^2*a^3*b*g^4*x + B^2*a^4*g^4)*log((d*x + c)*e/(b*x
 + a))^2 + 2*(A*B*b^4*g^4*x^4 + 4*A*B*a*b^3*g^4*x^3 + 6*A*B*a^2*b^2*g^4*x^2
 + 4*A*B*a^3*b*g^4*x + A*B*a^4*g^4)*log((d*x + c)*e/(b*x + a)), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)**4*(A+B*ln(e*(d*x+c)/(b*x+a)))**2,x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^4*(A+B*log(e*(d*x+c)/(b*x+a)))^2,x, algorithm="giac")
[Out] integrate((b*g*x + a*g)^4*(B*log((d*x + c)*e/(b*x + a)) + A)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a\,g+b\,g\,x)^4\,\left(A+B\,\ln\left(\frac{e\,(c+d\,x)}{a+b\,x}\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*g + b*g*x)^4*(A + B*log((e*(c + d*x))/(a + b*x)))^2,x)
[Out] int((a*g + b*g*x)^4*(A + B*log((e*(c + d*x))/(a + b*x)))^2, x)
```

3.183 $\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2 dx$

Optimal. Leaf size=420

$$-\frac{5B^2(bc-ad)^3g^3x}{12d^3}+\frac{B^2(bc-ad)^2g^3(a+bx)^2}{12bd^2}+\frac{11B^2(bc-ad)^4g^3\log(a+bx)}{12bd^4}+\frac{5B^2(bc-ad)^4g^3\log\left(\frac{c+dx}{a+bx}\right)}{12bd^4}-$$

```
[Out] -5/12*B^2*(-a*d+b*c)^3*g^3*x/d^3+1/12*B^2*(-a*d+b*c)^2*g^3*(b*x+a)^2/b/d^2+
11/12*B^2*(-a*d+b*c)^4*g^3*ln(b*x+a)/b/d^4+5/12*B^2*(-a*d+b*c)^4*g^3*ln((d*
x+c)/(b*x+a))/b/d^4-1/4*B*(-a*d+b*c)^2*g^3*(b*x+a)^2*(A+B*ln(e*(d*x+c)/(b*x
+a)))/b/d^2+1/6*B*(-a*d+b*c)*g^3*(b*x+a)^3*(A+B*ln(e*(d*x+c)/(b*x+a)))/b/d+
1/2*B*(-a*d+b*c)^3*g^3*(d*x+c)*(A+B*ln(e*(d*x+c)/(b*x+a)))/d^4+1/4*g^3*(b*x
+a)^4*(A+B*ln(e*(d*x+c)/(b*x+a)))^2/b+1/2*B*(-a*d+b*c)^4*g^3*(A+B*ln(e*(d*x
+c)/(b*x+a)))*ln(1-d*(b*x+a)/b/(d*x+c))/b/d^4-1/2*B^2*(-a*d+b*c)^4*g^3*poly
log(2,d*(b*x+a)/b/(d*x+c))/b/d^4
```

Rubi [A]

time = 0.34, antiderivative size = 420, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 8, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2552, 2356, 2389, 2379, 2438, 2351, 31, 46}

$$-\frac{B^2g^3(bc-ad)^4\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{2bd^4}+\frac{Bg^3(bc-ad)^4\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{2bd^4}+\frac{Bg^3(c+dx)(bc-ad)^3\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{2d^4}-\frac{Bg^3(a+bx)^2(bc-ad)^2\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{4bd^2}+\frac{Bg^3(a+bx)^3(bc-ad)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{6bd}+\frac{g^3(a+bx)^4\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}{4b}+\frac{11B^2g^3(bc-ad)^4\log(a+bx)}{12bd^4}+\frac{5B^2g^3(bc-ad)^4\log\left(\frac{c+dx}{a+bx}\right)}{12bd^4}-\frac{5B^2g^3x(bc-ad)^3}{12d^3}+\frac{B^2g^3(a+bx)^2(bc-ad)^2}{12bd^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*g + b*g*x)^3*(A + B*Log[(e*(c + d*x))/(a + b*x)])^2,x]
```

```
[Out] (-5*B^2*(b*c - a*d)^3*g^3*x)/(12*d^3) + (B^2*(b*c - a*d)^2*g^3*(a + b*x)^2)
/(12*b*d^2) + (11*B^2*(b*c - a*d)^4*g^3*Log[a + b*x])/(12*b*d^4) + (5*B^2*(
b*c - a*d)^4*g^3*Log[(c + d*x)/(a + b*x)])/(12*b*d^4) - (B*(b*c - a*d)^2*g^
3*(a + b*x)^2*(A + B*Log[(e*(c + d*x))/(a + b*x)]))/(4*b*d^2) + (B*(b*c - a
*d)*g^3*(a + b*x)^3*(A + B*Log[(e*(c + d*x))/(a + b*x)]))/(6*b*d) + (B*(b*c
 - a*d)^3*g^3*(c + d*x)*(A + B*Log[(e*(c + d*x))/(a + b*x)]))/(2*d^4) + (g^
3*(a + b*x)^4*(A + B*Log[(e*(c + d*x))/(a + b*x)])^2)/(4*b) + (B*(b*c - a*d
)^4*g^3*(A + B*Log[(e*(c + d*x))/(a + b*x)])*Log[1 - (d*(a + b*x))/(b*(c +
d*x))])/(2*b*d^4) - (B^2*(b*c - a*d)^4*g^3*PolyLog[2, (d*(a + b*x))/(b*(c +
 d*x))])/(2*b*d^4)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
```

```
NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
n + 2, 0])
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2552

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(
m + 1)*(g/d)^m, Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a
 + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[n
 + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[d*f
- c*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Rubi steps

$$\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2 dx = \frac{g^3(a+bx)^4\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{4b} - \frac{B\int \frac{(bc-ad)g^4(a+bx)^3\left(-\right.}{c+}}{2b}$$

$$= \frac{g^3(a+bx)^4\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{4b} - \frac{\left(B(bc-ad)g^3\right)\int \frac{(a+}{}}{}$$

$$= \frac{g^3(a+bx)^4\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{4b} - \frac{\left(B(bc-ad)g^3\right)\int \left(\frac{b}{-}\right.}{}$$

$$= \frac{g^3(a+bx)^4\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{4b} - \frac{\left(B(bc-ad)g^3\right)\int (a+}{}$$

$$= \frac{AB(bc-ad)^3g^3x}{2d^3} - \frac{B(bc-ad)^2g^3(a+bx)^2\left(A+B\log\left(\frac{e}{}\right.\right.}{4bd^2}$$

$$= \frac{AB(bc-ad)^3g^3x}{2d^3} + \frac{B^2(bc-ad)^3g^3(a+bx)\log\left(\frac{e(c+dx)}{a+bx}\right)}{2bd^3} -$$

$$= \frac{AB(bc-ad)^3g^3x}{2d^3} + \frac{B^2(bc-ad)^4g^3\log(c+dx)}{2bd^4} + \frac{B^2(bc-}{}$$

$$= \frac{AB(bc-ad)^3g^3x}{2d^3} - \frac{5B^2(bc-ad)^3g^3x}{12d^3} + \frac{B^2(bc-ad)^2g^3(a}{12bd^2}$$

$$= \frac{AB(bc-ad)^3g^3x}{2d^3} - \frac{5B^2(bc-ad)^3g^3x}{12d^3} + \frac{B^2(bc-ad)^2g^3(a}{12bd^2}$$

$$= \frac{AB(bc-ad)^3g^3x}{2d^3} - \frac{5B^2(bc-ad)^3g^3x}{12d^3} + \frac{B^2(bc-ad)^2g^3(a}{12bd^2}$$

$$= \frac{AB(bc-ad)^3g^3x}{2d^3} - \frac{5B^2(bc-ad)^3g^3x}{12d^3} + \frac{B^2(bc-ad)^2g^3(a}{12bd^2}$$

Mathematica [A]
time = 0.23, size = 392, normalized size = 0.93

$$\frac{g^3\left((a+bx)^4\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2+\frac{B(bc-ad)\left(6Abd(bc-ad)^2x+6B(bc-ad)^3\log(a+bx)-B(bc-ad)\left(2bd(bc-ad)x-d^2(a+bx)^2-2(bc-ad)^2\log(c+dx)\right)-3B(bc-ad)^2(bdx+(-bc+ad)\log(c+dx))+6bB(bc-ad)^2(c+dx)\log\left(\frac{e(c+dx)}{a+bx}\right)+3d^2(-bc+ad)(a+bx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)+2d^3(a+bx)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)-6(bc-ad)^3\log(c+dx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)-3B(bc-ad)^3\left(\left(2\log\left(\frac{d(a+bx)}{-bc+ad}\right)-\log(c+dx)\right)\log(c+dx)+2\text{Li}_2\left(\frac{b(c+dx)}{bc-ad}\right)\right)\right)}{3d^4}\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*g + b*g*x)^3*(A + B*Log[(e*(c + d*x))/(a + b*x)])^2,x]
```

```
[Out] (g^3*((a + b*x)^4*(A + B*Log[(e*(c + d*x))/(a + b*x)])^2 + (B*(b*c - a*d)*(
6*A*b*d*(b*c - a*d)^2*x + 6*B*(b*c - a*d)^3*Log[a + b*x] - B*(b*c - a*d)*(2
*b*d*(b*c - a*d)*x - d^2*(a + b*x)^2 - 2*(b*c - a*d)^2*Log[c + d*x]) - 3*B*
(b*c - a*d)^2*(b*d*x + (-(b*c) + a*d)*Log[c + d*x]) + 6*b*B*(b*c - a*d)^2*(
c + d*x)*Log[(e*(c + d*x))/(a + b*x)] + 3*d^2*(-(b*c) + a*d)*(a + b*x)^2*(A
 + B*Log[(e*(c + d*x))/(a + b*x)]) + 2*d^3*(a + b*x)^3*(A + B*Log[(e*(c + d
*x))/(a + b*x)]) - 6*(b*c - a*d)^3*Log[c + d*x]*(A + B*Log[(e*(c + d*x))/(a
 + b*x)]) - 3*B*(b*c - a*d)^3*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c
 + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(3*d^4))
)/(4*b)
```

Maple [F]
time = 0.22, size = 0, normalized size = 0.00

$$\int (bgx + ag)^3 \left(A + B \ln\left(\frac{e(dx+c)}{bx+a}\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*g*x+a*g)^3*(A+B*ln(e*(d*x+c)/(b*x+a)))^2,x)
```

```
[Out] int((b*g*x+a*g)^3*(A+B*ln(e*(d*x+c)/(b*x+a)))^2,x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1529 vs. 2(404) = 808.
time = 0.39, size = 1529, normalized size = 3.64

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^3*(A+B*log(e*(d*x+c)/(b*x+a)))^2,x, algorithm="maxima
")
```

```
[Out] 1/4*A^2*b^3*g^3*x^4 + A^2*a*b^2*g^3*x^3 + 3/2*A^2*a^2*b*g^3*x^2 + 2*(x*log(
d*x*e/(b*x + a) + c*e/(b*x + a)) - a*log(b*x + a)/b + c*log(d*x + c)/d)*A*B
*a^3*g^3 + 3*(x^2*log(d*x*e/(b*x + a) + c*e/(b*x + a)) + a^2*log(b*x + a)/b
^2 - c^2*log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d))*A*B*a^2*b*g^3 + (2*x^3*log
(d*x*e/(b*x + a) + c*e/(b*x + a)) - 2*a^3*log(b*x + a)/b^3 + 2*c^3*log(d*x
+ c)/d^3 + ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*A
*B*a*b^2*g^3 + 1/12*(6*x^4*log(d*x*e/(b*x + a) + c*e/(b*x + a)) + 6*a^4*log
(b*x + a)/b^4 - 6*c^4*log(d*x + c)/d^4 + (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3
*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3))*A*B*b^3*
g^3 + A^2*a^3*g^3*x + 1/12*(5*b^3*c^4*g^3 - 14*a*b^2*c^3*d*g^3 + 9*a^2*b*c^
2*d^2*g^3 + 6*a^3*c*d^3*g^3)*B^2*log(d*x + c)/d^4 + 1/2*(b^4*c^4*g^3 - 4*a*
b^3*c^3*d*g^3 + 6*a^2*b^2*c^2*d^2*g^3 - 4*a^3*b*c*d^3*g^3 + a^4*d^4*g^3)*(l
og(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c
- a*d)))*B^2/(b*d^4) + 1/12*(3*B^2*b^4*d^4*g^3*x^4 + 2*(b^4*c*d^3*g^3 + 5*a
```

```
*b^3*d^4*g^3)*B^2*x^3 - 2*(b^4*c^2*d^2*g^3 - 5*a*b^3*c*d^3*g^3 - 5*a^2*b^2*
d^4*g^3)*B^2*x^2 + (b^4*c^3*d*g^3 - 7*a*b^3*c^2*d^2*g^3 + 17*a^2*b^2*c*d^3*
g^3 + a^3*b*d^4*g^3)*B^2*x + 3*(B^2*b^4*d^4*g^3*x^4 + 4*B^2*a*b^3*d^4*g^3*x
^3 + 6*B^2*a^2*b^2*d^4*g^3*x^2 + 4*B^2*a^3*b*d^4*g^3*x + B^2*a^4*d^4*g^3)*l
og(b*x + a)^2 + 3*(B^2*b^4*d^4*g^3*x^4 + 4*B^2*a*b^3*d^4*g^3*x^3 + 6*B^2*a^
2*b^2*d^4*g^3*x^2 + 4*B^2*a^3*b*d^4*g^3*x - (b^4*c^4*g^3 - 4*a*b^3*c^3*d*g^
3 + 6*a^2*b^2*c^2*d^2*g^3 - 4*a^3*b*c*d^3*g^3)*B^2)*log(d*x + c)^2 - (6*B^2
*b^4*d^4*g^3*x^4 + 2*(b^4*c*d^3*g^3 + 11*a*b^3*d^4*g^3)*B^2*x^3 - 3*(b^4*c^
2*d^2*g^3 - 4*a*b^3*c*d^3*g^3 - 9*a^2*b^2*d^4*g^3)*B^2*x^2 + 6*(b^4*c^3*d*g
^3 - 4*a*b^3*c^2*d^2*g^3 + 6*a^2*b^2*c*d^3*g^3 + a^3*b*d^4*g^3)*B^2*x + (6*
a*b^3*c^3*d*g^3 - 21*a^2*b^2*c^2*d^2*g^3 + 26*a^3*b*c*d^3*g^3 - 5*a^4*d^4*g
^3)*B^2)*log(b*x + a) + (6*B^2*b^4*d^4*g^3*x^4 + 2*(b^4*c*d^3*g^3 + 11*a*b^
3*d^4*g^3)*B^2*x^3 - 3*(b^4*c^2*d^2*g^3 - 4*a*b^3*c*d^3*g^3 - 9*a^2*b^2*d^4
*g^3)*B^2*x^2 + 6*(b^4*c^3*d*g^3 - 4*a*b^3*c^2*d^2*g^3 + 6*a^2*b^2*c*d^3*g^
3 + a^3*b*d^4*g^3)*B^2*x - 6*(B^2*b^4*d^4*g^3*x^4 + 4*B^2*a*b^3*d^4*g^3*x^3
 + 6*B^2*a^2*b^2*d^4*g^3*x^2 + 4*B^2*a^3*b*d^4*g^3*x + B^2*a^4*d^4*g^3)*log
(b*x + a))*log(d*x + c))/(b*d^4)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^3*(A+B*log(e*(d*x+c)/(b*x+a)))^2,x, algorithm="fricas
")
```

```
[Out] integral(A^2*b^3*g^3*x^3 + 3*A^2*a*b^2*g^3*x^2 + 3*A^2*a^2*b*g^3*x + A^2*a^
3*g^3 + (B^2*b^3*g^3*x^3 + 3*B^2*a*b^2*g^3*x^2 + 3*B^2*a^2*b*g^3*x + B^2*a^
3*g^3)*log((d*x + c)*e/(b*x + a))^2 + 2*(A*B*b^3*g^3*x^3 + 3*A*B*a*b^2*g^3*
x^2 + 3*A*B*a^2*b*g^3*x + A*B*a^3*g^3)*log((d*x + c)*e/(b*x + a)), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)**3*(A+B*ln(e*(d*x+c)/(b*x+a)))**2,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^3*(A+B*log(e*(d*x+c)/(b*x+a)))^2,x, algorithm="giac")
[Out] integrate((b*g*x + a*g)^3*(B*log((d*x + c)*e/(b*x + a)) + A)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a\,g+b\,g\,x)^3\,\left(A+B\,\ln\left(\frac{e\,(c+d\,x)}{a+b\,x}\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*g + b*g*x)^3*(A + B*log((e*(c + d*x))/(a + b*x)))^2,x)
[Out] int((a*g + b*g*x)^3*(A + B*log((e*(c + d*x))/(a + b*x)))^2, x)
```

3.184 $\int (ag+bgx)^2 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2 dx$

Optimal. Leaf size=335

$$\frac{B^2(bc-ad)^2g^2x}{3d^2}-\frac{B^2(bc-ad)^3g^2\log(a+bx)}{bd^3}-\frac{B^2(bc-ad)^3g^2\log\left(\frac{c+dx}{a+bx}\right)}{3bd^3}+\frac{B(bc-ad)g^2(a+bx)^2\left(A+B\mathrm{l}\right.}{3bd}$$

```
[Out] 1/3*B^2*(-a*d+b*c)^2*g^2*x/d^2-B^2*(-a*d+b*c)^3*g^2*ln(b*x+a)/b/d^3-1/3*B^2
*(-a*d+b*c)^3*g^2*ln((d*x+c)/(b*x+a))/b/d^3+1/3*B*(-a*d+b*c)*g^2*(b*x+a)^2*
(A+B*ln(e*(d*x+c)/(b*x+a)))/b/d-2/3*B*(-a*d+b*c)^2*g^2*(d*x+c)*(A+B*ln(e*(d
*x+c)/(b*x+a)))/d^3+1/3*g^2*(b*x+a)^3*(A+B*ln(e*(d*x+c)/(b*x+a)))^2/b-2/3*B
*(-a*d+b*c)^3*g^2*(A+B*ln(e*(d*x+c)/(b*x+a)))*ln(1-d*(b*x+a)/b/(d*x+c))/b/d
^3+2/3*B^2*(-a*d+b*c)^3*g^2*polylog(2,d*(b*x+a)/b/(d*x+c))/b/d^3
```

Rubi [A]
time = 0.24, antiderivative size = 335, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2552, 2356, 2389, 2379, 2438, 2351, 31, 46}

$$\frac{2B^2g^2(bc-ad)^3\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{3bd^3}-\frac{2Bg^2(bc-ad)^3\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{3bd^3}-\frac{2Bg^2(c+dx)(bc-ad)^2\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{3d^3}+\frac{Bg^2(a+bx)^2(bc-ad)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{3bd}+\frac{g^2(a+bx)^3\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}{3b}-\frac{B^2g^2(bc-ad)^3\log(a+bx)}{bd^3}-\frac{B^2g^2(bc-ad)^3\log\left(\frac{c+dx}{a+bx}\right)}{3bd^3}+\frac{B^2g^2x(bc-ad)^2}{3d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*g + b*g*x)^2*(A + B*Log[(e*(c + d*x))/(a + b*x)])^2,x]
```

```
[Out] (B^2*(b*c - a*d)^2*g^2*x)/(3*d^2) - (B^2*(b*c - a*d)^3*g^2*Log[a + b*x])/(b
*d^3) - (B^2*(b*c - a*d)^3*g^2*Log[(c + d*x)/(a + b*x)])/(3*b*d^3) + (B*(b*
c - a*d)*g^2*(a + b*x)^2*(A + B*Log[(e*(c + d*x))/(a + b*x)]))/(3*b*d) - (2
*B*(b*c - a*d)^2*g^2*(c + d*x)*(A + B*Log[(e*(c + d*x))/(a + b*x)]))/(3*d^3
) + (g^2*(a + b*x)^3*(A + B*Log[(e*(c + d*x))/(a + b*x)])^2)/(3*b) - (2*B*(
b*c - a*d)^3*g^2*(A + B*Log[(e*(c + d*x))/(a + b*x)])*Log[1 - (d*(a + b*x))
/(b*(c + d*x))])/(3*b*d^3) + (2*B^2*(b*c - a*d)^3*g^2*PolyLog[2, (d*(a + b*
x))/(b*(c + d*x))])/(3*b*d^3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2552

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(
m + 1)*(g/d)^m, Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a
 + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[n
 + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[d*f
- c*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Rubi steps

$$\int (ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2\,dx = \frac{g^2(a+bx)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{3b} - \frac{(2B)\int \frac{(bc-ad)g^3(a+bx)}{}}{3}$$

$$= \frac{g^2(a+bx)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{3b} - \frac{(2B(bc-ad)g^2)\int \frac{(}{}}{}$$

$$= \frac{g^2(a+bx)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{3b} - \frac{(2B(bc-ad)g^2)\int \Big(}{}$$

$$= \frac{g^2(a+bx)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{3b} - \frac{(2B(bc-ad)g^2)\int (a}{}$$

$$= -\frac{2AB(bc-ad)^2g^2x}{3d^2} + \frac{B(bc-ad)g^2(a+bx)^2\Big(A+B\log}{3bd}$$

$$= -\frac{2AB(bc-ad)^2g^2x}{3d^2} - \frac{2B^2(bc-ad)^2g^2(a+bx)\log\left(\frac{e(c+dx}{a+bx}\right.}{3bd^2}$$

$$= -\frac{2AB(bc-ad)^2g^2x}{3d^2} - \frac{2B^2(bc-ad)^3g^2\log(c+dx)}{3bd^3} - \frac{2B^2}{}$$

$$= -\frac{2AB(bc-ad)^2g^2x}{3d^2} + \frac{B^2(bc-ad)^2g^2x}{3d^2} - \frac{B^2(bc-ad)^3g^2}{bd^3}$$

$$= -\frac{2AB(bc-ad)^2g^2x}{3d^2} + \frac{B^2(bc-ad)^2g^2x}{3d^2} - \frac{B^2(bc-ad)^3g^2}{bd^3}$$

$$= -\frac{2AB(bc-ad)^2g^2x}{3d^2} + \frac{B^2(bc-ad)^2g^2x}{3d^2} - \frac{B^2(bc-ad)^3g^2}{bd^3}$$

$$= -\frac{2AB(bc-ad)^2g^2x}{3d^2} + \frac{B^2(bc-ad)^2g^2x}{3d^2} - \frac{B^2(bc-ad)^3g^2}{bd^3}$$

Mathematica [A]
time = 0.16, size = 290, normalized size = 0.87

$$\frac{g^2\left((a+bx)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2-\frac{B(bc-ad)\left(2Abd(bc-ad)x+2B(bc-ad)^2\log(a+bx)-B(bc-ad)(bdx+(-bc+ad)\log(c+dx))+2bB(bc-ad)(c+dx)\log\left(\frac{e(c+dx)}{a+bx}\right)-d^2(a+bx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)-2(bc-ad)^2\log(c+dx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)-B(bc-ad)^2\left(\left(2\log\left(\frac{d(a+bx)}{-bc+ad}\right)-\log(c+dx)\right)\log(c+dx)+2\text{Li}_2\left(\frac{b(c+dx)}{bc-ad}\right)\right)\right)}{d^3}\right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*g + b*g*x)^2*(A + B*Log[(e*(c + d*x))/(a + b*x)])^2,x]
```

```
[Out] (g^2*((a + b*x)^3*(A + B*Log[(e*(c + d*x))/(a + b*x)])^2 - (B*(b*c - a*d)*(
2*A*b*d*(b*c - a*d)*x + 2*B*(b*c - a*d)^2*Log[a + b*x] - B*(b*c - a*d)*(b*d
```

```
*x + (-(b*c) + a*d)*Log[c + d*x]) + 2*b*B*(b*c - a*d)*(c + d*x)*Log[(e*(c +
 d*x))/(a + b*x)] - d^2*(a + b*x)^2*(A + B*Log[(e*(c + d*x))/(a + b*x)]) -
2*(b*c - a*d)^2*Log[c + d*x]*(A + B*Log[(e*(c + d*x))/(a + b*x)]) - B*(b*c
- a*d)^2*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x]
 + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/d^3))/(3*b)
```

Maple [F]
time = 0.16, size = 0, normalized size = 0.00

$$\int (bgx + ag)^2 \left(A + B\ln\left(\frac{e(dx+c)}{bx+a}\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*g*x+a*g)^2*(A+B*ln(e*(d*x+c)/(b*x+a)))^2,x)
```

```
[Out] int((b*g*x+a*g)^2*(A+B*ln(e*(d*x+c)/(b*x+a)))^2,x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1027 vs. $2(324) = 648$.
time = 0.38, size = 1027, normalized size = 3.07

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^2*(A+B*log(e*(d*x+c)/(b*x+a)))^2,x, algorithm="maxima
")
```

```
[Out] 1/3*A^2*b^2*g^2*x^3 + A^2*a*b*g^2*x^2 + 2*(x*log(d*x*e/(b*x + a) + c*e/(b*x
 + a)) - a*log(b*x + a)/b + c*log(d*x + c)/d)*A*B*a^2*g^2 + 2*(x^2*log(d*x*
e/(b*x + a) + c*e/(b*x + a)) + a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)/d^2
+ (b*c - a*d)*x/(b*d))*A*B*a*b*g^2 + 1/3*(2*x^3*log(d*x*e/(b*x + a) + c*e/(
b*x + a)) - 2*a^3*log(b*x + a)/b^3 + 2*c^3*log(d*x + c)/d^3 + ((b^2*c*d - a
*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*A*B*b^2*g^2 + A^2*a^2*g^2
*x - 1/3*(b^2*c^3*g^2 - a*b*c^2*d*g^2 - 2*a^2*c*d^2*g^2)*B^2*log(d*x + c)/d
^3 - 2/3*(b^3*c^3*g^2 - 3*a*b^2*c^2*d*g^2 + 3*a^2*b*c*d^2*g^2 - a^3*d^3*g^2
)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(
b*c - a*d)))*B^2/(b*d^3) + 1/3*(B^2*b^3*d^3*g^2*x^3 + (b^3*c*d^2*g^2 + 2*a*
b^2*d^3*g^2)*B^2*x^2 - (b^3*c^2*d*g^2 - 4*a*b^2*c*d^2*g^2)*B^2*x + (B^2*b^3
*d^3*g^2*x^3 + 3*B^2*a*b^2*d^3*g^2*x^2 + 3*B^2*a^2*b*d^3*g^2*x + B^2*a^3*d^
3*g^2)*log(b*x + a)^2 + (B^2*b^3*d^3*g^2*x^3 + 3*B^2*a*b^2*d^3*g^2*x^2 + 3*
B^2*a^2*b*d^3*g^2*x + (b^3*c^3*g^2 - 3*a*b^2*c^2*d*g^2 + 3*a^2*b*c*d^2*g^2)
*B^2)*log(d*x + c)^2 - (2*B^2*b^3*d^3*g^2*x^3 + (b^3*c*d^2*g^2 + 5*a*b^2*d^
3*g^2)*B^2*x^2 - 2*(b^3*c^2*d*g^2 - 3*a*b^2*c*d^2*g^2 - a^2*b*d^3*g^2)*B^2*
x - (2*a*b^2*c^2*d*g^2 - 5*a^2*b*c*d^2*g^2 + a^3*d^3*g^2)*B^2)*log(b*x + a)
 + (2*B^2*b^3*d^3*g^2*x^3 + (b^3*c*d^2*g^2 + 5*a*b^2*d^3*g^2)*B^2*x^2 - 2*(
b^3*c^2*d*g^2 - 3*a*b^2*c*d^2*g^2 - a^2*b*d^3*g^2)*B^2*x - 2*(B^2*b^3*d^3*g
```

```
^2*x^3 + 3*B^2*a*b^2*d^3*g^2*x^2 + 3*B^2*a^2*b*d^3*g^2*x + B^2*a^3*d^3*g^2)
*log(b*x + a))*log(d*x + c))/(b*d^3)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^2*(A+B*log(e*(d*x+c)/(b*x+a)))^2,x, algorithm="fricas
")
```

```
[Out] integral(A^2*b^2*g^2*x^2 + 2*A^2*a*b*g^2*x + A^2*a^2*g^2 + (B^2*b^2*g^2*x^2
 + 2*B^2*a*b*g^2*x + B^2*a^2*g^2)*log((d*x + c)*e/(b*x + a))^2 + 2*(A*B*b^2
*g^2*x^2 + 2*A*B*a*b*g^2*x + A*B*a^2*g^2)*log((d*x + c)*e/(b*x + a)), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)**2*(A+B*ln(e*(d*x+c)/(b*x+a)))**2,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^2*(A+B*log(e*(d*x+c)/(b*x+a)))^2,x, algorithm="giac")
```

```
[Out] integrate((b*g*x + a*g)^2*(B*log((d*x + c)*e/(b*x + a)) + A)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a\,g+b\,g\,x)^2\,\left(A+B\,\ln\left(\frac{e\,(c+d\,x)}{a+b\,x}\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*g + b*g*x)^2*(A + B*log((e*(c + d*x))/(a + b*x)))^2,x)
```

```
[Out] int((a*g + b*g*x)^2*(A + B*log((e*(c + d*x))/(a + b*x)))^2, x)
```

3.185 $\int (ag+bgx) \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2 dx$

Optimal. Leaf size=202

$$\frac{B^2(bc-ad)^2 g\log(a+bx)}{bd^2}+\frac{B(bc-ad)g(c+dx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{d^2}+\frac{g(a+bx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right.}{2b}$$

[Out] B^2*(-a*d+b*c)^2*g*ln(b*x+a)/b/d^2+B*(-a*d+b*c)*g*(d*x+c)*(A+B*ln(e*(d*x+c)/(b*x+a)))/d^2+1/2*g*(b*x+a)^2*(A+B*ln(e*(d*x+c)/(b*x+a)))^2/b+B*(-a*d+b*c)^2*g*(A+B*ln(e*(d*x+c)/(b*x+a)))*ln(1-d*(b*x+a)/b/(d*x+c))/b/d^2-B^2*(-a*d+b*c)^2*g*polylog(2,d*(b*x+a)/b/(d*x+c))/b/d^2

Rubi [A]
time = 0.15, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {2552, 2356, 2389, 2379, 2438, 2351, 31}

$$-\frac{B^2g(bc-ad)^2\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{bd^2}+\frac{Bg(bc-ad)^2\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{bd^2}+\frac{Bg(c+dx)(bc-ad)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{d^2}+\frac{g(a+bx)^2\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}{2b}+\frac{B^2g(bc-ad)^2\log(a+bx)}{bd^2}$$

Antiderivative was successfully verified.

[In] Int[(a*g + b*g*x)*(A + B*Log[(e*(c + d*x))/(a + b*x)])^2,x]

[Out] (B^2*(b*c - a*d)^2*g*Log[a + b*x])/(b*d^2) + (B*(b*c - a*d)*g*(c + d*x)*(A + B*Log[(e*(c + d*x))/(a + b*x)]))/d^2 + (g*(a + b*x)^2*(A + B*Log[(e*(c + d*x))/(a + b*x)])^2)/(2*b) + (B*(b*c - a*d)^2*g*(A + B*Log[(e*(c + d*x))/(a + b*x)])*Log[1 - (d*(a + b*x))/(b*(c + d*x))])/(b*d^2) - (B^2*(b*c - a*d)^2*g*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/(b*d^2)

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.), x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x] - Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
```

```
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2552

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(
m + 1)*(g/d)^m, Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a
 + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[n
 + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[d*f
- c*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Rubi steps

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2 dx = \frac{g(a+bx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{2b} - \frac{B\int \frac{(bc-ad)g^2(a+bx)\left(-A\right.}{c+d}}{bg}$$

$$= \frac{g(a+bx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{2b} - (B(bc-ad)g)\int \frac{(a+b}{}$$

$$= \frac{g(a+bx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{2b} - (B(bc-ad)g)\int \left(\frac{b(}{}\right.$$

$$= \frac{g(a+bx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{2b} - (B(bc-ad)g)\int \left(-A\right.$$

$$= \frac{AB(bc-ad)gx}{d} - \frac{B(bc-ad)^2 g\log(c+dx)\left(A+B\log\left(\frac{e}{}\right.\right.}{bd^2}$$

$$= \frac{AB(bc-ad)gx}{d} + \frac{B^2(bc-ad)g(a+bx)\log\left(\frac{e(c+dx)}{a+bx}\right)}{bd} - \frac{B}{}$$

$$= \frac{AB(bc-ad)gx}{d} + \frac{B^2(bc-ad)^2 g\log(c+dx)}{bd^2} + \frac{B^2(bc-ad}{}$$

$$= \frac{AB(bc-ad)gx}{d} + \frac{B^2(bc-ad)^2 g\log(c+dx)}{bd^2} + \frac{B^2(bc-ad}{}$$

$$= \frac{AB(bc-ad)gx}{d} + \frac{B^2(bc-ad)^2 g\log(c+dx)}{bd^2} - \frac{B^2(bc-ad}{}$$

$$= \frac{AB(bc-ad)gx}{d} + \frac{B^2(bc-ad)^2 g\log(c+dx)}{bd^2} - \frac{B^2(bc-ad}{}$$

$$= \frac{AB(bc-ad)gx}{d} + \frac{B^2(bc-ad)^2 g\log(c+dx)}{bd^2} - \frac{B^2(bc-ad}{}$$

Mathematica [A]
time = 0.13, size = 203, normalized size = 1.00

$$\frac{g\left((a+bx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2+\frac{B(bc-ad)\left(2Abdx+2B(bc-ad)\log(a+bx)+2bB(c+dx)\log\left(\frac{e(c+dx)}{a+bx}\right)-2(bc-ad)\log(c+dx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)-B(bc-ad)\left(\left(2\log\left(\frac{d(a+bx)}{-bc+ad}\right)-\log(c+dx)\right)\log(c+dx)+2\mathrm{Li}_2\left(\frac{b(c+dx)}{bc-ad}\right)\right)\right)}{d^2}\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*g + b*g*x)*(A + B*Log[(e*(c + d*x))/(a + b*x)])^2,x]
```

```
[Out] (g*((a + b*x)^2*(A + B*Log[(e*(c + d*x))/(a + b*x)])^2 + (B*(b*c - a*d)*(2*
A*b*d*x + 2*B*(b*c - a*d)*Log[a + b*x] + 2*b*B*(c + d*x)*Log[(e*(c + d*x))/
```

```
(a + b*x)] - 2*(b*c - a*d)*Log[c + d*x]*(A + B*Log[(e*(c + d*x))/(a + b*x)]
) - B*(b*c - a*d)*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log
[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/d^2))/(2*b)
```

Maple [F]
time = 0.10, size = 0, normalized size = 0.00

$$\int (bgx + ag) \left(A + B \ln \left(\frac{e(dx + c)}{bx + a}\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*g*x+a*g)*(A+B*ln(e*(d*x+c)/(b*x+a)))^2,x)
```

```
[Out] int((b*g*x+a*g)*(A+B*ln(e*(d*x+c)/(b*x+a)))^2,x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 534 vs. $2(202) = 404$.
time = 0.36, size = 534, normalized size = 2.64

$$\frac{1}{2} A^2bgx^2 + 2\left(x\log\left(\frac{dxe}{bx+a} + \frac{ce}{bx+a}\right) - \frac{a\log(bx+a)}{b} + \frac{c\log(dx+c)}{d}\right)ABag + \left(x^2\log\left(\frac{dxe}{bx+a} + \frac{ce}{bx+a}\right) + \frac{a^2\log(bx+a)}{b^2} - \frac{c^2\log(dx+c)}{d^2} + \frac{(bc-ad)x}{bd}\right)ABbg + A^2agx + \frac{B^2acg\log(dx+c)}{d} + \frac{(b^2c^2g - 2abcdg + a^2d^2g)\left(\log(bx+a)\log\left(\frac{bdx+ad}{bc-ad} + 1\right) + \mathrm{Li}_2\left(-\frac{bdx+ad}{bc-ad}\right)\right)B^2}{bd^2} + \frac{B^2b^2d^2gx^2 + 2B^2b^2cdgx + (B^2b^2d^2gx^2 + 2B^2abd^2gx + B^2a^2d^2g)\log(bx+a)^2 + (B^2b^2d^2gx^2 + 2B^2abd^2gx - (b^2c^2g - 2abcdg)B^2)\log(dx+c)^2 - 2(B^2b^2d^2gx^2 + B^2abcdg + (b^2cdg + abd^2g)B^2x)\log(bx+a) + 2(B^2b^2d^2gx^2 + (b^2cdg + abd^2g)B^2x - (B^2b^2d^2gx^2 + 2B^2abd^2gx + B^2a^2d^2g)\log(bx+a))\log(dx+c)}{2bd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)*(A+B*log(e*(d*x+c)/(b*x+a)))^2,x, algorithm="maxima")
```

```
[Out] 1/2*A^2*b*g*x^2 + 2*(x*log(d*x*e/(b*x + a) + c*e/(b*x + a)) - a*log(b*x + a
)/b + c*log(d*x + c)/d)*A*B*a*g + (x^2*log(d*x*e/(b*x + a) + c*e/(b*x + a))
 + a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d))*A*B*b
*g + A^2*a*g*x + B^2*a*c*g*log(d*x + c)/d + (b^2*c^2*g - 2*a*b*c*d*g + a^2*
d^2*g)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a
*d)/(b*c - a*d)))*B^2/(b*d^2) + 1/2*(B^2*b^2*d^2*g*x^2 + 2*B^2*b^2*c*d*g*x
+ (B^2*b^2*d^2*g*x^2 + 2*B^2*a*b*d^2*g*x + B^2*a^2*d^2*g)*log(b*x + a)^2 +
(B^2*b^2*d^2*g*x^2 + 2*B^2*a*b*d^2*g*x - (b^2*c^2*g - 2*a*b*c*d*g)*B^2)*log
(d*x + c)^2 - 2*(B^2*b^2*d^2*g*x^2 + B^2*a*b*c*d*g + (b^2*c*d*g + a*b*d^2*g
)*B^2*x)*log(b*x + a) + 2*(B^2*b^2*d^2*g*x^2 + (b^2*c*d*g + a*b*d^2*g)*B^2*
x - (B^2*b^2*d^2*g*x^2 + 2*B^2*a*b*d^2*g*x + B^2*a^2*d^2*g)*log(b*x + a))*l
og(d*x + c))/(b*d^2)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)*(A+B*log(e*(d*x+c)/(b*x+a)))^2,x, algorithm="fricas")
```

```
[Out] integral(A^2*b*g*x + A^2*a*g + (B^2*b*g*x + B^2*a*g)*log((d*x + c)*e/(b*x +
 a))^2 + 2*(A*B*b*g*x + A*B*a*g)*log((d*x + c)*e/(b*x + a)), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)*(A+B*ln(e*(d*x+c)/(b*x+a)))**2,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)*(A+B*log(e*(d*x+c)/(b*x+a)))^2,x, algorithm="giac")
```

```
[Out] integrate((b*g*x + a*g)*(B*log((d*x + c)*e/(b*x + a)) + A)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a\,g+b\,g\,x)\,\left(A+B\,\ln\left(\frac{e\,(c+d\,x)}{a+b\,x}\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*g + b*g*x)*(A + B*log((e*(c + d*x))/(a + b*x)))^2,x)
```

```
[Out] int((a*g + b*g*x)*(A + B*log((e*(c + d*x))/(a + b*x)))^2, x)
```

3.186 $\int \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{ag+bgx}\,dx$

Optimal. Leaf size=128

$$-\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{bg}-\frac{2B\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)\mathrm{Li}_2\left(\frac{b(c+dx)}{d(a+bx)}\right)}{bg}+\frac{2B^2\mathrm{Li}_3\left(\frac{b(c+dx)}{d(a+bx)}\right)}{bg}$$

```
[Out] -ln((a*d-b*c)/d/(b*x+a))*(A+B*ln(e*(d*x+c)/(b*x+a)))^2/b/g-2*B*(A+B*ln(e*(d
*x+c)/(b*x+a)))*polylog(2,b*(d*x+c)/d/(b*x+a))/b/g+2*B^2*polylog(3,b*(d*x+c
)/d/(b*x+a))/b/g
```

Rubi [A]
time = 0.08, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2552, 2354, 2421, 6724}

$$-\frac{2B\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{bg}+\frac{2B^2\text{PolyLog}\left(3,\frac{b(c+dx)}{d(a+bx)}\right)}{bg}-\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}{bg}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(c + d*x))/(a + b*x)])^2/(a*g + b*g*x),x]
```

```
[Out] -((Log[-((b*c - a*d)/(d*(a + b*x)))]*(A + B*Log[(e*(c + d*x))/(a + b*x)])^2
)/(b*g)) - (2*B*(A + B*Log[(e*(c + d*x))/(a + b*x)])*PolyLog[2, (b*(c + d*x
))/(d*(a + b*x))])/(b*g) + (2*B^2*PolyLog[3, (b*(c + d*x))/(d*(a + b*x))])/
(b*g)
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2552

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(
m + 1)*(g/d)^m, Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a
 + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[n
 + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[d*f
- c*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{ag+bgx}\,dx = \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2\log(ag+bgx)}{bg} - \frac{(2B)\int \frac{(a+bx)\left(\frac{de}{a+bx}-\frac{be(c+dx)}{(a+bx)^2}\right)\left(A+B\log \cdots\right.}{e(c+dx)}}{bg}\cdots$$

$$= \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2\log(ag+bgx)}{bg} - \frac{(2B)\int \frac{(a+bx)\left(\frac{de}{a+bx}-\frac{be(c+dx)}{(a+bx)^2}\right)\left(A+B\log \cdots\right.}{c+dx}}{beg}\cdots$$

$$= \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2\log(ag+bgx)}{bg} - \frac{(2B)\int \frac{\left(de-\frac{be(c+dx)}{a+bx}\right)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\cdots\right.}{c+dx}}{beg}\cdots$$

$$= \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2\log(ag+bgx)}{bg} - \frac{(2B)\int \frac{(bc-ad)e\left(-A-B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)\log\cdots}{(a+bx)(c+dx)}}{beg}\cdots$$

$$= \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2\log(ag+bgx)}{bg} - \frac{(2B(bc-ad))\int \frac{\left(-A-B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)\cdots}{(a+bx)(c+d\cdots}}{bg}\cdots$$

$$= \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2\log(ag+bgx)}{bg} - \frac{(2B(bc-ad))\int \left(\frac{b\left(-A-B\log\left(\frac{e(c+dx)}{a+bx}\cdots\right.\right.}{(bc-ad)(a\cdots}\cdots\right.}{\cdots}\cdots$$

$$= \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2\log(ag+bgx)}{bg} - \frac{(2B)\int \frac{\left(-A-B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)\log(ag+bgx)\cdots}{a+bx}}{g}\cdots$$

$$= \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2\log(ag+bgx)}{bg} - \frac{(2B)\int \left(\frac{A\log(ag+bgx)}{-a-bx}+\frac{B\log\left(\frac{e(c+dx)}{a+bx}\cdots\right.}{-a-\cdots}\cdots\right.}{g}\cdots$$

$$= \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2\log(ag+bgx)}{bg} - \frac{(2AB)\int \frac{\log(ag+bgx)}{-a-bx}\,dx}{g} - \frac{(2B^2)\int \cdots}{\cdots}\cdots$$

$$= -\frac{2AB\log\left(\frac{b(c+dx)}{bc-ad}\right)\log(ag+bgx)}{bg} + \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2\log(ag+bgx)}{bg}\cdots$$

$$= -\frac{2AB\log\left(\frac{b(c+dx)}{bc-ad}\right)\log(ag+bgx)}{bg} + \frac{2B^2\log\left(\frac{b(c+dx)}{bc-ad}\right)\left(\log\left(\frac{1}{a+bx}\right)+\log(c\cdots\right.}{bg\cdots}\cdots$$

$$= \frac{AB\log^2(g(a+bx))}{bg} - \frac{2B^2\log\left(\frac{1}{a+bx}\right)\log(g(a+bx))\log(c+dx)}{bg} - \frac{B^2\log(g\cdots}{\cdots}\cdots$$

$$= \frac{AB\log^2(g(a+bx))}{bg} - \frac{2B^2\log\left(\frac{1}{a+bx}\right)\log(g(a+bx))\log(c+dx)}{bg} + \frac{B^2\log\left(\cdots\right.}{\cdots}\cdots$$

$$= \frac{AB\log^2(g(a+bx))}{bg} + \frac{B^2\log^3(g(a+bx))}{3bg} - \frac{B^2\log^2\left(\frac{1}{a+bx}\right)\log(c+dx)}{bg} - \frac{2\cdots}{\cdots}\cdots$$

$$\frac{AB\log^2(g(a+bx))}{\cdots} + \frac{B^2\log^3(g(a+bx))}{\cdots} \quad \frac{B^2\log^2\left(\frac{1}{a+bx}\right)\log(c+dx)}{\cdots} \quad 2\cdots$$

Mathematica [A]
time = 0.13, size = 251, normalized size = 1.96

$$\frac{-AB\log^2\left(\frac{a}{b}+x\right)+A^2\log(a+bx)+2AB\log\left(\frac{a}{b}+x\right)\log(a+bx)-2AB\log\left(\frac{c}{d}+x\right)\log(a+bx)+2AB\log\left(\frac{c}{d}+x\right)\log\left(\frac{d(a+bx)}{-bc+ad}\right)+2AB\log(a+bx)\log\left(\frac{e(c+dx)}{a+bx}\right)-B^2\log\left(\frac{-bc+ad}{d(a+bx)}\right)\log^2\left(\frac{e(c+dx)}{a+bx}\right)+2AB\mathrm{Li}_2\left(\frac{b(c+dx)}{bc-ad}\right)-2B^2\log\left(\frac{e(c+dx)}{a+bx}\right)\mathrm{Li}_2\left(\frac{b(c+dx)}{d(a+bx)}\right)+2B^2\mathrm{Li}_3\left(\frac{b(c+dx)}{d(a+bx)}\right)}{bg}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(c + d*x))/(a + b*x)])^2/(a*g + b*g*x),x]
```

```
[Out] (-(A*B*Log[a/b + x]^2) + A^2*Log[a + b*x] + 2*A*B*Log[a/b + x]*Log[a + b*x]
 - 2*A*B*Log[c/d + x]*Log[a + b*x] + 2*A*B*Log[c/d + x]*Log[(d*(a + b*x))/(
-(b*c) + a*d)] + 2*A*B*Log[a + b*x]*Log[(e*(c + d*x))/(a + b*x)] - B^2*Log[
(-(b*c) + a*d)/(d*(a + b*x))]*Log[(e*(c + d*x))/(a + b*x)]^2 + 2*A*B*PolyLo
g[2, (b*(c + d*x))/(b*c - a*d)] - 2*B^2*Log[(e*(c + d*x))/(a + b*x)]*PolyLo
g[2, (b*(c + d*x))/(d*(a + b*x))] + 2*B^2*PolyLog[3, (b*(c + d*x))/(d*(a +
b*x))])/(b*g)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 473 vs. $2(127) = 254$.
time = 0.89, size = 474, normalized size = 3.70

method	result
risch	$\frac{A^2\ln(bx+a)}{gb}-\frac{B^2\ln\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)^2\ln\left(1-\frac{b\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)}{ed}\right)}{bg}-\frac{2B^2\ln\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)\operatorname{polylog}\left(2,\frac{b\left(\frac{de}{b}-\frac{e(ad-\cdots}{b(bx+\cdots}\right)}{ed}\right.\cdots}{bg}\cdots$
derivativedivides	$\frac{e(ad-cb)\left(-\frac{bA^2\ln\left(\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)b-ed\right)}{ge(ad-cb)}-\frac{bB^2\ln\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)^2\ln\left(1-\frac{b\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)}{ed}\right)}{ge(ad-cb)}-\frac{2bB^2\ln\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right.\cdots}{g\cdots}\cdots\right.}{\cdots}$
default	$\frac{e(ad-cb)\left(-\frac{bA^2\ln\left(\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)b-ed\right)}{ge(ad-cb)}-\frac{bB^2\ln\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)^2\ln\left(1-\frac{b\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)}{ed}\right)}{ge(ad-cb)}-\frac{2bB^2\ln\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right.\cdots}{g\cdots}\cdots\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*(d*x+c)/(b*x+a)))^2/(b*g*x+a*g),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^2*e*(a*d-b*c)*(-b/g/e/(a*d-b*c)*A^2*ln((d*e/b-e*(a*d-b*c)/b/(b*x+a))*b-
e*d)-b/g/e/(a*d-b*c)*B^2*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))^2*ln(1-b/e/d*(d*e/
b-e*(a*d-b*c)/b/(b*x+a)))-2*b/g/e/(a*d-b*c)*B^2*ln(d*e/b-e*(a*d-b*c)/b/(b*x
+a))*polylog(2,b/e/d*(d*e/b-e*(a*d-b*c)/b/(b*x+a)))+2*b/g/e/(a*d-b*c)*B^2*p
olylog(3,b/e/d*(d*e/b-e*(a*d-b*c)/b/(b*x+a)))-2*b/g/e/(a*d-b*c)*A*B*dilog(-
((d*e/b-e*(a*d-b*c)/b/(b*x+a))*b-e*d)/e/d)-2*b/g/e/(a*d-b*c)*A*B*ln(d*e/b-e
*(a*d-b*c)/b/(b*x+a))*ln(-((d*e/b-e*(a*d-b*c)/b/(b*x+a))*b-e*d)/e/d))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(d*x+c)/(b*x+a)))^2/(b*g*x+a*g),x, algorithm="maxima")
```

```
[Out] B^2*log(b*x + a)*log(d*x + c)^2/(b*g) + A^2*log(b*g*x + a*g)/(b*g) - integr
ate(-(2*A*B*b*c + B^2*b*c + (B^2*b*d*x + B^2*b*c)*log(b*x + a)^2 + (2*A*B*b
*d + B^2*b*d)*x - 2*(A*B*b*c + B^2*b*c + (A*B*b*d + B^2*b*d)*x)*log(b*x + a
) + 2*(A*B*b*c + B^2*b*c + (A*B*b*d + B^2*b*d)*x - (2*B^2*b*d*x + (b*c + a*
d)*B^2)*log(b*x + a))*log(d*x + c))/(b^2*d*g*x^2 + a*b*c*g + (b^2*c*g + a*b
*d*g)*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(d*x+c)/(b*x+a)))^2/(b*g*x+a*g),x, algorithm="fricas")
```

```
[Out] integral((B^2*log((d*x + c)*e/(b*x + a))^2 + 2*A*B*log((d*x + c)*e/(b*x + a
)) + A^2)/(b*g*x + a*g), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A^2}{a+bx}\,dx + \int \frac{B^2 \log\left(\frac{ce}{a+bx}+\frac{dex}{a+bx}\right)^2}{a+bx}\,dx + \int \frac{2AB \log\left(\frac{ce}{a+bx}+\frac{dex}{a+bx}\right)}{a+bx}\,dx}{g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(d*x+c)/(b*x+a)))**2/(b*g*x+a*g),x)
```

```
[Out] (Integral(A**2/(a + b*x), x) + Integral(B**2*log(c*e/(a + b*x) + d*e*x/(a +
 b*x))**2/(a + b*x), x) + Integral(2*A*B*log(c*e/(a + b*x) + d*e*x/(a + b*x
))/(a + b*x), x))/g
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(d*x+c)/(b*x+a)))^2/(b*g*x+a*g),x, algorithm="giac")
[Out] integrate((B*log((d*x + c)*e/(b*x + a)) + A)^2/(b*g*x + a*g), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(A+B\,\ln\left(\frac{e\,\left(c+d\,x\right)}{a+b\,x}\right)\right)}^2}{a\,g+b\,g\,x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(c + d*x))/(a + b*x)))^2/(a*g + b*g*x),x)
[Out] int((A + B*log((e*(c + d*x))/(a + b*x)))^2/(a*g + b*g*x), x)
```

3.187 $\int \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(ag+bgx)^2}\,dx$

Optimal. Leaf size=153

$$\frac{2AB(c+dx)}{(bc-ad)g^2(a+bx)}-\frac{2B^2(c+dx)}{(bc-ad)g^2(a+bx)}+\frac{2B^2(c+dx)\log\left(\frac{e(c+dx)}{a+bx}\right)}{(bc-ad)g^2(a+bx)}-\frac{(c+dx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(bc-ad)g^2(a+bx)}$$

```
[Out] 2*A*B*(d*x+c)/(-a*d+b*c)/g^2/(b*x+a)-2*B^2*(d*x+c)/(-a*d+b*c)/g^2/(b*x+a)+2
*B^2*(d*x+c)*ln(e*(d*x+c)/(b*x+a))/(-a*d+b*c)/g^2/(b*x+a)-(d*x+c)*(A+B*ln(e
*(d*x+c)/(b*x+a)))^2/(-a*d+b*c)/g^2/(b*x+a)
```

Rubi [A]
time = 0.05, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {2552, 2333, 2332}

$$-\frac{(c+dx)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}{g^2(a+bx)(bc-ad)}+\frac{2AB(c+dx)}{g^2(a+bx)(bc-ad)}+\frac{2B^2(c+dx)\log\left(\frac{e(c+dx)}{a+bx}\right)}{g^2(a+bx)(bc-ad)}-\frac{2B^2(c+dx)}{g^2(a+bx)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(c + d*x))/(a + b*x)])^2/(a*g + b*g*x)^2,x]
```

```
[Out] (2*A*B*(c + d*x))/((b*c - a*d)*g^2*(a + b*x)) - (2*B^2*(c + d*x))/((b*c - a
*d)*g^2*(a + b*x)) + (2*B^2*(c + d*x)*Log[(e*(c + d*x))/(a + b*x)])/((b*c -
 a*d)*g^2*(a + b*x)) - ((c + d*x)*(A + B*Log[(e*(c + d*x))/(a + b*x)])^2)/(
(b*c - a*d)*g^2*(a + b*x))
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2552

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(
m + 1)*(g/d)^m, Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a
 + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[n
```

```
 + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[d*f
- c*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Rubi steps

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(ag+bgx)^2}\,dx = -\frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{bg^2(a+bx)} + \frac{(2B)\int\frac{(bc-ad)\left(-A-B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{g(a+bx)^2(c+dx)}\,dx}{bg}$$

$$= -\frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{bg^2(a+bx)} + \frac{(2B(bc-ad))\int\frac{-A-B\log\left(\frac{e(c+dx)}{a+bx}\right)}{(a+bx)^2(c+dx)}\,dx}{bg^2}$$

$$= -\frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{bg^2(a+bx)} + \frac{(2B(bc-ad))\int\left(\frac{b\left(-A-B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{(bc-ad)(a+bx)^2} - \frac{bd(-A}{(b}\right.}{bg^2}$$

$$= -\frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{bg^2(a+bx)} + \frac{(2B)\int\frac{-A-B\log\left(\frac{e(c+dx)}{a+bx}\right)}{(a+bx)^2}\,dx}{g^2} - \frac{(2Bd)\int\frac{-A-B\log}{a}}{(bc-a}$$

$$= \frac{2B\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{bg^2(a+bx)} + \frac{2Bd\log(a+bx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{b(bc-ad)g^2} - 2B$$

$$= \frac{2B\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{bg^2(a+bx)} + \frac{2Bd\log(a+bx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{b(bc-ad)g^2} - 2B$$

$$= \frac{2B\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{bg^2(a+bx)} + \frac{2Bd\log(a+bx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{b(bc-ad)g^2} - 2B$$

$$= -\frac{2B^2}{bg^2(a+bx)} - \frac{2B^2d\log(a+bx)}{b(bc-ad)g^2} + \frac{2B^2d\log(c+dx)}{b(bc-ad)g^2} + \frac{2B\left(A+B\log\right.}{bg^2(a+b}$$

$$= -\frac{2B^2}{bg^2(a+bx)} - \frac{2B^2d\log(a+bx)}{b(bc-ad)g^2} + \frac{2B^2d\log(c+dx)}{b(bc-ad)g^2} - \frac{2B^2d\log\left(-\frac{d(a+}{bc-}\right.}{b(bc-}$$

$$= -\frac{2B^2}{bg^2(a+bx)} - \frac{2B^2d\log(a+bx)}{b(bc-ad)g^2} + \frac{B^2d\log^2(a+bx)}{b(bc-ad)g^2} + \frac{2B^2d\log(c+dx}{b(bc-ad)g^2}$$

$$= -\frac{2B^2}{bg^2(a+bx)} - \frac{2B^2d\log(a+bx)}{b(bc-ad)g^2} + \frac{B^2d\log^2(a+bx)}{b(bc-ad)g^2} + \frac{2B^2d\log(c+dx}{b(bc-ad)g^2}$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

time = 0.32, size = 314, normalized size = 2.05

$$-\frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2+\frac{B\left(2B(bc-ad+d(a+bx)\log(a+bx)-d(a+bx)\log(c+dx))-2(bc-ad)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)-2d(a+bx)\log(a+bx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)+2d(a+bx)\log(c+dx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)-Bd(a+bx)\left(\log(a+bx)\left(\log(a+bx)-2\log\left(\frac{b(c+dx)}{bc-ad}\right)\right)-2\text{Li}_2\left(\frac{d(a+bx)}{-bc+ad}\right)\right)+Bd(a+bx)\left(\left(2\log\left(\frac{d(a+bx)}{-bc+ad}\right)-\log(c+dx)\right)\log(c+dx)+2\text{Li}_2\left(\frac{b(c+dx)}{bc-ad}\right)\right)\right)}{bc-ad}}{bg^2(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(c + d*x))/(a + b*x)])^2/(a*g + b*g*x)^2,x]
```

```
[Out] -(((A + B*Log[(e*(c + d*x))/(a + b*x)])^2 + (B*(2*B*(b*c - a*d + d*(a + b*x
)*Log[a + b*x] - d*(a + b*x)*Log[c + d*x]) - 2*(b*c - a*d)*(A + B*Log[(e*(c
 + d*x))/(a + b*x)]) - 2*d*(a + b*x)*Log[a + b*x]*(A + B*Log[(e*(c + d*x))/
(a + b*x)]) + 2*d*(a + b*x)*Log[c + d*x]*(A + B*Log[(e*(c + d*x))/(a + b*x)
]) - B*d*(a + b*x)*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c -
 a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) + B*d*(a + b*x)*((2*L
og[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2
, (b*(c + d*x))/(b*c - a*d)])))/(b*c - a*d))/(b*g^2*(a + b*x)))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 345 vs. 2(153) = 306.
time = 0.39, size = 346, normalized size = 2.26

method	result
norman	$\frac{\frac{\left(A^2-2AB+2B^2\right)x}{ga}+\frac{B^2 c\ln\left(\frac{e(dx+c)}{bx+a}\right)^2}{g(ad-cb)}+\frac{B^2 dx\ln\left(\frac{e(dx+c)}{bx+a}\right)^2}{g(ad-cb)}+\frac{2(A-B)cB\ln\left(\frac{e(dx+c)}{bx+a}\right)}{g(ad-cb)}+\frac{2d(A-B)Bx\ln\left(\frac{e(dx+c)}{bx+a}\right)}{g(ad-cb)}}{g(bx+a)}$
derivativedivides	$\frac{e(ad-cb)\left(\frac{b^2A^2\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)}{(ad-cb)^2e^2g^2}+\frac{2b^2AB\left(\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)\ln\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)+\frac{e(ad-cb)}{b(bx+a)}-\frac{de}{b}\right)}{(ad-cb)^2e^2g^2}+ b^2B^2\left(\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)\ln\left(\right.\right.}{b^2}$
default	$\frac{e(ad-cb)\left(\frac{b^2A^2\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)}{(ad-cb)^2e^2g^2}+\frac{2b^2AB\left(\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)\ln\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)+\frac{e(ad-cb)}{b(bx+a)}-\frac{de}{b}\right)}{(ad-cb)^2e^2g^2}+ b^2B^2\left(\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)\ln\left(\right.\right.}{b^2}$
risch	$-\frac{A^2}{g^2(bx+a)b}+\frac{B^2\ln\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)^2 d}{g^2(ad-cb)b}-\frac{B^2\ln\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)^2 ad}{g^2(ad-cb)b(bx+a)}+\frac{B^2\ln\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)^2 c}{g^2(ad-cb)(bx+a)}-\frac{2B^2\ln\left(\frac{de}{b}-\frac{e(a}{b(b}\right.}{g^2(ad-cb}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*(d*x+c)/(b*x+a)))^2/(b*g*x+a*g)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^2*e*(a*d-b*c)*(b^2/(a*d-b*c)^2/e^2/g^2*A^2*(d*e/b-e*(a*d-b*c)/b/(b*x+a)
)+2*b^2/(a*d-b*c)^2/e^2/g^2*A*B*((d*e/b-e*(a*d-b*c)/b/(b*x+a))*ln(d*e/b-e*(
a*d-b*c)/b/(b*x+a))+e*(a*d-b*c)/b/(b*x+a)-d*e/b)+b^2/(a*d-b*c)^2/e^2/g^2*B^
2*((d*e/b-e*(a*d-b*c)/b/(b*x+a))*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))^2-2*(d*e/b
-e*(a*d-b*c)/b/(b*x+a))*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))-2*e*(a*d-b*c)/b/(b*
x+a)+2*d*e/b))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 422 vs. 2(155) = 310.

time = 0.29, size = 422, normalized size = 2.76

$$\left(2\left(\frac{1}{b^2g^2x+abg^2}+\frac{d\log\left(bx+a\right)}{(b^2c-abd)g^2}-\frac{d\log\left(dx+c\right)}{(b^2c-abd)g^2}\right)\log\left(\frac{dxe}{bx+a}+\frac{ce}{bx+a}\right)+\frac{(bdx+ad)\log\left(bx+a\right)^2+(bdx+ad)\log\left(dx+c\right)^2-2\,bc+2\,ad-2\,(bdx+ad)\log\left(bx+a\right)+2\,(bdx+ad-(bdx+ad)\log\left(bx+a\right))\log\left(dx+c\right)}{ab^2cg^2-a^2bdg^2+(b^3cg^2-ab^2dg^2)x}\right)B^2-2\,AB\left(\frac{\log\left(\frac{dxe}{bx+a}+\frac{ce}{bx+a}\right)}{b^2g^2x+abg^2}-\frac{1}{b^2g^2x+abg^2}-\frac{d\log\left(bx+a\right)}{(b^2c-abd)g^2}+\frac{d\log\left(dx+c\right)}{(b^2c-abd)g^2}\right)-\frac{B^2\log\left(\frac{dxe}{bx+a}+\frac{ce}{bx+a}\right)^2}{b^2g^2x+abg^2}-\frac{A^2}{b^2g^2x+abg^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(d*x+c)/(b*x+a)))^2/(b*g*x+a*g)^2,x, algorithm="maxima
")
```

```
[Out] (2*(1/(b^2*g^2*x + a*b*g^2) + d*log(b*x + a)/((b^2*c - a*b*d)*g^2) - d*log(
d*x + c)/((b^2*c - a*b*d)*g^2))*log(d*x*e/(b*x + a) + c*e/(b*x + a)) + ((b*
d*x + a*d)*log(b*x + a)^2 + (b*d*x + a*d)*log(d*x + c)^2 - 2*b*c + 2*a*d -
2*(b*d*x + a*d)*log(b*x + a) + 2*(b*d*x + a*d - (b*d*x + a*d)*log(b*x + a))
*log(d*x + c))/(a*b^2*c*g^2 - a^2*b*d*g^2 + (b^3*c*g^2 - a*b^2*d*g^2)*x))*B
^2 - 2*A*B*(log(d*x*e/(b*x + a) + c*e/(b*x + a))/(b^2*g^2*x + a*b*g^2) - 1/
(b^2*g^2*x + a*b*g^2) - d*log(b*x + a)/((b^2*c - a*b*d)*g^2) + d*log(d*x +
c)/((b^2*c - a*b*d)*g^2)) - B^2*log(d*x*e/(b*x + a) + c*e/(b*x + a))^2/(b^2
*g^2*x + a*b*g^2) - A^2/(b^2*g^2*x + a*b*g^2)
```

Fricas [**A**]
time = 0.34, size = 152, normalized size = 0.99

$$-\frac{(A^2-2\,AB+2\,B^2)bc-(A^2-2\,AB+2\,B^2)ad+(B^2bdx+B^2bc)\log\left(\frac{(dx+c)e}{bx+a}\right)^2+2\,((AB-B^2)bdx+(AB-B^2)bc)\log\left(\frac{(dx+c)e}{bx+a}\right)}{(b^3c-ab^2d)g^2x+(ab^2c-a^2bd)g^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(d*x+c)/(b*x+a)))^2/(b*g*x+a*g)^2,x, algorithm="fricas
")
```

```
[Out] -((A^2 - 2*A*B + 2*B^2)*b*c - (A^2 - 2*A*B + 2*B^2)*a*d + (B^2*b*d*x + B^2*
b*c)*log((d*x + c)*e/(b*x + a))^2 + 2*((A*B - B^2)*b*d*x + (A*B - B^2)*b*c)
*log((d*x + c)*e/(b*x + a)))/((b^3*c - a*b^2*d)*g^2*x + (a*b^2*c - a^2*b*d)
*g^2)
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 430 vs. $2(128) = 256$.
time = 1.34, size = 430, normalized size = 2.81

$$\frac{2Bd(A-B)\log\left(x+\frac{2ABad^2+2ABbcd-2B^2ad^2-2B^2bcd-\frac{2Ba^2d^3(A-B)}{ad-bc}+\frac{4Babcd^2(A-B)}{ad-bc}-\frac{2Bb^2c^2d(A-B)}{ad-bc}}{4ABbd^2-4B^2bd^2}\right)}{bg^2\left(ad-bc\right)}-\frac{2Bd(A-B)\log\left(x+\frac{2ABad^2+2ABbcd-2B^2ad^2-2B^2bcd+\frac{2Ba^2d^3(A-B)}{ad-bc}-\frac{4Babcd^2(A-B)}{ad-bc}+\frac{2Bb^2c^2d(A-B)}{ad-bc}}{4ABbd^2-4B^2bd^2}\right)}{bg^2\left(ad-bc\right)}+\frac{(-2AB+2B^2)\log\left(\frac{e(c+dx)}{a+bx}\right)}{abg^2+b^2g^2x}+\frac{(B^2c+B^2dx)\log\left(\frac{e(c+dx)}{a+bx}\right)^2}{a^2dg^2-abcg^2+abdg^2x-b^2cg^2x}+\frac{-A^2+2AB-2B^2}{abg^2+b^2g^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(d*x+c)/(b*x+a)))**2/(b*g*x+a*g)**2,x)
```

```
[Out] 2*B*d*(A - B)*log(x + (2*A*B*a*d**2 + 2*A*B*b*c*d - 2*B**2*a*d**2 - 2*B**2*
b*c*d - 2*B*a**2*d**3*(A - B)/(a*d - b*c) + 4*B*a*b*c*d**2*(A - B)/(a*d - b
*c) - 2*B*b**2*c**2*d*(A - B)/(a*d - b*c))/(4*A*B*b*d**2 - 4*B**2*b*d**2))/
(b*g**2*(a*d - b*c)) - 2*B*d*(A - B)*log(x + (2*A*B*a*d**2 + 2*A*B*b*c*d -
```

```
2*B**2*a*d**2 - 2*B**2*b*c*d + 2*B*a**2*d**3*(A - B)/(a*d - b*c) - 4*B*a*b*
c*d**2*(A - B)/(a*d - b*c) + 2*B*b**2*c**2*d*(A - B)/(a*d - b*c))/(4*A*B*b*
d**2 - 4*B**2*b*d**2))/(b*g**2*(a*d - b*c)) + (-2*A*B + 2*B**2)*log(e*(c +
d*x)/(a + b*x))/(a*b*g**2 + b**2*g**2*x) + (B**2*c + B**2*d*x)*log(e*(c + d
*x)/(a + b*x))**2/(a**2*d*g**2 - a*b*c*g**2 + a*b*d*g**2*x - b**2*c*g**2*x)
 + (-A**2 + 2*A*B - 2*B**2)/(a*b*g**2 + b**2*g**2*x)
```

Giac [A]
time = 4.80, size = 188, normalized size = 1.23

$$-\left(\frac{(dxe+ce)B^2\log\left(\frac{dxe+ce}{bx+a}\right)^2}{(bx+a)g^2}+\frac{2\,(dxe+ce)(AB-B^2)\log\left(\frac{dxe+ce}{bx+a}\right)}{(bx+a)g^2}+\frac{(dxe+ce)(A^2-2\,AB+2\,B^2)}{(bx+a)g^2}\right)\left(\frac{bc}{(bce-ade)(bc-ad)}-\frac{ad}{(bce-ade)(bc-ad)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(d*x+c)/(b*x+a)))^2/(b*g*x+a*g)^2,x, algorithm="giac")
```

```
[Out] -((d*x*e + c*e)*B^2*log((d*x*e + c*e)/(b*x + a))^2/((b*x + a)*g^2) + 2*(d*x
*e + c*e)*(A*B - B^2)*log((d*x*e + c*e)/(b*x + a))/((b*x + a)*g^2) + (d*x*e
 + c*e)*(A^2 - 2*A*B + 2*B^2)/((b*x + a)*g^2))*(b*c/((b*c*e - a*d*e)*(b*c -
 a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))
```

Mupad [B]
time = 6.38, size = 223, normalized size = 1.46

$$\frac{\ln\left(\frac{e\,(c+d\,x)}{a+b\,x}\right)\,\left(\frac{2\,B^2}{b^2\,d\,g^2}-\frac{2\,A\,B}{b^2\,d\,g^2}\right)}{\frac{x}{d}+\frac{a}{b\,d}}-\ln\left(\frac{e\,(c+d\,x)}{a+b\,x}\right)^2\,\left(\frac{B^2}{b^2\,g^2\,\left(x+\frac{a}{b}\right)}-\frac{B^2\,d}{b\,g^2\,(a\,d-b\,c)}\right)-\frac{A^2-2\,A\,B+2\,B^2}{x\,b^2\,g^2+a\,b\,g^2}+\frac{B\,d\,\mathrm{atan}\left(\frac{\left(2\,b\,d\,x+\frac{c\,b^2\,g^2+a\,d\,b\,g^2}{b\,g^2}\right)\,1\mathrm{i}}{a\,d-b\,c}\right)\,(A-B)\,4\mathrm{i}}{b\,g^2\,(a\,d-b\,c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(c + d*x))/(a + b*x)))^2/(a*g + b*g*x)^2,x)
```

```
[Out] (log((e*(c + d*x))/(a + b*x))*((2*B^2)/(b^2*d*g^2) - (2*A*B)/(b^2*d*g^2)))/
(x/d + a/(b*d)) - log((e*(c + d*x))/(a + b*x))^2*(B^2/(b^2*g^2*(x + a/b)) -
 (B^2*d)/(b*g^2*(a*d - b*c))) - (A^2 + 2*B^2 - 2*A*B)/(b^2*g^2*x + a*b*g^2)
 + (B*d*atan(((2*b*d*x + (b^2*c*g^2 + a*b*d*g^2)/(b*g^2))*1i)/(a*d - b*c))*
(A - B)*4i)/(b*g^2*(a*d - b*c))
```

3.188 $\int \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(ag+bgx)^3}\,dx$

Optimal. Leaf size=296

$$-\frac{2ABd(c+dx)}{(bc-ad)^2g^3(a+bx)}+\frac{2B^2d(c+dx)}{(bc-ad)^2g^3(a+bx)}-\frac{bB^2(c+dx)^2}{4(bc-ad)^2g^3(a+bx)^2}-\frac{2B^2d(c+dx)\log\left(\frac{e(c+dx)}{a+bx}\right)}{(bc-ad)^2g^3(a+bx)}+\frac{bB(c}{}$$

```
[Out] -2*A*B*d*(d*x+c)/(-a*d+b*c)^2/g^3/(b*x+a)+2*B^2*d*(d*x+c)/(-a*d+b*c)^2/g^3/
(b*x+a)-1/4*b*B^2*(d*x+c)^2/(-a*d+b*c)^2/g^3/(b*x+a)^2-2*B^2*d*(d*x+c)*ln(e
*(d*x+c)/(b*x+a))/(-a*d+b*c)^2/g^3/(b*x+a)+1/2*b*B*(d*x+c)^2*(A+B*ln(e*(d*x
+c)/(b*x+a)))/(-a*d+b*c)^2/g^3/(b*x+a)^2+d*(d*x+c)*(A+B*ln(e*(d*x+c)/(b*x+a
)))^2/(-a*d+b*c)^2/g^3/(b*x+a)-1/2*b*(d*x+c)^2*(A+B*ln(e*(d*x+c)/(b*x+a)))^
2/(-a*d+b*c)^2/g^3/(b*x+a)^2
```

Rubi [A]
time = 0.11, antiderivative size = 296, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {2552, 2367, 2333, 2332, 2342, 2341}

$$\frac{bB(c+dx)^2\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{2g^3(a+bx)^2(bc-ad)^2}-\frac{b(c+dx)^2\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}{2g^3(a+bx)^2(bc-ad)^2}+\frac{d(c+dx)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}{g^3(a+bx)(bc-ad)^2}-\frac{2ABd(c+dx)}{g^3(a+bx)(bc-ad)^2}-\frac{2B^2d(c+dx)\log\left(\frac{e(c+dx)}{a+bx}\right)}{g^3(a+bx)(bc-ad)^2}-\frac{bB^2(c+dx)^2}{4g^3(a+bx)^2(bc-ad)^2}+\frac{2B^2d(c+dx)}{g^3(a+bx)(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(c + d*x))/(a + b*x)])^2/(a*g + b*g*x)^3,x]
```

```
[Out] (-2*A*B*d*(c + d*x))/((b*c - a*d)^2*g^3*(a + b*x)) + (2*B^2*d*(c + d*x))/((
b*c - a*d)^2*g^3*(a + b*x)) - (b*B^2*(c + d*x)^2)/(4*(b*c - a*d)^2*g^3*(a +
 b*x)^2) - (2*B^2*d*(c + d*x)*Log[(e*(c + d*x))/(a + b*x)])/((b*c - a*d)^2*
g^3*(a + b*x)) + (b*B*(c + d*x)^2*(A + B*Log[(e*(c + d*x))/(a + b*x)]))/(2*
(b*c - a*d)^2*g^3*(a + b*x)^2) + (d*(c + d*x)*(A + B*Log[(e*(c + d*x))/(a +
 b*x)])^2)/((b*c - a*d)^2*g^3*(a + b*x)) - (b*(c + d*x)^2*(A + B*Log[(e*(c
+ d*x))/(a + b*x)])^2)/(2*(b*c - a*d)^2*g^3*(a + b*x)^2)
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2367

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^(r_.))^(
q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*x^n])^p, (d + e*x
^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
&& IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0] && IntegerQ[r]))
```

Rule 2552

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(
m + 1)*(g/d)^m, Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a
 + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[n
 + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[d*f
- c*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Rubi steps

$$
\int \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(ag+bgx)^3}\,dx = -\frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{2bg^3(a+bx)^2} + \frac{B\int\frac{(bc-ad)\left(-A-B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{g^2(a+bx)^3(c+dx)}\,dx}{bg}
$$

$$
= -\frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{2bg^3(a+bx)^2} + \frac{(B(bc-ad))\int\frac{-A-B\log\left(\frac{e(c+dx)}{a+bx}\right)}{(a+bx)^3(c+dx)}\,dx}{bg^3}
$$

$$
= -\frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{2bg^3(a+bx)^2} + \frac{(B(bc-ad))\int\left(\frac{b\left(-A-B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{(bc-ad)(a+bx)^3} - \frac{bd\left(-A-\cdots\right.}{(bc-\cdots}\right.}{\cdots}
$$

$$
= -\frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{2bg^3(a+bx)^2} + \frac{B\int\frac{-A-B\log\left(\frac{e(c+dx)}{a+bx}\right)}{(a+bx)^3}\,dx}{g^3} + \frac{(Bd^2)\int\frac{-A-B\log\left(\frac{e\cdots}{\cdots}\right.}{a+bx\cdots}}{(bc-ad)^2g\cdots}
$$

$$
= \frac{B\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{2bg^3(a+bx)^2} - \frac{Bd\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{b(bc-ad)g^3(a+bx)} - \frac{Bd^2\log(a+bx)\cdots}{b(bc\cdots}
$$

$$
= \frac{B\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{2bg^3(a+bx)^2} - \frac{Bd\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{b(bc-ad)g^3(a+bx)} - \frac{Bd^2\log(a+bx)\cdots}{b(bc\cdots}
$$

$$
= \frac{B\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{2bg^3(a+bx)^2} - \frac{Bd\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{b(bc-ad)g^3(a+bx)} - \frac{Bd^2\log(a+bx)\cdots}{b(bc\cdots}
$$

$$
= -\frac{B^2}{4bg^3(a+bx)^2} + \frac{3B^2d}{2b(bc-ad)g^3(a+bx)} + \frac{3B^2d^2\log(a+bx)}{2b(bc-ad)^2g^3} - \frac{3B^2d^2\,\mathrm{lo}\cdots}{2b(bc-\cdots}
$$

$$
= -\frac{B^2}{4bg^3(a+bx)^2} + \frac{3B^2d}{2b(bc-ad)g^3(a+bx)} + \frac{3B^2d^2\log(a+bx)}{2b(bc-ad)^2g^3} - \frac{3B^2d^2\,\mathrm{lo}\cdots}{2b(bc-\cdots}
$$

$$
= -\frac{B^2}{4bg^3(a+bx)^2} + \frac{3B^2d}{2b(bc-ad)g^3(a+bx)} + \frac{3B^2d^2\log(a+bx)}{2b(bc-ad)^2g^3} - \frac{B^2d^2\log\cdots}{2b(bc-\cdots}
$$

$$
= -\frac{B^2}{4bg^3(a+bx)^2} + \frac{3B^2d}{2b(bc-ad)g^3(a+bx)} + \frac{3B^2d^2\log(a+bx)}{2b(bc-ad)^2g^3} - \frac{B^2d^2\log\cdots}{2b(bc-\cdots}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 0.30, size = 444, normalized size = 1.50

$$
\frac{-2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2 + \frac{B\left(4Bd(a+bx)(bc-ad+d(a+bx)\log(a+bx)-d(a+bx)\log(c+dx)) - B\left((bc-ad)^2+2d(-bc+ad)(a+bx)-2d^2(a+bx)^2\log(a+bx)+2d^2(a+bx)^2\log(c+dx)\right)+2(bc-ad)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)+4d(-bc+ad)(a+bx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)-4d^2(a+bx)^2\log(a+bx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)+4d^2(a+bx)^2\log(c+dx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)-2Bd^2(a+bx)^2\left(\log(a+bx)\left(\log(a+bx)-2\log\left(\frac{b(c+dx)}{bc-ad}\right)\right)-2\,\mathrm{Li}_2\left(\frac{d(a+bx)}{-bc+ad}\right)\right)+2Bd^2(a+bx)^2\left(\left(2\log\left(\frac{d(a+bx)}{-bc+ad}\right)-\log(c+dx)\right)\log(c+dx)+2\,\mathrm{Li}_2\left(\frac{b(c+dx)}{bc-ad}\right)\right)\right)}{(bc-ad)^2}}{4bg^3(a+bx)^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(c + d*x))/(a + b*x)])^2/(a*g + b*g*x)^3,x]
```

```
[Out] (-2*(A + B*Log[(e*(c + d*x))/(a + b*x)])^2 + (B*(4*B*d*(a + b*x)*(b*c - a*d
 + d*(a + b*x)*Log[a + b*x] - d*(a + b*x)*Log[c + d*x]) - B*((b*c - a*d)^2
+ 2*d*(-(b*c) + a*d)*(a + b*x) - 2*d^2*(a + b*x)^2*Log[a + b*x] + 2*d^2*(a
+ b*x)^2*Log[c + d*x]) + 2*(b*c - a*d)^2*(A + B*Log[(e*(c + d*x))/(a + b*x)
]) + 4*d*(-(b*c) + a*d)*(a + b*x)*(A + B*Log[(e*(c + d*x))/(a + b*x)]) - 4*
d^2*(a + b*x)^2*Log[a + b*x]*(A + B*Log[(e*(c + d*x))/(a + b*x)]) + 4*d^2*(
a + b*x)^2*Log[c + d*x]*(A + B*Log[(e*(c + d*x))/(a + b*x)]) - 2*B*d^2*(a +
 b*x)^2*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2
*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) + 2*B*d^2*(a + b*x)^2*((2*Log[(d
*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*
(c + d*x))/(b*c - a*d)])))/(b*c - a*d)^2)/(4*b*g^3*(a + b*x)^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 700 vs. $2(290) = 580$.
time = 0.39, size = 701, normalized size = 2.37 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*(d*x+c)/(b*x+a)))^2/(b*g*x+a*g)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^2*e*(a*d-b*c)*(-1/2*b^3/(a*d-b*c)^3/e^3/g^3*A^2*(d*e/b-e*(a*d-b*c)/b/(b
*x+a))^2+b^2/(a*d-b*c)^3/e^2/g^3*A^2*d*(d*e/b-e*(a*d-b*c)/b/(b*x+a))-2*b^3/
(a*d-b*c)^3/e^3/g^3*A*B*(1/2*(d*e/b-e*(a*d-b*c)/b/(b*x+a))^2*ln(d*e/b-e*(a*
d-b*c)/b/(b*x+a))-1/4*(d*e/b-e*(a*d-b*c)/b/(b*x+a))^2)+2*b^2/(a*d-b*c)^3/e^
2/g^3*A*B*d*((d*e/b-e*(a*d-b*c)/b/(b*x+a))*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))+
e*(a*d-b*c)/b/(b*x+a)-d*e/b)-b^3/(a*d-b*c)^3/e^3/g^3*B^2*(1/2*(d*e/b-e*(a*d
-b*c)/b/(b*x+a))^2*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))^2-1/2*(d*e/b-e*(a*d-b*c)
/b/(b*x+a))^2*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))+1/4*(d*e/b-e*(a*d-b*c)/b/(b*x
+a))^2)+b^2/(a*d-b*c)^3/e^2/g^3*B^2*d*((d*e/b-e*(a*d-b*c)/b/(b*x+a))*ln(d*e
/b-e*(a*d-b*c)/b/(b*x+a))^2-2*(d*e/b-e*(a*d-b*c)/b/(b*x+a))*ln(d*e/b-e*(a*d
-b*c)/b/(b*x+a))-2*e*(a*d-b*c)/b/(b*x+a)+2*d*e/b))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 853 vs. $2(294) = 588$.
time = 0.33, size = 853, normalized size = 2.88

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(d*x+c)/(b*x+a)))^2/(b*g*x+a*g)^3,x, algorithm="maxima
")
```

```
[Out] -1/4*(2*((2*b*d*x - b*c + 3*a*d)/((b^4*c - a*b^3*d)*g^3*x^2 + 2*(a*b^3*c -
a^2*b^2*d)*g^3*x + (a^2*b^2*c - a^3*b*d)*g^3) + 2*d^2*log(b*x + a)/((b^3*c^
2 - 2*a*b^2*c*d + a^2*b*d^2)*g^3) - 2*d^2*log(d*x + c)/((b^3*c^2 - 2*a*b^2*
c*d + a^2*b*d^2)*g^3))*log(d*x*e/(b*x + a) + c*e/(b*x + a)) + (b^2*c^2 - 8*
```

```
a*b*c*d + 7*a^2*d^2 + 2*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*log(b*x + a)^
2 + 2*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*log(d*x + c)^2 - 6*(b^2*c*d - a
*b*d^2)*x - 6*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*log(b*x + a) + 2*(3*b^2
*d^2*x^2 + 6*a*b*d^2*x + 3*a^2*d^2 - 2*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2
)*log(b*x + a))*log(d*x + c))/(a^2*b^3*c^2*g^3 - 2*a^3*b^2*c*d*g^3 + a^4*b*
d^2*g^3 + (b^5*c^2*g^3 - 2*a*b^4*c*d*g^3 + a^2*b^3*d^2*g^3)*x^2 + 2*(a*b^4*
c^2*g^3 - 2*a^2*b^3*c*d*g^3 + a^3*b^2*d^2*g^3)*x))*B^2 - 1/2*A*B*((2*b*d*x
- b*c + 3*a*d)/((b^4*c - a*b^3*d)*g^3*x^2 + 2*(a*b^3*c - a^2*b^2*d)*g^3*x +
 (a^2*b^2*c - a^3*b*d)*g^3) + 2*log(d*x*e/(b*x + a) + c*e/(b*x + a))/(b^3*g
^3*x^2 + 2*a*b^2*g^3*x + a^2*b*g^3) + 2*d^2*log(b*x + a)/((b^3*c^2 - 2*a*b^
2*c*d + a^2*b*d^2)*g^3) - 2*d^2*log(d*x + c)/((b^3*c^2 - 2*a*b^2*c*d + a^2*
b*d^2)*g^3)) - 1/2*B^2*log(d*x*e/(b*x + a) + c*e/(b*x + a))^2/(b^3*g^3*x^2
+ 2*a*b^2*g^3*x + a^2*b*g^3) - 1/2*A^2/(b^3*g^3*x^2 + 2*a*b^2*g^3*x + a^2*b
*g^3)
```

Fricas [**A**]
time = 0.36, size = 371, normalized size = 1.25

$$-\frac{(2A^2-2AB+B^2)b^2c^2-4(A^2-2AB+2B^2)abcd+(2A^2-6AB+7B^2)a^2d^2-2(B^2b^2d^2x^2+2B^2abd^2x-B^2b^2c^2+2B^2abcd)\log\left(\frac{(dx+c)e}{bx+a}\right)^2+2((2AB-3B^2)b^2cd-(2AB-3B^2)abd^2)x-2((2AB-3B^2)b^2d^2x^2-(2AB-B^2)b^2c^2+4(AB-B^2)abcd-2(B^2b^2cd-2(AB-B^2)abd^2)x)\log\left(\frac{(dx+c)e}{bx+a}\right)}{4((b^5c^2-2ab^4cd+a^2b^3d^2)g^3x^2+2(ab^4c^2-2a^2b^3cd+a^3b^2d^2)g^3x+(a^2b^3c^2-2a^3b^2cd+a^4bd^2)g^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(d*x+c)/(b*x+a)))^2/(b*g*x+a*g)^3,x, algorithm="fricas
")
```

```
[Out] -1/4*((2*A^2 - 2*A*B + B^2)*b^2*c^2 - 4*(A^2 - 2*A*B + 2*B^2)*a*b*c*d + (2*
A^2 - 6*A*B + 7*B^2)*a^2*d^2 - 2*(B^2*b^2*d^2*x^2 + 2*B^2*a*b*d^2*x - B^2*b
^2*c^2 + 2*B^2*a*b*c*d)*log((d*x + c)*e/(b*x + a))^2 + 2*((2*A*B - 3*B^2)*b
^2*c*d - (2*A*B - 3*B^2)*a*b*d^2)*x - 2*((2*A*B - 3*B^2)*b^2*d^2*x^2 - (2*A
*B - B^2)*b^2*c^2 + 4*(A*B - B^2)*a*b*c*d - 2*(B^2*b^2*c*d - 2*(A*B - B^2)*
a*b*d^2)*x)*log((d*x + c)*e/(b*x + a)))/((b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d
^2)*g^3*x^2 + 2*(a*b^4*c^2 - 2*a^2*b^3*c*d + a^3*b^2*d^2)*g^3*x + (a^2*b^3*
c^2 - 2*a^3*b^2*c*d + a^4*b*d^2)*g^3)
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 892 vs. $2(269) = 538$.
time = 2.91, size = 892, normalized size = 3.01

$$\frac{Bd^2\cdot(2A-3B)\log\left(x+\frac{2ABad^3+2ABbcd^2-3B^2ad^3-3B^2bcd^2-\frac{Ba^3d^5\cdot(2A-3B)}{(ad-bc)^2}+\frac{3Ba^2bcd^4\cdot(2A-3B)}{(ad-bc)^2}-\frac{3Bab^2c^2d^3\cdot(2A-3B)}{(ad-bc)^2}+\frac{Bb^3c^3d^2\cdot(2A-3B)}{(ad-bc)^2}}{4ABbd^3-6B^2bd^3}\right)}{2bg^3(ad-bc)^2}-\frac{Bd^2\cdot(2A-3B)\log\left(x+\frac{2ABad^3+2ABbcd^2-3B^2ad^3-3B^2bcd^2+\frac{Ba^3d^5\cdot(2A-3B)}{(ad-bc)^2}-\frac{3Ba^2bcd^4\cdot(2A-3B)}{(ad-bc)^2}+\frac{3Bab^2c^2d^3\cdot(2A-3B)}{(ad-bc)^2}-\frac{Bb^3c^3d^2\cdot(2A-3B)}{(ad-bc)^2}}{4ABbd^3-6B^2bd^3}\right)}{2bg^3(ad-bc)^2}+\frac{(2B^2acd+2B^2ad^2x-B^2bc^2+B^2bd^2x^2)\log\left(\frac{e(c+dx)}{a+bx}\right)^2}{2a^4d^2g^3-4a^3bcdg^3+4a^3bd^2g^3x+2a^2b^2c^2g^3-8a^2b^2cdg^3x+2a^2b^2d^2g^3x^2+4ab^3c^2g^3x-4ab^3cdg^3x^2+2b^4c^2g^3x^2}+\frac{(-2ABad+2ABbc+3B^2ad-B^2bc+2B^2bdx)\log\left(\frac{e(c+dx)}{a+bx}\right)}{2a^3bdg^3-2a^2b^2cg^3+4a^2b^2dg^3x-4ab^3cg^3x+2ab^3dg^3x^2-2b^4cg^3x^2}+\frac{-2A^2ad+2A^2bc+6ABad-2ABbc-7B^2ad+B^2bc+x(4ABbd-6B^2bd)}{4a^3bdg^3-4a^2b^2cg^3+x^2\cdot(4ab^3dg^3-4b^4cg^3)+x(8a^2b^2dg^3-8ab^3cg^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(d*x+c)/(b*x+a)))**2/(b*g*x+a*g)**3,x)
```

```
[Out] B*d**2*(2*A - 3*B)*log(x + (2*A*B*a*d**3 + 2*A*B*b*c*d**2 - 3*B**2*a*d**3 -
 3*B**2*b*c*d**2 - B*a**3*d**5*(2*A - 3*B)/(a*d - b*c)**2 + 3*B*a**2*b*c*d*
*4*(2*A - 3*B)/(a*d - b*c)**2 - 3*B*a*b**2*c**2*d**3*(2*A - 3*B)/(a*d - b*c
)**2 + B*b**3*c**3*d**2*(2*A - 3*B)/(a*d - b*c)**2)/(4*A*B*b*d**3 - 6*B**2*
```

```
b*d**3))/(2*b*g**3*(a*d - b*c)**2) - B*d**2*(2*A - 3*B)*log(x + (2*A*B*a*d*
*3 + 2*A*B*b*c*d**2 - 3*B**2*a*d**3 - 3*B**2*b*c*d**2 + B*a**3*d**5*(2*A -
3*B)/(a*d - b*c)**2 - 3*B*a**2*b*c*d**4*(2*A - 3*B)/(a*d - b*c)**2 + 3*B*a*
b**2*c**2*d**3*(2*A - 3*B)/(a*d - b*c)**2 - B*b**3*c**3*d**2*(2*A - 3*B)/(a
*d - b*c)**2)/(4*A*B*b*d**3 - 6*B**2*b*d**3))/(2*b*g**3*(a*d - b*c)**2) + (
2*B**2*a*c*d + 2*B**2*a*d**2*x - B**2*b*c**2 + B**2*b*d**2*x**2)*log(e*(c +
 d*x)/(a + b*x))**2/(2*a**4*d**2*g**3 - 4*a**3*b*c*d*g**3 + 4*a**3*b*d**2*g
**3*x + 2*a**2*b**2*c**2*g**3 - 8*a**2*b**2*c*d*g**3*x + 2*a**2*b**2*d**2*g
**3*x**2 + 4*a*b**3*c**2*g**3*x - 4*a*b**3*c*d*g**3*x**2 + 2*b**4*c**2*g**3
*x**2) + (-2*A*B*a*d + 2*A*B*b*c + 3*B**2*a*d - B**2*b*c + 2*B**2*b*d*x)*lo
g(e*(c + d*x)/(a + b*x))/(2*a**3*b*d*g**3 - 2*a**2*b**2*c*g**3 + 4*a**2*b**
2*d*g**3*x - 4*a*b**3*c*g**3*x + 2*a*b**3*d*g**3*x**2 - 2*b**4*c*g**3*x**2)
 + (-2*A**2*a*d + 2*A**2*b*c + 6*A*B*a*d - 2*A*B*b*c - 7*B**2*a*d + B**2*b*
c + x*(4*A*B*b*d - 6*B**2*b*d))/(4*a**3*b*d*g**3 - 4*a**2*b**2*c*g**3 + x**
2*(4*a*b**3*d*g**3 - 4*b**4*c*g**3) + x*(8*a**2*b**2*d*g**3 - 8*a*b**3*c*g*
*3))
```

Giac [A]
time = 4.61, size = 493, normalized size = 1.67

$$\frac{\left(\frac{4\,(dxe+ce)B^2de\log\left(\frac{dxe+ce}{bx+a}\right)^2}{bx+a}+\frac{8\,(dxe+ce)ABde\log\left(\frac{dxe+ce}{bx+a}\right)}{bx+a}-\frac{8\,(dxe+ce)B^2de\log\left(\frac{dxe+ce}{bx+a}\right)}{bx+a}-\frac{2\,(dxe+ce)^2B^2b\log\left(\frac{dxe+ce}{bx+a}\right)^2}{(bx+a)^2}+\frac{4\,(dxe+ce)A^2de}{bx+a}-\frac{8\,(dxe+ce)ABde}{bx+a}+\frac{8\,(dxe+ce)B^2de}{bx+a}-\frac{4\,(dxe+ce)^2ABb\log\left(\frac{dxe+ce}{bx+a}\right)}{(bx+a)^2}+\frac{2\,(dxe+ce)^2B^2b\log\left(\frac{dxe+ce}{bx+a}\right)}{(bx+a)^2}-\frac{2\,(dxe+ce)^2A^2b}{(bx+a)^2}+\frac{2\,(dxe+ce)^2ABb}{(bx+a)^2}-\frac{(dxe+ce)^2B^2b}{(bx+a)^2}\right)\left(\frac{bc}{(bce-ade)(bc-ad)}-\frac{ad}{(bce-ade)(bc-ad)}\right)}{4\,(bcg^3e-adg^3e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(d*x+c)/(b*x+a)))^2/(b*g*x+a*g)^3,x, algorithm="giac")
```

```
[Out] 1/4*(4*(d*x*e + c*e)*B^2*d*e*log((d*x*e + c*e)/(b*x + a))^2/(b*x + a) + 8*(
d*x*e + c*e)*A*B*d*e*log((d*x*e + c*e)/(b*x + a))/(b*x + a) - 8*(d*x*e + c*
e)*B^2*d*e*log((d*x*e + c*e)/(b*x + a))/(b*x + a) - 2*(d*x*e + c*e)^2*B^2*b
*log((d*x*e + c*e)/(b*x + a))^2/(b*x + a)^2 + 4*(d*x*e + c*e)*A^2*d*e/(b*x
+ a) - 8*(d*x*e + c*e)*A*B*d*e/(b*x + a) + 8*(d*x*e + c*e)*B^2*d*e/(b*x + a
) - 4*(d*x*e + c*e)^2*A*B*b*log((d*x*e + c*e)/(b*x + a))/(b*x + a)^2 + 2*(d
*x*e + c*e)^2*B^2*b*log((d*x*e + c*e)/(b*x + a))/(b*x + a)^2 - 2*(d*x*e + c
*e)^2*A^2*b/(b*x + a)^2 + 2*(d*x*e + c*e)^2*A*B*b/(b*x + a)^2 - (d*x*e + c*
e)^2*B^2*b/(b*x + a)^2)*(b*c/((b*c*e - a*d*e)*(b*c - a*d)) - a*d/((b*c*e -
a*d*e)*(b*c - a*d)))/(b*c*g^3*e - a*d*g^3*e)
```

Mupad [B]
time = 6.00, size = 507, normalized size = 1.71

$$\frac{\ln\left(\frac{e\,(c+d\,x)}{a+b\,x}\right)\,\left(\frac{B^2\,x\,(a\,d-b\,c)}{b\,g^3\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}-\frac{A\,B}{b^2\,d\,g^3}+\frac{B^2\,d^2\,\left(\frac{2\,a^2\,d^2-3\,a\,b\,c\,d+b^2\,c^2}{2\,b\,d^3}+\frac{a\,(a\,d-b\,c)}{2\,b\,d^2}\right)}{b\,g^3\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}\right)}{\frac{b\,x^2}{d}+\frac{a^2}{b\,d}+\frac{2\,a\,x}{d}}-\ln\left(\frac{e\,(c+d\,x)}{a+b\,x}\right)^2\,\left(\frac{B^2}{2\,b^2\,g^3\,\left(2\,a\,x+b\,x^2+\frac{a^2}{b}\right)}-\frac{B^2\,d^2}{2\,b\,g^3\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}\right)-\frac{\frac{2\,A^2\,a\,d-2\,A^2\,b\,c+7\,B^2\,a\,d-B^2\,b\,c-6\,A\,B\,a\,d+2\,A\,B\,b\,c}{2\,(a\,d-b\,c)}+\frac{x\,(3\,B^2\,b\,d-2\,A\,B\,b\,d)}{a\,d-b\,c}}{2\,a^2\,b\,g^3+4\,a\,b^2\,g^3\,x+2\,b^3\,g^3\,x^2}-\frac{B\,d^2\,\mathrm{atan}\left(\frac{B\,d^2\,\left(2\,b\,d\,x-\frac{b^3\,c^2\,g^3-a^2\,b\,d^2\,g^3}{b\,g^3\,(a\,d-b\,c)}\right)\,(2\,A-3\,B)\,1{}\mathrm{i}}{(a\,d-b\,c)\,(3\,B^2\,d^2-2\,A\,B\,d^2)}\right)\,(2\,A-3\,B)\,1{}\mathrm{i}}{b\,g^3\,{(a\,d-b\,c)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(c + d*x))/(a + b*x)))^2/(a*g + b*g*x)^3,x)
```

```
[Out] (log((e*(c + d*x))/(a + b*x))*((B^2*x*(a*d - b*c))/(b*g^3*(a^2*d^2 + b^2*c^
2 - 2*a*b*c*d)) - (A*B)/(b^2*d*g^3) + (B^2*d^2*((2*a^2*d^2 + b^2*c^2 - 3*a*
```

```
b*c*d)/(2*b*d^3) + (a*(a*d - b*c))/(2*b*d^2)))/(b*g^3*(a^2*d^2 + b^2*c^2 -
2*a*b*c*d))))/((b*x^2)/d + a^2/(b*d) + (2*a*x)/d) - log((e*(c + d*x))/(a +
b*x))^2*(B^2/(2*b^2*g^3*(2*a*x + b*x^2 + a^2/b)) - (B^2*d^2)/(2*b*g^3*(a^2*
d^2 + b^2*c^2 - 2*a*b*c*d))) - ((2*A^2*a*d - 2*A^2*b*c + 7*B^2*a*d - B^2*b*
c - 6*A*B*a*d + 2*A*B*b*c)/(2*(a*d - b*c)) + (x*(3*B^2*b*d - 2*A*B*b*d))/(a
*d - b*c))/(2*a^2*b*g^3 + 2*b^3*g^3*x^2 + 4*a*b^2*g^3*x) - (B*d^2*atan((B*d
^2*(2*b*d*x - (b^3*c^2*g^3 - a^2*b*d^2*g^3)/(b*g^3*(a*d - b*c)))*(2*A - 3*B
)*1i)/((a*d - b*c)*(3*B^2*d^2 - 2*A*B*d^2)))*(2*A - 3*B)*1i)/(b*g^3*(a*d -
b*c)^2)
```

3.189 $\int \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(ag+bgx)^4}\,dx$

Optimal. Leaf size=399

$$-\frac{2B^2d^2(c+dx)}{(bc-ad)^3g^4(a+bx)}+\frac{bB^2d(c+dx)^2}{2(bc-ad)^3g^4(a+bx)^2}-\frac{2b^2B^2(c+dx)^3}{27(bc-ad)^3g^4(a+bx)^3}+\frac{B^2d^3\log^2\left(\frac{c+dx}{a+bx}\right)}{3b(bc-ad)^3g^4}+\frac{2Bd^2(c+dx)}{(bc-}$$

```
[Out] -2*B^2*d^2*(d*x+c)/(-a*d+b*c)^3/g^4/(b*x+a)+1/2*b*B^2*d*(d*x+c)^2/(-a*d+b*c
)^3/g^4/(b*x+a)^2-2/27*b^2*B^2*(d*x+c)^3/(-a*d+b*c)^3/g^4/(b*x+a)^3+1/3*B^2
*d^3*ln((d*x+c)/(b*x+a))^2/b/(-a*d+b*c)^3/g^4+2*B*d^2*(d*x+c)*(A+B*ln(e*(d*
x+c)/(b*x+a)))/(-a*d+b*c)^3/g^4/(b*x+a)-b*B*d*(d*x+c)^2*(A+B*ln(e*(d*x+c)/(
b*x+a)))/(-a*d+b*c)^3/g^4/(b*x+a)^2+2/9*b^2*B*(d*x+c)^3*(A+B*ln(e*(d*x+c)/(
b*x+a)))/(-a*d+b*c)^3/g^4/(b*x+a)^3-2/3*B*d^3*ln((d*x+c)/(b*x+a))*(A+B*ln(e
*(d*x+c)/(b*x+a)))/b/(-a*d+b*c)^3/g^4-1/3*(A+B*ln(e*(d*x+c)/(b*x+a)))^2/b/g
^4/(b*x+a)^3
```

Rubi [A]
time = 0.17, antiderivative size = 399, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {2552, 2356, 45, 2372, 2338}

$$\frac{2b^2B(c+dx)^3\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{9g^4(a+bx)^3(bc-ad)^3}-\frac{2Bd^3\log\left(\frac{c+dx}{a+bx}\right)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{3bg^4(bc-ad)^3}+\frac{2Bd^2(c+dx)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{g^4(a+bx)(bc-ad)^3}-\frac{bBd(c+dx)^2\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{g^4(a+bx)^2(bc-ad)^3}-\frac{\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}{3bg^4(a+bx)^3}-\frac{2b^2B^2(c+dx)^3}{27g^4(a+bx)^3(bc-ad)^3}+\frac{B^2d^3\log^2\left(\frac{c+dx}{a+bx}\right)}{3bg^4(bc-ad)^3}-\frac{2B^2d^2(c+dx)}{g^4(a+bx)(bc-ad)^3}+\frac{bB^2d(c+dx)^2}{2g^4(a+bx)^2(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(c + d*x))/(a + b*x)])^2/(a*g + b*g*x)^4,x]
```

```
[Out] (-2*B^2*d^2*(c + d*x))/((b*c - a*d)^3*g^4*(a + b*x)) + (b*B^2*d*(c + d*x)^2
)/(2*(b*c - a*d)^3*g^4*(a + b*x)^2) - (2*b^2*B^2*(c + d*x)^3)/(27*(b*c - a*
d)^3*g^4*(a + b*x)^3) + (B^2*d^3*Log[(c + d*x)/(a + b*x)]^2)/(3*b*(b*c - a*
d)^3*g^4) + (2*B*d^2*(c + d*x)*(A + B*Log[(e*(c + d*x))/(a + b*x)]))/((b*c
- a*d)^3*g^4*(a + b*x)) - (b*B*d*(c + d*x)^2*(A + B*Log[(e*(c + d*x))/(a +
b*x)]))/((b*c - a*d)^3*g^4*(a + b*x)^2) + (2*b^2*B*(c + d*x)^3*(A + B*Log[(
e*(c + d*x))/(a + b*x)]))/(9*(b*c - a*d)^3*g^4*(a + b*x)^3) - (2*B*d^3*Log[
(c + d*x)/(a + b*x)]*(A + B*Log[(e*(c + d*x))/(a + b*x)]))/(3*b*(b*c - a*d)
^3*g^4) - (A + B*Log[(e*(c + d*x))/(a + b*x)])^2/(3*b*g^4*(a + b*x)^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rule 2552

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(
m + 1)*(g/d)^m, Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a
 + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[n
 + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[d*f
- c*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Rubi steps

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(ag+bgx)^4}\,dx = -\frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{3bg^4(a+bx)^3} + \frac{(2B)\int \frac{(bc-ad)\left(-A-B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{g^3(a+bx)^4(c+dx)}\,dx}{3bg}$$

$$= -\frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{3bg^4(a+bx)^3} + \frac{(2B(bc-ad))\int \frac{-A-B\log\left(\frac{e(c+dx)}{a+bx}\right)}{(a+bx)^4(c+dx)}\,dx}{3bg^4}$$

$$= -\frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{3bg^4(a+bx)^3} + \frac{(2B(bc-ad))\int \left(\frac{b\left(-A-B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{(bc-ad)(a+bx)^4} - \frac{bd(-A-\ldots}{(bc-\ldots}\right.}{\ldots}$$

$$= -\frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{3bg^4(a+bx)^3} + \frac{(2B)\int \frac{-A-B\log\left(\frac{e(c+dx)}{a+bx}\right)}{(a+bx)^4}\,dx}{3g^4} - \frac{(2Bd^3)\int \frac{-A-B\log\ldots}{a\ldots}}{3(bc-ad\ldots}$$

$$= \frac{2B\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{9bg^4(a+bx)^3} - \frac{Bd\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{3b(bc-ad)g^4(a+bx)^2} + \frac{2Bd^2\left(A+B\log\ldots\right.}{3b(bc-ad)^2g^4\ldots}$$

$$= \frac{2B\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{9bg^4(a+bx)^3} - \frac{Bd\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{3b(bc-ad)g^4(a+bx)^2} + \frac{2Bd^2\left(A+B\log\ldots\right.}{3b(bc-ad)^2g^4\ldots}$$

$$= \frac{2B\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{9bg^4(a+bx)^3} - \frac{Bd\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{3b(bc-ad)g^4(a+bx)^2} + \frac{2Bd^2\left(A+B\log\ldots\right.}{3b(bc-ad)^2g^4\ldots}$$

$$= -\frac{2B^2}{27bg^4(a+bx)^3} + \frac{5B^2d}{18b(bc-ad)g^4(a+bx)^2} - \frac{11B^2d^2}{9b(bc-ad)^2g^4(a+bx)} - \ldots$$

$$= -\frac{2B^2}{27bg^4(a+bx)^3} + \frac{5B^2d}{18b(bc-ad)g^4(a+bx)^2} - \frac{11B^2d^2}{9b(bc-ad)^2g^4(a+bx)} - \ldots$$

$$= -\frac{2B^2}{27bg^4(a+bx)^3} + \frac{5B^2d}{18b(bc-ad)g^4(a+bx)^2} - \frac{11B^2d^2}{9b(bc-ad)^2g^4(a+bx)} - \ldots$$

$$= -\frac{2B^2}{27bg^4(a+bx)^3} + \frac{5B^2d}{18b(bc-ad)g^4(a+bx)^2} - \frac{11B^2d^2}{9b(bc-ad)^2g^4(a+bx)} - \ldots$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 0.45, size = 585, normalized size = 1.47

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(c + d*x))/(a + b*x)])^2/(a*g + b*g*x)^4,x]
```

```
[Out] -1/54*(18*(A + B*Log[(e*(c + d*x))/(a + b*x)])^2 + (B*(36*B*d^2*(a + b*x)^2
*(b*c - a*d + d*(a + b*x)*Log[a + b*x] - d*(a + b*x)*Log[c + d*x]) - 9*B*d*
(a + b*x)*((b*c - a*d)^2 + 2*d*(-(b*c) + a*d)*(a + b*x) - 2*d^2*(a + b*x)^2
*Log[a + b*x] + 2*d^2*(a + b*x)^2*Log[c + d*x]) + 2*B*(2*(b*c - a*d)^3 - 3*
d*(b*c - a*d)^2*(a + b*x) + 6*d^2*(b*c - a*d)*(a + b*x)^2 + 6*d^3*(a + b*x)
^3*Log[a + b*x] - 6*d^3*(a + b*x)^3*Log[c + d*x]) - 12*(b*c - a*d)^3*(A + B
*Log[(e*(c + d*x))/(a + b*x)]) + 18*d*(b*c - a*d)^2*(a + b*x)*(A + B*Log[(e
*(c + d*x))/(a + b*x)]) + 36*d^2*(-(b*c) + a*d)*(a + b*x)^2*(A + B*Log[(e*(
c + d*x))/(a + b*x)]) - 36*d^3*(a + b*x)^3*Log[a + b*x]*(A + B*Log[(e*(c +
d*x))/(a + b*x)]) + 36*d^3*(a + b*x)^3*Log[c + d*x]*(A + B*Log[(e*(c + d*x)
)/(a + b*x)]) - 18*B*d^3*(a + b*x)^3*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b
*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) + 1
8*B*d^3*(a + b*x)^3*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*L
og[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(b*c - a*d)^3)/(b*
g^4*(a + b*x)^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1060 vs. $2(387) = 774$.
time = 0.49, size = 1061, normalized size = 2.66 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*(d*x+c)/(b*x+a)))^2/(b*g*x+a*g)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^2*e*(a*d-b*c)*(1/3*b^4/(a*d-b*c)^4/e^4/g^4*A^2*(d*e/b-e*(a*d-b*c)/b/(b*
x+a))^3-b^3/(a*d-b*c)^4/e^3/g^4*A^2*d*(d*e/b-e*(a*d-b*c)/b/(b*x+a))^2+b^2/(
a*d-b*c)^4/e^2/g^4*A^2*d^2*(d*e/b-e*(a*d-b*c)/b/(b*x+a))+2*b^4/(a*d-b*c)^4/
e^4/g^4*A*B*(1/3*(d*e/b-e*(a*d-b*c)/b/(b*x+a))^3*ln(d*e/b-e*(a*d-b*c)/b/(b*
x+a))-1/9*(d*e/b-e*(a*d-b*c)/b/(b*x+a))^3)-4*b^3/(a*d-b*c)^4/e^3/g^4*A*B*d*
(1/2*(d*e/b-e*(a*d-b*c)/b/(b*x+a))^2*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))-1/4*(d
*e/b-e*(a*d-b*c)/b/(b*x+a))^2)+2*b^2/(a*d-b*c)^4/e^2/g^4*A*B*d^2*((d*e/b-e*
(a*d-b*c)/b/(b*x+a))*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))+e*(a*d-b*c)/b/(b*x+a)-
d*e/b)+b^4/(a*d-b*c)^4/e^4/g^4*B^2*(1/3*(d*e/b-e*(a*d-b*c)/b/(b*x+a))^3*ln(
d*e/b-e*(a*d-b*c)/b/(b*x+a))^2-2/9*(d*e/b-e*(a*d-b*c)/b/(b*x+a))^3*ln(d*e/b
-e*(a*d-b*c)/b/(b*x+a))+2/27*(d*e/b-e*(a*d-b*c)/b/(b*x+a))^3)-2*b^3/(a*d-b*
c)^4/e^3/g^4*B^2*d*(1/2*(d*e/b-e*(a*d-b*c)/b/(b*x+a))^2*ln(d*e/b-e*(a*d-b*c
)/b/(b*x+a))^2-1/2*(d*e/b-e*(a*d-b*c)/b/(b*x+a))^2*ln(d*e/b-e*(a*d-b*c)/b/(
b*x+a))+1/4*(d*e/b-e*(a*d-b*c)/b/(b*x+a))^2)+b^2/(a*d-b*c)^4/e^2/g^4*B^2*d^
2*((d*e/b-e*(a*d-b*c)/b/(b*x+a))*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))^2-2*(d*e/b
-e*(a*d-b*c)/b/(b*x+a))*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))-2*e*(a*d-b*c)/b/(b*
x+a)+2*d*e/b))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1426 vs. $2(392) = 784$.
time = 0.46, size = 1426, normalized size = 3.57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(d*x+c)/(b*x+a)))^2/(b*g*x+a*g)^4,x, algorithm="maxima
")
```

```
[Out] 1/54*(6*((6*b^2*d^2*x^2 + 2*b^2*c^2 - 7*a*b*c*d + 11*a^2*d^2 - 3*(b^2*c*d -
 5*a*b*d^2)*x)/((b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2)*g^4*x^3 + 3*(a*b^5*c^
2 - 2*a^2*b^4*c*d + a^3*b^3*d^2)*g^4*x^2 + 3*(a^2*b^4*c^2 - 2*a^3*b^3*c*d +
 a^4*b^2*d^2)*g^4*x + (a^3*b^3*c^2 - 2*a^4*b^2*c*d + a^5*b*d^2)*g^4) + 6*d^
3*log(b*x + a)/((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*g^4
) - 6*d^3*log(d*x + c)/((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*
d^3)*g^4))*log(d*x*e/(b*x + a) + c*e/(b*x + a)) - (4*b^3*c^3 - 27*a*b^2*c^2
*d + 108*a^2*b*c*d^2 - 85*a^3*d^3 + 66*(b^3*c*d^2 - a*b^2*d^3)*x^2 - 18*(b^
3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(b*x + a)^2 - 18*
(b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(d*x + c)^2 -
3*(5*b^3*c^2*d - 54*a*b^2*c*d^2 + 49*a^2*b*d^3)*x + 66*(b^3*d^3*x^3 + 3*a*b
^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(b*x + a) - 6*(11*b^3*d^3*x^3 + 33
*a*b^2*d^3*x^2 + 33*a^2*b*d^3*x + 11*a^3*d^3 - 6*(b^3*d^3*x^3 + 3*a*b^2*d^3
*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(b*x + a))*log(d*x + c))/(a^3*b^4*c^3*g^
4 - 3*a^4*b^3*c^2*d*g^4 + 3*a^5*b^2*c*d^2*g^4 - a^6*b*d^3*g^4 + (b^7*c^3*g^
4 - 3*a*b^6*c^2*d*g^4 + 3*a^2*b^5*c*d^2*g^4 - a^3*b^4*d^3*g^4)*x^3 + 3*(a*b
^6*c^3*g^4 - 3*a^2*b^5*c^2*d*g^4 + 3*a^3*b^4*c*d^2*g^4 - a^4*b^3*d^3*g^4)*x
^2 + 3*(a^2*b^5*c^3*g^4 - 3*a^3*b^4*c^2*d*g^4 + 3*a^4*b^3*c*d^2*g^4 - a^5*b
^2*d^3*g^4)*x))*B^2 + 1/9*A*B*((6*b^2*d^2*x^2 + 2*b^2*c^2 - 7*a*b*c*d + 11*
a^2*d^2 - 3*(b^2*c*d - 5*a*b*d^2)*x)/((b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2)
*g^4*x^3 + 3*(a*b^5*c^2 - 2*a^2*b^4*c*d + a^3*b^3*d^2)*g^4*x^2 + 3*(a^2*b^4
*c^2 - 2*a^3*b^3*c*d + a^4*b^2*d^2)*g^4*x + (a^3*b^3*c^2 - 2*a^4*b^2*c*d +
a^5*b*d^2)*g^4) - 6*log(d*x*e/(b*x + a) + c*e/(b*x + a))/(b^4*g^4*x^3 + 3*a
*b^3*g^4*x^2 + 3*a^2*b^2*g^4*x + a^3*b*g^4) + 6*d^3*log(b*x + a)/((b^4*c^3
- 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*g^4) - 6*d^3*log(d*x + c)/((
b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*g^4)) - 1/3*B^2*log(
d*x*e/(b*x + a) + c*e/(b*x + a))^2/(b^4*g^4*x^3 + 3*a*b^3*g^4*x^2 + 3*a^2*b
^2*g^4*x + a^3*b*g^4) - 1/3*A^2/(b^4*g^4*x^3 + 3*a*b^3*g^4*x^2 + 3*a^2*b^2*
g^4*x + a^3*b*g^4)
```

Fricas [A]
time = 0.46, size = 678, normalized size = 1.70

$$-\frac{2\,(9\,A^2-6\,AB+2\,B^2)b^3c^3-27\,(2\,A^2-2\,AB+B^2)ab^2c^2d+54\,(A^2-2\,AB+2\,B^2)a^2bcd^2-(18\,A^2-66\,AB+85\,B^2)a^3d^3-6\,((6\,AB-11\,B^2)b^3cd^2-(6\,AB-11\,B^2)ab^2d^3)x^2+18\,(B^2b^3d^3x^3+3\,B^2ab^2d^3x^2+3\,B^2a^2bd^3x+B^2b^3c^3-3\,B^2ab^2c^2d+3\,B^2a^2bcd^2)\log\left(\frac{(dx+c)e}{bx+a}\right)^2+3\,((6\,AB-5\,B^2)b^3c^2d-18\,(2\,AB-3\,B^2)ab^2cd^2+(30\,AB-49\,B^2)a^2bd^3)x+6\,((6\,AB-11\,B^2)b^3d^3x^3+2\,(3\,AB-B^2)b^3c^3-9\,(2\,AB-B^2)ab^2c^2d+18\,(AB-B^2)a^2bcd^2-3\,(2\,B^2b^3cd^2-3\,(2\,AB-3\,B^2)ab^2d^3)x^2+3\,(B^2b^3c^2d-6\,B^2ab^2cd^2+6\,(AB-B^2)a^2bd^3)x)\log\left(\frac{(dx+c)e}{bx+a}\right)}{54\,((b^7c^3-3\,ab^6c^2d+3\,a^2b^5cd^2-a^3b^4d^3)g^4x^3+3\,(ab^6c^3-3\,a^2b^5c^2d+3\,a^3b^4cd^2-a^4b^3d^3)g^4x^2+3\,(a^2b^5c^3-3\,a^3b^4c^2d+3\,a^4b^3cd^2-a^5b^2d^3)g^4x+(a^3b^4c^3-3\,a^4b^3c^2d+3\,a^5b^2cd^2-a^6bd^3)g^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(d*x+c)/(b*x+a)))^2/(b*g*x+a*g)^4,x, algorithm="fricas
")
```

```
[Out] -1/54*(2*(9*A^2 - 6*A*B + 2*B^2)*b^3*c^3 - 27*(2*A^2 - 2*A*B + B^2)*a*b^2*c
^2*d + 54*(A^2 - 2*A*B + 2*B^2)*a^2*b*c*d^2 - (18*A^2 - 66*A*B + 85*B^2)*a^
```

```
3*d^3 - 6*((6*A*B - 11*B^2)*b^3*c*d^2 - (6*A*B - 11*B^2)*a*b^2*d^3)*x^2 + 1
8*(B^2*b^3*d^3*x^3 + 3*B^2*a*b^2*d^3*x^2 + 3*B^2*a^2*b*d^3*x + B^2*b^3*c^3
 - 3*B^2*a*b^2*c^2*d + 3*B^2*a^2*b*c*d^2)*log((d*x + c)*e/(b*x + a))^2 + 3*(
(6*A*B - 5*B^2)*b^3*c^2*d - 18*(2*A*B - 3*B^2)*a*b^2*c*d^2 + (30*A*B - 49*B
^2)*a^2*b*d^3)*x + 6*((6*A*B - 11*B^2)*b^3*d^3*x^3 + 2*(3*A*B - B^2)*b^3*c^
3 - 9*(2*A*B - B^2)*a*b^2*c^2*d + 18*(A*B - B^2)*a^2*b*c*d^2 - 3*(2*B^2*b^3
*c*d^2 - 3*(2*A*B - 3*B^2)*a*b^2*d^3)*x^2 + 3*(B^2*b^3*c^2*d - 6*B^2*a*b^2*
c*d^2 + 6*(A*B - B^2)*a^2*b*d^3)*x)*log((d*x + c)*e/(b*x + a)))/((b^7*c^3 -
 3*a*b^6*c^2*d + 3*a^2*b^5*c*d^2 - a^3*b^4*d^3)*g^4*x^3 + 3*(a*b^6*c^3 - 3*
a^2*b^5*c^2*d + 3*a^3*b^4*c*d^2 - a^4*b^3*d^3)*g^4*x^2 + 3*(a^2*b^5*c^3 - 3
*a^3*b^4*c^2*d + 3*a^4*b^3*c*d^2 - a^5*b^2*d^3)*g^4*x + (a^3*b^4*c^3 - 3*a^
4*b^3*c^2*d + 3*a^5*b^2*c*d^2 - a^6*b*d^3)*g^4)
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1544 vs. $2(362) = 724$.
time = 18.41, size = 1544, normalized size = 3.87

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(d*x+c)/(b*x+a)))**2/(b*g*x+a*g)**4,x)
```

```
[Out] B*d**3*(6*A - 11*B)*log(x + (6*A*B*a*d**4 + 6*A*B*b*c*d**3 - 11*B**2*a*d**4
 - 11*B**2*b*c*d**3 - B*a**4*d**7*(6*A - 11*B)/(a*d - b*c)**3 + 4*B*a**3*b*
c*d**6*(6*A - 11*B)/(a*d - b*c)**3 - 6*B*a**2*b**2*c**2*d**5*(6*A - 11*B)/(
a*d - b*c)**3 + 4*B*a*b**3*c**3*d**4*(6*A - 11*B)/(a*d - b*c)**3 - B*b**4*c
**4*d**3*(6*A - 11*B)/(a*d - b*c)**3)/(12*A*B*b*d**4 - 22*B**2*b*d**4))/(9*
b*g**4*(a*d - b*c)**3) - B*d**3*(6*A - 11*B)*log(x + (6*A*B*a*d**4 + 6*A*B*
b*c*d**3 - 11*B**2*a*d**4 - 11*B**2*b*c*d**3 + B*a**4*d**7*(6*A - 11*B)/(a*
d - b*c)**3 - 4*B*a**3*b*c*d**6*(6*A - 11*B)/(a*d - b*c)**3 + 6*B*a**2*b**2
*c**2*d**5*(6*A - 11*B)/(a*d - b*c)**3 - 4*B*a*b**3*c**3*d**4*(6*A - 11*B)/
(a*d - b*c)**3 + B*b**4*c**4*d**3*(6*A - 11*B)/(a*d - b*c)**3)/(12*A*B*b*d*
*4 - 22*B**2*b*d**4))/(9*b*g**4*(a*d - b*c)**3) + (3*B**2*a**2*c*d**2 + 3*B
**2*a**2*d**3*x - 3*B**2*a*b*c**2*d + 3*B**2*a*b*d**3*x**2 + B**2*b**2*c**3
 + B**2*b**2*d**3*x**3)*log(e*(c + d*x)/(a + b*x))**2/(3*a**6*d**3*g**4 - 9
*a**5*b*c*d**2*g**4 + 9*a**5*b*d**3*g**4*x + 9*a**4*b**2*c**2*d*g**4 - 27*a
**4*b**2*c*d**2*g**4*x + 9*a**4*b**2*d**3*g**4*x**2 - 3*a**3*b**3*c**3*g**4
 + 27*a**3*b**3*c**2*d*g**4*x - 27*a**3*b**3*c*d**2*g**4*x**2 + 3*a**3*b**3
*d**3*g**4*x**3 - 9*a**2*b**4*c**3*g**4*x + 27*a**2*b**4*c**2*d*g**4*x**2 -
 9*a**2*b**4*c*d**2*g**4*x**3 - 9*a*b**5*c**3*g**4*x**2 + 9*a*b**5*c**2*d*g
**4*x**3 - 3*b**6*c**3*g**4*x**3) + (-6*A*B*a**2*d**2 + 12*A*B*a*b*c*d - 6*
A*B*b**2*c**2 + 11*B**2*a**2*d**2 - 7*B**2*a*b*c*d + 15*B**2*a*b*d**2*x + 2
*B**2*b**2*c**2 - 3*B**2*b**2*c*d*x + 6*B**2*b**2*d**2*x**2)*log(e*(c + d*x
)/(a + b*x))/(9*a**5*b*d**2*g**4 - 18*a**4*b**2*c*d*g**4 + 27*a**4*b**2*d**
2*g**4*x + 9*a**3*b**3*c**2*g**4 - 54*a**3*b**3*c*d*g**4*x + 27*a**3*b**3*d
```

```
**2*g**4*x**2 + 27*a**2*b**4*c**2*g**4*x - 54*a**2*b**4*c*d*g**4*x**2 + 9*a
**2*b**4*d**2*g**4*x**3 + 27*a*b**5*c**2*g**4*x**2 - 18*a*b**5*c*d*g**4*x**
3 + 9*b**6*c**2*g**4*x**3) - (18*A**2*a**2*d**2 - 36*A**2*a*b*c*d + 18*A**2
*b**2*c**2 - 66*A*B*a**2*d**2 + 42*A*B*a*b*c*d - 12*A*B*b**2*c**2 + 85*B**2
*a**2*d**2 - 23*B**2*a*b*c*d + 4*B**2*b**2*c**2 + x**2*(-36*A*B*b**2*d**2 +
 66*B**2*b**2*d**2) + x*(-90*A*B*a*b*d**2 + 18*A*B*b**2*c*d + 147*B**2*a*b*
d**2 - 15*B**2*b**2*c*d))/(54*a**5*b*d**2*g**4 - 108*a**4*b**2*c*d*g**4 + 5
4*a**3*b**3*c**2*g**4 + x**3*(54*a**2*b**4*d**2*g**4 - 108*a*b**5*c*d*g**4
+ 54*b**6*c**2*g**4) + x**2*(162*a**3*b**3*d**2*g**4 - 324*a**2*b**4*c*d*g*
*4 + 162*a*b**5*c**2*g**4) + x*(162*a**4*b**2*d**2*g**4 - 324*a**3*b**3*c*d
*g**4 + 162*a**2*b**4*c**2*g**4))
```

Giac [A]
time = 3.12, size = 760, normalized size = 1.90

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(d*x+c)/(b*x+a)))^2/(b*g*x+a*g)^4,x, algorithm="giac")
```

```
[Out] -1/54*(54*(d*x*e + c*e)*B^2*d^2*e^2*log((d*x*e + c*e)/(b*x + a))^2/(b*x + a
) - 54*(d*x*e + c*e)^2*B^2*b*d*e*log((d*x*e + c*e)/(b*x + a))^2/(b*x + a)^2
 + 108*(d*x*e + c*e)*A*B*d^2*e^2*log((d*x*e + c*e)/(b*x + a))/(b*x + a) - 1
08*(d*x*e + c*e)*B^2*d^2*e^2*log((d*x*e + c*e)/(b*x + a))/(b*x + a) - 108*(
d*x*e + c*e)^2*A*B*b*d*e*log((d*x*e + c*e)/(b*x + a))/(b*x + a)^2 + 54*(d*x
*e + c*e)^2*B^2*b*d*e*log((d*x*e + c*e)/(b*x + a))/(b*x + a)^2 + 18*(d*x*e
+ c*e)^3*B^2*b^2*log((d*x*e + c*e)/(b*x + a))^2/(b*x + a)^3 + 54*(d*x*e + c
*e)*A^2*d^2*e^2/(b*x + a) - 108*(d*x*e + c*e)*A*B*d^2*e^2/(b*x + a) + 108*(
d*x*e + c*e)*B^2*d^2*e^2/(b*x + a) - 54*(d*x*e + c*e)^2*A^2*b*d*e/(b*x + a)
^2 + 54*(d*x*e + c*e)^2*A*B*b*d*e/(b*x + a)^2 - 27*(d*x*e + c*e)^2*B^2*b*d*
e/(b*x + a)^2 + 36*(d*x*e + c*e)^3*A*B*b^2*log((d*x*e + c*e)/(b*x + a))/(b*
x + a)^3 - 12*(d*x*e + c*e)^3*B^2*b^2*log((d*x*e + c*e)/(b*x + a))/(b*x + a
)^3 + 18*(d*x*e + c*e)^3*A^2*b^2/(b*x + a)^3 - 12*(d*x*e + c*e)^3*A*B*b^2/(
b*x + a)^3 + 4*(d*x*e + c*e)^3*B^2*b^2/(b*x + a)^3)*(b*c/((b*c*e - a*d*e)*(
b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))/(b^2*c^2*g^4*e^2 - 2*a*b*c
*d*g^4*e^2 + a^2*d^2*g^4*e^2)
```

Mupad [B]
time = 7.70, size = 1064, normalized size = 2.67

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(c + d*x))/(a + b*x)))^2/(a*g + b*g*x)^4,x)
```

```
[Out] ((18*A^2*a^2*d^2 + 18*A^2*b^2*c^2 + 85*B^2*a^2*d^2 + 4*B^2*b^2*c^2 - 66*A*B
*a^2*d^2 - 12*A*B*b^2*c^2 - 36*A^2*a*b*c*d - 23*B^2*a*b*c*d + 42*A*B*a*b*c*
d)/(6*(a*d - b*c)) + (x*(49*B^2*a*b*d^2 - 5*B^2*b^2*c*d - 30*A*B*a*b*d^2 +
6*A*B*b^2*c*d))/(2*(a*d - b*c)) + (d*x^2*(11*B^2*b^2*d - 6*A*B*b^2*d))/(a*d
 - b*c))/(x*(27*a^2*b^3*c*g^4 - 27*a^3*b^2*d*g^4) - x^2*(27*a^2*b^3*d*g^4 -
 27*a*b^4*c*g^4) + x^3*(9*b^5*c*g^4 - 9*a*b^4*d*g^4) + 9*a^3*b^2*c*g^4 - 9*
a^4*b*d*g^4) - log((e*(c + d*x))/(a + b*x))^2*(B^2/(3*b^2*g^4*(3*a^2*x + a^
3/b + b^2*x^3 + 3*a*b*x^2)) - (B^2*d^3)/(3*b*g^4*(a^3*d^3 - b^3*c^3 + 3*a*b
^2*c^2*d - 3*a^2*b*c*d^2))) - (log((e*(c + d*x))/(a + b*x))*((2*A*B)/(3*b^2
*d*g^4) - (2*B^2*d^3*(a*((3*a^2*d^2 + b^2*c^2 - 4*a*b*c*d)/(6*b*d^3) + (a*(
a*d - b*c))/(3*b*d^2)) + (3*a^3*d^3 - b^3*c^3 + 4*a*b^2*c^2*d - 6*a^2*b*c*d
^2)/(3*b*d^4)))/(3*b*g^4*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2
)) + (2*B^2*d^3*x^2*((b^2*c - a*b*d)/(3*d^2) - (2*b*(a*d - b*c))/(3*d^2)))/
(3*b*g^4*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) - (2*B^2*d^3*
x*(b*((3*a^2*d^2 + b^2*c^2 - 4*a*b*c*d)/(6*b*d^3) + (a*(a*d - b*c))/(3*b*d^
2)) + (3*a^2*d^2 + b^2*c^2 - 4*a*b*c*d)/(3*d^3) + (2*a*(a*d - b*c))/(3*d^2)
))/(3*b*g^4*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))))/((3*a^2*
x)/d + a^3/(b*d) + (b^2*x^3)/d + (3*a*b*x^2)/d) - (B*d^3*atan((B*d^3*((b^4*
c^3*g^4 + a^3*b*d^3*g^4 - a*b^3*c^2*d*g^4 - a^2*b^2*c*d^2*g^4)/(b^3*c^2*g^4
 + a^2*b*d^2*g^4 - 2*a*b^2*c*d*g^4) + 2*b*d*x)*(6*A - 11*B)*(b^3*c^2*g^4 +
a^2*b*d^2*g^4 - 2*a*b^2*c*d*g^4)*1i)/(b*g^4*(a*d - b*c)^3*(11*B^2*d^3 - 6*A
*B*d^3)))*(6*A - 11*B)*2i)/(9*b*g^4*(a*d - b*c)^3)
```

3.190 $\int \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(ag+bgx)^5}\,dx$

Optimal. Leaf size=498

$$\frac{2B^2d^3(c+dx)}{(bc-ad)^4g^5(a+bx)}-\frac{3bB^2d^2(c+dx)^2}{4(bc-ad)^4g^5(a+bx)^2}+\frac{2b^2B^2d(c+dx)^3}{9(bc-ad)^4g^5(a+bx)^3}-\frac{b^3B^2(c+dx)^4}{32(bc-ad)^4g^5(a+bx)^4}-\frac{B^2d^4\log^2}{4b(bc-}$$

```
[Out] 2*B^2*d^3*(d*x+c)/(-a*d+b*c)^4/g^5/(b*x+a)-3/4*b*B^2*d^2*(d*x+c)^2/(-a*d+b*
c)^4/g^5/(b*x+a)^2+2/9*b^2*B^2*d*(d*x+c)^3/(-a*d+b*c)^4/g^5/(b*x+a)^3-1/32*
b^3*B^2*(d*x+c)^4/(-a*d+b*c)^4/g^5/(b*x+a)^4-1/4*B^2*d^4*ln((d*x+c)/(b*x+a)
)^2/b/(-a*d+b*c)^4/g^5-2*B*d^3*(d*x+c)*(A+B*ln(e*(d*x+c)/(b*x+a)))/(-a*d+b*
c)^4/g^5/(b*x+a)+3/2*b*B*d^2*(d*x+c)^2*(A+B*ln(e*(d*x+c)/(b*x+a)))/(-a*d+b*
c)^4/g^5/(b*x+a)^2-2/3*b^2*B*d*(d*x+c)^3*(A+B*ln(e*(d*x+c)/(b*x+a)))/(-a*d+
b*c)^4/g^5/(b*x+a)^3+1/8*b^3*B*(d*x+c)^4*(A+B*ln(e*(d*x+c)/(b*x+a)))/(-a*d+
b*c)^4/g^5/(b*x+a)^4+1/2*B*d^4*ln((d*x+c)/(b*x+a))*(A+B*ln(e*(d*x+c)/(b*x+a
)))/b/(-a*d+b*c)^4/g^5-1/4*(A+B*ln(e*(d*x+c)/(b*x+a)))^2/b/g^5/(b*x+a)^4
```

Rubi [A]
time = 0.20, antiderivative size = 498, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {2552, 2356, 45, 2372, 2338}

$$\frac{b^3B(c+dx)^4\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{8g^5(a+bx)^4(bc-ad)^4}-\frac{2b^2Bd(c+dx)^3\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{3g^5(a+bx)^3(bc-ad)^4}+\frac{Bd^4\log\left(\frac{c+dx}{a+bx}\right)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{2bg^5(bc-ad)^4}-\frac{2Bd^3(c+dx)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{g^5(a+bx)(bc-ad)^4}+\frac{3bBd^2(c+dx)^2\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{2g^5(a+bx)^2(bc-ad)^4}-\frac{\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}{4bg^5(a+bx)^4}-\frac{b^3B^2(c+dx)^4}{32g^5(a+bx)^4(bc-ad)^4}+\frac{2b^2B^2d(c+dx)^3}{9g^5(a+bx)^3(bc-ad)^4}-\frac{B^2d^4\log^2\left(\frac{c+dx}{a+bx}\right)}{4bg^5(bc-ad)^4}+\frac{2B^2d^3(c+dx)}{g^5(a+bx)(bc-ad)^4}-\frac{3bB^2d^2(c+dx)^2}{4g^5(a+bx)^2(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(c + d*x))/(a + b*x)])^2/(a*g + b*g*x)^5,x]
```

```
[Out] (2*B^2*d^3*(c + d*x))/((b*c - a*d)^4*g^5*(a + b*x)) - (3*b*B^2*d^2*(c + d*x
)^2)/(4*(b*c - a*d)^4*g^5*(a + b*x)^2) + (2*b^2*B^2*d*(c + d*x)^3)/(9*(b*c
- a*d)^4*g^5*(a + b*x)^3) - (b^3*B^2*(c + d*x)^4)/(32*(b*c - a*d)^4*g^5*(a
+ b*x)^4) - (B^2*d^4*Log[(c + d*x)/(a + b*x)]^2)/(4*b*(b*c - a*d)^4*g^5) -
(2*B*d^3*(c + d*x)*(A + B*Log[(e*(c + d*x))/(a + b*x)]))/((b*c - a*d)^4*g^5
*(a + b*x)) + (3*b*B*d^2*(c + d*x)^2*(A + B*Log[(e*(c + d*x))/(a + b*x)]))/
(2*(b*c - a*d)^4*g^5*(a + b*x)^2) - (2*b^2*B*d*(c + d*x)^3*(A + B*Log[(e*(c
 + d*x))/(a + b*x)]))/(3*(b*c - a*d)^4*g^5*(a + b*x)^3) + (b^3*B*(c + d*x)^
4*(A + B*Log[(e*(c + d*x))/(a + b*x)]))/(8*(b*c - a*d)^4*g^5*(a + b*x)^4) +
 (B*d^4*Log[(c + d*x)/(a + b*x)]*(A + B*Log[(e*(c + d*x))/(a + b*x)]))/(2*b
*(b*c - a*d)^4*g^5) - (A + B*Log[(e*(c + d*x))/(a + b*x)])^2/(4*b*g^5*(a +
b*x)^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rule 2552

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(
m + 1)*(g/d)^m, Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a
 + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[n
 + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[d*f
- c*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Rubi steps

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(ag+bgx)^5}\,dx = -\frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{4bg^5(a+bx)^4}+\frac{B\int\frac{(bc-ad)\left(-A-B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{g^4(a+bx)^5(c+dx)}dx}{2bg}$$

$$= -\frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{4bg^5(a+bx)^4}+\frac{(B(bc-ad))\int\frac{-A-B\log\left(\frac{e(c+dx)}{a+bx}\right)}{(a+bx)^5(c+dx)}dx}{2bg^5}$$

$$= -\frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{4bg^5(a+bx)^4}+\frac{(B(bc-ad))\int\left(\frac{b\left(-A-B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{(bc-ad)(a+bx)^5}-\frac{bd\left(-A-B\right.}{(bc-a}\right.}{}$$

$$= -\frac{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{4bg^5(a+bx)^4}+\frac{B\int\frac{-A-B\log\left(\frac{e(c+dx)}{a+bx}\right)}{(a+bx)^5}dx}{2g^5}+\frac{(Bd^4)\int\frac{-A-B\log\left(\frac{e(c}{a}\right)}{a+bx}}{2(bc-ad)^4g}$$

$$= \frac{B\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{8bg^5(a+bx)^4}-\frac{Bd\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{6b(bc-ad)g^5(a+bx)^3}+\frac{Bd^2\left(A+B\log\left(\right.\right.}{4b(bc-ad)^2g^5(a}$$

$$= \frac{B\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{8bg^5(a+bx)^4}-\frac{Bd\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{6b(bc-ad)g^5(a+bx)^3}+\frac{Bd^2\left(A+B\log\left(\right.\right.}{4b(bc-ad)^2g^5(a}$$

$$= \frac{B\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{8bg^5(a+bx)^4}-\frac{Bd\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{6b(bc-ad)g^5(a+bx)^3}+\frac{Bd^2\left(A+B\log\left(\right.\right.}{4b(bc-ad)^2g^5(a}$$

$$= -\frac{B^2}{32bg^5(a+bx)^4}+\frac{7B^2d}{72b(bc-ad)g^5(a+bx)^3}-\frac{13B^2d^2}{48b(bc-ad)^2g^5(a+bx)^2}+$$

$$= -\frac{B^2}{32bg^5(a+bx)^4}+\frac{7B^2d}{72b(bc-ad)g^5(a+bx)^3}-\frac{13B^2d^2}{48b(bc-ad)^2g^5(a+bx)^2}+$$

$$= -\frac{B^2}{32bg^5(a+bx)^4}+\frac{7B^2d}{72b(bc-ad)g^5(a+bx)^3}-\frac{13B^2d^2}{48b(bc-ad)^2g^5(a+bx)^2}+$$

$$= -\frac{B^2}{32bg^5(a+bx)^4}+\frac{7B^2d}{72b(bc-ad)g^5(a+bx)^3}-\frac{13B^2d^2}{48b(bc-ad)^2g^5(a+bx)^2}+$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 0.60, size = 748, normalized size = 1.50

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(c + d*x))/(a + b*x)])^2/(a*g + b*g*x)^5,x]
```

```
[Out] (-72*(A + B*Log[(e*(c + d*x))/(a + b*x)])^2 + (B*(144*B*d^3*(a + b*x)^3*(b*
c - a*d + d*(a + b*x)*Log[a + b*x] - d*(a + b*x)*Log[c + d*x]) - 36*B*d^2*(
a + b*x)^2*((b*c - a*d)^2 + 2*d*(-(b*c) + a*d)*(a + b*x) - 2*d^2*(a + b*x)^
2*Log[a + b*x] + 2*d^2*(a + b*x)^2*Log[c + d*x]) + 8*B*d*(a + b*x)*(2*(b*c
- a*d)^3 - 3*d*(b*c - a*d)^2*(a + b*x) + 6*d^2*(b*c - a*d)*(a + b*x)^2 + 6*
d^3*(a + b*x)^3*Log[a + b*x] - 6*d^3*(a + b*x)^3*Log[c + d*x]) - 3*B*(3*(b*
c - a*d)^4 + 4*d*(-(b*c) + a*d)^3*(a + b*x) + 6*d^2*(b*c - a*d)^2*(a + b*x)
^2 + 12*d^3*(-(b*c) + a*d)*(a + b*x)^3 - 12*d^4*(a + b*x)^4*Log[a + b*x] +
12*d^4*(a + b*x)^4*Log[c + d*x]) + 36*(b*c - a*d)^4*(A + B*Log[(e*(c + d*x)
)/(a + b*x)]) + 48*d*(-(b*c) + a*d)^3*(a + b*x)*(A + B*Log[(e*(c + d*x))/(a
 + b*x)]) + 72*d^2*(b*c - a*d)^2*(a + b*x)^2*(A + B*Log[(e*(c + d*x))/(a +
b*x)]) + 144*d^3*(-(b*c) + a*d)*(a + b*x)^3*(A + B*Log[(e*(c + d*x))/(a + b
*x)]) - 144*d^4*(a + b*x)^4*Log[a + b*x]*(A + B*Log[(e*(c + d*x))/(a + b*x)
]) + 144*d^4*(a + b*x)^4*Log[c + d*x]*(A + B*Log[(e*(c + d*x))/(a + b*x)])
- 72*B*d^4*(a + b*x)^4*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b
*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) + 72*B*d^4*(a + b
*x)^4*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] +
2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(b*c - a*d)^4)/(288*b*g^5*(a + b
*x)^4)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1421 vs. $2(480) = 960$.
time = 0.55, size = 1422, normalized size = 2.86

method	result	size
derivativedivides	Expression too large to display	1422
default	Expression too large to display	1422
norman	Expression too large to display	1796
risch	Expression too large to display	2601

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*(d*x+c)/(b*x+a)))^2/(b*g*x+a*g)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^2*e*(a*d-b*c)*(-1/4*b^5/(a*d-b*c)^5/e^5/g^5*A^2*(d*e/b-e*(a*d-b*c)/b/(b
*x+a))^4+b^4/(a*d-b*c)^5/e^4/g^5*A^2*d*(d*e/b-e*(a*d-b*c)/b/(b*x+a))^3-3/2*
b^3/(a*d-b*c)^5/e^3/g^5*A^2*d^2*(d*e/b-e*(a*d-b*c)/b/(b*x+a))^2+b^2/(a*d-b*
c)^5/e^2/g^5*A^2*d^3*(d*e/b-e*(a*d-b*c)/b/(b*x+a))-2*b^5/(a*d-b*c)^5/e^5/g^
5*A*B*(1/4*(d*e/b-e*(a*d-b*c)/b/(b*x+a))^4*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))-
1/16*(d*e/b-e*(a*d-b*c)/b/(b*x+a))^4)+6*b^4/(a*d-b*c)^5/e^4/g^5*A*B*d*(1/3*
(d*e/b-e*(a*d-b*c)/b/(b*x+a))^3*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))-1/9*(d*e/b-
e*(a*d-b*c)/b/(b*x+a))^3)-6*b^3/(a*d-b*c)^5/e^3/g^5*A*B*d^2*(1/2*(d*e/b-e*(
a*d-b*c)/b/(b*x+a))^2*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))-1/4*(d*e/b-e*(a*d-b*c
)/b/(b*x+a))^2)+2*b^2/(a*d-b*c)^5/e^2/g^5*A*B*d^3*((d*e/b-e*(a*d-b*c)/b/(b*
```

```
x+a))*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))+e*(a*d-b*c)/b/(b*x+a)-d*e/b)-b^5/(a*d
-b*c)^5/e^5/g^5*B^2*(1/4*(d*e/b-e*(a*d-b*c)/b/(b*x+a))^4*ln(d*e/b-e*(a*d-b*
c)/b/(b*x+a))^2-1/8*(d*e/b-e*(a*d-b*c)/b/(b*x+a))^4*ln(d*e/b-e*(a*d-b*c)/b/
(b*x+a))+1/32*(d*e/b-e*(a*d-b*c)/b/(b*x+a))^4)+3*b^4/(a*d-b*c)^5/e^4/g^5*B^
2*d*(1/3*(d*e/b-e*(a*d-b*c)/b/(b*x+a))^3*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))^2-
2/9*(d*e/b-e*(a*d-b*c)/b/(b*x+a))^3*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))+2/27*(d
*e/b-e*(a*d-b*c)/b/(b*x+a))^3)-3*b^3/(a*d-b*c)^5/e^3/g^5*B^2*d^2*(1/2*(d*e/
b-e*(a*d-b*c)/b/(b*x+a))^2*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))^2-1/2*(d*e/b-e*(
a*d-b*c)/b/(b*x+a))^2*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))+1/4*(d*e/b-e*(a*d-b*c
)/b/(b*x+a))^2)+b^2/(a*d-b*c)^5/e^2/g^5*B^2*d^3*((d*e/b-e*(a*d-b*c)/b/(b*x+
a))*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))^2-2*(d*e/b-e*(a*d-b*c)/b/(b*x+a))*ln(d*
e/b-e*(a*d-b*c)/b/(b*x+a))-2*e*(a*d-b*c)/b/(b*x+a)+2*d*e/b))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 2128 vs. $2(486) = 972$.
time = 0.53, size = 2128, normalized size = 4.27

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(d*x+c)/(b*x+a)))^2/(b*g*x+a*g)^5,x, algorithm="maxima
")
```

```
[Out] -1/288*(12*((12*b^3*d^3*x^3 - 3*b^3*c^3 + 13*a*b^2*c^2*d - 23*a^2*b*c*d^2 +
 25*a^3*d^3 - 6*(b^3*c*d^2 - 7*a*b^2*d^3)*x^2 + 4*(b^3*c^2*d - 5*a*b^2*c*d^
2 + 13*a^2*b*d^3)*x)/((b^8*c^3 - 3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 - a^3*b^5*
d^3)*g^5*x^4 + 4*(a*b^7*c^3 - 3*a^2*b^6*c^2*d + 3*a^3*b^5*c*d^2 - a^4*b^4*d
^3)*g^5*x^3 + 6*(a^2*b^6*c^3 - 3*a^3*b^5*c^2*d + 3*a^4*b^4*c*d^2 - a^5*b^3*
d^3)*g^5*x^2 + 4*(a^3*b^5*c^3 - 3*a^4*b^4*c^2*d + 3*a^5*b^3*c*d^2 - a^6*b^2
*d^3)*g^5*x + (a^4*b^4*c^3 - 3*a^5*b^3*c^2*d + 3*a^6*b^2*c*d^2 - a^7*b*d^3)
*g^5) + 12*d^4*log(b*x + a)/((b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 -
 4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5) - 12*d^4*log(d*x + c)/((b^5*c^4 - 4*a*b^
4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5))*log(d*x*e/
(b*x + a) + c*e/(b*x + a)) + (9*b^4*c^4 - 64*a*b^3*c^3*d + 216*a^2*b^2*c^2*
d^2 - 576*a^3*b*c*d^3 + 415*a^4*d^4 - 300*(b^4*c*d^3 - a*b^3*d^4)*x^3 + 6*(
13*b^4*c^2*d^2 - 176*a*b^3*c*d^3 + 163*a^2*b^2*d^4)*x^2 + 72*(b^4*d^4*x^4 +
 4*a*b^3*d^4*x^3 + 6*a^2*b^2*d^4*x^2 + 4*a^3*b*d^4*x + a^4*d^4)*log(b*x + a
)^2 + 72*(b^4*d^4*x^4 + 4*a*b^3*d^4*x^3 + 6*a^2*b^2*d^4*x^2 + 4*a^3*b*d^4*x
 + a^4*d^4)*log(d*x + c)^2 - 4*(7*b^4*c^3*d - 60*a*b^3*c^2*d^2 + 324*a^2*b^
2*c*d^3 - 271*a^3*b*d^4)*x - 300*(b^4*d^4*x^4 + 4*a*b^3*d^4*x^3 + 6*a^2*b^2
*d^4*x^2 + 4*a^3*b*d^4*x + a^4*d^4)*log(b*x + a) + 12*(25*b^4*d^4*x^4 + 100
*a*b^3*d^4*x^3 + 150*a^2*b^2*d^4*x^2 + 100*a^3*b*d^4*x + 25*a^4*d^4 - 12*(b
^4*d^4*x^4 + 4*a*b^3*d^4*x^3 + 6*a^2*b^2*d^4*x^2 + 4*a^3*b*d^4*x + a^4*d^4)
*log(b*x + a))*log(d*x + c))/(a^4*b^5*c^4*g^5 - 4*a^5*b^4*c^3*d*g^5 + 6*a^6
*b^3*c^2*d^2*g^5 - 4*a^7*b^2*c*d^3*g^5 + a^8*b*d^4*g^5 + (b^9*c^4*g^5 - 4*a
```

```
*b^8*c^3*d*g^5 + 6*a^2*b^7*c^2*d^2*g^5 - 4*a^3*b^6*c*d^3*g^5 + a^4*b^5*d^4*
g^5)*x^4 + 4*(a*b^8*c^4*g^5 - 4*a^2*b^7*c^3*d*g^5 + 6*a^3*b^6*c^2*d^2*g^5 -
 4*a^4*b^5*c*d^3*g^5 + a^5*b^4*d^4*g^5)*x^3 + 6*(a^2*b^7*c^4*g^5 - 4*a^3*b^
6*c^3*d*g^5 + 6*a^4*b^5*c^2*d^2*g^5 - 4*a^5*b^4*c*d^3*g^5 + a^6*b^3*d^4*g^5
)*x^2 + 4*(a^3*b^6*c^4*g^5 - 4*a^4*b^5*c^3*d*g^5 + 6*a^5*b^4*c^2*d^2*g^5 -
4*a^6*b^3*c*d^3*g^5 + a^7*b^2*d^4*g^5)*x))*B^2 - 1/24*A*B*((12*b^3*d^3*x^3
- 3*b^3*c^3 + 13*a*b^2*c^2*d - 23*a^2*b*c*d^2 + 25*a^3*d^3 - 6*(b^3*c*d^2 -
 7*a*b^2*d^3)*x^2 + 4*(b^3*c^2*d - 5*a*b^2*c*d^2 + 13*a^2*b*d^3)*x)/((b^8*c
^3 - 3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 - a^3*b^5*d^3)*g^5*x^4 + 4*(a*b^7*c^3
- 3*a^2*b^6*c^2*d + 3*a^3*b^5*c*d^2 - a^4*b^4*d^3)*g^5*x^3 + 6*(a^2*b^6*c^3
 - 3*a^3*b^5*c^2*d + 3*a^4*b^4*c*d^2 - a^5*b^3*d^3)*g^5*x^2 + 4*(a^3*b^5*c^
3 - 3*a^4*b^4*c^2*d + 3*a^5*b^3*c*d^2 - a^6*b^2*d^3)*g^5*x + (a^4*b^4*c^3 -
 3*a^5*b^3*c^2*d + 3*a^6*b^2*c*d^2 - a^7*b*d^3)*g^5) + 12*log(d*x*e/(b*x +
a) + c*e/(b*x + a))/(b^5*g^5*x^4 + 4*a*b^4*g^5*x^3 + 6*a^2*b^3*g^5*x^2 + 4*
a^3*b^2*g^5*x + a^4*b*g^5) + 12*d^4*log(b*x + a)/((b^5*c^4 - 4*a*b^4*c^3*d
+ 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5) - 12*d^4*log(d*x +
c)/((b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*
d^4)*g^5)) - 1/4*B^2*log(d*x*e/(b*x + a) + c*e/(b*x + a))^2/(b^5*g^5*x^4 +
4*a*b^4*g^5*x^3 + 6*a^2*b^3*g^5*x^2 + 4*a^3*b^2*g^5*x + a^4*b*g^5) - 1/4*A^
2/(b^5*g^5*x^4 + 4*a*b^4*g^5*x^3 + 6*a^2*b^3*g^5*x^2 + 4*a^3*b^2*g^5*x + a^
4*b*g^5)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1043 vs. $2(486) = 972$.
time = 0.44, size = 1043, normalized size = 2.09

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(d*x+c)/(b*x+a)))^2/(b*g*x+a*g)^5,x, algorithm="fricas
")
```

```
[Out] -1/288*(9*(8*A^2 - 4*A*B + B^2)*b^4*c^4 - 32*(9*A^2 - 6*A*B + 2*B^2)*a*b^3*
c^3*d + 216*(2*A^2 - 2*A*B + B^2)*a^2*b^2*c^2*d^2 - 288*(A^2 - 2*A*B + 2*B^
2)*a^3*b*c*d^3 + (72*A^2 - 300*A*B + 415*B^2)*a^4*d^4 + 12*((12*A*B - 25*B^
2)*b^4*c*d^3 - (12*A*B - 25*B^2)*a*b^3*d^4)*x^3 - 6*((12*A*B - 13*B^2)*b^4*
c^2*d^2 - 16*(6*A*B - 11*B^2)*a*b^3*c*d^3 + (84*A*B - 163*B^2)*a^2*b^2*d^4)
*x^2 - 72*(B^2*b^4*d^4*x^4 + 4*B^2*a*b^3*d^4*x^3 + 6*B^2*a^2*b^2*d^4*x^2 +
4*B^2*a^3*b*d^4*x - B^2*b^4*c^4 + 4*B^2*a*b^3*c^3*d - 6*B^2*a^2*b^2*c^2*d^2
 + 4*B^2*a^3*b*c*d^3)*log((d*x + c)*e/(b*x + a))^2 + 4*((12*A*B - 7*B^2)*b^
4*c^3*d - 12*(6*A*B - 5*B^2)*a*b^3*c^2*d^2 + 108*(2*A*B - 3*B^2)*a^2*b^2*c*
d^3 - (156*A*B - 271*B^2)*a^3*b*d^4)*x - 12*((12*A*B - 25*B^2)*b^4*d^4*x^4
- 3*(4*A*B - B^2)*b^4*c^4 + 16*(3*A*B - B^2)*a*b^3*c^3*d - 36*(2*A*B - B^2)
*a^2*b^2*c^2*d^2 + 48*(A*B - B^2)*a^3*b*c*d^3 - 4*(3*B^2*b^4*c*d^3 - 2*(6*A
*B - 11*B^2)*a*b^3*d^4)*x^3 + 6*(B^2*b^4*c^2*d^2 - 8*B^2*a*b^3*c*d^3 + 6*(2
```

```
*A*B - 3*B^2)*a^2*b^2*d^4)*x^2 - 4*(B^2*b^4*c^3*d - 6*B^2*a*b^3*c^2*d^2 + 1
8*B^2*a^2*b^2*c*d^3 - 12*(A*B - B^2)*a^3*b*d^4)*x)*log((d*x + c)*e/(b*x + a
)))/((b^9*c^4 - 4*a*b^8*c^3*d + 6*a^2*b^7*c^2*d^2 - 4*a^3*b^6*c*d^3 + a^4*b
^5*d^4)*g^5*x^4 + 4*(a*b^8*c^4 - 4*a^2*b^7*c^3*d + 6*a^3*b^6*c^2*d^2 - 4*a^
4*b^5*c*d^3 + a^5*b^4*d^4)*g^5*x^3 + 6*(a^2*b^7*c^4 - 4*a^3*b^6*c^3*d + 6*a
^4*b^5*c^2*d^2 - 4*a^5*b^4*c*d^3 + a^6*b^3*d^4)*g^5*x^2 + 4*(a^3*b^6*c^4 -
4*a^4*b^5*c^3*d + 6*a^5*b^4*c^2*d^2 - 4*a^6*b^3*c*d^3 + a^7*b^2*d^4)*g^5*x
+ (a^4*b^5*c^4 - 4*a^5*b^4*c^3*d + 6*a^6*b^3*c^2*d^2 - 4*a^7*b^2*c*d^3 + a^
8*b*d^4)*g^5)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(d*x+c)/(b*x+a)))**2/(b*g*x+a*g)**5,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1029 vs. 2(486) = 972.
time = 3.13, size = 1029, normalized size = 2.07

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(d*x+c)/(b*x+a)))^2/(b*g*x+a*g)^5,x, algorithm="giac")
```

```
[Out] 1/288*(288*(d*x*e + c*e)*B^2*d^3*e^3*log((d*x*e + c*e)/(b*x + a))^2/(b*x +
a) - 432*(d*x*e + c*e)^2*B^2*b*d^2*e^2*log((d*x*e + c*e)/(b*x + a))^2/(b*x
+ a)^2 + 288*(d*x*e + c*e)^3*B^2*b^2*d*e*log((d*x*e + c*e)/(b*x + a))^2/(b*
x + a)^3 + 576*(d*x*e + c*e)*A*B*d^3*e^3*log((d*x*e + c*e)/(b*x + a))/(b*x
+ a) - 576*(d*x*e + c*e)*B^2*d^3*e^3*log((d*x*e + c*e)/(b*x + a))/(b*x + a)
 - 864*(d*x*e + c*e)^2*A*B*b*d^2*e^2*log((d*x*e + c*e)/(b*x + a))/(b*x + a)
^2 + 432*(d*x*e + c*e)^2*B^2*b*d^2*e^2*log((d*x*e + c*e)/(b*x + a))/(b*x +
a)^2 + 576*(d*x*e + c*e)^3*A*B*b^2*d*e*log((d*x*e + c*e)/(b*x + a))/(b*x +
a)^3 - 192*(d*x*e + c*e)^3*B^2*b^2*d*e*log((d*x*e + c*e)/(b*x + a))/(b*x +
a)^3 - 72*(d*x*e + c*e)^4*B^2*b^3*log((d*x*e + c*e)/(b*x + a))^2/(b*x + a)^
4 + 288*(d*x*e + c*e)*A^2*d^3*e^3/(b*x + a) - 576*(d*x*e + c*e)*A*B*d^3*e^3
/(b*x + a) + 576*(d*x*e + c*e)*B^2*d^3*e^3/(b*x + a) - 432*(d*x*e + c*e)^2*
A^2*b*d^2*e^2/(b*x + a)^2 + 432*(d*x*e + c*e)^2*A*B*b*d^2*e^2/(b*x + a)^2 -
 216*(d*x*e + c*e)^2*B^2*b*d^2*e^2/(b*x + a)^2 + 288*(d*x*e + c*e)^3*A^2*b^
2*d*e/(b*x + a)^3 - 192*(d*x*e + c*e)^3*A*B*b^2*d*e/(b*x + a)^3 + 64*(d*x*e
 + c*e)^3*B^2*b^2*d*e/(b*x + a)^3 - 144*(d*x*e + c*e)^4*A*B*b^3*log((d*x*e
+ c*e)/(b*x + a))/(b*x + a)^4 + 36*(d*x*e + c*e)^4*B^2*b^3*log((d*x*e + c*e
```

```
)/(b*x + a))/(b*x + a)^4 - 72*(d*x*e + c*e)^4*A^2*b^3/(b*x + a)^4 + 36*(d*x
*e + c*e)^4*A*B*b^3/(b*x + a)^4 - 9*(d*x*e + c*e)^4*B^2*b^3/(b*x + a)^4)*(b
*c/((b*c*e - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))/(b^3*
c^3*g^5*e^3 - 3*a*b^2*c^2*d*g^5*e^3 + 3*a^2*b*c*d^2*g^5*e^3 - a^3*d^3*g^5*e
^3)
```

Mupad [B]
time = 10.94, size = 1880, normalized size = 3.78

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(c + d*x))/(a + b*x)))^2/(a*g + b*g*x)^5,x)
```

```
[Out] (log((e*(c + d*x))/(a + b*x))*((B^2*d^4*(a*(a*((4*a^2*d^2 + b^2*c^2 - 5*a*b
*c*d)/(12*b*d^3) + (a*(a*d - b*c))/(4*b*d^2)) + (6*a^3*d^3 - b^3*c^3 + 5*a*
b^2*c^2*d - 10*a^2*b*c*d^2)/(12*b*d^4)) + (4*a^4*d^4 + b^4*c^4 + 10*a^2*b^2
*c^2*d^2 - 5*a*b^3*c^3*d - 10*a^3*b*c*d^3)/(4*b*d^5)))/(2*b*g^5*(a^4*d^4 +
b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)) - (A*B)/(2*b^
2*d*g^5) + (B^2*d^4*x^2*(b*(b*((4*a^2*d^2 + b^2*c^2 - 5*a*b*c*d)/(12*b*d^3)
 + (a*(a*d - b*c))/(4*b*d^2)) + (4*a^2*d^2 + b^2*c^2 - 5*a*b*c*d)/(6*d^3) +
 (a*(a*d - b*c))/(2*d^2)) - a*((b^2*c - a*b*d)/(4*d^2) - (b*(a*d - b*c))/(2
*d^2)) + (b^3*c^2 + 4*a^2*b*d^2 - 5*a*b^2*c*d)/(4*d^3)))/(2*b*g^5*(a^4*d^4
+ b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)) - (B^2*d^4*
x^3*(b*((b^2*c - a*b*d)/(4*d^2) - (b*(a*d - b*c))/(2*d^2)) + (b^3*c - a*b^2
*d)/(4*d^2)))/(2*b*g^5*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3
*d - 4*a^3*b*c*d^3)) + (B^2*d^4*x*(b*(a*((4*a^2*d^2 + b^2*c^2 - 5*a*b*c*d)/
(12*b*d^3) + (a*(a*d - b*c))/(4*b*d^2)) + (6*a^3*d^3 - b^3*c^3 + 5*a*b^2*c^
2*d - 10*a^2*b*c*d^2)/(12*b*d^4)) + a*(b*((4*a^2*d^2 + b^2*c^2 - 5*a*b*c*d)
/(12*b*d^3) + (a*(a*d - b*c))/(4*b*d^2)) + (4*a^2*d^2 + b^2*c^2 - 5*a*b*c*d
)/(6*d^3) + (a*(a*d - b*c))/(2*d^2)) + (6*a^3*d^3 - b^3*c^3 + 5*a*b^2*c^2*d
 - 10*a^2*b*c*d^2)/(4*d^4)))/(2*b*g^5*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^
2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))))/((4*a^3*x)/d + a^4/(b*d) + (b^3*x^4)/
d + (6*a^2*b*x^2)/d + (4*a*b^2*x^3)/d) - log((e*(c + d*x))/(a + b*x))^2*(B^
2/(4*b^2*g^5*(4*a^3*x + a^4/b + b^3*x^4 + 6*a^2*b*x^2 + 4*a*b^2*x^3)) - (B^
2*d^4)/(4*b*g^5*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*
a^3*b*c*d^3))) - ((72*A^2*a^3*d^3 - 72*A^2*b^3*c^3 + 415*B^2*a^3*d^3 - 9*B^
2*b^3*c^3 - 300*A*B*a^3*d^3 + 36*A*B*b^3*c^3 + 216*A^2*a*b^2*c^2*d - 216*A^
2*a^2*b*c*d^2 + 55*B^2*a*b^2*c^2*d - 161*B^2*a^2*b*c*d^2 - 156*A*B*a*b^2*c^
2*d + 276*A*B*a^2*b*c*d^2)/(12*(a*d - b*c)) + (x^2*(163*B^2*a*b^2*d^3 - 13*
B^2*b^3*c*d^2 - 84*A*B*a*b^2*d^3 + 12*A*B*b^3*c*d^2))/(2*(a*d - b*c)) + (x*
(271*B^2*a^2*b*d^3 + 7*B^2*b^3*c^2*d - 53*B^2*a*b^2*c*d^2 - 156*A*B*a^2*b*d
^3 - 12*A*B*b^3*c^2*d + 60*A*B*a*b^2*c*d^2))/(3*(a*d - b*c)) + (d*x^3*(25*B
^2*b^3*d^2 - 12*A*B*b^3*d^2))/(a*d - b*c))/(x*(96*a^3*b^4*c^2*g^5 + 96*a^5*
b^2*d^2*g^5 - 192*a^4*b^3*c*d*g^5) + x^3*(96*a*b^6*c^2*g^5 + 96*a^3*b^4*d^2
```

```
*g^5 - 192*a^2*b^5*c*d*g^5) + x^4*(24*b^7*c^2*g^5 + 24*a^2*b^5*d^2*g^5 - 48
*a*b^6*c*d*g^5) + x^2*(144*a^2*b^5*c^2*g^5 + 144*a^4*b^3*d^2*g^5 - 288*a^3*
b^4*c*d*g^5) + 24*a^6*b*d^2*g^5 + 24*a^4*b^3*c^2*g^5 - 48*a^5*b^2*c*d*g^5)
+ (B*d^4*atan((B*d^4*(12*A - 25*B)*(24*b^5*c^4*g^5 - 24*a^4*b*d^4*g^5 - 48*
a*b^4*c^3*d*g^5 + 48*a^3*b^2*c*d^3*g^5)*1i)/(24*b*g^5*(a*d - b*c)^4*(25*B^2
*d^4 - 12*A*B*d^4)) + (B*d^5*x*(12*A - 25*B)*(b^4*c^3*g^5 - a^3*b*d^3*g^5 -
 3*a*b^3*c^2*d*g^5 + 3*a^2*b^2*c*d^2*g^5)*2i)/(g^5*(a*d - b*c)^4*(25*B^2*d^
4 - 12*A*B*d^4)))*(12*A - 25*B)*1i)/(12*b*g^5*(a*d - b*c)^4)
```

3.191 $\int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}\,dx$

Optimal. Leaf size=35

$$\mathrm{Int}\left(\frac{(ag+bgx)^2}{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)},x\right)$$

```
[Out] Unintegrable((b*g*x+a*g)^2/(A+B*ln(e*(d*x+c)/(b*x+a))),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a*g + b*g*x)^2/(A + B*Log[(e*(c + d*x))/(a + b*x)]),x]
[Out] Defer[Int][(a*g + b*g*x)^2/(A + B*Log[(e*(c + d*x))/(a + b*x)]), x]
```

Rubi steps

$$\int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}\,dx = \int \left(\frac{a^2g^2}{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}+\frac{2abg^2x}{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}+\frac{b^2g^2x^2}{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}\right)dx$$
$$= \left(a^2g^2\right)\int \frac{1}{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}\,dx+\left(2abg^2\right)\int \frac{x}{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}\,dx+\left(b^2g^2\right.$$

Mathematica [A]
time = 0.59, size = 0, normalized size = 0.00

$$\int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a*g + b*g*x)^2/(A + B*Log[(e*(c + d*x))/(a + b*x)]),x]
[Out] Integrate[(a*g + b*g*x)^2/(A + B*Log[(e*(c + d*x))/(a + b*x)]), x]
```

Maple [A]
time = 1.41, size = 0, normalized size = 0.00

$$\int \frac{(bgx+ag)^2}{A+B\ln\left(\frac{e(dx+c)}{bx+a}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*g*x+a*g)^2/(A+B*ln(e*(d*x+c)/(b*x+a))),x)
[Out] int((b*g*x+a*g)^2/(A+B*ln(e*(d*x+c)/(b*x+a))),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^2/(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="maxima")
[Out] integrate((b*g*x + a*g)^2/(B*log((d*x + c)*e/(b*x + a)) + A), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^2/(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="fricas")
[Out] integral((b^2*g^2*x^2 + 2*a*b*g^2*x + a^2*g^2)/(B*log((d*x + c)*e/(b*x + a)
) + A), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$g^2\left(\int \frac{a^2}{A+B\log\left(\frac{ce}{a+bx}+\frac{dex}{a+bx}\right)}\,dx+\int \frac{b^2x^2}{A+B\log\left(\frac{ce}{a+bx}+\frac{dex}{a+bx}\right)}\,dx+\int \frac{2abx}{A+B\log\left(\frac{ce}{a+bx}+\frac{dex}{a+bx}\right)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)**2/(A+B*ln(e*(d*x+c)/(b*x+a))),x)
[Out] g**2*(Integral(a**2/(A + B*log(c*e/(a + b*x) + d*e*x/(a + b*x))), x) + Inte
gral(b**2*x**2/(A + B*log(c*e/(a + b*x) + d*e*x/(a + b*x))), x) + Integral(
2*a*b*x/(A + B*log(c*e/(a + b*x) + d*e*x/(a + b*x))), x))
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^2/(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="giac")
[Out] integrate((b*g*x + a*g)^2/(B*log((d*x + c)*e/(b*x + a)) + A), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{(a\,g+b\,g\,x)^2}{A+B\,\ln\left(\frac{e\,(c+d\,x)}{a+b\,x}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*g + b*g*x)^2/(A + B*log((e*(c + d*x))/(a + b*x))),x)
[Out] int((a*g + b*g*x)^2/(A + B*log((e*(c + d*x))/(a + b*x))), x)
```

3.192 $\int \frac{ag+bgx}{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}\,dx$

Optimal. Leaf size=33

$$\text{Int}\left(\frac{ag+bgx}{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)},x\right)$$

`[Out] Unintegrable((b*g*x+a*g)/(A+B*ln(e*(d*x+c)/(b*x+a))),x)`

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{ag+bgx}{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}\,dx$$

Verification is not applicable to the result.

`[In] Int[(a*g + b*g*x)/(A + B*Log[(e*(c + d*x))/(a + b*x)]),x]`

`[Out] Defer[Int][(a*g + b*g*x)/(A + B*Log[(e*(c + d*x))/(a + b*x)]), x]`

Rubi steps

$$\begin{aligned}\int \frac{ag+bgx}{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}\,dx &= \int \left(\frac{ag}{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}+\frac{bgx}{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}\right)dx\\ &= (ag)\int \frac{1}{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}\,dx+(bg)\int \frac{x}{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}\,dx\end{aligned}$$

Mathematica [A]
time = 0.17, size = 0, normalized size = 0.00

$$\int \frac{ag+bgx}{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}\,dx$$

Verification is not applicable to the result.

`[In] Integrate[(a*g + b*g*x)/(A + B*Log[(e*(c + d*x))/(a + b*x)]),x]`

`[Out] Integrate[(a*g + b*g*x)/(A + B*Log[(e*(c + d*x))/(a + b*x)]), x]`

Maple [A]
time = 0.69, size = 0, normalized size = 0.00

$$\int \frac{bgx + ag}{A + B\ln\left(\frac{e(dx+c)}{bx+a}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*g*x+a*g)/(A+B*ln(e*(d*x+c)/(b*x+a))),x)
[Out] int((b*g*x+a*g)/(A+B*ln(e*(d*x+c)/(b*x+a))),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)/(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="maxima")
[Out] integrate((b*g*x + a*g)/(B*log((d*x + c)*e/(b*x + a)) + A), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)/(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="fricas")
[Out] integral((b*g*x + a*g)/(B*log((d*x + c)*e/(b*x + a)) + A), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$g\left(\int \frac{a}{A + B\log\left(\frac{ce}{a+bx} + \frac{dex}{a+bx}\right)}\,dx + \int \frac{bx}{A + B\log\left(\frac{ce}{a+bx} + \frac{dex}{a+bx}\right)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)/(A+B*ln(e*(d*x+c)/(b*x+a))),x)
[Out] g*(Integral(a/(A + B*log(c*e/(a + b*x) + d*e*x/(a + b*x))), x) + Integral(b
*x/(A + B*log(c*e/(a + b*x) + d*e*x/(a + b*x))), x))
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)/(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="giac")
[Out] integrate((b*g*x + a*g)/(B*log((d*x + c)*e/(b*x + a)) + A), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{a\,g+b\,g\,x}{A+B\,\ln\left(\frac{e\,\left(c+d\,x\right)}{a+b\,x}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*g + b*g*x)/(A + B*log((e*(c + d*x))/(a + b*x))),x)
[Out] int((a*g + b*g*x)/(A + B*log((e*(c + d*x))/(a + b*x))), x)
```

3.193 $\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}\,dx$

Optimal. Leaf size=35

$$\text{Int}\left(\frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)},x\right)$$

```
[Out] Unintegrable(1/(b*g*x+a*g)/(A+B*ln(e*(d*x+c)/(b*x+a))),x)
```

Rubi [A]

time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((a*g + b*g*x)*(A + B*Log[(e*(c + d*x))/(a + b*x)])),x]
[Out] Defer[Int][1/((a*g + b*g*x)*(A + B*Log[(e*(c + d*x))/(a + b*x)])), x]
```

Rubi steps

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}\,dx = \int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}\,dx$$

Mathematica [A]

time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((a*g + b*g*x)*(A + B*Log[(e*(c + d*x))/(a + b*x)])),x]
[Out] Integrate[1/((a*g + b*g*x)*(A + B*Log[(e*(c + d*x))/(a + b*x)])), x]
```

Maple [A]

time = 0.99, size = 0, normalized size = 0.00

$$\int \frac{1}{(bgx+ag)\left(A+B\ln\left(\frac{e(dx+c)}{bx+a}\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*g*x+a*g)/(A+B*ln(e*(d*x+c)/(b*x+a))),x)
```

```
[Out] int(1/(b*g*x+a*g)/(A+B*ln(e*(d*x+c)/(b*x+a))),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)/(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*g*x + a*g)*(B*log((d*x + c)*e/(b*x + a)) + A)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)/(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="fricas")
```

```
[Out] integral(1/(A*b*g*x + A*a*g + (B*b*g*x + B*a*g)*log((d*x + c)*e/(b*x + a)))
, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{Aa+Abx+Ba\log\left(\frac{ce}{a+bx}+\frac{dex}{a+bx}\right)+Bbx\log\left(\frac{ce}{a+bx}+\frac{dex}{a+bx}\right)}\,dx}{g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)/(A+B*ln(e*(d*x+c)/(b*x+a))),x)
```

```
[Out] Integral(1/(A*a + A*b*x + B*a*log(c*e/(a + b*x) + d*e*x/(a + b*x)) + B*b*x*
log(c*e/(a + b*x) + d*e*x/(a + b*x))), x)/g
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)/(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="giac")
```

```
[Out] integrate(1/((b*g*x + a*g)*(B*log((d*x + c)*e/(b*x + a)) + A)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{(a\,g+b\,g\,x)\,\left(A+B\,\ln\left(\frac{e\,(c+d\,x)}{a+b\,x}\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*g + b*g*x)*(A + B*log((e*(c + d*x))/(a + b*x)))),x)
[Out] int(1/((a*g + b*g*x)*(A + B*log((e*(c + d*x))/(a + b*x)))), x)
```

3.194 $\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}\,dx$

Optimal. Leaf size=53

$$-\frac{e^{-\frac{A}{B}}\operatorname{Ei}\left(\frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{B}\right)}{B(bc-ad)eg^2}$$

```
[Out] -Ei((A+B*ln(e*(d*x+c)/(b*x+a)))/B)/B/(-a*d+b*c)/e/exp(A/B)/g^2
```

Rubi [A]
time = 0.06, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {2552, 2336, 2209}

$$-\frac{e^{-\frac{A}{B}}\operatorname{Ei}\left(\frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{B}\right)}{Beg^2(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a*g + b*g*x)^2*(A + B*Log[(e*(c + d*x))/(a + b*x)])),x]
```

```
[Out] -(ExpIntegralEi[(A + B*Log[(e*(c + d*x))/(a + b*x)])/B]/(B*(b*c - a*d)*e*E^
(A/B)*g^2))
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2336

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[1/(n*c^(1
/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b,
 c, p}, x] && IntegerQ[1/n]
```

Rule 2552

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(
m + 1)*(g/d)^m, Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a
 + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[n
 + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[d*f
 - c*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Rubi steps

$$\int \frac{1}{(ag+bgx)^2 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}\,dx = \int \frac{1}{(ag+bgx)^2 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}\,dx$$

Mathematica [A]
time = 0.07, size = 50, normalized size = 0.94

$$\frac{e^{-\frac{A}{B}}\mathrm{Ei}\left(\frac{A}{B}+\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{B(-bc+ad)eg^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a*g + b*g*x)^2*(A + B*Log[(e*(c + d*x))/(a + b*x)])),x]
[Out] ExpIntegralEi[A/B + Log[(e*(c + d*x))/(a + b*x)]]/(B*(-(b*c) + a*d)*e*E^(A/
B)*g^2)
```

Maple [A]
time = 4.18, size = 69, normalized size = 1.30

method	result	size
derivativedivides	$-\frac{e^{-\frac{A}{B}}\,\mathrm{expIntegral}\left(1,-\ln\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)-\frac{A}{B}\right)}{e(ad-cb)g^2B}$	69
default	$-\frac{e^{-\frac{A}{B}}\,\mathrm{expIntegral}\left(1,-\ln\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)-\frac{A}{B}\right)}{e(ad-cb)g^2B}$	69
risch	$-\frac{e^{-\frac{A}{B}}\,\mathrm{expIntegral}\left(1,-\ln\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)-\frac{A}{B}\right)}{e(ad-cb)g^2B}$	69

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*g*x+a*g)^2/(A+B*ln(e*(d*x+c)/(b*x+a))),x,method=_RETURNVERBOSE)
[Out] -1/e/(a*d-b*c)/g^2/B*exp(-A/B)*Ei(1,-ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))-A/B)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="maxima
")
```

```
[Out] integrate(1/((b*g*x + a*g)^2*(B*log((d*x + c)*e/(b*x + a)) + A)), x)
```

Fricas [A]
time = 0.37, size = 48, normalized size = 0.91

$$-\frac{e^{\left(-\frac{A}{B}-1\right)}\operatorname{log\_integral}\left(\frac{(dx+c)e^{\left(\frac{A}{B}+1\right)}}{bx+a}\right)}{(Bbc-Bad)g^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="fricas
")
[Out] -e^(-A/B - 1)*log_integral((d*x + c)*e^(A/B + 1)/(b*x + a))/((B*b*c - B*a*d
)*g^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{1}{Aa^2+2Aabx+Ab^2x^2+Ba^2\log\left(\frac{ce}{a+bx}+\frac{dex}{a+bx}\right)+2Babx\log\left(\frac{ce}{a+bx}+\frac{dex}{a+bx}\right)+Bb^2x^2\log\left(\frac{ce}{a+bx}+\frac{dex}{a+bx}\right)}\,dx}{g^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)**2/(A+B*ln(e*(d*x+c)/(b*x+a))),x)
[Out] Integral(1/(A*a**2 + 2*A*a*b*x + A*b**2*x**2 + B*a**2*log(c*e/(a + b*x) + d
*e*x/(a + b*x)) + 2*B*a*b*x*log(c*e/(a + b*x) + d*e*x/(a + b*x)) + B*b**2*x
**2*log(c*e/(a + b*x) + d*e*x/(a + b*x))), x)/g**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="giac")
[Out] integrate(1/((b*g*x + a*g)^2*(B*log((d*x + c)*e/(b*x + a)) + A)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{1}{{\left(a\,g+b\,g\,x\right)}^2\,\left(A+B\,\ln\left(\frac{e\,\left(c+d\,x\right)}{a+b\,x}\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*g + b*g*x)^2*(A + B*log((e*(c + d*x))/(a + b*x)))),x)
[Out] int(1/((a*g + b*g*x)^2*(A + B*log((e*(c + d*x))/(a + b*x)))), x)
```

3.195 $\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}\,dx$

Optimal. Leaf size=109

$$\frac{de^{-\frac{A}{B}}\text{Ei}\left(\frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{B}\right)}{B(bc-ad)^2eg^3}-\frac{be^{-\frac{2A}{B}}\text{Ei}\left(\frac{2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{B}\right)}{B(bc-ad)^2e^2g^3}$$

```
[Out] d*Ei((A+B*ln(e*(d*x+c)/(b*x+a)))/B)/B/(-a*d+b*c)^2/e/exp(A/B)/g^3-b*Ei(2*(A
+B*ln(e*(d*x+c)/(b*x+a)))/B)/B/(-a*d+b*c)^2/e^2/exp(2*A/B)/g^3
```

Rubi [A]
time = 0.13, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {2552, 2367, 2336, 2209, 2346}

$$\frac{de^{-\frac{A}{B}}\text{Ei}\left(\frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{B}\right)}{Beg^3(bc-ad)^2}-\frac{be^{-\frac{2A}{B}}\text{Ei}\left(\frac{2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{B}\right)}{Be^2g^3(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a*g + b*g*x)^3*(A + B*Log[(e*(c + d*x))/(a + b*x)])),x]
```

```
[Out] (d*ExpIntegralEi[(A + B*Log[(e*(c + d*x))/(a + b*x)])/B])/(B*(b*c - a*d)^2*
e*E^(A/B)*g^3) - (b*ExpIntegralEi[(2*(A + B*Log[(e*(c + d*x))/(a + b*x)]))/
B])/(B*(b*c - a*d)^2*e^2*E^((2*A)/B)*g^3)
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2336

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[1/(n*c^(1
/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b,
 c, p}, x] && IntegerQ[1/n]
```

Rule 2346

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> Dist[1/c^
(m + 1), Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x] /; FreeQ
[{a, b, c, p}, x] && IntegerQ[m]
```

Rule 2367

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^(r_.))^(
q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*x^n])^p, (d + e*x
^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
&& IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0] && IntegerQ[r]))
```

Rule 2552

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(
m + 1)*(g/d)^m, Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a
 + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[n
 + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[d*f
- c*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Rubi steps

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}\,dx=\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}\,dx$$

Mathematica [A]
time = 0.15, size = 89, normalized size = 0.82

$$\frac{e^{-\frac{2A}{B}}\left(dee^{A/B}\text{Ei}\left(\frac{A}{B}+\log\left(\frac{e(c+dx)}{a+bx}\right)\right)-b\text{Ei}\left(\frac{2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{B}\right)\right)}{B(bc-ad)^2e^2g^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a*g + b*g*x)^3*(A + B*Log[(e*(c + d*x))/(a + b*x)])),x]
```

```
[Out] (d*e*E^(A/B)*ExpIntegralEi[A/B + Log[(e*(c + d*x))/(a + b*x)]] - b*ExpInteg
ralEi[(2*(A + B*Log[(e*(c + d*x))/(a + b*x)]))/B])/(B*(b*c - a*d)^2*e^2*E^(
(2*A)/B)*g^3)
```

Maple [A]
time = 6.17, size = 126, normalized size = 1.16

method	result	size
derivativedivides	$-\frac{-\frac{b\,e^{-\frac{2A}{B}}\operatorname{expIntegral}\left(1,-2\ln\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)-\frac{2A}{B}\right)}{B}+\frac{ed\,e^{-\frac{A}{B}}\operatorname{expIntegral}\left(1,-\ln\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)-\frac{A}{B}\right)}{B}}{e^{2}(ad-cb)^{2}g^{3}}$	126

default	$-\frac{-\frac{b\,\mathrm{e}^{-\frac{2A}{B}}\,\mathrm{expIntegral}\left(1,-2\ln\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)-\frac{2A}{B}\right)}{B}+\frac{ed\,\mathrm{e}^{-\frac{A}{B}}\,\mathrm{expIntegral}\left(1,-\ln\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)-\frac{A}{B}\right)}{B}}{e^{2}(ad-cb)^{2}g^{3}}$	126
risch	$\frac{b\,\mathrm{e}^{-\frac{2A}{B}}\,\mathrm{expIntegral}\left(1,-2\ln\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)-\frac{2A}{B}\right)}{g^{3}(ad-cb)^{2}e^{2}B}-\frac{d\,\mathrm{e}^{-\frac{A}{B}}\,\mathrm{expIntegral}\left(1,-\ln\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)-\frac{A}{B}\right)}{g^{3}(ad-cb)^{2}eB}$	139

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*g*x+a*g)^3/(A+B*ln(e*(d*x+c)/(b*x+a))),x,method=_RETURNVERBOSE)
```

```
[Out] -1/e^2/(a*d-b*c)^2/g^3*(-b/B*exp(-2*A/B)*Ei(1,-2*ln(d*e/b-e*(a*d-b*c)/b/(b*
x+a))-2*A/B)+e*d/B*exp(-A/B)*Ei(1,-ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))-A/B))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)^3/(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="maxima
")
```

```
[Out] integrate(1/((b*g*x + a*g)^3*(B*log((d*x + c)*e/(b*x + a)) + A)), x)
```

Fricas [A]
time = 0.40, size = 120, normalized size = 1.10

$$\frac{\left(de^{\left(\frac{A}{B}+1\right)}\,\mathrm{log\_integral}\left(\frac{(dx+c)e^{\left(\frac{A}{B}+1\right)}}{bx+a}\right)-b\,\mathrm{log\_integral}\left(\frac{(d^{2}x^{2}+2\,cdx+c^{2})e^{\left(\frac{2\,A}{B}+2\right)}}{b^{2}x^{2}+2\,abx+a^{2}}\right)\right)e^{\left(-\frac{2\,A}{B}-2\right)}}{(Bb^{2}c^{2}-2\,Babcd+Ba^{2}d^{2})g^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)^3/(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="fricas
")
```

```
[Out] (d*e^(A/B + 1)*log_integral((d*x + c)*e^(A/B + 1)/(b*x + a)) - b*log_integr
al((d^2*x^2 + 2*c*d*x + c^2)*e^(2*A/B + 2)/(b^2*x^2 + 2*a*b*x + a^2)))*e^(-
2*A/B - 2)/((B*b^2*c^2 - 2*B*a*b*c*d + B*a^2*d^2)*g^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{Aa^{3}+3Aa^{2}bx+3Aab^{2}x^{2}+Ab^{3}x^{3}+Ba^{3}\log\left(\frac{ce}{a+bx}+\frac{dex}{a+bx}\right)+3Ba^{2}bx\log\left(\frac{ce}{a+bx}+\frac{dex}{a+bx}\right)+3Bab^{2}x^{2}\log\left(\frac{ce}{a+bx}+\frac{dex}{a+bx}\right)+Bb^{3}x^{3}\log\left(\frac{ce}{a+bx}+\frac{dex}{a+bx}\right)}\,dx}{g^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)**3/(A+B*ln(e*(d*x+c)/(b*x+a))),x)
```

```
[Out] Integral(1/(A*a**3 + 3*A*a**2*b*x + 3*A*a*b**2*x**2 + A*b**3*x**3 + B*a**3*
log(c*e/(a + b*x) + d*e*x/(a + b*x)) + 3*B*a**2*b*x*log(c*e/(a + b*x) + d*e
*x/(a + b*x)) + 3*B*a*b**2*x**2*log(c*e/(a + b*x) + d*e*x/(a + b*x)) + B*b*
*3*x**3*log(c*e/(a + b*x) + d*e*x/(a + b*x))), x)/g**3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)^3/(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="giac")
```

```
[Out] integrate(1/((b*g*x + a*g)^3*(B*log((d*x + c)*e/(b*x + a)) + A)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a\,g+b\,g\,x\right)}^3\,\left(A+B\,\ln\left(\frac{e\,\left(c+d\,x\right)}{a+b\,x}\right)\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*g + b*g*x)^3*(A + B*log((e*(c + d*x))/(a + b*x)))),x)
```

```
[Out] int(1/((a*g + b*g*x)^3*(A + B*log((e*(c + d*x))/(a + b*x)))), x)
```

3.196 $\int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx$

Optimal. Leaf size=35

$$\text{Int}\left(\frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2},x\right)$$

```
[Out] Unintegrable((b*g*x+a*g)^2/(A+B*ln(e*(d*x+c)/(b*x+a)))^2,x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a*g + b*g*x)^2/(A + B*Log[(e*(c + d*x))/(a + b*x)])^2,x]
```

```
[Out] Defer[Int][(a*g + b*g*x)^2/(A + B*Log[(e*(c + d*x))/(a + b*x)])^2, x]
```

Rubi steps

$$\int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx = \int \left(\frac{a^2g^2}{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}+\frac{2abg^2x}{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}+\frac{b^2g^2x^2}{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\right)dx$$
$$= \left(a^2g^2\right)\int \frac{1}{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx + \left(2abg^2\right)\int \frac{x}{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}$$

Mathematica [A]
time = 1.13, size = 0, normalized size = 0.00

$$\int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a*g + b*g*x)^2/(A + B*Log[(e*(c + d*x))/(a + b*x)])^2,x]
```

```
[Out] Integrate[(a*g + b*g*x)^2/(A + B*Log[(e*(c + d*x))/(a + b*x)])^2, x]
```

Maple [A]
time = 1.40, size = 0, normalized size = 0.00

$$\int \frac{(bgx+ag)^2}{\left(A+B\ln\left(\frac{e(dx+c)}{bx+a}\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*g*x+a*g)^2/(A+B*ln(e*(d*x+c)/(b*x+a)))^2,x)
```

```
[Out] int((b*g*x+a*g)^2/(A+B*ln(e*(d*x+c)/(b*x+a)))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^2/(A+B*log(e*(d*x+c)/(b*x+a)))^2,x, algorithm="maxima
")
```

```
[Out] -(b^3*d*g^2*x^4 + a^3*c*g^2 + (b^3*c*g^2 + 3*a*b^2*d*g^2)*x^3 + 3*(a*b^2*c*
g^2 + a^2*b*d*g^2)*x^2 + (3*a^2*b*c*g^2 + a^3*d*g^2)*x)/((b*c - a*d)*B^2*lo
g(b*x + a) - (b*c - a*d)*B^2*log(d*x + c) - (b*c - a*d)*A*B - (b*c - a*d)*B
^2) + integrate((4*b^3*d*g^2*x^3 + 3*a^2*b*c*g^2 + a^3*d*g^2 + 3*(b^3*c*g^2
 + 3*a*b^2*d*g^2)*x^2 + 6*(a*b^2*c*g^2 + a^2*b*d*g^2)*x)/((b*c - a*d)*B^2*l
og(b*x + a) - (b*c - a*d)*B^2*log(d*x + c) - (b*c - a*d)*A*B - (b*c - a*d)*
B^2), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^2/(A+B*log(e*(d*x+c)/(b*x+a)))^2,x, algorithm="fricas
")
```

```
[Out] integral((b^2*g^2*x^2 + 2*a*b*g^2*x + a^2*g^2)/(B^2*log((d*x + c)*e/(b*x +
a))^2 + 2*A*B*log((d*x + c)*e/(b*x + a)) + A^2), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)**2/(A+B*ln(e*(d*x+c)/(b*x+a)))**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^2/(A+B*log(e*(d*x+c)/(b*x+a)))^2,x, algorithm="giac")
```

```
[Out] integrate((b*g*x + a*g)^2/(B*log((d*x + c)*e/(b*x + a)) + A)^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{(a\,g+b\,g\,x)^2}{\left(A+B\,\ln\left(\frac{e\,(c+d\,x)}{a+b\,x}\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*g + b*g*x)^2/(A + B*log((e*(c + d*x))/(a + b*x)))^2,x)
```

```
[Out] int((a*g + b*g*x)^2/(A + B*log((e*(c + d*x))/(a + b*x)))^2, x)
```

3.197 $\int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx$

Optimal. Leaf size=33

$$\text{Int}\left(\frac{ag+bgx}{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2},x\right)$$

```
[Out] Unintegrable((b*g*x+a*g)/(A+B*ln(e*(d*x+c)/(b*x+a)))^2,x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a*g + b*g*x)/(A + B*Log[(e*(c + d*x))/(a + b*x)])^2,x]
```

```
[Out] Defer[Int][(a*g + b*g*x)/(A + B*Log[(e*(c + d*x))/(a + b*x)])^2, x]
```

Rubi steps

$$\begin{aligned}\int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx &= \int \left(\frac{ag}{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}+\frac{bgx}{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\right)dx\\ &= (ag)\int \frac{1}{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx+(bg)\int \frac{x}{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx\end{aligned}$$

Mathematica [A]
time = 0.67, size = 0, normalized size = 0.00

$$\int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a*g + b*g*x)/(A + B*Log[(e*(c + d*x))/(a + b*x)])^2,x]
```

```
[Out] Integrate[(a*g + b*g*x)/(A + B*Log[(e*(c + d*x))/(a + b*x)])^2, x]
```

Maple [A]
time = 1.09, size = 0, normalized size = 0.00

$$\int \frac{bgx+ag}{\left(A+B\ln\left(\frac{e(dx+c)}{bx+a}\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*g*x+a*g)/(A+B*ln(e*(d*x+c)/(b*x+a)))^2,x)
```

```
[Out] int((b*g*x+a*g)/(A+B*ln(e*(d*x+c)/(b*x+a)))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)/(A+B*log(e*(d*x+c)/(b*x+a)))^2,x, algorithm="maxima")
```

```
[Out] -(b^2*d*g*x^3 + a^2*c*g + (b^2*c*g + 2*a*b*d*g)*x^2 + (2*a*b*c*g + a^2*d*g)
*x)/((b*c - a*d)*B^2*log(b*x + a) - (b*c - a*d)*B^2*log(d*x + c) - (b*c - a
*d)*A*B - (b*c - a*d)*B^2) + integrate((3*b^2*d*g*x^2 + 2*a*b*c*g + a^2*d*g
 + 2*(b^2*c*g + 2*a*b*d*g)*x)/((b*c - a*d)*B^2*log(b*x + a) - (b*c - a*d)*B
^2*log(d*x + c) - (b*c - a*d)*A*B - (b*c - a*d)*B^2), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)/(A+B*log(e*(d*x+c)/(b*x+a)))^2,x, algorithm="fricas")
```

```
[Out] integral((b*g*x + a*g)/(B^2*log((d*x + c)*e/(b*x + a))^2 + 2*A*B*log((d*x +
 c)*e/(b*x + a)) + A^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{-a^{2}cg-a^{2}dgx-2abcgx-2abdgx^{2}-b^{2}cgx^{2}-b^{2}dgx^{3}}{ABad-ABbc+\left(B^{2}ad-B^{2}bc\right)\log\left(\frac{e(c+dx)}{a+bx}\right)}+\frac{g\left(\int\frac{a^{2}d}{A+B\log\left(\frac{ce}{a+bx}+\frac{dex}{a+bx}\right)}\,dx+\int\frac{2abc}{A+B\log\left(\frac{ce}{a+bx}+\frac{dex}{a+bx}\right)}\,dx+\int\frac{2b^{2}cx}{A+B\log\left(\frac{ce}{a+bx}+\frac{dex}{a+bx}\right)}\,dx+\int\frac{3b^{2}dx^{2}}{A+B\log\left(\frac{ce}{a+bx}+\frac{dex}{a+bx}\right)}\,dx+\int\frac{4abdx}{A+B\log\left(\frac{ce}{a+bx}+\frac{dex}{a+bx}\right)}\,dx\right)}{B\left(ad-bc\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)/(A+B*ln(e*(d*x+c)/(b*x+a)))**2,x)
```

```
[Out] (-a**2*c*g - a**2*d*g*x - 2*a*b*c*g*x - 2*a*b*d*g*x**2 - b**2*c*g*x**2 - b*
*2*d*g*x**3)/(A*B*a*d - A*B*b*c + (B**2*a*d - B**2*b*c)*log(e*(c + d*x)/(a
+ b*x))) + g*(Integral(a**2*d/(A + B*log(c*e/(a + b*x) + d*e*x/(a + b*x))),
 x) + Integral(2*a*b*c/(A + B*log(c*e/(a + b*x) + d*e*x/(a + b*x))), x) + I
ntegral(2*b**2*c*x/(A + B*log(c*e/(a + b*x) + d*e*x/(a + b*x))), x) + Integ
ral(3*b**2*d*x**2/(A + B*log(c*e/(a + b*x) + d*e*x/(a + b*x))), x) + Integr
al(4*a*b*d*x/(A + B*log(c*e/(a + b*x) + d*e*x/(a + b*x))), x))/(B*(a*d - b*
c))
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)/(A+B*log(e*(d*x+c)/(b*x+a)))^2,x, algorithm="giac")
```

```
[Out] integrate((b*g*x + a*g)/(B*log((d*x + c)*e/(b*x + a)) + A)^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{a\,g+b\,g\,x}{{\left(A+B\,\ln\left(\frac{e\,\left(c+d\,x\right)}{a+b\,x}\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*g + b*g*x)/(A + B*log((e*(c + d*x))/(a + b*x)))^2,x)
```

```
[Out] int((a*g + b*g*x)/(A + B*log((e*(c + d*x))/(a + b*x)))^2, x)
```

3.198 $\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx$

Optimal. Leaf size=35

$$\text{Int}\left(\frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2},x\right)$$

[Out] Unintegrable(1/(b*g*x+a*g)/(A+B*ln(e*(d*x+c)/(b*x+a)))^2,x)

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx$$

Verification is not applicable to the result.

[In] Int[1/((a*g + b*g*x)*(A + B*Log[(e*(c + d*x))/(a + b*x)])^2),x]

[Out] Defer[Int][1/((a*g + b*g*x)*(A + B*Log[(e*(c + d*x))/(a + b*x)])^2), x]

Rubi steps

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx = \int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx$$

Mathematica [A]
time = 0.40, size = 0, normalized size = 0.00

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx$$

Verification is not applicable to the result.

[In] Integrate[1/((a*g + b*g*x)*(A + B*Log[(e*(c + d*x))/(a + b*x)])^2),x]

[Out] Integrate[1/((a*g + b*g*x)*(A + B*Log[(e*(c + d*x))/(a + b*x)])^2), x]

Maple [A]
time = 0.61, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bgx+ag\right)\left(A+B\ln\left(\frac{e(dx+c)}{bx+a}\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*g*x+a*g)/(A+B*ln(e*(d*x+c)/(b*x+a)))^2,x)
```

```
[Out] int(1/(b*g*x+a*g)/(A+B*ln(e*(d*x+c)/(b*x+a)))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)/(A+B*log(e*(d*x+c)/(b*x+a)))^2,x, algorithm="maxima
")
```

```
[Out] d*integrate(1/((b*c*g - a*d*g)*B^2*log(b*x + a) - (b*c*g - a*d*g)*B^2*log(d
*x + c) - (b*c*g - a*d*g)*A*B - (b*c*g - a*d*g)*B^2), x) - (d*x + c)/((b*c*
g - a*d*g)*B^2*log(b*x + a) - (b*c*g - a*d*g)*B^2*log(d*x + c) - (b*c*g - a
*d*g)*A*B - (b*c*g - a*d*g)*B^2)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)/(A+B*log(e*(d*x+c)/(b*x+a)))^2,x, algorithm="fricas
")
```

```
[Out] integral(1/(A^2*b*g*x + A^2*a*g + (B^2*b*g*x + B^2*a*g)*log((d*x + c)*e/(b*
x + a))^2 + 2*(A*B*b*g*x + A*B*a*g)*log((d*x + c)*e/(b*x + a))), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{-c-dx}{ABadg-ABbcg+\left(B^{2}adg-B^{2}bcg\right)\log\left(\frac{e(c+dx)}{a+bx}\right)}+\frac{d\int\frac{1}{A+B\log\left(\frac{ce}{a+bx}+\frac{dex}{a+bx}\right)}\,dx}{Bg\left(ad-bc\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)/(A+B*ln(e*(d*x+c)/(b*x+a)))**2,x)
```

```
[Out] (-c - d*x)/(A*B*a*d*g - A*B*b*c*g + (B**2*a*d*g - B**2*b*c*g)*log(e*(c + d*
x)/(a + b*x))) + d*Integral(1/(A + B*log(c*e/(a + b*x) + d*e*x/(a + b*x))),
 x)/(B*g*(a*d - b*c))
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)/(A+B*log(e*(d*x+c)/(b*x+a)))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((b*g*x + a*g)*(B*log((d*x + c)*e/(b*x + a)) + A)^2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{(a\,g+b\,g\,x)\,{\left(A+B\,\ln\left(\frac{e\,(c+d\,x)}{a+b\,x}\right)\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*g + b*g*x)*(A + B*log((e*(c + d*x))/(a + b*x)))^2),x)
```

```
[Out] int(1/((a*g + b*g*x)*(A + B*log((e*(c + d*x))/(a + b*x)))^2), x)
```

3.199 $\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx$

Optimal. Leaf size=104

$$-\frac{e^{-\frac{A}{B}}\operatorname{Ei}\left(\frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{B}\right)}{B^2(bc-ad)eg^2}+\frac{c+dx}{B(bc-ad)g^2(a+bx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}$$

```
[Out] -Ei((A+B*ln(e*(d*x+c)/(b*x+a)))/B)/B^2/(-a*d+b*c)/e/exp(A/B)/g^2+(d*x+c)/B/
(-a*d+b*c)/g^2/(b*x+a)/(A+B*ln(e*(d*x+c)/(b*x+a)))
```

Rubi [A]
time = 0.07, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2552, 2334, 2336, 2209}

$$\frac{c+dx}{Bg^2(a+bx)(bc-ad)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}-\frac{e^{-\frac{A}{B}}\operatorname{Ei}\left(\frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{B}\right)}{B^2eg^2(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a*g + b*g*x)^2*(A + B*Log[(e*(c + d*x))/(a + b*x)])^2),x]
```

```
[Out] -(ExpIntegralEi[(A + B*Log[(e*(c + d*x))/(a + b*x)])/B]/(B^2*(b*c - a*d)*e*
E^(A/B)*g^2)) + (c + d*x)/(B*(b*c - a*d)*g^2*(a + b*x)*(A + B*Log[(e*(c + d
*x))/(a + b*x)]))
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2334

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x*((a + b
*Log[c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Dist[1/(b*n*(p + 1)), Int[(a + b*
Log[c*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, n}, x] && LtQ[p, -1] && Inte
gerQ[2*p]
```

Rule 2336

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[1/(n*c^(1
/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b,
```

```
c, p}, x] && IntegerQ[1/n]
```

Rule 2552

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(
m + 1)*(g/d)^m, Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a
 + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[n
 + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[d*f
- c*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Rubi steps

$$\int \frac{1}{(ag+bgx)^2 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx = \int \frac{1}{(ag+bgx)^2 \left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx$$

Mathematica [A]
time = 0.11, size = 88, normalized size = 0.85

$$\frac{\frac{e^{-\frac{A}{B}}\operatorname{Ei}\left(\frac{A}{B}+\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{e}-\frac{B(c+dx)}{(a+bx)\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}}{B^2(-bc+ad)g^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a*g + b*g*x)^2*(A + B*Log[(e*(c + d*x))/(a + b*x)])^2),x]
```

```
[Out] (ExpIntegralEi[A/B + Log[(e*(c + d*x))/(a + b*x)]]/(e*E^(A/B)) - (B*(c + d*
x))/((a + b*x)*(A + B*Log[(e*(c + d*x))/(a + b*x)])))/(B^2*(-(b*c) + a*d)*g
^2)
```

Maple [A]
time = 2.72, size = 138, normalized size = 1.33

method	result	size
risch	$-\frac{dx+c}{(ad-cb)B(bx+a)g^2\left(A+B\ln\left(\frac{e(dx+c)}{bx+a}\right)\right)}-\frac{\mathrm{e}^{-\frac{A}{B}}\operatorname{expIntegral}\left(1,-\ln\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)-\frac{A}{B}\right)}{g^2B^2e(ad-cb)}$	121
derivativedivides	$\frac{-\frac{\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}}{\ln\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)+\frac{A}{B}}-\mathrm{e}^{-\frac{A}{B}}\operatorname{expIntegral}\left(1,-\ln\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)-\frac{A}{B}\right)}{e(ad-cb)g^2B^2}$	138

default	$\dfrac{-\frac{\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}}{\ln\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)+\frac{A}{B}}-\mathrm{e}^{-\frac{A}{B}}\,\mathrm{expIntegral}\left(1,-\ln\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)-\frac{A}{B}\right)}{e(ad-cb)\,g^{2}B^{2}}$	138

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*g*x+a*g)^2/(A+B*ln(e*(d*x+c)/(b*x+a)))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/e/(a*d-b*c)/g^2/B^2*(-(d*e/b-e*(a*d-b*c)/b/(b*x+a))/(ln(d*e/b-e*(a*d-b*c)
/b/(b*x+a))+A/B)-exp(-A/B)*Ei(1,-ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))-A/B))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(d*x+c)/(b*x+a)))^2,x, algorithm="maxi
ma")
```

```
[Out] (d*x + c)/((a*b*c*g^2 - a^2*d*g^2)*A*B + (a*b*c*g^2 - a^2*d*g^2)*B^2 + ((b^
2*c*g^2 - a*b*d*g^2)*A*B + (b^2*c*g^2 - a*b*d*g^2)*B^2)*x - ((b^2*c*g^2 - a
*b*d*g^2)*B^2*x + (a*b*c*g^2 - a^2*d*g^2)*B^2)*log(b*x + a) + ((b^2*c*g^2 -
 a*b*d*g^2)*B^2*x + (a*b*c*g^2 - a^2*d*g^2)*B^2)*log(d*x + c)) + integrate(
1/(A*B*a^2*g^2 + B^2*a^2*g^2 + (A*B*b^2*g^2 + B^2*b^2*g^2)*x^2 + 2*(A*B*a*b
*g^2 + B^2*a*b*g^2)*x - (B^2*b^2*g^2*x^2 + 2*B^2*a*b*g^2*x + B^2*a^2*g^2)*l
og(b*x + a) + (B^2*b^2*g^2*x^2 + 2*B^2*a*b*g^2*x + B^2*a^2*g^2)*log(d*x + c
)), x)
```

Fricas [A]
time = 0.35, size = 205, normalized size = 1.97

$$\frac{(Bdx+Bc)e^{\left(\frac{A}{B}+1\right)}-\left(Abx+Aa+(Bbx+Ba)\log\left(\frac{(dx+c)e}{bx+a}\right)\right)\mathrm{log\_integral}\left(\frac{(dx+c)e^{\left(\frac{A}{B}+1\right)}}{bx+a}\right)}{((B^{3}b^{2}c-B^{3}abd)g^{2}x+(B^{3}abc-B^{3}a^{2}d)g^{2})e^{\left(\frac{A}{B}+1\right)}\log\left(\frac{(dx+c)e}{bx+a}\right)+((AB^{2}b^{2}c-AB^{2}abd)g^{2}x+(AB^{2}abc-AB^{2}a^{2}d)g^{2})e^{\left(\frac{A}{B}+1\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(d*x+c)/(b*x+a)))^2,x, algorithm="fric
as")
```

```
[Out] ((B*d*x + B*c)*e^(A/B + 1) - (A*b*x + A*a + (B*b*x + B*a)*log((d*x + c)*e/(
b*x + a)))*log_integral((d*x + c)*e^(A/B + 1)/(b*x + a)))/(((B^3*b^2*c - B^
3*a*b*d)*g^2*x + (B^3*a*b*c - B^3*a^2*d)*g^2)*e^(A/B + 1)*log((d*x + c)*e/(
b*x + a)) + ((A*B^2*b^2*c - A*B^2*a*b*d)*g^2*x + (A*B^2*a*b*c - A*B^2*a^2*d
)*g^2)*e^(A/B + 1))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{-c-dx}{ABa^2dg^2-ABabcg^2+ABabdg^2x-ABb^2cg^2x+\left(B^2a^2dg^2-B^2abcg^2+B^2abdg^2x-B^2b^2cg^2x\right)\log\left(\frac{e(c+dx)}{a+bx}\right)}+\frac{\int\frac{1}{Aa^2+2Aabx+Ab^2x^2+Ba^2\log\left(\frac{ce}{a+bx}+\frac{dex}{a+bx}\right)+2Babx\log\left(\frac{ce}{a+bx}+\frac{dex}{a+bx}\right)+Bb^2x^2\log\left(\frac{ce}{a+bx}+\frac{dex}{a+bx}\right)}\,dx}{Bg^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)**2/(A+B*ln(e*(d*x+c)/(b*x+a)))**2,x)
```

```
[Out] (-c - d*x)/(A*B*a**2*d*g**2 - A*B*a*b*c*g**2 + A*B*a*b*d*g**2*x - A*B*b**2*
c*g**2*x + (B**2*a**2*d*g**2 - B**2*a*b*c*g**2 + B**2*a*b*d*g**2*x - B**2*b
**2*c*g**2*x)*log(e*(c + d*x)/(a + b*x))) + Integral(1/(A*a**2 + 2*A*a*b*x
+ A*b**2*x**2 + B*a**2*log(c*e/(a + b*x) + d*e*x/(a + b*x)) + 2*B*a*b*x*log
(c*e/(a + b*x) + d*e*x/(a + b*x)) + B*b**2*x**2*log(c*e/(a + b*x) + d*e*x/(
a + b*x))), x)/(B*g**2)
```

Giac [A]
time = 4.46, size = 152, normalized size = 1.46

$$\left(\frac{bc}{(bce-ade)(bc-ad)}-\frac{ad}{(bce-ade)(bc-ad)}\right)\left(\frac{dxe+ce}{\left(B^2g^2\log\left(\frac{dxe+ce}{bx+a}\right)+ABg^2\right)(bx+a)}-\frac{\mathrm{Ei}\left(\frac{A}{B}+\log\left(\frac{dxe+ce}{bx+a}\right)\right)e^{\left(-\frac{A}{B}\right)}}{B^2g^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(d*x+c)/(b*x+a)))^2,x, algorithm="giac
")
```

```
[Out] (b*c/((b*c*e - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))*((d
*x*e + c*e)/((B^2*g^2*log((d*x*e + c*e)/(b*x + a)) + A*B*g^2)*(b*x + a)) -
Ei(A/B + log((d*x*e + c*e)/(b*x + a)))*e^(-A/B)/(B^2*g^2))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{(a\,g+b\,g\,x)}^2\,{\left(A+B\,\ln\left(\frac{e\,(c+d\,x)}{a+b\,x}\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*g + b*g*x)^2*(A + B*log((e*(c + d*x))/(a + b*x)))^2),x)
```

```
[Out] int(1/((a*g + b*g*x)^2*(A + B*log((e*(c + d*x))/(a + b*x)))^2), x)
```

3.200 $\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx$

Optimal. Leaf size=159

$$\frac{de^{-\frac{A}{B}}\text{Ei}\left(\frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{B}\right)}{B^2(bc-ad)^2eg^3}-\frac{2be^{-\frac{2A}{B}}\text{Ei}\left(\frac{2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{B}\right)}{B^2(bc-ad)^2e^2g^3}+\frac{c+dx}{B(bc-ad)g^3(a+bx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}$$

```
[Out] d*Ei((A+B*ln(e*(d*x+c)/(b*x+a)))/B)/B^2/(-a*d+b*c)^2/e/exp(A/B)/g^3-2*b*Ei(
2*(A+B*ln(e*(d*x+c)/(b*x+a)))/B)/B^2/(-a*d+b*c)^2/e^2/exp(2*A/B)/g^3+(d*x+c
)/B/(-a*d+b*c)/g^3/(b*x+a)^2/(A+B*ln(e*(d*x+c)/(b*x+a)))
```

Rubi [A]
time = 0.16, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2552, 2357, 2367, 2336, 2209, 2346}

$$-\frac{2be^{-\frac{2A}{B}}\text{Ei}\left(\frac{2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{B}\right)}{B^2e^2g^3(bc-ad)^2}+\frac{de^{-\frac{A}{B}}\text{Ei}\left(\frac{A+B\log\left(\frac{e(c+dx)}{a+bx}\right)}{B}\right)}{B^2eg^3(bc-ad)^2}+\frac{c+dx}{Bg^3(a+bx)^2(bc-ad)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a*g + b*g*x)^3*(A + B*Log[(e*(c + d*x))/(a + b*x)])^2),x]
```

```
[Out] (d*ExpIntegralEi[(A + B*Log[(e*(c + d*x))/(a + b*x)])/B])/(B^2*(b*c - a*d)^
2*e*E^(A/B)*g^3) - (2*b*ExpIntegralEi[(2*(A + B*Log[(e*(c + d*x))/(a + b*x)
]))/B])/(B^2*(b*c - a*d)^2*e^2*E^((2*A)/B)*g^3) + (c + d*x)/(B*(b*c - a*d)*
g^3*(a + b*x)^2*(A + B*Log[(e*(c + d*x))/(a + b*x)]))
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2336

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[1/(n*c^(1
/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b,
 c, p}, x] && IntegerQ[1/n]
```

Rule 2346

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> Dist[1/c^
(m + 1), Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x] /; FreeQ
```

```
[{a, b, c, p}, x] && IntegerQ[m]
```

Rule 2357

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x
_Symbol] :> Simp[x*(d + e*x)^q*((a + b*Log[c*x^n])^(p + 1)/(b*n*(p + 1))),
x] + (-Dist[(q + 1)/(b*n*(p + 1)), Int[(d + e*x)^q*(a + b*Log[c*x^n])^(p +
1), x], x] + Dist[d*(q/(b*n*(p + 1))), Int[(d + e*x)^(q - 1)*(a + b*Log[c*x
^n])^(p + 1), x], x]) /; FreeQ[{a, b, c, d, e, n}, x] && LtQ[p, -1] && GtQ[
q, 0]
```

Rule 2367

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^(r_.))^(
q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*x^n])^p, (d + e*x
^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
&& IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0] && IntegerQ[r]))
```

Rule 2552

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(
m + 1)*(g/d)^m, Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a
 + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[n
 + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[d*f
- c*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Rubi steps

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx=\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}\,dx$$

Mathematica [A]
time = 0.31, size = 135, normalized size = 0.85

$$\frac{\frac{de^{-\frac{A}{B}}\operatorname{Ei}\left(\frac{A}{B}+\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{e}-\frac{2be^{-\frac{2A}{B}}\operatorname{Ei}\left(\frac{2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{B}\right)}{e^2}+\frac{B(bc-ad)(c+dx)}{(a+bx)^2\left(A+B\log\left(\frac{e(c+dx)}{a+bx}\right)\right)}}{B^2(bc-ad)^2g^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a*g + b*g*x)^3*(A + B*Log[(e*(c + d*x))/(a + b*x)])^2),x]
```

```
[Out] ((d*ExpIntegralEi[A/B + Log[(e*(c + d*x))/(a + b*x)]])/(e*E^(A/B)) - (2*b*E
xpIntegralEi[(2*(A + B*Log[(e*(c + d*x))/(a + b*x)]))/B])/(e^2*E^((2*A)/B))
 + (B*(b*c - a*d)*(c + d*x))/((a + b*x)^2*(A + B*Log[(e*(c + d*x))/(a + b*x
)]))))/(B^2*(b*c - a*d)^2*g^3)
```

Maple [A]
time = 3.96, size = 268, normalized size = 1.69

method	result
derivativedivides	$-\frac{\frac{b\left(-\frac{\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)^2}{\ln\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)+\frac{A}{B}}-2\,e^{-\frac{2A}{B}}\,\mathrm{expIntegral}\left(1,-2\ln\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)-\frac{2A}{B}\right)\right)}{B^2}-\frac{ed\left(-\frac{\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}}{\ln\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)+\frac{A}{B}}-e^{-\frac{A}{B}}\,\mathrm{expI}\cdots\right.}{B^2}}{e^2(ad-cb)^2g^3}$
default	$-\frac{\frac{b\left(-\frac{\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)^2}{\ln\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)+\frac{A}{B}}-2\,e^{-\frac{2A}{B}}\,\mathrm{expIntegral}\left(1,-2\ln\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)-\frac{2A}{B}\right)\right)}{B^2}-\frac{ed\left(-\frac{\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}}{\ln\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)+\frac{A}{B}}-e^{-\frac{A}{B}}\,\mathrm{expI}\cdots\right.}{B^2}}{e^2(ad-cb)^2g^3}$
risch	$-\frac{dx+c}{(ad-cb)B(bx+a)^2g^3\left(A+B\ln\left(\frac{e(dx+c)}{bx+a}\right)\right)}+\frac{bcd\,e^{-\frac{A}{B}}\,\mathrm{expIntegral}\left(1,-\ln\left(\frac{de}{b}-\frac{e(ad-cb)}{b(bx+a)}\right)-\frac{A}{B}\right)}{e\,g^3B^2(ad-cb)^3}-\frac{2cb^2e^{-\frac{2A}{B}}\,\mathrm{expIn}\cdots}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*g*x+a*g)^3/(A+B*ln(e*(d*x+c)/(b*x+a)))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/e^2/(a*d-b*c)^2/g^3*(b/B^2*(-(d*e/b-e*(a*d-b*c)/b/(b*x+a))^2/(ln(d*e/b-e
*(a*d-b*c)/b/(b*x+a))+A/B)-2*exp(-2*A/B)*Ei(1,-2*ln(d*e/b-e*(a*d-b*c)/b/(b*
x+a))-2*A/B))-e*d/B^2*(-(d*e/b-e*(a*d-b*c)/b/(b*x+a))/(ln(d*e/b-e*(a*d-b*c)
/b/(b*x+a))+A/B)-exp(-A/B)*Ei(1,-ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))-A/B)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)^3/(A+B*log(e*(d*x+c)/(b*x+a)))^2,x, algorithm="maxi
ma")
```

```
[Out] (d*x + c)/((a^2*b*c*g^3 - a^3*d*g^3)*A*B + (a^2*b*c*g^3 - a^3*d*g^3)*B^2 +
((b^3*c*g^3 - a*b^2*d*g^3)*A*B + (b^3*c*g^3 - a*b^2*d*g^3)*B^2)*x^2 + 2*((a
*b^2*c*g^3 - a^2*b*d*g^3)*A*B + (a*b^2*c*g^3 - a^2*b*d*g^3)*B^2)*x - ((b^3*
c*g^3 - a*b^2*d*g^3)*B^2*x^2 + 2*(a*b^2*c*g^3 - a^2*b*d*g^3)*B^2*x + (a^2*b
*c*g^3 - a^3*d*g^3)*B^2)*log(b*x + a) + ((b^3*c*g^3 - a*b^2*d*g^3)*B^2*x^2
+ 2*(a*b^2*c*g^3 - a^2*b*d*g^3)*B^2*x + (a^2*b*c*g^3 - a^3*d*g^3)*B^2)*log(
d*x + c)) - integrate(-(b*d*x + 2*b*c - a*d)/(((b^4*c*g^3 - a*b^3*d*g^3)*A*
B + (b^4*c*g^3 - a*b^3*d*g^3)*B^2)*x^3 + (a^3*b*c*g^3 - a^4*d*g^3)*A*B + (a
```

```
^3*b*c*g^3 - a^4*d*g^3)*B^2 + 3*((a*b^3*c*g^3 - a^2*b^2*d*g^3)*A*B + (a*b^3
*c*g^3 - a^2*b^2*d*g^3)*B^2)*x^2 + 3*((a^2*b^2*c*g^3 - a^3*b*d*g^3)*A*B + (
a^2*b^2*c*g^3 - a^3*b*d*g^3)*B^2)*x - ((b^4*c*g^3 - a*b^3*d*g^3)*B^2*x^3 +
3*(a*b^3*c*g^3 - a^2*b^2*d*g^3)*B^2*x^2 + 3*(a^2*b^2*c*g^3 - a^3*b*d*g^3)*B
^2*x + (a^3*b*c*g^3 - a^4*d*g^3)*B^2)*log(b*x + a) + ((b^4*c*g^3 - a*b^3*d*
g^3)*B^2*x^3 + 3*(a*b^3*c*g^3 - a^2*b^2*d*g^3)*B^2*x^2 + 3*(a^2*b^2*c*g^3 -
 a^3*b*d*g^3)*B^2*x + (a^3*b*c*g^3 - a^4*d*g^3)*B^2)*log(d*x + c)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 550 vs. $2(158) = 316$.
time = 0.36, size = 550, normalized size = 3.46

$$\frac{(Bbc^2 - Bacd + (Bbcd - Bad^2)x)e^{\left(\frac{2A}{B}+2\right)} - 2\left(Ab^3x^2 + 2Aab^2x + Aa^2b + (Bb^3x^2 + 2Bab^2x + Ba^2b)\log\left(\frac{(dx+c)e}{bx+a}\right)\right)\text{log\_integral}\left(\frac{(d^2x^2+2cdx+c^2)e^{\left(\frac{2A}{B}+2\right)}}{b^2x^2+2abx+a^2}\right) + \left((Bb^2dx^2 + 2Babdx + Ba^2d)e^{\left(\frac{A}{B}+1\right)}\log\left(\frac{(dx+c)e}{bx+a}\right) + (Ab^2dx^2 + 2Aabdx + Aa^2d)e^{\left(\frac{A}{B}+1\right)}\right)\text{log\_integral}\left(\frac{(dx+c)e^{\left(\frac{A}{B}+1\right)}}{bx+a}\right)}{((B^3b^4c^2 - 2B^3ab^3cd + B^3a^2b^2d^2)g^3x^2 + 2(B^3ab^3c^2 - 2B^3a^2b^2cd + B^3a^3bd^2)g^3x + (B^3a^2b^2c^2 - 2B^3a^3bcd + B^3a^4d^2)g^3)e^{\left(\frac{2A}{B}+2\right)}\log\left(\frac{(dx+c)e}{bx+a}\right) + ((AB^2b^4c^2 - 2AB^2ab^3cd + AB^2a^2b^2d^2)g^3x^2 + 2(AB^2ab^3c^2 - 2AB^2a^2b^2cd + AB^2a^3bd^2)g^3x + (AB^2a^2b^2c^2 - 2AB^2a^3bcd + AB^2a^4d^2)g^3)e^{\left(\frac{2A}{B}+2\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)^3/(A+B*log(e*(d*x+c)/(b*x+a)))^2,x, algorithm="fric
as")
```

```
[Out] ((B*b*c^2 - B*a*c*d + (B*b*c*d - B*a*d^2)*x)*e^(2*A/B + 2) - 2*(A*b^3*x^2 +
 2*A*a*b^2*x + A*a^2*b + (B*b^3*x^2 + 2*B*a*b^2*x + B*a^2*b)*log((d*x + c)*
e/(b*x + a)))*log_integral((d^2*x^2 + 2*c*d*x + c^2)*e^(2*A/B + 2)/(b^2*x^2
 + 2*a*b*x + a^2)) + ((B*b^2*d*x^2 + 2*B*a*b*d*x + B*a^2*d)*e^(A/B + 1)*log
((d*x + c)*e/(b*x + a)) + (A*b^2*d*x^2 + 2*A*a*b*d*x + A*a^2*d)*e^(A/B + 1)
)*log_integral((d*x + c)*e^(A/B + 1)/(b*x + a)))/(((B^3*b^4*c^2 - 2*B^3*a*b
^3*c*d + B^3*a^2*b^2*d^2)*g^3*x^2 + 2*(B^3*a*b^3*c^2 - 2*B^3*a^2*b^2*c*d +
B^3*a^3*b*d^2)*g^3*x + (B^3*a^2*b^2*c^2 - 2*B^3*a^3*b*c*d + B^3*a^4*d^2)*g^
3)*e^(2*A/B + 2)*log((d*x + c)*e/(b*x + a)) + ((A*B^2*b^4*c^2 - 2*A*B^2*a*b
^3*c*d + A*B^2*a^2*b^2*d^2)*g^3*x^2 + 2*(A*B^2*a*b^3*c^2 - 2*A*B^2*a^2*b^2*
c*d + A*B^2*a^3*b*d^2)*g^3*x + (A*B^2*a^2*b^2*c^2 - 2*A*B^2*a^3*b*c*d + A*B
^2*a^4*d^2)*g^3)*e^(2*A/B + 2))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)**3/(A+B*ln(e*(d*x+c)/(b*x+a)))**2,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 317 vs. $2(158) = 316$.
time = 5.14, size = 317, normalized size = 1.99

$$\left(\frac{d\text{Ei}\left(\frac{A}{B} + \log\left(\frac{dxe+ce}{bx+a}\right)\right)e^{\left(-\frac{A}{B}+1\right)}}{B^2bcg^3e - B^2adg^3e} - \frac{2\,b\text{Ei}\left(\frac{2A}{B} + 2\log\left(\frac{dxe+ce}{bx+a}\right)\right)e^{\left(-\frac{2A}{B}\right)}}{B^2bcg^3e - B^2adg^3e} - \frac{\frac{(dxe+ce)de}{bx+a} - \frac{(dxe+ce)^2b}{(bx+a)^2}}{B^2bcg^3e\log\left(\frac{dxe+ce}{bx+a}\right) - B^2adg^3e\log\left(\frac{dxe+ce}{bx+a}\right) + ABbcg^3e - ABadg^3e}\right)\left(\frac{bc}{(bce-ade)(bc-ad)} - \frac{ad}{(bce-ade)(bc-ad)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)^3/(A+B*log(e*(d*x+c)/(b*x+a)))^2,x, algorithm="giac
")
```

```
[Out] (d*Ei(A/B + log((d*x*e + c*e)/(b*x + a)))*e^(-A/B + 1)/(B^2*b*c*g^3*e - B^2
*a*d*g^3*e) - 2*b*Ei(2*A/B + 2*log((d*x*e + c*e)/(b*x + a)))*e^(-2*A/B)/(B^
2*b*c*g^3*e - B^2*a*d*g^3*e) - ((d*x*e + c*e)*d*e/(b*x + a) - (d*x*e + c*e)
^2*b/(b*x + a)^2)/(B^2*b*c*g^3*e*log((d*x*e + c*e)/(b*x + a)) - B^2*a*d*g^3
*e*log((d*x*e + c*e)/(b*x + a)) + A*B*b*c*g^3*e - A*B*a*d*g^3*e))*(b*c/((b*
c*e - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a\,g+b\,g\,x)^3\,\left(A+B\,\ln\left(\frac{e\,(c+d\,x)}{a+b\,x}\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*g + b*g*x)^3*(A + B*log((e*(c + d*x))/(a + b*x)))^2),x)
```

```
[Out] int(1/((a*g + b*g*x)^3*(A + B*log((e*(c + d*x))/(a + b*x)))^2), x)
```

3.201 $\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right) dx$

Optimal. Leaf size=182

$$-\frac{2B(bc-ad)^4g^4x}{5d^4}+\frac{B(bc-ad)^3g^4(a+bx)^2}{5bd^3}-\frac{2B(bc-ad)^2g^4(a+bx)^3}{15bd^2}+\frac{B(bc-ad)g^4(a+bx)^4}{10bd}+\frac{2B(bc-}{}$$

```
[Out] -2/5*B*(-a*d+b*c)^4*g^4*x/d^4+1/5*B*(-a*d+b*c)^3*g^4*(b*x+a)^2/b/d^3-2/15*B
*(-a*d+b*c)^2*g^4*(b*x+a)^3/b/d^2+1/10*B*(-a*d+b*c)*g^4*(b*x+a)^4/b/d+2/5*B
*(-a*d+b*c)^5*g^4*ln(d*x+c)/b/d^5+1/5*g^4*(b*x+a)^5*(A+B*ln(e*(d*x+c)^2/(b*
x+a)^2))/b
```

Rubi [A]
time = 0.08, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {2548, 21, 45}

$$\frac{g^4(a+bx)^5\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{5b}+\frac{2Bg^4(bc-ad)^5\log(c+dx)}{5bd^5}-\frac{2Bg^4x(bc-ad)^4}{5d^4}+\frac{Bg^4(a+bx)^2(bc-ad)^3}{5bd^3}-\frac{2Bg^4(a+bx)^3(bc-ad)^2}{15bd^2}+\frac{Bg^4(a+bx)^4(bc-ad)}{10bd}$$

Antiderivative was successfully verified.

```
[In] Int[(a*g + b*g*x)^4*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]),x]
```

```
[Out] (-2*B*(b*c - a*d)^4*g^4*x)/(5*d^4) + (B*(b*c - a*d)^3*g^4*(a + b*x)^2)/(5*b
*d^3) - (2*B*(b*c - a*d)^2*g^4*(a + b*x)^3)/(15*b*d^2) + (B*(b*c - a*d)*g^4
*(a + b*x)^4)/(10*b*d) + (2*B*(b*c - a*d)^5*g^4*Log[c + d*x])/(5*b*d^5) + (
g^4*(a + b*x)^5*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]))/(5*b)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2548

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
```

```
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Dist[B*n*((b*c
- a*d)/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /;
 FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c -
 a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Rubi steps

$$\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right) dx = \frac{g^4(a+bx)^5\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{5b} - \frac{B\int \frac{2(-bc+ad)g^5(a+bx)^4}{c+dx}}{5bg}$$
$$= \frac{g^4(a+bx)^5\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{5b} + \frac{(2B(bc-ad)g^4)\int \frac{(a}{c}}{5b}$$
$$= \frac{g^4(a+bx)^5\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{5b} + \frac{(2B(bc-ad)g^4)\int (}{}$$
$$= -\frac{2B(bc-ad)^4g^4x}{5d^4} + \frac{B(bc-ad)^3g^4(a+bx)^2}{5bd^3} - \frac{2B(bc-a}{1}$$

Mathematica [A]
time = 0.07, size = 144, normalized size = 0.79

$$\frac{g^4\left(-\frac{B(-bc+ad)(-12bd(bc-ad)^3x+6d^2(bc-ad)^2(a+bx)^2+4d^3(-bc+ad)(a+bx)^3+3d^4(a+bx)^4+12(bc-ad)^4\log(c+dx))}{6d^5}+(a+bx)^5\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)\right)}{5b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*g + b*g*x)^4*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]),x]
```

```
[Out] (g^4*(-1/6*(B*(-(b*c) + a*d)*(-12*b*d*(b*c - a*d)^3*x + 6*d^2*(b*c - a*d)^2
*(a + b*x)^2 + 4*d^3*(-(b*c) + a*d)*(a + b*x)^3 + 3*d^4*(a + b*x)^4 + 12*(b
*c - a*d)^4*Log[c + d*x]))/d^5 + (a + b*x)^5*(A + B*Log[(e*(c + d*x)^2)/(a
+ b*x)^2])))/(5*b)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 717 vs. $2(170) = 340$.
time = 0.37, size = 718, normalized size = 3.95 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*g*x+a*g)^4*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/b*(2/5*g^4*B*b^4*c^4/d^4*(b*x+a)-8/5*g^4*B*a^3/d*(b*x+a)*b*c+2*g^4*B*a/d
^4*ln(a*d/(b*x+a)-b*c/(b*x+a)-d)*c^4*b^4-1/5*g^4*B*b^3*c^3/d^3*(b*x+a)^2+2/
15*g^4*B*b^2*c^2/d^2*(b*x+a)^3-1/10*g^4*B*b*c/d*(b*x+a)^4+2/5*g^4*B*b^5*c^5
/d^5*ln(1/(b*x+a))-8/5*g^4*B*a/d^3*(b*x+a)*b^3*c^3+2*g^4*B*a^4/d*ln(1/(b*x+
```

```
a))*c*b-2/5*g^4*B*b^5*c^5/d^5*ln(a*d/(b*x+a)-b*c/(b*x+a)-d)-4/15*g^4*B*a/d*
(b*x+a)^3*b*c+12/5*g^4*B*a^2/d^2*(b*x+a)*b^2*c^2-2*g^4*B*a/d^4*ln(1/(b*x+a)
)*c^4*b^4-2*g^4*B*a^4/d*ln(a*d/(b*x+a)-b*c/(b*x+a)-d)*c*b+4*g^4*B*a^3/d^2*l
n(a*d/(b*x+a)-b*c/(b*x+a)-d)*c^2*b^2-4*g^4*B*a^2/d^3*ln(a*d/(b*x+a)-b*c/(b*
x+a)-d)*c^3*b^3+4*g^4*B*a^2/d^3*ln(1/(b*x+a))*c^3*b^3-4*g^4*B*a^3/d^2*ln(1/
(b*x+a))*c^2*b^2+1/5*g^4*B*a^3*(b*x+a)^2+2/15*g^4*B*a^2*(b*x+a)^3+1/10*g^4*
B*a*(b*x+a)^4+2/5*g^4*B*a^4*(b*x+a)-2/5*g^4*B*a^5*ln(1/(b*x+a))+2/5*g^4*B*a
^5*ln(a*d/(b*x+a)-b*c/(b*x+a)-d)-1/5*g^4*B*(b*x+a)^5*ln(e*(a*d/(b*x+a)-b*c/
(b*x+a)-d)^2/b^2)-1/5*g^4*A*(b*x+a)^5-3/5*g^4*B*a^2/d*(b*x+a)^2*b*c+3/5*g^4
*B*a/d^2*(b*x+a)^2*b^2*c^2)
```

<u>Maxima</u> [**B**] Leaf count of result is larger than twice the leaf count of optimal. 897 vs. $2(171) = 342$.
time = 0.33, size = 897, normalized size = 4.93

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^4*(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="maxi
ma")
```

```
[Out] 1/5*A*b^4*g^4*x^5 + A*a*b^3*g^4*x^4 + 2*A*a^2*b^2*g^4*x^3 + 2*A*a^3*b*g^4*x
^2 + (x*log(d^2*x^2*e/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*x*e/(b^2*x^2 + 2*a*
b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2)) - 2*a*log(b*x + a)/b + 2*c*lo
g(d*x + c)/d)*B*a^4*g^4 + 2*(x^2*log(d^2*x^2*e/(b^2*x^2 + 2*a*b*x + a^2) +
2*c*d*x*e/(b^2*x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2)) + 2*
a^2*log(b*x + a)/b^2 - 2*c^2*log(d*x + c)/d^2 + 2*(b*c - a*d)*x/(b*d))*B*a^
3*b*g^4 + 2*(x^3*log(d^2*x^2*e/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*x*e/(b^2*x
^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2)) - 2*a^3*log(b*x + a)
/b^3 + 2*c^3*log(d*x + c)/d^3 + ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2
*d^2)*x)/(b^2*d^2))*B*a^2*b^2*g^4 + 1/3*(3*x^4*log(d^2*x^2*e/(b^2*x^2 + 2*a
*b*x + a^2) + 2*c*d*x*e/(b^2*x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*
x + a^2)) + 6*a^4*log(b*x + a)/b^4 - 6*c^4*log(d*x + c)/d^4 + (2*(b^3*c*d^2
 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x
)/(b^3*d^3))*B*a*b^3*g^4 + 1/30*(6*x^5*log(d^2*x^2*e/(b^2*x^2 + 2*a*b*x + a
^2) + 2*c*d*x*e/(b^2*x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2)
) - 12*a^5*log(b*x + a)/b^5 + 12*c^5*log(d*x + c)/d^5 + (3*(b^4*c*d^3 - a*b
^3*d^4)*x^4 - 4*(b^4*c^2*d^2 - a^2*b^2*d^4)*x^3 + 6*(b^4*c^3*d - a^3*b*d^4)
*x^2 - 12*(b^4*c^4 - a^4*d^4)*x)/(b^4*d^4))*B*b^4*g^4 + A*a^4*g^4*x
```

<u>Fricas</u> [**B**] Leaf count of result is larger than twice the leaf count of optimal. 455 vs. $2(171) = 342$.
time = 0.40, size = 455, normalized size = 2.50

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^4*(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="fric
as")
```

```
[Out] 1/30*(6*A*b^5*d^5*g^4*x^5 - 12*B*a^5*d^5*g^4*log(b*x + a) + 3*(B*b^5*c*d^4
+ (10*A - B)*a*b^4*d^5)*g^4*x^4 - 4*(B*b^5*c^2*d^3 - 5*B*a*b^4*c*d^4 - (15*
A - 4*B)*a^2*b^3*d^5)*g^4*x^3 + 6*(B*b^5*c^3*d^2 - 5*B*a*b^4*c^2*d^3 + 10*B
*a^2*b^3*c*d^4 + 2*(5*A - 3*B)*a^3*b^2*d^5)*g^4*x^2 - 6*(2*B*b^5*c^4*d - 10
*B*a*b^4*c^3*d^2 + 20*B*a^2*b^3*c^2*d^3 - 20*B*a^3*b^2*c*d^4 - (5*A - 8*B)*
a^4*b*d^5)*g^4*x + 12*(B*b^5*c^5 - 5*B*a*b^4*c^4*d + 10*B*a^2*b^3*c^3*d^2 -
 10*B*a^3*b^2*c^2*d^3 + 5*B*a^4*b*c*d^4)*g^4*log(d*x + c) + 6*(B*b^5*d^5*g^
4*x^5 + 5*B*a*b^4*d^5*g^4*x^4 + 10*B*a^2*b^3*d^5*g^4*x^3 + 10*B*a^3*b^2*d^5
*g^4*x^2 + 5*B*a^4*b*d^5*g^4*x)*log((d^2*x^2 + 2*c*d*x + c^2)*e/(b^2*x^2 +
2*a*b*x + a^2)))/(b*d^5)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 998 vs. $2(163) = 326$.
time = 5.31, size = 998, normalized size = 5.48

$$\frac{Ab^4g^4x^5}{5} - \frac{2Ba^5g^4\log\left(x + \frac{\frac{2Ba^6d^5g^4}{b} + 10Ba^5cd^4g^4 - 20Ba^4bc^2d^3g^4 + 20Ba^3b^2c^3d^2g^4 - 10Ba^2b^3c^4dg^4 + 2Bab^4c^5g^4}{2Ba^5d^5g^4 + 10Ba^4bcd^4g^4 - 20Ba^3b^2c^2d^3g^4 + 20Ba^2b^3c^3d^2g^4 - 10Bab^4c^4dg^4 + 2Bb^5c^5g^4}\right)}{5b} + \frac{2Bcg^4\cdot\left(5a^4d^4 - 10a^3bcd^3 + 10a^2b^2c^2d^2 - 5ab^3c^3d + b^4c^4\right)\log\left(x + \frac{12Ba^5cd^4g^4 - 20Ba^4bc^2d^3g^4 + 20Ba^3b^2c^3d^2g^4 - 10Ba^2b^3c^4dg^4 + 2Bab^4c^5g^4 - 2Bacg^4\cdot\left(5a^4d^4 - 10a^3bcd^3 + 10a^2b^2c^2d^2 - 5ab^3c^3d + b^4c^4\right) + \frac{2Bbc^2g^4\cdot\left(5a^4d^4 - 10a^3bcd^3 + 10a^2b^2c^2d^2 - 5ab^3c^3d + b^4c^4\right)}{d}}{2Ba^5d^5g^4 + 10Ba^4bcd^4g^4 - 20Ba^3b^2c^2d^3g^4 + 20Ba^2b^3c^3d^2g^4 - 10Bab^4c^4dg^4 + 2Bb^5c^5g^4}\right)}{5d^5} + x^4\left(Aab^3g^4 - \frac{Bab^3g^4}{10} + \frac{Bb^4cg^4}{10d}\right) + x^3\cdot\left(2Aa^2b^2g^4 - \frac{8Ba^2b^2g^4}{15} + \frac{2Bab^3cg^4}{3d} - \frac{2Bb^4c^2g^4}{15d^2}\right) + x^2\cdot\left(2Aa^3bg^4 - \frac{6Ba^3bg^4}{5} + \frac{2Ba^2b^2cg^4}{d} - \frac{Bab^3c^2g^4}{d^2} + \frac{Bb^4c^3g^4}{5d^3}\right) + x\left(Aa^4g^4 - \frac{8Ba^4g^4}{5} + \frac{4Ba^3bcg^4}{d} - \frac{4Ba^2b^2c^2g^4}{d^2} + \frac{2Bab^3c^3g^4}{d^3} - \frac{2Bb^4c^4g^4}{5d^4}\right) + \left(Ba^4g^4x + 2Ba^3bg^4x^2 + 2Ba^2b^2g^4x^3 + Bab^3g^4x^4 + \frac{Bb^4g^4x^5}{5}\right)\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)**4*(A+B*ln(e*(d*x+c)**2/(b*x+a)**2)),x)
```

```
[Out] A*b**4*g**4*x**5/5 - 2*B*a**5*g**4*log(x + (2*B*a**6*d**5*g**4/b + 10*B*a**
5*c*d**4*g**4 - 20*B*a**4*b*c**2*d**3*g**4 + 20*B*a**3*b**2*c**3*d**2*g**4
- 10*B*a**2*b**3*c**4*d*g**4 + 2*B*a*b**4*c**5*g**4)/(2*B*a**5*d**5*g**4 +
10*B*a**4*b*c*d**4*g**4 - 20*B*a**3*b**2*c**2*d**3*g**4 + 20*B*a**2*b**3*c*
*3*d**2*g**4 - 10*B*a*b**4*c**4*d*g**4 + 2*B*b**5*c**5*g**4))/(5*b) + 2*B*c
*g**4*(5*a**4*d**4 - 10*a**3*b*c*d**3 + 10*a**2*b**2*c**2*d**2 - 5*a*b**3*c
**3*d + b**4*c**4)*log(x + (12*B*a**5*c*d**4*g**4 - 20*B*a**4*b*c**2*d**3*g
**4 + 20*B*a**3*b**2*c**3*d**2*g**4 - 10*B*a**2*b**3*c**4*d*g**4 + 2*B*a*b*
*4*c**5*g**4 - 2*B*a*c*g**4*(5*a**4*d**4 - 10*a**3*b*c*d**3 + 10*a**2*b**2*
c**2*d**2 - 5*a*b**3*c**3*d + b**4*c**4) + 2*B*b*c**2*g**4*(5*a**4*d**4 - 1
0*a**3*b*c*d**3 + 10*a**2*b**2*c**2*d**2 - 5*a*b**3*c**3*d + b**4*c**4)/d)/
(2*B*a**5*d**5*g**4 + 10*B*a**4*b*c*d**4*g**4 - 20*B*a**3*b**2*c**2*d**3*g*
*4 + 20*B*a**2*b**3*c**3*d**2*g**4 - 10*B*a*b**4*c**4*d*g**4 + 2*B*b**5*c**
5*g**4))/(5*d**5) + x**4*(A*a*b**3*g**4 - B*a*b**3*g**4/10 + B*b**4*c*g**4/
(10*d)) + x**3*(2*A*a**2*b**2*g**4 - 8*B*a**2*b**2*g**4/15 + 2*B*a*b**3*c*g
**4/(3*d) - 2*B*b**4*c**2*g**4/(15*d**2)) + x**2*(2*A*a**3*b*g**4 - 6*B*a**
3*b*g**4/5 + 2*B*a**2*b**2*c*g**4/d - B*a*b**3*c**2*g**4/d**2 + B*b**4*c**3
*g**4/(5*d**3)) + x*(A*a**4*g**4 - 8*B*a**4*g**4/5 + 4*B*a**3*b*c*g**4/d -
4*B*a**2*b**2*c**2*g**4/d**2 + 2*B*a*b**3*c**3*g**4/d**3 - 2*B*b**4*c**4*g*
*4/(5*d**4)) + (B*a**4*g**4*x + 2*B*a**3*b*g**4*x**2 + 2*B*a**2*b**2*g**4*x
**3 + B*a*b**3*g**4*x**4 + B*b**4*g**4*x**5/5)*log(e*(c + d*x)**2/(a + b*x)
**2)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^4*(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="giac
")
[Out] Timed out
```

Mupad [B]
time = 4.79, size = 1024, normalized size = 5.63

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*g + b*g*x)^4*(A + B*log((e*(c + d*x)^2)/(a + b*x)^2)),x)
[Out] x^2*(((5*a*d + 5*b*c)*((((b^3*g^4*(25*A*a*d + 5*A*b*c - 2*B*a*d + 2*B*b*c))
/(5*d) - (A*b^3*g^4*(5*a*d + 5*b*c))/(5*d))*(5*a*d + 5*b*c))/(5*b*d) - (a*b
^2*g^4*(10*A*a*d + 5*A*b*c - 2*B*a*d + 2*B*b*c))/d + (A*a*b^3*c*g^4)/d))/(1
0*b*d) + (a^2*b*g^4*(5*A*a*d + 5*A*b*c - 2*B*a*d + 2*B*b*c))/d - (a*c*((b^3
*g^4*(25*A*a*d + 5*A*b*c - 2*B*a*d + 2*B*b*c))/(5*d) - (A*b^3*g^4*(5*a*d +
5*b*c))/(5*d)))/(2*b*d)) - x^3*((((b^3*g^4*(25*A*a*d + 5*A*b*c - 2*B*a*d +
2*B*b*c))/(5*d) - (A*b^3*g^4*(5*a*d + 5*b*c))/(5*d))*(5*a*d + 5*b*c))/(15*b
*d) - (a*b^2*g^4*(10*A*a*d + 5*A*b*c - 2*B*a*d + 2*B*b*c))/(3*d) + (A*a*b^3
*c*g^4)/(3*d)) + x*((a^3*g^4*(5*A*a*d + 10*A*b*c - 4*B*a*d + 4*B*b*c))/d -
((5*a*d + 5*b*c)*(((5*a*d + 5*b*c)*((((b^3*g^4*(25*A*a*d + 5*A*b*c - 2*B*a*
d + 2*B*b*c))/(5*d) - (A*b^3*g^4*(5*a*d + 5*b*c))/(5*d))*(5*a*d + 5*b*c))/(
5*b*d) - (a*b^2*g^4*(10*A*a*d + 5*A*b*c - 2*B*a*d + 2*B*b*c))/d + (A*a*b^3*
c*g^4)/d))/(5*b*d) + (2*a^2*b*g^4*(5*A*a*d + 5*A*b*c - 2*B*a*d + 2*B*b*c))/
d - (a*c*((b^3*g^4*(25*A*a*d + 5*A*b*c - 2*B*a*d + 2*B*b*c))/(5*d) - (A*b^3
*g^4*(5*a*d + 5*b*c))/(5*d)))/(b*d)))/(5*b*d) + (a*c*((((b^3*g^4*(25*A*a*d
+ 5*A*b*c - 2*B*a*d + 2*B*b*c))/(5*d) - (A*b^3*g^4*(5*a*d + 5*b*c))/(5*d))*
(5*a*d + 5*b*c))/(5*b*d) - (a*b^2*g^4*(10*A*a*d + 5*A*b*c - 2*B*a*d + 2*B*b
*c))/d + (A*a*b^3*c*g^4)/d))/(b*d)) + log((e*(c + d*x)^2)/(a + b*x)^2)*((B*
b^4*g^4*x^5)/5 + B*a^4*g^4*x + 2*B*a^3*b*g^4*x^2 + B*a*b^3*g^4*x^4 + 2*B*a^
2*b^2*g^4*x^3) + x^4*((b^3*g^4*(25*A*a*d + 5*A*b*c - 2*B*a*d + 2*B*b*c))/(2
0*d) - (A*b^3*g^4*(5*a*d + 5*b*c))/(20*d)) + (log(c + d*x)*((2*B*b^4*c^5*g^
4)/5 + 2*B*a^4*c*d^4*g^4 - 4*B*a^3*b*c^2*d^3*g^4 + 4*B*a^2*b^2*c^3*d^2*g^4
- 2*B*a*b^3*c^4*d*g^4))/d^5 + (A*b^4*g^4*x^5)/5 - (2*B*a^5*g^4*log(a + b*x)
)/(5*b)
```

3.202 $\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right) dx$

Optimal. Leaf size=151

$$\frac{B(bc-ad)^3g^3x}{2d^3}-\frac{B(bc-ad)^2g^3(a+bx)^2}{4bd^2}+\frac{B(bc-ad)g^3(a+bx)^3}{6bd}-\frac{B(bc-ad)^4g^3\log(c+dx)}{2bd^4}+\frac{g^3(a+bx)^4}{}$$

```
[Out] 1/2*B*(-a*d+b*c)^3*g^3*x/d^3-1/4*B*(-a*d+b*c)^2*g^3*(b*x+a)^2/b/d^2+1/6*B*(
-a*d+b*c)*g^3*(b*x+a)^3/b/d-1/2*B*(-a*d+b*c)^4*g^3*ln(d*x+c)/b/d^4+1/4*g^3*
(b*x+a)^4*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))/b
```

Rubi [A]
time = 0.06, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {2548, 21, 45}

$$\frac{g^3(a+bx)^4\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{4b}-\frac{Bg^3(bc-ad)^4\log(c+dx)}{2bd^4}+\frac{Bg^3x(bc-ad)^3}{2d^3}-\frac{Bg^3(a+bx)^2(bc-ad)^2}{4bd^2}+\frac{Bg^3(a+bx)^3(bc-ad)}{6bd}$$

Antiderivative was successfully verified.

```
[In] Int[(a*g + b*g*x)^3*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]),x]
```

```
[Out] (B*(b*c - a*d)^3*g^3*x)/(2*d^3) - (B*(b*c - a*d)^2*g^3*(a + b*x)^2)/(4*b*d^
2) + (B*(b*c - a*d)*g^3*(a + b*x)^3)/(6*b*d) - (B*(b*c - a*d)^4*g^3*Log[c +
 d*x])/(2*b*d^4) + (g^3*(a + b*x)^4*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]
))/(4*b)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2548

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Dist[B*n*((b*c
```

```
- a*d)/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /;
 FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c -
 a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Rubi steps

$$\int (ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)dx = \frac{g^3(a+bx)^4\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{4b} - \frac{B\int \frac{2(-bc+ad)g^4(a+bx}{c+dx}}{4bg}$$

$$= \frac{g^3(a+bx)^4\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{4b} + \frac{(B(bc-ad)g^3)\int \frac{(}{}}{2b}$$

$$= \frac{g^3(a+bx)^4\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{4b} + \frac{(B(bc-ad)g^3)\int (}{}$$

$$= \frac{B(bc-ad)^3g^3x}{2d^3} - \frac{B(bc-ad)^2g^3(a+bx)^2}{4bd^2} + \frac{B(bc-ad)g}{6b}$$

Mathematica [A]
time = 0.06, size = 122, normalized size = 0.81

$$\frac{g^3\left(\frac{B(bc-ad)\left(6bd(bc-ad)^2x+3d^2(-bc+ad)(a+bx)^2+2d^3(a+bx)^3-6(bc-ad)^3\log(c+dx)\right)}{3d^4}+(a+bx)^4\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*g + b*g*x)^3*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]),x]
```

```
[Out] (g^3*((B*(b*c - a*d)*(6*b*d*(b*c - a*d)^2*x + 3*d^2*(-(b*c) + a*d)*(a + b*x
)^2 + 2*d^3*(a + b*x)^3 - 6*(b*c - a*d)^3*Log[c + d*x]))/(3*d^4) + (a + b*x
)^4*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])))/(4*b)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 542 vs. $2(141) = 282$.
time = 0.36, size = 543, normalized size = 3.60

method	result
risch	$\frac{g^3(bx+a)^4B\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)}{4b} + \frac{g^3b^3Ax^4}{4} + g^3b^2Aax^3 - \frac{g^3b^2Bax^3}{6} + \frac{g^3b^3Bcx^3}{6d} + \frac{3g^3bAa^2x^2}{2} - \frac{3g^3bBa^2}{4}$
derivativedivides	$-\frac{-\frac{g^3A(bx+a)^4}{4} - \frac{g^3B(bx+a)^4\ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^2}{b^2}\right)}{4} - \frac{g^3Bb^4c^4\ln\left(\frac{1}{bx+a}\right)}{2d^4} + \frac{g^3Bb^2c^2(bx+a)^2}{4d^2} - \frac{g^3Bbc(bx+a)^3}{6d} - \frac{g^3Bb^3}{2}}{}$

default	$-\frac{-\frac{g^3 A (bx+a)^4}{4}-\frac{g^3 B (bx+a)^4 \ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^2}{b^2}\right)}{4}-\frac{g^3 B\, b^4 c^4 \ln\left(\frac{1}{bx+a}\right)}{2d^4}+\frac{g^3 B\, b^2 c^2 (bx+a)^2}{4d^2}-\frac{g^3 B bc (bx+a)^3}{6d}-\frac{g^3 B\, b^3 c^3 \cdots}{2d^3}}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*g*x+a*g)^3*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/b*(-1/4*g^3*A*(b*x+a)^4-1/4*g^3*B*(b*x+a)^4*ln(e*(a*d/(b*x+a)-b*c/(b*x+a
)-d)^2/b^2)-1/2*g^3*B*b^4*c^4/d^4*ln(1/(b*x+a))+1/4*g^3*B*b^2*c^2/d^2*(b*x+
a)^2-1/6*g^3*B*b*c/d*(b*x+a)^3-1/2*g^3*B*b^3*c^3/d^3*(b*x+a)+1/2*g^3*B*b^4*
c^4/d^4*ln(a*d/(b*x+a)-b*c/(b*x+a)-d)+2*g^3*B*a^3/d*ln(1/(b*x+a))*b*c-3*g^3
*B*a^2/d^2*ln(1/(b*x+a))*b^2*c^2+2*g^3*B*a/d^3*ln(1/(b*x+a))*b^3*c^3-1/2*g^
3*B*a/d*(b*x+a)^2*b*c-3/2*g^3*B*a^2/d*(b*x+a)*b*c+3/2*g^3*B*a/d^2*(b*x+a)*b
^2*c^2-2*g^3*B*a^3/d*ln(a*d/(b*x+a)-b*c/(b*x+a)-d)*b*c+3*g^3*B*a^2/d^2*ln(a
*d/(b*x+a)-b*c/(b*x+a)-d)*b^2*c^2-2*g^3*B*a/d^3*ln(a*d/(b*x+a)-b*c/(b*x+a)-
d)*b^3*c^3+1/4*g^3*B*a^2*(b*x+a)^2+1/6*g^3*B*a*(b*x+a)^3+1/2*g^3*B*a^3*(b*x
+a)+1/2*g^3*B*a^4*ln(a*d/(b*x+a)-b*c/(b*x+a)-d)-1/2*g^3*B*a^4*ln(1/(b*x+a))
)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 657 vs. $2(142) = 284$.
time = 0.31, size = 657, normalized size = 4.35

$$\frac{1}{4} A b^3 g^3 x^4 + A a b^2 g^3 x^3 + \frac{3}{2} A a^2 b g^3 x^2 + \left(x \log\left(\frac{d^2 x^2 e}{b^2 x^2 + 2 abx + a^2} + \frac{2 cdxe}{b^2 x^2 + 2 abx + a^2} + \frac{c^2 e}{b^2 x^2 + 2 abx + a^2}\right) - \frac{2 a \log(bx+a)}{b} + \frac{2 c \log(dx+c)}{d}\right) B a^3 g^3 + \frac{3}{2}\left(x^2 \log\left(\frac{d^2 x^2 e}{b^2 x^2 + 2 abx + a^2} + \frac{2 cdxe}{b^2 x^2 + 2 abx + a^2} + \frac{c^2 e}{b^2 x^2 + 2 abx + a^2}\right) + \frac{2 a^2 \log(bx+a)}{b^2} - \frac{2 c^2 \log(dx+c)}{d^2} + \frac{2 (bc - ad) x}{bd}\right) B a^2 b g^3 + \left(x^3 \log\left(\frac{d^2 x^2 e}{b^2 x^2 + 2 abx + a^2} + \frac{2 cdxe}{b^2 x^2 + 2 abx + a^2} + \frac{c^2 e}{b^2 x^2 + 2 abx + a^2}\right) - \frac{2 a^3 \log(bx+a)}{b^3} + \frac{2 c^3 \log(dx+c)}{d^3} + \frac{(b^2 cd - abd^2) x^2 - 2 (b^2 c^2 - a^2 d^2) x}{b^2 d^2}\right) B a b^2 g^3 + \frac{1}{12}\left(3 x^4 \log\left(\frac{d^2 x^2 e}{b^2 x^2 + 2 abx + a^2} + \frac{2 cdxe}{b^2 x^2 + 2 abx + a^2} + \frac{c^2 e}{b^2 x^2 + 2 abx + a^2}\right) + \frac{6 a^4 \log(bx+a)}{b^4} - \frac{6 c^4 \log(dx+c)}{d^4} + \frac{2 (b^3 cd^2 - ab^2 d^3) x^3 - 3 (b^3 c^2 d - a^2 b d^3) x^2 + 6 (b^3 c^3 - a^3 d^3) x}{b^3 d^3}\right) B b^3 g^3 + A a^3 g^3 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^3*(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="maxi
ma")
```

```
[Out] 1/4*A*b^3*g^3*x^4 + A*a*b^2*g^3*x^3 + 3/2*A*a^2*b*g^3*x^2 + (x*log(d^2*x^2*
e/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*x*e/(b^2*x^2 + 2*a*b*x + a^2) + c^2*e/(
b^2*x^2 + 2*a*b*x + a^2)) - 2*a*log(b*x + a)/b + 2*c*log(d*x + c)/d)*B*a^3*
g^3 + 3/2*(x^2*log(d^2*x^2*e/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*x*e/(b^2*x^2
 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2)) + 2*a^2*log(b*x + a)/b
^2 - 2*c^2*log(d*x + c)/d^2 + 2*(b*c - a*d)*x/(b*d))*B*a^2*b*g^3 + (x^3*log
(d^2*x^2*e/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*x*e/(b^2*x^2 + 2*a*b*x + a^2)
+ c^2*e/(b^2*x^2 + 2*a*b*x + a^2)) - 2*a^3*log(b*x + a)/b^3 + 2*c^3*log(d*x
 + c)/d^3 + ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*
B*a*b^2*g^3 + 1/12*(3*x^4*log(d^2*x^2*e/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*x
*e/(b^2*x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2)) + 6*a^4*log
(b*x + a)/b^4 - 6*c^4*log(d*x + c)/d^4 + (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3
*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3))*B*b^3*g^
3 + A*a^3*g^3*x
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 341 vs. $2(142) = 284$.

time = 0.40, size = 341, normalized size = 2.26

$$\frac{3\,Ab^{4}d^{4}g^{3}x^{4}-6\,Ba^{4}d^{4}g^{3}\log\left(bx+a\right)+2\,\left(Bb^{4}cd^{3}+\left(6\,A-B\right)ab^{3}d^{4}\right)g^{3}x^{3}-3\,\left(Bb^{4}c^{2}d^{2}-4\,Bab^{3}cd^{3}-3\,\left(2\,A-B\right)a^{2}b^{2}d^{4}\right)g^{3}x^{2}+6\,\left(Bb^{4}c^{3}d-4\,Bab^{3}c^{2}d^{2}+6\,Ba^{2}b^{2}cd^{3}+\left(2\,A-3\,B\right)a^{3}bd^{4}\right)g^{3}x-6\,\left(Bb^{4}c^{4}-4\,Bab^{3}c^{3}d+6\,Ba^{2}b^{2}c^{2}d^{2}-4\,Ba^{3}bcd^{3}\right)g^{3}\log\left(dx+c\right)+3\,\left(Bb^{4}d^{4}g^{3}x^{4}+4\,Bab^{3}d^{4}g^{3}x^{3}+6\,Ba^{2}b^{2}d^{4}g^{3}x^{2}+4\,Ba^{3}bd^{4}g^{3}x\right)\log\left(\frac{\left(d^{2}x^{2}+2\,cdx+c^{2}\right)e}{b^{2}x^{2}+2\,abx+a^{2}}\right)}{12\,bd^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^3*(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="fric
as")
```

```
[Out] 1/12*(3*A*b^4*d^4*g^3*x^4 - 6*B*a^4*d^4*g^3*log(b*x + a) + 2*(B*b^4*c*d^3 +
 (6*A - B)*a*b^3*d^4)*g^3*x^3 - 3*(B*b^4*c^2*d^2 - 4*B*a*b^3*c*d^3 - 3*(2*A
 - B)*a^2*b^2*d^4)*g^3*x^2 + 6*(B*b^4*c^3*d - 4*B*a*b^3*c^2*d^2 + 6*B*a^2*b
^2*c*d^3 + (2*A - 3*B)*a^3*b*d^4)*g^3*x - 6*(B*b^4*c^4 - 4*B*a*b^3*c^3*d +
6*B*a^2*b^2*c^2*d^2 - 4*B*a^3*b*c*d^3)*g^3*log(d*x + c) + 3*(B*b^4*d^4*g^3*
x^4 + 4*B*a*b^3*d^4*g^3*x^3 + 6*B*a^2*b^2*d^4*g^3*x^2 + 4*B*a^3*b*d^4*g^3*x
)*log((d^2*x^2 + 2*c*d*x + c^2)*e/(b^2*x^2 + 2*a*b*x + a^2)))/(b*d^4)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 707 vs. $2(131) = 262$.
time = 3.11, size = 707, normalized size = 4.68

$$\frac{Ab^{3}g^{3}x^{4}}{4}-\frac{Ba^{4}g^{3}\log\left(x+\frac{\frac{Ba^{5}d^{4}g^{3}}{b}+4Ba^{4}cd^{3}g^{3}-6Ba^{3}bc^{2}d^{2}g^{3}+4Ba^{2}b^{2}c^{3}dg^{3}-Bab^{3}c^{4}g^{3}}{Ba^{4}d^{4}g^{3}+4Ba^{3}bcd^{3}g^{3}-6Ba^{2}b^{2}c^{2}d^{2}g^{3}+4Bab^{3}c^{3}dg^{3}-Bb^{4}c^{4}g^{3}}\right)}{2b}+\frac{Bcg^{3}\cdot\left(2ad-bc\right)\left(2a^{2}d^{2}-2abcd+b^{2}c^{2}\right)\log\left(x+\frac{5Ba^{4}cd^{3}g^{3}-6Ba^{3}bc^{2}d^{2}g^{3}+4Ba^{2}b^{2}c^{3}dg^{3}-Bab^{3}c^{4}g^{3}-Bacg^{3}\cdot\left(2ad-bc\right)\left(2a^{2}d^{2}-2abcd+b^{2}c^{2}\right)+\frac{Bbc^{2}g^{3}\cdot\left(2ad-bc\right)\left(2a^{2}d^{2}-2abcd+b^{2}c^{2}\right)}{d}}{Ba^{4}d^{4}g^{3}+4Ba^{3}bcd^{3}g^{3}-6Ba^{2}b^{2}c^{2}d^{2}g^{3}+4Bab^{3}c^{3}dg^{3}-Bb^{4}c^{4}g^{3}}\right)}{2d^{4}}+x^{3}\left(Aab^{2}g^{3}-\frac{Bab^{2}g^{3}}{6}+\frac{Bb^{3}cg^{3}}{6d}\right)+x^{2}\cdot\left(\frac{3Aa^{2}bg^{3}}{2}-\frac{3Ba^{2}bg^{3}}{4}+\frac{Bab^{2}cg^{3}}{d}-\frac{Bb^{3}c^{2}g^{3}}{4d^{2}}\right)+x\left(Aa^{3}g^{3}-\frac{3Ba^{3}g^{3}}{2}+\frac{3Ba^{2}bcg^{3}}{d}-\frac{2Bab^{2}c^{2}g^{3}}{d^{2}}+\frac{Bb^{3}c^{3}g^{3}}{2d^{3}}\right)+\left(Ba^{3}g^{3}x+\frac{3Ba^{2}bg^{3}x^{2}}{2}+Bab^{2}g^{3}x^{3}+\frac{Bb^{3}g^{3}x^{4}}{4}\right)\log\left(\frac{e\left(c+dx\right)^{2}}{\left(a+bx\right)^{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)**3*(A+B*ln(e*(d*x+c)**2/(b*x+a)**2)),x)
```

```
[Out] A*b**3*g**3*x**4/4 - B*a**4*g**3*log(x + (B*a**5*d**4*g**3/b + 4*B*a**4*c*d
**3*g**3 - 6*B*a**3*b*c**2*d**2*g**3 + 4*B*a**2*b**2*c**3*d*g**3 - B*a*b**3
*c**4*g**3)/(B*a**4*d**4*g**3 + 4*B*a**3*b*c*d**3*g**3 - 6*B*a**2*b**2*c**2
*d**2*g**3 + 4*B*a*b**3*c**3*d*g**3 - B*b**4*c**4*g**3))/(2*b) + B*c*g**3*(
2*a*d - b*c)*(2*a**2*d**2 - 2*a*b*c*d + b**2*c**2)*log(x + (5*B*a**4*c*d**3
*g**3 - 6*B*a**3*b*c**2*d**2*g**3 + 4*B*a**2*b**2*c**3*d*g**3 - B*a*b**3*c*
*4*g**3 - B*a*c*g**3*(2*a*d - b*c)*(2*a**2*d**2 - 2*a*b*c*d + b**2*c**2) +
B*b*c**2*g**3*(2*a*d - b*c)*(2*a**2*d**2 - 2*a*b*c*d + b**2*c**2)/d)/(B*a**
4*d**4*g**3 + 4*B*a**3*b*c*d**3*g**3 - 6*B*a**2*b**2*c**2*d**2*g**3 + 4*B*a
*b**3*c**3*d*g**3 - B*b**4*c**4*g**3))/(2*d**4) + x**3*(A*a*b**2*g**3 - B*a
*b**2*g**3/6 + B*b**3*c*g**3/(6*d)) + x**2*(3*A*a**2*b*g**3/2 - 3*B*a**2*b*
g**3/4 + B*a*b**2*c*g**3/d - B*b**3*c**2*g**3/(4*d**2)) + x*(A*a**3*g**3 -
3*B*a**3*g**3/2 + 3*B*a**2*b*c*g**3/d - 2*B*a*b**2*c**2*g**3/d**2 + B*b**3*
c**3*g**3/(2*d**3)) + (B*a**3*g**3*x + 3*B*a**2*b*g**3*x**2/2 + B*a*b**2*g*
*3*x**3 + B*b**3*g**3*x**4/4)*log(e*(c + d*x)**2/(a + b*x)**2)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 364 vs. $2(142) = 284$.
time = 68.09, size = 364, normalized size = 2.41

$$-\frac{Ba^{4}g^{3}\log\left(bx+a\right)}{2\,b}+\frac{1}{4}\,\left(Ab^{3}g^{3}+Bb^{3}g^{3}\right)x^{4}+\frac{\left(Bb^{3}cg^{3}+6\,Aab^{2}dg^{3}+5\,Bab^{2}dg^{3}\right)x^{3}}{6\,d}+\frac{1}{4}\,\left(Bb^{3}g^{3}x^{4}+4\,Bab^{2}g^{3}x^{3}+6\,Ba^{2}bg^{3}x^{2}+4\,Ba^{3}g^{3}x\right)\log\left(\frac{d^{2}x^{2}+2\,cdx+c^{2}}{b^{2}x^{2}+2\,abx+a^{2}}\right)-\frac{\left(Bb^{3}c^{2}g^{3}-4\,Bab^{2}cdg^{3}-6\,Aa^{2}bd^{2}g^{3}-3\,Ba^{2}bd^{2}g^{3}\right)x^{2}}{4\,d^{2}}+\frac{\left(Bb^{3}c^{3}g^{3}-4\,Bab^{2}c^{2}dg^{3}+6\,Ba^{2}bcd^{2}g^{3}+2\,Aa^{3}d^{3}g^{3}-Ba^{3}d^{3}g^{3}\right)x}{2\,d^{3}}-\frac{\left(Bb^{3}c^{4}g^{3}-4\,Bab^{2}c^{3}dg^{3}+6\,Ba^{2}bc^{2}d^{2}g^{3}-4\,Ba^{3}cd^{3}g^{3}\right)\log\left(-dx-c\right)}{2\,d^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^3*(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="giac
")
```

```
[Out] -1/2*B*a^4*g^3*log(b*x + a)/b + 1/4*(A*b^3*g^3 + B*b^3*g^3)*x^4 + 1/6*(B*b^
3*c*g^3 + 6*A*a*b^2*d*g^3 + 5*B*a*b^2*d*g^3)*x^3/d + 1/4*(B*b^3*g^3*x^4 + 4
*B*a*b^2*g^3*x^3 + 6*B*a^2*b*g^3*x^2 + 4*B*a^3*g^3*x)*log((d^2*x^2 + 2*c*d*
x + c^2)/(b^2*x^2 + 2*a*b*x + a^2)) - 1/4*(B*b^3*c^2*g^3 - 4*B*a*b^2*c*d*g^
3 - 6*A*a^2*b*d^2*g^3 - 3*B*a^2*b*d^2*g^3)*x^2/d^2 + 1/2*(B*b^3*c^3*g^3 - 4
*B*a*b^2*c^2*d*g^3 + 6*B*a^2*b*c*d^2*g^3 + 2*A*a^3*d^3*g^3 - B*a^3*d^3*g^3)
*x/d^3 - 1/2*(B*b^3*c^4*g^3 - 4*B*a*b^2*c^3*d*g^3 + 6*B*a^2*b*c^2*d^2*g^3 -
 4*B*a^3*c*d^3*g^3)*log(-d*x - c)/d^4
```

Mupad [B]
time = 4.85, size = 567, normalized size = 3.75

$$\ln\left(\frac{e\,(c+d\,x)^2}{(a+b\,x)^2}\right)\left(B\,a^3\,g^3\,x+\frac{3\,B\,a^2\,b\,g^3\,x^2}{2}+B\,a\,b^2\,g^3\,x^3+\frac{B\,b^3\,g^3\,x^4}{4}\right)-x^2\left(\frac{\left(\frac{b^2\,g^3\,(8\,A\,a\,d+2\,A\,b\,c-B\,a\,d+B\,b\,c)}{2\,d}-\frac{A\,b^2\,g^3\,(2\,a\,d+2\,b\,c)}{2\,d}\right)(2\,a\,d+2\,b\,c)}{4\,b\,d}-\frac{a\,b\,g^3\,(3\,A\,a\,d+2\,A\,b\,c-B\,a\,d+B\,b\,c)}{d}+\frac{A\,a\,b^2\,c\,g^3}{2\,d}\right)+x\left(\frac{(2\,a\,d+2\,b\,c)\left(\frac{\left(\frac{b^2\,g^3\,(8\,A\,a\,d+2\,A\,b\,c-B\,a\,d+B\,b\,c)}{2\,d}-\frac{A\,b^2\,g^3\,(2\,a\,d+2\,b\,c)}{2\,d}\right)(2\,a\,d+2\,b\,c)}{2\,b\,d}-\frac{2\,a\,b\,g^3\,(3\,A\,a\,d+2\,A\,b\,c-B\,a\,d+B\,b\,c)}{d}+\frac{A\,a\,b^2\,c\,g^3}{d}\right)}{2\,b\,d}+\frac{a^2\,g^3\,(4\,A\,a\,d+6\,A\,b\,c-3\,B\,a\,d+3\,B\,b\,c)}{d}-\frac{a\,c\left(\frac{b^2\,g^3\,(8\,A\,a\,d+2\,A\,b\,c-B\,a\,d+B\,b\,c)}{2\,d}-\frac{A\,b^2\,g^3\,(2\,a\,d+2\,b\,c)}{2\,d}\right)}{b\,d}\right)+x^3\left(\frac{b^2\,g^3\,(8\,A\,a\,d+2\,A\,b\,c-B\,a\,d+B\,b\,c)}{6\,d}-\frac{A\,b^2\,g^3\,(2\,a\,d+2\,b\,c)}{6\,d}\right)-\frac{\ln(c+d\,x)\,\left(-4\,B\,a^3\,c\,d^3\,g^3+6\,B\,a^2\,b\,c^2\,d^2\,g^3-4\,B\,a\,b^2\,c^3\,d\,g^3+B\,b^3\,c^4\,g^3\right)}{2\,d^4}+\frac{A\,b^3\,g^3\,x^4}{4}-\frac{B\,a^4\,g^3\,\ln(a+b\,x)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*g + b*g*x)^3*(A + B*log((e*(c + d*x)^2)/(a + b*x)^2)),x)
```

```
[Out] log((e*(c + d*x)^2)/(a + b*x)^2)*((B*b^3*g^3*x^4)/4 + B*a^3*g^3*x + (3*B*a^
2*b*g^3*x^2)/2 + B*a*b^2*g^3*x^3) - x^2*((((b^2*g^3*(8*A*a*d + 2*A*b*c - B*
a*d + B*b*c))/(2*d) - (A*b^2*g^3*(2*a*d + 2*b*c))/(2*d))*(2*a*d + 2*b*c))/(
4*b*d) - (a*b*g^3*(3*A*a*d + 2*A*b*c - B*a*d + B*b*c))/d + (A*a*b^2*c*g^3)/
(2*d)) + x*(((2*a*d + 2*b*c)*((((b^2*g^3*(8*A*a*d + 2*A*b*c - B*a*d + B*b*c
))/(2*d) - (A*b^2*g^3*(2*a*d + 2*b*c))/(2*d))*(2*a*d + 2*b*c))/(2*b*d) - (2
*a*b*g^3*(3*A*a*d + 2*A*b*c - B*a*d + B*b*c))/d + (A*a*b^2*c*g^3)/d))/(2*b*
d) + (a^2*g^3*(4*A*a*d + 6*A*b*c - 3*B*a*d + 3*B*b*c))/d - (a*c*((b^2*g^3*(
8*A*a*d + 2*A*b*c - B*a*d + B*b*c))/(2*d) - (A*b^2*g^3*(2*a*d + 2*b*c))/(2*
d)))/(b*d)) + x^3*((b^2*g^3*(8*A*a*d + 2*A*b*c - B*a*d + B*b*c))/(6*d) - (A
*b^2*g^3*(2*a*d + 2*b*c))/(6*d)) - (log(c + d*x)*(B*b^3*c^4*g^3 - 4*B*a^3*c
*d^3*g^3 + 6*B*a^2*b*c^2*d^2*g^3 - 4*B*a*b^2*c^3*d*g^3))/(2*d^4) + (A*b^3*g
^3*x^4)/4 - (B*a^4*g^3*log(a + b*x))/(2*b)
```

3.203 $\int (ag+bgx)^2 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right) dx$

Optimal. Leaf size=120

$$-\frac{2B(bc-ad)^2g^2x}{3d^2}+\frac{B(bc-ad)g^2(a+bx)^2}{3bd}+\frac{2B(bc-ad)^3g^2\log(c+dx)}{3bd^3}+\frac{g^2(a+bx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{3b}$$

```
[Out] -2/3*B*(-a*d+b*c)^2*g^2*x/d^2+1/3*B*(-a*d+b*c)*g^2*(b*x+a)^2/b/d+2/3*B*(-a*
d+b*c)^3*g^2*ln(d*x+c)/b/d^3+1/3*g^2*(b*x+a)^3*(A+B*ln(e*(d*x+c)^2/(b*x+a)^
2))/b
```

Rubi [A]
time = 0.05, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {2548, 21, 45}

$$\frac{g^2(a+bx)^3\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{3b}+\frac{2Bg^2(bc-ad)^3\log(c+dx)}{3bd^3}-\frac{2Bg^2x(bc-ad)^2}{3d^2}+\frac{Bg^2(a+bx)^2(bc-ad)}{3bd}$$

Antiderivative was successfully verified.

```
[In] Int[(a*g + b*g*x)^2*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]),x]
```

```
[Out] (-2*B*(b*c - a*d)^2*g^2*x)/(3*d^2) + (B*(b*c - a*d)*g^2*(a + b*x)^2)/(3*b*d
) + (2*B*(b*c - a*d)^3*g^2*Log[c + d*x])/(3*b*d^3) + (g^2*(a + b*x)^3*(A +
B*Log[(e*(c + d*x)^2)/(a + b*x)^2]))/(3*b)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2548

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Dist[B*n*((b*c
```

```
- a*d)/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /;
 FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c -
 a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Rubi steps

$$\int (ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)dx=\frac{g^2(a+bx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{3b}-\frac{B\int\frac{2(-bc+ad)g^3(a+bx)^2}{c+dx}}{3bg}$$

$$=\frac{g^2(a+bx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{3b}+\frac{(2B(bc-ad)g^2)\int\frac{(a}{c}}{3b}$$

$$=\frac{g^2(a+bx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{3b}+\frac{(2B(bc-ad)g^2)\int\Big(}{}$$

$$=-\frac{2B(bc-ad)^2g^2x}{3d^2}+\frac{B(bc-ad)g^2(a+bx)^2}{3bd}+\frac{2B(bc-ad}{}$$

Mathematica [A]
time = 0.04, size = 98, normalized size = 0.82

$$\frac{g^2\left(\frac{B(bc-ad)\left(d\left(a^2d+4abdx+b^2x(-2c+dx)\right)\right)+2(bc-ad)^2\log(c+dx)}{d^3}+(a+bx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)\right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*g + b*g*x)^2*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]),x]
[Out] (g^2*((B*(b*c - a*d)*(d*(a^2*d + 4*a*b*d*x + b^2*x*(-2*c + d*x)) + 2*(b*c -
 a*d)^2*Log[c + d*x]))/d^3 + (a + b*x)^3*(A + B*Log[(e*(c + d*x)^2)/(a + b*
x)^2])))/(3*b)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 392 vs. 2(112) = 224.
time = 0.38, size = 393, normalized size = 3.28

method	result
risch	$\frac{g^2(bx+a)^3B\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)}{3b}+\frac{g^2b^2Ax^3}{3}+g^2bAa\,x^2-\frac{g^2bBa\,x^2}{3}+\frac{g^2b^2Bc\,x^2}{3d}+g^2A\,a^2x-\frac{2g^2B\ln(dx+c)}{3b}$
derivativedivides	$-\frac{-\frac{g^2A(bx+a)^3}{3}-\frac{g^2B(bx+a)^3\ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^2}{b^2}\right)}{3}+\frac{2g^2B\,a^3\ln\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)}{3}-\frac{2g^2B\,a^2\ln\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)bc}{d}+2g}{}$

default	$-\frac{-\frac{g^{2}A(bx+a)^{3}}{3}-\frac{g^{2}B(bx+a)^{3}\ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^{2}}{b^{2}}\right)}{3}+\frac{2g^{2}B\,a^{3}\ln\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)}{3}-\frac{2g^{2}B\,a^{2}\ln\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)bc}{d}+\cdots}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*g*x+a*g)^2*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/b*(-1/3*g^2*A*(b*x+a)^3-1/3*g^2*B*(b*x+a)^3*ln(e*(a*d/(b*x+a)-b*c/(b*x+a
)-d)^2/b^2)+2/3*g^2*B*a^3*ln(a*d/(b*x+a)-b*c/(b*x+a)-d)-2*g^2*B*a^2/d*ln(a*
d/(b*x+a)-b*c/(b*x+a)-d)*b*c+2*g^2*B*a/d^2*ln(a*d/(b*x+a)-b*c/(b*x+a)-d)*b^
2*c^2-2/3*g^2*B*a^3*ln(1/(b*x+a))+2*g^2*B*a^2/d*ln(1/(b*x+a))*b*c-2*g^2*B*a
/d^2*ln(1/(b*x+a))*b^2*c^2+1/3*g^2*B*a*(b*x+a)^2+2/3*g^2*B*a^2*(b*x+a)-4/3*
g^2*B*a/d*(b*x+a)*b*c-2/3*g^2*B*b^3*c^3/d^3*ln(a*d/(b*x+a)-b*c/(b*x+a)-d)+2
/3*g^2*B*b^3*c^3/d^3*ln(1/(b*x+a))-1/3*g^2*B*b*c/d*(b*x+a)^2+2/3*g^2*B*b^2*
c^2/d^2*(b*x+a))
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 445 vs. $2(113) = 226$.
time = 0.31, size = 445, normalized size = 3.71

$$\frac{1}{3}Ab^{2}g^{2}x^{3}+Aabg^{2}x^{2}+\left(x\log\left(\frac{d^{2}x^{2}e}{b^{2}x^{2}+2\,abx+a^{2}}+\frac{2\,cdxe}{b^{2}x^{2}+2\,abx+a^{2}}+\frac{c^{2}e}{b^{2}x^{2}+2\,abx+a^{2}}\right)-\frac{2\,a\log\left(bx+a\right)}{b}+\frac{2\,c\log\left(dx+c\right)}{d}\right)Ba^{2}g^{2}+\left(x^{2}\log\left(\frac{d^{2}x^{2}e}{b^{2}x^{2}+2\,abx+a^{2}}+\frac{2\,cdxe}{b^{2}x^{2}+2\,abx+a^{2}}+\frac{c^{2}e}{b^{2}x^{2}+2\,abx+a^{2}}\right)+\frac{2\,a^{2}\log\left(bx+a\right)}{b^{2}}-\frac{2\,c^{2}\log\left(dx+c\right)}{d^{2}}+\frac{2\,(bc-ad)x}{bd}\right)Babg^{2}+\frac{1}{3}\left(x^{3}\log\left(\frac{d^{2}x^{2}e}{b^{2}x^{2}+2\,abx+a^{2}}+\frac{2\,cdxe}{b^{2}x^{2}+2\,abx+a^{2}}+\frac{c^{2}e}{b^{2}x^{2}+2\,abx+a^{2}}\right)-\frac{2\,a^{3}\log\left(bx+a\right)}{b^{3}}+\frac{2\,c^{3}\log\left(dx+c\right)}{d^{3}}+\frac{(b^{2}cd-abd^{2})x^{2}-2\,(b^{2}c^{2}-a^{2}d^{2})x}{b^{2}d^{2}}\right)Bb^{2}g^{2}+Aa^{2}g^{2}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^2*(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="maxi
ma")
```

```
[Out] 1/3*A*b^2*g^2*x^3 + A*a*b*g^2*x^2 + (x*log(d^2*x^2*e/(b^2*x^2 + 2*a*b*x + a
^2) + 2*c*d*x*e/(b^2*x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2)
) - 2*a*log(b*x + a)/b + 2*c*log(d*x + c)/d)*B*a^2*g^2 + (x^2*log(d^2*x^2*e
/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*x*e/(b^2*x^2 + 2*a*b*x + a^2) + c^2*e/(b
^2*x^2 + 2*a*b*x + a^2)) + 2*a^2*log(b*x + a)/b^2 - 2*c^2*log(d*x + c)/d^2
+ 2*(b*c - a*d)*x/(b*d))*B*a*b*g^2 + 1/3*(x^3*log(d^2*x^2*e/(b^2*x^2 + 2*a*
b*x + a^2) + 2*c*d*x*e/(b^2*x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x
 + a^2)) - 2*a^3*log(b*x + a)/b^3 + 2*c^3*log(d*x + c)/d^3 + ((b^2*c*d - a*
b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*B*b^2*g^2 + A*a^2*g^2*x
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 243 vs. $2(113) = 226$.
time = 0.38, size = 243, normalized size = 2.02

$$\frac{Ab^{3}d^{3}g^{2}x^{3}-2\,Ba^{3}d^{3}g^{2}\log\left(bx+a\right)+(Bb^{3}cd^{2}+(3\,A-B)ab^{2}d^{3})g^{2}x^{2}-(2\,Bb^{3}c^{2}d-6\,Bab^{2}cd^{2}-(3\,A-4\,B)a^{2}bd^{3})g^{2}x+2\,(Bb^{3}c^{3}-3\,Bab^{2}c^{2}d+3\,Ba^{2}bcd^{2})g^{2}\log\left(dx+c\right)+(Bb^{3}d^{3}g^{2}x^{3}+3\,Bab^{2}d^{3}g^{2}x^{2}+3\,Ba^{2}bd^{3}g^{2}x)\log\left(\frac{(d^{2}x^{2}+2\,cdx+c^{2})e}{b^{2}x^{2}+2\,abx+a^{2}}\right)}{3\,bd^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^2*(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="fric
as")
```

```
[Out] 1/3*(A*b^3*d^3*g^2*x^3 - 2*B*a^3*d^3*g^2*log(b*x + a) + (B*b^3*c*d^2 + (3*A
 - B)*a*b^2*d^3)*g^2*x^2 - (2*B*b^3*c^2*d - 6*B*a*b^2*c*d^2 - (3*A - 4*B)*a
^2*b*d^3)*g^2*x + 2*(B*b^3*c^3 - 3*B*a*b^2*c^2*d + 3*B*a^2*b*c*d^2)*g^2*log
(d*x + c) + (B*b^3*d^3*g^2*x^3 + 3*B*a*b^2*d^3*g^2*x^2 + 3*B*a^2*b*d^3*g^2*
x)*log((d^2*x^2 + 2*c*d*x + c^2)*e/(b^2*x^2 + 2*a*b*x + a^2)))/(b*d^3)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 517 vs. $2(107) = 214$.
time = 1.92, size = 517, normalized size = 4.31

$$\frac{Ab^2g^2x^3}{3} - \frac{2Ba^3g^2\log\left(x + \frac{\frac{2Ba^4d^3g^2}{b} + 6Ba^3cd^2g^2 - 6Ba^2bc^2dg^2 + 2Bab^2c^3g^2}{2Ba^3d^3g^2 + 6Ba^2bcd^2g^2 - 6Bab^2c^2dg^2 + 2Bb^3c^3g^2}\right)}{3b} + \frac{2Bcg^2\cdot(3a^2d^2 - 3abcd + b^2c^2)\log\left(x + \frac{8Ba^3cd^2g^2 - 6Ba^2bc^2dg^2 + 2Bab^2c^3g^2 - 2Bacg^2\cdot(3a^2d^2 - 3abcd + b^2c^2) + \frac{2Bbc^2g^2\cdot(3a^2d^2 - 3abcd + b^2c^2)}{d}}{2Ba^3d^3g^2 + 6Ba^2bcd^2g^2 - 6Bab^2c^2dg^2 + 2Bb^3c^3g^2}\right)}{3d^3} + x^2\left(Aabg^2 - \frac{Babg^2}{3} + \frac{Bb^2cg^2}{3d}\right) + x\left(Aa^2g^2 - \frac{4Ba^2g^2}{3} + \frac{2Babcg^2}{d} - \frac{2Bb^2c^2g^2}{3d^2}\right) + \left(Ba^2g^2x + Babg^2x^2 + \frac{Bb^2g^2x^3}{3}\right)\log\left(\frac{e(c + dx)^2}{(a + bx)^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)**2*(A+B*ln(e*(d*x+c)**2/(b*x+a)**2)),x)
```

```
[Out] A*b**2*g**2*x**3/3 - 2*B*a**3*g**2*log(x + (2*B*a**4*d**3*g**2/b + 6*B*a**3
*c*d**2*g**2 - 6*B*a**2*b*c**2*d*g**2 + 2*B*a*b**2*c**3*g**2)/(2*B*a**3*d**
3*g**2 + 6*B*a**2*b*c*d**2*g**2 - 6*B*a*b**2*c**2*d*g**2 + 2*B*b**3*c**3*g*
*2))/(3*b) + 2*B*c*g**2*(3*a**2*d**2 - 3*a*b*c*d + b**2*c**2)*log(x + (8*B*
a**3*c*d**2*g**2 - 6*B*a**2*b*c**2*d*g**2 + 2*B*a*b**2*c**3*g**2 - 2*B*a*c*
g**2*(3*a**2*d**2 - 3*a*b*c*d + b**2*c**2) + 2*B*b*c**2*g**2*(3*a**2*d**2 -
 3*a*b*c*d + b**2*c**2)/d)/(2*B*a**3*d**3*g**2 + 6*B*a**2*b*c*d**2*g**2 - 6
*B*a*b**2*c**2*d*g**2 + 2*B*b**3*c**3*g**2))/(3*d**3) + x**2*(A*a*b*g**2 -
B*a*b*g**2/3 + B*b**2*c*g**2/(3*d)) + x*(A*a**2*g**2 - 4*B*a**2*g**2/3 + 2*
B*a*b*c*g**2/d - 2*B*b**2*c**2*g**2/(3*d**2)) + (B*a**2*g**2*x + B*a*b*g**2
*x**2 + B*b**2*g**2*x**3/3)*log(e*(c + d*x)**2/(a + b*x)**2)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 248 vs. $2(113) = 226$.
time = 13.76, size = 248, normalized size = 2.07

$$-\frac{2\,Ba^3g^2\log\left(bx + a\right)}{3\,b} + \frac{1}{3}\,(Ab^2g^2 + Bb^2g^2)x^3 + \frac{(Bb^2cg^2 + 3\,Aabdg^2 + 2\,Babdg^2)x^2}{3\,d} + \frac{1}{3}\,(Bb^2g^2x^3 + 3\,Babg^2x^2 + 3\,Ba^2g^2x)\log\left(\frac{d^2x^2 + 2\,cdx + c^2}{b^2x^2 + 2\,abx + a^2}\right) - \frac{(2\,Bb^2c^2g^2 - 6\,Babcdg^2 - 3\,Aa^2d^2g^2 + Ba^2d^2g^2)x}{3\,d^2} + \frac{2\,(Bb^2c^3g^2 - 3\,Babc^2dg^2 + 3\,Ba^2cd^2g^2)\log\left(dx + c\right)}{3\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^2*(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="giac
")
```

```
[Out] -2/3*B*a^3*g^2*log(b*x + a)/b + 1/3*(A*b^2*g^2 + B*b^2*g^2)*x^3 + 1/3*(B*b^
2*c*g^2 + 3*A*a*b*d*g^2 + 2*B*a*b*d*g^2)*x^2/d + 1/3*(B*b^2*g^2*x^3 + 3*B*a
*b*g^2*x^2 + 3*B*a^2*g^2*x)*log((d^2*x^2 + 2*c*d*x + c^2)/(b^2*x^2 + 2*a*b*
x + a^2)) - 1/3*(2*B*b^2*c^2*g^2 - 6*B*a*b*c*d*g^2 - 3*A*a^2*d^2*g^2 + B*a^
2*d^2*g^2)*x/d^2 + 2/3*(B*b^2*c^3*g^2 - 3*B*a*b*c^2*d*g^2 + 3*B*a^2*c*d^2*g
^2)*log(d*x + c)/d^3
```

Mupad [B]
time = 4.65, size = 296, normalized size = 2.47

$$x^2\left(\frac{b\,g^2\,(9\,A\,a\,d + 3\,A\,b\,c - 2\,B\,a\,d + 2\,B\,b\,c)}{6\,d} - \frac{A\,b\,g^2\,(3\,a\,d + 3\,b\,c)}{6\,d}\right) - x\left(\frac{(3\,a\,d + 3\,b\,c)\,\left(\frac{b\,g^2\,(9\,A\,a\,d + 3\,A\,b\,c - 2\,B\,a\,d + 2\,B\,b\,c)}{3\,d} - \frac{A\,b\,g^2\,(3\,a\,d + 3\,b\,c)}{3\,d}\right)}{3\,b\,d} - \frac{a\,g^2\,(3\,A\,a\,d + 3\,A\,b\,c - 2\,B\,a\,d + 2\,B\,b\,c)}{d} + \frac{A\,a\,b\,c\,g^2}{d}\right) + \ln\left(\frac{e\,(c + d\,x)^2}{(a + b\,x)^2}\right)\,\left(B\,a^2\,g^2\,x + B\,a\,b\,g^2\,x^2 + \frac{B\,b^2\,g^2\,x^3}{3}\right) + \frac{\ln(c + d\,x)\,(6\,B\,a^2\,c\,d^2\,g^2 - 6\,B\,a\,b\,c^2\,d\,g^2 + 2\,B\,b^2\,c^3\,g^2)}{3\,d^3} + \frac{A\,b^2\,g^2\,x^3}{3} - \frac{2\,B\,a^3\,g^2\,\ln(a + b\,x)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*g + b*g*x)^2*(A + B*log((e*(c + d*x)^2)/(a + b*x)^2)),x)
```

```
[Out] x^2*((b*g^2*(9*A*a*d + 3*A*b*c - 2*B*a*d + 2*B*b*c))/(6*d) - (A*b*g^2*(3*a*
d + 3*b*c))/(6*d)) - x*(((3*a*d + 3*b*c)*((b*g^2*(9*A*a*d + 3*A*b*c - 2*B*a
*d + 2*B*b*c))/(3*d) - (A*b*g^2*(3*a*d + 3*b*c))/(3*d)))/(3*b*d) - (a*g^2*(
3*A*a*d + 3*A*b*c - 2*B*a*d + 2*B*b*c))/d + (A*a*b*c*g^2)/d) + log((e*(c +
d*x)^2)/(a + b*x)^2)*((B*b^2*g^2*x^3)/3 + B*a^2*g^2*x + B*a*b*g^2*x^2) + (l
og(c + d*x)*(2*B*b^2*c^3*g^2 + 6*B*a^2*c*d^2*g^2 - 6*B*a*b*c^2*d*g^2))/(3*d
^3) + (A*b^2*g^2*x^3)/3 - (2*B*a^3*g^2*log(a + b*x))/(3*b)
```

3.204 $\int (ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)\,dx$

Optimal. Leaf size=78

$$\frac{B(bc-ad)gx}{d}-\frac{B(bc-ad)^2g\log(c+dx)}{bd^2}+\frac{g(a+bx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{2b}$$

```
[Out] B*(-a*d+b*c)*g*x/d-B*(-a*d+b*c)^2*g*ln(d*x+c)/b/d^2+1/2*g*(b*x+a)^2*(A+B*ln
(e*(d*x+c)^2/(b*x+a)^2))/b
```

Rubi [A]

time = 0.03, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2548, 21, 45}

$$\frac{g(a+bx)^2\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{2b}-\frac{Bg(bc-ad)^2\log(c+dx)}{bd^2}+\frac{Bgx(bc-ad)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a*g + b*g*x)*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]),x]
```

```
[Out] (B*(b*c - a*d)*g*x)/d - (B*(b*c - a*d)^2*g*Log[c + d*x])/(b*d^2) + (g*(a +
b*x)^2*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]))/(2*b)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2548

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Dist[B*n*((b*c
- a*d)/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /;
 FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c -
```

```
a*d, 0] && NeQ[m, -1] && !(EqQ[m, -2] && IntegerQ[n])
```

Rubi steps

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)dx=\frac{g(a+bx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{2b}-\frac{B\int\frac{2(bc-ad)g^2(-a-bx)}{c+dx}d}{2bg}$$
$$=\frac{g(a+bx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{2b}-\frac{(B(bc-ad)g)\int\frac{-a-b}{c+d}}{b}$$
$$=\frac{g(a+bx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{2b}-\frac{(B(bc-ad)g)\int\left(-\frac{b}{a}\right.}{b}$$
$$=\frac{B(bc-ad)gx}{d}-\frac{B(bc-ad)^2g\log(c+dx)}{bd^2}+\frac{g(a+bx)^2\left(A\right.}{}$$

Mathematica [A]
time = 0.03, size = 72, normalized size = 0.92

$$\frac{g\left(-\frac{2B(-bc+ad)(bdx+(-bc+ad)\log(c+dx))}{d^2}+(a+bx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*g + b*g*x)*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]),x]
[Out] (g*((-2*B*(-(b*c) + a*d)*(b*d*x + (-(b*c) + a*d)*Log[c + d*x]))/d^2 + (a +
b*x)^2*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])))/(2*b)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 244 vs. $2(76) = 152$.
time = 0.27, size = 245, normalized size = 3.14

method	result
risch	$\frac{gBx(bx+2a)\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)}{2}+\frac{gbA\,x^2}{2}+gAax+\frac{2gB\ln(-dx-c)ac}{d}-\frac{gbB\ln(-dx-c)c^2}{d^2}-\frac{B\,a^2g\ln(bx+a)}{b}-$
derivativedivides	$-\frac{-\frac{gA(bx+a)^2}{2}-\frac{gB(bx+a)^2\ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^2}{b^2}\right)}{2}-gB\ln\left(\frac{1}{bx+a}\right)a^2+\frac{2gB\ln\left(\frac{1}{bx+a}\right)abc}{d}-\frac{gB\ln\left(\frac{1}{bx+a}\right)b^2c^2}{d^2}+gB(bx+}{b}$
default	$-\frac{-\frac{gA(bx+a)^2}{2}-\frac{gB(bx+a)^2\ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^2}{b^2}\right)}{2}-gB\ln\left(\frac{1}{bx+a}\right)a^2+\frac{2gB\ln\left(\frac{1}{bx+a}\right)abc}{d}-\frac{gB\ln\left(\frac{1}{bx+a}\right)b^2c^2}{d^2}+gB(bx+}{b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*g*x+a*g)*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/b*(-1/2*g*A*(b*x+a)^2-1/2*g*B*(b*x+a)^2*ln(e*(a*d/(b*x+a)-b*c/(b*x+a)-d)
^2/b^2)-g*B*ln(1/(b*x+a))*a^2+2*g*B/d*ln(1/(b*x+a))*a*b*c-g*B/d^2*ln(1/(b*x
+a))*b^2*c^2+g*B*(b*x+a)*a-g*B/d*(b*x+a)*b*c+g*B*ln(a*d/(b*x+a)-b*c/(b*x+a)
-d)*a^2-2*g*B/d*ln(a*d/(b*x+a)-b*c/(b*x+a)-d)*a*b*c+g*B/d^2*ln(a*d/(b*x+a)-
b*c/(b*x+a)-d)*b^2*c^2)
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 256 vs. $2(77) = 154$.
time = 0.29, size = 256, normalized size = 3.28

$$\frac{1}{2}\,Abgx^2 + \left(x\log\left(\frac{d^2x^2e}{b^2x^2 + 2\,abx + a^2} + \frac{2\,cdxe}{b^2x^2 + 2\,abx + a^2} + \frac{c^2e}{b^2x^2 + 2\,abx + a^2}\right) - \frac{2\,a\log\left(bx + a\right)}{b} + \frac{2\,c\log\left(dx + c\right)}{d}\right)Bag + \frac{1}{2}\left(x^2\log\left(\frac{d^2x^2e}{b^2x^2 + 2\,abx + a^2} + \frac{2\,cdxe}{b^2x^2 + 2\,abx + a^2} + \frac{c^2e}{b^2x^2 + 2\,abx + a^2}\right) + \frac{2\,a^2\log\left(bx + a\right)}{b^2} - \frac{2\,c^2\log\left(dx + c\right)}{d^2} + \frac{2\,(bc - ad)x}{bd}\right)Bbg + Aagx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)*(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="maxima
")
```

```
[Out] 1/2*A*b*g*x^2 + (x*log(d^2*x^2*e/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*x*e/(b^2
*x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2)) - 2*a*log(b*x + a)
/b + 2*c*log(d*x + c)/d)*B*a*g + 1/2*(x^2*log(d^2*x^2*e/(b^2*x^2 + 2*a*b*x
+ a^2) + 2*c*d*x*e/(b^2*x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a
^2)) + 2*a^2*log(b*x + a)/b^2 - 2*c^2*log(d*x + c)/d^2 + 2*(b*c - a*d)*x/(b
*d))*B*b*g + A*a*g*x
```

Fricas [**A**]
time = 0.46, size = 147, normalized size = 1.88

$$\frac{Ab^2d^2gx^2 - 2\,Ba^2d^2g\log\left(bx + a\right) + 2\,(Bb^2cd + (A - B)abd^2)gx - 2\,(Bb^2c^2 - 2\,Babcd)g\log\left(dx + c\right) + (Bb^2d^2gx^2 + 2\,Babd^2gx)\log\left(\frac{(d^2x^2 + 2\,cdx + c^2)e}{b^2x^2 + 2\,abx + a^2}\right)}{2\,bd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)*(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="fricas
")
```

```
[Out] 1/2*(A*b^2*d^2*g*x^2 - 2*B*a^2*d^2*g*log(b*x + a) + 2*(B*b^2*c*d + (A - B)*
a*b*d^2)*g*x - 2*(B*b^2*c^2 - 2*B*a*b*c*d)*g*log(d*x + c) + (B*b^2*d^2*g*x^
2 + 2*B*a*b*d^2*g*x)*log((d^2*x^2 + 2*c*d*x + c^2)*e/(b^2*x^2 + 2*a*b*x + a
^2)))/(b*d^2)
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 250 vs. $2(68) = 136$.
time = 1.44, size = 250, normalized size = 3.21

$$\frac{Abgx^2}{2} - \frac{Ba^2g\log\left(x + \frac{\frac{Ba^3d^2g}{b} + 2Ba^2cdg - Babc^2g}{Ba^2d^2g + 2Babcdg - Bb^2c^2g}\right)}{b} + \frac{Bcg(2ad - bc)\log\left(x + \frac{3Ba^2cdg - Babc^2g - Bacg(2ad - bc) + \frac{Bbc^2g(2ad - bc)}{d}}{Ba^2d^2g + 2Babcdg - Bb^2c^2g}\right)}{d^2} + x\left(Aag - Bag + \frac{Bbcg}{d}\right) + \left(Bagx + \frac{Bbgx^2}{2}\right)\log\left(\frac{e(c + dx)^2}{(a + bx)^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)*(A+B*ln(e*(d*x+c)**2/(b*x+a)**2)),x)
```

```
[Out] A*b*g*x**2/2 - B*a**2*g*log(x + (B*a**3*d**2*g/b + 2*B*a**2*c*d*g - B*a*b*c
**2*g)/(B*a**2*d**2*g + 2*B*a*b*c*d*g - B*b**2*c**2*g))/b + B*c*g*(2*a*d -
b*c)*log(x + (3*B*a**2*c*d*g - B*a*b*c**2*g - B*a*c*g*(2*a*d - b*c) + B*b*c
**2*g*(2*a*d - b*c)/d)/(B*a**2*d**2*g + 2*B*a*b*c*d*g - B*b**2*c**2*g))/d**
2 + x*(A*a*g - B*a*g + B*b*c*g/d) + (B*a*g*x + B*b*g*x**2/2)*log(e*(c + d*x
)**2/(a + b*x)**2)
```

Giac [A]
time = 7.09, size = 128, normalized size = 1.64

$$-\frac{Ba^2g\log(bx+a)}{b}+\frac{1}{2}(Abg+Bbg)x^2+\frac{1}{2}\left(Bbgx^2+2\,Bagx\right)\log\left(\frac{d^2x^2+2\,cdx+c^2}{b^2x^2+2\,abx+a^2}\right)+\frac{(Bbcg+Aadg)x}{d}-\frac{(Bbc^2g-2\,Bacdg)\log(-dx-c)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)*(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="giac")
```

```
[Out] -B*a^2*g*log(b*x + a)/b + 1/2*(A*b*g + B*b*g)*x^2 + 1/2*(B*b*g*x^2 + 2*B*a*
g*x)*log((d^2*x^2 + 2*c*d*x + c^2)/(b^2*x^2 + 2*a*b*x + a^2)) + (B*b*c*g +
A*a*d*g)*x/d - (B*b*c^2*g - 2*B*a*c*d*g)*log(-d*x - c)/d^2
```

Mupad [B]
time = 4.38, size = 120, normalized size = 1.54

$$x\left(\frac{g\,(2\,A\,a\,d+A\,b\,c-B\,a\,d+B\,b\,c)}{d}-\frac{A\,g\,(a\,d+b\,c)}{d}\right)+\ln\left(\frac{e\,{(c+d\,x)}^2}{{(a+b\,x)}^2}\right)\,\left(\frac{B\,b\,g\,x^2}{2}+B\,a\,g\,x\right)+\frac{A\,b\,g\,x^2}{2}-\frac{B\,a^2\,g\,\ln(a+b\,x)}{b}+\frac{B\,c\,g\,\ln(c+d\,x)\,(2\,a\,d-b\,c)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*g + b*g*x)*(A + B*log((e*(c + d*x)^2)/(a + b*x)^2)),x)
```

```
[Out] x*((g*(2*A*a*d + A*b*c - B*a*d + B*b*c))/d - (A*g*(a*d + b*c))/d) + log((e*
(c + d*x)^2)/(a + b*x)^2)*((B*b*g*x^2)/2 + B*a*g*x) + (A*b*g*x^2)/2 - (B*a^
2*g*log(a + b*x))/b + (B*c*g*log(c + d*x)*(2*a*d - b*c))/d^2
```

3.205 $\int \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{ag+bgx}\,dx$

Optimal. Leaf size=83

$$-\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{bg}-\frac{2B\text{Li}_2\left(1+\frac{bc-ad}{d(a+bx)}\right)}{bg}$$

```
[Out] -ln((a*d-b*c)/d/(b*x+a))*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))/b/g-2*B*polylog(2,
1+(-a*d+b*c)/d/(b*x+a))/b/g
```

Rubi [A]
time = 0.15, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {2542, 2458, 2378, 2370, 2352}

$$-\frac{2B\text{PolyLog}\left(2,\frac{bc-ad}{d(a+bx)}+1\right)}{bg}-\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{bg}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])/(a*g + b*g*x),x]
```

```
[Out] -((Log[-((b*c - a*d)/(d*(a + b*x)))]*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2
]))/(b*g)) - (2*B*PolyLog[2, 1 + (b*c - a*d)/(d*(a + b*x))])/(b*g)
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2370

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)/(x_))^(q_.)*(
x_)^(m_.), x_Symbol] :> Int[(e + d*x)^q*(a + b*Log[c*x^n])^p, x] /; FreeQ[{
a, b, c, d, e, m, n, p}, x] && EqQ[m, q] && IntegerQ[q]
```

Rule 2378

```
Int[((a_.) + Log[(c_.)*(x_)^(n_)]*(b_.))/((x_)*((d_) + (e_.)*(x_)^(r_.))),
x_Symbol] :> Dist[1/n, Subst[Int[(a + b*Log[c*x])/(x*(d + e*x^(r/n))), x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IntegerQ[r/n]
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
```

```
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2542

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))/((f_.) + (g_.)*(x_)), x_Symbol] :> Simp[(-Log[-(b*c - a*d)/(d*(a
+ b*x))])*((A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/g), x] + Dist[B*n*((b*c
 - a*d)/g), Int[Log[-(b*c - a*d)/(d*(a + b*x))]/((a + b*x)*(c + d*x)), x],
x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
 - a*d, 0] && EqQ[b*f - a*g, 0]
```

Rubi steps

$$\int \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{ag+bgx}\,dx = \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)\log(ag+bgx)}{bg} - \frac{B\int \frac{(a+bx)^2\left(\frac{2de(c+dx)}{(a+bx)^2}-\frac{2be(c+dx)^2}{(a+bx)^3}\right)\log(ag+}{e(c+dx)^2}}{bg}$$

$$= \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)\log(ag+bgx)}{bg} - \frac{B\int \frac{(a+bx)^2\left(\frac{2de(c+dx)}{(a+bx)^2}-\frac{2be(c+dx)^2}{(a+bx)^3}\right)\log(ag+}{(c+dx)^2}}{beg}$$

$$= \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)\log(ag+bgx)}{bg} - \frac{B\int\left(-\frac{2be\log(ag+bgx)}{a+bx}+\frac{2de\log(ag+bgx)}{c+dx}\right.}{beg}$$

$$= \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)\log(ag+bgx)}{bg} + \frac{(2B)\int\frac{\log(ag+bgx)}{a+bx}\,dx}{g} - \frac{(2Bd)\int\frac{\log(a}{c}}{bg}$$

$$= -\frac{2B\log\left(\frac{b(c+dx)}{bc-ad}\right)\log(ag+bgx)}{bg} + \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)\log(ag+bgx)}{bg} + ($$

$$= -\frac{2B\log\left(\frac{b(c+dx)}{bc-ad}\right)\log(ag+bgx)}{bg} + \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)\log(ag+bgx)}{bg} + $$

$$= \frac{B\log^2(g(a+bx))}{bg} - \frac{2B\log\left(\frac{b(c+dx)}{bc-ad}\right)\log(ag+bgx)}{bg} + \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right.}{bg}$$

Mathematica [A]
time = 0.03, size = 87, normalized size = 1.05

$$\frac{\log(a+bx)\left(A+B\log(a+bx)-2B\log\left(\frac{b(c+dx)}{bc-ad}\right)+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)-2B\text{Li}_2\left(\frac{d(a+bx)}{-bc+ad}\right)}{bg}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])/(a*g + b*g*x),x]
```

```
[Out] (Log[a + b*x]*(A + B*Log[a + b*x] - 2*B*Log[(b*(c + d*x))/(b*c - a*d)] + B*
Log[(e*(c + d*x)^2)/(a + b*x)^2]) - 2*B*PolyLog[2, (d*(a + b*x))/(-(b*c) +
a*d)])/(b*g)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 257 vs. $2(82) = 164$.
time = 0.39, size = 258, normalized size = 3.11

method	result
derivativedivides	$-\frac{\frac{A\ln\left(\frac{1}{bx+a}\right)}{g}+\frac{B\ln\left(\frac{1}{bx+a}\right)\ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^{2}}{b^{2}}\right)}{g}-\frac{2B\,\mathrm{dilog}\left(-\frac{\frac{ad-cb}{bx+a}-d}{d}\right)ad}{g(ad-cb)}+\frac{2B\,\mathrm{dilog}\left(-\frac{\frac{ad-cb}{bx+a}-d}{d}\right)bc}{g(ad-cb)}-\frac{2B\ln\left(\frac{1}{bx+a}\right)\cdots}{g\cdots}}{b}$
default	$-\frac{\frac{A\ln\left(\frac{1}{bx+a}\right)}{g}+\frac{B\ln\left(\frac{1}{bx+a}\right)\ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^{2}}{b^{2}}\right)}{g}-\frac{2B\,\mathrm{dilog}\left(-\frac{\frac{ad-cb}{bx+a}-d}{d}\right)ad}{g(ad-cb)}+\frac{2B\,\mathrm{dilog}\left(-\frac{\frac{ad-cb}{bx+a}-d}{d}\right)bc}{g(ad-cb)}-\frac{2B\ln\left(\frac{1}{bx+a}\right)\cdots}{g\cdots}}{b}$
risch	$\frac{A\ln(bx+a)}{gb}-\frac{B\ln\left(\frac{1}{bx+a}\right)\ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^{2}}{b^{2}}\right)}{bg}+\frac{2B\,\mathrm{dilog}\left(-\frac{\frac{ad-cb}{bx+a}-d}{d}\right)ad}{bg(ad-cb)}-\frac{2B\,\mathrm{dilog}\left(-\frac{\frac{ad-cb}{bx+a}-d}{d}\right)c}{g(ad-cb)}+\cdots$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*(d*x+c)^2/(b*x+a)^2))/(b*g*x+a*g),x,method=_RETURNVERBOSE)
```

```
[Out] -1/b*(1/g*A*ln(1/(b*x+a))+1/g*B*ln(1/(b*x+a))*ln(e*(a*d/(b*x+a)-b*c/(b*x+a)
-d)^2/b^2)-2/g*B*dilog(-((a*d-b*c)/(b*x+a)-d)/d)/(a*d-b*c)*a*d+2/g*B*dilog(
-((a*d-b*c)/(b*x+a)-d)/d)/(a*d-b*c)*b*c-2/g*B*ln(1/(b*x+a))*ln(-((a*d-b*c)/
(b*x+a)-d)/d)/(a*d-b*c)*a*d+2/g*B*ln(1/(b*x+a))*ln(-((a*d-b*c)/(b*x+a)-d)/d
)/(a*d-b*c)*b*c)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))/(b*g*x+a*g),x, algorithm="maxima
")
```

```
[Out] B*(2*log(b*x + a)*log(d*x + c)/(b*g) - integrate(-(b*d*x + b*c - 2*(2*b*d*x
 + b*c + a*d)*log(b*x + a))/(b^2*d*g*x^2 + a*b*c*g + (b^2*c*g + a*b*d*g)*x)
, x)) + A*log(b*g*x + a*g)/(b*g)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))/(b*g*x+a*g),x, algorithm="fricas
")
```

```
[Out] integral((B*log((d^2*x^2 + 2*c*d*x + c^2)*e/(b^2*x^2 + 2*a*b*x + a^2)) + A)
/(b*g*x + a*g), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A}{a+bx}\,dx + \int \frac{B\log\left(\frac{c^2e}{a^2+2abx+b^2x^2}+\frac{2cdex}{a^2+2abx+b^2x^2}+\frac{d^2ex^2}{a^2+2abx+b^2x^2}\right)}{a+bx}\,dx}{g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(d*x+c)**2/(b*x+a)**2))/(b*g*x+a*g),x)
```

```
[Out] (Integral(A/(a + b*x), x) + Integral(B*log(c**2*e/(a**2 + 2*a*b*x + b**2*x*
*2) + 2*c*d*e*x/(a**2 + 2*a*b*x + b**2*x**2) + d**2*e*x**2/(a**2 + 2*a*b*x
+ b**2*x**2))/(a + b*x), x))/g
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))/(b*g*x+a*g),x, algorithm="giac")
```

```
[Out] integrate((B*log((d*x + c)^2*e/(b*x + a)^2) + A)/(b*g*x + a*g), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\,\ln\left(\frac{e\,{\left(c+d\,x\right)}^2}{{\left(a+b\,x\right)}^2}\right)}{a\,g+b\,g\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(c + d*x)^2)/(a + b*x)^2))/(a*g + b*g*x),x)
```

```
[Out] int((A + B*log((e*(c + d*x)^2)/(a + b*x)^2))/(a*g + b*g*x), x)
```

3.206 $\int \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(ag+bgx)^2}\,dx$

Optimal. Leaf size=102

$$-\frac{A(c+dx)}{(bc-ad)g^2(a+bx)}+\frac{2B(c+dx)}{(bc-ad)g^2(a+bx)}-\frac{B(c+dx)\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(bc-ad)g^2(a+bx)}$$

```
[Out] -A*(d*x+c)/(-a*d+b*c)/g^2/(b*x+a)+2*B*(d*x+c)/(-a*d+b*c)/g^2/(b*x+a)-B*(d*x
+c)*ln(e*(d*x+c)^2/(b*x+a)^2)/(-a*d+b*c)/g^2/(b*x+a)
```

Rubi [A]
time = 0.03, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {2552, 2332}

$$-\frac{A(c+dx)}{g^2(a+bx)(bc-ad)}-\frac{B(c+dx)\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{g^2(a+bx)(bc-ad)}+\frac{2B(c+dx)}{g^2(a+bx)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])/(a*g + b*g*x)^2,x]
```

```
[Out] -((A*(c + d*x))/((b*c - a*d)*g^2*(a + b*x))) + (2*B*(c + d*x))/((b*c - a*d)
*g^2*(a + b*x)) - (B*(c + d*x)*Log[(e*(c + d*x)^2)/(a + b*x)^2])/((b*c - a*
d)*g^2*(a + b*x))
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2552

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(
m + 1)*(g/d)^m, Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a
 + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[n
 + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[d*f
- c*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Rubi steps

$$\int \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(ag+bgx)^2}\,dx = -\frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{bg^2(a+bx)} + \frac{B\int \frac{2(-bc+ad)}{g(a+bx)^2(c+dx)}\,dx}{bg}$$
$$= -\frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{bg^2(a+bx)} - \frac{(2B(bc-ad))\int \frac{1}{(a+bx)^2(c+dx)}\,dx}{bg^2}$$
$$= -\frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{bg^2(a+bx)} - \frac{(2B(bc-ad))\int \left(\frac{b}{(bc-ad)(a+bx)^2} - \frac{bd}{(bc-ad)^2(a+bx)} + \frac{}{(bc}\right.}{bg^2}$$
$$= \frac{2B}{bg^2(a+bx)} + \frac{2Bd\log(a+bx)}{b(bc-ad)g^2} - \frac{2Bd\log(c+dx)}{b(bc-ad)g^2} - \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{bg^2(a+bx)}$$

Mathematica [A]
time = 0.04, size = 89, normalized size = 0.87

$$\frac{2Bd(a+bx)\log(a+bx) - 2Bd(a+bx)\log(c+dx) - (bc-ad)\left(A-2B+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{b(bc-ad)g^2(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])/(a*g + b*g*x)^2,x]
```

```
[Out] (2*B*d*(a + b*x)*Log[a + b*x] - 2*B*d*(a + b*x)*Log[c + d*x] - (b*c - a*d)*
(A - 2*B + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]))/(b*(b*c - a*d)*g^2*(a + b*x
))
```

Maple [A]
time = 0.36, size = 205, normalized size = 2.01

method	result
norman	$\frac{\frac{(A-2B)x}{ga}+\frac{Bc\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)}{g(ad-cb)}+\frac{dBx\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)}{g(ad-cb)}}{g(bx+a)}$
risch	$-\frac{B\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)}{b\,g^2(bx+a)} - \frac{2B\ln(bx+a)bdx-2B\ln(-dx-c)bdx+2B\ln(bx+a)ad-2B\ln(-dx-c)ad+Aad-Abc-2Bad+2B}{g^2(bx+a)b(ad-cb)}$
derivativedivides	$-\frac{\frac{A}{(bx+a)g^2}+\frac{B\ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^2}{b^2}\right)}{g^2(bx+a)}-\frac{2Bad}{g^2(ad-cb)(bx+a)}+\frac{2Bbc}{g^2(ad-cb)(bx+a)}-\frac{2B\,d^2\ln\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)a}{g^2(ad-cb)^2}+\frac{2Bd\ln\left(\frac{ad}{bx+}\right.}{g^2(}}{b}$
default	$-\frac{\frac{A}{(bx+a)g^2}+\frac{B\ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^2}{b^2}\right)}{g^2(bx+a)}-\frac{2Bad}{g^2(ad-cb)(bx+a)}+\frac{2Bbc}{g^2(ad-cb)(bx+a)}-\frac{2B\,d^2\ln\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)a}{g^2(ad-cb)^2}+\frac{2Bd\ln\left(\frac{ad}{bx+}\right.}{g^2(}}{b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*(d*x+c)^2/(b*x+a)^2))/(b*g*x+a*g)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/b*(1/(b*x+a)/g^2*A+1/g^2*B/(b*x+a)*ln(e*(a*d/(b*x+a)-b*c/(b*x+a)-d)^2/b^
2)-2/g^2*B/(a*d-b*c)/(b*x+a)*a*d+2/g^2*B/(a*d-b*c)/(b*x+a)*b*c-2/g^2*B*d^2/
(a*d-b*c)^2*ln(a*d/(b*x+a)-b*c/(b*x+a)-d)*a+2/g^2*B*d/(a*d-b*c)^2*ln(a*d/(b
*x+a)-b*c/(b*x+a)-d)*b*c)
```

Maxima [A]
time = 0.29, size = 190, normalized size = 1.86

$$-B\left(\frac{\log\left(\frac{d^2x^2e}{b^2x^2+2\,abx+a^2}+\frac{2\,cdxe}{b^2x^2+2\,abx+a^2}+\frac{c^2e}{b^2x^2+2\,abx+a^2}\right)}{b^2g^2x+abg^2}-\frac{2}{b^2g^2x+abg^2}-\frac{2\,d\log\left(bx+a\right)}{(b^2c-abd)g^2}+\frac{2\,d\log\left(dx+c\right)}{(b^2c-abd)g^2}\right)-\frac{A}{b^2g^2x+abg^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))/(b*g*x+a*g)^2,x, algorithm="maxi
ma")
```

```
[Out] -B*(log(d^2*x^2*e/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*x*e/(b^2*x^2 + 2*a*b*x
+ a^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2))/(b^2*g^2*x + a*b*g^2) - 2/(b^2*g^
2*x + a*b*g^2) - 2*d*log(b*x + a)/((b^2*c - a*b*d)*g^2) + 2*d*log(d*x + c)/
((b^2*c - a*b*d)*g^2)) - A/(b^2*g^2*x + a*b*g^2)
```

Fricas [A]
time = 0.37, size = 108, normalized size = 1.06

$$-\frac{(A-2\,B)bc-(A-2\,B)ad+(Bbdx+Bbc)\log\left(\frac{(d^2x^2+2\,cdx+c^2)e}{b^2x^2+2\,abx+a^2}\right)}{(b^3c-ab^2d)g^2x+(ab^2c-a^2bd)g^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))/(b*g*x+a*g)^2,x, algorithm="fric
as")
```

```
[Out] -((A - 2*B)*b*c - (A - 2*B)*a*d + (B*b*d*x + B*b*c)*log((d^2*x^2 + 2*c*d*x
+ c^2)*e/(b^2*x^2 + 2*a*b*x + a^2)))/((b^3*c - a*b^2*d)*g^2*x + (a*b^2*c -
a^2*b*d)*g^2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 253 vs. $2(83) = 166$.
time = 0.77, size = 253, normalized size = 2.48

$$-\frac{B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{abg^2+b^2g^2x}+\frac{2Bd\log\left(x+\frac{-\frac{2Ba^2d^3}{ad-bc}+\frac{4Babcd^2}{ad-bc}+2Bad^2-\frac{2Bb^2c^2d}{ad-bc}+2Bbcd}{4Bbd^2}\right)}{bg^2\left(ad-bc\right)}-\frac{2Bd\log\left(x+\frac{\frac{2Ba^2d^3}{ad-bc}-\frac{4Babcd^2}{ad-bc}+2Bad^2+\frac{2Bb^2c^2d}{ad-bc}+2Bbcd}{4Bbd^2}\right)}{bg^2\left(ad-bc\right)}+\frac{-A+2B}{abg^2+b^2g^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(d*x+c)**2/(b*x+a)**2))/(b*g*x+a*g)**2,x)
```

```
[Out] -B*log(e*(c + d*x)**2/(a + b*x)**2)/(a*b*g**2 + b**2*g**2*x) + 2*B*d*log(x
+ (-2*B*a**2*d**3/(a*d - b*c) + 4*B*a*b*c*d**2/(a*d - b*c) + 2*B*a*d**2 - 2
*B*b**2*c**2*d/(a*d - b*c) + 2*B*b*c*d)/(4*B*b*d**2))/(b*g**2*(a*d - b*c))
- 2*B*d*log(x + (2*B*a**2*d**3/(a*d - b*c) - 4*B*a*b*c*d**2/(a*d - b*c) + 2
*B*a*d**2 + 2*B*b**2*c**2*d/(a*d - b*c) + 2*B*b*c*d)/(4*B*b*d**2))/(b*g**2*
(a*d - b*c)) + (-A + 2*B)/(a*b*g**2 + b**2*g**2*x)
```

Giac [A]
time = 4.28, size = 188, normalized size = 1.84

$$-\left(2\left(b^{2}cg^{2}-abdg^{2}\right)\left(\frac{d\log\left(\left|\frac{bcg}{bgx+ag}-\frac{adg}{bgx+ag}+d\right|\right)}{b^{4}c^{2}g^{4}-2\,ab^{3}cdg^{4}+a^{2}b^{2}d^{2}g^{4}}-\frac{1}{(b^{2}cg^{2}-abdg^{2})(bgx+ag)bg}\right)+\frac{\log\left(\frac{(dx+c)^{2}e}{(bx+a)^{2}}\right)}{(bgx+ag)bg}\right)B-\frac{A}{(bgx+ag)bg}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))/(b*g*x+a*g)^2,x, algorithm="giac
")
```

```
[Out] -(2*(b^2*c*g^2 - a*b*d*g^2)*(d*log(abs(b*c*g/(b*g*x + a*g) - a*d*g/(b*g*x +
 a*g) + d))/(b^4*c^2*g^4 - 2*a*b^3*c*d*g^4 + a^2*b^2*d^2*g^4) - 1/((b^2*c*g
^2 - a*b*d*g^2)*(b*g*x + a*g)*b*g)) + log((d*x + c)^2*e/(b*x + a)^2)/((b*g*
x + a*g)*b*g))*B - A/((b*g*x + a*g)*b*g)
```

Mupad [B]
time = 5.94, size = 108, normalized size = 1.06

$$-\frac{A-2\,B}{x\,b^{2}\,g^{2}+a\,b\,g^{2}}-\frac{B\,\ln\left(\frac{e\,(c+d\,x)^{2}}{(a+b\,x)^{2}}\right)}{b^{2}\,g^{2}\,\left(x+\frac{a}{b}\right)}+\frac{B\,d\,\mathrm{atan}\left(\frac{b\,c\,2\mathrm{i}+b\,d\,x\,2\mathrm{i}}{a\,d-b\,c}+1\mathrm{i}\right)\,4\mathrm{i}}{b\,g^{2}\,\left(a\,d-b\,c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(c + d*x)^2)/(a + b*x)^2))/(a*g + b*g*x)^2,x)
```

```
[Out] (B*d*atan((b*c*2i + b*d*x*2i)/(a*d - b*c) + 1i)*4i)/(b*g^2*(a*d - b*c)) - (
B*log((e*(c + d*x)^2)/(a + b*x)^2))/(b^2*g^2*(x + a/b)) - (A - 2*B)/(b^2*g^
2*x + a*b*g^2)
```

3.207 $\int \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(ag+bgx)^3}\,dx$

Optimal. Leaf size=139

$$\frac{B}{2bg^3(a+bx)^2}-\frac{Bd}{b(bc-ad)g^3(a+bx)}-\frac{Bd^2\log(a+bx)}{b(bc-ad)^2g^3}+\frac{Bd^2\log(c+dx)}{b(bc-ad)^2g^3}-\frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{2bg^3(a+bx)^2}$$

```
[Out] 1/2*B/b/g^3/(b*x+a)^2-B*d/b/(-a*d+b*c)/g^3/(b*x+a)-B*d^2*ln(b*x+a)/b/(-a*d+
b*c)^2/g^3+B*d^2*ln(d*x+c)/b/(-a*d+b*c)^2/g^3+1/2*(-A-B*ln(e*(d*x+c)^2/(b*x
+a)^2))/b/g^3/(b*x+a)^2
```

Rubi [A]
time = 0.07, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {2548, 21, 46}

$$-\frac{B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A}{2bg^3(a+bx)^2}-\frac{Bd^2\log(a+bx)}{bg^3(bc-ad)^2}+\frac{Bd^2\log(c+dx)}{bg^3(bc-ad)^2}-\frac{Bd}{bg^3(a+bx)(bc-ad)}+\frac{B}{2bg^3(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])/(a*g + b*g*x)^3,x]
```

```
[Out] B/(2*b*g^3*(a + b*x)^2) - (B*d)/(b*(b*c - a*d)*g^3*(a + b*x)) - (B*d^2*Log[
a + b*x])/(b*(b*c - a*d)^2*g^3) + (B*d^2*Log[c + d*x])/(b*(b*c - a*d)^2*g^3
) - (A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])/(2*b*g^3*(a + b*x)^2)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2548

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
```

```
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Dist[B*n*((b*c
- a*d)/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /;
 FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c -
 a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Rubi steps

$$\int \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(ag+bgx)^3}\,dx = -\frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{2bg^3(a+bx)^2}+\frac{B\int\frac{2(-bc+ad)}{g^2(a+bx)^3(c+dx)}\,dx}{2bg}$$
$$=-\frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{2bg^3(a+bx)^2}-\frac{(B(bc-ad))\int\frac{1}{(a+bx)^3(c+dx)}\,dx}{bg^3}$$
$$=-\frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{2bg^3(a+bx)^2}-\frac{(B(bc-ad))\int\left(\frac{b}{(bc-ad)(a+bx)^3}-\frac{bd}{(bc-ad)^2(a+bx)^2}+\frac{}{(bc\cdot}\right.}{bg^3}$$
$$=\frac{B}{2bg^3(a+bx)^2}-\frac{Bd}{b(bc-ad)g^3(a+bx)}-\frac{Bd^2\log(a+bx)}{b(bc-ad)^2g^3}+\frac{Bd^2\log(c+dx)}{b(bc-ad)^2g^3}$$

Mathematica [A]
time = 0.06, size = 128, normalized size = 0.92

$$-\frac{2Bd^2(a+bx)^2\log(a+bx)-2Bd^2(a+bx)^2\log(c+dx)+(bc-ad)\left(Abc-bBc-aAd+3aBd+2bBdx+B(bc-ad)\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{2b(bc-ad)^2g^3(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])/(a*g + b*g*x)^3,x]
[Out] -1/2*(2*B*d^2*(a + b*x)^2*Log[a + b*x] - 2*B*d^2*(a + b*x)^2*Log[c + d*x] +
 (b*c - a*d)*(A*b*c - b*B*c - a*A*d + 3*a*B*d + 2*b*B*d*x + B*(b*c - a*d)*L
og[(e*(c + d*x)^2)/(a + b*x)^2]))/(b*(b*c - a*d)^2*g^3*(a + b*x)^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 293 vs. $2(138) = 276$.
time = 0.40, size = 294, normalized size = 2.12

method	result
norman	$\frac{\frac{dBx}{g(ad-cb)}+\frac{Ba\,d^2x\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)}{\left(a^2d^2-2abcd+b^2c^2\right)g}-\frac{Aabd-A\,b^2c-3Babd+B\,b^2c}{2g\,b^2(ad-cb)}+\frac{Bc(2ad-cb)\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)}{2g\left(a^2d^2-2abcd+b^2c^2\right)}+\frac{d^2Bb\,x^2\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)}{2\left(a^2d^2-2abcd+b^2c^2\right)g}}{g^2(bx+a)^2}$
risch	$-\frac{B\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)}{2b\,g^3(bx+a)^2}-\frac{2B\ln(bx+a)b^2d^2x^2-2B\ln(-dx-c)b^2d^2x^2+4B\ln(bx+a)ab\,d^2x-4B\ln(-dx-c)ab\,d^2x+2B\,a^2\,\text{I}}{2(a^2d^2-2}$

derivativedivides	$-\frac{\frac{A}{2(bx+a)^2g^3}+\frac{B\ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^2}{b^2}\right)}{2g^3(bx+a)^2}-\frac{B\,a^2d^2}{2g^3(ad-cb)^2(bx+a)^2}+\frac{Badbc}{g^3(ad-cb)^2(bx+a)^2}-\frac{B\,b^2c^2}{2g^3(ad-cb)^2(bx+a)^2}-\frac{B\,d^2a}{g^3(ad-cb)^2(}}{b}$
default	$-\frac{\frac{A}{2(bx+a)^2g^3}+\frac{B\ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^2}{b^2}\right)}{2g^3(bx+a)^2}-\frac{B\,a^2d^2}{2g^3(ad-cb)^2(bx+a)^2}+\frac{Badbc}{g^3(ad-cb)^2(bx+a)^2}-\frac{B\,b^2c^2}{2g^3(ad-cb)^2(bx+a)^2}-\frac{B\,d^2a}{g^3(ad-cb)^2(}}{b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*(d*x+c)^2/(b*x+a)^2))/(b*g*x+a*g)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/b*(1/2/(b*x+a)^2/g^3*A+1/2/g^3*B/(b*x+a)^2*ln(e*(a*d/(b*x+a)-b*c/(b*x+a)
-d)^2/b^2)-1/2/g^3*B/(a*d-b*c)^2*a^2*d^2/(b*x+a)^2+1/g^3*B/(a*d-b*c)^2*a*d/
(b*x+a)^2*b*c-1/2/g^3*B/(a*d-b*c)^2*b^2*c^2/(b*x+a)^2-1/g^3*B/(a*d-b*c)^2*d
^2/(b*x+a)*a+1/g^3*B/(a*d-b*c)^2*d/(b*x+a)*b*c-1/g^3*B*d^3/(a*d-b*c)^3*ln(a
*d/(b*x+a)-b*c/(b*x+a)-d)*a+1/g^3*B*d^2/(a*d-b*c)^3*ln(a*d/(b*x+a)-b*c/(b*x
+a)-d)*b*c)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 309 vs. $2(136) = 272$.
time = 0.32, size = 309, normalized size = 2.22

$$-\frac{1}{2}B\left(\frac{2\,bdx-bc+3\,ad}{(b^4c-ab^3d)g^3x^2+2\,(ab^3c-a^2b^2d)g^3x+(a^2b^2c-a^3bd)g^3}+\frac{\log\left(\frac{d^2x^2e}{b^2x^2+2\,abx+a^2}+\frac{2\,cdxe}{b^2x^2+2\,abx+a^2}+\frac{c^2e}{b^2x^2+2\,abx+a^2}\right)}{b^3g^3x^2+2\,ab^2g^3x+a^2bg^3}+\frac{2\,d^2\log(bx+a)}{(b^3c^2-2\,ab^2cd+a^2bd^2)g^3}-\frac{2\,d^2\log(dx+c)}{(b^3c^2-2\,ab^2cd+a^2bd^2)g^3}\right)-\frac{A}{2\,(b^3g^3x^2+2\,ab^2g^3x+a^2bg^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))/(b*g*x+a*g)^3,x, algorithm="maxi
ma")
```

```
[Out] -1/2*B*((2*b*d*x - b*c + 3*a*d)/((b^4*c - a*b^3*d)*g^3*x^2 + 2*(a*b^3*c - a
^2*b^2*d)*g^3*x + (a^2*b^2*c - a^3*b*d)*g^3) + log(d^2*x^2*e/(b^2*x^2 + 2*a
*b*x + a^2) + 2*c*d*x*e/(b^2*x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*
x + a^2))/(b^3*g^3*x^2 + 2*a*b^2*g^3*x + a^2*b*g^3) + 2*d^2*log(b*x + a)/((
b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*g^3) - 2*d^2*log(d*x + c)/((b^3*c^2 - 2*
a*b^2*c*d + a^2*b*d^2)*g^3)) - 1/2*A/(b^3*g^3*x^2 + 2*a*b^2*g^3*x + a^2*b*g
^3)
```

Fricas [A]
time = 0.35, size = 238, normalized size = 1.71

$$-\frac{(A-B)b^2c^2-2\,(A-2\,B)abcd+(A-3\,B)a^2d^2+2\,(Bb^2cd-Babd^2)x-(Bb^2d^2x^2+2\,Babd^2x-Bb^2c^2+2\,Babcd)\log\left(\frac{(d^2x^2+2\,cdx+c^2)e}{b^2x^2+2\,abx+a^2}\right)}{2\,((b^5c^2-2\,ab^4cd+a^2b^3d^2)g^3x^2+2\,(ab^4c^2-2\,a^2b^3cd+a^3b^2d^2)g^3x+(a^2b^3c^2-2\,a^3b^2cd+a^4bd^2)g^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))/(b*g*x+a*g)^3,x, algorithm="fric
as")
```

```
[Out] -1/2*((A - B)*b^2*c^2 - 2*(A - 2*B)*a*b*c*d + (A - 3*B)*a^2*d^2 + 2*(B*b^2*
c*d - B*a*b*d^2)*x - (B*b^2*d^2*x^2 + 2*B*a*b*d^2*x - B*b^2*c^2 + 2*B*a*b*c
*d)*log((d^2*x^2 + 2*c*d*x + c^2)*e/(b^2*x^2 + 2*a*b*x + a^2)))/((b^5*c^2 -
 2*a*b^4*c*d + a^2*b^3*d^2)*g^3*x^2 + 2*(a*b^4*c^2 - 2*a^2*b^3*c*d + a^3*b^
2*d^2)*g^3*x + (a^2*b^3*c^2 - 2*a^3*b^2*c*d + a^4*b*d^2)*g^3)
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 418 vs. $2(122) = 244$.

time = 1.60, size = 418, normalized size = 3.01

$$-\frac{B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{2a^2bg^3+4ab^2g^3x+2b^3g^3x^2}+\frac{Bd^2\log\left(x+\frac{-\frac{Ba^3d^5}{(ad-bc)^2}+\frac{3Ba^2bcd^4}{(ad-bc)^2}-\frac{3Bab^2c^2d^3}{(ad-bc)^2}+Bad^3+\frac{Bb^3c^3d^2}{(ad-bc)^2}+Bbcd^2}{2Bbd^3}\right)}{bg^3(ad-bc)^2}-\frac{Bd^2\log\left(x+\frac{\frac{Ba^3d^5}{(ad-bc)^2}-\frac{3Ba^2bcd^4}{(ad-bc)^2}+\frac{3Bab^2c^2d^3}{(ad-bc)^2}+Bad^3-\frac{Bb^3c^3d^2}{(ad-bc)^2}+Bbcd^2}{2Bbd^3}\right)}{bg^3(ad-bc)^2}+\frac{-Aad+Abc+3Bad-Bbc+2Bbdx}{2a^3bdg^3-2a^2b^2cg^3+x^2\cdot(2ab^3dg^3-2b^4cg^3)+x(4a^2b^2dg^3-4ab^3cg^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(d*x+c)**2/(b*x+a)**2))/(b*g*x+a*g)**3,x)
```

```
[Out] -B*log(e*(c + d*x)**2/(a + b*x)**2)/(2*a**2*b*g**3 + 4*a*b**2*g**3*x + 2*b*
*3*g**3*x**2) + B*d**2*log(x + (-B*a**3*d**5/(a*d - b*c)**2 + 3*B*a**2*b*c*
d**4/(a*d - b*c)**2 - 3*B*a*b**2*c**2*d**3/(a*d - b*c)**2 + B*a*d**3 + B*b*
*3*c**3*d**2/(a*d - b*c)**2 + B*b*c*d**2)/(2*B*b*d**3))/(b*g**3*(a*d - b*c)
**2) - B*d**2*log(x + (B*a**3*d**5/(a*d - b*c)**2 - 3*B*a**2*b*c*d**4/(a*d
- b*c)**2 + 3*B*a*b**2*c**2*d**3/(a*d - b*c)**2 + B*a*d**3 - B*b**3*c**3*d*
*2/(a*d - b*c)**2 + B*b*c*d**2)/(2*B*b*d**3))/(b*g**3*(a*d - b*c)**2) + (-A
*a*d + A*b*c + 3*B*a*d - B*b*c + 2*B*b*d*x)/(2*a**3*b*d*g**3 - 2*a**2*b**2*
c*g**3 + x**2*(2*a*b**3*d*g**3 - 2*b**4*c*g**3) + x*(4*a**2*b**2*d*g**3 - 4
*a*b**3*c*g**3))
```

Giac [**A**]

time = 2.66, size = 259, normalized size = 1.86

$$-\frac{Bd^2\log(bx+a)}{b^3c^2g^3-2\,ab^2cdg^3+a^2bd^2g^3}+\frac{Bd^2\log(dx+c)}{b^3c^2g^3-2\,ab^2cdg^3+a^2bd^2g^3}-\frac{B\log\left(\frac{d^2x^2+2\,cdx+c^2}{b^2x^2+2\,abx+a^2}\right)}{2\,(b^3g^3x^2+2\,ab^2g^3x+a^2bg^3)}-\frac{2\,Bbdx+Abc-Aad+2\,Bad}{2\,(b^4cg^3x^2-ab^3dg^3x^2+2\,ab^3cg^3x-2\,a^2b^2dg^3x+a^2b^2cg^3-a^3bdg^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))/(b*g*x+a*g)^3,x, algorithm="giac
")
```

```
[Out] -B*d^2*log(b*x + a)/(b^3*c^2*g^3 - 2*a*b^2*c*d*g^3 + a^2*b*d^2*g^3) + B*d^2
*log(d*x + c)/(b^3*c^2*g^3 - 2*a*b^2*c*d*g^3 + a^2*b*d^2*g^3) - 1/2*B*log((
d^2*x^2 + 2*c*d*x + c^2)/(b^2*x^2 + 2*a*b*x + a^2))/(b^3*g^3*x^2 + 2*a*b^2*
g^3*x + a^2*b*g^3) - 1/2*(2*B*b*d*x + A*b*c - A*a*d + 2*B*a*d)/(b^4*c*g^3*x
^2 - a*b^3*d*g^3*x^2 + 2*a*b^3*c*g^3*x - 2*a^2*b^2*d*g^3*x + a^2*b^2*c*g^3
- a^3*b*d*g^3)
```

Mupad [**B**]

time = 5.93, size = 206, normalized size = 1.48

$$\frac{2\,B\,d^2\,\mathrm{atanh}\left(\frac{b^3\,c^2\,g^3-a^2\,b\,d^2\,g^3}{b\,g^3\,{(a\,d-b\,c)}^2}-\frac{2\,b\,d\,x}{a\,d-b\,c}\right)}{b\,g^3\,{(a\,d-b\,c)}^2}-\frac{B\,\ln\left(\frac{e\,{(c+d\,x)}^2}{{(a+b\,x)}^2}\right)}{2\,b^2\,g^3\,\left(2\,a\,x+b\,x^2+\frac{a^2}{b}\right)}-\frac{\frac{A\,a\,d-A\,b\,c-3\,B\,a\,d+B\,b\,c}{2\,(a\,d-b\,c)}-\frac{B\,b\,d\,x}{a\,d-b\,c}}{a^2\,b\,g^3+2\,a\,b^2\,g^3\,x+b^3\,g^3\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(c + d*x)^2)/(a + b*x)^2))/(a*g + b*g*x)^3,x)
```

```
[Out] (2*B*d^2*atanh((b^3*c^2*g^3 - a^2*b*d^2*g^3)/(b*g^3*(a*d - b*c)^2) - (2*b*d
*x)/(a*d - b*c)))/(b*g^3*(a*d - b*c)^2) - (B*log((e*(c + d*x)^2)/(a + b*x)^
2))/(2*b^2*g^3*(2*a*x + b*x^2 + a^2/b)) - ((A*a*d - A*b*c - 3*B*a*d + B*b*c
)/(2*(a*d - b*c)) - (B*b*d*x)/(a*d - b*c))/(a^2*b*g^3 + b^3*g^3*x^2 + 2*a*b
^2*g^3*x)
```

3.208 $\int \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(ag+bgx)^4}\,dx$

Optimal. Leaf size=177

$$\frac{2B}{9bg^4(a+bx)^3}-\frac{Bd}{3b(bc-ad)g^4(a+bx)^2}+\frac{2Bd^2}{3b(bc-ad)^2g^4(a+bx)}+\frac{2Bd^3\log(a+bx)}{3b(bc-ad)^3g^4}-\frac{2Bd^3\log(c+dx)}{3b(bc-ad)^3g^4}-\frac{A}{}$$

[Out] 2/9*B/b/g^4/(b*x+a)^3-1/3*B*d/b/(-a*d+b*c)/g^4/(b*x+a)^2+2/3*B*d^2/b/(-a*d+b*c)^2/g^4/(b*x+a)+2/3*B*d^3*ln(b*x+a)/b/(-a*d+b*c)^3/g^4-2/3*B*d^3*ln(d*x+c)/b/(-a*d+b*c)^3/g^4+1/3*(-A-B*ln(e*(d*x+c)^2/(b*x+a)^2))/b/g^4/(b*x+a)^3

Rubi [A]
time = 0.09, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {2548, 21, 46}

$$-\frac{B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A}{3bg^4(a+bx)^3}+\frac{2Bd^3\log(a+bx)}{3bg^4(bc-ad)^3}-\frac{2Bd^3\log(c+dx)}{3bg^4(bc-ad)^3}+\frac{2Bd^2}{3bg^4(a+bx)(bc-ad)^2}-\frac{Bd}{3bg^4(a+bx)^2(bc-ad)}+\frac{2B}{9bg^4(a+bx)^3}$$

Antiderivative was successfully verified.

[In] Int[(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])/(a*g + b*g*x)^4,x]

[Out] (2*B)/(9*b*g^4*(a + b*x)^3) - (B*d)/(3*b*(b*c - a*d)*g^4*(a + b*x)^2) + (2*B*d^2)/(3*b*(b*c - a*d)^2*g^4*(a + b*x)) + (2*B*d^3*Log[a + b*x])/(3*b*(b*c - a*d)^3*g^4) - (2*B*d^3*Log[c + d*x])/(3*b*(b*c - a*d)^3*g^4) - (A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])/(3*b*g^4*(a + b*x)^3)

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2548

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
```

```
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Dist[B*n*((b*c
- a*d)/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /;
 FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c -
 a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Rubi steps

$$\int \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(ag+bgx)^4}\,dx = -\frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{3bg^4(a+bx)^3}+\frac{B\int\frac{2(-bc+ad)}{g^3(a+bx)^4(c+dx)}\,dx}{3bg}$$
$$= -\frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{3bg^4(a+bx)^3}-\frac{(2B(bc-ad))\int\frac{1}{(a+bx)^4(c+dx)}\,dx}{3bg^4}$$
$$= -\frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{3bg^4(a+bx)^3}-\frac{(2B(bc-ad))\int\left(\frac{b}{(bc-ad)(a+bx)^4}-\frac{bd}{(bc-ad)^2(a+bx)^3}+\frac{}{(bc-}\right.}{3bg^4}$$
$$= \frac{2B}{9bg^4(a+bx)^3}-\frac{Bd}{3b(bc-ad)g^4(a+bx)^2}+\frac{2Bd^2}{3b(bc-ad)^2g^4(a+bx)}+\frac{2Bd^3\log}{3b(bc-}$$

Mathematica [A]
time = 0.08, size = 140, normalized size = 0.79

$$\frac{\frac{B\left(2(bc-ad)^3-3d(bc-ad)^2(a+bx)+6d^2(bc-ad)(a+bx)^2+6d^3(a+bx)^3\log(a+bx)-6d^3(a+bx)^3\log(c+dx)\right)}{(bc-ad)^3}-3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{9bg^4(a+bx)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])/(a*g + b*g*x)^4,x]
```

```
[Out] ((B*(2*(b*c - a*d)^3 - 3*d*(b*c - a*d)^2*(a + b*x) + 6*d^2*(b*c - a*d)*(a +
 b*x)^2 + 6*d^3*(a + b*x)^3*Log[a + b*x] - 6*d^3*(a + b*x)^3*Log[c + d*x]))
/(b*c - a*d)^3 - 3*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]))/(9*b*g^4*(a +
b*x)^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 421 vs. 2(168) = 336.
time = 0.43, size = 422, normalized size = 2.38

method	result
risch	$-\frac{B\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)}{3b\,g^4(bx+a)^3}-\frac{-6B\ln(-dx-c)b^3d^3x^3+6B\ln(bx+a)b^3d^3x^3-18B\ln(-dx-c)a\,b^2d^3x^2+18B\ln(bx+a)a\,b^2d^3x^2-}{}$

derivativedivides	$-\frac{\frac{A}{3(bx+a)^3g^4}+\frac{B\ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^2}{b^2}\right)}{3g^4(bx+a)^3}-\frac{2B\,a^3d^3}{9g^4(ad-cb)^3(bx+a)^3}+\frac{2B\,a^2d^2bc}{3g^4(ad-cb)^3(bx+a)^3}-\frac{2Bad\,b^2c^2}{3g^4(ad-cb)^3(bx+a)^3}-\frac{B}{3g^4(ad-}}{}$
default	$-\frac{\frac{A}{3(bx+a)^3g^4}+\frac{B\ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^2}{b^2}\right)}{3g^4(bx+a)^3}-\frac{2B\,a^3d^3}{9g^4(ad-cb)^3(bx+a)^3}+\frac{2B\,a^2d^2bc}{3g^4(ad-cb)^3(bx+a)^3}-\frac{2Bad\,b^2c^2}{3g^4(ad-cb)^3(bx+a)^3}-\frac{B}{3g^4(ad-}}{}$
norman	$\frac{B\,a^2d^3x\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)}{(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3)g}+\frac{Bab\,d^3x^2\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)}{(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3)g}+\frac{(3A\,a^2d^2-6Aabcd+3A\,b^2c^2-6B\,a^2d^2+6Babcd-2B\,b^2}{3ga(a^2d^2-2abcd+b^2c^2)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*(d*x+c)^2/(b*x+a)^2))/(b*g*x+a*g)^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/b*(1/3/(b*x+a)^3/g^4*A+1/3/g^4*B/(b*x+a)^3*ln(e*(a*d/(b*x+a)-b*c/(b*x+a)
-d)^2/b^2)-2/9/g^4*B*a^3*d^3/(a*d-b*c)^3/(b*x+a)^3+2/3/g^4*B*a^2*d^2/(a*d-b
*c)^3*b*c/(b*x+a)^3-2/3/g^4*B*a*d/(a*d-b*c)^3*b^2*c^2/(b*x+a)^3-1/3/g^4*B*a
^2*d^3/(a*d-b*c)^3/(b*x+a)^2+2/3/g^4*B*a*d^2/(a*d-b*c)^3*b*c/(b*x+a)^2-2/3/
g^4*B*a*d^3/(a*d-b*c)^3/(b*x+a)-2/3/g^4*B*a*d^4/(a*d-b*c)^4*ln(a*d/(b*x+a)-
b*c/(b*x+a)-d)+2/9/g^4*B*b^3*c^3/(a*d-b*c)^3/(b*x+a)^3-1/3/g^4*B*b^2*c^2/(a
*d-b*c)^3*d/(b*x+a)^2+2/3/g^4*B*b*c/(a*d-b*c)^3*d^2/(b*x+a)+2/3/g^4*B*b*c*d
^3/(a*d-b*c)^4*ln(a*d/(b*x+a)-b*c/(b*x+a)-d))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 483 vs. $2(166) = 332$.
time = 0.32, size = 483, normalized size = 2.73

$$\frac{1}{9}B\left(\frac{6\,b^2d^2x^2+2\,b^2c^2-7\,abcd+11\,a^2d^2-3\,(b^2cd-5\,abd^2)x}{(b^6c^2-2\,ab^5cd+a^2b^4d^2)g^4x^3+3\,(ab^5c^2-2\,a^2b^4cd+a^3b^3d^2)g^4x^2+3\,(a^2b^4c^2-2\,a^3b^3cd+a^4b^2d^2)g^4x+(a^3b^3c^2-2\,a^4b^2cd+a^5bd^2)g^4}-\frac{3\,\log\left(\frac{d^2x^2e}{b^2x^2+2\,abx+a^2}+\frac{2\,cdxe}{b^2x^2+2\,abx+a^2}+\frac{c^2e}{b^2x^2+2\,abx+a^2}\right)}{b^4g^4x^3+3\,ab^3g^4x^2+3\,a^2b^2g^4x+a^3bg^4}+\frac{6\,d^3\log(bx+a)}{(b^4c^3-3\,ab^3c^2d+3\,a^2b^2cd^2-a^3bd^3)g^4}-\frac{6\,d^3\log(dx+c)}{(b^4c^3-3\,ab^3c^2d+3\,a^2b^2cd^2-a^3bd^3)g^4}\right)-\frac{A}{3\,(b^4g^4x^3+3\,ab^3g^4x^2+3\,a^2b^2g^4x+a^3bg^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))/(b*g*x+a*g)^4,x, algorithm="maxi
ma")
```

```
[Out] 1/9*B*((6*b^2*d^2*x^2 + 2*b^2*c^2 - 7*a*b*c*d + 11*a^2*d^2 - 3*(b^2*c*d - 5
*a*b*d^2)*x)/((b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2)*g^4*x^3 + 3*(a*b^5*c^2
- 2*a^2*b^4*c*d + a^3*b^3*d^2)*g^4*x^2 + 3*(a^2*b^4*c^2 - 2*a^3*b^3*c*d + a
^4*b^2*d^2)*g^4*x + (a^3*b^3*c^2 - 2*a^4*b^2*c*d + a^5*b*d^2)*g^4) - 3*log(
d^2*x^2*e/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*x*e/(b^2*x^2 + 2*a*b*x + a^2) +
 c^2*e/(b^2*x^2 + 2*a*b*x + a^2))/(b^4*g^4*x^3 + 3*a*b^3*g^4*x^2 + 3*a^2*b^
2*g^4*x + a^3*b*g^4) + 6*d^3*log(b*x + a)/((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2
*b^2*c*d^2 - a^3*b*d^3)*g^4) - 6*d^3*log(d*x + c)/((b^4*c^3 - 3*a*b^3*c^2*d
 + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*g^4)) - 1/3*A/(b^4*g^4*x^3 + 3*a*b^3*g^4*x^
2 + 3*a^2*b^2*g^4*x + a^3*b*g^4)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 430 vs. $2(166) = 332$.

time = 0.38, size = 430, normalized size = 2.43

$$-\frac{(3A-2B)b^3c^3-9(A-B)ab^2c^2d+9(A-2B)a^2bcd^2-(3A-11B)a^3d^3-6(Bb^3cd^2-Bab^2d^3)x^2+3(Bb^3c^2d-6Bab^2cd^2+5Ba^2bd^3)x+3(Bb^3d^3x^3+3Bab^2d^3x^2+3Ba^2bd^3x+Bb^3c^3-3Bab^2c^2d+3Ba^2bcd^2)\log\left(\frac{(d^2x^2+2cdx+c^2)e}{b^2x^2+2abx+a^2}\right)}{9\left((b^7c^3-3ab^6c^2d+3a^2b^5cd^2-a^3b^4d^3)g^4x^3+3(ab^6c^3-3a^2b^5c^2d+3a^3b^4cd^2-a^4b^3d^3)g^4x^2+3(a^2b^5c^3-3a^3b^4c^2d+3a^4b^3cd^2-a^5b^2d^3)g^4x+(a^3b^4c^3-3a^4b^3c^2d+3a^5b^2cd^2-a^6bd^3)g^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))/(b*g*x+a*g)^4,x, algorithm="fric
as")
```

```
[Out] -1/9*((3*A - 2*B)*b^3*c^3 - 9*(A - B)*a*b^2*c^2*d + 9*(A - 2*B)*a^2*b*c*d^2
 - (3*A - 11*B)*a^3*d^3 - 6*(B*b^3*c*d^2 - B*a*b^2*d^3)*x^2 + 3*(B*b^3*c^2*
d - 6*B*a*b^2*c*d^2 + 5*B*a^2*b*d^3)*x + 3*(B*b^3*d^3*x^3 + 3*B*a*b^2*d^3*x
^2 + 3*B*a^2*b*d^3*x + B*b^3*c^3 - 3*B*a*b^2*c^2*d + 3*B*a^2*b*c*d^2)*log((
d^2*x^2 + 2*c*d*x + c^2)*e/(b^2*x^2 + 2*a*b*x + a^2)))/((b^7*c^3 - 3*a*b^6*
c^2*d + 3*a^2*b^5*c*d^2 - a^3*b^4*d^3)*g^4*x^3 + 3*(a*b^6*c^3 - 3*a^2*b^5*c
^2*d + 3*a^3*b^4*c*d^2 - a^4*b^3*d^3)*g^4*x^2 + 3*(a^2*b^5*c^3 - 3*a^3*b^4*
c^2*d + 3*a^4*b^3*c*d^2 - a^5*b^2*d^3)*g^4*x + (a^3*b^4*c^3 - 3*a^4*b^3*c^2
*d + 3*a^5*b^2*c*d^2 - a^6*b*d^3)*g^4)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 677 vs. $2(162) = 324$.
time = 3.29, size = 677, normalized size = 3.82

$$-\frac{B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{3a^3bg^4+9a^2b^2g^4x+9ab^3g^4x^2+3b^4g^4x^3}+\frac{2Bd^3\log\left(x+\frac{-\frac{2Ba^4d^7}{(ad-bc)^3}+\frac{8Ba^3bcd^6}{(ad-bc)^3}-\frac{12Ba^2b^2c^2d^5}{(ad-bc)^3}+\frac{8Bab^3c^3d^4}{(ad-bc)^3}+2Bad^4-\frac{2Bb^4c^4d^3}{(ad-bc)^3}+2Bbcd^3}{4Bbd^4}\right)}{3bg^4(ad-bc)^3}-\frac{2Bd^3\log\left(x+\frac{\frac{2Ba^4d^7}{(ad-bc)^3}-\frac{8Ba^3bcd^6}{(ad-bc)^3}+\frac{12Ba^2b^2c^2d^5}{(ad-bc)^3}-\frac{8Bab^3c^3d^4}{(ad-bc)^3}+2Bad^4+\frac{2Bb^4c^4d^3}{(ad-bc)^3}+2Bbcd^3}{4Bbd^4}\right)}{3bg^4(ad-bc)^3}+\frac{-3Aa^2d^2+6Aabcd-3Ab^2c^2+11Ba^2d^2-7Babcd+2Bb^2c^2+6Bb^2d^2x^2+x(15Babd^2-3Bb^2cd)}{9a^5bd^2g^4-18a^4b^2cdg^4+9a^3b^3c^2g^4+x^3\cdot(9a^2b^4d^2g^4-18ab^5cdg^4+9b^6c^2g^4)+x^2\cdot(27a^3b^3d^2g^4-54a^2b^4cdg^4+27ab^5c^2g^4)+x\left(27a^4b^2d^2g^4-54a^3b^3cdg^4+27a^2b^4c^2g^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(d*x+c)**2/(b*x+a)**2))/(b*g*x+a*g)**4,x)
```

```
[Out] -B*log(e*(c + d*x)**2/(a + b*x)**2)/(3*a**3*b*g**4 + 9*a**2*b**2*g**4*x + 9
*a*b**3*g**4*x**2 + 3*b**4*g**4*x**3) + 2*B*d**3*log(x + (-2*B*a**4*d**7/(a
*d - b*c)**3 + 8*B*a**3*b*c*d**6/(a*d - b*c)**3 - 12*B*a**2*b**2*c**2*d**5/
(a*d - b*c)**3 + 8*B*a*b**3*c**3*d**4/(a*d - b*c)**3 + 2*B*a*d**4 - 2*B*b**
4*c**4*d**3/(a*d - b*c)**3 + 2*B*b*c*d**3)/(4*B*b*d**4))/(3*b*g**4*(a*d - b
*c)**3) - 2*B*d**3*log(x + (2*B*a**4*d**7/(a*d - b*c)**3 - 8*B*a**3*b*c*d**
6/(a*d - b*c)**3 + 12*B*a**2*b**2*c**2*d**5/(a*d - b*c)**3 - 8*B*a*b**3*c**
3*d**4/(a*d - b*c)**3 + 2*B*a*d**4 + 2*B*b**4*c**4*d**3/(a*d - b*c)**3 + 2*
B*b*c*d**3)/(4*B*b*d**4))/(3*b*g**4*(a*d - b*c)**3) + (-3*A*a**2*d**2 + 6*A
*a*b*c*d - 3*A*b**2*c**2 + 11*B*a**2*d**2 - 7*B*a*b*c*d + 2*B*b**2*c**2 + 6
*B*b**2*d**2*x**2 + x*(15*B*a*b*d**2 - 3*B*b**2*c*d))/(9*a**5*b*d**2*g**4 -
 18*a**4*b**2*c*d*g**4 + 9*a**3*b**3*c**2*g**4 + x**3*(9*a**2*b**4*d**2*g**
4 - 18*a*b**5*c*d*g**4 + 9*b**6*c**2*g**4) + x**2*(27*a**3*b**3*d**2*g**4 -
 54*a**2*b**4*c*d*g**4 + 27*a*b**5*c**2*g**4) + x*(27*a**4*b**2*d**2*g**4 -
 54*a**3*b**3*c*d*g**4 + 27*a**2*b**4*c**2*g**4))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 473 vs. $2(166) = 332$.
time = 2.66, size = 473, normalized size = 2.67

$$\frac{2Bd^3\log(bx+a)}{3(b^4c^3g^4-3ab^3c^2dg^4+3a^2b^2cd^2g^4-a^3bd^3g^4)}-\frac{2Bd^3\log(dx+c)}{3(b^4c^3g^4-3ab^3c^2dg^4+3a^2b^2cd^2g^4-a^3bd^3g^4)}-\frac{B\log\left(\frac{d^2x^2+2cdx+c^2}{b^2x^2+2abx+a^2}\right)}{3(b^4g^4x^3+3ab^3g^4x^2+3a^2b^2g^4x+a^3bg^4)}+\frac{6Bb^2d^2x^2-3Bb^2cdx+15Babd^2x-3Ab^2c^2-Bb^2c^2+6Aabcd-Babcd-3Aa^2d^2+8Ba^2d^2}{9(b^6c^2g^4x^3-2ab^5cdg^4x^3+a^2b^4d^2g^4x^3+3ab^5c^2g^4x^2-6a^2b^4cdg^4x^2+3a^3b^3d^2g^4x^2+3a^2b^4c^2g^4x-6a^3b^3cdg^4x+3a^4b^2d^2g^4x+a^3b^3c^2g^4-2a^4b^2cdg^4+a^5bd^2g^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))/(b*g*x+a*g)^4,x, algorithm="giac
")
```

```
[Out] 2/3*B*d^3*log(b*x + a)/(b^4*c^3*g^4 - 3*a*b^3*c^2*d*g^4 + 3*a^2*b^2*c*d^2*g
^4 - a^3*b*d^3*g^4) - 2/3*B*d^3*log(d*x + c)/(b^4*c^3*g^4 - 3*a*b^3*c^2*d*g
^4 + 3*a^2*b^2*c*d^2*g^4 - a^3*b*d^3*g^4) - 1/3*B*log((d^2*x^2 + 2*c*d*x +
c^2)/(b^2*x^2 + 2*a*b*x + a^2))/(b^4*g^4*x^3 + 3*a*b^3*g^4*x^2 + 3*a^2*b^2*
g^4*x + a^3*b*g^4) + 1/9*(6*B*b^2*d^2*x^2 - 3*B*b^2*c*d*x + 15*B*a*b*d^2*x
- 3*A*b^2*c^2 - B*b^2*c^2 + 6*A*a*b*c*d - B*a*b*c*d - 3*A*a^2*d^2 + 8*B*a^2
*d^2)/(b^6*c^2*g^4*x^3 - 2*a*b^5*c*d*g^4*x^3 + a^2*b^4*d^2*g^4*x^3 + 3*a*b^
5*c^2*g^4*x^2 - 6*a^2*b^4*c*d*g^4*x^2 + 3*a^3*b^3*d^2*g^4*x^2 + 3*a^2*b^4*c
^2*g^4*x - 6*a^3*b^3*c*d*g^4*x + 3*a^4*b^2*d^2*g^4*x + a^3*b^3*c^2*g^4 - 2*
a^4*b^2*c*d*g^4 + a^5*b*d^2*g^4)
```

Mupad [B]
time = 6.73, size = 341, normalized size = 1.93

$$\frac{2\,B\,b\,c^2}{9\,g^4\,{\left(a\,d-b\,c\right)}^2\,{\left(a+b\,x\right)}^3}-\frac{A\,b\,c^2}{3\,g^4\,{\left(a\,d-b\,c\right)}^2\,{\left(a+b\,x\right)}^3}-\frac{B\,\ln\left(\frac{e\,{\left(c+d\,x\right)}^2}{{\left(a+b\,x\right)}^2}\right)}{3\,b\,g^4\,{\left(a+b\,x\right)}^3}-\frac{A\,a^2\,d^2}{3\,b\,g^4\,{\left(a\,d-b\,c\right)}^2\,{\left(a+b\,x\right)}^3}+\frac{11\,B\,a^2\,d^2}{9\,b\,g^4\,{\left(a\,d-b\,c\right)}^2\,{\left(a+b\,x\right)}^3}+\frac{5\,B\,a\,d^2\,x}{3\,g^4\,{\left(a\,d-b\,c\right)}^2\,{\left(a+b\,x\right)}^3}+\frac{2\,B\,b\,d^2\,x^2}{3\,g^4\,{\left(a\,d-b\,c\right)}^2\,{\left(a+b\,x\right)}^3}+\frac{2\,A\,a\,c\,d}{3\,g^4\,{\left(a\,d-b\,c\right)}^2\,{\left(a+b\,x\right)}^3}-\frac{7\,B\,a\,c\,d}{9\,g^4\,{\left(a\,d-b\,c\right)}^2\,{\left(a+b\,x\right)}^3}-\frac{B\,b\,c\,d\,x}{3\,g^4\,{\left(a\,d-b\,c\right)}^2\,{\left(a+b\,x\right)}^3}+\frac{B\,d^3\,\mathrm{atan}\left(\frac{a\,d\,1{}\mathrm{i}+b\,c\,1{}\mathrm{i}+b\,d\,x\,2{}\mathrm{i}}{a\,d-b\,c}\right)\,4{}\mathrm{i}}{3\,b\,g^4\,{\left(a\,d-b\,c\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(c + d*x)^2)/(a + b*x)^2))/(a*g + b*g*x)^4,x)
```

```
[Out] (B*d^3*atan((a*d*1i + b*c*1i + b*d*x*2i)/(a*d - b*c))*4i)/(3*b*g^4*(a*d - b
*c)^3) - (B*log((e*(c + d*x)^2)/(a + b*x)^2))/(3*b*g^4*(a + b*x)^3) - (A*b*
c^2)/(3*g^4*(a*d - b*c)^2*(a + b*x)^3) + (2*B*b*c^2)/(9*g^4*(a*d - b*c)^2*(
a + b*x)^3) - (A*a^2*d^2)/(3*b*g^4*(a*d - b*c)^2*(a + b*x)^3) + (11*B*a^2*d
^2)/(9*b*g^4*(a*d - b*c)^2*(a + b*x)^3) + (5*B*a*d^2*x)/(3*g^4*(a*d - b*c)^
2*(a + b*x)^3) + (2*B*b*d^2*x^2)/(3*g^4*(a*d - b*c)^2*(a + b*x)^3) + (2*A*a
*c*d)/(3*g^4*(a*d - b*c)^2*(a + b*x)^3) - (7*B*a*c*d)/(9*g^4*(a*d - b*c)^2*
(a + b*x)^3) - (B*b*c*d*x)/(3*g^4*(a*d - b*c)^2*(a + b*x)^3)
```

3.209 $\int \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(ag+bgx)^5}\,dx$

Optimal. Leaf size=208

$$\frac{B}{8bg^5(a+bx)^4}-\frac{Bd}{6b(bc-ad)g^5(a+bx)^3}+\frac{Bd^2}{4b(bc-ad)^2g^5(a+bx)^2}-\frac{Bd^3}{2b(bc-ad)^3g^5(a+bx)}-\frac{Bd^4\log(a+bx)}{2b(bc-ad)^4g^5}$$

```
[Out] 1/8*B/b/g^5/(b*x+a)^4-1/6*B*d/b/(-a*d+b*c)/g^5/(b*x+a)^3+1/4*B*d^2/b/(-a*d+
b*c)^2/g^5/(b*x+a)^2-1/2*B*d^3/b/(-a*d+b*c)^3/g^5/(b*x+a)-1/2*B*d^4*ln(b*x+
a)/b/(-a*d+b*c)^4/g^5+1/2*B*d^4*ln(d*x+c)/b/(-a*d+b*c)^4/g^5+1/4*(-A-B*ln(e
*(d*x+c)^2/(b*x+a)^2))/b/g^5/(b*x+a)^4
```

Rubi [A]
time = 0.11, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {2548, 21, 46}

$$-\frac{B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A}{4bg^5(a+bx)^4}-\frac{Bd^4\log(a+bx)}{2bg^5(bc-ad)^4}+\frac{Bd^4\log(c+dx)}{2bg^5(bc-ad)^4}-\frac{Bd^3}{2bg^5(a+bx)(bc-ad)^3}+\frac{Bd^2}{4bg^5(a+bx)^2(bc-ad)^2}-\frac{Bd}{6bg^5(a+bx)^3(bc-ad)}+\frac{B}{8bg^5(a+bx)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])/(a*g + b*g*x)^5,x]
```

```
[Out] B/(8*b*g^5*(a + b*x)^4) - (B*d)/(6*b*(b*c - a*d)*g^5*(a + b*x)^3) + (B*d^2)
/(4*b*(b*c - a*d)^2*g^5*(a + b*x)^2) - (B*d^3)/(2*b*(b*c - a*d)^3*g^5*(a +
b*x)) - (B*d^4*Log[a + b*x])/(2*b*(b*c - a*d)^4*g^5) + (B*d^4*Log[c + d*x])
/(2*b*(b*c - a*d)^4*g^5) - (A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])/(4*b*g^
5*(a + b*x)^4)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2548

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Dist[B*n*((b*c
- a*d)/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /;
 FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c -
 a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Rubi steps

$$\int \frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(ag+bgx)^5}\,dx = -\frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{4bg^5(a+bx)^4}+\frac{B\int\frac{2(-bc+ad)}{g^4(a+bx)^5(c+dx)}\,dx}{4bg}$$

$$=-\frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{4bg^5(a+bx)^4}-\frac{(B(bc-ad))\int\frac{1}{(a+bx)^5(c+dx)}\,dx}{2bg^5}$$

$$=-\frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{4bg^5(a+bx)^4}-\frac{(B(bc-ad))\int\left(\frac{b}{(bc-ad)(a+bx)^5}-\frac{bd}{(bc-ad)^2(a+bx)^4}+\frac{}{(bc\cdots}\right.}{}$$

$$=\frac{B}{8bg^5(a+bx)^4}-\frac{Bd}{6b(bc-ad)g^5(a+bx)^3}+\frac{Bd^2}{4b(bc-ad)^2g^5(a+bx)^2}-\frac{}{2b(bc-\cdots}$$

Mathematica [**A**]
time = 0.12, size = 162, normalized size = 0.78

$$\frac{\frac{B\left(3(bc-ad)^4+4d(-bc+ad)^3(a+bx)+6d^2(bc-ad)^2(a+bx)^2+12d^3(-bc+ad)(a+bx)^3-12d^4(a+bx)^4\log(a+bx)+12d^4(a+bx)^4\log(c+dx)\right)}{(bc-ad)^4}-6\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{24bg^5(a+bx)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])/(a*g + b*g*x)^5,x]
```

```
[Out] ((B*(3*(b*c - a*d)^4 + 4*d*(-(b*c) + a*d)^3*(a + b*x) + 6*d^2*(b*c - a*d)^2
*(a + b*x)^2 + 12*d^3*(-(b*c) + a*d)*(a + b*x)^3 - 12*d^4*(a + b*x)^4*Log[a
 + b*x] + 12*d^4*(a + b*x)^4*Log[c + d*x]))/(b*c - a*d)^4 - 6*(A + B*Log[(e
*(c + d*x)^2)/(a + b*x)^2]))/(24*b*g^5*(a + b*x)^4)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 581 vs. 2(197) = 394.
time = 0.49, size = 582, normalized size = 2.80 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*(d*x+c)^2/(b*x+a)^2))/(b*g*x+a*g)^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/b*(1/4/(b*x+a)^4/g^5*A+1/4/g^5*B/(b*x+a)^4*ln(e*(a*d/(b*x+a)-b*c/(b*x+a)
-d)^2/b^2)-1/8/g^5*B*a^4*d^4/(a*d-b*c)^4/(b*x+a)^4+1/2/g^5*B*a^3*d^3/(a*d-b
```

```
*c)^4*b*c/(b*x+a)^4-3/4/g^5*B*a^2*d^2/(a*d-b*c)^4*b^2*c^2/(b*x+a)^4+1/2/g^5
*B*a*d/(a*d-b*c)^4*b^3*c^3/(b*x+a)^4-1/6/g^5*B*a^3*d^4/(a*d-b*c)^4/(b*x+a)^
3+1/2/g^5*B*a^2*d^3/(a*d-b*c)^4*b*c/(b*x+a)^3-1/2/g^5*B*a*d^2/(a*d-b*c)^4*b
^2*c^2/(b*x+a)^3-1/4/g^5*B*a^2*d^4/(a*d-b*c)^4/(b*x+a)^2+1/2/g^5*B*a*d^3/(a
*d-b*c)^4*b*c/(b*x+a)^2-1/2/g^5*B*a*d^4/(a*d-b*c)^4/(b*x+a)-1/2/g^5*B*a*d^5
/(a*d-b*c)^5*ln(a*d/(b*x+a)-b*c/(b*x+a)-d)-1/8/g^5*B*b^4*c^4/(a*d-b*c)^4/(b
*x+a)^4+1/6/g^5*B*b^3*c^3/(a*d-b*c)^4*d/(b*x+a)^3-1/4/g^5*B*b^2*c^2/(a*d-b*
c)^4*d^2/(b*x+a)^2+1/2/g^5*B*b*c/(a*d-b*c)^4*d^3/(b*x+a)+1/2/g^5*B*b*c*d^4/
(a*d-b*c)^5*ln(a*d/(b*x+a)-b*c/(b*x+a)-d))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 702 vs. $2(195) = 390$.
time = 0.33, size = 702, normalized size = 3.38

$$-\frac{1}{24}\,B\left(\frac{12\,b^3d^3x^3 - 3\,b^3c^3 + 13\,ab^2c^2d - 23\,a^2bcd^2 + 25\,a^3d^3 - 6\,(b^3cd^2 - 7\,ab^2d^3)x^2 + 4\,(b^3c^2d - 5\,ab^2cd^2 + 13\,a^2bd^3)x}{(b^8c^3 - 3\,ab^7c^2d + 3\,a^2b^6cd^2 - a^3b^5d^3)g^5x^4 + 4\,(ab^7c^3 - 3\,a^2b^6c^2d + 3\,a^3b^5cd^2 - a^4b^4d^3)g^5x^3 + 6\,(a^2b^6c^3 - 3\,a^3b^5c^2d + 3\,a^4b^4cd^2 - a^5b^3d^3)g^5x^2 + 4\,(a^3b^5c^3 - 3\,a^4b^4c^2d + 3\,a^5b^3cd^2 - a^6b^2d^3)g^5x + (a^4b^4c^3 - 3\,a^5b^3c^2d + 3\,a^6b^2cd^2 - a^7bd^3)g^5} + \frac{6\,\log\left(\frac{d^2x^2e}{b^2x^2+2\,abx+a^2} + \frac{2\,cdxe}{b^2x^2+2\,abx+a^2} + \frac{c^2e}{b^2x^2+2\,abx+a^2}\right)}{b^5g^5x^4 + 4\,ab^4g^5x^3 + 6\,a^2b^3g^5x^2 + 4\,a^3b^2g^5x + a^4bg^5} + \frac{12\,d^4\log\left(bx + a\right)}{(b^5c^4 - 4\,ab^4c^3d + 6\,a^2b^3c^2d^2 - 4\,a^3b^2cd^3 + a^4bd^4)g^5} - \frac{12\,d^4\log\left(dx + c\right)}{(b^5c^4 - 4\,ab^4c^3d + 6\,a^2b^3c^2d^2 - 4\,a^3b^2cd^3 + a^4bd^4)g^5}\right) - \frac{A}{4\,(b^5g^5x^4 + 4\,ab^4g^5x^3 + 6\,a^2b^3g^5x^2 + 4\,a^3b^2g^5x + a^4bg^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))/(b*g*x+a*g)^5,x, algorithm="maxi
ma")
```

```
[Out] -1/24*B*((12*b^3*d^3*x^3 - 3*b^3*c^3 + 13*a*b^2*c^2*d - 23*a^2*b*c*d^2 + 25
*a^3*d^3 - 6*(b^3*c*d^2 - 7*a*b^2*d^3)*x^2 + 4*(b^3*c^2*d - 5*a*b^2*c*d^2 +
 13*a^2*b*d^3)*x)/((b^8*c^3 - 3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 - a^3*b^5*d^3
)*g^5*x^4 + 4*(a*b^7*c^3 - 3*a^2*b^6*c^2*d + 3*a^3*b^5*c*d^2 - a^4*b^4*d^3)
*g^5*x^3 + 6*(a^2*b^6*c^3 - 3*a^3*b^5*c^2*d + 3*a^4*b^4*c*d^2 - a^5*b^3*d^3
)*g^5*x^2 + 4*(a^3*b^5*c^3 - 3*a^4*b^4*c^2*d + 3*a^5*b^3*c*d^2 - a^6*b^2*d^
3)*g^5*x + (a^4*b^4*c^3 - 3*a^5*b^3*c^2*d + 3*a^6*b^2*c*d^2 - a^7*b*d^3)*g^
5) + 6*log(d^2*x^2*e/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*x*e/(b^2*x^2 + 2*a*b
*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2))/(b^5*g^5*x^4 + 4*a*b^4*g^5*x^3
 + 6*a^2*b^3*g^5*x^2 + 4*a^3*b^2*g^5*x + a^4*b*g^5) + 12*d^4*log(b*x + a)/(
(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)
*g^5) - 12*d^4*log(d*x + c)/((b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 -
 4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5)) - 1/4*A/(b^5*g^5*x^4 + 4*a*b^4*g^5*x^3
+ 6*a^2*b^3*g^5*x^2 + 4*a^3*b^2*g^5*x + a^4*b*g^5)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 656 vs. $2(195) = 390$.
time = 0.36, size = 656, normalized size = 3.15

$$-\frac{3\,(2\,A - B)b^4c^4 - 8\,(3\,A - 2\,B)ab^3c^3d + 36\,(A - B)a^2b^2c^2d^2 - 24\,(A - 2\,B)a^3bcd^3 + (6\,A - 25\,B)a^4d^4 + 12\,(Bb^4cd^3 - Bab^3d^4)x^3 - 6\,(Bb^4c^2d^2 - 8\,Bab^3cd^3 + 7\,Ba^2b^2d^4)x^2 + 4\,(Bb^4c^3d - 6\,Bab^3c^2d^2 + 18\,Ba^2b^2cd^3 - 13\,Ba^3bd^4)x - 6\,(Bb^4d^4x^4 + 4\,Bab^3d^4x^3 + 6\,Ba^2b^2d^4x^2 + 4\,Ba^3bd^4x - Bb^4c^4 + 4\,Bab^3c^3d - 6\,Ba^2b^2c^2d^2 + 4\,Ba^3bcd^3)\log\left(\frac{(d^2x^2+2\,cdx+c^2)e}{b^2x^2+2\,abx+a^2}\right)}{24\,((b^9c^4 - 4\,ab^8c^3d + 6\,a^2b^7c^2d^2 - 4\,a^3b^6cd^3 + a^4b^5d^4)g^5x^4 + 4\,(ab^8c^4 - 4\,a^2b^7c^3d + 6\,a^3b^6c^2d^2 - 4\,a^4b^5cd^3 + a^5b^4d^4)g^5x^3 + 6\,(a^2b^7c^4 - 4\,a^3b^6c^3d + 6\,a^4b^5c^2d^2 - 4\,a^5b^4cd^3 + a^6b^3d^4)g^5x^2 + 4\,(a^3b^6c^4 - 4\,a^4b^5c^3d + 6\,a^5b^4c^2d^2 - 4\,a^6b^3cd^3 + a^7b^2d^4)g^5x + (a^4b^5c^4 - 4\,a^5b^4c^3d + 6\,a^6b^3c^2d^2 - 4\,a^7b^2cd^3 + a^8bd^4)g^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))/(b*g*x+a*g)^5,x, algorithm="fric
as")
```

```
[Out] -1/24*(3*(2*A - B)*b^4*c^4 - 8*(3*A - 2*B)*a*b^3*c^3*d + 36*(A - B)*a^2*b^2
*c^2*d^2 - 24*(A - 2*B)*a^3*b*c*d^3 + (6*A - 25*B)*a^4*d^4 + 12*(B*b^4*c*d^
3 - B*a*b^3*d^4)*x^3 - 6*(B*b^4*c^2*d^2 - 8*B*a*b^3*c*d^3 + 7*B*a^2*b^2*d^4
)*x^2 + 4*(B*b^4*c^3*d - 6*B*a*b^3*c^2*d^2 + 18*B*a^2*b^2*c*d^3 - 13*B*a^3*
b*d^4)*x - 6*(B*b^4*d^4*x^4 + 4*B*a*b^3*d^4*x^3 + 6*B*a^2*b^2*d^4*x^2 + 4*B
*a^3*b*d^4*x - B*b^4*c^4 + 4*B*a*b^3*c^3*d - 6*B*a^2*b^2*c^2*d^2 + 4*B*a^3*
b*c*d^3)*log((d^2*x^2 + 2*c*d*x + c^2)*e/(b^2*x^2 + 2*a*b*x + a^2)))/((b^9*
c^4 - 4*a*b^8*c^3*d + 6*a^2*b^7*c^2*d^2 - 4*a^3*b^6*c*d^3 + a^4*b^5*d^4)*g^
5*x^4 + 4*(a*b^8*c^4 - 4*a^2*b^7*c^3*d + 6*a^3*b^6*c^2*d^2 - 4*a^4*b^5*c*d^
3 + a^5*b^4*d^4)*g^5*x^3 + 6*(a^2*b^7*c^4 - 4*a^3*b^6*c^3*d + 6*a^4*b^5*c^2
*d^2 - 4*a^5*b^4*c*d^3 + a^6*b^3*d^4)*g^5*x^2 + 4*(a^3*b^6*c^4 - 4*a^4*b^5*
c^3*d + 6*a^5*b^4*c^2*d^2 - 4*a^6*b^3*c*d^3 + a^7*b^2*d^4)*g^5*x + (a^4*b^5
*c^4 - 4*a^5*b^4*c^3*d + 6*a^6*b^3*c^2*d^2 - 4*a^7*b^2*c*d^3 + a^8*b*d^4)*g
^5)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 947 vs. $2(182) = 364$.
time = 3.38, size = 947, normalized size = 4.55

$$-\frac{B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{4a^4bg^5+16a^3b^2g^5x+24a^2b^3g^5x^2+16ab^4g^5x^3+4b^5g^5x^4}+\frac{Bd^4\log\left(x+\frac{-\frac{Ba^5d^9}{(ad-bc)^4}+\frac{5Ba^4bcd^8}{(ad-bc)^4}-\frac{10Ba^3b^2c^2d^7}{(ad-bc)^4}+\frac{10Ba^2b^3c^3d^6}{(ad-bc)^4}-\frac{5Bab^4c^4d^5}{(ad-bc)^4}+Bad^5+\frac{Bb^5c^5d^4}{(ad-bc)^4}+Bbcd^4}{2Bbd^5}\right)}{2bg^5(ad-bc)^4}-\frac{Bd^4\log\left(x+\frac{\frac{Ba^5d^9}{(ad-bc)^4}-\frac{5Ba^4bcd^8}{(ad-bc)^4}+\frac{10Ba^3b^2c^2d^7}{(ad-bc)^4}-\frac{10Ba^2b^3c^3d^6}{(ad-bc)^4}+\frac{5Bab^4c^4d^5}{(ad-bc)^4}+Bad^5-\frac{Bb^5c^5d^4}{(ad-bc)^4}+Bbcd^4}{2Bbd^5}\right)}{2bg^5(ad-bc)^4}+\frac{-6Aa^3d^3+18Aa^2bcd^2-18Aab^2c^2d+6Ab^3c^3+25Ba^3d^3-23Ba^2bcd^2+13Bab^2c^2d-3Bb^3c^3+12Bb^3d^3x^3+x^2\cdot(42Bab^2d^3-6Bb^3cd^2)+x(52Ba^2bd^3-20Bab^2cd^2+4Bb^3c^2d)}{24a^7bd^3g^5-72a^6b^2cd^2g^5+72a^5b^3c^2dg^5-24a^4b^4c^3g^5+x^4\cdot(24a^3b^5d^3g^5-72a^2b^6cd^2g^5+72ab^7c^2dg^5-24b^8c^3g^5)+x^3\cdot(96a^4b^4d^3g^5-288a^3b^5cd^2g^5+288a^2b^6c^2dg^5-96ab^7c^3g^5)+x^2\cdot(144a^5b^3d^3g^5-432a^4b^4cd^2g^5+432a^3b^5c^2dg^5-144a^2b^6c^3g^5)+x(96a^6b^2d^3g^5-288a^5b^3cd^2g^5+288a^4b^4c^2dg^5-96a^3b^5c^3g^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(d*x+c)**2/(b*x+a)**2))/(b*g*x+a*g)**5,x)
```

```
[Out] -B*log(e*(c + d*x)**2/(a + b*x)**2)/(4*a**4*b*g**5 + 16*a**3*b**2*g**5*x +
24*a**2*b**3*g**5*x**2 + 16*a*b**4*g**5*x**3 + 4*b**5*g**5*x**4) + B*d**4*l
og(x + (-B*a**5*d**9/(a*d - b*c)**4 + 5*B*a**4*b*c*d**8/(a*d - b*c)**4 - 10
*B*a**3*b**2*c**2*d**7/(a*d - b*c)**4 + 10*B*a**2*b**3*c**3*d**6/(a*d - b*c
)**4 - 5*B*a*b**4*c**4*d**5/(a*d - b*c)**4 + B*a*d**5 + B*b**5*c**5*d**4/(a
*d - b*c)**4 + B*b*c*d**4)/(2*B*b*d**5))/(2*b*g**5*(a*d - b*c)**4) - B*d**4
*log(x + (B*a**5*d**9/(a*d - b*c)**4 - 5*B*a**4*b*c*d**8/(a*d - b*c)**4 + 1
0*B*a**3*b**2*c**2*d**7/(a*d - b*c)**4 - 10*B*a**2*b**3*c**3*d**6/(a*d - b*
c)**4 + 5*B*a*b**4*c**4*d**5/(a*d - b*c)**4 + B*a*d**5 - B*b**5*c**5*d**4/(
a*d - b*c)**4 + B*b*c*d**4)/(2*B*b*d**5))/(2*b*g**5*(a*d - b*c)**4) + (-6*A
*a**3*d**3 + 18*A*a**2*b*c*d**2 - 18*A*a*b**2*c**2*d + 6*A*b**3*c**3 + 25*B
*a**3*d**3 - 23*B*a**2*b*c*d**2 + 13*B*a*b**2*c**2*d - 3*B*b**3*c**3 + 12*B
*b**3*d**3*x**3 + x**2*(42*B*a*b**2*d**3 - 6*B*b**3*c*d**2) + x*(52*B*a**2*
b*d**3 - 20*B*a*b**2*c*d**2 + 4*B*b**3*c**2*d))/(24*a**7*b*d**3*g**5 - 72*a
**6*b**2*c*d**2*g**5 + 72*a**5*b**3*c**2*d*g**5 - 24*a**4*b**4*c**3*g**5 +
x**4*(24*a**3*b**5*d**3*g**5 - 72*a**2*b**6*c*d**2*g**5 + 72*a*b**7*c**2*d*
g**5 - 24*b**8*c**3*g**5) + x**3*(96*a**4*b**4*d**3*g**5 - 288*a**3*b**5*c*
d**2*g**5 + 288*a**2*b**6*c**2*d*g**5 - 96*a*b**7*c**3*g**5) + x**2*(144*a*
*5*b**3*d**3*g**5 - 432*a**4*b**4*c*d**2*g**5 + 432*a**3*b**5*c**2*d*g**5 -
 144*a**2*b**6*c**3*g**5) + x*(96*a**6*b**2*d**3*g**5 - 288*a**5*b**3*c*d**
2*g**5 + 288*a**4*b**4*c**2*d*g**5 - 96*a**3*b**5*c**3*g**5))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 416 vs. 2(195) = 390.
time = 3.80, size = 416, normalized size = 2.00

$$\frac{Bd^4\log\left(-\frac{bcg}{bgx+ag}+\frac{adg}{bgx+ag}-d\right)}{2\,(b^5c^4g^5-4\,ab^4c^3dg^5+6\,a^2b^3c^2d^2g^5-4\,a^3b^2cd^3g^5+a^4bd^4g^5)}-\frac{Bd^3}{2\,(b^3c^3g^3-3\,ab^2c^2dg^3+3\,a^2bcd^2g^3-a^3d^3g^3)(bgx+ag)bg}+\frac{Bd^2}{4\,(b^2c^2g-2\,abcdg+a^2d^2g)(bgx+ag)^2bg^2}-\frac{B\log\left(\frac{\frac{b^2c^2g^2}{(bgx+ag)^2}-\frac{2\,abcdg^2}{(bgx+ag)^2}+\frac{a^2d^2g^2}{(bgx+ag)^2}+\frac{2\,bcdg}{bgx+ag}-\frac{2\,ad^2g}{bgx+ag}+d^2}{b^2}\right)}{4\,(bgx+ag)^4bg}-\frac{Bd}{6\,(bgx+ag)^3(bc-ad)bg^2}-\frac{2\,Ab^3g^3+Bb^3g^3}{8\,(bgx+ag)^4b^4g^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))/(b*g*x+a*g)^5,x, algorithm="giac
")
```

```
[Out] 1/2*B*d^4*log(-b*c*g/(b*g*x + a*g) + a*d*g/(b*g*x + a*g) - d)/(b^5*c^4*g^5
- 4*a*b^4*c^3*d*g^5 + 6*a^2*b^3*c^2*d^2*g^5 - 4*a^3*b^2*c*d^3*g^5 + a^4*b*d
^4*g^5) - 1/2*B*d^3/((b^3*c^3*g^3 - 3*a*b^2*c^2*d*g^3 + 3*a^2*b*c*d^2*g^3 -
 a^3*d^3*g^3)*(b*g*x + a*g)*b*g) + 1/4*B*d^2/((b^2*c^2*g - 2*a*b*c*d*g + a^
2*d^2*g)*(b*g*x + a*g)^2*b*g^2) - 1/4*B*log((b^2*c^2*g^2/(b*g*x + a*g)^2 -
2*a*b*c*d*g^2/(b*g*x + a*g)^2 + a^2*d^2*g^2/(b*g*x + a*g)^2 + 2*b*c*d*g/(b*
g*x + a*g) - 2*a*d^2*g/(b*g*x + a*g) + d^2)/b^2)/((b*g*x + a*g)^4*b*g) - 1/
6*B*d/((b*g*x + a*g)^3*(b*c - a*d)*b*g^2) - 1/8*(2*A*b^3*g^3 + B*b^3*g^3)/(
(b*g*x + a*g)^4*b^4*g^4)
```

Mupad [B]
time = 7.90, size = 579, normalized size = 2.78

$$\frac{B\,d^4\,\mathrm{atanh}\left(\frac{-2\,a^4\,b\,d^4\,g^5+4\,a^3\,b^2\,c\,d^3\,g^5-4\,a\,b^4\,c^3\,d\,g^5+2\,b^5\,c^4\,g^5}{2\,b\,g^5\,{\left(a\,d-b\,c\right)}^4}-\frac{2\,b\,d\,x\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}{{\left(a\,d-b\,c\right)}^4}\right)}{b\,g^5\,{\left(a\,d-b\,c\right)}^4}-\frac{B\,\ln\left(\frac{e\,{\left(c+d\,x\right)}^2}{{\left(a+b\,x\right)}^2}\right)}{4\,b^2\,g^5\,\left(4\,a^3\,x+\frac{a^4}{b}+b^3\,x^4+6\,a^2\,b\,x^2+4\,a\,b^2\,x^3\right)}-\frac{\frac{6\,A\,a^3\,d^3-6\,A\,b^3\,c^3-25\,B\,a^3\,d^3+3\,B\,b^3\,c^3+18\,A\,a\,b^2\,c^2\,d-18\,A\,a^2\,b\,c\,d^2-13\,B\,a\,b^2\,c^2\,d+23\,B\,a^2\,b\,c\,d^2}{12\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}+\frac{d^2\,x^2\,\left(B\,b^3\,c-7\,B\,a\,b^2\,d\right)}{2\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}-\frac{d\,x\,\left(13\,B\,a^2\,b\,d^2-5\,B\,a\,b^2\,c\,d+B\,b^3\,c^2\right)}{3\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}-\frac{B\,b^3\,d^3\,x^3}{a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3}}{2\,a^4\,b\,g^5+8\,a^3\,b^2\,g^5\,x+12\,a^2\,b^3\,g^5\,x^2+8\,a\,b^4\,g^5\,x^3+2\,b^5\,g^5\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(c + d*x)^2)/(a + b*x)^2))/(a*g + b*g*x)^5,x)
```

```
[Out] (B*d^4*atanh((2*b^5*c^4*g^5 - 2*a^4*b*d^4*g^5 - 4*a*b^4*c^3*d*g^5 + 4*a^3*b
^2*c*d^3*g^5)/(2*b*g^5*(a*d - b*c)^4) - (2*b*d*x*(a^3*d^3 - b^3*c^3 + 3*a*b
^2*c^2*d - 3*a^2*b*c*d^2))/(a*d - b*c)^4))/(b*g^5*(a*d - b*c)^4) - (B*log((
e*(c + d*x)^2)/(a + b*x)^2))/(4*b^2*g^5*(4*a^3*x + a^4/b + b^3*x^4 + 6*a^2*
b*x^2 + 4*a*b^2*x^3)) - ((6*A*a^3*d^3 - 6*A*b^3*c^3 - 25*B*a^3*d^3 + 3*B*b^
3*c^3 + 18*A*a*b^2*c^2*d - 18*A*a^2*b*c*d^2 - 13*B*a*b^2*c^2*d + 23*B*a^2*b
*c*d^2)/(12*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) + (d^2*x^2
*(B*b^3*c - 7*B*a*b^2*d))/(2*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c
*d^2)) - (d*x*(B*b^3*c^2 + 13*B*a^2*b*d^2 - 5*B*a*b^2*c*d))/(3*(a^3*d^3 - b
^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) - (B*b^3*d^3*x^3)/(a^3*d^3 - b^3*c
^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))/(2*a^4*b*g^5 + 2*b^5*g^5*x^4 + 8*a^3*b
^2*g^5*x + 8*a*b^4*g^5*x^3 + 12*a^2*b^3*g^5*x^2)
```

3.210 $\int (ag+bgx)^4 \left(A+B\log \left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2 \, dx$

Optimal. Leaf size=515

$$\frac{26B^2(bc-ad)^4g^4x}{15d^4}-\frac{7B^2(bc-ad)^3g^4(a+bx)^2}{15bd^3}+\frac{2B^2(bc-ad)^2g^4(a+bx)^3}{15bd^2}-\frac{10B^2(bc-ad)^5g^4\log(a+bx)}{3bd^5}$$

```
[Out] 26/15*B^2*(-a*d+b*c)^4*g^4*x/d^4-7/15*B^2*(-a*d+b*c)^3*g^4*(b*x+a)^2/b/d^3+
2/15*B^2*(-a*d+b*c)^2*g^4*(b*x+a)^3/b/d^2-10/3*B^2*(-a*d+b*c)^5*g^4*ln(b*x+
a)/b/d^5-26/15*B^2*(-a*d+b*c)^5*g^4*ln((d*x+c)/(b*x+a))/b/d^5+2/5*B*(-a*d+b
*c)^3*g^4*(b*x+a)^2*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))/b/d^3-4/15*B*(-a*d+b*c)
^2*g^4*(b*x+a)^3*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))/b/d^2+1/5*B*(-a*d+b*c)*g^4
*(b*x+a)^4*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))/b/d-4/5*B*(-a*d+b*c)^4*g^4*(d*x+
c)*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))/d^5+1/5*g^4*(b*x+a)^5*(A+B*ln(e*(d*x+c)^
2/(b*x+a)^2))^2/b-4/5*B*(-a*d+b*c)^5*g^4*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))*ln
(1-d*(b*x+a)/b/(d*x+c))/b/d^5+8/5*B^2*(-a*d+b*c)^5*g^4*polylog(2,d*(b*x+a)/
b/(d*x+c))/b/d^5
```

Rubi [A]

time = 0.46, antiderivative size = 515, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 8, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2552, 2356, 2389, 2379, 2438, 2351, 31, 46}

$$\frac{8B^2g^4(bc-ad)^5\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{5bd^5}-\frac{4Bg^4(bc-ad)^5\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{5bd^5}-\frac{4Bg^4(c+dx)(bc-ad)^4\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{5d^5}+\frac{2Bg^4(a+bx)^2(bc-ad)^3\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{5bd^3}-\frac{4Bg^4(a+bx)^3(bc-ad)^2\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{15bd^2}+\frac{Bg^4(a+bx)^4(bc-ad)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{5bd}+\frac{g^4(a+bx)^5\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{5b}-\frac{10B^2g^4(bc-ad)^5\log(a+bx)}{3bd^5}-\frac{26B^2g^4(bc-ad)^5\log\left(\frac{c+dx}{a+bx}\right)}{15bd^5}+\frac{26B^2g^4x(bc-ad)^4}{15d^4}-\frac{7B^2g^4(a+bx)^2(bc-ad)^3}{15bd^3}+\frac{2B^2g^4(a+bx)^3(bc-ad)^2}{15bd^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*g + b*g*x)^4*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2,x]
```

```
[Out] (26*B^2*(b*c - a*d)^4*g^4*x)/(15*d^4) - (7*B^2*(b*c - a*d)^3*g^4*(a + b*x)^
2)/(15*b*d^3) + (2*B^2*(b*c - a*d)^2*g^4*(a + b*x)^3)/(15*b*d^2) - (10*B^2*
(b*c - a*d)^5*g^4*Log[a + b*x])/(3*b*d^5) - (26*B^2*(b*c - a*d)^5*g^4*Log[(
c + d*x)/(a + b*x)])/(15*b*d^5) + (2*B*(b*c - a*d)^3*g^4*(a + b*x)^2*(A + B
*Log[(e*(c + d*x)^2)/(a + b*x)^2]))/(5*b*d^3) - (4*B*(b*c - a*d)^2*g^4*(a +
 b*x)^3*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]))/(15*b*d^2) + (B*(b*c - a*
d)*g^4*(a + b*x)^4*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]))/(5*b*d) - (4*B
*(b*c - a*d)^4*g^4*(c + d*x)*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]))/(5*d
^5) + (g^4*(a + b*x)^5*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2)/(5*b) -
(4*B*(b*c - a*d)^5*g^4*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])*Log[1 - (d*
(a + b*x))/(b*(c + d*x))])/(5*b*d^5) + (8*B^2*(b*c - a*d)^5*g^4*PolyLog[2,
(d*(a + b*x))/(b*(c + d*x))])/(5*b*d^5)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2552

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(
m + 1)*(g/d)^m, Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a
```

```
 + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[n
 + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[d*f
- c*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Rubi steps

$$\int (ag+bgx)^4 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2 dx = \frac{g^4(a+bx)^5\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{5b} - \frac{(2B)\int \frac{2(bc-ad)g^5(\ldots}{\ldots}}{\ldots}$$

$$= \frac{g^4(a+bx)^5\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{5b} - \frac{(4B(bc-ad)g^4)\ldots}{\ldots}$$

$$= \frac{g^4(a+bx)^5\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{5b} - \frac{(4B(bc-ad)g^4)\ldots}{\ldots}$$

$$= \frac{g^4(a+bx)^5\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{5b} - \frac{(4B(bc-ad)g^4)\ldots}{\ldots}$$

$$= -\frac{4AB(bc-ad)^4g^4x}{5d^4} + \frac{2B(bc-ad)^3g^4(a+bx)^2\left(A+B\ldots\right.}{5bd^3}$$

$$= -\frac{4AB(bc-ad)^4g^4x}{5d^4} - \frac{4B^2(bc-ad)^4g^4(a+bx)\log\left(\frac{e(c\ldots}{(a\ldots}\right.}{5bd^4}$$

$$= -\frac{4AB(bc-ad)^4g^4x}{5d^4} - \frac{8B^2(bc-ad)^5g^4\log(c+dx)}{5bd^5} - \frac{4\ldots}{\ldots}$$

$$= -\frac{4AB(bc-ad)^4g^4x}{5d^4} + \frac{26B^2(bc-ad)^4g^4x}{15d^4} - \frac{7B^2(bc-\ldots}{\ldots}$$

$$= -\frac{4AB(bc-ad)^4g^4x}{5d^4} + \frac{26B^2(bc-ad)^4g^4x}{15d^4} - \frac{7B^2(bc-\ldots}{\ldots}$$

$$= -\frac{4AB(bc-ad)^4g^4x}{5d^4} + \frac{26B^2(bc-ad)^4g^4x}{15d^4} - \frac{7B^2(bc-\ldots}{\ldots}$$

$$= -\frac{4AB(bc-ad)^4g^4x}{5d^4} + \frac{26B^2(bc-ad)^4g^4x}{15d^4} - \frac{7B^2(bc-\ldots}{\ldots}$$

Mathematica [A]

time = 0.32, size = 524, normalized size = 1.02

$$\frac{g^4\left((a+bx)^5\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2-\frac{B(bc-ad)\left(12Abd(bc-ad)^3x+24B(bc-ad)^4\log(c+dx)-4B(bc-ad)^2\left(2bd(bc-ad)x-d^2(a+bx)^2-2(bc-ad)^2\log(c+dx)\right)-B(bc-ad)\left(6bd(bc-ad)^2x+3d^2(-bc+ad)(a+bx)^2+2d^3(a+bx)^3-6(bc-ad)^3\log(c+dx)\right)-12B(bc-ad)^3(bdx+(-bc+ad)\log(c+dx))+12Bd(bc-ad)^3(a+bx)\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)-6d^2(bc-ad)^2(a+bx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)+4d^3(bc-ad)(a+bx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)-3d^4(a+bx)^4\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)-12(bc-ad)^4\log(c+dx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)-12B(bc-ad)^4\left(\left(2\log\left(\frac{d(a+bx)}{-bc+ad}\right)-\log(c+dx)\right)\log(c+dx)+2\mathrm{Li}_2\left(\frac{b(c+dx)}{bc-ad}\right)\right)\right)}{3d^5}\right)}{5b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*g + b*g*x)^4*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2,x]
```

```
[Out] (g^4*((a + b*x)^5*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2 - (B*(b*c - a*
d)*(12*A*b*d*(b*c - a*d)^3*x + 24*B*(b*c - a*d)^4*Log[c + d*x] - 4*B*(b*c -
 a*d)^2*(2*b*d*(b*c - a*d)*x - d^2*(a + b*x)^2 - 2*(b*c - a*d)^2*Log[c + d*
x]) - B*(b*c - a*d)*(6*b*d*(b*c - a*d)^2*x + 3*d^2*(-(b*c) + a*d)*(a + b*x)
^2 + 2*d^3*(a + b*x)^3 - 6*(b*c - a*d)^3*Log[c + d*x]) - 12*B*(b*c - a*d)^3
*(b*d*x + (-(b*c) + a*d)*Log[c + d*x]) + 12*B*d*(b*c - a*d)^3*(a + b*x)*Log
[(e*(c + d*x)^2)/(a + b*x)^2] - 6*d^2*(b*c - a*d)^2*(a + b*x)^2*(A + B*Log[
(e*(c + d*x)^2)/(a + b*x)^2]) + 4*d^3*(b*c - a*d)*(a + b*x)^3*(A + B*Log[(e
*(c + d*x)^2)/(a + b*x)^2]) - 3*d^4*(a + b*x)^4*(A + B*Log[(e*(c + d*x)^2)/
(a + b*x)^2]) - 12*(b*c - a*d)^4*Log[c + d*x]*(A + B*Log[(e*(c + d*x)^2)/(a
 + b*x)^2]) - 12*B*(b*c - a*d)^4*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Lo
g[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(3*d^
5)))/(5*b)
```

Maple [F]
time = 0.17, size = 0, normalized size = 0.00

$$\int (bgx+ag)^4\left(A+B\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*g*x+a*g)^4*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))^2,x)
```

```
[Out] int((b*g*x+a*g)^4*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))^2,x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 2371 vs. 2(496) = 992.
time = 0.45, size = 2371, normalized size = 4.60

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^4*(A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2,x, algorithm="ma
xima")
```

```
[Out] 1/5*A^2*b^4*g^4*x^5 + A^2*a*b^3*g^4*x^4 + 2*A^2*a^2*b^2*g^4*x^3 + 2*A^2*a^3
*b*g^4*x^2 + 2*(x*log(d^2*x^2*e/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*x*e/(b^2*
x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2)) - 2*a*log(b*x + a)/
```

b + 2*c*log(d*x + c)/d)*A*B*a^4*g^4 + 4*(x^2*log(d^2*x^2*e/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*x*e/(b^2*x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2)) + 2*a^2*log(b*x + a)/b^2 - 2*c^2*log(d*x + c)/d^2 + 2*(b*c - a*d)*x/(b*d))*A*B*a^3*b*g^4 + 4*(x^3*log(d^2*x^2*e/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*x*e/(b^2*x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2)) - 2*a^3*log(b*x + a)/b^3 + 2*c^3*log(d*x + c)/d^3 + ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*A*B*a^2*b^2*g^4 + 2/3*(3*x^4*log(d^2*x^2*e/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*x*e/(b^2*x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2)) + 6*a^4*log(b*x + a)/b^4 - 6*c^4*log(d*x + c)/d^4 + (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3))*A*B*a*b^3*g^4 + 1/15*(6*x^5*log(d^2*x^2*e/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*x*e/(b^2*x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2)) - 12*a^5*log(b*x + a)/b^5 + 12*c^5*log(d*x + c)/d^5 + (3*(b^4*c*d^3 - a*b^3*d^4)*x^4 - 4*(b^4*c^2*d^2 - a^2*b^2*d^4)*x^3 + 6*(b^4*c^3*d - a^3*b*d^4)*x^2 - 12*(b^4*c^4 - a^4*d^4)*x)/(b^4*d^4))*A*B*b^4*g^4 + A^2*a^4*g^4*x - 2/15*(19*b^4*c^5*g^4 - 83*a*b^3*c^4*d*g^4 + 136*a^2*b^2*c^3*d^2*g^4 - 96*a^3*b*c^2*d^3*g^4 + 18*a^4*c*d^4*g^4)*B^2*log(d*x + c)/d^5 - 8/5*(b^5*c^5*g^4 - 5*a*b^4*c^4*d*g^4 + 10*a^2*b^3*c^3*d^2*g^4 - 10*a^3*b^2*c^2*d^3*g^4 + 5*a^4*b*c*d^4*g^4 - a^5*d^5*g^4)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B^2/(b*d^5) + 1/15*(3*B^2*b^5*d^5*g^4*x^5 + 3*(b^5*c*d^4*g^4 + 4*a*b^4*d^5*g^4)*B^2*x^4 - 2*(b^5*c^2*d^3*g^4 - 8*a*b^4*c*d^4*g^4 - 8*a^2*b^3*d^5*g^4)*B^2*x^3 - (b^5*c^3*d^2*g^4 + 3*a*b^4*c^2*d^3*g^4 - 27*a^2*b^3*c*d^4*g^4 - 7*a^3*b^2*d^5*g^4)*B^2*x^2 + (14*b^5*c^4*d*g^4 - 58*a*b^4*c^3*d^2*g^4 + 84*a^2*b^3*c^2*d^3*g^4 - 38*a^3*b^2*c*d^4*g^4 + 13*a^4*b*d^5*g^4)*B^2*x + 12*(B^2*b^5*d^5*g^4*x^5 + 5*B^2*a*b^4*d^5*g^4*x^4 + 10*B^2*a^2*b^3*d^5*g^4*x^3 + 10*B^2*a^3*b^2*d^5*g^4*x^2 + 5*B^2*a^4*b*d^5*g^4*x + B^2*a^5*d^5*g^4)*log(b*x + a)^2 + 12*(B^2*b^5*d^5*g^4*x^5 + 5*B^2*a*b^4*d^5*g^4*x^4 + 10*B^2*a^2*b^3*d^5*g^4*x^3 + 10*B^2*a^3*b^2*d^5*g^4*x^2 + 5*B^2*a^4*b*d^5*g^4*x + (b^5*c^5*g^4 - 5*a*b^4*c^4*d*g^4 + 10*a^2*b^3*c^3*d^2*g^4 - 10*a^3*b^2*c^2*d^3*g^4 + 5*a^4*b*c*d^4*g^4)*B^2)*log(d*x + c)^2 - 2*(6*B^2*b^5*d^5*g^4*x^5 + 3*(b^5*c*d^4*g^4 + 9*a*b^4*d^5*g^4)*B^2*x^4 - 4*(b^5*c^2*d^3*g^4 - 5*a*b^4*c*d^4*g^4 - 11*a^2*b^3*d^5*g^4)*B^2*x^3 + 6*(b^5*c^3*d^2*g^4 - 5*a*b^4*c^2*d^3*g^4 + 10*a^2*b^3*c*d^4*g^4 + 4*a^3*b^2*d^5*g^4)*B^2*x^2 - 6*(2*b^5*c^4*d*g^4 - 10*a*b^4*c^3*d^2*g^4 + 20*a^2*b^3*c^2*d^3*g^4 - 20*a^3*b^2*c*d^4*g^4 + 3*a^4*b*d^5*g^4)*B^2*x - (12*a*b^4*c^4*d*g^4 - 54*a^2*b^3*c^3*d^2*g^4 + 94*a^3*b^2*c^2*d^3*g^4 - 77*a^4*b*c*d^4*g^4 + 19*a^5*d^5*g^4)*B^2)*log(b*x + a) + 2*(6*B^2*b^5*d^5*g^4*x^5 + 3*(b^5*c*d^4*g^4 + 9*a*b^4*d^5*g^4)*B^2*x^4 - 4*(b^5*c^2*d^3*g^4 - 5*a*b^4*c*d^4*g^4 - 11*a^2*b^3*d^5*g^4)*B^2*x^3 + 6*(b^5*c^3*d^2*g^4 - 5*a*b^4*c^2*d^3*g^4 + 10*a^2*b^3*c*d^4*g^4 + 4*a^3*b^2*d^5*g^4)*B^2*x^2 - 6*(2*b^5*c^4*d*g^4 - 10*a*b^4*c^3*d^2*g^4 + 20*a^2*b^3*c^2*d^3*g^4 - 20*a^3*b^2*c*d^4*g^4 + 3*a^4*b*d^5*g^4)*B^2*x - 12*(B^2*b^5*d^5*g^4*x^5 + 5*B^2*a*b^4*d^5*g^4*x^4 + 10*B^2*a^2*b^3*d^5*g^4*x^3 + 10*B^2*a^3*b^2*d^5*g^4*x^2 + 5*B^2*a^4*b*d^5*g^4*x + B^2*a^5*d^5*g^4)*log(b*x + a))*log(d*x + c))/(b*d^5)

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^4*(A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2,x, algorithm="fr
icas")
```

```
[Out] integral(A^2*b^4*g^4*x^4 + 4*A^2*a*b^3*g^4*x^3 + 6*A^2*a^2*b^2*g^4*x^2 + 4*
A^2*a^3*b*g^4*x + A^2*a^4*g^4 + (B^2*b^4*g^4*x^4 + 4*B^2*a*b^3*g^4*x^3 + 6*
B^2*a^2*b^2*g^4*x^2 + 4*B^2*a^3*b*g^4*x + B^2*a^4*g^4)*log((d^2*x^2 + 2*c*d
*x + c^2)*e/(b^2*x^2 + 2*a*b*x + a^2))^2 + 2*(A*B*b^4*g^4*x^4 + 4*A*B*a*b^3
*g^4*x^3 + 6*A*B*a^2*b^2*g^4*x^2 + 4*A*B*a^3*b*g^4*x + A*B*a^4*g^4)*log((d^
2*x^2 + 2*c*d*x + c^2)*e/(b^2*x^2 + 2*a*b*x + a^2)), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)**4*(A+B*ln(e*(d*x+c)**2/(b*x+a)**2))**2,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^4*(A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2,x, algorithm="gi
ac")
```

```
[Out] integrate((b*g*x + a*g)^4*(B*log((d*x + c)^2*e/(b*x + a)^2) + A)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a\,g+b\,g\,x)^4\,\left(A+B\,\ln\left(\frac{e\,{(c+d\,x)}^2}{{(a+b\,x)}^2}\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*g + b*g*x)^4*(A + B*log((e*(c + d*x)^2)/(a + b*x)^2))^2,x)
```

```
[Out] int((a*g + b*g*x)^4*(A + B*log((e*(c + d*x)^2)/(a + b*x)^2))^2, x)
```

3.211 $\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2 dx$

Optimal. Leaf size=422

$$-\frac{5B^2(bc-ad)^3g^3x}{3d^3}+\frac{B^2(bc-ad)^2g^3(a+bx)^2}{3bd^2}+\frac{11B^2(bc-ad)^4g^3\log(a+bx)}{3bd^4}+\frac{5B^2(bc-ad)^4g^3\log\left(\frac{c+dx}{a+bx}\right)}{3bd^4}$$

```
[Out] -5/3*B^2*(-a*d+b*c)^3*g^3*x/d^3+1/3*B^2*(-a*d+b*c)^2*g^3*(b*x+a)^2/b/d^2+11
/3*B^2*(-a*d+b*c)^4*g^3*ln(b*x+a)/b/d^4+5/3*B^2*(-a*d+b*c)^4*g^3*ln((d*x+c)
/(b*x+a))/b/d^4-1/2*B*(-a*d+b*c)^2*g^3*(b*x+a)^2*(A+B*ln(e*(d*x+c)^2/(b*x+a
)^2))/b/d^2+1/3*B*(-a*d+b*c)*g^3*(b*x+a)^3*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))/
b/d+B*(-a*d+b*c)^3*g^3*(d*x+c)*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))/d^4+1/4*g^3*
(b*x+a)^4*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))^2/b+B*(-a*d+b*c)^4*g^3*(A+B*ln(e*
(d*x+c)^2/(b*x+a)^2))*ln(1-d*(b*x+a)/b/(d*x+c))/b/d^4-2*B^2*(-a*d+b*c)^4*g^
3*polylog(2,d*(b*x+a)/b/(d*x+c))/b/d^4
```

Rubi [A]
time = 0.33, antiderivative size = 422, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 8, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {2552, 2356, 2389, 2379, 2438, 2351, 31, 46}

$$-\frac{2B^2g^3(bc-ad)^4\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{bd^4}+\frac{Bg^3(bc-ad)^4\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{bd^4}+\frac{Bg^3(c+dx)(bc-ad)^3\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{d^4}-\frac{Bg^3(a+bx)^2(bc-ad)^2\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{2bd^2}+\frac{Bg^3(a+bx)^3(bc-ad)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{3bd}+\frac{g^3(a+bx)^4\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{4b}+\frac{11B^2g^3(bc-ad)^4\log(a+bx)}{3bd^4}+\frac{5B^2g^3(bc-ad)^4\log\left(\frac{c+dx}{a+bx}\right)}{3bd^4}-\frac{5B^2g^3x(bc-ad)^3}{3d^3}+\frac{B^2g^3(a+bx)^2(bc-ad)^2}{3bd^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*g + b*g*x)^3*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2,x]
```

```
[Out] (-5*B^2*(b*c - a*d)^3*g^3*x)/(3*d^3) + (B^2*(b*c - a*d)^2*g^3*(a + b*x)^2)/
(3*b*d^2) + (11*B^2*(b*c - a*d)^4*g^3*Log[a + b*x])/(3*b*d^4) + (5*B^2*(b*c
 - a*d)^4*g^3*Log[(c + d*x)/(a + b*x)])/(3*b*d^4) - (B*(b*c - a*d)^2*g^3*(a
 + b*x)^2*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]))/(2*b*d^2) + (B*(b*c - a
*d)*g^3*(a + b*x)^3*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]))/(3*b*d) + (B*
(b*c - a*d)^3*g^3*(c + d*x)*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]))/d^4 +
 (g^3*(a + b*x)^4*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2)/(4*b) + (B*(b
*c - a*d)^4*g^3*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])*Log[1 - (d*(a + b*
x))/(b*(c + d*x))])/(b*d^4) - (2*B^2*(b*c - a*d)^4*g^3*PolyLog[2, (d*(a + b
*x))/(b*(c + d*x))])/(b*d^4)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
```

```
NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
n + 2, 0])
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2552

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(
m + 1)*(g/d)^m, Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a
 + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[n
 + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[d*f
- c*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Rubi steps

$$\int (ag+bgx)^3 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2 dx = \frac{g^3(a+bx)^4\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{4b} - \frac{B\int \frac{2(bc-ad)g^4(a+b}{}}{}$$

$$= \frac{g^3(a+bx)^4\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{4b} - \frac{(B(bc-ad)g^3)\int}{}$$

$$= \frac{g^3(a+bx)^4\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{4b} - \frac{(B(bc-ad)g^3)\int}{}$$

$$= \frac{g^3(a+bx)^4\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{4b} - \frac{(B(bc-ad)g^3)\int}{}$$

$$= \frac{AB(bc-ad)^3g^3x}{d^3} - \frac{B(bc-ad)^2g^3(a+bx)^2\left(A+B\log\right.}{2bd^2}$$

$$= \frac{AB(bc-ad)^3g^3x}{d^3} + \frac{B^2(bc-ad)^3g^3(a+bx)\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right.}{bd^3}$$

$$= \frac{AB(bc-ad)^3g^3x}{d^3} + \frac{2B^2(bc-ad)^4g^3\log(c+dx)}{bd^4} + \frac{B^2(b}{}$$

$$= \frac{AB(bc-ad)^3g^3x}{d^3} - \frac{5B^2(bc-ad)^3g^3x}{3d^3} + \frac{B^2(bc-ad)^2g^3}{3bd^2}$$

$$= \frac{AB(bc-ad)^3g^3x}{d^3} - \frac{5B^2(bc-ad)^3g^3x}{3d^3} + \frac{B^2(bc-ad)^2g^3}{3bd^2}$$

$$= \frac{AB(bc-ad)^3g^3x}{d^3} - \frac{5B^2(bc-ad)^3g^3x}{3d^3} + \frac{B^2(bc-ad)^2g^3}{3bd^2}$$

$$= \frac{AB(bc-ad)^3g^3x}{d^3} - \frac{5B^2(bc-ad)^3g^3x}{3d^3} + \frac{B^2(bc-ad)^2g^3}{3bd^2}$$

Mathematica [A]
time = 0.22, size = 402, normalized size = 0.95

$$\frac{g^3\left((a+bx)^4\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2+\frac{2B(bc-ad)\left(6Abd(bc-ad)^2x+12B(bc-ad)^3\log(c+dx)-2B(bc-ad)\left(2bd(bc-ad)x-d^2(a+bx)^2-2(bc-ad)^2\log(c+dx)\right)-6B(bc-ad)^2(bdx+(-bc+ad)\log(c+dx))+6Bd(bc-ad)^2(a+bx)\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+3d^2(-bc+ad)(a+bx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)+2d^3(a+bx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)-6(bc-ad)^3\log(c+dx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)-6B(bc-ad)^3\left(\left(2\log\left(\frac{d(a+bx)}{-bc+ad}\right)-\log(c+dx)\right)\log(c+dx)+2\text{Li}_2\left(\frac{b(c+dx)}{bc-ad}\right)\right)\right)}{3d^4}\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*g + b*g*x)^3*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2,x]
```

```
[Out] (g^3*((a + b*x)^4*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2 + (2*B*(b*c -
a*d)*(6*A*b*d*(b*c - a*d)^2*x + 12*B*(b*c - a*d)^3*Log[c + d*x] - 2*B*(b*c
- a*d)*(2*b*d*(b*c - a*d)*x - d^2*(a + b*x)^2 - 2*(b*c - a*d)^2*Log[c + d*x
]) - 6*B*(b*c - a*d)^2*(b*d*x + (-(b*c) + a*d)*Log[c + d*x]) + 6*B*d*(b*c -
 a*d)^2*(a + b*x)*Log[(e*(c + d*x)^2)/(a + b*x)^2] + 3*d^2*(-(b*c) + a*d)*(
a + b*x)^2*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]) + 2*d^3*(a + b*x)^3*(A
+ B*Log[(e*(c + d*x)^2)/(a + b*x)^2]) - 6*(b*c - a*d)^3*Log[c + d*x]*(A + B
*Log[(e*(c + d*x)^2)/(a + b*x)^2]) - 6*B*(b*c - a*d)^3*((2*Log[(d*(a + b*x)
)/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))
/(b*c - a*d)])))/(3*d^4)))/(4*b)
```

Maple [F]
time = 0.19, size = 0, normalized size = 0.00

$$\int (bgx + ag)^3 \left(A + B\ln\left(\frac{e(dx + c)^2}{(bx + a)^2}\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*g*x+a*g)^3*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))^2,x)
```

```
[Out] int((b*g*x+a*g)^3*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))^2,x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1749 vs. $2(412) = 824$.
time = 0.44, size = 1749, normalized size = 4.14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^3*(A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2,x, algorithm="ma
xima")
```

```
[Out] 1/4*A^2*b^3*g^3*x^4 + A^2*a*b^2*g^3*x^3 + 3/2*A^2*a^2*b*g^3*x^2 + 2*(x*log(
d^2*x^2*e/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*x*e/(b^2*x^2 + 2*a*b*x + a^2) +
 c^2*e/(b^2*x^2 + 2*a*b*x + a^2)) - 2*a*log(b*x + a)/b + 2*c*log(d*x + c)/d
)*A*B*a^3*g^3 + 3*(x^2*log(d^2*x^2*e/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*x*e/
(b^2*x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2)) + 2*a^2*log(b*
x + a)/b^2 - 2*c^2*log(d*x + c)/d^2 + 2*(b*c - a*d)*x/(b*d))*A*B*a^2*b*g^3
+ 2*(x^3*log(d^2*x^2*e/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*x*e/(b^2*x^2 + 2*a
*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2)) - 2*a^3*log(b*x + a)/b^3 + 2
*c^3*log(d*x + c)/d^3 + ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)
/(b^2*d^2))*A*B*a*b^2*g^3 + 1/6*(3*x^4*log(d^2*x^2*e/(b^2*x^2 + 2*a*b*x + a
^2) + 2*c*d*x*e/(b^2*x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2)
) + 6*a^4*log(b*x + a)/b^4 - 6*c^4*log(d*x + c)/d^4 + (2*(b^3*c*d^2 - a*b^2
```

```
*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d
^3))*A*B*b^3*g^3 + A^2*a^3*g^3*x + 1/3*(8*b^3*c^4*g^3 - 26*a*b^2*c^3*d*g^3
+ 27*a^2*b*c^2*d^2*g^3 - 6*a^3*c*d^3*g^3)*B^2*log(d*x + c)/d^4 + 2*(b^4*c^4
*g^3 - 4*a*b^3*c^3*d*g^3 + 6*a^2*b^2*c^2*d^2*g^3 - 4*a^3*b*c*d^3*g^3 + a^4*
d^4*g^3)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x +
 a*d)/(b*c - a*d)))*B^2/(b*d^4) + 1/12*(3*B^2*b^4*d^4*g^3*x^4 + 4*(b^4*c*d^
3*g^3 + 2*a*b^3*d^4*g^3)*B^2*x^3 - 2*(b^4*c^2*d^2*g^3 - 8*a*b^3*c*d^3*g^3 -
 2*a^2*b^2*d^4*g^3)*B^2*x^2 - 4*(2*b^4*c^3*d*g^3 - 5*a*b^3*c^2*d^2*g^3 + a^
2*b^2*c*d^3*g^3 - a^3*b*d^4*g^3)*B^2*x + 12*(B^2*b^4*d^4*g^3*x^4 + 4*B^2*a*
b^3*d^4*g^3*x^3 + 6*B^2*a^2*b^2*d^4*g^3*x^2 + 4*B^2*a^3*b*d^4*g^3*x + B^2*a
^4*d^4*g^3)*log(b*x + a)^2 + 12*(B^2*b^4*d^4*g^3*x^4 + 4*B^2*a*b^3*d^4*g^3*
x^3 + 6*B^2*a^2*b^2*d^4*g^3*x^2 + 4*B^2*a^3*b*d^4*g^3*x - (b^4*c^4*g^3 - 4*
a*b^3*c^3*d*g^3 + 6*a^2*b^2*c^2*d^2*g^3 - 4*a^3*b*c*d^3*g^3)*B^2)*log(d*x +
 c)^2 - 4*(3*B^2*b^4*d^4*g^3*x^4 + 2*(b^4*c*d^3*g^3 + 5*a*b^3*d^4*g^3)*B^2*
x^3 - 3*(b^4*c^2*d^2*g^3 - 4*a*b^3*c*d^3*g^3 - 3*a^2*b^2*d^4*g^3)*B^2*x^2 +
 6*(b^4*c^3*d*g^3 - 4*a*b^3*c^2*d^2*g^3 + 6*a^2*b^2*c*d^3*g^3 - a^3*b*d^4*g
^3)*B^2*x + (6*a*b^3*c^3*d*g^3 - 21*a^2*b^2*c^2*d^2*g^3 + 26*a^3*b*c*d^3*g^
3 - 8*a^4*d^4*g^3)*B^2)*log(b*x + a) + 4*(3*B^2*b^4*d^4*g^3*x^4 + 2*(b^4*c*
d^3*g^3 + 5*a*b^3*d^4*g^3)*B^2*x^3 - 3*(b^4*c^2*d^2*g^3 - 4*a*b^3*c*d^3*g^3
 - 3*a^2*b^2*d^4*g^3)*B^2*x^2 + 6*(b^4*c^3*d*g^3 - 4*a*b^3*c^2*d^2*g^3 + 6*
a^2*b^2*c*d^3*g^3 - a^3*b*d^4*g^3)*B^2*x - 6*(B^2*b^4*d^4*g^3*x^4 + 4*B^2*a
*b^3*d^4*g^3*x^3 + 6*B^2*a^2*b^2*d^4*g^3*x^2 + 4*B^2*a^3*b*d^4*g^3*x + B^2*
a^4*d^4*g^3)*log(b*x + a))*log(d*x + c))/(b*d^4)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^3*(A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2,x, algorithm="fr
icas")
```

```
[Out] integral(A^2*b^3*g^3*x^3 + 3*A^2*a*b^2*g^3*x^2 + 3*A^2*a^2*b*g^3*x + A^2*a^
3*g^3 + (B^2*b^3*g^3*x^3 + 3*B^2*a*b^2*g^3*x^2 + 3*B^2*a^2*b*g^3*x + B^2*a^
3*g^3)*log((d^2*x^2 + 2*c*d*x + c^2)*e/(b^2*x^2 + 2*a*b*x + a^2))^2 + 2*(A*
B*b^3*g^3*x^3 + 3*A*B*a*b^2*g^3*x^2 + 3*A*B*a^2*b*g^3*x + A*B*a^3*g^3)*log(
(d^2*x^2 + 2*c*d*x + c^2)*e/(b^2*x^2 + 2*a*b*x + a^2)), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)**3*(A+B*ln(e*(d*x+c)**2/(b*x+a)**2))**2,x)
```

[Out] Timed out

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^3*(A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2,x, algorithm="gi
ac")
```

```
[Out] integrate((b*g*x + a*g)^3*(B*log((d*x + c)^2*e/(b*x + a)^2) + A)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a\,g+b\,g\,x)^3\,\left(A+B\,\ln\left(\frac{e\,{(c+d\,x)}^2}{{(a+b\,x)}^2}\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*g + b*g*x)^3*(A + B*log((e*(c + d*x)^2)/(a + b*x)^2))^2,x)
```

```
[Out] int((a*g + b*g*x)^3*(A + B*log((e*(c + d*x)^2)/(a + b*x)^2))^2, x)
```

3.212 $\int (ag+bgx)^2 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2 dx$

Optimal. Leaf size=343

$$\frac{4B^2(bc-ad)^2g^2x}{3d^2}-\frac{4B^2(bc-ad)^3g^2\log(a+bx)}{bd^3}-\frac{4B^2(bc-ad)^3g^2\log\left(\frac{c+dx}{a+bx}\right)}{3bd^3}+\frac{2B(bc-ad)g^2(a+bx)^2\left(A\right.}{3bd}$$

```
[Out] 4/3*B^2*(-a*d+b*c)^2*g^2*x/d^2-4*B^2*(-a*d+b*c)^3*g^2*ln(b*x+a)/b/d^3-4/3*B
^2*(-a*d+b*c)^3*g^2*ln((d*x+c)/(b*x+a))/b/d^3+2/3*B*(-a*d+b*c)*g^2*(b*x+a)^
2*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))/b/d-4/3*B*(-a*d+b*c)^2*g^2*(d*x+c)*(A+B*l
n(e*(d*x+c)^2/(b*x+a)^2))/d^3+1/3*g^2*(b*x+a)^3*(A+B*ln(e*(d*x+c)^2/(b*x+a)
^2))^2/b-4/3*B*(-a*d+b*c)^3*g^2*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))*ln(1-d*(b*x
+a)/b/(d*x+c))/b/d^3+8/3*B^2*(-a*d+b*c)^3*g^2*polylog(2,d*(b*x+a)/b/(d*x+c)
)/b/d^3
```

Rubi [A]
time = 0.25, antiderivative size = 343, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {2552, 2356, 2389, 2379, 2438, 2351, 31, 46}

$$\frac{8B^2g^2(bc-ad)^3\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{3bd^3}-\frac{4Bg^2(bc-ad)^3\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{3bd^3}-\frac{4Bg^2(c+dx)(bc-ad)^2\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{3d^3}+\frac{2Bg^2(a+bx)^2(bc-ad)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{3bd}+\frac{g^2(a+bx)^3\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{3b}-\frac{4B^2g^2(bc-ad)^3\log(a+bx)}{bd^3}-\frac{4B^2g^2(bc-ad)^3\log\left(\frac{c+dx}{a+bx}\right)}{3bd^3}+\frac{4B^2g^2x(bc-ad)^2}{3d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*g + b*g*x)^2*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2,x]
```

```
[Out] (4*B^2*(b*c - a*d)^2*g^2*x)/(3*d^2) - (4*B^2*(b*c - a*d)^3*g^2*Log[a + b*x]
)/(b*d^3) - (4*B^2*(b*c - a*d)^3*g^2*Log[(c + d*x)/(a + b*x)])/(3*b*d^3) +
(2*B*(b*c - a*d)*g^2*(a + b*x)^2*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]))/
(3*b*d) - (4*B*(b*c - a*d)^2*g^2*(c + d*x)*(A + B*Log[(e*(c + d*x)^2)/(a +
b*x)^2]))/(3*d^3) + (g^2*(a + b*x)^3*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2
])^2)/(3*b) - (4*B*(b*c - a*d)^3*g^2*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2
])*Log[1 - (d*(a + b*x))/(b*(c + d*x))])/(3*b*d^3) + (8*B^2*(b*c - a*d)^3*g
^2*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/(3*b*d^3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
```

```
n + 2, 0])
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2552

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(
m + 1)*(g/d)^m, Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a
 + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[n
 + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[d*f
- c*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Rubi steps

$$
\begin{aligned}
\int (ag+bgx)^2 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2 dx &= \frac{g^2(a+bx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{3b} - (2B)\int \frac{2(bc-ad)g^3(\cdots}{\cdots} \\
&= \frac{g^2(a+bx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{3b} - \frac{(4B(bc-ad)g^2)\cdots}{\cdots} \\
&= \frac{g^2(a+bx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{3b} - \frac{(4B(bc-ad)g^2)\cdots}{\cdots} \\
&= \frac{g^2(a+bx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{3b} - \frac{(4B(bc-ad)g^2)\cdots}{\cdots} \\
&= -\frac{4AB(bc-ad)^2g^2x}{3d^2} + \frac{2B(bc-ad)g^2(a+bx)^2\left(A+B\cdots\right.}{3bd} \\
&= -\frac{4AB(bc-ad)^2g^2x}{3d^2} - \frac{4B^2(bc-ad)^2g^2(a+bx)\log\left(\frac{e(c\cdots}{(a\cdots}\right.}{3bd^2} \\
&= -\frac{4AB(bc-ad)^2g^2x}{3d^2} - \frac{8B^2(bc-ad)^3g^2\log(c+dx)}{3bd^3} - \frac{4\cdots}{\cdots} \\
&= -\frac{4AB(bc-ad)^2g^2x}{3d^2} + \frac{4B^2(bc-ad)^2g^2x}{3d^2} - \frac{4B^2(bc-a\cdots}{\cdots} \\
&= -\frac{4AB(bc-ad)^2g^2x}{3d^2} + \frac{4B^2(bc-ad)^2g^2x}{3d^2} - \frac{4B^2(bc-a\cdots}{\cdots} \\
&= -\frac{4AB(bc-ad)^2g^2x}{3d^2} + \frac{4B^2(bc-ad)^2g^2x}{3d^2} - \frac{4B^2(bc-a\cdots}{\cdots} \\
&= -\frac{4AB(bc-ad)^2g^2x}{3d^2} + \frac{4B^2(bc-ad)^2g^2x}{3d^2} - \frac{4B^2(bc-a\cdots}{\cdots}
\end{aligned}
$$

Mathematica [A]
time = 0.16, size = 298, normalized size = 0.87

$$
\frac{g^2\left((a+bx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2-\frac{2B(bc-ad)\left(2Abd(bc-ad)x+4B(bc-ad)^2\log(c+dx)-2B(bc-ad)(bdx+(-bc+ad)\log(c+dx))+2Bd(bc-ad)(a+bx)\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)-d^2(a+bx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)-2(bc-ad)^2\log(c+dx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)-2B(bc-ad)^2\left(\left(2\log\left(\frac{d(a+bx)}{-bc+ad}\right)-\log(c+dx)\right)\log(c+dx)+2\operatorname{Li}_2\left(\frac{b(c+dx)}{bc-ad}\right)\right)\right)}{d^3}\right)}{3b}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*g + b*g*x)^2*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2,x]
```

```
[Out] (g^2*((a + b*x)^3*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2 - (2*B*(b*c -
a*d)*(2*A*b*d*(b*c - a*d)*x + 4*B*(b*c - a*d)^2*Log[c + d*x] - 2*B*(b*c - a
*d)*(b*d*x + (-(b*c) + a*d)*Log[c + d*x]) + 2*B*d*(b*c - a*d)*(a + b*x)*Log
[(e*(c + d*x)^2)/(a + b*x)^2] - d^2*(a + b*x)^2*(A + B*Log[(e*(c + d*x)^2)/
(a + b*x)^2]) - 2*(b*c - a*d)^2*Log[c + d*x]*(A + B*Log[(e*(c + d*x)^2)/(a
+ b*x)^2]) - 2*B*(b*c - a*d)^2*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[
c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/d^3))/(
3*b)
```

Maple [**F**]
time = 0.16, size = 0, normalized size = 0.00

$$\int \left(b g x +a g \right)^{2} \left(A +B \ln \left(\frac{e \left(d x +c \right)^{2}}{\left(b x +a \right)^{2}}\right)\right)^{2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*g*x+a*g)^2*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))^2,x)
```

```
[Out] int((b*g*x+a*g)^2*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))^2,x)
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1190 vs. 2(332) = 664.
time = 0.43, size = 1190, normalized size = 3.47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^2*(A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2,x, algorithm="ma
xima")
```

```
[Out] 1/3*A^2*b^2*g^2*x^3 + A^2*a*b*g^2*x^2 + 2*(x*log(d^2*x^2*e/(b^2*x^2 + 2*a*b
*x + a^2) + 2*c*d*x*e/(b^2*x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x
+ a^2)) - 2*a*log(b*x + a)/b + 2*c*log(d*x + c)/d)*A*B*a^2*g^2 + 2*(x^2*log
(d^2*x^2*e/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*x*e/(b^2*x^2 + 2*a*b*x + a^2)
+ c^2*e/(b^2*x^2 + 2*a*b*x + a^2)) + 2*a^2*log(b*x + a)/b^2 - 2*c^2*log(d*x
 + c)/d^2 + 2*(b*c - a*d)*x/(b*d))*A*B*a*b*g^2 + 2/3*(x^3*log(d^2*x^2*e/(b^
2*x^2 + 2*a*b*x + a^2) + 2*c*d*x*e/(b^2*x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x
^2 + 2*a*b*x + a^2)) - 2*a^3*log(b*x + a)/b^3 + 2*c^3*log(d*x + c)/d^3 + ((
b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*A*B*b^2*g^2 +
A^2*a^2*g^2*x - 4/3*(2*b^2*c^3*g^2 - 4*a*b*c^2*d*g^2 + a^2*c*d^2*g^2)*B^2*l
og(d*x + c)/d^3 - 8/3*(b^3*c^3*g^2 - 3*a*b^2*c^2*d*g^2 + 3*a^2*b*c*d^2*g^2
- a^3*d^3*g^2)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b
*d*x + a*d)/(b*c - a*d)))*B^2/(b*d^3) + 1/3*(B^2*b^3*d^3*g^2*x^3 + (2*b^3*c
*d^2*g^2 + a*b^2*d^3*g^2)*B^2*x^2 + (4*a*b^2*c*d^2*g^2 - a^2*b*d^3*g^2)*B^2
```

```
*x + 4*(B^2*b^3*d^3*g^2*x^3 + 3*B^2*a*b^2*d^3*g^2*x^2 + 3*B^2*a^2*b*d^3*g^2
*x + B^2*a^3*d^3*g^2)*log(b*x + a)^2 + 4*(B^2*b^3*d^3*g^2*x^3 + 3*B^2*a*b^2
*d^3*g^2*x^2 + 3*B^2*a^2*b*d^3*g^2*x + (b^3*c^3*g^2 - 3*a*b^2*c^2*d*g^2 + 3
*a^2*b*c*d^2*g^2)*B^2)*log(d*x + c)^2 - 4*(B^2*b^3*d^3*g^2*x^3 + (b^3*c*d^2
*g^2 + 2*a*b^2*d^3*g^2)*B^2*x^2 - (2*b^3*c^2*d*g^2 - 6*a*b^2*c*d^2*g^2 + a^
2*b*d^3*g^2)*B^2*x - (2*a*b^2*c^2*d*g^2 - 5*a^2*b*c*d^2*g^2 + 2*a^3*d^3*g^2
)*B^2)*log(b*x + a) + 4*(B^2*b^3*d^3*g^2*x^3 + (b^3*c*d^2*g^2 + 2*a*b^2*d^3
*g^2)*B^2*x^2 - (2*b^3*c^2*d*g^2 - 6*a*b^2*c*d^2*g^2 + a^2*b*d^3*g^2)*B^2*x
 - 2*(B^2*b^3*d^3*g^2*x^3 + 3*B^2*a*b^2*d^3*g^2*x^2 + 3*B^2*a^2*b*d^3*g^2*x
 + B^2*a^3*d^3*g^2)*log(b*x + a))*log(d*x + c))/(b*d^3)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^2*(A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2,x, algorithm="fr
icas")
```

```
[Out] integral(A^2*b^2*g^2*x^2 + 2*A^2*a*b*g^2*x + A^2*a^2*g^2 + (B^2*b^2*g^2*x^2
 + 2*B^2*a*b*g^2*x + B^2*a^2*g^2)*log((d^2*x^2 + 2*c*d*x + c^2)*e/(b^2*x^2
+ 2*a*b*x + a^2))^2 + 2*(A*B*b^2*g^2*x^2 + 2*A*B*a*b*g^2*x + A*B*a^2*g^2)*l
og((d^2*x^2 + 2*c*d*x + c^2)*e/(b^2*x^2 + 2*a*b*x + a^2)), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)**2*(A+B*ln(e*(d*x+c)**2/(b*x+a)**2))**2,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^2*(A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2,x, algorithm="gi
ac")
```

```
[Out] integrate((b*g*x + a*g)^2*(B*log((d*x + c)^2*e/(b*x + a)^2) + A)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a\,g+b\,g\,x)^2\,\left(A+B\,\ln\left(\frac{e\,(c+d\,x)^2}{(a+b\,x)^2}\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*g + b*g*x)^2*(A + B*log((e*(c + d*x)^2)/(a + b*x)^2))^2,x)
[Out] int((a*g + b*g*x)^2*(A + B*log((e*(c + d*x)^2)/(a + b*x)^2))^2, x)
```

3.213 $\int (ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2 dx$

Optimal. Leaf size=211

$$\frac{4B^2(bc-ad)^2g\log(a+bx)}{bd^2}+\frac{2B(bc-ad)g(c+dx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{d^2}+\frac{g(a+bx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{2b}$$

```
[Out] 4*B^2*(-a*d+b*c)^2*g*ln(b*x+a)/b/d^2+2*B*(-a*d+b*c)*g*(d*x+c)*(A+B*ln(e*(d*
x+c)^2/(b*x+a)^2))/d^2+1/2*g*(b*x+a)^2*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))^2/b+
2*B*(-a*d+b*c)^2*g*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))*ln(1-d*(b*x+a)/b/(d*x+c)
)/b/d^2-4*B^2*(-a*d+b*c)^2*g*polylog(2,d*(b*x+a)/b/(d*x+c))/b/d^2
```

Rubi [A]
time = 0.16, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.219$, Rules used = {2552, 2356, 2389, 2379, 2438, 2351, 31}

$$-\frac{4B^2g(bc-ad)^2\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{bd^2}+\frac{2Bg(bc-ad)^2\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{bd^2}+\frac{2Bg(c+dx)(bc-ad)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{d^2}+\frac{g(a+bx)^2\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{2b}+\frac{4B^2g(bc-ad)^2\log(a+bx)}{bd^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*g + b*g*x)*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2,x]
```

```
[Out] (4*B^2*(b*c - a*d)^2*g*Log[a + b*x])/(b*d^2) + (2*B*(b*c - a*d)*g*(c + d*x)
*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]))/d^2 + (g*(a + b*x)^2*(A + B*Log[
(e*(c + d*x)^2)/(a + b*x)^2])^2)/(2*b) + (2*B*(b*c - a*d)^2*g*(A + B*Log[(e
*(c + d*x)^2)/(a + b*x)^2])*Log[1 - (d*(a + b*x))/(b*(c + d*x))])/(b*d^2) -
 (4*B^2*(b*c - a*d)^2*g*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/(b*d^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
```

1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q, -1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] && !IGtQ[q, 0]) || (EqQ[p, 2] && NeQ[q, 1]))

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2552

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(
m + 1)*(g/d)^m, Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a
 + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[n
 + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[d*f
- c*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Rubi steps

$$\int (ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2 dx = \frac{g(a+bx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{2b} - B\int \frac{2(bc-ad)g^2(a+bx)(}{}$$

$$= \frac{g(a+bx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{2b} - (2B(bc-ad)g)\int$$

$$= \frac{g(a+bx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{2b} - \frac{(2B(bc-ad)g)\int}{}$$

$$= \frac{g(a+bx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{2b} - \frac{(2B(bc-ad)g)\int}{}$$

$$= \frac{2AB(bc-ad)gx}{d} - \frac{2B(bc-ad)^2g\log(c+dx)\left(A+B\log\right.}{bd^2}$$

$$= \frac{2AB(bc-ad)gx}{d} + \frac{2B^2(bc-ad)g(a+bx)\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{bd}$$

$$= \frac{2AB(bc-ad)gx}{d} + \frac{4B^2(bc-ad)^2g\log(c+dx)}{bd^2} + \frac{2B^2(bc}{}$$

$$= \frac{2AB(bc-ad)gx}{d} + \frac{4B^2(bc-ad)^2g\log(c+dx)}{bd^2} + \frac{2B^2(bc}{}$$

$$= \frac{2AB(bc-ad)gx}{d} + \frac{4B^2(bc-ad)^2g\log(c+dx)}{bd^2} - \frac{4B^2(bc}{}$$

$$= \frac{2AB(bc-ad)gx}{d} + \frac{4B^2(bc-ad)^2g\log(c+dx)}{bd^2} - \frac{4B^2(bc}{}$$

$$= \frac{2AB(bc-ad)gx}{d} + \frac{4B^2(bc-ad)^2g\log(c+dx)}{bd^2} - \frac{4B^2(bc}{}$$

Mathematica [A]
time = 0.13, size = 195, normalized size = 0.92

$$\frac{g\left((a+bx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2+\frac{4B(bc-ad)\left(Abdx+B(bc-ad)\log^2(c+dx)+Bd(a+bx)\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)-(bc-ad)\log(c+dx)\left(A-2B+2B\log\left(\frac{d(a+bx)}{-bc+ad}\right)+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)+(-2bBc+2aBd)\text{Li}_2\left(\frac{b(c+dx)}{bc-ad}\right)\right)}{d^2}\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*g + b*g*x)*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2,x]
```

```
[Out] (g*((a + b*x)^2*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2 + (4*B*(b*c - a*
d)*(A*b*d*x + B*(b*c - a*d)*Log[c + d*x]^2 + B*d*(a + b*x)*Log[(e*(c + d*x)
^2)/(a + b*x)^2] - (b*c - a*d)*Log[c + d*x]*(A - 2*B + 2*B*Log[(d*(a + b*x)
)/(-(b*c) + a*d)] + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]) + (-2*b*B*c + 2*a*B
*d)*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]))/d^2))/(2*b)
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int \left(bgx + ag\right) \left(A + B \ln \left(\frac{e \left(dx + c\right)^{2}}{\left(bx + a\right)^{2}}\right)\right)^{2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*g*x+a*g)*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))^2,x)
```

```
[Out] int((b*g*x+a*g)*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))^2,x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 656 vs. $2(211) = 422$.
time = 0.40, size = 656, normalized size = 3.11

$$\frac{1}{2} A^{2} b g x^{2} + 2 \left(x \log\left(\frac{d^{2} x^{2} e}{b^{2} x^{2} + 2 abx + a^{2}} + \frac{2 cdxe}{b^{2} x^{2} + 2 abx + a^{2}} + \frac{c^{2} e}{b^{2} x^{2} + 2 abx + a^{2}}\right) - \frac{2 a \log\left(bx + a\right)}{b} + \frac{2 c \log\left(dx + c\right)}{d}\right) ABag + \left(x^{2} \log\left(\frac{d^{2} x^{2} e}{b^{2} x^{2} + 2 abx + a^{2}} + \frac{2 cdxe}{b^{2} x^{2} + 2 abx + a^{2}} + \frac{c^{2} e}{b^{2} x^{2} + 2 abx + a^{2}}\right) + \frac{2 a^{2} \log\left(bx + a\right)}{b^{2}} - \frac{2 c^{2} \log\left(dx + c\right)}{d^{2}} + \frac{2 \left(bc - ad\right) x}{bd}\right) ABbg + A^{2} agx + \frac{2 B^{2} bc^{2} g \log\left(dx + c\right)}{d^{2}} + \frac{4 \left(b^{2} c^{2} g - 2 abcdg + a^{2} d^{2} g\right) \left(\log\left(bx + a\right) \log\left(\frac{bdx + ad}{bc - ad} + 1\right) + \operatorname{Li}_{2}\left(-\frac{bdx + ad}{bc - ad}\right)\right) B^{2}}{bd^{2}} + \frac{B^{2} b^{2} d^{2} g x^{2} + 2 \left(2 b^{2} cdg - abd^{2} g\right) B^{2} x + 4 \left(B^{2} b^{2} d^{2} g x^{2} + 2 B^{2} abd^{2} gx + B^{2} a^{2} d^{2} g\right) \log\left(bx + a\right)^{2} + 4 \left(B^{2} b^{2} d^{2} g x^{2} + 2 B^{2} abd^{2} gx - \left(b^{2} c^{2} g - 2 abcdg\right) B^{2}\right) \log\left(dx + c\right)^{2} - 4 \left(B^{2} b^{2} d^{2} g x^{2} + 2 B^{2} b^{2} cdgx + \left(2 abcdg - a^{2} d^{2} g\right) B^{2}\right) \log\left(bx + a\right) + 4 \left(B^{2} b^{2} d^{2} g x^{2} + 2 B^{2} b^{2} cdgx - 2 \left(B^{2} b^{2} d^{2} g x^{2} + 2 B^{2} abd^{2} gx + B^{2} a^{2} d^{2} g\right) \log\left(bx + a\right)\right) \log\left(dx + c\right)}{2 bd^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)*(A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2,x, algorithm="maxi
ma")
```

```
[Out] 1/2*A^2*b*g*x^2 + 2*(x*log(d^2*x^2*e/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*x*e/
(b^2*x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2)) - 2*a*log(b*x
+ a)/b + 2*c*log(d*x + c)/d)*A*B*a*g + (x^2*log(d^2*x^2*e/(b^2*x^2 + 2*a*b*
x + a^2) + 2*c*d*x*e/(b^2*x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x +
 a^2)) + 2*a^2*log(b*x + a)/b^2 - 2*c^2*log(d*x + c)/d^2 + 2*(b*c - a*d)*x/
(b*d))*A*B*b*g + A^2*a*g*x + 2*B^2*b*c^2*g*log(d*x + c)/d^2 + 4*(b^2*c^2*g
- 2*a*b*c*d*g + a^2*d^2*g)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1)
 + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B^2/(b*d^2) + 1/2*(B^2*b^2*d^2*g*x^2
+ 2*(2*b^2*c*d*g - a*b*d^2*g)*B^2*x + 4*(B^2*b^2*d^2*g*x^2 + 2*B^2*a*b*d^2*
g*x + B^2*a^2*d^2*g)*log(b*x + a)^2 + 4*(B^2*b^2*d^2*g*x^2 + 2*B^2*a*b*d^2*
g*x - (b^2*c^2*g - 2*a*b*c*d*g)*B^2)*log(d*x + c)^2 - 4*(B^2*b^2*d^2*g*x^2
+ 2*B^2*b^2*c*d*g*x + (2*a*b*c*d*g - a^2*d^2*g)*B^2)*log(b*x + a) + 4*(B^2*
b^2*d^2*g*x^2 + 2*B^2*b^2*c*d*g*x - 2*(B^2*b^2*d^2*g*x^2 + 2*B^2*a*b*d^2*g*
x + B^2*a^2*d^2*g)*log(b*x + a))*log(d*x + c))/(b*d^2)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)*(A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2,x, algorithm="fric
as")
```

```
[Out] integral(A^2*b*g*x + A^2*a*g + (B^2*b*g*x + B^2*a*g)*log((d^2*x^2 + 2*c*d*x
 + c^2)*e/(b^2*x^2 + 2*a*b*x + a^2))^2 + 2*(A*B*b*g*x + A*B*a*g)*log((d^2*x
^2 + 2*c*d*x + c^2)*e/(b^2*x^2 + 2*a*b*x + a^2)), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)*(A+B*ln(e*(d*x+c)**2/(b*x+a)**2))**2,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)*(A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2,x, algorithm="giac
")
```

```
[Out] integrate((b*g*x + a*g)*(B*log((d*x + c)^2*e/(b*x + a)^2) + A)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a\,g+b\,g\,x)\,\left(A+B\,\ln\left(\frac{e\,{\left(c+d\,x\right)}^2}{{\left(a+b\,x\right)}^2}\right)\right)^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*g + b*g*x)*(A + B*log((e*(c + d*x)^2)/(a + b*x)^2))^2,x)
```

```
[Out] int((a*g + b*g*x)*(A + B*log((e*(c + d*x)^2)/(a + b*x)^2))^2, x)
```

3.214 $\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{ag+bgx}\,dx$

Optimal. Leaf size=132

$$-\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{bg}-\frac{4B\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)\text{Li}_2\left(\frac{b(c+dx)}{d(a+bx)}\right)}{bg}+\frac{8B^2\text{Li}_3\left(\frac{b(c+dx)}{d(a+bx)}\right)}{bg}$$

```
[Out] -ln((a*d-b*c)/d/(b*x+a))*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))^2/b/g-4*B*(A+B*ln(
e*(d*x+c)^2/(b*x+a)^2))*polylog(2,b*(d*x+c)/d/(b*x+a))/b/g+8*B^2*polylog(3,
b*(d*x+c)/d/(b*x+a))/b/g
```

Rubi [A]

time = 0.08, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2552, 2354, 2421, 6724}

$$-\frac{4B\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{bg}+\frac{8B^2\text{PolyLog}\left(3,\frac{b(c+dx)}{d(a+bx)}\right)}{bg}-\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{bg}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2/(a*g + b*g*x),x]
```

```
[Out] -((Log[-((b*c - a*d)/(d*(a + b*x)))]*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2
])^2)/(b*g)) - (4*B*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])*PolyLog[2, (b*
(c + d*x))/(d*(a + b*x))])/(b*g) + (8*B^2*PolyLog[3, (b*(c + d*x))/(d*(a +
b*x))])/(b*g)
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2552

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(
m + 1)*(g/d)^m, Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a
 + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[n
 + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[d*f
- c*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{ag+bgx}\,dx = \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2\log(ag+bgx)}{bg} - \frac{(2B)\int \frac{(a+bx)^2\left(\frac{2de(c+dx)}{(a+bx)^2}-\frac{2be(c+dx)^2}{(a+bx)^3}\right)\cdots}{e(c\cdots}}{bg\cdots}$$

$$= \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2\log(ag+bgx)}{bg} - \frac{(2B)\int \frac{(a+bx)^2\left(\frac{2de(c+dx)}{(a+bx)^2}-\frac{2be(c+dx)^2}{(a+bx)^3}\right)\cdots}{(c\cdots}}{beg\cdots}$$

$$= \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2\log(ag+bgx)}{bg} - \frac{(2B)\int \frac{2(bc-ad)e\left(-A-B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\cdots\right.}{(a+bx)(c+dx)}}{beg}\cdots$$

$$= \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2\log(ag+bgx)}{bg} - \frac{(4B(bc-ad))\int \frac{\left(-A-B\log\left(\frac{e(c+dx)}{(a+bx)}\cdots\right.\right.}{(a+bx)(c\cdots}}{bg}\cdots$$

$$= \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2\log(ag+bgx)}{bg} - \frac{(4B(bc-ad))\int \left(\frac{b\left(-A-B\log\left(\frac{e(c+\cdots}{(a+\cdots}\right.\right.}{(bc-a\cdots}\right.}{\cdots}$$

$$= \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2\log(ag+bgx)}{bg} - \frac{(4B)\int \frac{\left(-A-B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)\log(ag+\cdots}{a+bx}}{g}\cdots$$

$$= \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2\log(ag+bgx)}{bg} - \frac{(4B)\int \left(\frac{A\log(ag+bgx)}{-a-bx}+\frac{B\log\left(\frac{e(c+\cdots}{(a+b\cdots}\right.}{\cdots}\right.}{g}\cdots$$

$$= \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2\log(ag+bgx)}{bg} - \frac{(4AB)\int \frac{\log(ag+bgx)}{-a-bx}\,dx}{g} - \frac{(4B^2)\int\cdots}{\cdots}$$

$$= -\frac{4AB\log\left(\frac{b(c+dx)}{bc-ad}\right)\log(ag+bgx)}{bg} + \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2\log(ag+bg\cdots}{bg}\cdots$$

$$= -\frac{4AB\log\left(\frac{b(c+dx)}{bc-ad}\right)\log(ag+bgx)}{bg} + \frac{4B^2\log\left(\frac{b(c+dx)}{bc-ad}\right)\left(\log\left(\frac{1}{(a+bx)^2}\right)+\log\cdots\right.}{\cdots}$$

$$= \frac{2AB\log^2(g(a+bx))}{bg} - \frac{4B^2\log\left(\frac{1}{(a+bx)^2}\right)\log(g(a+bx))\log(c+dx)}{bg} - \frac{B^2\cdots}{\cdots}$$

$$= \frac{2AB\log^2(g(a+bx))}{bg} - \frac{4B^2\log\left(\frac{1}{(a+bx)^2}\right)\log(g(a+bx))\log(c+dx)}{bg} + \frac{B^2\cdots}{\cdots}$$

$$= \frac{2AB\log^2(g(a+bx))}{bg} + \frac{4B^2\log^3(g(a+bx))}{3bg} - \frac{B^2\log^2\left(\frac{1}{(a+bx)^2}\right)\log(c+dx\cdots}{bg}\cdots$$

$$= \frac{2AB\log^2(g(a+bx))}{bg} + \frac{4B^2\log^3(g(a+bx))}{\cdots} - \frac{B^2\log^2\left(\frac{1}{\cdots}\right)\log(c+d\cdots}{\cdots}$$

Mathematica [A]
time = 0.16, size = 257, normalized size = 1.95

$$\frac{-2AB\log^2\left(\frac{a}{b}+x\right)+A^2\log(a+bx)+4AB\log\left(\frac{a}{b}+x\right)\log(a+bx)-4AB\log\left(\frac{c}{d}+x\right)\log(a+bx)+4AB\log\left(\frac{c}{d}+x\right)\log\left(\frac{d(a+bx)}{-bc+ad}\right)+2AB\log(a+bx)\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)-B^2\log\left(\frac{-bc+ad}{d(a+bx)}\right)\log^2\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+4AB\text{Li}_2\left(\frac{b(c+dx)}{bc-ad}\right)-4B^2\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\text{Li}_2\left(\frac{b(c+dx)}{d(a+bx)}\right)+8B^2\text{Li}_3\left(\frac{b(c+dx)}{d(a+bx)}\right)}{bg}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2/(a*g + b*g*x),x]
```

```
[Out] (-2*A*B*Log[a/b + x]^2 + A^2*Log[a + b*x] + 4*A*B*Log[a/b + x]*Log[a + b*x]
 - 4*A*B*Log[c/d + x]*Log[a + b*x] + 4*A*B*Log[c/d + x]*Log[(d*(a + b*x))/(
-(b*c) + a*d)] + 2*A*B*Log[a + b*x]*Log[(e*(c + d*x)^2)/(a + b*x)^2] - B^2*
Log[(-(b*c) + a*d)/(d*(a + b*x))]*Log[(e*(c + d*x)^2)/(a + b*x)^2]^2 + 4*A*
B*PolyLog[2, (b*(c + d*x))/(b*c - a*d)] - 4*B^2*Log[(e*(c + d*x)^2)/(a + b*
x)^2]*PolyLog[2, (b*(c + d*x))/(d*(a + b*x))] + 8*B^2*PolyLog[3, (b*(c + d*
x))/(d*(a + b*x))])/(b*g)
```

Maple [F]
time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{\left(A+B\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)\right)^2}{bgx+ag}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*(d*x+c)^2/(b*x+a)^2))^2/(b*g*x+a*g),x)
```

```
[Out] int((A+B*ln(e*(d*x+c)^2/(b*x+a)^2))^2/(b*g*x+a*g),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2/(b*g*x+a*g),x, algorithm="maxi
ma")
```

```
[Out] 4*B^2*log(b*x + a)*log(d*x + c)^2/(b*g) + A^2*log(b*g*x + a*g)/(b*g) - inte
grate(-(2*A*B*b*c + B^2*b*c + 4*(B^2*b*d*x + B^2*b*c)*log(b*x + a)^2 + (2*A
*B*b*d + B^2*b*d)*x - 4*(A*B*b*c + B^2*b*c + (A*B*b*d + B^2*b*d)*x)*log(b*x
 + a) + 4*(A*B*b*c + B^2*b*c + (A*B*b*d + B^2*b*d)*x - 2*(2*B^2*b*d*x + (b*
c + a*d)*B^2)*log(b*x + a))*log(d*x + c))/(b^2*d*g*x^2 + a*b*c*g + (b^2*c*g
 + a*b*d*g)*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2/(b*g*x+a*g),x, algorithm="fric
as")
```

```
[Out] integral((B^2*log((d^2*x^2 + 2*c*d*x + c^2)*e/(b^2*x^2 + 2*a*b*x + a^2))^2
+ 2*A*B*log((d^2*x^2 + 2*c*d*x + c^2)*e/(b^2*x^2 + 2*a*b*x + a^2)) + A^2)/(
b*g*x + a*g), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A^2}{a+bx}\,dx + \int \frac{B^2 \log\left(\frac{c^2e}{a^2+2abx+b^2x^2}+\frac{2cdex}{a^2+2abx+b^2x^2}+\frac{d^2ex^2}{a^2+2abx+b^2x^2}\right)^2}{a+bx}\,dx + \int \frac{2AB \log\left(\frac{c^2e}{a^2+2abx+b^2x^2}+\frac{2cdex}{a^2+2abx+b^2x^2}+\frac{d^2ex^2}{a^2+2abx+b^2x^2}\right)}{a+bx}\,dx}{g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(d*x+c)**2/(b*x+a)**2))**2/(b*g*x+a*g),x)
```

```
[Out] (Integral(A**2/(a + b*x), x) + Integral(B**2*log(c**2*e/(a**2 + 2*a*b*x + b
**2*x**2) + 2*c*d*e*x/(a**2 + 2*a*b*x + b**2*x**2) + d**2*e*x**2/(a**2 + 2*
a*b*x + b**2*x**2))**2/(a + b*x), x) + Integral(2*A*B*log(c**2*e/(a**2 + 2*
a*b*x + b**2*x**2) + 2*c*d*e*x/(a**2 + 2*a*b*x + b**2*x**2) + d**2*e*x**2/(
a**2 + 2*a*b*x + b**2*x**2))/(a + b*x), x))/g
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2/(b*g*x+a*g),x, algorithm="giac
")
```

```
[Out] integrate((B*log((d*x + c)^2*e/(b*x + a)^2) + A)^2/(b*g*x + a*g), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(A+B\,\ln\left(\frac{e\,{\left(c+d\,x\right)}^2}{{\left(a+b\,x\right)}^2}\right)\right)}^2}{a\,g+b\,g\,x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(c + d*x)^2)/(a + b*x)^2))^2/(a*g + b*g*x),x)
```

```
[Out] int((A + B*log((e*(c + d*x)^2)/(a + b*x)^2))^2/(a*g + b*g*x), x)
```

3.215 $\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(ag+bgx)^2}\,dx$

Optimal. Leaf size=157

$$\frac{4AB(c+dx)}{(bc-ad)g^2(a+bx)}-\frac{8B^2(c+dx)}{(bc-ad)g^2(a+bx)}+\frac{4B^2(c+dx)\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(bc-ad)g^2(a+bx)}-\frac{(c+dx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(bc-ad)g^2(a+bx)}$$

```
[Out] 4*A*B*(d*x+c)/(-a*d+b*c)/g^2/(b*x+a)-8*B^2*(d*x+c)/(-a*d+b*c)/g^2/(b*x+a)+4
*B^2*(d*x+c)*ln(e*(d*x+c)^2/(b*x+a)^2)/(-a*d+b*c)/g^2/(b*x+a)-(d*x+c)*(A+B*
ln(e*(d*x+c)^2/(b*x+a)^2))^2/(-a*d+b*c)/g^2/(b*x+a)
```

Rubi [A]
time = 0.05, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.088$, Rules used = {2552, 2333, 2332}

$$-\frac{(c+dx)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{g^2(a+bx)(bc-ad)}+\frac{4AB(c+dx)}{g^2(a+bx)(bc-ad)}+\frac{4B^2(c+dx)\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{g^2(a+bx)(bc-ad)}-\frac{8B^2(c+dx)}{g^2(a+bx)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2/(a*g + b*g*x)^2,x]
```

```
[Out] (4*A*B*(c + d*x))/((b*c - a*d)*g^2*(a + b*x)) - (8*B^2*(c + d*x))/((b*c - a
*d)*g^2*(a + b*x)) + (4*B^2*(c + d*x)*Log[(e*(c + d*x)^2)/(a + b*x)^2])/((b
*c - a*d)*g^2*(a + b*x)) - ((c + d*x)*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^
2])^2)/((b*c - a*d)*g^2*(a + b*x))
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2552

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(
m + 1)*(g/d)^m, Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a
```

```
 + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[n
 + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[d*f
- c*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Rubi steps

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(ag+bgx)^2}\,dx = -\frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{bg^2(a+bx)}+\frac{(2B)\int\frac{2(bc-ad)\left(-A-B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{g(a+bx)^2(c+dx)}\,dx}{bg}$$

$$= -\frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{bg^2(a+bx)}+\frac{(4B(bc-ad))\int\frac{-A-B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(a+bx)^2(c+dx)}\,dx}{bg^2}$$

$$= -\frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{bg^2(a+bx)}+\frac{(4B(bc-ad))\int\left(\frac{b\left(-A-B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{(bc-ad)(a+bx)^2}-\frac{bd(}{}\right.}{}$$

$$= -\frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{bg^2(a+bx)}+\frac{(4B)\int\frac{-A-B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(a+bx)^2}\,dx}{g^2}-\frac{(4Bd)\int\frac{-A-B}{}}{(bc-}$$

$$= \frac{4B\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{bg^2(a+bx)}+\frac{4Bd\log(a+bx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{b(bc-ad)g^2}-\frac{4}{}$$

$$= \frac{4B\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{bg^2(a+bx)}+\frac{4Bd\log(a+bx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{b(bc-ad)g^2}-\frac{4}{}$$

$$= \frac{4B\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{bg^2(a+bx)}+\frac{4Bd\log(a+bx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{b(bc-ad)g^2}-\frac{4}{}$$

$$= -\frac{8B^2}{bg^2(a+bx)}-\frac{8B^2d\log(a+bx)}{b(bc-ad)g^2}+\frac{8B^2d\log(c+dx)}{b(bc-ad)g^2}+\frac{4B\left(A+B\log\left(\right.\right.}{bg^2(a+b}$$

$$= -\frac{8B^2}{bg^2(a+bx)}-\frac{8B^2d\log(a+bx)}{b(bc-ad)g^2}+\frac{8B^2d\log(c+dx)}{b(bc-ad)g^2}-\frac{8B^2d\log\left(-\frac{d(a+b}{bc-a}\right.}{b(bc-}$$

$$= -\frac{8B^2}{bg^2(a+bx)}-\frac{8B^2d\log(a+bx)}{b(bc-ad)g^2}+\frac{4B^2d\log^2(a+bx)}{b(bc-ad)g^2}+\frac{8B^2d\log(c+dx}{b(bc-ad)g^2}$$

$$= -\frac{8B^2}{bg^2(a+bx)}-\frac{8B^2d\log(a+bx)}{b(bc-ad)g^2}+\frac{4B^2d\log^2(a+bx)}{b(bc-ad)g^2}+\frac{8B^2d\log(c+dx}{b(bc-ad)g^2}$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 0.32, size = 322, normalized size = 2.05

$$-\frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2+\frac{4B\left(2B(bc-ad+d(a+bx)\log(a+bx)-d(a+bx)\log(c+dx))-(bc-ad)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)-d(a+bx)\log(a+bx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)+d(a+bx)\log(c+dx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)-Bd(a+bx)\left(\log(a+bx)\left(\log(a+bx)-2\log\left(\frac{b(c+dx)}{bc-ad}\right)\right)-2\text{Li}_2\left(\frac{d(a+bx)}{-bc+ad}\right)\right)+Bd(a+bx)\left(\left(2\log\left(\frac{d(a+bx)}{-bc+ad}\right)-\log(c+dx)\right)\log(c+dx)+2\text{Li}_2\left(\frac{b(c+dx)}{bc-ad}\right)\right)\right)}{bc-ad}}{bg^2(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2/(a*g + b*g*x)^2,x]
```

```
[Out] -(((A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2 + (4*B*(2*B*(b*c - a*d + d*(a
 + b*x)*Log[a + b*x] - d*(a + b*x)*Log[c + d*x]) - (b*c - a*d)*(A + B*Log[(
e*(c + d*x)^2)/(a + b*x)^2]) - d*(a + b*x)*Log[a + b*x]*(A + B*Log[(e*(c +
d*x)^2)/(a + b*x)^2]) + d*(a + b*x)*Log[c + d*x]*(A + B*Log[(e*(c + d*x)^2)
/(a + b*x)^2]) - B*d*(a + b*x)*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c +
d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) + B*d*(a
+ b*x)*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] +
 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(b*c - a*d))/(b*g^2*(a + b*x)))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 430 vs. $2(157) = 314$.
time = 0.42, size = 431, normalized size = 2.75

method	result
norman	$\frac{\frac{(A^2-4AB+8B^2)x}{ga}+\frac{B^2c\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)^2}{g(ad-cb)}+\frac{B^2dx\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)^2}{g(ad-cb)}+\frac{2(A-2B)cB\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)}{g(ad-cb)}+\frac{2d(A-2B)Bx\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)}{g(ad-cb)}}{g(bx+a)}$
risch	$-\frac{A^2}{g^2(bx+a)b}+\frac{\frac{8B^2x}{ag}+\frac{B^2c\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)^2}{g(ad-cb)}+\frac{B^2dx\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)^2}{g(ad-cb)}-\frac{4B^2c\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)}{g(ad-cb)}-\frac{4B^2dx\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)}{g(ad-cb)}}{g(bx+a)}-\frac{2A}{}$
derivativedivides	$-\frac{\frac{A^2}{(bx+a)g^2}+\frac{B^2\ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^2}{b^2}\right)^2}{g^2(bx+a)}+\frac{8B^2}{g^2(bx+a)}-\frac{4B^2\ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^2}{b^2}\right)}{g^2(bx+a)}+\frac{4B^2d\ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^2}{b^2}\right)}{g^2(ad-cb)}}{}$
default	$-\frac{\frac{A^2}{(bx+a)g^2}+\frac{B^2\ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^2}{b^2}\right)^2}{g^2(bx+a)}+\frac{8B^2}{g^2(bx+a)}-\frac{4B^2\ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^2}{b^2}\right)}{g^2(bx+a)}+\frac{4B^2d\ln\left(\frac{e\left(\frac{ad}{bx+a}-\frac{bc}{bx+a}-d\right)^2}{b^2}\right)}{g^2(ad-cb)}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*(d*x+c)^2/(b*x+a)^2))^2/(b*g*x+a*g)^2,x,method=_RETURNVERBOSE
)
```

```
[Out] -1/b*(1/(b*x+a)/g^2*A^2+1/g^2*B^2/(b*x+a)*ln(e*(a*d/(b*x+a)-b*c/(b*x+a)-d)^
2/b^2)^2+8/g^2*B^2/(b*x+a)-4/g^2*B^2/(b*x+a)*ln(e*(a*d/(b*x+a)-b*c/(b*x+a)-
d)^2/b^2)+4/g^2*B^2*d/(a*d-b*c)*ln(e*(a*d/(b*x+a)-b*c/(b*x+a)-d)^2/b^2)-1/g
^2*B^2*d/(a*d-b*c)*ln(e*(a*d/(b*x+a)-b*c/(b*x+a)-d)^2/b^2)^2+2/g^2*A*B/(b*x
```

```
+a)*ln(e*(a*d/(b*x+a)-b*c/(b*x+a)-d)^2/b^2)-4/g^2*A*B/(a*d-b*c)/(b*x+a)*a*d
+4/g^2*A*B/(a*d-b*c)/(b*x+a)*b*c-4/g^2*A*B*d^2/(a*d-b*c)^2*ln(a*d/(b*x+a)-b
*c/(b*x+a)-d)*a+4/g^2*A*B*d/(a*d-b*c)^2*ln(a*d/(b*x+a)-b*c/(b*x+a)-d)*b*c)
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 582 vs. $2(159) = 318$.
time = 0.33, size = 582, normalized size = 3.71

$$4\left(\left(\frac{1}{b^2g^2x+abg^2}+\frac{d\log\left(bx+a\right)}{(b^2c-abd)g^2}-\frac{d\log\left(dx+c\right)}{(b^2c-abd)g^2}\right)\log\left(\frac{d^2x^2e}{b^2x^2+2\,abx+a^2}+\frac{2\,cdxe}{b^2x^2+2\,abx+a^2}+\frac{c^2e}{b^2x^2+2\,abx+a^2}\right)+\frac{(bdx+ad)\log\left(bx+a\right)^2+(bdx+ad)\log\left(dx+c\right)^2-2\,bc+2\,ad-2\,(bdx+ad)\log\left(bx+a\right)+2\,(bdx+ad-(bdx+ad)\log\left(bx+a\right))\log\left(dx+c\right)}{ab^2cg^2-a^2bdg^2+(b^3cg^2-ab^2dg^2)x}\right)B^2-2\,AB\left(\frac{\log\left(\frac{d^2x^2e}{b^2x^2+2\,abx+a^2}+\frac{2\,cdxe}{b^2x^2+2\,abx+a^2}+\frac{c^2e}{b^2x^2+2\,abx+a^2}\right)}{b^2g^2x+abg^2}-\frac{2}{b^2g^2x+abg^2}-\frac{2\,d\log\left(bx+a\right)}{(b^2c-abd)g^2}+\frac{2\,d\log\left(dx+c\right)}{(b^2c-abd)g^2}\right)-\frac{B^2\log\left(\frac{d^2x^2e}{b^2x^2+2\,abx+a^2}+\frac{2\,cdxe}{b^2x^2+2\,abx+a^2}+\frac{c^2e}{b^2x^2+2\,abx+a^2}\right)^2}{b^2g^2x+abg^2}-\frac{A^2}{b^2g^2x+abg^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2/(b*g*x+a*g)^2,x, algorithm="ma
xima")
```

```
[Out] 4*((1/(b^2*g^2*x + a*b*g^2) + d*log(b*x + a)/((b^2*c - a*b*d)*g^2) - d*log(
d*x + c)/((b^2*c - a*b*d)*g^2))*log(d^2*x^2*e/(b^2*x^2 + 2*a*b*x + a^2) + 2
*c*d*x*e/(b^2*x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2)) + ((b
*d*x + a*d)*log(b*x + a)^2 + (b*d*x + a*d)*log(d*x + c)^2 - 2*b*c + 2*a*d -
 2*(b*d*x + a*d)*log(b*x + a) + 2*(b*d*x + a*d - (b*d*x + a*d)*log(b*x + a)
)*log(d*x + c))/(a*b^2*c*g^2 - a^2*b*d*g^2 + (b^3*c*g^2 - a*b^2*d*g^2)*x))*
B^2 - 2*A*B*(log(d^2*x^2*e/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*x*e/(b^2*x^2 +
 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2))/(b^2*g^2*x + a*b*g^2) -
2/(b^2*g^2*x + a*b*g^2) - 2*d*log(b*x + a)/((b^2*c - a*b*d)*g^2) + 2*d*log(
d*x + c)/((b^2*c - a*b*d)*g^2)) - B^2*log(d^2*x^2*e/(b^2*x^2 + 2*a*b*x + a^
2) + 2*c*d*x*e/(b^2*x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2))
^2/(b^2*g^2*x + a*b*g^2) - A^2/(b^2*g^2*x + a*b*g^2)
```

Fricas [**A**]
time = 0.39, size = 196, normalized size = 1.25

$$-\frac{(A^2-4\,AB+8\,B^2)bc-(A^2-4\,AB+8\,B^2)ad+(B^2bdx+B^2bc)\log\left(\frac{(d^2x^2+2\,cdx+c^2)e}{b^2x^2+2\,abx+a^2}\right)^2+2\,((AB-2\,B^2)bdx+(AB-2\,B^2)bc)\log\left(\frac{(d^2x^2+2\,cdx+c^2)e}{b^2x^2+2\,abx+a^2}\right)}{(b^3c-ab^2d)g^2x+(ab^2c-a^2bd)g^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2/(b*g*x+a*g)^2,x, algorithm="fr
icas")
```

```
[Out] -((A^2 - 4*A*B + 8*B^2)*b*c - (A^2 - 4*A*B + 8*B^2)*a*d + (B^2*b*d*x + B^2*
b*c)*log((d^2*x^2 + 2*c*d*x + c^2)*e/(b^2*x^2 + 2*a*b*x + a^2))^2 + 2*((A*B
 - 2*B^2)*b*d*x + (A*B - 2*B^2)*b*c)*log((d^2*x^2 + 2*c*d*x + c^2)*e/(b^2*x
^2 + 2*a*b*x + a^2)))/((b^3*c - a*b^2*d)*g^2*x + (a*b^2*c - a^2*b*d)*g^2)
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 450 vs. $2(134) = 268$.
time = 2.75, size = 450, normalized size = 2.87

$$\frac{4Bd(A-2B)\log\left(x+\frac{4ABad^2+4ABbcd-8B^2ad^2-8B^2bcd-\frac{4Ba^2d^3(A-2B)}{ad-bc}+\frac{8Babcd^2(A-2B)}{ad-bc}-\frac{4Bb^2c^2d(A-2B)}{ad-bc}}{8ABbd^2-16B^2bd^2}\right)}{bg^2\left(ad-bc\right)}-\frac{4Bd(A-2B)\log\left(x+\frac{4ABad^2+4ABbcd-8B^2ad^2-8B^2bcd+\frac{4Ba^2d^3(A-2B)}{ad-bc}-\frac{8Babcd^2(A-2B)}{ad-bc}+\frac{4Bb^2c^2d(A-2B)}{ad-bc}}{8ABbd^2-16B^2bd^2}\right)}{bg^2\left(ad-bc\right)}+\frac{(-2AB+4B^2)\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{abg^2+b^2g^2x}+\frac{(B^2c+B^2dx)\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)^2}{a^2dg^2-abcg^2+abdg^2x-b^2cg^2x}+\frac{-A^2+4AB-8B^2}{abg^2+b^2g^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(d*x+c)**2/(b*x+a)**2))**2/(b*g*x+a*g)**2,x)
```

```
[Out] 4*B*d*(A - 2*B)*log(x + (4*A*B*a*d**2 + 4*A*B*b*c*d - 8*B**2*a*d**2 - 8*B**
2*b*c*d - 4*B*a**2*d**3*(A - 2*B)/(a*d - b*c) + 8*B*a*b*c*d**2*(A - 2*B)/(a
*d - b*c) - 4*B*b**2*c**2*d*(A - 2*B)/(a*d - b*c))/(8*A*B*b*d**2 - 16*B**2*
b*d**2))/(b*g**2*(a*d - b*c)) - 4*B*d*(A - 2*B)*log(x + (4*A*B*a*d**2 + 4*A
*B*b*c*d - 8*B**2*a*d**2 - 8*B**2*b*c*d + 4*B*a**2*d**3*(A - 2*B)/(a*d - b*
c) - 8*B*a*b*c*d**2*(A - 2*B)/(a*d - b*c) + 4*B*b**2*c**2*d*(A - 2*B)/(a*d
- b*c))/(8*A*B*b*d**2 - 16*B**2*b*d**2))/(b*g**2*(a*d - b*c)) + (-2*A*B + 4
*B**2)*log(e*(c + d*x)**2/(a + b*x)**2)/(a*b*g**2 + b**2*g**2*x) + (B**2*c
+ B**2*d*x)*log(e*(c + d*x)**2/(a + b*x)**2)**2/(a**2*d*g**2 - a*b*c*g**2 +
 a*b*d*g**2*x - b**2*c*g**2*x) + (-A**2 + 4*A*B - 8*B**2)/(a*b*g**2 + b**2*
g**2*x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 374 vs. 2(159) = 318.
time = 3.15, size = 374, normalized size = 2.38

$$-\left(\frac{B^2 d}{b^2 c g^2 - abdg^2} + \frac{B^2}{(bgx + ag)bg}\right) \log\left(\frac{\frac{b^2c^2g^2}{(bgx+ag)^2} - \frac{2\,abcdg^2}{(bgx+ag)^2} + \frac{a^2d^2g^2}{(bgx+ag)^2} + \frac{2\,bcdg}{bgx+ag} - \frac{2\,ad^2g}{bgx+ag} + d^2}{b^2}\right)^2 - \frac{4\,(ABd - B^2d)\log\left(\frac{bcg}{bgx+ag} - \frac{adg}{bgx+ag} + d\right)}{b^2cg^2 - abdg^2} - \frac{2\,(AB - B^2)\log\left(\frac{\frac{b^2c^2g^2}{(bgx+ag)^2} - \frac{2\,abcdg^2}{(bgx+ag)^2} + \frac{a^2d^2g^2}{(bgx+ag)^2} + \frac{2\,bcdg}{bgx+ag} - \frac{2\,ad^2g}{bgx+ag} + d^2}{b^2}\right)}{(bgx + ag)bg} - \frac{A^2 - 2\,AB + 5\,B^2}{(bgx + ag)bg}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2/(b*g*x+a*g)^2,x, algorithm="gi
ac")
```

```
[Out] -(B^2*d/(b^2*c*g^2 - a*b*d*g^2) + B^2/((b*g*x + a*g)*b*g))*log((b^2*c^2*g^2
/(b*g*x + a*g)^2 - 2*a*b*c*d*g^2/(b*g*x + a*g)^2 + a^2*d^2*g^2/(b*g*x + a*g
)^2 + 2*b*c*d*g/(b*g*x + a*g) - 2*a*d^2*g/(b*g*x + a*g) + d^2)/b^2)^2 - 4*(
A*B*d - B^2*d)*log(b*c*g/(b*g*x + a*g) - a*d*g/(b*g*x + a*g) + d)/(b^2*c*g^
2 - a*b*d*g^2) - 2*(A*B - B^2)*log((b^2*c^2*g^2/(b*g*x + a*g)^2 - 2*a*b*c*d
*g^2/(b*g*x + a*g)^2 + a^2*d^2*g^2/(b*g*x + a*g)^2 + 2*b*c*d*g/(b*g*x + a*g
) - 2*a*d^2*g/(b*g*x + a*g) + d^2)/b^2)/((b*g*x + a*g)*b*g) - (A^2 - 2*A*B
+ 5*B^2)/((b*g*x + a*g)*b*g)
```

Mupad [B]
time = 6.49, size = 227, normalized size = 1.45

$$\frac{\ln\left(\frac{e\,(c+d\,x)^2}{(a+b\,x)^2}\right)\,\left(\frac{4\,B^2}{b^2\,d\,g^2} - \frac{2\,A\,B}{b^2\,d\,g^2}\right)}{\frac{x}{d} + \frac{a}{b\,d}} - \frac{A^2 - 4\,A\,B + 8\,B^2}{x\,b^2\,g^2 + a\,b\,g^2} - \ln\left(\frac{e\,(c + d\,x)^2}{(a + b\,x)^2}\right)^2\,\left(\frac{B^2}{b^2\,g^2\,\left(x + \frac{a}{b}\right)} - \frac{B^2\,d}{b\,g^2\,(a\,d - b\,c)}\right) + \frac{B\,d\,\mathrm{atan}\left(\frac{\left(2\,b\,d\,x + \frac{c\,b^2\,g^2 + a\,d\,b\,g^2}{b\,g^2}\right)\,1{}\mathrm{i}}{a\,d - b\,c}\right)\,(A - 2\,B)\,8{}\mathrm{i}}{b\,g^2\,(a\,d - b\,c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(c + d*x)^2)/(a + b*x)^2))^2/(a*g + b*g*x)^2,x)
```

```
[Out] (log((e*(c + d*x)^2)/(a + b*x)^2)*((4*B^2)/(b^2*d*g^2) - (2*A*B)/(b^2*d*g^2
)))/(x/d + a/(b*d)) - (A^2 + 8*B^2 - 4*A*B)/(b^2*g^2*x + a*b*g^2) - log((e*
```

```
(c + d*x)^2)/(a + b*x)^2)^2*(B^2/(b^2*g^2*(x + a/b)) - (B^2*d)/(b*g^2*(a*d
- b*c))) + (B*d*atan(((2*b*d*x + (b^2*c*g^2 + a*b*d*g^2)/(b*g^2))*1i)/(a*d
- b*c))*(A - 2*B)*8i)/(b*g^2*(a*d - b*c))
```

3.216 $\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(ag+bgx)^3}\,dx$

Optimal. Leaf size=299

$$-\frac{4ABd(c+dx)}{(bc-ad)^2g^3(a+bx)}+\frac{8B^2d(c+dx)}{(bc-ad)^2g^3(a+bx)}-\frac{bB^2(c+dx)^2}{(bc-ad)^2g^3(a+bx)^2}-\frac{4B^2d(c+dx)\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(bc-ad)^2g^3(a+bx)}+\frac{bB(c}{}$$

```
[Out] -4*A*B*d*(d*x+c)/(-a*d+b*c)^2/g^3/(b*x+a)+8*B^2*d*(d*x+c)/(-a*d+b*c)^2/g^3/
(b*x+a)-b*B^2*(d*x+c)^2/(-a*d+b*c)^2/g^3/(b*x+a)^2-4*B^2*d*(d*x+c)*ln(e*(d*
x+c)^2/(b*x+a)^2)/(-a*d+b*c)^2/g^3/(b*x+a)+b*B*(d*x+c)^2*(A+B*ln(e*(d*x+c)^
2/(b*x+a)^2))/(-a*d+b*c)^2/g^3/(b*x+a)^2+d*(d*x+c)*(A+B*ln(e*(d*x+c)^2/(b*x
+a)^2))^2/(-a*d+b*c)^2/g^3/(b*x+a)-1/2*b*(d*x+c)^2*(A+B*ln(e*(d*x+c)^2/(b*x
+a)^2))^2/(-a*d+b*c)^2/g^3/(b*x+a)^2
```

Rubi [A]
time = 0.11, antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {2552, 2367, 2333, 2332, 2342, 2341}

$$\frac{bB(c+dx)^2\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{g^3(a+bx)^2(bc-ad)^2}-\frac{b(c+dx)^2\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{2g^3(a+bx)^2(bc-ad)^2}+\frac{d(c+dx)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{g^3(a+bx)(bc-ad)^2}-\frac{4ABd(c+dx)}{g^3(a+bx)(bc-ad)^2}-\frac{4B^2d(c+dx)\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{g^3(a+bx)(bc-ad)^2}-\frac{bB^2(c+dx)^2}{g^3(a+bx)^2(bc-ad)^2}+\frac{8B^2d(c+dx)}{g^3(a+bx)(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2/(a*g + b*g*x)^3,x]
```

```
[Out] (-4*A*B*d*(c + d*x))/((b*c - a*d)^2*g^3*(a + b*x)) + (8*B^2*d*(c + d*x))/((
b*c - a*d)^2*g^3*(a + b*x)) - (b*B^2*(c + d*x)^2)/((b*c - a*d)^2*g^3*(a + b
*x)^2) - (4*B^2*d*(c + d*x)*Log[(e*(c + d*x)^2)/(a + b*x)^2])/((b*c - a*d)^
2*g^3*(a + b*x)) + (b*B*(c + d*x)^2*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]
))/((b*c - a*d)^2*g^3*(a + b*x)^2) + (d*(c + d*x)*(A + B*Log[(e*(c + d*x)^2
)/(a + b*x)^2])^2)/((b*c - a*d)^2*g^3*(a + b*x)) - (b*(c + d*x)^2*(A + B*Lo
g[(e*(c + d*x)^2)/(a + b*x)^2])^2)/(2*(b*c - a*d)^2*g^3*(a + b*x)^2)
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2367

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^(r_.))^(
q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*x^n])^p, (d + e*x
^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
&& IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0] && IntegerQ[r]))
```

Rule 2552

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(
m + 1)*(g/d)^m, Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a
 + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[n
 + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[d*f
- c*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Rubi steps

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(ag+bgx)^3}\,dx = -\frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{2bg^3(a+bx)^2} + \frac{B\int \frac{2(bc-ad)\left(-A-B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{g^2(a+bx)^3(c+dx)}dx}{bg}$$

$$= -\frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{2bg^3(a+bx)^2} + \frac{(2B(bc-ad))\int \frac{-A-B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(a+bx)^3(c+dx)}dx}{bg^3}$$

$$= -\frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{2bg^3(a+bx)^2} + \frac{(2B(bc-ad))\int \left(\frac{b\left(-A-B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{(bc-ad)(a+bx)^3} - \cdots\right.}{\cdots}$$

$$= -\frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{2bg^3(a+bx)^2} + \frac{(2B)\int \frac{-A-B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(a+bx)^3}dx}{g^3} + \frac{(2Bd^2)\int \frac{-A\cdots}{\cdots}}{(bc\cdots}$$

$$= \frac{B\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{bg^3(a+bx)^2} - \frac{2Bd\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{b(bc-ad)g^3(a+bx)} - \frac{2Bd^2\log(a+\cdots}{\cdots}$$

$$= \frac{B\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{bg^3(a+bx)^2} - \frac{2Bd\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{b(bc-ad)g^3(a+bx)} - \frac{2Bd^2\log(a+\cdots}{\cdots}$$

$$= \frac{B\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{bg^3(a+bx)^2} - \frac{2Bd\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{b(bc-ad)g^3(a+bx)} - \frac{2Bd^2\log(a+\cdots}{\cdots}$$

$$= -\frac{B^2}{bg^3(a+bx)^2} + \frac{6B^2d}{b(bc-ad)g^3(a+bx)} + \frac{6B^2d^2\log(a+bx)}{b(bc-ad)^2g^3} - \frac{6B^2d^2\log\cdots}{b(bc-\cdots}$$

$$= -\frac{B^2}{bg^3(a+bx)^2} + \frac{6B^2d}{b(bc-ad)g^3(a+bx)} + \frac{6B^2d^2\log(a+bx)}{b(bc-ad)^2g^3} - \frac{6B^2d^2\log\cdots}{b(bc-\cdots}$$

$$= -\frac{B^2}{bg^3(a+bx)^2} + \frac{6B^2d}{b(bc-ad)g^3(a+bx)} + \frac{6B^2d^2\log(a+bx)}{b(bc-ad)^2g^3} - \frac{2B^2d^2\log^2\cdots}{b(bc-\cdots}$$

$$= -\frac{B^2}{bg^3(a+bx)^2} + \frac{6B^2d}{b(bc-ad)g^3(a+bx)} + \frac{6B^2d^2\log(a+bx)}{b(bc-ad)^2g^3} - \frac{2B^2d^2\log^2\cdots}{b(bc-\cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 0.31, size = 452, normalized size = 1.51

$$-\frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2 - \frac{2B\left(4Bd(a+bx)(bc-ad+d(a+bx)\log(a+bx)-d(a+bx)\log(c+dx))-B\left((bc-ad)^2+2d(-bc+ad)(a+bx)-2d^2(a+bx)^2\log(a+bx)+2d^2(a+bx)^2\log(c+dx)\right)+(bc-ad)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)+2d(-bc+ad)(a+bx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)-2d^2(a+bx)^2\log(a+bx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)+2d^2(a+bx)^2\log(c+dx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)-2Bd^2(a+bx)^2\left(\log(a+bx)\left(\log(a+bx)-2\log\left(\frac{b(c+dx)}{bc-ad}\right)\right)-2\text{Li}_2\left(\frac{d(a+bx)}{-bc+ad}\right)\right)+2Bd^2(a+bx)^2\left(\left(2\log\left(\frac{d(a+bx)}{-bc+ad}\right)-\log(c+dx)\right)\log(c+dx)+2\text{Li}_2\left(\frac{b(c+dx)}{bc-ad}\right)\right)\right)}{(bc-ad)^2}}{2bg^3(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2/(a*g + b*g*x)^3,x]
```

```
[Out] -1/2*((A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2 - (2*B*(4*B*d*(a + b*x)*(b
*c - a*d + d*(a + b*x)*Log[a + b*x] - d*(a + b*x)*Log[c + d*x]) - B*((b*c -
 a*d)^2 + 2*d*(-(b*c) + a*d)*(a + b*x) - 2*d^2*(a + b*x)^2*Log[a + b*x] + 2
*d^2*(a + b*x)^2*Log[c + d*x]) + (b*c - a*d)^2*(A + B*Log[(e*(c + d*x)^2)/(
a + b*x)^2]) + 2*d*(-(b*c) + a*d)*(a + b*x)*(A + B*Log[(e*(c + d*x)^2)/(a +
 b*x)^2]) - 2*d^2*(a + b*x)^2*Log[a + b*x]*(A + B*Log[(e*(c + d*x)^2)/(a +
b*x)^2]) + 2*d^2*(a + b*x)^2*Log[c + d*x]*(A + B*Log[(e*(c + d*x)^2)/(a + b
*x)^2]) - 2*B*d^2*(a + b*x)^2*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d
*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) + 2*B*d^2*
(a + b*x)^2*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d
*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(b*c - a*d)^2)/(b*g^3*(a +
 b*x)^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 638 vs. $2(297) = 594$.
time = 0.56, size = 639, normalized size = 2.14 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*(d*x+c)^2/(b*x+a)^2))^2/(b*g*x+a*g)^3,x,method=_RETURNVERBOSE
)
```

```
[Out] -1/b*(1/2/(b*x+a)^2/g^3*A^2+1/g^3*B^2/(b*x+a)^2-1/g^3*B^2/(b*x+a)^2*ln(e*(a
*d/(b*x+a)-b*c/(b*x+a)-d)^2/b^2)+1/2/g^3*B^2/(b*x+a)^2*ln(e*(a*d/(b*x+a)-b*
c/(b*x+a)-d)^2/b^2)^2+6/g^3*B^2*d/(a*d-b*c)/(b*x+a)+3/g^3*B^2*d^2/(a^2*d^2-
2*a*b*c*d+b^2*c^2)*ln(e*(a*d/(b*x+a)-b*c/(b*x+a)-d)^2/b^2)-1/2/g^3*B^2*d^2/
(a^2*d^2-2*a*b*c*d+b^2*c^2)*ln(e*(a*d/(b*x+a)-b*c/(b*x+a)-d)^2/b^2)^2-2/g^3
*B^2*d/(a*d-b*c)/(b*x+a)*ln(e*(a*d/(b*x+a)-b*c/(b*x+a)-d)^2/b^2)+1/g^3*A*B/
(b*x+a)^2*ln(e*(a*d/(b*x+a)-b*c/(b*x+a)-d)^2/b^2)-1/g^3*A*B/(a*d-b*c)^2*a^2
*d^2/(b*x+a)^2+2/g^3*A*B/(a*d-b*c)^2*a*d/(b*x+a)^2*b*c-1/g^3*A*B/(a*d-b*c)^
2*b^2*c^2/(b*x+a)^2-2/g^3*A*B/(a*d-b*c)^2*d^2/(b*x+a)*a+2/g^3*A*B/(a*d-b*c)
^2*d/(b*x+a)*b*c-2/g^3*A*B*d^3/(a*d-b*c)^3*ln(a*d/(b*x+a)-b*c/(b*x+a)-d)*a+
2/g^3*A*B*d^2/(a*d-b*c)^3*ln(a*d/(b*x+a)-b*c/(b*x+a)-d)*b*c)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1010 vs. $2(301) = 602$.
time = 0.37, size = 1010, normalized size = 3.38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2/(b*g*x+a*g)^3,x, algorithm="ma
xima")
```

```
[Out] -((((2*b*d*x - b*c + 3*a*d)/((b^4*c - a*b^3*d)*g^3*x^2 + 2*(a*b^3*c - a^2*b^
2*d)*g^3*x + (a^2*b^2*c - a^3*b*d)*g^3) + 2*d^2*log(b*x + a)/((b^3*c^2 - 2*
a*b^2*c*d + a^2*b*d^2)*g^3) - 2*d^2*log(d*x + c)/((b^3*c^2 - 2*a*b^2*c*d +
a^2*b*d^2)*g^3))*log(d^2*x^2*e/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*x*e/(b^2*x
^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2)) + (b^2*c^2 - 8*a*b*c
*d + 7*a^2*d^2 + 2*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*log(b*x + a)^2 + 2
*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*log(d*x + c)^2 - 6*(b^2*c*d - a*b*d^
2)*x - 6*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*log(b*x + a) + 2*(3*b^2*d^2*
x^2 + 6*a*b*d^2*x + 3*a^2*d^2 - 2*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*log
(b*x + a))*log(d*x + c))/(a^2*b^3*c^2*g^3 - 2*a^3*b^2*c*d*g^3 + a^4*b*d^2*g
^3 + (b^5*c^2*g^3 - 2*a*b^4*c*d*g^3 + a^2*b^3*d^2*g^3)*x^2 + 2*(a*b^4*c^2*g
^3 - 2*a^2*b^3*c*d*g^3 + a^3*b^2*d^2*g^3)*x))*B^2 - A*B*((2*b*d*x - b*c + 3
*a*d)/((b^4*c - a*b^3*d)*g^3*x^2 + 2*(a*b^3*c - a^2*b^2*d)*g^3*x + (a^2*b^2
*c - a^3*b*d)*g^3) + log(d^2*x^2*e/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*x*e/(b
^2*x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2))/(b^3*g^3*x^2 + 2
*a*b^2*g^3*x + a^2*b*g^3) + 2*d^2*log(b*x + a)/((b^3*c^2 - 2*a*b^2*c*d + a^
2*b*d^2)*g^3) - 2*d^2*log(d*x + c)/((b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*g^3
)) - 1/2*B^2*log(d^2*x^2*e/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*x*e/(b^2*x^2 +
 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2))^2/(b^3*g^3*x^2 + 2*a*b^2
*g^3*x + a^2*b*g^3) - 1/2*A^2/(b^3*g^3*x^2 + 2*a*b^2*g^3*x + a^2*b*g^3)
```

Fricas [**A**]
time = 0.34, size = 409, normalized size = 1.37

$$-\frac{(A^2-2\,AB+2\,B^2)b^2c^2-2\,(A^2-4\,AB+8\,B^2)abcd+(A^2-6\,AB+14\,B^2)a^2d^2-(B^2b^2d^2x^2+2\,B^2abd^2x-B^2b^2c^2+2\,B^2abcd)\log\left(\frac{(d^2x^2+2\,cdx+c^2)e}{b^2x^2+2\,abx+a^2}\right)^2+4\,((AB-3\,B^2)b^2cd-(AB-3\,B^2)abd^2)x-2\,((AB-3\,B^2)b^2d^2x^2-(AB-B^2)b^2c^2+2\,(AB-2\,B^2)abcd-2\,(B^2b^2cd-(AB-2\,B^2)abd^2)x)\log\left(\frac{(d^2x^2+2\,cdx+c^2)e}{b^2x^2+2\,abx+a^2}\right)}{2\,((b^5c^2-2\,ab^4cd+a^2b^3d^2)g^3x^2+2\,(ab^4c^2-2\,a^2b^3cd+a^3b^2d^2)g^3x+(a^2b^3c^2-2\,a^3b^2cd+a^4bd^2)g^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2/(b*g*x+a*g)^3,x, algorithm="fr
icas")
```

```
[Out] -1/2*((A^2 - 2*A*B + 2*B^2)*b^2*c^2 - 2*(A^2 - 4*A*B + 8*B^2)*a*b*c*d + (A^
2 - 6*A*B + 14*B^2)*a^2*d^2 - (B^2*b^2*d^2*x^2 + 2*B^2*a*b*d^2*x - B^2*b^2*
c^2 + 2*B^2*a*b*c*d)*log((d^2*x^2 + 2*c*d*x + c^2)*e/(b^2*x^2 + 2*a*b*x + a
^2))^2 + 4*((A*B - 3*B^2)*b^2*c*d - (A*B - 3*B^2)*a*b*d^2)*x - 2*((A*B - 3*
B^2)*b^2*d^2*x^2 - (A*B - B^2)*b^2*c^2 + 2*(A*B - 2*B^2)*a*b*c*d - 2*(B^2*b
^2*c*d - (A*B - 2*B^2)*a*b*d^2)*x)*log((d^2*x^2 + 2*c*d*x + c^2)*e/(b^2*x^2
 + 2*a*b*x + a^2)))/((b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*g^3*x^2 + 2*(a*b
^4*c^2 - 2*a^2*b^3*c*d + a^3*b^2*d^2)*g^3*x + (a^2*b^3*c^2 - 2*a^3*b^2*c*d
+ a^4*b*d^2)*g^3)
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 877 vs. $2(279) = 558$.
time = 5.22, size = 877, normalized size = 2.93

$$\frac{2Bd^2(A-3B)\log\left(x+\frac{2ABad^3+2ABbcd^2-6B^2ad^3-6B^2bcd^2-\frac{2Ba^3d^5(A-3B)}{(ad-bc)^2}+\frac{6Ba^2bcd^4(A-3B)}{(ad-bc)^2}-\frac{6Bab^2c^2d^3(A-3B)}{(ad-bc)^2}+\frac{2Bb^3c^3d^2(A-3B)}{(ad-bc)^2}}{4ABbd^3-12B^2bd^3}\right)}{bg^3\left(ad-bc\right)^2}-\frac{2Bd^2(A-3B)\log\left(x+\frac{2ABad^3+2ABbcd^2-6B^2ad^3-6B^2bcd^2+\frac{2Ba^3d^5(A-3B)}{(ad-bc)^2}-\frac{6Ba^2bcd^4(A-3B)}{(ad-bc)^2}+\frac{6Bab^2c^2d^3(A-3B)}{(ad-bc)^2}-\frac{2Bb^3c^3d^2(A-3B)}{(ad-bc)^2}}{4ABbd^3-12B^2bd^3}\right)}{bg^3\left(ad-bc\right)^2}+\frac{(2B^2acd+2B^2ad^2x-B^2bc^2+B^2bd^2x^2)\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)^2}{2a^4d^2g^3-4a^3bcdg^3+4a^3bd^2g^3x+2a^2b^2c^2g^3-8a^2b^2cdg^3x+2a^2b^2d^2g^3x^2+4ab^3c^2g^3x-4ab^3cdg^3x^2+2b^4c^2g^3x^2}+\frac{(-ABad+ABbc+3B^2ad-B^2bc+2B^2bdx)\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{a^3bdg^3-a^2b^2cg^3+2a^2b^2dg^3x-2ab^3cg^3x+ab^3dg^3x^2-b^4cg^3x^2}+\frac{-A^2ad+A^2bc+6ABad-2ABbc-14B^2ad+2B^2bc+x(4ABbd-12B^2bd)}{2a^3bdg^3-2a^2b^2cg^3+x^2\cdot(2ab^3dg^3-2b^4cg^3)+x\left(4a^2b^2dg^3-4ab^3cg^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(d*x+c)**2/(b*x+a)**2))**2/(b*g*x+a*g)**3,x)
```

```
[Out] 2*B*d**2*(A - 3*B)*log(x + (2*A*B*a*d**3 + 2*A*B*b*c*d**2 - 6*B**2*a*d**3 -
 6*B**2*b*c*d**2 - 2*B*a**3*d**5*(A - 3*B)/(a*d - b*c)**2 + 6*B*a**2*b*c*d*
*4*(A - 3*B)/(a*d - b*c)**2 - 6*B*a*b**2*c**2*d**3*(A - 3*B)/(a*d - b*c)**2
 + 2*B*b**3*c**3*d**2*(A - 3*B)/(a*d - b*c)**2)/(4*A*B*b*d**3 - 12*B**2*b*d
**3))/(b*g**3*(a*d - b*c)**2) - 2*B*d**2*(A - 3*B)*log(x + (2*A*B*a*d**3 +
2*A*B*b*c*d**2 - 6*B**2*a*d**3 - 6*B**2*b*c*d**2 + 2*B*a**3*d**5*(A - 3*B)/
(a*d - b*c)**2 - 6*B*a**2*b*c*d**4*(A - 3*B)/(a*d - b*c)**2 + 6*B*a*b**2*c*
*2*d**3*(A - 3*B)/(a*d - b*c)**2 - 2*B*b**3*c**3*d**2*(A - 3*B)/(a*d - b*c)
**2)/(4*A*B*b*d**3 - 12*B**2*b*d**3))/(b*g**3*(a*d - b*c)**2) + (2*B**2*a*c
*d + 2*B**2*a*d**2*x - B**2*b*c**2 + B**2*b*d**2*x**2)*log(e*(c + d*x)**2/(
a + b*x)**2)**2/(2*a**4*d**2*g**3 - 4*a**3*b*c*d*g**3 + 4*a**3*b*d**2*g**3*
x + 2*a**2*b**2*c**2*g**3 - 8*a**2*b**2*c*d*g**3*x + 2*a**2*b**2*d**2*g**3*
x**2 + 4*a*b**3*c**2*g**3*x - 4*a*b**3*c*d*g**3*x**2 + 2*b**4*c**2*g**3*x**
2) + (-A*B*a*d + A*B*b*c + 3*B**2*a*d - B**2*b*c + 2*B**2*b*d*x)*log(e*(c +
 d*x)**2/(a + b*x)**2)/(a**3*b*d*g**3 - a**2*b**2*c*g**3 + 2*a**2*b**2*d*g*
*3*x - 2*a*b**3*c*g**3*x + a*b**3*d*g**3*x**2 - b**4*c*g**3*x**2) + (-A**2*
a*d + A**2*b*c + 6*A*B*a*d - 2*A*B*b*c - 14*B**2*a*d + 2*B**2*b*c + x*(4*A*
B*b*d - 12*B**2*b*d))/(2*a**3*b*d*g**3 - 2*a**2*b**2*c*g**3 + x**2*(2*a*b**
3*d*g**3 - 2*b**4*c*g**3) + x*(4*a**2*b**2*d*g**3 - 4*a*b**3*c*g**3))
```

<u>Giac</u> [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2/(b*g*x+a*g)^3,x, algorithm="gi
ac")
```

```
[Out] integrate((B*log((d*x + c)^2*e/(b*x + a)^2) + A)^2/(b*g*x + a*g)^3, x)
```

<u>Mupad</u> [B]
time = 6.71, size = 504, normalized size = 1.69

$$\frac{\ln\left(\frac{e\,{(c+d\,x)}^2}{{(a+b\,x)}^2}\right)\,\left(\frac{2\,B^2\,x\,(a\,d-b\,c)}{b\,g^3\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}-\frac{A\,B}{b^2\,d\,g^3}+\frac{B^2\,d^2\,\left(\frac{2\,a^2\,d^2-3\,a\,b\,c\,d+b^2\,c^2}{b\,d^3}+\frac{a\,(a\,d-b\,c)}{b\,d^2}\right)}{b\,g^3\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}\right)}{\frac{b\,x^2}{d}+\frac{a^2}{b\,d}+\frac{2\,a\,x}{d}}-\ln\left(\frac{e\,{(c+d\,x)}^2}{{(a+b\,x)}^2}\right)^2\,\left(\frac{B^2}{2\,b^2\,g^3\,\left(2\,a\,x+b\,x^2+\frac{a^2}{b}\right)}-\frac{B^2\,d^2}{2\,b\,g^3\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}\right)-\frac{\frac{A^2\,a\,d-A^2\,b\,c+14\,B^2\,a\,d-2\,B^2\,b\,c-6\,A\,B\,a\,d+2\,A\,B\,b\,c}{2\,(a\,d-b\,c)}+\frac{2\,x\,(3\,B^2\,b\,d-A\,B\,b\,d)}{a\,d-b\,c}}{a^2\,b\,g^3+2\,a\,b^2\,g^3\,x+b^3\,g^3\,x^2}-\frac{B\,d^2\,\mathrm{atan}\left(\frac{B\,d^2\,\left(2\,b\,d\,x-\frac{b^3\,c^2\,g^3-a^2\,b\,d^2\,g^3}{b\,g^3\,(a\,d-b\,c)}\right)\,(A-3\,B)\,2{}\mathrm{i}}{(a\,d-b\,c)\,(6\,B^2\,d^2-2\,A\,B\,d^2)}\right)\,(A-3\,B)\,4{}\mathrm{i}}{b\,g^3\,{(a\,d-b\,c)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(c + d*x)^2)/(a + b*x)^2))^2/(a*g + b*g*x)^3,x)
```

```
[Out] (log((e*(c + d*x)^2)/(a + b*x)^2)*((2*B^2*x*(a*d - b*c))/(b*g^3*(a^2*d^2 +
b^2*c^2 - 2*a*b*c*d)) - (A*B)/(b^2*d*g^3) + (B^2*d^2*((2*a^2*d^2 + b^2*c^2
- 3*a*b*c*d)/(b*d^3) + (a*(a*d - b*c))/(b*d^2)))/(b*g^3*(a^2*d^2 + b^2*c^2
- 2*a*b*c*d))))/((b*x^2)/d + a^2/(b*d) + (2*a*x)/d) - log((e*(c + d*x)^2)/(
a + b*x)^2)^2*(B^2/(2*b^2*g^3*(2*a*x + b*x^2 + a^2/b)) - (B^2*d^2)/(2*b*g^3
```

```
*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))) - ((A^2*a*d - A^2*b*c + 14*B^2*a*d - 2*B
^2*b*c - 6*A*B*a*d + 2*A*B*b*c)/(2*(a*d - b*c)) + (2*x*(3*B^2*b*d - A*B*b*d
))/(a*d - b*c))/(a^2*b*g^3 + b^3*g^3*x^2 + 2*a*b^2*g^3*x) - (B*d^2*atan((B*
d^2*(2*b*d*x - (b^3*c^2*g^3 - a^2*b*d^2*g^3)/(b*g^3*(a*d - b*c)))*(A - 3*B)
*2i)/((a*d - b*c)*(6*B^2*d^2 - 2*A*B*d^2)))*(A - 3*B)*4i)/(b*g^3*(a*d - b*c
)^2)
```

3.217 $\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(ag+bgx)^4}\,dx$

Optimal. Leaf size=407

$$-\frac{8B^2d^2(c+dx)}{(bc-ad)^3g^4(a+bx)}+\frac{2bB^2d(c+dx)^2}{(bc-ad)^3g^4(a+bx)^2}-\frac{8b^2B^2(c+dx)^3}{27(bc-ad)^3g^4(a+bx)^3}+\frac{4B^2d^3\log^2\left(\frac{c+dx}{a+bx}\right)}{3b(bc-ad)^3g^4}+\frac{4Bd^2(c+dx)}{(bc-}$$

```
[Out] -8*B^2*d^2*(d*x+c)/(-a*d+b*c)^3/g^4/(b*x+a)+2*b*B^2*d*(d*x+c)^2/(-a*d+b*c)^
3/g^4/(b*x+a)^2-8/27*b^2*B^2*(d*x+c)^3/(-a*d+b*c)^3/g^4/(b*x+a)^3+4/3*B^2*d
^3*ln((d*x+c)/(b*x+a))^2/b/(-a*d+b*c)^3/g^4+4*B*d^2*(d*x+c)*(A+B*ln(e*(d*x+
c)^2/(b*x+a)^2))/(-a*d+b*c)^3/g^4/(b*x+a)-2*b*B*d*(d*x+c)^2*(A+B*ln(e*(d*x+
c)^2/(b*x+a)^2))/(-a*d+b*c)^3/g^4/(b*x+a)^2+4/9*b^2*B*(d*x+c)^3*(A+B*ln(e*(
d*x+c)^2/(b*x+a)^2))/(-a*d+b*c)^3/g^4/(b*x+a)^3-4/3*B*d^3*ln((d*x+c)/(b*x+a
))*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))/b/(-a*d+b*c)^3/g^4-1/3*(A+B*ln(e*(d*x+c)
^2/(b*x+a)^2))^2/b/g^4/(b*x+a)^3
```

Rubi [A]
time = 0.16, antiderivative size = 407, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.147$, Rules used = {2552, 2356, 45, 2372, 2338}

$$\frac{4b^2B(c+dx)^3\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{9g^4(a+bx)^3(bc-ad)^3}-\frac{4Bd^3\log\left(\frac{c+dx}{a+bx}\right)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{3bg^4(bc-ad)^3}+\frac{4Bd^2(c+dx)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{g^4(a+bx)(bc-ad)^3}-\frac{2bBd(c+dx)^2\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{g^4(a+bx)^2(bc-ad)^3}-\frac{\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{3bg^4(a+bx)^3}-\frac{8b^2B^2(c+dx)^3}{27g^4(a+bx)^3(bc-ad)^3}+\frac{4B^2d^3\log^2\left(\frac{c+dx}{a+bx}\right)}{3bg^4(bc-ad)^3}-\frac{8B^2d^2(c+dx)}{g^4(a+bx)(bc-ad)^3}+\frac{2bB^2d(c+dx)^2}{g^4(a+bx)^2(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2/(a*g + b*g*x)^4,x]
```

```
[Out] (-8*B^2*d^2*(c + d*x))/((b*c - a*d)^3*g^4*(a + b*x)) + (2*b*B^2*d*(c + d*x)
^2)/((b*c - a*d)^3*g^4*(a + b*x)^2) - (8*b^2*B^2*(c + d*x)^3)/(27*(b*c - a*
d)^3*g^4*(a + b*x)^3) + (4*B^2*d^3*Log[(c + d*x)/(a + b*x)]^2)/(3*b*(b*c -
a*d)^3*g^4) + (4*B*d^2*(c + d*x)*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]))/
((b*c - a*d)^3*g^4*(a + b*x)) - (2*b*B*d*(c + d*x)^2*(A + B*Log[(e*(c + d*x
)^2)/(a + b*x)^2]))/((b*c - a*d)^3*g^4*(a + b*x)^2) + (4*b^2*B*(c + d*x)^3*
(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]))/(9*(b*c - a*d)^3*g^4*(a + b*x)^3)
 - (4*B*d^3*Log[(c + d*x)/(a + b*x)]*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2
]))/(3*b*(b*c - a*d)^3*g^4) - (A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2/(3
*b*g^4*(a + b*x)^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rule 2552

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(
m + 1)*(g/d)^m, Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a
 + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[n
 + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[d*f
- c*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Rubi steps

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(ag+bgx)^4}\,dx = -\frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{3bg^4(a+bx)^3} + \frac{(2B)\int \frac{2(bc-ad)\left(-A-B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{g^3(a+bx)^4(c+dx)}\,dx}{3bg}$$

$$= -\frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{3bg^4(a+bx)^3} + \frac{(4B(bc-ad))\int \frac{-A-B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(a+bx)^4(c+dx)}\,dx}{3bg^4}$$

$$= -\frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{3bg^4(a+bx)^3} + \frac{(4B(bc-ad))\int \left(\frac{b\left(-A-B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{(bc-ad)(a+bx)^4} - \frac{bd\left(\cdots\right.}{\cdots}\right.}{\cdots}$$

$$= -\frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{3bg^4(a+bx)^3} + \frac{(4B)\int \frac{-A-B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(a+bx)^4}\,dx}{3g^4} - \frac{(4Bd^3)\int \frac{-A-\cdots}{\cdots}}{3(bc-\cdots}$$

$$= \frac{4B\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{9bg^4(a+bx)^3} - \frac{2Bd\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{3b(bc-ad)g^4(a+bx)^2} + \frac{4Bd^2\left(A+B\cdots\right.}{3b(bc-ad\cdots}$$

$$= \frac{4B\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{9bg^4(a+bx)^3} - \frac{2Bd\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{3b(bc-ad)g^4(a+bx)^2} + \frac{4Bd^2\left(A+B\cdots\right.}{3b(bc-ad\cdots}$$

$$= \frac{4B\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{9bg^4(a+bx)^3} - \frac{2Bd\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{3b(bc-ad)g^4(a+bx)^2} + \frac{4Bd^2\left(A+B\cdots\right.}{3b(bc-ad\cdots}$$

$$= -\frac{8B^2}{27bg^4(a+bx)^3} + \frac{10B^2d}{9b(bc-ad)g^4(a+bx)^2} - \frac{44B^2d^2}{9b(bc-ad)^2g^4(a+bx)} - \cdots$$

$$= -\frac{8B^2}{27bg^4(a+bx)^3} + \frac{10B^2d}{9b(bc-ad)g^4(a+bx)^2} - \frac{44B^2d^2}{9b(bc-ad)^2g^4(a+bx)} - \cdots$$

$$= -\frac{8B^2}{27bg^4(a+bx)^3} + \frac{10B^2d}{9b(bc-ad)g^4(a+bx)^2} - \frac{44B^2d^2}{9b(bc-ad)^2g^4(a+bx)} - \cdots$$

$$= -\frac{8B^2}{27bg^4(a+bx)^3} + \frac{10B^2d}{9b(bc-ad)g^4(a+bx)^2} - \frac{44B^2d^2}{9b(bc-ad)^2g^4(a+bx)} - \cdots$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 0.43, size = 598, normalized size = 1.47

$$-\frac{9\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2+\frac{2B\left(36Bd^2(a+bx)^2(bc-ad+d(a+bx)\log(a+bx)-d(a+bx)\log(c+dx))-9Bd(a+bx)\left((bc-ad)^2+2d(-bc+ad)(a+bx)-2d^2(a+bx)^2\log(a+bx)+2d^2(a+bx)^2\log(c+dx)\right)+2B\left(2(bc-ad)^3-3d(bc-ad)^2(a+bx)+6d^2(bc-ad)(a+bx)^2+6d^3(a+bx)^3\log(a+bx)-6d^3(a+bx)^3\log(c+dx)\right)-6(bc-ad)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)+9d(bc-ad)^2(a+bx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)+18d^2(-bc+ad)(a+bx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)-18d^3(a+bx)^3\log(a+bx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)+18d^3(a+bx)^3\log(c+dx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)-18Bd^3(a+bx)^3\left(\log(a+bx)\left(\log(a+bx)-2\log\left(\frac{b(c+dx)}{bc-ad}\right)\right)-2\text{Li}_2\left(\frac{d(a+bx)}{-bc+ad}\right)\right)+18Bd^3(a+bx)^3\left(\left(2\log\left(\frac{d(a+bx)}{-bc+ad}\right)-\log(c+dx)\right)\log(c+dx)+2\text{Li}_2\left(\frac{b(c+dx)}{bc-ad}\right)\right)\right)}{(bc-ad)^3}}{27bg^4(a+bx)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2/(a*g + b*g*x)^4,x]
```

```
[Out] -1/27*(9*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2 + (2*B*(36*B*d^2*(a + b
*x)^2*(b*c - a*d + d*(a + b*x)*Log[a + b*x] - d*(a + b*x)*Log[c + d*x]) - 9
*B*d*(a + b*x)*((b*c - a*d)^2 + 2*d*(-(b*c) + a*d)*(a + b*x) - 2*d^2*(a + b
*x)^2*Log[a + b*x] + 2*d^2*(a + b*x)^2*Log[c + d*x]) + 2*B*(2*(b*c - a*d)^3
 - 3*d*(b*c - a*d)^2*(a + b*x) + 6*d^2*(b*c - a*d)*(a + b*x)^2 + 6*d^3*(a +
 b*x)^3*Log[a + b*x] - 6*d^3*(a + b*x)^3*Log[c + d*x]) - 6*(b*c - a*d)^3*(A
 + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]) + 9*d*(b*c - a*d)^2*(a + b*x)*(A + B
*Log[(e*(c + d*x)^2)/(a + b*x)^2]) + 18*d^2*(-(b*c) + a*d)*(a + b*x)^2*(A +
 B*Log[(e*(c + d*x)^2)/(a + b*x)^2]) - 18*d^3*(a + b*x)^3*Log[a + b*x]*(A +
 B*Log[(e*(c + d*x)^2)/(a + b*x)^2]) + 18*d^3*(a + b*x)^3*Log[c + d*x]*(A +
 B*Log[(e*(c + d*x)^2)/(a + b*x)^2]) - 18*B*d^3*(a + b*x)^3*(Log[a + b*x]*(
Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x
))/(-(b*c) + a*d)]) + 18*B*d^3*(a + b*x)^3*((2*Log[(d*(a + b*x))/(-(b*c) +
a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)
])))/(b*c - a*d)^3)/(b*g^4*(a + b*x)^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 914 vs. $2(397) = 794$.

time = 0.67, size = 915, normalized size = 2.25 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*(d*x+c)^2/(b*x+a)^2))^2/(b*g*x+a*g)^4,x,method=_RETURNVERBOSE
)
```

```
[Out] -1/b*(1/3/(b*x+a)^3/g^4*A^2+8/27/g^4*B^2/(b*x+a)^3-4/9/g^4*B^2/(b*x+a)^3*ln
(e*(a*d/(b*x+a)-b*c/(b*x+a)-d)^2/b^2)+1/3/g^4*B^2/(b*x+a)^3*ln(e*(a*d/(b*x+
a)-b*c/(b*x+a)-d)^2/b^2)^2+10/9/g^4*B^2*d/(a*d-b*c)/(b*x+a)^2+44/9/g^4*B^2*
d^2/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(b*x+a)+22/9/g^4*d^3*B^2/(a^3*d^3-3*a^2*b*c
*d^2+3*a*b^2*c^2*d-b^3*c^3)*ln(e*(a*d/(b*x+a)-b*c/(b*x+a)-d)^2/b^2)-1/3/g^4
*d^3*B^2/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)*ln(e*(a*d/(b*x+a)-b*
c/(b*x+a)-d)^2/b^2)^2-2/3/g^4*B^2*d/(a*d-b*c)/(b*x+a)^2*ln(e*(a*d/(b*x+a)-b
*c/(b*x+a)-d)^2/b^2)-4/3/g^4*B^2*d^2/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(b*x+a)*ln
(e*(a*d/(b*x+a)-b*c/(b*x+a)-d)^2/b^2)+2/3/g^4*A*B/(b*x+a)^3*ln(e*(a*d/(b*x+
a)-b*c/(b*x+a)-d)^2/b^2)-4/9/g^4*A*B*a^3*d^3/(a*d-b*c)^3/(b*x+a)^3+4/3/g^4*
A*B*a^2*d^2/(a*d-b*c)^3*b*c/(b*x+a)^3-4/3/g^4*A*B*a*d/(a*d-b*c)^3*b^2*c^2/(
b*x+a)^3-2/3/g^4*A*B*a^2*d^3/(a*d-b*c)^3/(b*x+a)^2+4/3/g^4*A*B*a*d^2/(a*d-b
*c)^3*b*c/(b*x+a)^2-4/3/g^4*A*B*a*d^3/(a*d-b*c)^3/(b*x+a)-4/3/g^4*A*B*a*d^4
/(a*d-b*c)^4*ln(a*d/(b*x+a)-b*c/(b*x+a)-d)+4/9/g^4*A*B*b^3*c^3/(a*d-b*c)^3/
(b*x+a)^3-2/3/g^4*A*B*b^2*c^2/(a*d-b*c)^3*d/(b*x+a)^2+4/3/g^4*A*B*b*c/(a*d-
b*c)^3*d^2/(b*x+a)+4/3/g^4*A*B*b*c*d^3/(a*d-b*c)^4*ln(a*d/(b*x+a)-b*c/(b*x+
a)-d))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1585 vs. $2(402) = 804$.

time = 0.46, size = 1585, normalized size = 3.89

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2/(b*g*x+a*g)^4,x, algorithm="ma
xima")
```

```
[Out] 2/27*(3*((6*b^2*d^2*x^2 + 2*b^2*c^2 - 7*a*b*c*d + 11*a^2*d^2 - 3*(b^2*c*d -
 5*a*b*d^2)*x)/((b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2)*g^4*x^3 + 3*(a*b^5*c^
2 - 2*a^2*b^4*c*d + a^3*b^3*d^2)*g^4*x^2 + 3*(a^2*b^4*c^2 - 2*a^3*b^3*c*d +
 a^4*b^2*d^2)*g^4*x + (a^3*b^3*c^2 - 2*a^4*b^2*c*d + a^5*b*d^2)*g^4) + 6*d^
3*log(b*x + a)/((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*g^4
) - 6*d^3*log(d*x + c)/((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*
d^3)*g^4))*log(d^2*x^2*e/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*x*e/(b^2*x^2 + 2
*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2)) - (4*b^3*c^3 - 27*a*b^2*c^
2*d + 108*a^2*b*c*d^2 - 85*a^3*d^3 + 66*(b^3*c*d^2 - a*b^2*d^3)*x^2 - 18*(b
^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(b*x + a)^2 - 18
*(b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(d*x + c)^2 -
 3*(5*b^3*c^2*d - 54*a*b^2*c*d^2 + 49*a^2*b*d^3)*x + 66*(b^3*d^3*x^3 + 3*a*
b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(b*x + a) - 6*(11*b^3*d^3*x^3 + 3
3*a*b^2*d^3*x^2 + 33*a^2*b*d^3*x + 11*a^3*d^3 - 6*(b^3*d^3*x^3 + 3*a*b^2*d^
3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(b*x + a))*log(d*x + c))/(a^3*b^4*c^3*g
^4 - 3*a^4*b^3*c^2*d*g^4 + 3*a^5*b^2*c*d^2*g^4 - a^6*b*d^3*g^4 + (b^7*c^3*g
^4 - 3*a*b^6*c^2*d*g^4 + 3*a^2*b^5*c*d^2*g^4 - a^3*b^4*d^3*g^4)*x^3 + 3*(a*
b^6*c^3*g^4 - 3*a^2*b^5*c^2*d*g^4 + 3*a^3*b^4*c*d^2*g^4 - a^4*b^3*d^3*g^4)*
x^2 + 3*(a^2*b^5*c^3*g^4 - 3*a^3*b^4*c^2*d*g^4 + 3*a^4*b^3*c*d^2*g^4 - a^5*
b^2*d^3*g^4)*x))*B^2 + 2/9*A*B*((6*b^2*d^2*x^2 + 2*b^2*c^2 - 7*a*b*c*d + 11
*a^2*d^2 - 3*(b^2*c*d - 5*a*b*d^2)*x)/((b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2
)*g^4*x^3 + 3*(a*b^5*c^2 - 2*a^2*b^4*c*d + a^3*b^3*d^2)*g^4*x^2 + 3*(a^2*b^
4*c^2 - 2*a^3*b^3*c*d + a^4*b^2*d^2)*g^4*x + (a^3*b^3*c^2 - 2*a^4*b^2*c*d +
 a^5*b*d^2)*g^4) - 3*log(d^2*x^2*e/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*x*e/(b
^2*x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2))/(b^4*g^4*x^3 + 3
*a*b^3*g^4*x^2 + 3*a^2*b^2*g^4*x + a^3*b*g^4) + 6*d^3*log(b*x + a)/((b^4*c^
3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*g^4) - 6*d^3*log(d*x + c)/
((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*g^4)) - 1/3*B^2*lo
g(d^2*x^2*e/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*x*e/(b^2*x^2 + 2*a*b*x + a^2)
 + c^2*e/(b^2*x^2 + 2*a*b*x + a^2))^2/(b^4*g^4*x^3 + 3*a*b^3*g^4*x^2 + 3*a^
2*b^2*g^4*x + a^3*b*g^4) - 1/3*A^2/(b^4*g^4*x^3 + 3*a*b^3*g^4*x^2 + 3*a^2*b
^2*g^4*x + a^3*b*g^4)
```

Fricas [**A**]
time = 0.37, size = 717, normalized size = 1.76

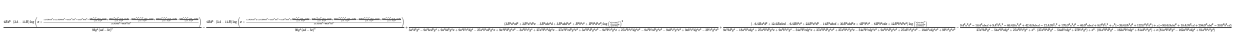
[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2/(b*g*x+a*g)^4,x, algorithm="fr
icas")
```

```
[Out] -1/27*((9*A^2 - 12*A*B + 8*B^2)*b^3*c^3 - 27*(A^2 - 2*A*B + 2*B^2)*a*b^2*c^
2*d + 27*(A^2 - 4*A*B + 8*B^2)*a^2*b*c*d^2 - (9*A^2 - 66*A*B + 170*B^2)*a^3
*d^3 - 12*((3*A*B - 11*B^2)*b^3*c*d^2 - (3*A*B - 11*B^2)*a*b^2*d^3)*x^2 + 9
*(B^2*b^3*d^3*x^3 + 3*B^2*a*b^2*d^3*x^2 + 3*B^2*a^2*b*d^3*x + B^2*b^3*c^3 -
 3*B^2*a*b^2*c^2*d + 3*B^2*a^2*b*c*d^2)*log((d^2*x^2 + 2*c*d*x + c^2)*e/(b^
2*x^2 + 2*a*b*x + a^2))^2 + 6*((3*A*B - 5*B^2)*b^3*c^2*d - 18*(A*B - 3*B^2)
*a*b^2*c*d^2 + (15*A*B - 49*B^2)*a^2*b*d^3)*x + 6*((3*A*B - 11*B^2)*b^3*d^3
*x^3 + (3*A*B - 2*B^2)*b^3*c^3 - 9*(A*B - B^2)*a*b^2*c^2*d + 9*(A*B - 2*B^2
)*a^2*b*c*d^2 - 3*(2*B^2*b^3*c*d^2 - 3*(A*B - 3*B^2)*a*b^2*d^3)*x^2 + 3*(B^
2*b^3*c^2*d - 6*B^2*a*b^2*c*d^2 + 3*(A*B - 2*B^2)*a^2*b*d^3)*x)*log((d^2*x^
2 + 2*c*d*x + c^2)*e/(b^2*x^2 + 2*a*b*x + a^2)))/((b^7*c^3 - 3*a*b^6*c^2*d
+ 3*a^2*b^5*c*d^2 - a^3*b^4*d^3)*g^4*x^3 + 3*(a*b^6*c^3 - 3*a^2*b^5*c^2*d +
 3*a^3*b^4*c*d^2 - a^4*b^3*d^3)*g^4*x^2 + 3*(a^2*b^5*c^3 - 3*a^3*b^4*c^2*d
+ 3*a^4*b^3*c*d^2 - a^5*b^2*d^3)*g^4*x + (a^3*b^4*c^3 - 3*a^4*b^3*c^2*d + 3
*a^5*b^2*c*d^2 - a^6*b*d^3)*g^4)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1561 vs. $2(382) = 764$.
time = 28.99, size = 1561, normalized size = 3.84



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(d*x+c)**2/(b*x+a)**2))**2/(b*g*x+a*g)**4,x)
```

```
[Out] 4*B*d**3*(3*A - 11*B)*log(x + (12*A*B*a*d**4 + 12*A*B*b*c*d**3 - 44*B**2*a*
d**4 - 44*B**2*b*c*d**3 - 4*B*a**4*d**7*(3*A - 11*B)/(a*d - b*c)**3 + 16*B*
a**3*b*c*d**6*(3*A - 11*B)/(a*d - b*c)**3 - 24*B*a**2*b**2*c**2*d**5*(3*A -
 11*B)/(a*d - b*c)**3 + 16*B*a*b**3*c**3*d**4*(3*A - 11*B)/(a*d - b*c)**3 -
 4*B*b**4*c**4*d**3*(3*A - 11*B)/(a*d - b*c)**3)/(24*A*B*b*d**4 - 88*B**2*b
*d**4))/(9*b*g**4*(a*d - b*c)**3) - 4*B*d**3*(3*A - 11*B)*log(x + (12*A*B*a
*d**4 + 12*A*B*b*c*d**3 - 44*B**2*a*d**4 - 44*B**2*b*c*d**3 + 4*B*a**4*d**7
*(3*A - 11*B)/(a*d - b*c)**3 - 16*B*a**3*b*c*d**6*(3*A - 11*B)/(a*d - b*c)*
*3 + 24*B*a**2*b**2*c**2*d**5*(3*A - 11*B)/(a*d - b*c)**3 - 16*B*a*b**3*c**
3*d**4*(3*A - 11*B)/(a*d - b*c)**3 + 4*B*b**4*c**4*d**3*(3*A - 11*B)/(a*d -
 b*c)**3)/(24*A*B*b*d**4 - 88*B**2*b*d**4))/(9*b*g**4*(a*d - b*c)**3) + (3*
B**2*a**2*c*d**2 + 3*B**2*a**2*d**3*x - 3*B**2*a*b*c**2*d + 3*B**2*a*b*d**3
*x**2 + B**2*b**2*c**3 + B**2*b**2*d**3*x**3)*log(e*(c + d*x)**2/(a + b*x)*
*2)**2/(3*a**6*d**3*g**4 - 9*a**5*b*c*d**2*g**4 + 9*a**5*b*d**3*g**4*x + 9*
a**4*b**2*c**2*d*g**4 - 27*a**4*b**2*c*d**2*g**4*x + 9*a**4*b**2*d**3*g**4*
x**2 - 3*a**3*b**3*c**3*g**4 + 27*a**3*b**3*c**2*d*g**4*x - 27*a**3*b**3*c*
```

```
d**2*g**4*x**2 + 3*a**3*b**3*d**3*g**4*x**3 - 9*a**2*b**4*c**3*g**4*x + 27*
a**2*b**4*c**2*d*g**4*x**2 - 9*a**2*b**4*c*d**2*g**4*x**3 - 9*a*b**5*c**3*g
**4*x**2 + 9*a*b**5*c**2*d*g**4*x**3 - 3*b**6*c**3*g**4*x**3) + (-6*A*B*a**
2*d**2 + 12*A*B*a*b*c*d - 6*A*B*b**2*c**2 + 22*B**2*a**2*d**2 - 14*B**2*a*b
*c*d + 30*B**2*a*b*d**2*x + 4*B**2*b**2*c**2 - 6*B**2*b**2*c*d*x + 12*B**2*
b**2*d**2*x**2)*log(e*(c + d*x)**2/(a + b*x)**2)/(9*a**5*b*d**2*g**4 - 18*a
**4*b**2*c*d*g**4 + 27*a**4*b**2*d**2*g**4*x + 9*a**3*b**3*c**2*g**4 - 54*a
**3*b**3*c*d*g**4*x + 27*a**3*b**3*d**2*g**4*x**2 + 27*a**2*b**4*c**2*g**4*
x - 54*a**2*b**4*c*d*g**4*x**2 + 9*a**2*b**4*d**2*g**4*x**3 + 27*a*b**5*c**
2*g**4*x**2 - 18*a*b**5*c*d*g**4*x**3 + 9*b**6*c**2*g**4*x**3) - (9*A**2*a*
*2*d**2 - 18*A**2*a*b*c*d + 9*A**2*b**2*c**2 - 66*A*B*a**2*d**2 + 42*A*B*a*
b*c*d - 12*A*B*b**2*c**2 + 170*B**2*a**2*d**2 - 46*B**2*a*b*c*d + 8*B**2*b*
*2*c**2 + x**2*(-36*A*B*b**2*d**2 + 132*B**2*b**2*d**2) + x*(-90*A*B*a*b*d*
*2 + 18*A*B*b**2*c*d + 294*B**2*a*b*d**2 - 30*B**2*b**2*c*d))/(27*a**5*b*d*
*2*g**4 - 54*a**4*b**2*c*d*g**4 + 27*a**3*b**3*c**2*g**4 + x**3*(27*a**2*b*
*4*d**2*g**4 - 54*a*b**5*c*d*g**4 + 27*b**6*c**2*g**4) + x**2*(81*a**3*b**3
*d**2*g**4 - 162*a**2*b**4*c*d*g**4 + 81*a*b**5*c**2*g**4) + x*(81*a**4*b**
2*d**2*g**4 - 162*a**3*b**3*c*d*g**4 + 81*a**2*b**4*c**2*g**4))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2/(b*g*x+a*g)^4,x, algorithm="gi
ac")
```

```
[Out] integrate((B*log((d*x + c)^2*e/(b*x + a)^2) + A)^2/(b*g*x + a*g)^4, x)
```

Mupad [**B**]
time = 9.08, size = 1069, normalized size = 2.63

$$\frac{\frac{9A^2a^2d^2-18A^2abcd+9A^2b^2c^2-66ABa^2d^2+42ABabcd-12ABb^2c^2+170B^2a^2d^2-46B^2abcd+8B^2b^2c^2}{3(ad-bc)}+\frac{2x\left(-5cB^2b^2d+49aB^2bd^2+3AcBb^2d-15AaBbd^2\right)}{ad-bc}+\frac{4dx^2\left(11B^2b^2d-3ABb^2d\right)}{ad-bc}}{x\left(27a^2b^3cg^4-27a^3b^2dg^4\right)-x^2\left(27a^2b^3dg^4-27ab^4cg^4\right)+x^3\left(9b^5cg^4-9ab^4dg^4\right)+9a^3b^2cg^4-9a^4bdg^4}-\ln\left(\frac{e\left(c+dx\right)^2}{\left(a+bx\right)^2}\right)^2\left(\frac{B^2}{3b^2g^4\left(3a^2x+\frac{a^3}{b}+b^2x^3+3abx^2\right)}-\frac{B^2d^3}{3bg^4\left(a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3\right)}\right)-\frac{\ln\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\left(\frac{2AB}{3b^2dg^4}-\frac{2B^2d^3\left(a\left(\frac{3a^2d^2-4abcd+b^2c^2}{3bd^3}+\frac{2a(ad-bc)}{3bd^2}\right)+\frac{2\left(3a^3d^3-6a^2bcd^2+4ab^2c^2d-b^3c^3\right)}{3bd^4}\right)}{3bg^4\left(a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3\right)}+\frac{2B^2d^3x^2\left(\frac{2\left(b^2c-abd\right)}{3d^2}-\frac{4b(ad-bc)}{3d^2}\right)}{3bg^4\left(a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3\right)}-\frac{2B^2d^3x\left(b\left(\frac{3a^2d^2-4abcd+b^2c^2}{3bd^3}+\frac{2a(ad-bc)}{3bd^2}\right)+\frac{2\left(3a^2d^2-4abcd+b^2c^2\right)}{3d^3}+\frac{4a(ad-bc)}{3d^2}\right)}{3bg^4\left(a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3\right)}\right)}{\frac{3a^2x}{d}+\frac{a^3}{bd}+\frac{b^2x^3}{d}+\frac{3abx^2}{d}}-\frac{B\,d^3\,\mathrm{atan}\left(\frac{B\,d^3\left(\frac{a^3bd^2g^4-a^2b^2cd^2g^4-ab^3c^2dg^4+b^4c^3g^4}{a^2bd^2g^4-2ab^2cdg^4+b^3c^2g^4}+2bdx\right)\left(3A-11B\right)\left(a^2bd^2g^4-2ab^2cdg^4+b^3c^2g^4\right)4\mathrm{i}}{bg^4\left(ad-bc\right)^3\left(44B^2d^3-12ABd^3\right)}\right)\left(3A-11B\right)8\mathrm{i}}{9bg^4\left(ad-bc\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(c + d*x)^2)/(a + b*x)^2))^2/(a*g + b*g*x)^4,x)
```

```
[Out] ((9*A^2*a^2*d^2 + 9*A^2*b^2*c^2 + 170*B^2*a^2*d^2 + 8*B^2*b^2*c^2 - 66*A*B*
a^2*d^2 - 12*A*B*b^2*c^2 - 18*A^2*a*b*c*d - 46*B^2*a*b*c*d + 42*A*B*a*b*c*d
)/(3*(a*d - b*c)) + (2*x*(49*B^2*a*b*d^2 - 5*B^2*b^2*c*d - 15*A*B*a*b*d^2 +
 3*A*B*b^2*c*d))/(a*d - b*c) + (4*d*x^2*(11*B^2*b^2*d - 3*A*B*b^2*d))/(a*d
- b*c))/(x*(27*a^2*b^3*c*g^4 - 27*a^3*b^2*d*g^4) - x^2*(27*a^2*b^3*d*g^4 -
27*a*b^4*c*g^4) + x^3*(9*b^5*c*g^4 - 9*a*b^4*d*g^4) + 9*a^3*b^2*c*g^4 - 9*a
^4*b*d*g^4) - log((e*(c + d*x)^2)/(a + b*x)^2)^2*(B^2/(3*b^2*g^4*(3*a^2*x +
 a^3/b + b^2*x^3 + 3*a*b*x^2)) - (B^2*d^3)/(3*b*g^4*(a^3*d^3 - b^3*c^3 + 3*
```

```
a*b^2*c^2*d - 3*a^2*b*c*d^2))) - (log((e*(c + d*x)^2)/(a + b*x)^2)*((2*A*B)
/(3*b^2*d*g^4) - (2*B^2*d^3*(a*((3*a^2*d^2 + b^2*c^2 - 4*a*b*c*d)/(3*b*d^3)
 + (2*a*(a*d - b*c))/(3*b*d^2)) + (2*(3*a^3*d^3 - b^3*c^3 + 4*a*b^2*c^2*d -
 6*a^2*b*c*d^2))/(3*b*d^4)))/(3*b*g^4*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d -
3*a^2*b*c*d^2)) + (2*B^2*d^3*x^2*((2*(b^2*c - a*b*d))/(3*d^2) - (4*b*(a*d -
 b*c))/(3*d^2)))/(3*b*g^4*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^
2)) - (2*B^2*d^3*x*(b*((3*a^2*d^2 + b^2*c^2 - 4*a*b*c*d)/(3*b*d^3) + (2*a*(
a*d - b*c))/(3*b*d^2)) + (2*(3*a^2*d^2 + b^2*c^2 - 4*a*b*c*d))/(3*d^3) + (4
*a*(a*d - b*c))/(3*d^2)))/(3*b*g^4*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a
^2*b*c*d^2))))/((3*a^2*x)/d + a^3/(b*d) + (b^2*x^3)/d + (3*a*b*x^2)/d) - (B
*d^3*atan((B*d^3*((b^4*c^3*g^4 + a^3*b*d^3*g^4 - a*b^3*c^2*d*g^4 - a^2*b^2*
c*d^2*g^4)/(b^3*c^2*g^4 + a^2*b*d^2*g^4 - 2*a*b^2*c*d*g^4) + 2*b*d*x)*(3*A
- 11*B)*(b^3*c^2*g^4 + a^2*b*d^2*g^4 - 2*a*b^2*c*d*g^4)*4i)/(b*g^4*(a*d - b
*c)^3*(44*B^2*d^3 - 12*A*B*d^3)))*(3*A - 11*B)*8i)/(9*b*g^4*(a*d - b*c)^3)
```

3.218 $\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(ag+bgx)^5}\,dx$

Optimal. Leaf size=501

$$\frac{8B^2d^3(c+dx)}{(bc-ad)^4g^5(a+bx)}-\frac{3bB^2d^2(c+dx)^2}{(bc-ad)^4g^5(a+bx)^2}+\frac{8b^2B^2d(c+dx)^3}{9(bc-ad)^4g^5(a+bx)^3}-\frac{b^3B^2(c+dx)^4}{8(bc-ad)^4g^5(a+bx)^4}-\frac{B^2d^4\log^2\left(\right.}{b(bc-ad}$$

```
[Out] 8*B^2*d^3*(d*x+c)/(-a*d+b*c)^4/g^5/(b*x+a)-3*b*B^2*d^2*(d*x+c)^2/(-a*d+b*c)
^4/g^5/(b*x+a)^2+8/9*b^2*B^2*d*(d*x+c)^3/(-a*d+b*c)^4/g^5/(b*x+a)^3-1/8*b^3
*B^2*(d*x+c)^4/(-a*d+b*c)^4/g^5/(b*x+a)^4-B^2*d^4*ln((d*x+c)/(b*x+a))^2/b/(
-a*d+b*c)^4/g^5-4*B*d^3*(d*x+c)*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))/(-a*d+b*c)^
4/g^5/(b*x+a)+3*b*B*d^2*(d*x+c)^2*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))/(-a*d+b*c
)^4/g^5/(b*x+a)^2-4/3*b^2*B*d*(d*x+c)^3*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))/(-a
*d+b*c)^4/g^5/(b*x+a)^3+1/4*b^3*B*(d*x+c)^4*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))
/(-a*d+b*c)^4/g^5/(b*x+a)^4+B*d^4*ln((d*x+c)/(b*x+a))*(A+B*ln(e*(d*x+c)^2/(
b*x+a)^2))/b/(-a*d+b*c)^4/g^5-1/4*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))^2/b/g^5/(
b*x+a)^4
```

Rubi [A]
time = 0.20, antiderivative size = 501, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.147$, Rules used = {2552, 2356, 45, 2372, 2338}

$$\frac{b^3B(c+dx)^4\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{4g^5(a+bx)^4(bc-ad)^4}-\frac{4b^2Bd(c+dx)^3\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{3g^5(a+bx)^3(bc-ad)^4}+\frac{Bd^4\log\left(\frac{c+dx}{a+bx}\right)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{bg^5(bc-ad)^4}-\frac{4Bd^3(c+dx)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{g^5(a+bx)(bc-ad)^4}+\frac{3bBd^2(c+dx)^2\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{g^5(a+bx)^2(bc-ad)^4}-\frac{\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{4bg^5(a+bx)^4}-\frac{b^3B^2(c+dx)^4}{8g^5(a+bx)^4(bc-ad)^4}+\frac{8b^2B^2d(c+dx)^3}{9g^5(a+bx)^3(bc-ad)^4}-\frac{B^2d^4\log^2\left(\frac{c+dx}{a+bx}\right)}{bg^5(bc-ad)^4}+\frac{8B^2d^3(c+dx)}{g^5(a+bx)(bc-ad)^4}-\frac{3bB^2d^2(c+dx)^2}{g^5(a+bx)^2(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2/(a*g + b*g*x)^5,x]
```

```
[Out] (8*B^2*d^3*(c + d*x))/((b*c - a*d)^4*g^5*(a + b*x)) - (3*b*B^2*d^2*(c + d*x
)^2)/((b*c - a*d)^4*g^5*(a + b*x)^2) + (8*b^2*B^2*d*(c + d*x)^3)/(9*(b*c -
a*d)^4*g^5*(a + b*x)^3) - (b^3*B^2*(c + d*x)^4)/(8*(b*c - a*d)^4*g^5*(a + b
*x)^4) - (B^2*d^4*Log[(c + d*x)/(a + b*x)]^2)/(b*(b*c - a*d)^4*g^5) - (4*B*
d^3*(c + d*x)*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]))/((b*c - a*d)^4*g^5*
(a + b*x)) + (3*b*B*d^2*(c + d*x)^2*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]
))/((b*c - a*d)^4*g^5*(a + b*x)^2) - (4*b^2*B*d*(c + d*x)^3*(A + B*Log[(e*(
c + d*x)^2)/(a + b*x)^2]))/(3*(b*c - a*d)^4*g^5*(a + b*x)^3) + (b^3*B*(c +
d*x)^4*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]))/(4*(b*c - a*d)^4*g^5*(a +
b*x)^4) + (B*d^4*Log[(c + d*x)/(a + b*x)]*(A + B*Log[(e*(c + d*x)^2)/(a + b
*x)^2]))/(b*(b*c - a*d)^4*g^5) - (A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2
/(4*b*g^5*(a + b*x)^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rule 2552

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(
m + 1)*(g/d)^m, Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a
 + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[n
 + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[d*f
- c*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Rubi steps

$$\int \frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(ag+bgx)^5}\,dx = -\frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{4bg^5(a+bx)^4} + \frac{B\int \frac{2(bc-ad)\left(-A-B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{g^4(a+bx)^5(c+dx)}\,dx}{2bg}$$

$$= -\frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{4bg^5(a+bx)^4} + \frac{(B(bc-ad))\int \frac{-A-B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(a+bx)^5(c+dx)}\,dx}{bg^5}$$

$$= -\frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{4bg^5(a+bx)^4} + \frac{(B(bc-ad))\int \left(\frac{b\left(-A-B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{(bc-ad)(a+bx)^5} - \frac{bd(-\cdots}{\cdots}\right.}{\cdots}$$

$$= -\frac{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{4bg^5(a+bx)^4} + \frac{B\int \frac{-A-B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(a+bx)^5}\,dx}{g^5} + \frac{(Bd^4)\int \frac{-A-B\log(\cdots}{a+\cdots}}{(bc-ad\cdots}$$

$$= \frac{B\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{4bg^5(a+bx)^4} - \frac{Bd\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{3b(bc-ad)g^5(a+bx)^3} + \frac{Bd^2\left(A+B\log\cdots\right.}{2b(bc-ad)^2g\cdots}$$

$$= \frac{B\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{4bg^5(a+bx)^4} - \frac{Bd\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{3b(bc-ad)g^5(a+bx)^3} + \frac{Bd^2\left(A+B\log\cdots\right.}{2b(bc-ad)^2g\cdots}$$

$$= \frac{B\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{4bg^5(a+bx)^4} - \frac{Bd\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{3b(bc-ad)g^5(a+bx)^3} + \frac{Bd^2\left(A+B\log\cdots\right.}{2b(bc-ad)^2g\cdots}$$

$$= -\frac{B^2}{8bg^5(a+bx)^4} + \frac{7B^2d}{18b(bc-ad)g^5(a+bx)^3} - \frac{13B^2d^2}{12b(bc-ad)^2g^5(a+bx)^2} + \cdots$$

$$= -\frac{B^2}{8bg^5(a+bx)^4} + \frac{7B^2d}{18b(bc-ad)g^5(a+bx)^3} - \frac{13B^2d^2}{12b(bc-ad)^2g^5(a+bx)^2} + \cdots$$

$$= -\frac{B^2}{8bg^5(a+bx)^4} + \frac{7B^2d}{18b(bc-ad)g^5(a+bx)^3} - \frac{13B^2d^2}{12b(bc-ad)^2g^5(a+bx)^2} + \cdots$$

$$= -\frac{B^2}{8bg^5(a+bx)^4} + \frac{7B^2d}{18b(bc-ad)g^5(a+bx)^3} - \frac{13B^2d^2}{12b(bc-ad)^2g^5(a+bx)^2} + \cdots$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

time = 0.58, size = 762, normalized size = 1.52

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2/(a*g + b*g*x)^5,x]
```

```
[Out] (-18*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2 + (B*(144*B*d^3*(a + b*x)^3
*(b*c - a*d + d*(a + b*x)*Log[a + b*x] - d*(a + b*x)*Log[c + d*x]) - 36*B*d
^2*(a + b*x)^2*((b*c - a*d)^2 + 2*d*(-(b*c) + a*d)*(a + b*x) - 2*d^2*(a + b
*x)^2*Log[a + b*x] + 2*d^2*(a + b*x)^2*Log[c + d*x]) + 8*B*d*(a + b*x)*(2*(
b*c - a*d)^3 - 3*d*(b*c - a*d)^2*(a + b*x) + 6*d^2*(b*c - a*d)*(a + b*x)^2
+ 6*d^3*(a + b*x)^3*Log[a + b*x] - 6*d^3*(a + b*x)^3*Log[c + d*x]) - 3*B*(3
*(b*c - a*d)^4 + 4*d*(-(b*c) + a*d)^3*(a + b*x) + 6*d^2*(b*c - a*d)^2*(a +
b*x)^2 + 12*d^3*(-(b*c) + a*d)*(a + b*x)^3 - 12*d^4*(a + b*x)^4*Log[a + b*x
] + 12*d^4*(a + b*x)^4*Log[c + d*x]) + 18*(b*c - a*d)^4*(A + B*Log[(e*(c +
d*x)^2)/(a + b*x)^2]) + 24*d*(-(b*c) + a*d)^3*(a + b*x)*(A + B*Log[(e*(c +
d*x)^2)/(a + b*x)^2]) + 36*d^2*(b*c - a*d)^2*(a + b*x)^2*(A + B*Log[(e*(c +
 d*x)^2)/(a + b*x)^2]) + 72*d^3*(-(b*c) + a*d)*(a + b*x)^3*(A + B*Log[(e*(c
 + d*x)^2)/(a + b*x)^2]) - 72*d^4*(a + b*x)^4*Log[a + b*x]*(A + B*Log[(e*(c
 + d*x)^2)/(a + b*x)^2]) + 72*d^4*(a + b*x)^4*Log[c + d*x]*(A + B*Log[(e*(c
 + d*x)^2)/(a + b*x)^2]) - 72*B*d^4*(a + b*x)^4*(Log[a + b*x]*(Log[a + b*x]
 - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) +
 a*d)]) + 72*B*d^4*(a + b*x)^4*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[
c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(b*c -
a*d)^4)/(72*b*g^5*(a + b*x)^4)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1244 vs. $2(491) = 982$.
time = 0.90, size = 1245, normalized size = 2.49 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*(d*x+c)^2/(b*x+a)^2))^2/(b*g*x+a*g)^5,x,method=_RETURNVERBOSE
)
```

```
[Out] -1/b*(-1/2/g^5*d^2*B^2/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(b*x+a)^2*ln(e*(a*d/(b*x
+a)-b*c/(b*x+a)-d)^2/b^2)-1/g^5*d^3*B^2/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*
d-b^3*c^3)/(b*x+a)*ln(e*(a*d/(b*x+a)-b*c/(b*x+a)-d)^2/b^2)-1/3/g^5*d*B^2/(a
*d-b*c)/(b*x+a)^3*ln(e*(a*d/(b*x+a)-b*c/(b*x+a)-d)^2/b^2)+7/18/g^5*d*B^2/(a
*d-b*c)/(b*x+a)^3+1/g^5*A*B*b*c/(a*d-b*c)^4*d^3/(b*x+a)+1/g^5*A*B*b*c*d^4/(
a*d-b*c)^5*ln(a*d/(b*x+a)-b*c/(b*x+a)-d)+1/3/g^5*A*B*b^3*c^3/(a*d-b*c)^4*d/
(b*x+a)^3-1/2/g^5*A*B*b^2*c^2/(a*d-b*c)^4*d^2/(b*x+a)^2-1/g^5*A*B*a*d^5/(a*
d-b*c)^5*ln(a*d/(b*x+a)-b*c/(b*x+a)-d)-1/2/g^5*A*B*a^2*d^4/(a*d-b*c)^4/(b*x
+a)^2-1/g^5*A*B*a*d^4/(a*d-b*c)^4/(b*x+a)+13/12/g^5*d^2*B^2/(a^2*d^2-2*a*b*
c*d+b^2*c^2)/(b*x+a)^2+25/6/g^5*d^3*B^2/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*
d-b^3*c^3)/(b*x+a)+25/12/g^5*d^4*B^2/(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d
^2-4*a*b^3*c^3*d+b^4*c^4)*ln(e*(a*d/(b*x+a)-b*c/(b*x+a)-d)^2/b^2)-1/4/g^5*d
^4*B^2/(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)*ln(e
*(a*d/(b*x+a)-b*c/(b*x+a)-d)^2/b^2)^2+1/2/g^5*A*B/(b*x+a)^4*ln(e*(a*d/(b*x+
a)-b*c/(b*x+a)-d)^2/b^2)+1/g^5*A*B*a^3*d^3/(a*d-b*c)^4*b*c/(b*x+a)^4+1/g^5*
```

```
A*B*a*d^3/(a*d-b*c)^4*b*c/(b*x+a)^2+1/g^5*A*B*a*d/(a*d-b*c)^4*b^3*c^3/(b*x+
a)^4+1/g^5*A*B*a^2*d^3/(a*d-b*c)^4*b*c/(b*x+a)^3-1/g^5*A*B*a*d^2/(a*d-b*c)^
4*b^2*c^2/(b*x+a)^3-3/2/g^5*A*B*a^2*d^2/(a*d-b*c)^4*b^2*c^2/(b*x+a)^4-1/4/g
^5*B^2/(b*x+a)^4*ln(e*(a*d/(b*x+a)-b*c/(b*x+a)-d)^2/b^2)+1/4/g^5*B^2/(b*x+a
)^4*ln(e*(a*d/(b*x+a)-b*c/(b*x+a)-d)^2/b^2)^2+1/4/(b*x+a)^4/g^5*A^2+1/8/g^5
*B^2/(b*x+a)^4-1/4/g^5*A*B*b^4*c^4/(a*d-b*c)^4/(b*x+a)^4-1/4/g^5*A*B*a^4*d^
4/(a*d-b*c)^4/(b*x+a)^4-1/3/g^5*A*B*a^3*d^4/(a*d-b*c)^4/(b*x+a)^3)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 2287 vs. $2(497) = 994$.
time = 0.56, size = 2287, normalized size = 4.56

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2/(b*g*x+a*g)^5,x, algorithm="ma
xima")
```

```
[Out] -1/72*(6*((12*b^3*d^3*x^3 - 3*b^3*c^3 + 13*a*b^2*c^2*d - 23*a^2*b*c*d^2 + 2
5*a^3*d^3 - 6*(b^3*c*d^2 - 7*a*b^2*d^3)*x^2 + 4*(b^3*c^2*d - 5*a*b^2*c*d^2
+ 13*a^2*b*d^3)*x)/((b^8*c^3 - 3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 - a^3*b^5*d^
3)*g^5*x^4 + 4*(a*b^7*c^3 - 3*a^2*b^6*c^2*d + 3*a^3*b^5*c*d^2 - a^4*b^4*d^3
)*g^5*x^3 + 6*(a^2*b^6*c^3 - 3*a^3*b^5*c^2*d + 3*a^4*b^4*c*d^2 - a^5*b^3*d^
3)*g^5*x^2 + 4*(a^3*b^5*c^3 - 3*a^4*b^4*c^2*d + 3*a^5*b^3*c*d^2 - a^6*b^2*d
^3)*g^5*x + (a^4*b^4*c^3 - 3*a^5*b^3*c^2*d + 3*a^6*b^2*c*d^2 - a^7*b*d^3)*g
^5) + 12*d^4*log(b*x + a)/((b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4
*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5) - 12*d^4*log(d*x + c)/((b^5*c^4 - 4*a*b^4*
c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5))*log(d^2*x^2*
e/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*x*e/(b^2*x^2 + 2*a*b*x + a^2) + c^2*e/(
b^2*x^2 + 2*a*b*x + a^2)) + (9*b^4*c^4 - 64*a*b^3*c^3*d + 216*a^2*b^2*c^2*d
^2 - 576*a^3*b*c*d^3 + 415*a^4*d^4 - 300*(b^4*c*d^3 - a*b^3*d^4)*x^3 + 6*(1
3*b^4*c^2*d^2 - 176*a*b^3*c*d^3 + 163*a^2*b^2*d^4)*x^2 + 72*(b^4*d^4*x^4 +
4*a*b^3*d^4*x^3 + 6*a^2*b^2*d^4*x^2 + 4*a^3*b*d^4*x + a^4*d^4)*log(b*x + a)
^2 + 72*(b^4*d^4*x^4 + 4*a*b^3*d^4*x^3 + 6*a^2*b^2*d^4*x^2 + 4*a^3*b*d^4*x
+ a^4*d^4)*log(d*x + c)^2 - 4*(7*b^4*c^3*d - 60*a*b^3*c^2*d^2 + 324*a^2*b^2
*c*d^3 - 271*a^3*b*d^4)*x - 300*(b^4*d^4*x^4 + 4*a*b^3*d^4*x^3 + 6*a^2*b^2*
d^4*x^2 + 4*a^3*b*d^4*x + a^4*d^4)*log(b*x + a) + 12*(25*b^4*d^4*x^4 + 100*
a*b^3*d^4*x^3 + 150*a^2*b^2*d^4*x^2 + 100*a^3*b*d^4*x + 25*a^4*d^4 - 12*(b^
4*d^4*x^4 + 4*a*b^3*d^4*x^3 + 6*a^2*b^2*d^4*x^2 + 4*a^3*b*d^4*x + a^4*d^4)*
log(b*x + a))*log(d*x + c))/(a^4*b^5*c^4*g^5 - 4*a^5*b^4*c^3*d*g^5 + 6*a^6*
b^3*c^2*d^2*g^5 - 4*a^7*b^2*c*d^3*g^5 + a^8*b*d^4*g^5 + (b^9*c^4*g^5 - 4*a*
b^8*c^3*d*g^5 + 6*a^2*b^7*c^2*d^2*g^5 - 4*a^3*b^6*c*d^3*g^5 + a^4*b^5*d^4*g
^5)*x^4 + 4*(a*b^8*c^4*g^5 - 4*a^2*b^7*c^3*d*g^5 + 6*a^3*b^6*c^2*d^2*g^5 -
4*a^4*b^5*c*d^3*g^5 + a^5*b^4*d^4*g^5)*x^3 + 6*(a^2*b^7*c^4*g^5 - 4*a^3*b^6
*c^3*d*g^5 + 6*a^4*b^5*c^2*d^2*g^5 - 4*a^5*b^4*c*d^3*g^5 + a^6*b^3*d^4*g^5)
```

```
*x^2 + 4*(a^3*b^6*c^4*g^5 - 4*a^4*b^5*c^3*d*g^5 + 6*a^5*b^4*c^2*d^2*g^5 - 4
*a^6*b^3*c*d^3*g^5 + a^7*b^2*d^4*g^5)*x))*B^2 - 1/12*A*B*((12*b^3*d^3*x^3 -
 3*b^3*c^3 + 13*a*b^2*c^2*d - 23*a^2*b*c*d^2 + 25*a^3*d^3 - 6*(b^3*c*d^2 -
7*a*b^2*d^3)*x^2 + 4*(b^3*c^2*d - 5*a*b^2*c*d^2 + 13*a^2*b*d^3)*x)/((b^8*c^
3 - 3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 - a^3*b^5*d^3)*g^5*x^4 + 4*(a*b^7*c^3 -
 3*a^2*b^6*c^2*d + 3*a^3*b^5*c*d^2 - a^4*b^4*d^3)*g^5*x^3 + 6*(a^2*b^6*c^3
- 3*a^3*b^5*c^2*d + 3*a^4*b^4*c*d^2 - a^5*b^3*d^3)*g^5*x^2 + 4*(a^3*b^5*c^3
 - 3*a^4*b^4*c^2*d + 3*a^5*b^3*c*d^2 - a^6*b^2*d^3)*g^5*x + (a^4*b^4*c^3 -
3*a^5*b^3*c^2*d + 3*a^6*b^2*c*d^2 - a^7*b*d^3)*g^5) + 6*log(d^2*x^2*e/(b^2*
x^2 + 2*a*b*x + a^2) + 2*c*d*x*e/(b^2*x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2
 + 2*a*b*x + a^2))/(b^5*g^5*x^4 + 4*a*b^4*g^5*x^3 + 6*a^2*b^3*g^5*x^2 + 4*a
^3*b^2*g^5*x + a^4*b*g^5) + 12*d^4*log(b*x + a)/((b^5*c^4 - 4*a*b^4*c^3*d +
 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5) - 12*d^4*log(d*x + c
)/((b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d
^4)*g^5)) - 1/4*B^2*log(d^2*x^2*e/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*x*e/(b^
2*x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2))^2/(b^5*g^5*x^4 +
4*a*b^4*g^5*x^3 + 6*a^2*b^3*g^5*x^2 + 4*a^3*b^2*g^5*x + a^4*b*g^5) - 1/4*A^
2/(b^5*g^5*x^4 + 4*a*b^4*g^5*x^3 + 6*a^2*b^3*g^5*x^2 + 4*a^3*b^2*g^5*x + a^
4*b*g^5)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1084 vs. $2(497) = 994$.
time = 0.36, size = 1084, normalized size = 2.16

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2/(b*g*x+a*g)^5,x, algorithm="fr
icas")
```

```
[Out] -1/72*(9*(2*A^2 - 2*A*B + B^2)*b^4*c^4 - 8*(9*A^2 - 12*A*B + 8*B^2)*a*b^3*c
^3*d + 108*(A^2 - 2*A*B + 2*B^2)*a^2*b^2*c^2*d^2 - 72*(A^2 - 4*A*B + 8*B^2)
*a^3*b*c*d^3 + (18*A^2 - 150*A*B + 415*B^2)*a^4*d^4 + 12*((6*A*B - 25*B^2)*
b^4*c*d^3 - (6*A*B - 25*B^2)*a*b^3*d^4)*x^3 - 6*((6*A*B - 13*B^2)*b^4*c^2*d
^2 - 16*(3*A*B - 11*B^2)*a*b^3*c*d^3 + (42*A*B - 163*B^2)*a^2*b^2*d^4)*x^2
- 18*(B^2*b^4*d^4*x^4 + 4*B^2*a*b^3*d^4*x^3 + 6*B^2*a^2*b^2*d^4*x^2 + 4*B^2
*a^3*b*d^4*x - B^2*b^4*c^4 + 4*B^2*a*b^3*c^3*d - 6*B^2*a^2*b^2*c^2*d^2 + 4*
B^2*a^3*b*c*d^3)*log((d^2*x^2 + 2*c*d*x + c^2)*e/(b^2*x^2 + 2*a*b*x + a^2))
^2 + 4*((6*A*B - 7*B^2)*b^4*c^3*d - 12*(3*A*B - 5*B^2)*a*b^3*c^2*d^2 + 108*
(A*B - 3*B^2)*a^2*b^2*c*d^3 - (78*A*B - 271*B^2)*a^3*b*d^4)*x - 6*((6*A*B -
 25*B^2)*b^4*d^4*x^4 - 3*(2*A*B - B^2)*b^4*c^4 + 8*(3*A*B - 2*B^2)*a*b^3*c^
3*d - 36*(A*B - B^2)*a^2*b^2*c^2*d^2 + 24*(A*B - 2*B^2)*a^3*b*c*d^3 - 4*(3*
B^2*b^4*c*d^3 - 2*(3*A*B - 11*B^2)*a*b^3*d^4)*x^3 + 6*(B^2*b^4*c^2*d^2 - 8*
B^2*a*b^3*c*d^3 + 6*(A*B - 3*B^2)*a^2*b^2*d^4)*x^2 - 4*(B^2*b^4*c^3*d - 6*B
^2*a*b^3*c^2*d^2 + 18*B^2*a^2*b^2*c*d^3 - 6*(A*B - 2*B^2)*a^3*b*d^4)*x)*log
```

```
((d^2*x^2 + 2*c*d*x + c^2)*e/(b^2*x^2 + 2*a*b*x + a^2)))/((b^9*c^4 - 4*a*b^
8*c^3*d + 6*a^2*b^7*c^2*d^2 - 4*a^3*b^6*c*d^3 + a^4*b^5*d^4)*g^5*x^4 + 4*(a
*b^8*c^4 - 4*a^2*b^7*c^3*d + 6*a^3*b^6*c^2*d^2 - 4*a^4*b^5*c*d^3 + a^5*b^4*
d^4)*g^5*x^3 + 6*(a^2*b^7*c^4 - 4*a^3*b^6*c^3*d + 6*a^4*b^5*c^2*d^2 - 4*a^5
*b^4*c*d^3 + a^6*b^3*d^4)*g^5*x^2 + 4*(a^3*b^6*c^4 - 4*a^4*b^5*c^3*d + 6*a^
5*b^4*c^2*d^2 - 4*a^6*b^3*c*d^3 + a^7*b^2*d^4)*g^5*x + (a^4*b^5*c^4 - 4*a^5
*b^4*c^3*d + 6*a^6*b^3*c^2*d^2 - 4*a^7*b^2*c*d^3 + a^8*b*d^4)*g^5)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(d*x+c)**2/(b*x+a)**2))**2/(b*g*x+a*g)**5,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 6.03, size = 868, normalized size = 1.73

$$\frac{1}{4}\left(\frac{B^2d^4}{b^5c^4g^5 - 4ab^4c^3dg^5 + 6a^2b^3c^2d^2g^5 - 4a^3b^2cd^3g^5 + a^4bd^4g^5} - \frac{B^2}{(bgx+ag)^4bg}\right)\log\left(\frac{\frac{b^2c^2g^2}{(bgx+ag)^2} - \frac{2abcdg^2}{(bgx+ag)^2} + \frac{a^2d^2g^2}{(bgx+ag)^2} + \frac{2bcdg}{bgx+ag} - \frac{2ad^2g}{bgx+ag} + d^2}{b^2}\right)^2 - \frac{1}{12}\left(\frac{12B^2d^3}{(b^3c^3g^3 - 3ab^2c^2dg^3 + 3a^2bcd^2g^3 - a^3d^3g^3)(bgx+ag)bg} - \frac{6B^2d^2}{(b^2c^2g - 2abcdg + a^2d^2g)(bgx+ag)^2bg^2} + \frac{4B^2d}{(bgx+ag)^3(bc-ad)bg^2} + \frac{3(2ABb^3g^3 + B^2b^3g^3)}{(bgx+ag)^4b^4g^4}\right)\log\left(\frac{\frac{b^2c^2g^2}{(bgx+ag)^2} - \frac{2abcdg^2}{(bgx+ag)^2} + \frac{a^2d^2g^2}{(bgx+ag)^2} + \frac{2bcdg}{bgx+ag} - \frac{2ad^2g}{bgx+ag} + d^2}{b^2}\right) + \frac{(6ABd^4 - 19B^2d^4)\log\left(-\frac{bcg}{bgx+ag} + \frac{adg}{bgx+ag} - d\right)}{6(b^5c^4g^5 - 4ab^4c^3dg^5 + 6a^2b^3c^2d^2g^5 - 4a^3b^2cd^3g^5 + a^4bd^4g^5)} - \frac{6ABd^3 - 19B^2d^3}{6(b^3c^3g^3 - 3ab^2c^2dg^3 + 3a^2bcd^2g^3 - a^3d^3g^3)(bgx+ag)bg} + \frac{6ABbd^2 - 7B^2bd^2}{12(b^2c^2g - 2abcdg + a^2d^2g)(bgx+ag)^2b^2g^2} - \frac{6ABb^2dg - B^2b^2dg}{18(bgx+ag)^3(bc-ad)b^3g^3} - \frac{2A^2b^3g^3 + 2ABb^3g^3 + B^2b^3g^3}{8(bgx+ag)^4b^4g^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2/(b*g*x+a*g)^5,x, algorithm="gi
ac")
```

```
[Out] 1/4*(B^2*d^4/(b^5*c^4*g^5 - 4*a*b^4*c^3*d*g^5 + 6*a^2*b^3*c^2*d^2*g^5 - 4*a
^3*b^2*c*d^3*g^5 + a^4*b*d^4*g^5) - B^2/((b*g*x + a*g)^4*b*g))*log((b^2*c^2
*g^2/(b*g*x + a*g)^2 - 2*a*b*c*d*g^2/(b*g*x + a*g)^2 + a^2*d^2*g^2/(b*g*x +
 a*g)^2 + 2*b*c*d*g/(b*g*x + a*g) - 2*a*d^2*g/(b*g*x + a*g) + d^2)/b^2)^2 -
 1/12*(12*B^2*d^3/((b^3*c^3*g^3 - 3*a*b^2*c^2*d*g^3 + 3*a^2*b*c*d^2*g^3 - a
^3*d^3*g^3)*(b*g*x + a*g)*b*g) - 6*B^2*d^2/((b^2*c^2*g - 2*a*b*c*d*g + a^2*
d^2*g)*(b*g*x + a*g)^2*b*g^2) + 4*B^2*d/((b*g*x + a*g)^3*(b*c - a*d)*b*g^2)
 + 3*(2*A*B*b^3*g^3 + B^2*b^3*g^3)/((b*g*x + a*g)^4*b^4*g^4))*log((b^2*c^2*
g^2/(b*g*x + a*g)^2 - 2*a*b*c*d*g^2/(b*g*x + a*g)^2 + a^2*d^2*g^2/(b*g*x +
a*g)^2 + 2*b*c*d*g/(b*g*x + a*g) - 2*a*d^2*g/(b*g*x + a*g) + d^2)/b^2) + 1/
6*(6*A*B*d^4 - 19*B^2*d^4)*log(-b*c*g/(b*g*x + a*g) + a*d*g/(b*g*x + a*g) -
 d)/(b^5*c^4*g^5 - 4*a*b^4*c^3*d*g^5 + 6*a^2*b^3*c^2*d^2*g^5 - 4*a^3*b^2*c*
d^3*g^5 + a^4*b*d^4*g^5) - 1/6*(6*A*B*d^3 - 19*B^2*d^3)/((b^3*c^3*g^3 - 3*a
*b^2*c^2*d*g^3 + 3*a^2*b*c*d^2*g^3 - a^3*d^3*g^3)*(b*g*x + a*g)*b*g) + 1/12
*(6*A*B*b*d^2 - 7*B^2*b*d^2)/((b^2*c^2*g - 2*a*b*c*d*g + a^2*d^2*g)*(b*g*x
+ a*g)^2*b^2*g^2) - 1/18*(6*A*B*b^2*d*g - B^2*b^2*d*g)/((b*g*x + a*g)^3*(b*
c - a*d)*b^3*g^3) - 1/8*(2*A^2*b^3*g^3 + 2*A*B*b^3*g^3 + B^2*b^3*g^3)/((b*g
*x + a*g)^4*b^4*g^4)
```

Mupad [B]
time = 12.11, size = 1882, normalized size = 3.76

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(c + d*x)^2)/(a + b*x)^2))^2/(a*g + b*g*x)^5,x)
```

```
[Out] (log((e*(c + d*x)^2)/(a + b*x)^2)*((B^2*d^4*(a*(a*((4*a^2*d^2 + b^2*c^2 - 5
*a*b*c*d)/(6*b*d^3) + (a*(a*d - b*c))/(2*b*d^2)) + (6*a^3*d^3 - b^3*c^3 + 5
*a*b^2*c^2*d - 10*a^2*b*c*d^2)/(6*b*d^4)) + (4*a^4*d^4 + b^4*c^4 + 10*a^2*b
^2*c^2*d^2 - 5*a*b^3*c^3*d - 10*a^3*b*c*d^3)/(2*b*d^5)))/(2*b*g^5*(a^4*d^4
+ b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)) - (A*B)/(2*
b^2*d*g^5) + (B^2*d^4*x^2*(b*(b*((4*a^2*d^2 + b^2*c^2 - 5*a*b*c*d)/(6*b*d^3
) + (a*(a*d - b*c))/(2*b*d^2)) + (4*a^2*d^2 + b^2*c^2 - 5*a*b*c*d)/(3*d^3)
+ (a*(a*d - b*c))/d^2) - a*((b^2*c - a*b*d)/(2*d^2) - (b*(a*d - b*c))/d^2)
+ (b^3*c^2 + 4*a^2*b*d^2 - 5*a*b^2*c*d)/(2*d^3)))/(2*b*g^5*(a^4*d^4 + b^4*c
^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)) - (B^2*d^4*x^3*(b*
((b^2*c - a*b*d)/(2*d^2) - (b*(a*d - b*c))/d^2) + (b^3*c - a*b^2*d)/(2*d^2)
))/(2*b*g^5*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*
b*c*d^3)) + (B^2*d^4*x*(b*(a*((4*a^2*d^2 + b^2*c^2 - 5*a*b*c*d)/(6*b*d^3) +
 (a*(a*d - b*c))/(2*b*d^2)) + (6*a^3*d^3 - b^3*c^3 + 5*a*b^2*c^2*d - 10*a^2
*b*c*d^2)/(6*b*d^4)) + a*(b*((4*a^2*d^2 + b^2*c^2 - 5*a*b*c*d)/(6*b*d^3) +
(a*(a*d - b*c))/(2*b*d^2)) + (4*a^2*d^2 + b^2*c^2 - 5*a*b*c*d)/(3*d^3) + (a
*(a*d - b*c))/d^2) + (6*a^3*d^3 - b^3*c^3 + 5*a*b^2*c^2*d - 10*a^2*b*c*d^2)
/(2*d^4)))/(2*b*g^5*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d
- 4*a^3*b*c*d^3))))/((4*a^3*x)/d + a^4/(b*d) + (b^3*x^4)/d + (6*a^2*b*x^2)/
d + (4*a*b^2*x^3)/d) - log((e*(c + d*x)^2)/(a + b*x)^2)^2*(B^2/(4*b^2*g^5*(
4*a^3*x + a^4/b + b^3*x^4 + 6*a^2*b*x^2 + 4*a*b^2*x^3)) - (B^2*d^4)/(4*b*g^
5*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)))
 - ((18*A^2*a^3*d^3 - 18*A^2*b^3*c^3 + 415*B^2*a^3*d^3 - 9*B^2*b^3*c^3 - 15
0*A*B*a^3*d^3 + 18*A*B*b^3*c^3 + 54*A^2*a*b^2*c^2*d - 54*A^2*a^2*b*c*d^2 +
55*B^2*a*b^2*c^2*d - 161*B^2*a^2*b*c*d^2 - 78*A*B*a*b^2*c^2*d + 138*A*B*a^2
*b*c*d^2)/(12*(a*d - b*c)) + (x^2*(163*B^2*a*b^2*d^3 - 13*B^2*b^3*c*d^2 - 4
2*A*B*a*b^2*d^3 + 6*A*B*b^3*c*d^2))/(2*(a*d - b*c)) + (x*(271*B^2*a^2*b*d^3
 + 7*B^2*b^3*c^2*d - 53*B^2*a*b^2*c*d^2 - 78*A*B*a^2*b*d^3 - 6*A*B*b^3*c^2*
d + 30*A*B*a*b^2*c*d^2))/(3*(a*d - b*c)) + (d*x^3*(25*B^2*b^3*d^2 - 6*A*B*b
^3*d^2))/(a*d - b*c))/(x*(24*a^3*b^4*c^2*g^5 + 24*a^5*b^2*d^2*g^5 - 48*a^4*
b^3*c*d*g^5) + x^3*(24*a*b^6*c^2*g^5 + 24*a^3*b^4*d^2*g^5 - 48*a^2*b^5*c*d*
g^5) + x^4*(6*b^7*c^2*g^5 + 6*a^2*b^5*d^2*g^5 - 12*a*b^6*c*d*g^5) + x^2*(36
*a^2*b^5*c^2*g^5 + 36*a^4*b^3*d^2*g^5 - 72*a^3*b^4*c*d*g^5) + 6*a^6*b*d^2*g
^5 + 6*a^4*b^3*c^2*g^5 - 12*a^5*b^2*c*d*g^5) + (B*d^4*atan((B*d^4*(6*A - 25
*B)*(6*b^5*c^4*g^5 - 6*a^4*b*d^4*g^5 - 12*a*b^4*c^3*d*g^5 + 12*a^3*b^2*c*d^
3*g^5)*1i)/(6*b*g^5*(a*d - b*c)^4*(25*B^2*d^4 - 6*A*B*d^4)) + (B*d^5*x*(6*A
 - 25*B)*(b^4*c^3*g^5 - a^3*b*d^3*g^5 - 3*a*b^3*c^2*d*g^5 + 3*a^2*b^2*c*d^2
```

```
*g^5)*2i)/(g^5*(a*d - b*c)^4*(25*B^2*d^4 - 6*A*B*d^4)))*(6*A - 25*B)*1i)/(3
*b*g^5*(a*d - b*c)^4)
```

3.219 $\int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}\,dx$

Optimal. Leaf size=37

$$\text{Int}\left(\frac{(ag+bgx)^2}{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)},x\right)$$

```
[Out] Unintegrable((b*g*x+a*g)^2/(A+B*ln(e*(d*x+c)^2/(b*x+a)^2)),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a*g + b*g*x)^2/(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]),x]
[Out] Defer[Int][(a*g + b*g*x)^2/(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]), x]
```

Rubi steps

$$\int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}\,dx = \int \left(\frac{a^2g^2}{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}+\frac{2abg^2x}{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}+\frac{b^2g^2x^2}{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}\right)$$
$$= (a^2g^2)\int \frac{1}{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}\,dx+(2abg^2)\int \frac{x}{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}\,dx+(b$$

Mathematica [A]
time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{(ag+bgx)^2}{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a*g + b*g*x)^2/(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]),x]
```

[Out] Integrate[(a*g + b*g*x)^2/(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]), x]

Maple [A]
time = 0.65, size = 0, normalized size = 0.00

$$\int \frac{(bgx+ag)^2}{A+B\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*g*x+a*g)^2/(A+B*ln(e*(d*x+c)^2/(b*x+a)^2)),x)

[Out] int((b*g*x+a*g)^2/(A+B*ln(e*(d*x+c)^2/(b*x+a)^2)),x)

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*g*x+a*g)^2/(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="maxima")

[Out] integrate((b*g*x + a*g)^2/(B*log((d*x + c)^2*e/(b*x + a)^2) + A), x)

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*g*x+a*g)^2/(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="fricas")

[Out] integral((b^2*g^2*x^2 + 2*a*b*g^2*x + a^2*g^2)/(B*log((d^2*x^2 + 2*c*d*x + c^2)*e/(b^2*x^2 + 2*a*b*x + a^2)) + A), x)

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$g^2\left(\int \frac{a^2}{A+B\log\left(\frac{c^2e}{a^2+2abx+b^2x^2}+\frac{2cdex}{a^2+2abx+b^2x^2}+\frac{d^2ex^2}{a^2+2abx+b^2x^2}\right)}\,dx+\int \frac{b^2x^2}{A+B\log\left(\frac{c^2e}{a^2+2abx+b^2x^2}+\frac{2cdex}{a^2+2abx+b^2x^2}+\frac{d^2ex^2}{a^2+2abx+b^2x^2}\right)}\,dx+\int \frac{2abx}{A+B\log\left(\frac{c^2e}{a^2+2abx+b^2x^2}+\frac{2cdex}{a^2+2abx+b^2x^2}+\frac{d^2ex^2}{a^2+2abx+b^2x^2}\right)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*g*x+a*g)**2/(A+B*ln(e*(d*x+c)**2/(b*x+a)**2)),x)

[Out] g**2*(Integral(a**2/(A + B*log(c**2*e/(a**2 + 2*a*b*x + b**2*x**2) + 2*c*d*e*x/(a**2 + 2*a*b*x + b**2*x**2) + d**2*e*x**2/(a**2 + 2*a*b*x + b**2*x**2)

```
)), x) + Integral(b**2*x**2/(A + B*log(c**2*e/(a**2 + 2*a*b*x + b**2*x**2)
+ 2*c*d*e*x/(a**2 + 2*a*b*x + b**2*x**2) + d**2*e*x**2/(a**2 + 2*a*b*x + b*
*2*x**2))), x) + Integral(2*a*b*x/(A + B*log(c**2*e/(a**2 + 2*a*b*x + b**2*
x**2) + 2*c*d*e*x/(a**2 + 2*a*b*x + b**2*x**2) + d**2*e*x**2/(a**2 + 2*a*b*
x + b**2*x**2))), x))
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^2/(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="giac
")
```

```
[Out] integrate((b*g*x + a*g)^2/(B*log((d*x + c)^2*e/(b*x + a)^2) + A), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{{\left(a\,g+b\,g\,x\right)}^2}{A+B\,\ln\left(\frac{e\,{\left(c+d\,x\right)}^2}{{\left(a+b\,x\right)}^2}\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*g + b*g*x)^2/(A + B*log((e*(c + d*x)^2)/(a + b*x)^2)),x)
```

```
[Out] int((a*g + b*g*x)^2/(A + B*log((e*(c + d*x)^2)/(a + b*x)^2)), x)
```

3.220 $\int \frac{ag+bgx}{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}\,dx$

Optimal. Leaf size=35

$$\text{Int}\left(\frac{ag+bgx}{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)},x\right)$$

```
[Out] Unintegrable((b*g*x+a*g)/(A+B*ln(e*(d*x+c)^2/(b*x+a)^2)),x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{ag+bgx}{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a*g + b*g*x)/(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]),x]
[Out] Defer[Int][(a*g + b*g*x)/(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]), x]
```

Rubi steps

$$\begin{aligned}\int \frac{ag+bgx}{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}\,dx &= \int \left(\frac{ag}{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}+\frac{bgx}{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}\right)dx\\ &= (ag)\int \frac{1}{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}\,dx+(bg)\int \frac{x}{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}\,dx\end{aligned}$$

Mathematica [A]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{ag+bgx}{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a*g + b*g*x)/(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]),x]
[Out] Integrate[(a*g + b*g*x)/(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]), x]
```

Maple [A]
time = 0.56, size = 0, normalized size = 0.00

$$\int \frac{bgx + ag}{A + B\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*g*x+a*g)/(A+B*ln(e*(d*x+c)^2/(b*x+a)^2)),x)
```

```
[Out] int((b*g*x+a*g)/(A+B*ln(e*(d*x+c)^2/(b*x+a)^2)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)/(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="maxima
")
```

```
[Out] integrate((b*g*x + a*g)/(B*log((d*x + c)^2*e/(b*x + a)^2) + A), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)/(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="fricas
")
```

```
[Out] integral((b*g*x + a*g)/(B*log((d^2*x^2 + 2*c*d*x + c^2)*e/(b^2*x^2 + 2*a*b*
x + a^2)) + A), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$g\left(\int \frac{a}{A + B\log\left(\frac{c^2e}{a^2+2abx+b^2x^2} + \frac{2cdex}{a^2+2abx+b^2x^2} + \frac{d^2ex^2}{a^2+2abx+b^2x^2}\right)}\, dx + \int \frac{bx}{A + B\log\left(\frac{c^2e}{a^2+2abx+b^2x^2} + \frac{2cdex}{a^2+2abx+b^2x^2} + \frac{d^2ex^2}{a^2+2abx+b^2x^2}\right)}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)/(A+B*ln(e*(d*x+c)**2/(b*x+a)**2)),x)
```

```
[Out] g*(Integral(a/(A + B*log(c**2*e/(a**2 + 2*a*b*x + b**2*x**2) + 2*c*d*e*x/(a
**2 + 2*a*b*x + b**2*x**2) + d**2*e*x**2/(a**2 + 2*a*b*x + b**2*x**2))), x)
 + Integral(b*x/(A + B*log(c**2*e/(a**2 + 2*a*b*x + b**2*x**2) + 2*c*d*e*x/
```

```
(a**2 + 2*a*b*x + b**2*x**2) + d**2*e*x**2/(a**2 + 2*a*b*x + b**2*x**2))),
x))
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)/(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="giac")
[Out] integrate((b*g*x + a*g)/(B*log((d*x + c)^2*e/(b*x + a)^2) + A), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{a\,g+b\,g\,x}{A+B\,\ln\left(\frac{e\,{\left(c+d\,x\right)}^2}{{\left(a+b\,x\right)}^2}\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*g + b*g*x)/(A + B*log((e*(c + d*x)^2)/(a + b*x)^2)),x)
[Out] int((a*g + b*g*x)/(A + B*log((e*(c + d*x)^2)/(a + b*x)^2)), x)
```

3.221 $\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}\,dx$

Optimal. Leaf size=37

$$\text{Int}\left(\frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)},x\right)$$

```
[Out] Unintegrable(1/(b*g*x+a*g)/(A+B*ln(e*(d*x+c)^2/(b*x+a)^2)),x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((a*g + b*g*x)*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])),x]
[Out] Defer[Int][1/((a*g + b*g*x)*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])), x]
```

Rubi steps

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}\,dx=\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}\,dx$$

Mathematica [A]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((a*g + b*g*x)*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])),x]
[Out] Integrate[1/((a*g + b*g*x)*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])), x]
```

Maple [A]
time = 0.52, size = 0, normalized size = 0.00

$$\int \frac{1}{(bgx+ag)\left(A+B\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*g*x+a*g)/(A+B*ln(e*(d*x+c)^2/(b*x+a)^2)),x)
```

```
[Out] int(1/(b*g*x+a*g)/(A+B*ln(e*(d*x+c)^2/(b*x+a)^2)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)/(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="maxi
ma")
```

```
[Out] integrate(1/((b*g*x + a*g)*(B*log((d*x + c)^2*e/(b*x + a)^2) + A)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)/(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="fric
as")
```

```
[Out] integral(1/(A*b*g*x + A*a*g + (B*b*g*x + B*a*g)*log((d^2*x^2 + 2*c*d*x + c^
2)*e/(b^2*x^2 + 2*a*b*x + a^2))), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{Aa+Abx+Ba\log\left(\frac{c^2e}{a^2+2abx+b^2x^2}+\frac{2cdex}{a^2+2abx+b^2x^2}+\frac{d^2ex^2}{a^2+2abx+b^2x^2}\right)+Bbx\log\left(\frac{c^2e}{a^2+2abx+b^2x^2}+\frac{2cdex}{a^2+2abx+b^2x^2}+\frac{d^2ex^2}{a^2+2abx+b^2x^2}\right)}\,dx}{g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)/(A+B*ln(e*(d*x+c)**2/(b*x+a)**2)),x)
```

```
[Out] Integral(1/(A*a + A*b*x + B*a*log(c**2*e/(a**2 + 2*a*b*x + b**2*x**2) + 2*c
*d*e*x/(a**2 + 2*a*b*x + b**2*x**2) + d**2*e*x**2/(a**2 + 2*a*b*x + b**2*x*
```

```
*2)) + B*b*x*log(c**2*e/(a**2 + 2*a*b*x + b**2*x**2) + 2*c*d*e*x/(a**2 + 2*
a*b*x + b**2*x**2) + d**2*e*x**2/(a**2 + 2*a*b*x + b**2*x**2))), x)/g
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)/(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="giac
")
```

```
[Out] integrate(1/((b*g*x + a*g)*(B*log((d*x + c)^2*e/(b*x + a)^2) + A)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{(a\,g+b\,g\,x)\,\left(A+B\,\ln\left(\frac{e\,{(c+d\,x)}^2}{{(a+b\,x)}^2}\right)\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*g + b*g*x)*(A + B*log((e*(c + d*x)^2)/(a + b*x)^2))),x)
```

```
[Out] int(1/((a*g + b*g*x)*(A + B*log((e*(c + d*x)^2)/(a + b*x)^2))), x)
```

3.222 $\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}\,dx$

Optimal. Leaf size=91

$$-\frac{e^{-\frac{A}{2B}}(c+dx)\text{Ei}\left(\frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{2B}\right)}{2B(bc-ad)g^2(a+bx)\sqrt{\frac{e(c+dx)^2}{(a+bx)^2}}}$$

```
[Out] -1/2*(d*x+c)*Ei(1/2*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))/B)/B/(-a*d+b*c)/exp(1/2
*A/B)/g^2/(b*x+a)/(e*(d*x+c)^2/(b*x+a)^2)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.088$, Rules used = {2552, 2337, 2209}

$$-\frac{e^{-\frac{A}{2B}}(c+dx)\text{Ei}\left(\frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{2B}\right)}{2Bg^2(a+bx)(bc-ad)\sqrt{\frac{e(c+dx)^2}{(a+bx)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a*g + b*g*x)^2*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])),x]
```

```
[Out] -1/2*((c + d*x)*ExpIntegralEi[(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])/(2*B
)])/(B*(b*c - a*d)*E^(A/(2*B))*g^2*(a + b*x)*Sqrt[(e*(c + d*x)^2)/(a + b*x)
^2])
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2552

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(
m + 1)*(g/d)^m, Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a
 + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[n
 + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[d*f
- c*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Rubi steps

$$\int \frac{1}{(ag+bgx)^2 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}\,dx = \int \frac{1}{(ag+bgx)^2 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}\,dx$$

Mathematica [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{(ag+bgx)^2 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((a*g + b*g*x)^2*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])),x]
[Out] Integrate[1/((a*g + b*g*x)^2*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])), x]
```

Maple [F]
time = 0.59, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bgx+ag\right)^{2} \left(A+B\ln\left(\frac{e\left(dx+c\right)^{2}}{\left(bx+a\right)^{2}}\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*g*x+a*g)^2/(A+B*ln(e*(d*x+c)^2/(b*x+a)^2)),x)
[Out] int(1/(b*g*x+a*g)^2/(A+B*ln(e*(d*x+c)^2/(b*x+a)^2)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="ma
xima")
```

```
[Out] integrate(1/((b*g*x + a*g)^2*(B*log((d*x + c)^2*e/(b*x + a)^2) + A)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="fr
icas")
```

```
[Out] integral(1/(A*b^2*g^2*x^2 + 2*A*a*b*g^2*x + A*a^2*g^2 + (B*b^2*g^2*x^2 + 2*
B*a*b*g^2*x + B*a^2*g^2)*log((d^2*x^2 + 2*c*d*x + c^2)*e/(b^2*x^2 + 2*a*b*x
 + a^2))), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{Aa^2+2Aabx+Ab^2x^2+Ba^2\log\left(\frac{c^2e}{a^2+2abx+b^2x^2}+\frac{2cdex}{a^2+2abx+b^2x^2}+\frac{d^2ex^2}{a^2+2abx+b^2x^2}\right)+2Babx\log\left(\frac{c^2e}{a^2+2abx+b^2x^2}+\frac{2cdex}{a^2+2abx+b^2x^2}+\frac{d^2ex^2}{a^2+2abx+b^2x^2}\right)+Bb^2x^2\log\left(\frac{c^2e}{a^2+2abx+b^2x^2}+\frac{2cdex}{a^2+2abx+b^2x^2}+\frac{d^2ex^2}{a^2+2abx+b^2x^2}\right)}\,dx}{g^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)**2/(A+B*ln(e*(d*x+c)**2/(b*x+a)**2)),x)
```

```
[Out] Integral(1/(A*a**2 + 2*A*a*b*x + A*b**2*x**2 + B*a**2*log(c**2*e/(a**2 + 2*
a*b*x + b**2*x**2) + 2*c*d*e*x/(a**2 + 2*a*b*x + b**2*x**2) + d**2*e*x**2/(
a**2 + 2*a*b*x + b**2*x**2)) + 2*B*a*b*x*log(c**2*e/(a**2 + 2*a*b*x + b**2*
x**2) + 2*c*d*e*x/(a**2 + 2*a*b*x + b**2*x**2) + d**2*e*x**2/(a**2 + 2*a*b*
x + b**2*x**2)) + B*b**2*x**2*log(c**2*e/(a**2 + 2*a*b*x + b**2*x**2) + 2*c
*d*e*x/(a**2 + 2*a*b*x + b**2*x**2) + d**2*e*x**2/(a**2 + 2*a*b*x + b**2*x*
*2))), x)/g**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="gi
ac")
```

```
[Out] integrate(1/((b*g*x + a*g)^2*(B*log((d*x + c)^2*e/(b*x + a)^2) + A)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a\,g+b\,g\,x\right)}^2\,\left(A+B\,\ln\left(\frac{e\,{\left(c+d\,x\right)}^2}{{\left(a+b\,x\right)}^2}\right)\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*g + b*g*x)^2*(A + B*log((e*(c + d*x)^2)/(a + b*x)^2))),x)
[Out] int(1/((a*g + b*g*x)^2*(A + B*log((e*(c + d*x)^2)/(a + b*x)^2))), x)
```

3.223 $\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}\,dx$

Optimal. Leaf size=151

$$\frac{de^{-\frac{A}{2B}}(c+dx)\text{Ei}\left(\frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{2B}\right)}{2B(bc-ad)^2g^3(a+bx)\sqrt{\frac{e(c+dx)^2}{(a+bx)^2}}}-\frac{be^{-\frac{A}{B}}\text{Ei}\left(\frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{B}\right)}{2B(bc-ad)^2eg^3}$$

```
[Out] -1/2*b*Ei((A+B*ln(e*(d*x+c)^2/(b*x+a)^2))/B)/B/(-a*d+b*c)^2/e/exp(A/B)/g^3+
1/2*d*(d*x+c)*Ei(1/2*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))/B)/B/(-a*d+b*c)^2/exp(
1/2*A/B)/g^3/(b*x+a)/(e*(d*x+c)^2/(b*x+a)^2)^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.147, Rules used = {2552, 2367, 2337, 2209, 2347}

$$\frac{de^{-\frac{A}{2B}}(c+dx)\text{Ei}\left(\frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{2B}\right)}{2Bg^3(a+bx)(bc-ad)^2\sqrt{\frac{e(c+dx)^2}{(a+bx)^2}}}-\frac{be^{-\frac{A}{B}}\text{Ei}\left(\frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{B}\right)}{2Beg^3(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a*g + b*g*x)^3*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])),x]
```

```
[Out] (d*(c + d*x)*ExpIntegralEi[(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])/(2*B)])
/(2*B*(b*c - a*d)^2*E^(A/(2*B))*g^3*(a + b*x)*Sqrt[(e*(c + d*x)^2)/(a + b*x
)^2]) - (b*ExpIntegralEi[(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])/B])/(2*B*
(b*c - a*d)^2*e*E^(A/B)*g^3)
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2367

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^(r_.))^(
q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*x^n])^p, (d + e*x
^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
&& IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0] && IntegerQ[r]))
```

Rule 2552

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(
m + 1)*(g/d)^m, Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a
 + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[n
 + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[d*f
- c*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Rubi steps

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}\,dx=\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}\,dx$$

Mathematica [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((a*g + b*g*x)^3*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])),x]
[Out] Integrate[1/((a*g + b*g*x)^3*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])), x]
```

Maple [F]
time = 0.65, size = 0, normalized size = 0.00

$$\int \frac{1}{(bgx+ag)^3\left(A+B\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*g*x+a*g)^3/(A+B*ln(e*(d*x+c)^2/(b*x+a)^2)),x)
```

```
[Out] int(1/(b*g*x+a*g)^3/(A+B*ln(e*(d*x+c)^2/(b*x+a)^2)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)^3/(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="ma
xima")
```

```
[Out] integrate(1/((b*g*x + a*g)^3*(B*log((d*x + c)^2*e/(b*x + a)^2) + A)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)^3/(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="fr
icas")
```

```
[Out] integral(1/(A*b^3*g^3*x^3 + 3*A*a*b^2*g^3*x^2 + 3*A*a^2*b*g^3*x + A*a^3*g^3
 + (B*b^3*g^3*x^3 + 3*B*a*b^2*g^3*x^2 + 3*B*a^2*b*g^3*x + B*a^3*g^3)*log((d
^2*x^2 + 2*c*d*x + c^2)*e/(b^2*x^2 + 2*a*b*x + a^2))), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{Aa^3+3Aa^2bx+3Aab^2x^2+Ab^3x^3+Ba^3\log\left(\frac{c^2e}{a^2+2abx+b^2x^2}+\frac{2cdex}{a^2+2abx+b^2x^2}+\frac{d^2ex^2}{a^2+2abx+b^2x^2}\right)+3Ba^2bx\log\left(\frac{c^2e}{a^2+2abx+b^2x^2}+\frac{2cdex}{a^2+2abx+b^2x^2}+\frac{d^2ex^2}{a^2+2abx+b^2x^2}\right)+3Bab^2x^2\log\left(\frac{c^2e}{a^2+2abx+b^2x^2}+\frac{2cdex}{a^2+2abx+b^2x^2}+\frac{d^2ex^2}{a^2+2abx+b^2x^2}\right)+Bb^3x^3\log\left(\frac{c^2e}{a^2+2abx+b^2x^2}+\frac{2cdex}{a^2+2abx+b^2x^2}+\frac{d^2ex^2}{a^2+2abx+b^2x^2}\right)}\,dx}{g^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)**3/(A+B*ln(e*(d*x+c)**2/(b*x+a)**2)),x)
```

```
[Out] Integral(1/(A*a**3 + 3*A*a**2*b*x + 3*A*a*b**2*x**2 + A*b**3*x**3 + B*a**3*
log(c**2*e/(a**2 + 2*a*b*x + b**2*x**2) + 2*c*d*e*x/(a**2 + 2*a*b*x + b**2*
x**2) + d**2*e*x**2/(a**2 + 2*a*b*x + b**2*x**2)) + 3*B*a**2*b*x*log(c**2*e
/(a**2 + 2*a*b*x + b**2*x**2) + 2*c*d*e*x/(a**2 + 2*a*b*x + b**2*x**2) + d*
*2*e*x**2/(a**2 + 2*a*b*x + b**2*x**2)) + 3*B*a*b**2*x**2*log(c**2*e/(a**2
+ 2*a*b*x + b**2*x**2) + 2*c*d*e*x/(a**2 + 2*a*b*x + b**2*x**2) + d**2*e*x*
*2/(a**2 + 2*a*b*x + b**2*x**2)) + B*b**3*x**3*log(c**2*e/(a**2 + 2*a*b*x +
 b**2*x**2) + 2*c*d*e*x/(a**2 + 2*a*b*x + b**2*x**2) + d**2*e*x**2/(a**2 +
2*a*b*x + b**2*x**2))), x)/g**3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)^3/(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="gi
ac")
[Out] integrate(1/((b*g*x + a*g)^3*(B*log((d*x + c)^2*e/(b*x + a)^2) + A)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a\,g+b\,g\,x)^3\,\left(A+B\,\ln\left(\frac{e\,(c+d\,x)^2}{(a+b\,x)^2}\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*g + b*g*x)^3*(A + B*log((e*(c + d*x)^2)/(a + b*x)^2))),x)
[Out] int(1/((a*g + b*g*x)^3*(A + B*log((e*(c + d*x)^2)/(a + b*x)^2))), x)
```

3.224 $\int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx$

Optimal. Leaf size=37

$$\text{Int}\left(\frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2},x\right)$$

```
[Out] Unintegrable((b*g*x+a*g)^2/(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))^2,x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a*g + b*g*x)^2/(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2,x]
[Out] Defer[Int][(a*g + b*g*x)^2/(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2, x]
```

Rubi steps

$$\int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx = \int \left(\frac{a^2g^2}{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}+\frac{2abg^2x}{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}+\frac{b^2g^2x}{\left(A+B\log\left(\frac{e}{}\right.\right.}\right.$$

$$= \left(a^2g^2\right)\int \frac{1}{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx+\left(2abg^2\right)\int \frac{x}{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}$$

Mathematica [A]
time = 0.26, size = 0, normalized size = 0.00

$$\int \frac{(ag+bgx)^2}{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a*g + b*g*x)^2/(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2,x]
[Out] Integrate[(a*g + b*g*x)^2/(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2, x]
```

Maple [A]
time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{(bgx+ag)^2}{\left(A+B\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*g*x+a*g)^2/(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))^2,x)
[Out] int((b*g*x+a*g)^2/(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^2/(A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2,x, algorithm="ma
xima")
[Out] -1/2*(b^3*d*g^2*x^4 + a^3*c*g^2 + (b^3*c*g^2 + 3*a*b^2*d*g^2)*x^3 + 3*(a*b^
2*c*g^2 + a^2*b*d*g^2)*x^2 + (3*a^2*b*c*g^2 + a^3*d*g^2)*x)/(2*(b*c - a*d)*
B^2*log(b*x + a) - 2*(b*c - a*d)*B^2*log(d*x + c) - (b*c - a*d)*A*B - (b*c
- a*d)*B^2) + integrate(1/2*(4*b^3*d*g^2*x^3 + 3*a^2*b*c*g^2 + a^3*d*g^2 +
3*(b^3*c*g^2 + 3*a*b^2*d*g^2)*x^2 + 6*(a*b^2*c*g^2 + a^2*b*d*g^2)*x)/(2*(b*
c - a*d)*B^2*log(b*x + a) - 2*(b*c - a*d)*B^2*log(d*x + c) - (b*c - a*d)*A*
B - (b*c - a*d)*B^2), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^2/(A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2,x, algorithm="fr
icas")
[Out] integral((b^2*g^2*x^2 + 2*a*b*g^2*x + a^2*g^2)/(B^2*log((d^2*x^2 + 2*c*d*x
+ c^2)*e/(b^2*x^2 + 2*a*b*x + a^2))^2 + 2*A*B*log((d^2*x^2 + 2*c*d*x + c^2)
*e/(b^2*x^2 + 2*a*b*x + a^2)) + A^2), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)**2/(A+B*ln(e*(d*x+c)**2/(b*x+a)**2))**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^2/(A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2,x, algorithm="gi
ac")
```

```
[Out] integrate((b*g*x + a*g)^2/(B*log((d*x + c)^2*e/(b*x + a)^2) + A)^2, x)
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{{\left(a\,g+b\,g\,x\right)}^2}{{\left(A+B\,\ln\left(\frac{e\,{\left(c+d\,x\right)}^2}{{\left(a+b\,x\right)}^2}\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*g + b*g*x)^2/(A + B*log((e*(c + d*x)^2)/(a + b*x)^2))^2,x)
```

```
[Out] int((a*g + b*g*x)^2/(A + B*log((e*(c + d*x)^2)/(a + b*x)^2))^2, x)
```

3.225 $\int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx$

Optimal. Leaf size=35

$$\text{Int}\left(\frac{ag+bgx}{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2},x\right)$$

```
[Out] Unintegrable((b*g*x+a*g)/(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))^2,x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a*g + b*g*x)/(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2,x]
[Out] Defer[Int][(a*g + b*g*x)/(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2, x]
```

Rubi steps

$$\begin{aligned}\int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx &= \int \left(\frac{ag}{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}+\frac{bgx}{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\right)dx\\ &= (ag)\int \frac{1}{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx+(bg)\int \frac{x}{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx\end{aligned}$$

Mathematica [A]
time = 0.21, size = 0, normalized size = 0.00

$$\int \frac{ag+bgx}{\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a*g + b*g*x)/(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2,x]
```

[Out] Integrate[(a*g + b*g*x)/(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2, x]

Maple [A]
time = 0.65, size = 0, normalized size = 0.00

$$\int \frac{bgx+ag}{\left(A+B\ln\left(\frac{e(dx+c)^{2}}{(bx+a)^{2}}\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*g*x+a*g)/(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))^2,x)

[Out] int((b*g*x+a*g)/(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))^2,x)

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*g*x+a*g)/(A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2,x, algorithm="maxima")

[Out] -1/2*(b^2*d*g*x^3 + a^2*c*g + (b^2*c*g + 2*a*b*d*g)*x^2 + (2*a*b*c*g + a^2*d*g)*x)/(2*(b*c - a*d)*B^2*log(b*x + a) - 2*(b*c - a*d)*B^2*log(d*x + c) - (b*c - a*d)*A*B - (b*c - a*d)*B^2) + integrate(1/2*(3*b^2*d*g*x^2 + 2*a*b*c*g + a^2*d*g + 2*(b^2*c*g + 2*a*b*d*g)*x)/(2*(b*c - a*d)*B^2*log(b*x + a) - 2*(b*c - a*d)*B^2*log(d*x + c) - (b*c - a*d)*A*B - (b*c - a*d)*B^2), x)

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*g*x+a*g)/(A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2,x, algorithm="fricas")

[Out] integral((b*g*x + a*g)/(B^2*log((d^2*x^2 + 2*c*d*x + c^2)*e/(b^2*x^2 + 2*a*b*x + a^2))^2 + 2*A*B*log((d^2*x^2 + 2*c*d*x + c^2)*e/(b^2*x^2 + 2*a*b*x + a^2)) + A^2), x)

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{-a^{2}cg-a^{2}dgx-2abcgx-2abdgx^{2}-b^{2}cgx^{2}-b^{2}dgx^{3}}{2ABad-2ABbc+(2B^{2}ad-2B^{2}bc)\log\left(\frac{e(c+dx)^{2}}{(a+bx)^{2}}\right)}+\frac{g\left(\int\frac{a^{2}d}{A+B\log\left(\frac{c^{2}e}{a^{2}+2abx+b^{2}x^{2}}+\frac{2cdex}{a^{2}+2abx+b^{2}x^{2}}+\frac{d^{2}ex^{2}}{a^{2}+2abx+b^{2}x^{2}}\right)}\,dx+\int\frac{2abc}{A+B\log\left(\frac{c^{2}e}{a^{2}+2abx+b^{2}x^{2}}+\frac{2cdex}{a^{2}+2abx+b^{2}x^{2}}+\frac{d^{2}ex^{2}}{a^{2}+2abx+b^{2}x^{2}}\right)}\,dx+\int\frac{2b^{2}cx}{A+B\log\left(\frac{c^{2}e}{a^{2}+2abx+b^{2}x^{2}}+\frac{2cdex}{a^{2}+2abx+b^{2}x^{2}}+\frac{d^{2}ex^{2}}{a^{2}+2abx+b^{2}x^{2}}\right)}\,dx+\int\frac{3b^{2}dx^{2}}{A+B\log\left(\frac{c^{2}e}{a^{2}+2abx+b^{2}x^{2}}+\frac{2cdex}{a^{2}+2abx+b^{2}x^{2}}+\frac{d^{2}ex^{2}}{a^{2}+2abx+b^{2}x^{2}}\right)}\,dx+\int\frac{4abdx}{A+B\log\left(\frac{c^{2}e}{a^{2}+2abx+b^{2}x^{2}}+\frac{2cdex}{a^{2}+2abx+b^{2}x^{2}}+\frac{d^{2}ex^{2}}{a^{2}+2abx+b^{2}x^{2}}\right)}\,dx\right)}{2B\,(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)/(A+B*ln(e*(d*x+c)**2/(b*x+a)**2))**2,x)
```

```
[Out] (-a**2*c*g - a**2*d*g*x - 2*a*b*c*g*x - 2*a*b*d*g*x**2 - b**2*c*g*x**2 - b*
*2*d*g*x**3)/(2*A*B*a*d - 2*A*B*b*c + (2*B**2*a*d - 2*B**2*b*c)*log(e*(c +
d*x)**2/(a + b*x)**2)) + g*(Integral(a**2*d/(A + B*log(c**2*e/(a**2 + 2*a*b
*x + b**2*x**2) + 2*c*d*e*x/(a**2 + 2*a*b*x + b**2*x**2) + d**2*e*x**2/(a**
2 + 2*a*b*x + b**2*x**2))), x) + Integral(2*a*b*c/(A + B*log(c**2*e/(a**2 +
 2*a*b*x + b**2*x**2) + 2*c*d*e*x/(a**2 + 2*a*b*x + b**2*x**2) + d**2*e*x**
2/(a**2 + 2*a*b*x + b**2*x**2))), x) + Integral(2*b**2*c*x/(A + B*log(c**2*
e/(a**2 + 2*a*b*x + b**2*x**2) + 2*c*d*e*x/(a**2 + 2*a*b*x + b**2*x**2) + d
**2*e*x**2/(a**2 + 2*a*b*x + b**2*x**2))), x) + Integral(3*b**2*d*x**2/(A +
 B*log(c**2*e/(a**2 + 2*a*b*x + b**2*x**2) + 2*c*d*e*x/(a**2 + 2*a*b*x + b*
*2*x**2) + d**2*e*x**2/(a**2 + 2*a*b*x + b**2*x**2))), x) + Integral(4*a*b*
d*x/(A + B*log(c**2*e/(a**2 + 2*a*b*x + b**2*x**2) + 2*c*d*e*x/(a**2 + 2*a*
b*x + b**2*x**2) + d**2*e*x**2/(a**2 + 2*a*b*x + b**2*x**2))), x))/(2*B*(a*
d - b*c))
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)/(A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2,x, algorithm="giac
")
```

```
[Out] integrate((b*g*x + a*g)/(B*log((d*x + c)^2*e/(b*x + a)^2) + A)^2, x)
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{a\,g+b\,g\,x}{{\left(A+B\,\ln\left(\frac{e\,{\left(c+d\,x\right)}^2}{{\left(a+b\,x\right)}^2}\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*g + b*g*x)/(A + B*log((e*(c + d*x)^2)/(a + b*x)^2))^2,x)
```

```
[Out] int((a*g + b*g*x)/(A + B*log((e*(c + d*x)^2)/(a + b*x)^2))^2, x)
```

3.226 $\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx$

Optimal. Leaf size=37

$$\text{Int}\left(\frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2},x\right)$$

```
[Out] Unintegrable(1/(b*g*x+a*g)/(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))^2,x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((a*g + b*g*x)*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2),x]
[Out] Defer[Int][1/((a*g + b*g*x)*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2), x]
```

Rubi steps

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx=\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx$$

Mathematica [A]
time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{1}{(ag+bgx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((a*g + b*g*x)*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2),x]
[Out] Integrate[1/((a*g + b*g*x)*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2), x]
```

Maple [A]
time = 0.53, size = 0, normalized size = 0.00

$$\int \frac{1}{(bgx+ag)\left(A+B\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*g*x+a*g)/(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))^2,x)
```

```
[Out] int(1/(b*g*x+a*g)/(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)/(A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2,x, algorithm="ma
xima")
```

```
[Out] d*integrate(1/2/(2*(b*c*g - a*d*g)*B^2*log(b*x + a) - 2*(b*c*g - a*d*g)*B^2
*log(d*x + c) - (b*c*g - a*d*g)*A*B - (b*c*g - a*d*g)*B^2), x) - 1/2*(d*x +
 c)/(2*(b*c*g - a*d*g)*B^2*log(b*x + a) - 2*(b*c*g - a*d*g)*B^2*log(d*x + c
) - (b*c*g - a*d*g)*A*B - (b*c*g - a*d*g)*B^2)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)/(A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2,x, algorithm="fr
icas")
```

```
[Out] integral(1/(A^2*b*g*x + A^2*a*g + (B^2*b*g*x + B^2*a*g)*log((d^2*x^2 + 2*c*
d*x + c^2)*e/(b^2*x^2 + 2*a*b*x + a^2))^2 + 2*(A*B*b*g*x + A*B*a*g)*log((d^
2*x^2 + 2*c*d*x + c^2)*e/(b^2*x^2 + 2*a*b*x + a^2))), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{-c-dx}{2ABadg-2ABbcg+(2B^2adg-2B^2bcg)\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}+\frac{d\int\frac{1}{A+B\log\left(\frac{c^2e}{a^2+2abx+b^2x^2}+\frac{2cdex}{a^2+2abx+b^2x^2}+\frac{d^2ex^2}{a^2+2abx+b^2x^2}\right)}\,dx}{2Bg\,(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)/(A+B*ln(e*(d*x+c)**2/(b*x+a)**2))**2,x)
```

```
[Out] (-c - d*x)/(2*A*B*a*d*g - 2*A*B*b*c*g + (2*B**2*a*d*g - 2*B**2*b*c*g)*log(e
*(c + d*x)**2/(a + b*x)**2)) + d*Integral(1/(A + B*log(c**2*e/(a**2 + 2*a*b
*x + b**2*x**2) + 2*c*d*e*x/(a**2 + 2*a*b*x + b**2*x**2) + d**2*e*x**2/(a**
2 + 2*a*b*x + b**2*x**2))), x)/(2*B*g*(a*d - b*c))
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)/(A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2,x, algorithm="gi
ac")
```

```
[Out] integrate(1/((b*g*x + a*g)*(B*log((d*x + c)^2*e/(b*x + a)^2) + A)^2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{(a\,g+b\,g\,x)\,\left(A+B\,\ln\left(\frac{e\,{(c+d\,x)}^2}{{(a+b\,x)}^2}\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*g + b*g*x)*(A + B*log((e*(c + d*x)^2)/(a + b*x)^2))^2),x)
```

```
[Out] int(1/((a*g + b*g*x)*(A + B*log((e*(c + d*x)^2)/(a + b*x)^2))^2), x)
```

3.227 $\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx$

Optimal. Leaf size=147

$$-\frac{e^{-\frac{A}{2B}}(c+dx)\text{Ei}\left(\frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{2B}\right)}{4B^2(bc-ad)g^2(a+bx)\sqrt{\frac{e(c+dx)^2}{(a+bx)^2}}}+\frac{c+dx}{2B(bc-ad)g^2(a+bx)\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}$$

```
[Out] 1/2*(d*x+c)/B/(-a*d+b*c)/g^2/(b*x+a)/(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))-1/4*(d
*x+c)*Ei(1/2*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))/B)/B^2/(-a*d+b*c)/exp(1/2*A/B)
/g^2/(b*x+a)/(e*(d*x+c)^2/(b*x+a)^2)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2552, 2334, 2337, 2209}

$$\frac{c+dx}{2Bg^2(a+bx)(bc-ad)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}-\frac{e^{-\frac{A}{2B}}(c+dx)\text{Ei}\left(\frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{2B}\right)}{4B^2g^2(a+bx)(bc-ad)\sqrt{\frac{e(c+dx)^2}{(a+bx)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a*g + b*g*x)^2*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2),x]
```

```
[Out] -1/4*((c + d*x)*ExpIntegralEi[(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])/(2*B
)])/(B^2*(b*c - a*d)*E^(A/(2*B))*g^2*(a + b*x)*Sqrt[(e*(c + d*x)^2)/(a + b*
x)^2]) + (c + d*x)/(2*B*(b*c - a*d)*g^2*(a + b*x)*(A + B*Log[(e*(c + d*x)^2
)/(a + b*x)^2]))
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

Rule 2334

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x*((a + b
*Log[c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Dist[1/(b*n*(p + 1)), Int[(a + b*
Log[c*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, n}, x] && LtQ[p, -1] && Inte
```

```
gerQ[2*p]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2552

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(
m + 1)*(g/d)^m, Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a
 + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[n
 + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[d*f
- c*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Rubi steps

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx = \int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx$$

Mathematica [F]
time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((a*g + b*g*x)^2*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2),x]
[Out] Integrate[1/((a*g + b*g*x)^2*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2), x
]
```

Maple [F]
time = 0.55, size = 0, normalized size = 0.00

$$\int \frac{1}{(bgx+ag)^2\left(A+B\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*g*x+a*g)^2/(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))^2,x)
```

```
[Out] int(1/(b*g*x+a*g)^2/(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2,x, algorithm="
maxima")
```

```
[Out] 1/2*(d*x + c)/((a*b*c*g^2 - a^2*d*g^2)*A*B + (a*b*c*g^2 - a^2*d*g^2)*B^2 +
((b^2*c*g^2 - a*b*d*g^2)*A*B + (b^2*c*g^2 - a*b*d*g^2)*B^2)*x - 2*((b^2*c*g
^2 - a*b*d*g^2)*B^2*x + (a*b*c*g^2 - a^2*d*g^2)*B^2)*log(b*x + a) + 2*((b^2
*c*g^2 - a*b*d*g^2)*B^2*x + (a*b*c*g^2 - a^2*d*g^2)*B^2)*log(d*x + c)) + in
tegrate(1/2/(A*B*a^2*g^2 + B^2*a^2*g^2 + (A*B*b^2*g^2 + B^2*b^2*g^2)*x^2 +
2*(A*B*a*b*g^2 + B^2*a*b*g^2)*x - 2*(B^2*b^2*g^2*x^2 + 2*B^2*a*b*g^2*x + B^
2*a^2*g^2)*log(b*x + a) + 2*(B^2*b^2*g^2*x^2 + 2*B^2*a*b*g^2*x + B^2*a^2*g^
2)*log(d*x + c)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2,x, algorithm="
fricas")
```

```
[Out] integral(1/(A^2*b^2*g^2*x^2 + 2*A^2*a*b*g^2*x + A^2*a^2*g^2 + (B^2*b^2*g^2*
x^2 + 2*B^2*a*b*g^2*x + B^2*a^2*g^2)*log((d^2*x^2 + 2*c*d*x + c^2)*e/(b^2*x
^2 + 2*a*b*x + a^2))^2 + 2*(A*B*b^2*g^2*x^2 + 2*A*B*a*b*g^2*x + A*B*a^2*g^2
)*log((d^2*x^2 + 2*c*d*x + c^2)*e/(b^2*x^2 + 2*a*b*x + a^2))), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{-c-dx}{2ABa^2dg^2-2ABabcg^2+2ABabdg^2x-2ABb^2cg^2x+(2B^2a^2dg^2-2B^2abcg^2+2B^2abdg^2x-2B^2b^2cg^2x)\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}+\frac{\int\frac{1}{Aa^2+2Aabx+Ab^2x^2+Ba^2\log\left(\frac{c^2e}{a^2+2abx+b^2x^2}+\frac{2cdex}{a^2+2abx+b^2x^2}+\frac{d^2ex^2}{a^2+2abx+b^2x^2}\right)+2Babx\log\left(\frac{c^2e}{a^2+2abx+b^2x^2}+\frac{2cdex}{a^2+2abx+b^2x^2}+\frac{d^2ex^2}{a^2+2abx+b^2x^2}\right)+Bb^2x^2\log\left(\frac{c^2e}{a^2+2abx+b^2x^2}+\frac{2cdex}{a^2+2abx+b^2x^2}+\frac{d^2ex^2}{a^2+2abx+b^2x^2}\right)}\,dx}{2Bg^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)**2/(A+B*ln(e*(d*x+c)**2/(b*x+a)**2))**2,x)
```

```
[Out] (-c - d*x)/(2*A*B*a**2*d*g**2 - 2*A*B*a*b*c*g**2 + 2*A*B*a*b*d*g**2*x - 2*A
*B*b**2*c*g**2*x + (2*B**2*a**2*d*g**2 - 2*B**2*a*b*c*g**2 + 2*B**2*a*b*d*g
```

```
**2*x - 2*B**2*b**2*c*g**2*x)*log(e*(c + d*x)**2/(a + b*x)**2)) + Integral(
1/(A*a**2 + 2*A*a*b*x + A*b**2*x**2 + B*a**2*log(c**2*e/(a**2 + 2*a*b*x + b
**2*x**2) + 2*c*d*e*x/(a**2 + 2*a*b*x + b**2*x**2) + d**2*e*x**2/(a**2 + 2*
a*b*x + b**2*x**2)) + 2*B*a*b*x*log(c**2*e/(a**2 + 2*a*b*x + b**2*x**2) + 2
*c*d*e*x/(a**2 + 2*a*b*x + b**2*x**2) + d**2*e*x**2/(a**2 + 2*a*b*x + b**2*
x**2)) + B*b**2*x**2*log(c**2*e/(a**2 + 2*a*b*x + b**2*x**2) + 2*c*d*e*x/(a
**2 + 2*a*b*x + b**2*x**2) + d**2*e*x**2/(a**2 + 2*a*b*x + b**2*x**2))), x)
/(2*B*g**2)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2,x, algorithm="
giac")
```

```
[Out] integrate(1/((b*g*x + a*g)^2*(B*log((d*x + c)^2*e/(b*x + a)^2) + A)^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a\,g+b\,g\,x\right)}^2\,{\left(A+B\,\ln\left(\frac{e\,{\left(c+d\,x\right)}^2}{{\left(a+b\,x\right)}^2}\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*g + b*g*x)^2*(A + B*log((e*(c + d*x)^2)/(a + b*x)^2))^2),x)
```

```
[Out] int(1/((a*g + b*g*x)^2*(A + B*log((e*(c + d*x)^2)/(a + b*x)^2))^2), x)
```

3.228 $\int \frac{1}{(ag+bgx)^3\left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx$

Optimal. Leaf size=206

$$\frac{de^{-\frac{A}{2B}}(c+dx)\text{Ei}\left(\frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{2B}\right)}{4B^2(bc-ad)^2g^3(a+bx)\sqrt{\frac{e(c+dx)^2}{(a+bx)^2}}}-\frac{be^{-\frac{A}{B}}\text{Ei}\left(\frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{B}\right)}{2B^2(bc-ad)^2eg^3}+\frac{c+dx}{2B(bc-ad)g^3(a+bx)^2\left(A+B\log\right.}$$

```
[Out] -1/2*b*Ei((A+B*ln(e*(d*x+c)^2/(b*x+a)^2))/B)/B^2/(-a*d+b*c)^2/e/exp(A/B)/g^
3+1/2*(d*x+c)/B/(-a*d+b*c)/g^3/(b*x+a)^2/(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))+1/
4*d*(d*x+c)*Ei(1/2*(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))/B)/B^2/(-a*d+b*c)^2/exp(
1/2*A/B)/g^3/(b*x+a)/(e*(d*x+c)^2/(b*x+a)^2)^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {2552, 2357, 2367, 2337, 2209, 2347}

$$\frac{de^{-\frac{A}{2B}}(c+dx)\text{Ei}\left(\frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{2B}\right)}{4B^2g^3(a+bx)(bc-ad)^2\sqrt{\frac{e(c+dx)^2}{(a+bx)^2}}}-\frac{be^{-\frac{A}{B}}\text{Ei}\left(\frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{B}\right)}{2B^2eg^3(bc-ad)^2}+\frac{c+dx}{2Bg^3(a+bx)^2(bc-ad)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a*g + b*g*x)^3*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2),x]
```

```
[Out] (d*(c + d*x)*ExpIntegralEi[(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])/(2*B)])
/(4*B^2*(b*c - a*d)^2*E^(A/(2*B))*g^3*(a + b*x)*Sqrt[(e*(c + d*x)^2)/(a + b
*x)^2]) - (b*ExpIntegralEi[(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])/B])/(2*
B^2*(b*c - a*d)^2*e*E^(A/B)*g^3) + (c + d*x)/(2*B*(b*c - a*d)*g^3*(a + b*x)
^2*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]))
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
```

```
{a, b, c, n, p}, x]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2357

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x
_Symbol] :> Simp[x*(d + e*x)^q*((a + b*Log[c*x^n])^(p + 1)/(b*n*(p + 1))),
x] + (-Dist[(q + 1)/(b*n*(p + 1)), Int[(d + e*x)^q*(a + b*Log[c*x^n])^(p +
1), x], x] + Dist[d*(q/(b*n*(p + 1))), Int[(d + e*x)^(q - 1)*(a + b*Log[c*x
^n])^(p + 1), x], x]) /; FreeQ[{a, b, c, d, e, n}, x] && LtQ[p, -1] && GtQ[
q, 0]
```

Rule 2367

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^(r_.))^(
q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*x^n])^p, (d + e*x
^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
&& IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0] && IntegerQ[r]))
```

Rule 2552

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(
m + 1)*(g/d)^m, Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a
 + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[n
 + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[d*f
- c*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Rubi steps

$$\int \frac{1}{(ag+bgx)^3 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx = \int \frac{1}{(ag+bgx)^3 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx$$

Mathematica [F]
time = 0.22, size = 0, normalized size = 0.00

$$\int \frac{1}{(ag+bgx)^3 \left(A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((a*g + b*g*x)^3*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2),x]
```

```
[Out] Integrate[1/((a*g + b*g*x)^3*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2), x
]
```

Maple [F]
time = 0.66, size = 0, normalized size = 0.00

$$\int \frac{1}{(bgx+ag)^3 \left(A+B\ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*g*x+a*g)^3/(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))^2,x)
```

```
[Out] int(1/(b*g*x+a*g)^3/(A+B*ln(e*(d*x+c)^2/(b*x+a)^2))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)^3/(A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2,x, algorithm="
maxima")
```

```
[Out] 1/2*(d*x + c)/((a^2*b*c*g^3 - a^3*d*g^3)*A*B + (a^2*b*c*g^3 - a^3*d*g^3)*B^
2 + ((b^3*c*g^3 - a*b^2*d*g^3)*A*B + (b^3*c*g^3 - a*b^2*d*g^3)*B^2)*x^2 + 2
*((a*b^2*c*g^3 - a^2*b*d*g^3)*A*B + (a*b^2*c*g^3 - a^2*b*d*g^3)*B^2)*x - 2*
((b^3*c*g^3 - a*b^2*d*g^3)*B^2*x^2 + 2*(a*b^2*c*g^3 - a^2*b*d*g^3)*B^2*x +
(a^2*b*c*g^3 - a^3*d*g^3)*B^2)*log(b*x + a) + 2*((b^3*c*g^3 - a*b^2*d*g^3)*
B^2*x^2 + 2*(a*b^2*c*g^3 - a^2*b*d*g^3)*B^2*x + (a^2*b*c*g^3 - a^3*d*g^3)*B
^2)*log(d*x + c)) - integrate(-1/2*(b*d*x + 2*b*c - a*d)/(((b^4*c*g^3 - a*b
^3*d*g^3)*A*B + (b^4*c*g^3 - a*b^3*d*g^3)*B^2)*x^3 + (a^3*b*c*g^3 - a^4*d*g
^3)*A*B + (a^3*b*c*g^3 - a^4*d*g^3)*B^2 + 3*((a*b^3*c*g^3 - a^2*b^2*d*g^3)*
A*B + (a*b^3*c*g^3 - a^2*b^2*d*g^3)*B^2)*x^2 + 3*((a^2*b^2*c*g^3 - a^3*b*d*
g^3)*A*B + (a^2*b^2*c*g^3 - a^3*b*d*g^3)*B^2)*x - 2*((b^4*c*g^3 - a*b^3*d*g
^3)*B^2*x^3 + 3*(a*b^3*c*g^3 - a^2*b^2*d*g^3)*B^2*x^2 + 3*(a^2*b^2*c*g^3 -
a^3*b*d*g^3)*B^2*x + (a^3*b*c*g^3 - a^4*d*g^3)*B^2)*log(b*x + a) + 2*((b^4*
c*g^3 - a*b^3*d*g^3)*B^2*x^3 + 3*(a*b^3*c*g^3 - a^2*b^2*d*g^3)*B^2*x^2 + 3*
(a^2*b^2*c*g^3 - a^3*b*d*g^3)*B^2*x + (a^3*b*c*g^3 - a^4*d*g^3)*B^2)*log(d*
x + c)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)^3/(A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2,x, algorithm="
fricas")
```

```
[Out] integral(1/(A^2*b^3*g^3*x^3 + 3*A^2*a*b^2*g^3*x^2 + 3*A^2*a^2*b*g^3*x + A^2
*a^3*g^3 + (B^2*b^3*g^3*x^3 + 3*B^2*a*b^2*g^3*x^2 + 3*B^2*a^2*b*g^3*x + B^2
*a^3*g^3)*log((d^2*x^2 + 2*c*d*x + c^2)*e/(b^2*x^2 + 2*a*b*x + a^2))^2 + 2*
(A*B*b^3*g^3*x^3 + 3*A*B*a*b^2*g^3*x^2 + 3*A*B*a^2*b*g^3*x + A*B*a^3*g^3)*l
og((d^2*x^2 + 2*c*d*x + c^2)*e/(b^2*x^2 + 2*a*b*x + a^2))), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)**3/(A+B*ln(e*(d*x+c)**2/(b*x+a)**2))**2,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)^3/(A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2,x, algorithm="
giac")
```

```
[Out] integrate(1/((b*g*x + a*g)^3*(B*log((d*x + c)^2*e/(b*x + a)^2) + A)^2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(a\,g+b\,g\,x\right)}^3\,{\left(A+B\,\ln\left(\frac{e\,{\left(c+d\,x\right)}^2}{{\left(a+b\,x\right)}^2}\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*g + b*g*x)^3*(A + B*log((e*(c + d*x)^2)/(a + b*x)^2))^2),x)
```

```
[Out] int(1/((a*g + b*g*x)^3*(A + B*log((e*(c + d*x)^2)/(a + b*x)^2))^2), x)
```

3.229 $\int \frac{1}{(ag+bgx)^2(A+B\log(e(a+bx)^n(c+dx)^{-n}))}\,dx$

Optimal. Leaf size=96

$$\frac{e^{\frac{A}{Bn}}(c+dx)\left(e(a+bx)^n(c+dx)^{-n}\right)^{\frac{1}{n}}\operatorname{Ei}\left(-\frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{Bn}\right)}{B(bc-ad)g^2n(a+bx)}$$

```
[Out] exp(A/B/n)*(d*x+c)*(e*(b*x+a)^n/((d*x+c)^n))^(1/n)*Ei((-A-B*ln(e*(b*x+a)^n/
((d*x+c)^n)))/B/n)/B/(-a*d+b*c)/g^2/n/(b*x+a)
```

Rubi [A]
time = 0.10, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2573, 2549, 2347, 2209}

$$\frac{e^{\frac{A}{Bn}}(c+dx)\left(e(a+bx)^n(c+dx)^{-n}\right)^{\frac{1}{n}}\operatorname{Ei}\left(-\frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{Bn}\right)}{Bg^2n(a+bx)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])),x]
```

```
[Out] (E^(A/(B*n))*(c + d*x)*((e*(a + b*x)^n)/(c + d*x)^n)^n^(-1)*ExpIntegralEi[-
((A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(B*n))])/(B*(b*c - a*d)*g^2*n*(a
+ b*x))
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2549

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(m +
1)*(g/b)^m, Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x], x,
(a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ
[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[b*f - a*g, 0] && (GtQ[p, 0] || LtQ
[m, -1])
```

Rule 2573

```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

Rubi steps

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx=\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,d$$

Mathematica [**F**]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{1}{(ag+bgx)^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])),x]
[Out] Integrate[1/((a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])), x]
```

Maple [**F**]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bgx+ag\right)^{2}\left(A+B\ln\left(e\left(bx+a\right)^{n}\left(dx+c\right)^{-n}\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*g*x+a*g)^2/(A+B*ln(e*(b*x+a)^n/((d*x+c)^n))),x)
[Out] int(1/(b*g*x+a*g)^2/(A+B*ln(e*(b*x+a)^n/((d*x+c)^n))),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="
maxima")
[Out] integrate(1/((b*g*x + a*g)^2*(B*log((b*x + a)^n*e/(d*x + c)^n) + A)), x)
```

Fricas [**A**]
time = 0.36, size = 56, normalized size = 0.58

$$\frac{e^{\left(\frac{A+B}{Bn}\right)}\,\mathrm{log\_integral}\left(\frac{(dx+c)e^{\left(-\frac{A+B}{Bn}\right)}}{bx+a}\right)}{(Bbc-Bad)g^2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="
fricas")
```

```
[Out] e^((A + B)/(B*n))*log_integral((d*x + c)*e^(-(A + B)/(B*n))/(b*x + a))/((B*
b*c - B*a*d)*g^2*n)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)**2/(A+B*ln(e*(b*x+a)**n/((d*x+c)**n))),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="
giac")
```

```
[Out] integrate(1/((b*g*x + a*g)^2*(B*log((b*x + a)^n*e/(d*x + c)^n) + A)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a\,g+b\,g\,x\right)}^2\,\left(A+B\,\ln\left(\frac{e\,{\left(a+b\,x\right)}^n}{{\left(c+d\,x\right)}^n}\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*g + b*g*x)^2*(A + B*log((e*(a + b*x)^n)/(c + d*x)^n))),x)
```

```
[Out] int(1/((a*g + b*g*x)^2*(A + B*log((e*(a + b*x)^n)/(c + d*x)^n))), x)
```

3.230 $\int (f+gx)^4 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$

Optimal. Leaf size=355

$$\frac{B(bc-ad)g(a^3d^3g^3-a^2bd^2g^2(5df-cg)+ab^2dg(10d^2f^2-5cdfg+c^2g^2)-b^3(10d^3f^3-10cd^2f^2g+5c^2dfg^2}{5b^4d^4}$$

```
[Out] 1/5*B*(-a*d+b*c)*g*(a^3*d^3*g^3-a^2*b*d^2*g^2*(-c*g+5*d*f)+a*b^2*d*g*(c^2*g
^2-5*c*d*f*g+10*d^2*f^2)-b^3*(-c^3*g^3+5*c^2*d*f*g^2-10*c*d^2*f^2*g+10*d^3*
f^3))*x/b^4/d^4-1/10*B*(-a*d+b*c)*g^2*(a^2*d^2*g^2-a*b*d*g*(-c*g+5*d*f)+b^2
*(c^2*g^2-5*c*d*f*g+10*d^2*f^2))*x^2/b^3/d^3-1/15*B*(-a*d+b*c)*g^3*(-a*d*g-
b*c*g+5*b*d*f)*x^3/b^2/d^2-1/20*B*(-a*d+b*c)*g^4*x^4/b/d-1/5*B*(-a*g+b*f)^5
*ln(b*x+a)/b^5/g+1/5*(g*x+f)^5*(A+B*ln(e*(b*x+a)/(d*x+c)))/g+1/5*B*(-c*g+d*
f)^5*ln(d*x+c)/d^5/g
```

Rubi [A]
time = 0.34, antiderivative size = 355, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {2548, 84}

$$-\frac{Bg^2x^2(bc-ad)\left(a^2d^2g^2-abdg(5df-cg)+b^2(c^2g^2-5cdfg+10d^2f^2)\right)}{10b^3d^3}+\frac{Bgx(bc-ad)\left(a^3d^3g^3-a^2bd^2g^2(5df-cg)+ab^2dg(c^2g^2-5cdfg+10d^2f^2)-(b^3(-c^3g^3+5c^2dfg^2-10cd^2f^2g+10d^3f^3))\right)}{5b^4d^4}+\frac{(f+gx)^5\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{5g}-\frac{B(bf-ag)^5\log(a+bx)}{5b^5g}-\frac{Bg^3x^3(bc-ad)(-adg-bcg+5bdf)}{15b^2d^2}-\frac{Bg^4x^4(bc-ad)}{20bd}+\frac{B(df-cg)^5\log(c+dx)}{5d^5g}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
```

```
[Out] (B*(b*c - a*d)*g*(a^3*d^3*g^3 - a^2*b*d^2*g^2*(5*d*f - c*g) + a*b^2*d*g*(10
*d^2*f^2 - 5*c*d*f*g + c^2*g^2) - b^3*(10*d^3*f^3 - 10*c*d^2*f^2*g + 5*c^2*
d*f*g^2 - c^3*g^3))*x)/(5*b^4*d^4) - (B*(b*c - a*d)*g^2*(a^2*d^2*g^2 - a*b*
d*g*(5*d*f - c*g) + b^2*(10*d^2*f^2 - 5*c*d*f*g + c^2*g^2))*x^2)/(10*b^3*d^
3) - (B*(b*c - a*d)*g^3*(5*b*d*f - b*c*g - a*d*g)*x^3)/(15*b^2*d^2) - (B*(b
*c - a*d)*g^4*x^4)/(20*b*d) - (B*(b*f - a*g)^5*Log[a + b*x])/(5*b^5*g) + ((
f + g*x)^5*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(5*g) + (B*(d*f - c*g)^5*L
og[c + d*x])/(5*d^5*g)
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 2548

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Dist[B*n*((b*c
```

```
- a*d)/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /;
 FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c -
 a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Rubi steps

$$\int (f+gx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)dx=\frac{(f+gx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{5g}-\frac{B\int\frac{(bc-ad)(f+gx)^5}{(a+bx)(c+dx)}\,dx}{5g}$$

$$=\frac{(f+gx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{5g}-\frac{(B(bc-ad))\int\frac{(f+gx)^5}{(a+bx)(c+d}}{5g}$$

$$=\frac{(f+gx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{5g}-\frac{(B(bc-ad))\int\left(\frac{g^2(-a^3d^3}{}\right.}{}$$

$$=\frac{B(bc-ad)g(a^3d^3g^3-a^2bd^2g^2(5df-cg)+ab^2dg(10d^2f^2-5}{5b^4d}$$

Mathematica [A]
time = 0.40, size = 279, normalized size = 0.79

$$\frac{\frac{B(-bc+ad)g^2x\left(-12a^3d^3g^3+6a^2bd^2g^2(10df-2cg+dgx)-2ab^2dg\left(6c^2g^2-3cdg(10f+gx)+d^2\left(60f^2+15fgx+2g^2x^2\right)\right)+b^3\left(-12c^3g^3+6c^2dg^2(10f+gx)-2cd^2g\left(60f^2+15fgx+2g^2x^2\right)+d^3\left(120f^3+60f^2gx+20fg^2x^2+3g^3x^3\right)\right)\right)}{12b^4d^4}-\frac{B(bf-ag)^5\log(a+bx)}{b^5}+(f+gx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)+\frac{B(df-cg)^5\log(c+dx)}{d^5}}{5g}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
```

```
[Out] ((B*(-(b*c) + a*d)*g^2*x*(-12*a^3*d^3*g^3 + 6*a^2*b*d^2*g^2*(10*d*f - 2*c*g
 + d*g*x) - 2*a*b^2*d*g*(6*c^2*g^2 - 3*c*d*g*(10*f + g*x) + d^2*(60*f^2 + 1
5*f*g*x + 2*g^2*x^2)) + b^3*(-12*c^3*g^3 + 6*c^2*d*g^2*(10*f + g*x) - 2*c*d
^2*g*(60*f^2 + 15*f*g*x + 2*g^2*x^2) + d^3*(120*f^3 + 60*f^2*g*x + 20*f*g^2
*x^2 + 3*g^3*x^3))))/(12*b^4*d^4) - (B*(b*f - a*g)^5*Log[a + b*x])/b^5 + (f
 + g*x)^5*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + (B*(d*f - c*g)^5*Log[c + d
*x])/d^5)/(5*g)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 14847 vs. $2(341) = 682$.
time = 0.46, size = 14848, normalized size = 41.83

method	result
risch	$-\frac{g^4Ba^4x}{5b^4}+\frac{g^4Bc^4x}{5d^4}-\frac{g^3B\ln(-bx-a)a^4f}{b^4}+\frac{2g^2B\ln(-bx-a)a^3f^2}{b^3}-\frac{2gB\ln(-bx-a)a^2f^3}{b^2}+\frac{g^3B\ln(dx+c)c}{d^4}$
derivativedivides	Expression too large to display

default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^4*(A+B*ln(e*(b*x+a)/(d*x+c))),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [A]
time = 0.30, size = 603, normalized size = 1.70

$$\frac{1}{5}\,Ag^4x^5+Afg^3x^4+2\,Af^2g^2x^3+2\,Af^3gx^2+\left(x\log\left(\frac{bxe}{dx+c}+\frac{ae}{dx+c}\right)+\frac{a\log\left(bx+a\right)}{b}-\frac{c\log\left(dx+c\right)}{d}\right)Bf^4+2\left(x^2\log\left(\frac{bxe}{dx+c}+\frac{ae}{dx+c}\right)-\frac{a^2\log\left(bx+a\right)}{b^2}+\frac{c^2\log\left(dx+c\right)}{d^2}-\frac{(bc-ad)x}{bd}\right)Bf^3g+\left(2\,x^3\log\left(\frac{bxe}{dx+c}+\frac{ae}{dx+c}\right)+\frac{2\,a^3\log\left(bx+a\right)}{b^3}-\frac{2\,c^3\log\left(dx+c\right)}{d^3}-\frac{(b^2cd-abd^2)x^2-2\,(b^2c^2-a^2d^2)x}{b^2d^2}\right)Bf^2g^2+\frac{1}{6}\left(6\,x^4\log\left(\frac{bxe}{dx+c}+\frac{ae}{dx+c}\right)-\frac{6\,a^4\log\left(bx+a\right)}{b^4}+\frac{6\,c^4\log\left(dx+c\right)}{d^4}-\frac{2\,(b^3cd^2-ab^2d^3)x^3-3\,(b^3c^2d-a^2bd^3)x^2+6\,(b^3c^3-a^3d^3)x}{b^3d^3}\right)Bfg^3+\frac{1}{60}\left(12\,x^5\log\left(\frac{bxe}{dx+c}+\frac{ae}{dx+c}\right)+\frac{12\,a^5\log\left(bx+a\right)}{b^5}-\frac{12\,c^5\log\left(dx+c\right)}{d^5}-\frac{3\,(b^4cd^3-ab^3d^4)x^4-4\,(b^4c^2d^2-a^2b^2d^4)x^3+6\,(b^4c^3d-a^3bd^4)x^2-12\,(b^4c^4-a^4d^4)x}{b^4d^4}\right)Bg^4+Af^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^4*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="maxima")
```

```
[Out] 1/5*A*g^4*x^5 + A*f*g^3*x^4 + 2*A*f^2*g^2*x^3 + 2*A*f^3*g*x^2 + (x*log(b*x*
e/(d*x + c) + a*e/(d*x + c)) + a*log(b*x + a)/b - c*log(d*x + c)/d)*B*f^4 +
 2*(x^2*log(b*x*e/(d*x + c) + a*e/(d*x + c)) - a^2*log(b*x + a)/b^2 + c^2*l
og(d*x + c)/d^2 - (b*c - a*d)*x/(b*d))*B*f^3*g + (2*x^3*log(b*x*e/(d*x + c)
 + a*e/(d*x + c)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2
*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*B*f^2*g^2 + 1/6*(
6*x^4*log(b*x*e/(d*x + c) + a*e/(d*x + c)) - 6*a^4*log(b*x + a)/b^4 + 6*c^4
*log(d*x + c)/d^4 - (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d
^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3))*B*f*g^3 + 1/60*(12*x^5*log(b*
x*e/(d*x + c) + a*e/(d*x + c)) + 12*a^5*log(b*x + a)/b^5 - 12*c^5*log(d*x +
 c)/d^5 - (3*(b^4*c*d^3 - a*b^3*d^4)*x^4 - 4*(b^4*c^2*d^2 - a^2*b^2*d^4)*x^
3 + 6*(b^4*c^3*d - a^3*b*d^4)*x^2 - 12*(b^4*c^4 - a^4*d^4)*x)/(b^4*d^4))*B*
g^4 + A*f^4*x
```

Fricas [A]
time = 0.78, size = 635, normalized size = 1.79

$$\frac{12\,Ab^5d^5g^4x^5+3\,(20\,Ab^5d^5fg^3-(Bb^5cd^4-Bab^4d^5)g^4)x^4+4\,(30\,Ab^5d^5f^2g^2-5\,(Bb^5cd^4-Bab^4d^5)fg^3+(Bb^5c^2d^3-Ba^2b^3d^5)g^4)x^3+6\,(20\,Ab^5d^5f^3g-10\,(Bb^5cd^4-Bab^4d^5)f^2g^2+5\,(Bb^5c^2d^3-Ba^2b^3d^5)fg^3-(Bb^5c^3d^2-Ba^3b^2d^5)g^4)x^2+12\,(5\,Ab^5d^5f^4-10\,(Bb^5cd^4-Bab^4d^5)f^3g+10\,(Bb^5c^2d^3-Ba^2b^3d^5)f^2g^2-5\,(Bb^5c^3d^2-Ba^3b^2d^5)fg^3+(Bb^5c^4d-Ba^4bd^5)g^4)x+12\,(5\,Bab^4d^5f^4-10\,Ba^2b^3d^5f^3g+10\,Ba^3b^2d^5f^2g^2-5\,Ba^4bd^5fg^3+Ba^5d^5g^4)\log\left(bx+a\right)-12\,(5\,Bb^5cd^4f^4-10\,Bb^5c^2d^3f^3g+10\,Bb^5c^3d^2f^2g^2-5\,Bb^5c^4dfg^3+Bb^5c^5g^4)\log\left(dx+c\right)+12\,(Bb^5d^5g^4x^5+5\,Bb^5d^5fg^3x^4+10\,Bb^5d^5f^2g^2x^3+10\,Bb^5d^5f^3gx^2+5\,Bb^5d^5f^4x)\log\left(\frac{(bx+a)e}{dx+c}\right)}{60\,b^5d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^4*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="fricas")
```

```
[Out] 1/60*(12*A*b^5*d^5*g^4*x^5 + 3*(20*A*b^5*d^5*f*g^3 - (B*b^5*c*d^4 - B*a*b^4
*d^5)*g^4)*x^4 + 4*(30*A*b^5*d^5*f^2*g^2 - 5*(B*b^5*c*d^4 - B*a*b^4*d^5)*f*
g^3 + (B*b^5*c^2*d^3 - B*a^2*b^3*d^5)*g^4)*x^3 + 6*(20*A*b^5*d^5*f^3*g - 10
*(B*b^5*c*d^4 - B*a*b^4*d^5)*f^2*g^2 + 5*(B*b^5*c^2*d^3 - B*a^2*b^3*d^5)*f*
g^3 - (B*b^5*c^3*d^2 - B*a^3*b^2*d^5)*g^4)*x^2 + 12*(5*A*b^5*d^5*f^4 - 10*(
B*b^5*c*d^4 - B*a*b^4*d^5)*f^3*g + 10*(B*b^5*c^2*d^3 - B*a^2*b^3*d^5)*f^2*g
^2 - 5*(B*b^5*c^3*d^2 - B*a^3*b^2*d^5)*f*g^3 + (B*b^5*c^4*d - B*a^4*b*d^5)*
g^4)*x + 12*(5*B*a*b^4*d^5*f^4 - 10*B*a^2*b^3*d^5*f^3*g + 10*B*a^3*b^2*d^5*
```

```
f^2*g^2 - 5*B*a^4*b*d^5*f*g^3 + B*a^5*d^5*g^4)*log(b*x + a) - 12*(5*B*b^5*c
*d^4*f^4 - 10*B*b^5*c^2*d^3*f^3*g + 10*B*b^5*c^3*d^2*f^2*g^2 - 5*B*b^5*c^4*
d*f*g^3 + B*b^5*c^5*g^4)*log(d*x + c) + 12*(B*b^5*d^5*g^4*x^5 + 5*B*b^5*d^5
*f*g^3*x^4 + 10*B*b^5*d^5*f^2*g^2*x^3 + 10*B*b^5*d^5*f^3*g*x^2 + 5*B*b^5*d^
5*f^4*x)*log((b*x + a)*e/(d*x + c)))/(b^5*d^5)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1436 vs. $2(337) = 674$.
time = 89.01, size = 1436, normalized size = 4.05

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**4*(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

```
[Out] A*g**4*x**5/5 + B*a*(a**4*g**4 - 5*a**3*b*f*g**3 + 10*a**2*b**2*f**2*g**2 -
 10*a*b**3*f**3*g + 5*b**4*f**4)*log(x + (B*a**5*c*d**4*g**4 - 5*B*a**4*b*c
*d**4*f*g**3 + 10*B*a**3*b**2*c*d**4*f**2*g**2 - 10*B*a**2*b**3*c*d**4*f**3
*g + B*a**2*d**5*(a**4*g**4 - 5*a**3*b*f*g**3 + 10*a**2*b**2*f**2*g**2 - 10
*a*b**3*f**3*g + 5*b**4*f**4)/b + B*a*b**4*c**5*g**4 - 5*B*a*b**4*c**4*d*f*
g**3 + 10*B*a*b**4*c**3*d**2*f**2*g**2 - 10*B*a*b**4*c**2*d**3*f**3*g + 10*
B*a*b**4*c*d**4*f**4 - B*a*c*d**4*(a**4*g**4 - 5*a**3*b*f*g**3 + 10*a**2*b*
*2*f**2*g**2 - 10*a*b**3*f**3*g + 5*b**4*f**4))/(B*a**5*d**5*g**4 - 5*B*a**
4*b*d**5*f*g**3 + 10*B*a**3*b**2*d**5*f**2*g**2 - 10*B*a**2*b**3*d**5*f**3*
g + 5*B*a*b**4*d**5*f**4 + B*b**5*c**5*g**4 - 5*B*b**5*c**4*d*f*g**3 + 10*B
*b**5*c**3*d**2*f**2*g**2 - 10*B*b**5*c**2*d**3*f**3*g + 5*B*b**5*c*d**4*f*
*4))/(5*b**5) - B*c*(c**4*g**4 - 5*c**3*d*f*g**3 + 10*c**2*d**2*f**2*g**2 -
 10*c*d**3*f**3*g + 5*d**4*f**4)*log(x + (B*a**5*c*d**4*g**4 - 5*B*a**4*b*c
*d**4*f*g**3 + 10*B*a**3*b**2*c*d**4*f**2*g**2 - 10*B*a**2*b**3*c*d**4*f**3
*g + B*a*b**4*c**5*g**4 - 5*B*a*b**4*c**4*d*f*g**3 + 10*B*a*b**4*c**3*d**2*
f**2*g**2 - 10*B*a*b**4*c**2*d**3*f**3*g + 10*B*a*b**4*c*d**4*f**4 - B*a*b*
*4*c*(c**4*g**4 - 5*c**3*d*f*g**3 + 10*c**2*d**2*f**2*g**2 - 10*c*d**3*f**3
*g + 5*d**4*f**4) + B*b**5*c**2*(c**4*g**4 - 5*c**3*d*f*g**3 + 10*c**2*d**2
*f**2*g**2 - 10*c*d**3*f**3*g + 5*d**4*f**4)/d)/(B*a**5*d**5*g**4 - 5*B*a**
4*b*d**5*f*g**3 + 10*B*a**3*b**2*d**5*f**2*g**2 - 10*B*a**2*b**3*d**5*f**3*
g + 5*B*a*b**4*d**5*f**4 + B*b**5*c**5*g**4 - 5*B*b**5*c**4*d*f*g**3 + 10*B
*b**5*c**3*d**2*f**2*g**2 - 10*B*b**5*c**2*d**3*f**3*g + 5*B*b**5*c*d**4*f*
*4))/(5*d**5) + x**4*(A*f*g**3 + B*a*g**4/(20*b) - B*c*g**4/(20*d)) + x**3*
(2*A*f**2*g**2 - B*a**2*g**4/(15*b**2) + B*a*f*g**3/(3*b) + B*c**2*g**4/(15
*d**2) - B*c*f*g**3/(3*d)) + x**2*(2*A*f**3*g + B*a**3*g**4/(10*b**3) - B*a
**2*f*g**3/(2*b**2) + B*a*f**2*g**2/b - B*c**3*g**4/(10*d**3) + B*c**2*f*g*
*3/(2*d**2) - B*c*f**2*g**2/d) + x*(A*f**4 - B*a**4*g**4/(5*b**4) + B*a**3*
f*g**3/b**3 - 2*B*a**2*f**2*g**2/b**2 + 2*B*a*f**3*g/b + B*c**4*g**4/(5*d**
4) - B*c**3*f*g**3/d**3 + 2*B*c**2*f**2*g**2/d**2 - 2*B*c*f**3*g/d) + (B*f*
*4*x + 2*B*f**3*g*x**2 + 2*B*f**2*g**2*x**3 + B*f*g**3*x**4 + B*g**4*x**5/5
)*log(e*(a + b*x)/(c + d*x))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 19084 vs. $2(342) = 684$.
time = 4.90, size = 19084, normalized size = 53.76

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^4*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="giac")
```

```
[Out] 1/60*(60*B*b^11*c^2*d^4*f^4*e^6*log(-b*e + (b*x*e + a*e)*d/(d*x + c)) - 120
*B*a*b^10*c*d^5*f^4*e^6*log(-b*e + (b*x*e + a*e)*d/(d*x + c)) + 60*B*a^2*b^
9*d^6*f^4*e^6*log(-b*e + (b*x*e + a*e)*d/(d*x + c)) - 120*B*b^11*c^3*d^3*f^
3*g*e^6*log(-b*e + (b*x*e + a*e)*d/(d*x + c)) + 120*B*a*b^10*c^2*d^4*f^3*g*
e^6*log(-b*e + (b*x*e + a*e)*d/(d*x + c)) + 120*B*a^2*b^9*c*d^5*f^3*g*e^6*l
og(-b*e + (b*x*e + a*e)*d/(d*x + c)) - 120*B*a^3*b^8*d^6*f^3*g*e^6*log(-b*e
 + (b*x*e + a*e)*d/(d*x + c)) + 120*B*b^11*c^4*d^2*f^2*g^2*e^6*log(-b*e + (
b*x*e + a*e)*d/(d*x + c)) - 120*B*a*b^10*c^3*d^3*f^2*g^2*e^6*log(-b*e + (b*
x*e + a*e)*d/(d*x + c)) - 120*B*a^3*b^8*c*d^5*f^2*g^2*e^6*log(-b*e + (b*x*e
 + a*e)*d/(d*x + c)) + 120*B*a^4*b^7*d^6*f^2*g^2*e^6*log(-b*e + (b*x*e + a*
e)*d/(d*x + c)) - 60*B*b^11*c^5*d*f*g^3*e^6*log(-b*e + (b*x*e + a*e)*d/(d*x
 + c)) + 60*B*a*b^10*c^4*d^2*f*g^3*e^6*log(-b*e + (b*x*e + a*e)*d/(d*x + c)
) + 60*B*a^4*b^7*c*d^5*f*g^3*e^6*log(-b*e + (b*x*e + a*e)*d/(d*x + c)) - 60
*B*a^5*b^6*d^6*f*g^3*e^6*log(-b*e + (b*x*e + a*e)*d/(d*x + c)) + 12*B*b^11*
c^6*g^4*e^6*log(-b*e + (b*x*e + a*e)*d/(d*x + c)) - 12*B*a*b^10*c^5*d*g^4*e
^6*log(-b*e + (b*x*e + a*e)*d/(d*x + c)) - 12*B*a^5*b^6*c*d^5*g^4*e^6*log(-
b*e + (b*x*e + a*e)*d/(d*x + c)) + 12*B*a^6*b^5*d^6*g^4*e^6*log(-b*e + (b*x
*e + a*e)*d/(d*x + c)) - 300*(b*x*e + a*e)*B*b^10*c^2*d^5*f^4*e^5*log(-b*e
+ (b*x*e + a*e)*d/(d*x + c))/(d*x + c) + 600*(b*x*e + a*e)*B*a*b^9*c*d^6*f^
4*e^5*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c) - 300*(b*x*e + a*e)*B
*a^2*b^8*d^7*f^4*e^5*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c) + 600*
(b*x*e + a*e)*B*b^10*c^3*d^4*f^3*g*e^5*log(-b*e + (b*x*e + a*e)*d/(d*x + c)
)/(d*x + c) - 600*(b*x*e + a*e)*B*a*b^9*c^2*d^5*f^3*g*e^5*log(-b*e + (b*x*e
 + a*e)*d/(d*x + c))/(d*x + c) - 600*(b*x*e + a*e)*B*a^2*b^8*c*d^6*f^3*g*e^
5*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c) + 600*(b*x*e + a*e)*B*a^3
*b^7*d^7*f^3*g*e^5*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c) - 600*(b
*x*e + a*e)*B*b^10*c^4*d^3*f^2*g^2*e^5*log(-b*e + (b*x*e + a*e)*d/(d*x + c)
)/(d*x + c) + 600*(b*x*e + a*e)*B*a*b^9*c^3*d^4*f^2*g^2*e^5*log(-b*e + (b*x
*e + a*e)*d/(d*x + c))/(d*x + c) + 600*(b*x*e + a*e)*B*a^3*b^7*c*d^6*f^2*g^
2*e^5*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c) - 600*(b*x*e + a*e)*B
*a^4*b^6*d^7*f^2*g^2*e^5*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c) +
300*(b*x*e + a*e)*B*b^10*c^5*d^2*f*g^3*e^5*log(-b*e + (b*x*e + a*e)*d/(d*x
+ c))/(d*x + c) - 300*(b*x*e + a*e)*B*a*b^9*c^4*d^3*f*g^3*e^5*log(-b*e + (b
*x*e + a*e)*d/(d*x + c))/(d*x + c) - 300*(b*x*e + a*e)*B*a^4*b^6*c*d^6*f*g^
3*e^5*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c) + 300*(b*x*e + a*e)*B
*a^5*b^5*d^7*f*g^3*e^5*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c) - 60
```

```
*(b*x*e + a*e)*B*b^10*c^6*d*g^4*e^5*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(
d*x + c) + 60*(b*x*e + a*e)*B*a*b^9*c^5*d^2*g^4*e^5*log(-b*e + (b*x*e + a*e
)*d/(d*x + c))/(d*x + c) + 60*(b*x*e + a*e)*B*a^5*b^5*c*d^6*g^4*e^5*log(-b*
e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c) - 60*(b*x*e + a*e)*B*a^6*b^4*d^7*g
^4*e^5*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c) + 600*(b*x*e + a*e)^
2*B*b^9*c^2*d^6*f^4*e^4*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^2 -
 1200*(b*x*e + a*e)^2*B*a*b^8*c*d^7*f^4*e^4*log(-b*e + (b*x*e + a*e)*d/(d*x
 + c))/(d*x + c)^2 + 600*(b*x*e + a*e)^2*B*a^2*b^7*d^8*f^4*e^4*log(-b*e + (
b*x*e + a*e)*d/(d*x + c))/(d*x + c)^2 - 1200*(b*x*e + a*e)^2*B*b^9*c^3*d^5*
f^3*g*e^4*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^2 + 1200*(b*x*e +
 a*e)^2*B*a*b^8*c^2*d^6*f^3*g*e^4*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*
x + c)^2 + 1200*(b*x*e + a*e)^2*B*a^2*b^7*c*d^7*f^3*g*e^4*log(-b*e + (b*x*e
 + a*e)*d/(d*x + c))/(d*x + c)^2 - 1200*(b*x*e + a*e)^2*B*a^3*b^6*d^8*f^3*g
*e^4*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^2 + 1200*(b*x*e + a*e)
^2*B*b^9*c^4*d^4*f^2*g^2*e^4*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c
)^2 - 1200*(b*x*e + a*e)^2*B*a*b^8*c^3*d^5*f^2*g^2*e^4*log(-b*e + (b*x*e +
a*e)*d/(d*x + c))/(d*x + c)^2 - 1200*(b*x*e + a*e)^2*B*a^3*b^6*c*d^7*f^2*g^
2*e^4*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^2 + 1200*(b*x*e + a*e
)^2*B*a^4*b^5*d^8*f^2*g^2*e^4*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x +
c)^2 - 600*(b*x*e + a*e)^2*B*b^9*c^5*d^3*f*g^3*e^4*log(-b*e + (b*x*e + a*e)
*d/(d*x + c))/(d*x + c)^2 + 600*(b*x*e + a*e)^2*B*a*b^8*c^4*d^4*f*g^3*e^4*l
og(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^2 + 600*(b*x*e + a*e)^2*B*a^
4*b^5*c*d^7*f*g^3*e^4*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^2 - 6
00*(b*x*e + a*e)^2*B*a^5*b^4*d^8*f*g^3*e^4*log(-b*e + (b*x*e + a*e)*d/(d*x
+ c))/(d*x + c)^2 + 120*(b*x*e + a*e)^2*B*b^9*c^6*d^2*g^4*e^4*log(-b*e + (b
*x*e + a*e)*d/(d*x + c))/(d*x + c)^2 - 120*(b*x*e + a*e)^2*B*a*b^8*c^5*d^3*
g^4*e^4*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^2 - 120*(b*x*e + a*
e)^2*B*a^5*b^4*c*d^7*g^4*e^4*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c
)^2 + 120*(b*x*e + a*e)^2*B*a^6*b^3*d^8*g^4*e^4*log(-b*e + (b*x*e + a*e)*d/
(d*x + c))/(d*x + c)^2 - 600*(b*x*e + a*e)^3*B*b^8*c^2*d^7*f^4*e^3*log(-b*e
 + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^3 + 120...
```

Mupad [B]
time = 5.34, size = 1392, normalized size = 3.92

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^4*(A + B*log((e*(a + b*x))/(c + d*x))),x)
```

```
[Out] x^2*((20*A*a*c*f*g^3 + 20*A*b*d*f^3*g + 30*A*a*d*f^2*g^2 + 30*A*b*c*f^2*g^2
 + 10*B*a*d*f^2*g^2 - 10*B*b*c*f^2*g^2)/(10*b*d) + ((5*a*d + 5*b*c)*((((5*A
*a*d*g^4 + 5*A*b*c*g^4 + B*a*d*g^4 - B*b*c*g^4 + 20*A*b*d*f*g^3)/(5*b*d) -
(A*g^4*(5*a*d + 5*b*c))/(5*b*d))*(5*a*d + 5*b*c))/(5*b*d) - (5*A*a*c*g^4 +
20*A*a*d*f*g^3 + 20*A*b*c*f*g^3 + 5*B*a*d*f*g^3 - 5*B*b*c*f*g^3 + 30*A*b*d*
```

```
f^2*g^2)/(5*b*d) + (A*a*c*g^4)/(b*d)))/(10*b*d) - (a*c*((5*A*a*d*g^4 + 5*A*
b*c*g^4 + B*a*d*g^4 - B*b*c*g^4 + 20*A*b*d*f*g^3)/(5*b*d) - (A*g^4*(5*a*d +
 5*b*c))/(5*b*d)))/(2*b*d)) + x^4*((5*A*a*d*g^4 + 5*A*b*c*g^4 + B*a*d*g^4 -
 B*b*c*g^4 + 20*A*b*d*f*g^3)/(20*b*d) - (A*g^4*(5*a*d + 5*b*c))/(20*b*d)) +
 log((e*(a + b*x))/(c + d*x))*((B*g^4*x^5)/5 + B*f^4*x + 2*B*f^2*g^2*x^3 +
2*B*f^3*g*x^2 + B*f*g^3*x^4) + x*((5*A*b*d*f^4 + 20*A*a*d*f^3*g + 20*A*b*c*
f^3*g + 10*B*a*d*f^3*g - 10*B*b*c*f^3*g + 30*A*a*c*f^2*g^2)/(5*b*d) - ((5*a
*d + 5*b*c)*((20*A*a*c*f*g^3 + 20*A*b*d*f^3*g + 30*A*a*d*f^2*g^2 + 30*A*b*c
*f^2*g^2 + 10*B*a*d*f^2*g^2 - 10*B*b*c*f^2*g^2)/(5*b*d) + ((5*a*d + 5*b*c)*
((((5*A*a*d*g^4 + 5*A*b*c*g^4 + B*a*d*g^4 - B*b*c*g^4 + 20*A*b*d*f*g^3)/(5*
b*d) - (A*g^4*(5*a*d + 5*b*c))/(5*b*d))*(5*a*d + 5*b*c))/(5*b*d) - (5*A*a*c
*g^4 + 20*A*a*d*f*g^3 + 20*A*b*c*f*g^3 + 5*B*a*d*f*g^3 - 5*B*b*c*f*g^3 + 30
*A*b*d*f^2*g^2)/(5*b*d) + (A*a*c*g^4)/(b*d)))/(5*b*d) - (a*c*((5*A*a*d*g^4
+ 5*A*b*c*g^4 + B*a*d*g^4 - B*b*c*g^4 + 20*A*b*d*f*g^3)/(5*b*d) - (A*g^4*(5
*a*d + 5*b*c))/(5*b*d)))/(b*d)))/(5*b*d) + (a*c*((((5*A*a*d*g^4 + 5*A*b*c*g
^4 + B*a*d*g^4 - B*b*c*g^4 + 20*A*b*d*f*g^3)/(5*b*d) - (A*g^4*(5*a*d + 5*b*
c))/(5*b*d))*(5*a*d + 5*b*c))/(5*b*d) - (5*A*a*c*g^4 + 20*A*a*d*f*g^3 + 20*
A*b*c*f*g^3 + 5*B*a*d*f*g^3 - 5*B*b*c*f*g^3 + 30*A*b*d*f^2*g^2)/(5*b*d) + (
A*a*c*g^4)/(b*d)))/(b*d)) - x^3*((((5*A*a*d*g^4 + 5*A*b*c*g^4 + B*a*d*g^4 -
 B*b*c*g^4 + 20*A*b*d*f*g^3)/(5*b*d) - (A*g^4*(5*a*d + 5*b*c))/(5*b*d))*(5*
a*d + 5*b*c))/(15*b*d) - (5*A*a*c*g^4 + 20*A*a*d*f*g^3 + 20*A*b*c*f*g^3 + 5
*B*a*d*f*g^3 - 5*B*b*c*f*g^3 + 30*A*b*d*f^2*g^2)/(15*b*d) + (A*a*c*g^4)/(3*
b*d)) + (A*g^4*x^5)/5 + (log(a + b*x)*((B*a^5*g^4)/5 + B*a*b^4*f^4 - 2*B*a^
2*b^3*f^3*g + 2*B*a^3*b^2*f^2*g^2 - B*a^4*b*f*g^3))/b^5 - (log(c + d*x)*(B*
c^5*g^4 + 5*B*c*d^4*f^4 - 10*B*c^2*d^3*f^3*g + 10*B*c^3*d^2*f^2*g^2 - 5*B*c
^4*d*f*g^3))/(5*d^5)
```

3.231 $\int (f+gx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\, dx$

Optimal. Leaf size=227

$$-\frac{B(bc-ad)g(a^2d^2g^2-abdg(4df-cg)+b^2(6d^2f^2-4cdfg+c^2g^2))\,x}{4b^3d^3}-\frac{B(bc-ad)g^2(4bdf-bcg-adg)x^2}{8b^2d^2}$$

```
[Out] -1/4*B*(-a*d+b*c)*g*(a^2*d^2*g^2-a*b*d*g*(-c*g+4*d*f)+b^2*(c^2*g^2-4*c*d*f*
g+6*d^2*f^2))*x/b^3/d^3-1/8*B*(-a*d+b*c)*g^2*(-a*d*g-b*c*g+4*b*d*f)*x^2/b^2
/d^2-1/12*B*(-a*d+b*c)*g^3*x^3/b/d-1/4*B*(-a*g+b*f)^4*ln(b*x+a)/b^4/g+1/4*(
g*x+f)^4*(A+B*ln(e*(b*x+a)/(d*x+c)))/g+1/4*B*(-c*g+d*f)^4*ln(d*x+c)/d^4/g
```

Rubi [A]
time = 0.21, antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {2548, 84}

$$-\frac{Bgx(bc-ad)\left(a^2d^2g^2-abdg(4df-cg)+b^2(c^2g^2-4cdfg+6d^2f^2)\right)}{4b^3d^3}+\frac{(f+gx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{4g}-\frac{B(bf-ag)^4\log(a+bx)}{4b^4g}-\frac{Bg^2x^2(bc-ad)(-adg-bcg+4bdf)}{8b^2d^2}-\frac{Bg^3x^3(bc-ad)}{12bd}+\frac{B(df-cg)^4\log(c+dx)}{4d^4g}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
```

```
[Out] -1/4*(B*(b*c - a*d)*g*(a^2*d^2*g^2 - a*b*d*g*(4*d*f - c*g) + b^2*(6*d^2*f^2
 - 4*c*d*f*g + c^2*g^2))*x)/(b^3*d^3) - (B*(b*c - a*d)*g^2*(4*b*d*f - b*c*g
 - a*d*g)*x^2)/(8*b^2*d^2) - (B*(b*c - a*d)*g^3*x^3)/(12*b*d) - (B*(b*f - a
*g)^4*Log[a + b*x])/(4*b^4*g) + ((f + g*x)^4*(A + B*Log[(e*(a + b*x))/(c +
d*x)]))/(4*g) + (B*(d*f - c*g)^4*Log[c + d*x])/(4*d^4*g)
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 2548

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Dist[B*n*((b*c
- a*d)/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /;
 FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c -
 a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Rubi steps

$$\int (f+gx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right) dx = \frac{(f+gx)^4 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4g} - \frac{B\int \frac{(bc-ad)(f+gx)^4}{(a+bx)(c+dx)} dx}{4g}$$

$$= \frac{(f+gx)^4 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4g} - \frac{(B(bc-ad))\int \frac{(f+gx)^4}{(a+bx)(c+dx)}$$

$$= \frac{(f+gx)^4 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4g} - \frac{(B(bc-ad))\int \left(\frac{g^2(a^2d^2g^2-$$

$$= -\frac{B(bc-ad)g(a^2d^2g^2-abdg(4df-cg)+b^2(6d^2f^2-4cdfg+}{4b^3d^3}$$

Mathematica [A]
time = 0.18, size = 215, normalized size = 0.95

$$\frac{(f+gx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)-\frac{B(6bd(bc-ad)g^2(a^2d^2g^2+abdg(-4df+cg)+b^2(6d^2f^2-4cdfg+c^2g^2))x+3b^2d^2(bc-ad)g^3(4bdf-bcg-adg)x^2+2b^3d^3(bc-ad)g^4x^3+6d^4(bf-ag)^4\log(a+bx)-6b^4(df-cg)^4\log(c+dx))}{6b^4d^4}}{4g}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
```

```
[Out] ((f + g*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - (B*(6*b*d*(b*c - a*d)*g
^2*(a^2*d^2*g^2 + a*b*d*g*(-4*d*f + c*g) + b^2*(6*d^2*f^2 - 4*c*d*f*g + c^2
*g^2))*x + 3*b^2*d^2*(b*c - a*d)*g^3*(4*b*d*f - b*c*g - a*d*g)*x^2 + 2*b^3*
d^3*(b*c - a*d)*g^4*x^3 + 6*d^4*(b*f - a*g)^4*Log[a + b*x] - 6*b^4*(d*f - c
*g)^4*Log[c + d*x]))/(6*b^4*d^4))/(4*g)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 7243 vs. $2(215) = 430$.
time = 0.42, size = 7244, normalized size = 31.91

method	result
risch	$\frac{(gx+f)^4 B\ln\left(\frac{e(bx+a)}{dx+c}\right)}{4g} - \frac{B\ln(bx+a)f^4}{4g} + \frac{B\ln(-dx-c)f^4}{4g} + g^2Afx^3 + \frac{g^2Bafx^2}{2b} - \frac{g^2Bcfx^2}{2d} + \frac{g^3Ax^4}{4} +$
derivativedivides	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^3*(A+B*ln(e*(b*x+a)/(d*x+c))),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [A]
time = 0.29, size = 423, normalized size = 1.86

$$\frac{1}{4}\,Ag^3x^4 + Afg^2x^3 + \frac{3}{2}\,Af^2gx^2 + \left(x\log\left(\frac{bxe}{dx+c} + \frac{ae}{dx+c}\right) + \frac{a\log\left(bx+a\right)}{b} - \frac{c\log\left(dx+c\right)}{d}\right)Bf^3 + \frac{3}{2}\left(x^2\log\left(\frac{bxe}{dx+c} + \frac{ae}{dx+c}\right) - \frac{a^2\log\left(bx+a\right)}{b^2} + \frac{c^2\log\left(dx+c\right)}{d^2} - \frac{(bc-ad)x}{bd}\right)Bf^2g + \frac{1}{2}\left(2\,x^3\log\left(\frac{bxe}{dx+c} + \frac{ae}{dx+c}\right) + \frac{2\,a^3\log\left(bx+a\right)}{b^3} - \frac{2\,c^3\log\left(dx+c\right)}{d^3} - \frac{(b^2cd-abd^2)x^2 - 2\,(b^2c^2-a^2d^2)x}{b^2d^2}\right)Bfg^2 + \frac{1}{24}\left(6\,x^4\log\left(\frac{bxe}{dx+c} + \frac{ae}{dx+c}\right) - \frac{6\,a^4\log\left(bx+a\right)}{b^4} + \frac{6\,c^4\log\left(dx+c\right)}{d^4} - \frac{2\,(b^3cd^2-ab^2d^3)x^3 - 3\,(b^3c^2d-a^2bd^3)x^2 + 6\,(b^3c^3-a^3d^3)x}{b^3d^3}\right)Bg^3 + Af^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="maxima")
```

```
[Out] 1/4*A*g^3*x^4 + A*f*g^2*x^3 + 3/2*A*f^2*g*x^2 + (x*log(b*x*e/(d*x + c) + a*
e/(d*x + c)) + a*log(b*x + a)/b - c*log(d*x + c)/d)*B*f^3 + 3/2*(x^2*log(b*
x*e/(d*x + c) + a*e/(d*x + c)) - a^2*log(b*x + a)/b^2 + c^2*log(d*x + c)/d^
2 - (b*c - a*d)*x/(b*d))*B*f^2*g + 1/2*(2*x^3*log(b*x*e/(d*x + c) + a*e/(d*
x + c)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b
*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*B*f*g^2 + 1/24*(6*x^4*log(b
*x*e/(d*x + c) + a*e/(d*x + c)) - 6*a^4*log(b*x + a)/b^4 + 6*c^4*log(d*x +
c)/d^4 - (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6
*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3))*B*g^3 + A*f^3*x
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 444 vs. 2(216) = 432.
time = 0.48, size = 444, normalized size = 1.96

$$\frac{6\,Ab^4d^4g^3x^4 + 2\,(12\,Ab^4d^4fg^2 - (Bb^4cd^3 - Bab^3d^4)g^3)x^3 + 3\,(12\,Ab^4d^4f^2g - 4\,(Bb^4cd^3 - Bab^3d^4)fg^2 + (Bb^4c^2d^2 - Ba^2b^2d^4)g^3)x^2 + 6\,(4\,Ab^4d^4f^3 - 6\,(Bb^4cd^3 - Bab^3d^4)f^2g + 4\,(Bb^4c^2d^2 - Ba^2b^2d^4)fg^2 - (Bb^4c^3d - Ba^3bd^4)g^3)x + 6\,(4\,Bab^3d^4f^3 - 6\,Ba^2b^2d^4f^2g + 4\,Ba^3bd^4fg^2 - Ba^4d^4g^3)\log\left(bx+a\right) - 6\,(4\,Bb^4cd^3f^3 - 6\,Bb^4c^2d^2f^2g + 4\,Bb^4c^3dfg^2 - Bb^4c^4g^3)\log\left(dx+c\right) + 6\,(Bb^4d^4g^3x^4 + 4\,Bb^4d^4fg^2x^3 + 6\,Bb^4d^4f^2gx^2 + 4\,Bb^4d^4f^3x)\log\left(\frac{(bx+a)e}{dx+c}\right)}{24\,b^4d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="fricas")
```

```
[Out] 1/24*(6*A*b^4*d^4*g^3*x^4 + 2*(12*A*b^4*d^4*f*g^2 - (B*b^4*c*d^3 - B*a*b^3*
d^4)*g^3)*x^3 + 3*(12*A*b^4*d^4*f^2*g - 4*(B*b^4*c*d^3 - B*a*b^3*d^4)*f*g^2
 + (B*b^4*c^2*d^2 - B*a^2*b^2*d^4)*g^3)*x^2 + 6*(4*A*b^4*d^4*f^3 - 6*(B*b^4
*c*d^3 - B*a*b^3*d^4)*f^2*g + 4*(B*b^4*c^2*d^2 - B*a^2*b^2*d^4)*f*g^2 - (B*
b^4*c^3*d - B*a^3*b*d^4)*g^3)*x + 6*(4*B*a*b^3*d^4*f^3 - 6*B*a^2*b^2*d^4*f^
2*g + 4*B*a^3*b*d^4*f*g^2 - B*a^4*d^4*g^3)*log(b*x + a) - 6*(4*B*b^4*c*d^3*
f^3 - 6*B*b^4*c^2*d^2*f^2*g + 4*B*b^4*c^3*d*f*g^2 - B*b^4*c^4*g^3)*log(d*x
+ c) + 6*(B*b^4*d^4*g^3*x^4 + 4*B*b^4*d^4*f*g^2*x^3 + 6*B*b^4*d^4*f^2*g*x^2
 + 4*B*b^4*d^4*f^3*x)*log((b*x + a)*e/(d*x + c)))/(b^4*d^4)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 998 vs. 2(207) = 414.
time = 15.12, size = 998, normalized size = 4.40

$$\frac{Ag^3x^4}{4} - \frac{Ba(ag-2bf)\left(a^2g^2-2abfg+2b^2f^2\right)\log\left(x + \frac{Ba^4cd^3g^3 - 4Ba^3bcd^3fg^2 + 6Ba^2b^2cd^3f^2g + \frac{Ba^2d^4(ag-2bf)\left(a^2g^2-2abfg+2b^2f^2\right)}{b} + Bab^3c^4g^3 - 4Bab^3c^3dfg^2 + 6Bab^3c^2d^2f^2g - 8Bab^3cd^3f^3 - Bacd^3(ag-2bf)(a^2g^2-2abfg+2b^2f^2)}{Ba^4d^4g^3 - 4Ba^3bd^4fg^2 + 6Ba^2b^2d^4f^2g - 4Bab^3d^4f^3 + Bb^4c^4g^3 - 4Bb^4c^3dfg^2 + 6Bb^4c^2d^2f^2g - 4Bb^4cd^3f^3}\right)}{4b^4} + \frac{Bc(cg-2df)\left(c^2g^2-2cdfg+2d^2f^2\right)\log\left(x + \frac{Ba^4cd^3g^3 - 4Ba^3bcd^3fg^2 + 6Ba^2b^2cd^3f^2g + Bab^3c^4g^3 - 4Bab^3c^3dfg^2 + 6Bab^3c^2d^2f^2g - 8Bab^3cd^3f^3 - Bab^3c(cg-2df)(c^2g^2-2cdfg+2d^2f^2) + \frac{Bb^4c^2(cg-2df)\left(c^2g^2-2cdfg+2d^2f^2\right)}{d}}{Ba^4d^4g^3 - 4Ba^3bd^4fg^2 + 6Ba^2b^2d^4f^2g - 4Bab^3d^4f^3 + Bb^4c^4g^3 - 4Bb^4c^3dfg^2 + 6Bb^4c^2d^2f^2g - 4Bb^4cd^3f^3}\right)}{4d^4} + x^3\left(Afg^2 + \frac{Bag^3}{12b} - \frac{Bcg^3}{12d}\right) + x^2\cdot\left(\frac{3Af^2g}{2} - \frac{Ba^2g^3}{8b^2} + \frac{Bafg^2}{2b} + \frac{Bc^2g^3}{8d^2} - \frac{Bcfg^2}{2d}\right) + x\left(Af^3 + \frac{Ba^3g^3}{4b^3} - \frac{Ba^2fg^2}{b^2} + \frac{3Baf^2g}{2b} - \frac{Bc^3g^3}{4d^3} + \frac{Bc^2fg^2}{d^2} - \frac{3Bcf^2g}{2d}\right) + \left(Bf^3x + \frac{3Bf^2gx^2}{2} + Bfg^2x^3 + \frac{Bg^3x^4}{4}\right)\log\left(\frac{e(a+bx)}{c+dx}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**3*(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

```
[Out] A*g**3*x**4/4 - B*a*(a*g - 2*b*f)*(a**2*g**2 - 2*a*b*f*g + 2*b**2*f**2)*log
(x + (B*a**4*c*d**3*g**3 - 4*B*a**3*b*c*d**3*f*g**2 + 6*B*a**2*b**2*c*d**3*
f**2*g + B*a**2*d**4*(a*g - 2*b*f)*(a**2*g**2 - 2*a*b*f*g + 2*b**2*f**2)/b
+ B*a*b**3*c**4*g**3 - 4*B*a*b**3*c**3*d*f*g**2 + 6*B*a*b**3*c**2*d**2*f**2
*g - 8*B*a*b**3*c*d**3*f**3 - B*a*c*d**3*(a*g - 2*b*f)*(a**2*g**2 - 2*a*b*f
*g + 2*b**2*f**2))/(B*a**4*d**4*g**3 - 4*B*a**3*b*d**4*f*g**2 + 6*B*a**2*b*
*2*d**4*f**2*g - 4*B*a*b**3*d**4*f**3 + B*b**4*c**4*g**3 - 4*B*b**4*c**3*d*
f*g**2 + 6*B*b**4*c**2*d**2*f**2*g - 4*B*b**4*c*d**3*f**3))/(4*b**4) + B*c*
(c*g - 2*d*f)*(c**2*g**2 - 2*c*d*f*g + 2*d**2*f**2)*log(x + (B*a**4*c*d**3*
g**3 - 4*B*a**3*b*c*d**3*f*g**2 + 6*B*a**2*b**2*c*d**3*f**2*g + B*a*b**3*c*
*4*g**3 - 4*B*a*b**3*c**3*d*f*g**2 + 6*B*a*b**3*c**2*d**2*f**2*g - 8*B*a*b*
*3*c*d**3*f**3 - B*a*b**3*c*(c*g - 2*d*f)*(c**2*g**2 - 2*c*d*f*g + 2*d**2*f
**2) + B*b**4*c**2*(c*g - 2*d*f)*(c**2*g**2 - 2*c*d*f*g + 2*d**2*f**2)/d)/(
B*a**4*d**4*g**3 - 4*B*a**3*b*d**4*f*g**2 + 6*B*a**2*b**2*d**4*f**2*g - 4*B
*a*b**3*d**4*f**3 + B*b**4*c**4*g**3 - 4*B*b**4*c**3*d*f*g**2 + 6*B*b**4*c*
*2*d**2*f**2*g - 4*B*b**4*c*d**3*f**3))/(4*d**4) + x**3*(A*f*g**2 + B*a*g**
3/(12*b) - B*c*g**3/(12*d)) + x**2*(3*A*f**2*g/2 - B*a**2*g**3/(8*b**2) + B
*a*f*g**2/(2*b) + B*c**2*g**3/(8*d**2) - B*c*f*g**2/(2*d)) + x*(A*f**3 + B*
a**3*g**3/(4*b**3) - B*a**2*f*g**2/b**2 + 3*B*a*f**2*g/(2*b) - B*c**3*g**3/
(4*d**3) + B*c**2*f*g**2/d**2 - 3*B*c*f**2*g/(2*d)) + (B*f**3*x + 3*B*f**2*
g*x**2/2 + B*f*g**2*x**3 + B*g**3*x**4/4)*log(e*(a + b*x)/(c + d*x))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 11299 vs. $2(216) = 432$.
time = 5.80, size = 11299, normalized size = 49.78

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="giac")
```

```
[Out] 1/24*(24*B*b^9*c^2*d^3*f^3*e^5*log(-b*e + (b*x*e + a*e)*d/(d*x + c)) - 48*B
*a*b^8*c*d^4*f^3*e^5*log(-b*e + (b*x*e + a*e)*d/(d*x + c)) + 24*B*a^2*b^7*d
^5*f^3*e^5*log(-b*e + (b*x*e + a*e)*d/(d*x + c)) - 36*B*b^9*c^3*d^2*f^2*g*e
^5*log(-b*e + (b*x*e + a*e)*d/(d*x + c)) + 36*B*a*b^8*c^2*d^3*f^2*g*e^5*log
(-b*e + (b*x*e + a*e)*d/(d*x + c)) + 36*B*a^2*b^7*c*d^4*f^2*g*e^5*log(-b*e
+ (b*x*e + a*e)*d/(d*x + c)) - 36*B*a^3*b^6*d^5*f^2*g*e^5*log(-b*e + (b*x*e
 + a*e)*d/(d*x + c)) + 24*B*b^9*c^4*d*f*g^2*e^5*log(-b*e + (b*x*e + a*e)*d/
(d*x + c)) - 24*B*a*b^8*c^3*d^2*f*g^2*e^5*log(-b*e + (b*x*e + a*e)*d/(d*x +
 c)) - 24*B*a^3*b^6*c*d^4*f*g^2*e^5*log(-b*e + (b*x*e + a*e)*d/(d*x + c)) +
 24*B*a^4*b^5*d^5*f*g^2*e^5*log(-b*e + (b*x*e + a*e)*d/(d*x + c)) - 6*B*b^9
*c^5*g^3*e^5*log(-b*e + (b*x*e + a*e)*d/(d*x + c)) + 6*B*a*b^8*c^4*d*g^3*e^
5*log(-b*e + (b*x*e + a*e)*d/(d*x + c)) + 6*B*a^4*b^5*c*d^4*g^3*e^5*log(-b*
e + (b*x*e + a*e)*d/(d*x + c)) - 6*B*a^5*b^4*d^5*g^3*e^5*log(-b*e + (b*x*e
+ a*e)*d/(d*x + c)) - 96*(b*x*e + a*e)*B*b^8*c^2*d^4*f^3*e^4*log(-b*e + (b*
```

x*e + a*e)*d/(d*x + c))/(d*x + c) + 192*(b*x*e + a*e)*B*a*b^7*c*d^5*f^3*e^4
*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c) - 96*(b*x*e + a*e)*B*a^2*b
^6*d^6*f^3*e^4*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c) + 144*(b*x*e
 + a*e)*B*b^8*c^3*d^3*f^2*g*e^4*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x
+ c) - 144*(b*x*e + a*e)*B*a*b^7*c^2*d^4*f^2*g*e^4*log(-b*e + (b*x*e + a*e)
*d/(d*x + c))/(d*x + c) - 144*(b*x*e + a*e)*B*a^2*b^6*c*d^5*f^2*g*e^4*log(-
b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c) + 144*(b*x*e + a*e)*B*a^3*b^5*d^
6*f^2*g*e^4*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c) - 96*(b*x*e + a
*e)*B*b^8*c^4*d^2*f*g^2*e^4*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)
 + 96*(b*x*e + a*e)*B*a*b^7*c^3*d^3*f*g^2*e^4*log(-b*e + (b*x*e + a*e)*d/(d
*x + c))/(d*x + c) + 96*(b*x*e + a*e)*B*a^3*b^5*c*d^5*f*g^2*e^4*log(-b*e +
(b*x*e + a*e)*d/(d*x + c))/(d*x + c) - 96*(b*x*e + a*e)*B*a^4*b^4*d^6*f*g^2
*e^4*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c) + 24*(b*x*e + a*e)*B*b
^8*c^5*d*g^3*e^4*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c) - 24*(b*x*
e + a*e)*B*a*b^7*c^4*d^2*g^3*e^4*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x
 + c) - 24*(b*x*e + a*e)*B*a^4*b^4*c*d^5*g^3*e^4*log(-b*e + (b*x*e + a*e)*d
/(d*x + c))/(d*x + c) + 24*(b*x*e + a*e)*B*a^5*b^3*d^6*g^3*e^4*log(-b*e + (
b*x*e + a*e)*d/(d*x + c))/(d*x + c) + 144*(b*x*e + a*e)^2*B*b^7*c^2*d^5*f^3
*e^3*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^2 - 288*(b*x*e + a*e)^
2*B*a*b^6*c*d^6*f^3*e^3*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^2 +
 144*(b*x*e + a*e)^2*B*a^2*b^5*d^7*f^3*e^3*log(-b*e + (b*x*e + a*e)*d/(d*x
+ c))/(d*x + c)^2 - 216*(b*x*e + a*e)^2*B*b^7*c^3*d^4*f^2*g*e^3*log(-b*e +
(b*x*e + a*e)*d/(d*x + c))/(d*x + c)^2 + 216*(b*x*e + a*e)^2*B*a*b^6*c^2*d^
5*f^2*g*e^3*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^2 + 216*(b*x*e
+ a*e)^2*B*a^2*b^5*c*d^6*f^2*g*e^3*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d
x + c)^2 - 216(b*x*e + a*e)^2*B*a^3*b^4*d^7*f^2*g*e^3*log(-b*e + (b*x*e +
 a*e)*d/(d*x + c))/(d*x + c)^2 + 144*(b*x*e + a*e)^2*B*b^7*c^4*d^3*f*g^2*e^
3*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^2 - 144*(b*x*e + a*e)^2*B
*a*b^6*c^3*d^4*f*g^2*e^3*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^2
- 144*(b*x*e + a*e)^2*B*a^3*b^4*c*d^6*f*g^2*e^3*log(-b*e + (b*x*e + a*e)*d/
(d*x + c))/(d*x + c)^2 + 144*(b*x*e + a*e)^2*B*a^4*b^3*d^7*f*g^2*e^3*log(-b
*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^2 - 36*(b*x*e + a*e)^2*B*b^7*c^5*
d^2*g^3*e^3*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^2 + 36*(b*x*e +
 a*e)^2*B*a*b^6*c^4*d^3*g^3*e^3*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x
+ c)^2 + 36*(b*x*e + a*e)^2*B*a^4*b^3*c*d^6*g^3*e^3*log(-b*e + (b*x*e + a*e
)*d/(d*x + c))/(d*x + c)^2 - 36*(b*x*e + a*e)^2*B*a^5*b^2*d^7*g^3*e^3*log(-
b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^2 - 96*(b*x*e + a*e)^3*B*b^6*c^2
*d^6*f^3*e^2*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^3 + 192*(b*x*e
 + a*e)^3*B*a*b^5*c*d^7*f^3*e^2*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x
+ c)^3 - 96*(b*x*e + a*e)^3*B*a^2*b^4*d^8*f^3*e^2*log(-b*e + (b*x*e + a*e)*
d/(d*x + c))/(d*x + c)^3 + 144*(b*x*e + a*e)^3*B*b^6*c^3*d^5*f^2*g*e^2*log(
-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^3 - 144*(b*x*e + a*e)^3*B*a*b^5
*c^2*d^6*f^2*g*e^2*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^3 - 144*
(b*x*e + a*e)^3*B*a^2*b^4*c*d^7*f^2*g*e^2*log(-b*e + (b*x*e + a*e)*d/(d*x +
 c))/(d*x + c)^3 + 144*(b*x*e + a*e)^3*B*a^3*b^3*d^8*f^2*g*e^2*log(-b*e + (

```
b*x*e + a*e)*d/(d*x + c))/(d*x + c)^3 - 96*(b*x*e + a*e)^3*B*b^6*c^4*d^4*f*
g^2*e^2*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^3 + 96*(b*x*e + a*e
)^3*B*a*b^5*c^3*d^5*f*g^2*e^2*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x +
c)^3 + 96*(b*x*e + a*e)^3*B*a^3*b^3*c*d^7*f*g^2*e^2*log(-b*e + (b*x*e + a*e
)*d/(d*x + c))/(d*x + c)^3 - 96*(b*x*e + a*e)^3*B*a^4*b^2*d^8*f*g^2*e^2*log
(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^3 + 24*(b*x*e + a*e)^3*B*b^6*c
^5*d^3*g^3*e^2*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^3 - 24*(b*x*
e + a*e)^3*B*a*b^5*c^4*d^4*g^3*e^2*log(-b*e + (...
```

Mupad [B]
time = 4.69, size = 741, normalized size = 3.26

$$x\left(\frac{4\,A\,b\,d\,f^3+12\,A\,a\,c\,f\,g^2+12\,A\,a\,d\,f^2\,g+12\,A\,b\,c\,f^2\,g+6\,B\,a\,d\,f^2\,g-6\,B\,b\,c\,f^2\,g}{4\,b\,d}+\frac{\left(4\,a\,d+4\,b\,c\right)\,\left(\frac{\left(\frac{4\,A\,a\,d\,g^3+4\,A\,b\,c\,g^3+B\,a\,d\,g^3-B\,b\,c\,g^3+12\,A\,b\,d\,f\,g^2}{4\,b\,d}-\frac{A\,g^3\,\left(4\,a\,d+4\,b\,c\right)}{4\,b\,d}\right)\,\left(4\,a\,d+4\,b\,c\right)}{4\,b\,d}-\frac{4\,A\,a\,c\,g^3+12\,A\,a\,d\,f\,g^2+12\,A\,b\,c\,f\,g^2+12\,A\,b\,d\,f^2\,g+4\,B\,a\,d\,f\,g^2-4\,B\,b\,c\,f\,g^2}{4\,b\,d}+\frac{A\,a\,c\,g^3}{b\,d}\right)}{4\,b\,d}-\frac{a\,c\,\left(\frac{4\,A\,a\,d\,g^3+4\,A\,b\,c\,g^3+B\,a\,d\,g^3-B\,b\,c\,g^3+12\,A\,b\,d\,f\,g^2}{4\,b\,d}-\frac{A\,g^3\,\left(4\,a\,d+4\,b\,c\right)}{4\,b\,d}\right)}{b\,d}\right)-x^2\,\left(\frac{\left(\frac{4\,A\,a\,d\,g^3+4\,A\,b\,c\,g^3+B\,a\,d\,g^3-B\,b\,c\,g^3+12\,A\,b\,d\,f\,g^2}{4\,b\,d}-\frac{A\,g^3\,\left(4\,a\,d+4\,b\,c\right)}{4\,b\,d}\right)\,\left(4\,a\,d+4\,b\,c\right)}{8\,b\,d}-\frac{4\,A\,a\,c\,g^3+12\,A\,a\,d\,f\,g^2+12\,A\,b\,c\,f\,g^2+12\,A\,b\,d\,f^2\,g+4\,B\,a\,d\,f\,g^2-4\,B\,b\,c\,f\,g^2}{8\,b\,d}+\frac{A\,a\,c\,g^3}{2\,b\,d}\right)+\ln\left(\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right)\,\left(\frac{B\,g^3\,x^4}{4}+B\,f^3\,x+\frac{3\,B\,f^2\,g\,x^2}{2}+B\,f\,g^2\,x^3\right)+x^3\,\left(\frac{4\,A\,a\,d\,g^3+4\,A\,b\,c\,g^3+B\,a\,d\,g^3-B\,b\,c\,g^3+12\,A\,b\,d\,f\,g^2}{12\,b\,d}-\frac{A\,g^3\,\left(4\,a\,d+4\,b\,c\right)}{12\,b\,d}\right)+\frac{A\,g^3\,x^4}{4}-\frac{\ln\left(a+b\,x\right)\,\left(B\,a^4\,g^3-4\,B\,a\,b^3\,f^3+6\,B\,a^2\,b^2\,f^2\,g-4\,B\,a^3\,b\,f\,g^2\right)}{4\,b^4}+\frac{\ln\left(c+d\,x\right)\,\left(B\,c^4\,g^3-4\,B\,c\,d^3\,f^3+6\,B\,c^2\,d^2\,f^2\,g-4\,B\,c^3\,d\,f\,g^2\right)}{4\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^3*(A + B*log((e*(a + b*x))/(c + d*x))),x)
```

```
[Out] x*((4*A*b*d*f^3 + 12*A*a*c*f*g^2 + 12*A*a*d*f^2*g + 12*A*b*c*f^2*g + 6*B*a*
d*f^2*g - 6*B*b*c*f^2*g)/(4*b*d) + ((4*a*d + 4*b*c)*((((4*A*a*d*g^3 + 4*A*b
*c*g^3 + B*a*d*g^3 - B*b*c*g^3 + 12*A*b*d*f*g^2)/(4*b*d) - (A*g^3*(4*a*d +
4*b*c))/(4*b*d))*(4*a*d + 4*b*c))/(4*b*d) - (4*A*a*c*g^3 + 12*A*a*d*f*g^2 +
 12*A*b*c*f*g^2 + 12*A*b*d*f^2*g + 4*B*a*d*f*g^2 - 4*B*b*c*f*g^2)/(4*b*d) +
 (A*a*c*g^3)/(b*d)))/(4*b*d) - (a*c*((4*A*a*d*g^3 + 4*A*b*c*g^3 + B*a*d*g^3
 - B*b*c*g^3 + 12*A*b*d*f*g^2)/(4*b*d) - (A*g^3*(4*a*d + 4*b*c))/(4*b*d)))/
(b*d)) - x^2*((((4*A*a*d*g^3 + 4*A*b*c*g^3 + B*a*d*g^3 - B*b*c*g^3 + 12*A*b
*d*f*g^2)/(4*b*d) - (A*g^3*(4*a*d + 4*b*c))/(4*b*d))*(4*a*d + 4*b*c))/(8*b*
d) - (4*A*a*c*g^3 + 12*A*a*d*f*g^2 + 12*A*b*c*f*g^2 + 12*A*b*d*f^2*g + 4*B*
a*d*f*g^2 - 4*B*b*c*f*g^2)/(8*b*d) + (A*a*c*g^3)/(2*b*d)) + log((e*(a + b*x
))/(c + d*x))*((B*g^3*x^4)/4 + B*f^3*x + (3*B*f^2*g*x^2)/2 + B*f*g^2*x^3) +
 x^3*((4*A*a*d*g^3 + 4*A*b*c*g^3 + B*a*d*g^3 - B*b*c*g^3 + 12*A*b*d*f*g^2)/
(12*b*d) - (A*g^3*(4*a*d + 4*b*c))/(12*b*d)) + (A*g^3*x^4)/4 - (log(a + b*x
)*(B*a^4*g^3 - 4*B*a*b^3*f^3 + 6*B*a^2*b^2*f^2*g - 4*B*a^3*b*f*g^2))/(4*b^4
) + (log(c + d*x)*(B*c^4*g^3 - 4*B*c*d^3*f^3 + 6*B*c^2*d^2*f^2*g - 4*B*c^3*
d*f*g^2))/(4*d^4)
```

3.232 $\int (f+gx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right) dx$

Optimal. Leaf size=150

$$-\frac{B(bc-ad)g(3bdf-bcg-adg)x}{3b^2d^2}-\frac{B(bc-ad)g^2x^2}{6bd}-\frac{B(bf-ag)^3\log(a+bx)}{3b^3g}+\frac{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3g}$$

```
[Out] -1/3*B*(-a*d+b*c)*g*(-a*d*g-b*c*g+3*b*d*f)*x/b^2/d^2-1/6*B*(-a*d+b*c)*g^2*x
^2/b/d-1/3*B*(-a*g+b*f)^3*ln(b*x+a)/b^3/g+1/3*(g*x+f)^3*(A+B*ln(e*(b*x+a)/(
d*x+c)))/g+1/3*B*(-c*g+d*f)^3*ln(d*x+c)/d^3/g
```

Rubi [A]
time = 0.10, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {2548, 84}

$$\frac{(f+gx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3g}-\frac{B(bf-ag)^3\log(a+bx)}{3b^3g}-\frac{Bgx(bc-ad)(-adg-bcg+3bdf)}{3b^2d^2}-\frac{Bg^2x^2(bc-ad)}{6bd}+\frac{B(df-cg)^3\log(c+dx)}{3d^3g}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
```

```
[Out] -1/3*(B*(b*c - a*d)*g*(3*b*d*f - b*c*g - a*d*g)*x)/(b^2*d^2) - (B*(b*c - a*
d)*g^2*x^2)/(6*b*d) - (B*(b*f - a*g)^3*Log[a + b*x])/(3*b^3*g) + ((f + g*x)
^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(3*g) + (B*(d*f - c*g)^3*Log[c + d
*x])/(3*d^3*g)
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 2548

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Dist[B*n*((b*c
- a*d)/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /;
 FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c -
 a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Rubi steps

$$\int (f+gx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right) dx = \frac{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3g} - \frac{B\int \frac{(bc-ad)(f+gx)^3}{(a+bx)(c+dx)}dx}{3g}$$

$$= \frac{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3g} - \frac{(B(bc-ad))\int \frac{(f+gx)^3}{(a+bx)(c+dx)}$$

$$= \frac{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3g} - \frac{(B(bc-ad))\int \left(\frac{g^2(3bdf-bcg}{b^2d^2}\right.$$

$$= -\frac{B(bc-ad)g(3bdf-bcg-adg)x}{3b^2d^2} - \frac{B(bc-ad)g^2x^2}{6bd} - \frac{B(bf$$

Mathematica [A]
time = 0.09, size = 142, normalized size = 0.95

$$\frac{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right) - \frac{B(2bd(bc-ad)g^2(3bdf-bcg-adg)x+b^2d^2(bc-ad)g^3x^2+2d^3(bf-ag)^3\log(a+bx)-2b^3(df-cg)^3\log(c+dx))}{2b^3d^3}}{3g}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
```

```
[Out] ((f + g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - (B*(2*b*d*(b*c - a*d)*g
^2*(3*b*d*f - b*c*g - a*d*g)*x + b^2*d^2*(b*c - a*d)*g^3*x^2 + 2*d^3*(b*f -
 a*g)^3*Log[a + b*x] - 2*b^3*(d*f - c*g)^3*Log[c + d*x]))/(2*b^3*d^3))/(3*g
)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2997 vs. 2(140) = 280.
time = 0.39, size = 2998, normalized size = 19.99

method	result
risch	$\frac{(gx+f)^3 B\ln\left(\frac{e(bx+a)}{dx+c}\right)}{3g} + \frac{g^2Ax^3}{3} + gAfx^2 + \frac{g^2Bax^2}{6b} - \frac{g^2Bcx^2}{6d} + Af^2x + \frac{g^2B\ln(-bx-a)a^3}{3b^3} - \frac{gB\ln(}{}$
derivativedivides	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2*(A+B*ln(e*(b*x+a)/(d*x+c))),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d^2*e*(a*d-b*c)*(-B*d^3*g^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*(b*e/d+(a*d-
b*c)*e/d/(d*x+c))^2/b^2/(b*e-(b*e/d+(a*d-b*c)*e/d/(d*x+c))*d)^3*a^2+B*ln(b*
```

```
e/d+(a*d-b*c)*e/d/(d*x+c))*(b*e/d+(a*d-b*c)*e/d/(d*x+c))/b/e/(b*e-(b*e/d+(a
*d-b*c)*e/d/(d*x+c))*d)*c^2*g^2+2/3*B/d*g^2/(b*e-(b*e/d+(a*d-b*c)*e/d/(d*x+
c))*d)*c^2-B*g/(b*e-(b*e/d+(a*d-b*c)*e/d/(d*x+c))*d)*c*f+B*d^2*ln(b*e/d+(a*
d-b*c)*e/d/(d*x+c))*(b*e/d+(a*d-b*c)*e/d/(d*x+c))/b/e/(b*e-(b*e/d+(a*d-b*c)
*e/d/(d*x+c))*d)*f^2+B*e*g^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*(b*e/d+(a*d-b*
c)*e/d/(d*x+c))*b/(b*e-(b*e/d+(a*d-b*c)*e/d/(d*x+c))*d)^3*c^2+2*B*d*g*ln(b*
e/d+(a*d-b*c)*e/d/(d*x+c))*(b*e/d+(a*d-b*c)*e/d/(d*x+c))/(b*e-(b*e/d+(a*d-b
*c)*e/d/(d*x+c))*d)^2*c*f-B*d/e*g/b^2*ln(b*e-(b*e/d+(a*d-b*c)*e/d/(d*x+c))*
d)*a*f-2/3*B*d^3/e*g^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*(b*e/d+(a*d-b*c)*e/d
/(d*x+c))^3/b^2/(b*e-(b*e/d+(a*d-b*c)*e/d/(d*x+c))*d)^3*a*c-B*d^2/e*g^2*ln(
b*e/d+(a*d-b*c)*e/d/(d*x+c))*(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2/b^2/(b*e-(b*e/
d+(a*d-b*c)*e/d/(d*x+c))*d)^2*a*c+B*d^3/e*g*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))
*(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2/b^2/(b*e-(b*e/d+(a*d-b*c)*e/d/(d*x+c))*d)^
2*a*f-B*d^2/e*g*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*(b*e/d+(a*d-b*c)*e/d/(d*x+c
))^2/b/(b*e-(b*e/d+(a*d-b*c)*e/d/(d*x+c))*d)^2*c*f-2*B*d*ln(b*e/d+(a*d-b*c)
*e/d/(d*x+c))*(b*e/d+(a*d-b*c)*e/d/(d*x+c))/b/e/(b*e-(b*e/d+(a*d-b*c)*e/d/(
d*x+c))*d)*c*f*g+A*d^2*(1/3*e^2*g^2*(a^2*d^2-2*a*b*c*d+b^2*c^2)/d^3/(b*e-(b
*e/d+(a*d-b*c)*e/d/(d*x+c))*d)^3+e*g*(a*c*d*g-a*d^2*f-b*c^2*g+b*c*d*f)/d^3/
(b*e-(b*e/d+(a*d-b*c)*e/d/(d*x+c))*d)^2+(c^2*g^2-2*c*d*f*g+d^2*f^2)/d^3/(b*
e-(b*e/d+(a*d-b*c)*e/d/(d*x+c))*d))+1/3*B*d/e*g^2/b^3*ln(b*e-(b*e/d+(a*d-b*
c)*e/d/(d*x+c))*d)*a^2-1/6*B*d*e*g^2/b/(b*e-(b*e/d+(a*d-b*c)*e/d/(d*x+c))*d
)^2*a^2-1/6*B/d*e*g^2*b/(b*e-(b*e/d+(a*d-b*c)*e/d/(d*x+c))*d)^2*c^2+1/3*B/e
*g^2/b^2*ln(b*e-(b*e/d+(a*d-b*c)*e/d/(d*x+c))*d)*a*c+1/3*B/d/e*g^2/b*ln(b*e
-(b*e/d+(a*d-b*c)*e/d/(d*x+c))*d)*c^2-B/e*g/b*ln(b*e-(b*e/d+(a*d-b*c)*e/d/(
d*x+c))*d)*c*f-B*d*g^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*(b*e/d+(a*d-b*c)*e/d
/(d*x+c))^2/(b*e-(b*e/d+(a*d-b*c)*e/d/(d*x+c))*d)^3*c^2+1/3*B*d^4/e*g^2*ln(
b*e/d+(a*d-b*c)*e/d/(d*x+c))*(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3/b^3/(b*e-(b*e/
d+(a*d-b*c)*e/d/(d*x+c))*d)^3*a^2+1/3*B*d^2/e*g^2*ln(b*e/d+(a*d-b*c)*e/d/(d
*x+c))*(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3/b/(b*e-(b*e/d+(a*d-b*c)*e/d/(d*x+c))
*d)^3*c^2+B*d/e*g^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*(b*e/d+(a*d-b*c)*e/d/(d
*x+c))^2/b/(b*e-(b*e/d+(a*d-b*c)*e/d/(d*x+c))*d)^2*c^2-2*B*d*e*g^2*ln(b*e/d
+(a*d-b*c)*e/d/(d*x+c))*(b*e/d+(a*d-b*c)*e/d/(d*x+c))/(b*e-(b*e/d+(a*d-b*c)
*e/d/(d*x+c))*d)^3*a*c+2*B*d*g^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*(b*e/d+(a*
d-b*c)*e/d/(d*x+c))/b/(b*e-(b*e/d+(a*d-b*c)*e/d/(d*x+c))*d)^2*a*c-2*B*d^2*g
*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*(b*e/d+(a*d-b*c)*e/d/(d*x+c))/b/(b*e-(b*e/
d+(a*d-b*c)*e/d/(d*x+c))*d)^2*a*f+2*B*d^2*g^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c
))*(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2/b/(b*e-(b*e/d+(a*d-b*c)*e/d/(d*x+c))*d)^
3*a*c+B*d*g/b/(b*e-(b*e/d+(a*d-b*c)*e/d/(d*x+c))*d)*a*f+B*d^2*e*g^2*ln(b*e/
d+(a*d-b*c)*e/d/(d*x+c))*(b*e/d+(a*d-b*c)*e/d/(d*x+c))/b/(b*e-(b*e/d+(a*d-b
*c)*e/d/(d*x+c))*d)^3*a^2+B*d/b/e*ln(b*e-(b*e/d+(a*d-b*c)*e/d/(d*x+c))*d)*f
^2+1/3*B*e*g^2/(b*e-(b*e/d+(a*d-b*c)*e/d/(d*x+c))*d)^2*a*c-1/3*B*g^2/b/(b*e
-(b*e/d+(a*d-b*c)*e/d/(d*x+c))*d)*a*c-1/3*B*d*g^2/b^2/(b*e-(b*e/d+(a*d-b*c)
*e/d/(d*x+c))*d)*a^2-2*B*g^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*(b*e/d+(a*d-b*
c)*e/d/(d*x+c))/(b*e-(b*e/d+(a*d-b*c)*e/d/(d*x+c))*d)^2*c^2)
```

Maxima [A]
time = 0.28, size = 268, normalized size = 1.79

$$\frac{1}{3}Ag^{2}x^{3}+Afgx^{2}+\left(x\log\left(\frac{bxe}{dx+c}+\frac{ae}{dx+c}\right)+\frac{a\log\left(bx+a\right)}{b}-\frac{c\log\left(dx+c\right)}{d}\right)Bf^{2}+\left(x^{2}\log\left(\frac{bxe}{dx+c}+\frac{ae}{dx+c}\right)-\frac{a^{2}\log\left(bx+a\right)}{b^{2}}+\frac{c^{2}\log\left(dx+c\right)}{d^{2}}-\frac{(bc-ad)x}{bd}\right)Bfg+\frac{1}{6}\left(2x^{3}\log\left(\frac{bxe}{dx+c}+\frac{ae}{dx+c}\right)+\frac{2a^{3}\log\left(bx+a\right)}{b^{3}}-\frac{2c^{3}\log\left(dx+c\right)}{d^{3}}-\frac{(b^{2}cd-abd^{2})x^{2}-2(b^{2}c^{2}-a^{2}d^{2})x}{b^{2}d^{2}}\right)Bg^{2}+Af^{2}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="maxima")
```

```
[Out] 1/3*A*g^2*x^3 + A*f*g*x^2 + (x*log(b*x*e/(d*x + c) + a*e/(d*x + c)) + a*log
(b*x + a)/b - c*log(d*x + c)/d)*B*f^2 + (x^2*log(b*x*e/(d*x + c) + a*e/(d*x
 + c)) - a^2*log(b*x + a)/b^2 + c^2*log(d*x + c)/d^2 - (b*c - a*d)*x/(b*d))
*B*f*g + 1/6*(2*x^3*log(b*x*e/(d*x + c) + a*e/(d*x + c)) + 2*a^3*log(b*x +
a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a
^2*d^2)*x)/(b^2*d^2))*B*g^2 + A*f^2*x
```

Fricas [A]
time = 0.39, size = 279, normalized size = 1.86

$$\frac{2Ab^{3}d^{3}g^{2}x^{3}+(6Ab^{3}d^{3}fg-(Bb^{3}cd^{2}-Bab^{2}d^{3})g^{2})x^{2}+2(3Ab^{3}d^{3}f^{2}-3(Bb^{3}cd^{2}-Bab^{2}d^{3})fg+(Bb^{3}c^{2}d-Ba^{2}bd^{3})g^{2})x+2(3Bab^{2}d^{3}f^{2}-3Ba^{2}bd^{3}fg+Ba^{3}d^{3}g^{2})\log\left(bx+a\right)-2(3Bb^{3}cd^{2}f^{2}-3Bb^{3}c^{2}dfg+Bb^{3}c^{3}g^{2})\log\left(dx+c\right)+2(Bb^{3}d^{3}g^{2}x^{3}+3Bb^{3}d^{3}fgx^{2}+3Bb^{3}d^{3}f^{2}x)\log\left(\frac{(bx+a)e}{dx+c}\right)}{6b^{3}d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="fricas")
```

```
[Out] 1/6*(2*A*b^3*d^3*g^2*x^3 + (6*A*b^3*d^3*f*g - (B*b^3*c*d^2 - B*a*b^2*d^3)*g
^2)*x^2 + 2*(3*A*b^3*d^3*f^2 - 3*(B*b^3*c*d^2 - B*a*b^2*d^3)*f*g + (B*b^3*c
^2*d - B*a^2*b*d^3)*g^2)*x + 2*(3*B*a*b^2*d^3*f^2 - 3*B*a^2*b*d^3*f*g + B*a
^3*d^3*g^2)*log(b*x + a) - 2*(3*B*b^3*c*d^2*f^2 - 3*B*b^3*c^2*d*f*g + B*b^3
*c^3*g^2)*log(d*x + c) + 2*(B*b^3*d^3*g^2*x^3 + 3*B*b^3*d^3*f*g*x^2 + 3*B*b
^3*d^3*f^2*x)*log((b*x + a)*e/(d*x + c)))/(b^3*d^3)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 658 vs. $2(131) = 262$.
time = 8.04, size = 658, normalized size = 4.39

$$\frac{Ag^{2}x^{3}}{3}+\frac{Ba(a^{2}g^{2}-3abfg+3b^{2}f^{2})\log\left(x+\frac{Ba^{3}cd^{2}g^{2}-3Ba^{2}bcd^{2}fg+\frac{Ba^{2}d^{3}(a^{2}g^{2}-3abfg+3b^{2}f^{2})}{b}+Bab^{2}c^{3}g^{2}-3Bab^{2}c^{2}dfg+6Bab^{2}cd^{2}f^{2}-Bacd^{2}(a^{2}g^{2}-3abfg+3b^{2}f^{2})}{Ba^{3}d^{3}g^{2}-3Ba^{2}bd^{3}fg+3Bab^{2}d^{3}f^{2}+Bb^{3}c^{3}g^{2}-3Bb^{3}c^{2}dfg+3Bb^{3}cd^{2}f^{2}}\right)}{3b^{3}}-\frac{Bc(c^{2}g^{2}-3cdfg+3d^{2}f^{2})\log\left(x+\frac{Ba^{3}cd^{2}g^{2}-3Ba^{2}bcd^{2}fg+Bab^{2}c^{3}g^{2}-3Bab^{2}c^{2}dfg+6Bab^{2}cd^{2}f^{2}-Bab^{2}c(c^{2}g^{2}-3cdfg+3d^{2}f^{2})+\frac{Bb^{3}c^{2}(c^{2}g^{2}-3cdfg+3d^{2}f^{2})}{d}}{Ba^{3}d^{3}g^{2}-3Ba^{2}bd^{3}fg+3Bab^{2}d^{3}f^{2}+Bb^{3}c^{3}g^{2}-3Bb^{3}c^{2}dfg+3Bb^{3}cd^{2}f^{2}}\right)}{3d^{3}}+x^{2}\left(Afg+\frac{Bag^{2}}{6b}-\frac{Bcg^{2}}{6d}\right)+x\left(Af^{2}-\frac{Ba^{2}g^{2}}{3b^{2}}+\frac{Bafg}{b}+\frac{Bc^{2}g^{2}}{3d^{2}}-\frac{Bcfg}{d}\right)+\left(Bf^{2}x+Bfgx^{2}+\frac{Bg^{2}x^{3}}{3}\right)\log\left(\frac{e(a+bx)}{c+dx}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2*(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

```
[Out] A*g**2*x**3/3 + B*a*(a**2*g**2 - 3*a*b*f*g + 3*b**2*f**2)*log(x + (B*a**3*c
*d**2*g**2 - 3*B*a**2*b*c*d**2*f*g + B*a**2*d**3*(a**2*g**2 - 3*a*b*f*g + 3
*b**2*f**2)/b + B*a*b**2*c**3*g**2 - 3*B*a*b**2*c**2*d*f*g + 6*B*a*b**2*c*d
**2*f**2 - B*a*c*d**2*(a**2*g**2 - 3*a*b*f*g + 3*b**2*f**2))/(B*a**3*d**3*g
**2 - 3*B*a**2*b*d**3*f*g + 3*B*a*b**2*d**3*f**2 + B*b**3*c**3*g**2 - 3*B*b
**3*c**2*d*f*g + 3*B*b**3*c*d**2*f**2))/(3*b**3) - B*c*(c**2*g**2 - 3*c*d*f
```

```
*g + 3*d**2*f**2)*log(x + (B*a**3*c*d**2*g**2 - 3*B*a**2*b*c*d**2*f*g + B*a
*b**2*c**3*g**2 - 3*B*a*b**2*c**2*d*f*g + 6*B*a*b**2*c*d**2*f**2 - B*a*b**2
*c*(c**2*g**2 - 3*c*d*f*g + 3*d**2*f**2) + B*b**3*c**2*(c**2*g**2 - 3*c*d*f
*g + 3*d**2*f**2)/d)/(B*a**3*d**3*g**2 - 3*B*a**2*b*d**3*f*g + 3*B*a*b**2*d
**3*f**2 + B*b**3*c**3*g**2 - 3*B*b**3*c**2*d*f*g + 3*B*b**3*c*d**2*f**2))/
(3*d**3) + x**2*(A*f*g + B*a*g**2/(6*b) - B*c*g**2/(6*d)) + x*(A*f**2 - B*a
**2*g**2/(3*b**2) + B*a*f*g/b + B*c**2*g**2/(3*d**2) - B*c*f*g/d) + (B*f**2
*x + B*f*g*x**2 + B*g**2*x**3/3)*log(e*(a + b*x)/(c + d*x))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 5950 vs. $2(141) = 282$.
time = 5.43, size = 5950, normalized size = 39.67

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="giac")
```

```
[Out] 1/6*(6*B*b^7*c^2*d^2*f^2*e^4*log(-b*e + (b*x*e + a*e)*d/(d*x + c)) - 12*B*a
*b^6*c*d^3*f^2*e^4*log(-b*e + (b*x*e + a*e)*d/(d*x + c)) + 6*B*a^2*b^5*d^4*
f^2*e^4*log(-b*e + (b*x*e + a*e)*d/(d*x + c)) - 6*B*b^7*c^3*d*f*g*e^4*log(-
b*e + (b*x*e + a*e)*d/(d*x + c)) + 6*B*a*b^6*c^2*d^2*f*g*e^4*log(-b*e + (b*
x*e + a*e)*d/(d*x + c)) + 6*B*a^2*b^5*c*d^3*f*g*e^4*log(-b*e + (b*x*e + a*e
)*d/(d*x + c)) - 6*B*a^3*b^4*d^4*f*g*e^4*log(-b*e + (b*x*e + a*e)*d/(d*x +
c)) + 2*B*b^7*c^4*g^2*e^4*log(-b*e + (b*x*e + a*e)*d/(d*x + c)) - 2*B*a*b^6
*c^3*d*g^2*e^4*log(-b*e + (b*x*e + a*e)*d/(d*x + c)) - 2*B*a^3*b^4*c*d^3*g^
2*e^4*log(-b*e + (b*x*e + a*e)*d/(d*x + c)) + 2*B*a^4*b^3*d^4*g^2*e^4*log(-
b*e + (b*x*e + a*e)*d/(d*x + c)) - 18*(b*x*e + a*e)*B*b^6*c^2*d^3*f^2*e^3*l
og(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c) + 36*(b*x*e + a*e)*B*a*b^5*c
*d^4*f^2*e^3*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c) - 18*(b*x*e +
a*e)*B*a^2*b^4*d^5*f^2*e^3*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)
+ 18*(b*x*e + a*e)*B*b^6*c^3*d^2*f*g*e^3*log(-b*e + (b*x*e + a*e)*d/(d*x +
c))/(d*x + c) - 18*(b*x*e + a*e)*B*a*b^5*c^2*d^3*f*g*e^3*log(-b*e + (b*x*e
+ a*e)*d/(d*x + c))/(d*x + c) - 18*(b*x*e + a*e)*B*a^2*b^4*c*d^4*f*g*e^3*lo
g(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c) + 18*(b*x*e + a*e)*B*a^3*b^3*
d^5*f*g*e^3*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c) - 6*(b*x*e + a*
e)*B*b^6*c^4*d*g^2*e^3*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c) + 6*
(b*x*e + a*e)*B*a*b^5*c^3*d^2*g^2*e^3*log(-b*e + (b*x*e + a*e)*d/(d*x + c))
/(d*x + c) + 6*(b*x*e + a*e)*B*a^3*b^3*c*d^4*g^2*e^3*log(-b*e + (b*x*e + a*
e)*d/(d*x + c))/(d*x + c) - 6*(b*x*e + a*e)*B*a^4*b^2*d^5*g^2*e^3*log(-b*e
+ (b*x*e + a*e)*d/(d*x + c))/(d*x + c) + 18*(b*x*e + a*e)^2*B*b^5*c^2*d^4*f
^2*e^2*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^2 - 36*(b*x*e + a*e)
^2*B*a*b^4*c*d^5*f^2*e^2*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^2
+ 18*(b*x*e + a*e)^2*B*a^2*b^3*d^6*f^2*e^2*log(-b*e + (b*x*e + a*e)*d/(d*x
+ c))/(d*x + c)^2 - 18*(b*x*e + a*e)^2*B*b^5*c^3*d^3*f*g*e^2*log(-b*e + (b*
```

x*e + a*e)*d/(d*x + c))/(d*x + c)^2 + 18*(b*x*e + a*e)^2*B*a*b^4*c^2*d^4*f*g*e^2*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^2 + 18*(b*x*e + a*e)^2*B*a^2*b^3*c*d^5*f*g*e^2*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^2 - 18*(b*x*e + a*e)^2*B*a^3*b^2*d^6*f*g*e^2*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^2 + 6*(b*x*e + a*e)^2*B*b^5*c^4*d^2*g^2*e^2*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^2 - 6*(b*x*e + a*e)^2*B*a*b^4*c^3*d^3*g^2*e^2*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^2 - 6*(b*x*e + a*e)^2*B*a^3*b^2*c*d^5*g^2*e^2*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^2 + 6*(b*x*e + a*e)^2*B*a^4*b*d^6*g^2*e^2*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^2 - 6*(b*x*e + a*e)^3*B*b^4*c^2*d^5*f^2*e*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^3 + 12*(b*x*e + a*e)^3*B*a*b^3*c*d^6*f^2*e*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^3 - 6*(b*x*e + a*e)^3*B*a^2*b^2*d^7*f^2*e*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^3 + 6*(b*x*e + a*e)^3*B*b^4*c^3*d^4*f*g*e*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^3 - 6*(b*x*e + a*e)^3*B*a*b^3*c^2*d^5*f*g*e*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^3 - 6*(b*x*e + a*e)^3*B*a^2*b^2*c*d^6*f*g*e*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^3 + 6*(b*x*e + a*e)^3*B*a^3*b*d^7*f*g*e*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^3 - 2*(b*x*e + a*e)^3*B*b^4*c^4*d^3*g^2*e*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^3 + 2*(b*x*e + a*e)^3*B*a*b^3*c^3*d^4*g^2*e*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^3 + 2*(b*x*e + a*e)^3*B*a^3*b*c*d^6*g^2*e*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^3 - 2*(b*x*e + a*e)^3*B*a^4*d^7*g^2*e*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^3 + 6*(b*x*e + a*e)*B*b^6*c^2*d^3*f^2*e^3*log((b*x*e + a*e)/(d*x + c))/(d*x + c) - 12*(b*x*e + a*e)*B*a*b^5*c*d^4*f^2*e^3*log((b*x*e + a*e)/(d*x + c))/(d*x + c) + 6*(b*x*e + a*e)*B*a^2*b^4*d^5*f^2*e^3*log((b*x*e + a*e)/(d*x + c))/(d*x + c) - 12*(b*x*e + a*e)*B*a*b^5*c^2*d^3*f*g*e^3*log((b*x*e + a*e)/(d*x + c))/(d*x + c) + 24*(b*x*e + a*e)*B*a^2*b^4*c*d^4*f*g*e^3*log((b*x*e + a*e)/(d*x + c))/(d*x + c) - 12*(b*x*e + a*e)*B*a^3*b^3*d^5*f*g*e^3*log((b*x*e + a*e)/(d*x + c))/(d*x + c) + 6*(b*x*e + a*e)*B*a^2*b^4*c^2*d^3*g^2*e^3*log((b*x*e + a*e)/(d*x + c))/(d*x + c) - 12*(b*x*e + a*e)*B*a^3*b^3*c*d^4*g^2*e^3*log((b*x*e + a*e)/(d*x + c))/(d*x + c) + 6*(b*x*e + a*e)*B*a^4*b^2*d^5*g^2*e^3*log((b*x*e + a*e)/(d*x + c))/(d*x + c) - 12*(b*x*e + a*e)^2*B*b^5*c^2*d^4*f^2*e^2*log((b*x*e + a*e)/(d*x + c))/(d*x + c)^2 + 24*(b*x*e + a*e)^2*B*a*b^4*c*d^5*f^2*e^2*log((b*x*e + a*e)/(d*x + c))/(d*x + c)^2 - 12*(b*x*e + a*e)^2*B*a^2*b^3*d^6*f^2*e^2*log((b*x*e + a*e)/(d*x + c))/(d*x + c)^2 + 6*(b*x*e + a*e)^2*B*b^5*c^3*d^3*f*g*e^2*log((b*x*e + a*e)/(d*x + c))/(d*x + c)^2 + 6*(b*x*e + a*e)^2*B*a*b^4*c^2*d^4*f*g*e^2*log((b*x*e + a*e)/(d*x + c))/(d*x + c)^2 - 30*(b*x*e + a*e)^2*B*a^2*b^3*c*d^5*f*g*e^2*log((b*x*e + a*e)/(d*x + c))/(d*x + c)^2 + 18*(b*x*e + a*e)^2*B*a^3*b^2*d^6*f*g*e^2*log((b*x*e + a*e)...

Mupad [B]

time = 4.73, size = 356, normalized size = 2.37

$$x^2\left(\frac{3\,A\,a\,d\,g^2+3\,A\,b\,c\,g^2+B\,a\,d\,g^2-B\,b\,c\,g^2+6\,A\,b\,d\,f\,g}{6\,b\,d}-\frac{A\,g^2\,\left(3\,a\,d+3\,b\,c\right)}{6\,b\,d}\right)+\ln\left(\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right)\,\left(B\,f^2\,x+B\,f\,g\,x^2+\frac{B\,g^2\,x^3}{3}\right)-x\,\left(\frac{\left(\frac{3\,A\,a\,d\,g^2+3\,A\,b\,c\,g^2+B\,a\,d\,g^2-B\,b\,c\,g^2+6\,A\,b\,d\,f\,g}{3\,b\,d}-\frac{A\,g^2\,\left(3\,a\,d+3\,b\,c\right)}{3\,b\,d}\right)\,\left(3\,a\,d+3\,b\,c\right)}{3\,b\,d}-\frac{3\,A\,a\,c\,g^2+3\,A\,b\,d\,f^2+6\,A\,a\,d\,f\,g+6\,A\,b\,c\,f\,g+3\,B\,a\,d\,f\,g-3\,B\,b\,c\,f\,g}{3\,b\,d}+\frac{A\,a\,c\,g^2}{b\,d}\right)+\frac{\ln\left(a+b\,x\right)\,\left(B\,a^3\,g^2-3\,B\,a^2\,b\,f\,g+3\,B\,a\,b^2\,f^2\right)}{3\,b^3}-\frac{\ln\left(c+d\,x\right)\,\left(B\,c^3\,g^2-3\,B\,c^2\,d\,f\,g+3\,B\,c\,d^2\,f^2\right)}{3\,d^3}+\frac{A\,g^2\,x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^2*(A + B*log((e*(a + b*x))/(c + d*x))),x)
```

```
[Out] x^2*((3*A*a*d*g^2 + 3*A*b*c*g^2 + B*a*d*g^2 - B*b*c*g^2 + 6*A*b*d*f*g)/(6*b
*d) - (A*g^2*(3*a*d + 3*b*c))/(6*b*d)) + log((e*(a + b*x))/(c + d*x))*((B*g
^2*x^3)/3 + B*f^2*x + B*f*g*x^2) - x*((((3*A*a*d*g^2 + 3*A*b*c*g^2 + B*a*d*
g^2 - B*b*c*g^2 + 6*A*b*d*f*g)/(3*b*d) - (A*g^2*(3*a*d + 3*b*c))/(3*b*d))*(
3*a*d + 3*b*c))/(3*b*d) - (3*A*a*c*g^2 + 3*A*b*d*f^2 + 6*A*a*d*f*g + 6*A*b*
c*f*g + 3*B*a*d*f*g - 3*B*b*c*f*g)/(3*b*d) + (A*a*c*g^2)/(b*d)) + (log(a +
b*x)*(B*a^3*g^2 + 3*B*a*b^2*f^2 - 3*B*a^2*b*f*g))/(3*b^3) - (log(c + d*x)*(
B*c^3*g^2 + 3*B*c*d^2*f^2 - 3*B*c^2*d*f*g))/(3*d^3) + (A*g^2*x^3)/3
```

3.233 $\int (f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$

Optimal. Leaf size=109

$$-\frac{B(bc-ad)gx}{2bd}-\frac{B(bf-ag)^2\log(a+bx)}{2b^2g}+\frac{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2g}+\frac{B(df-cg)^2\log(c+dx)}{2d^2g}$$

```
[Out] -1/2*B*(-a*d+b*c)*g*x/b/d-1/2*B*(-a*g+b*f)^2*ln(b*x+a)/b^2/g+1/2*(g*x+f)^2*
(A+B*ln(e*(b*x+a)/(d*x+c)))/g+1/2*B*(-c*g+d*f)^2*ln(d*x+c)/d^2/g
```

Rubi [A]
time = 0.06, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2548, 84}

$$\frac{(f+gx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2g}-\frac{B(bf-ag)^2\log(a+bx)}{2b^2g}-\frac{Bgx(bc-ad)}{2bd}+\frac{B(df-cg)^2\log(c+dx)}{2d^2g}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
```

```
[Out] -1/2*(B*(b*c - a*d)*g*x)/(b*d) - (B*(b*f - a*g)^2*Log[a + b*x])/(2*b^2*g) +
 ((f + g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(2*g) + (B*(d*f - c*g)^
2*Log[c + d*x])/(2*d^2*g)
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 2548

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Dist[B*n*((b*c
- a*d)/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /;
 FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c -
 a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Rubi steps

$$\int (f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)dx = \frac{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2g} - \frac{B\int \frac{(bc-ad)(f+gx)^2}{(a+bx)(c+dx)}dx}{2g}$$
$$= \frac{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2g} - \frac{(B(bc-ad))\int \frac{(f+gx)^2}{(a+bx)(c+dx}$$
$$= \frac{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2g} - \frac{(B(bc-ad))\int\left(\frac{g^2}{bd}+\frac{(}{b(bc-}\right.}{}$$
$$= -\frac{B(bc-ad)gx}{2bd} - \frac{B(bf-ag)^2\log(a+bx)}{2b^2g} + \frac{(f+gx)^2\left(A+\right.}{}$$

Mathematica [**A**]
time = 0.08, size = 114, normalized size = 1.05

$$\frac{-Bd^2(bf-ag)^2\log(a+bx)+b\left(d(B(-bc+ad)g^2x+Abd(f+gx)^2)+bBd^2(f+gx)^2\log\left(\frac{e(a+bx)}{c+dx}\right)+bB(df-cg)^2\log(c+dx)\right)}{2b^2d^2g}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
```

```
[Out] (-(B*d^2*(b*f - a*g)^2*Log[a + b*x]) + b*(d*(B*(-(b*c) + a*d)*g^2*x + A*b*d
*(f + g*x)^2) + b*B*d^2*(f + g*x)^2*Log[(e*(a + b*x))/(c + d*x)] + b*B*(d*f
 - c*g)^2*Log[c + d*x]))/(2*b^2*d^2*g)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 963 vs. $2(101) = 202$.
time = 0.38, size = 964, normalized size = 8.84 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)*(A+B*ln(e*(b*x+a)/(d*x+c))),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d^2*e*(a*d-b*c)*(-A*d^2*(1/2*e*g*(a*d-b*c)/d^2/(b*e-(b*e/d+(a*d-b*c)*e/d
/(d*x+c))*d)^2+(c*g-d*f)/d^2/(b*e-(b*e/d+(a*d-b*c)*e/d/(d*x+c))*d))-1/2*B*d
/e*g/b^2*ln(b*e-(b*e/d+(a*d-b*c)*e/d/(d*x+c))*d)*a-1/2*B/e*g/b*ln(b*e-(b*e/
d+(a*d-b*c)*e/d/(d*x+c))*d)*c+1/2*B*d*g/b/(b*e-(b*e/d+(a*d-b*c)*e/d/(d*x+c)
)*d)*a-1/2*B*g/(b*e-(b*e/d+(a*d-b*c)*e/d/(d*x+c))*d)*c-B*d^2*g*ln(b*e/d+(a*
d-b*c)*e/d/(d*x+c))*(b*e/d+(a*d-b*c)*e/d/(d*x+c))/b/(b*e-(b*e/d+(a*d-b*c)*e
/d/(d*x+c))*d)^2*a+B*d*g*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*(b*e/d+(a*d-b*c)*e
/d/(d*x+c))/(b*e-(b*e/d+(a*d-b*c)*e/d/(d*x+c))*d)^2*c+1/2*B*d^3/e*g*ln(b*e/
d+(a*d-b*c)*e/d/(d*x+c))*(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2/b^2/(b*e-(b*e/d+(a
*d-b*c)*e/d/(d*x+c))*d)^2*a-1/2*B*d^2/e*g*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*(
b*e/d+(a*d-b*c)*e/d/(d*x+c))^2/b/(b*e-(b*e/d+(a*d-b*c)*e/d/(d*x+c))*d)^2*c+
```

```
B*d/b/e*ln(b*e-(b*e/d+(a*d-b*c)*e/d/(d*x+c))*d)*f-B*d*ln(b*e/d+(a*d-b*c)*e/
d/(d*x+c))*(b*e/d+(a*d-b*c)*e/d/(d*x+c))/b/e/(b*e-(b*e/d+(a*d-b*c)*e/d/(d*x
+c))*d)*c*g+B*d^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*(b*e/d+(a*d-b*c)*e/d/(d*x
+c))/b/e/(b*e-(b*e/d+(a*d-b*c)*e/d/(d*x+c))*d)*f)
```

Maxima [**A**]
time = 0.30, size = 144, normalized size = 1.32

$$\frac{1}{2}Agx^2+\left(x\log\left(\frac{bxe}{dx+c}+\frac{ae}{dx+c}\right)+\frac{a\log(bx+a)}{b}-\frac{c\log(dx+c)}{d}\right)Bf+\frac{1}{2}\left(x^2\log\left(\frac{bxe}{dx+c}+\frac{ae}{dx+c}\right)-\frac{a^2\log(bx+a)}{b^2}+\frac{c^2\log(dx+c)}{d^2}-\frac{(bc-ad)x}{bd}\right)Bg+Afx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="maxima")
```

```
[Out] 1/2*A*g*x^2 + (x*log(b*x*e/(d*x + c) + a*e/(d*x + c)) + a*log(b*x + a)/b -
c*log(d*x + c)/d)*B*f + 1/2*(x^2*log(b*x*e/(d*x + c) + a*e/(d*x + c)) - a^2
*log(b*x + a)/b^2 + c^2*log(d*x + c)/d^2 - (b*c - a*d)*x/(b*d))*B*g + A*f*x
```

Fricas [**A**]
time = 0.36, size = 149, normalized size = 1.37

$$\frac{Ab^2d^2gx^2+(2\,Ab^2d^2f-(Bb^2cd-Babd^2)g)x+(2\,Babd^2f-Ba^2d^2g)\log(bx+a)-(2\,Bb^2cdf-Bb^2c^2g)\log(dx+c)+(Bb^2d^2gx^2+2\,Bb^2d^2fx)\log\left(\frac{(bx+a)e}{dx+c}\right)}{2\,b^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="fricas")
```

```
[Out] 1/2*(A*b^2*d^2*g*x^2 + (2*A*b^2*d^2*f - (B*b^2*c*d - B*a*b*d^2)*g)*x + (2*B
*a*b*d^2*f - B*a^2*d^2*g)*log(b*x + a) - (2*B*b^2*c*d*f - B*b^2*c^2*g)*log(
d*x + c) + (B*b^2*d^2*g*x^2 + 2*B*b^2*d^2*f*x)*log((b*x + a)*e/(d*x + c)))/
(b^2*d^2)
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 318 vs. $2(90) = 180$.
time = 3.38, size = 318, normalized size = 2.92

$$\frac{Agx^2}{2}-\frac{Ba(ag-2bf)\log\left(x+\frac{Ba^2cdg+\frac{Ba^2d^2(ag-2bf)}{b}+Babc^2g-4Babcdf-Bacd(ag-2bf)}{Ba^2d^2g-2Babd^2f+Bb^2c^2g-2Bb^2cdf}\right)}{2b^2}+\frac{Bc(cg-2df)\log\left(x+\frac{Ba^2cdg+Babc^2g-4Babcdf-Babc(cg-2df)+\frac{Bb^2c^2(cg-2df)}{d}}{Ba^2d^2g-2Babd^2f+Bb^2c^2g-2Bb^2cdf}\right)}{2d^2}+x\left(Af+\frac{Bag}{2b}-\frac{Bcg}{2d}\right)+\left(Bfx+\frac{Bgx^2}{2}\right)\log\left(\frac{e(a+bx)}{c+dx}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

```
[Out] A*g*x**2/2 - B*a*(a*g - 2*b*f)*log(x + (B*a**2*c*d*g + B*a**2*d**2*(a*g - 2
*b*f)/b + B*a*b*c**2*g - 4*B*a*b*c*d*f - B*a*c*d*(a*g - 2*b*f))/(B*a**2*d**
2*g - 2*B*a*b*d**2*f + B*b**2*c**2*g - 2*B*b**2*c*d*f))/(2*b**2) + B*c*(c*g
 - 2*d*f)*log(x + (B*a**2*c*d*g + B*a*b*c**2*g - 4*B*a*b*c*d*f - B*a*b*c*(c
*g - 2*d*f) + B*b**2*c**2*(c*g - 2*d*f)/d)/(B*a**2*d**2*g - 2*B*a*b*d**2*f
+ B*b**2*c**2*g - 2*B*b**2*c*d*f))/(2*d**2) + x*(A*f + B*a*g/(2*b) - B*c*g/
(2*d)) + (B*f*x + B*g*x**2/2)*log(e*(a + b*x)/(c + d*x))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2355 vs. $2(102) = 204$.
time = 3.55, size = 2355, normalized size = 21.61

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="giac")
```

```
[Out] 1/2*(2*B*b^5*c^2*d*f*e^3*log(-b*e + (b*x*e + a*e)*d/(d*x + c)) - 4*B*a*b^4*
c*d^2*f*e^3*log(-b*e + (b*x*e + a*e)*d/(d*x + c)) + 2*B*a^2*b^3*d^3*f*e^3*l
og(-b*e + (b*x*e + a*e)*d/(d*x + c)) - B*b^5*c^3*g*e^3*log(-b*e + (b*x*e +
a*e)*d/(d*x + c)) + B*a*b^4*c^2*d*g*e^3*log(-b*e + (b*x*e + a*e)*d/(d*x + c
)) + B*a^2*b^3*c*d^2*g*e^3*log(-b*e + (b*x*e + a*e)*d/(d*x + c)) - B*a^3*b^
2*d^3*g*e^3*log(-b*e + (b*x*e + a*e)*d/(d*x + c)) - 4*(b*x*e + a*e)*B*b^4*c
^2*d^2*f*e^2*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c) + 8*(b*x*e + a
*e)*B*a*b^3*c*d^3*f*e^2*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c) - 4
*(b*x*e + a*e)*B*a^2*b^2*d^4*f*e^2*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d
*x + c) + 2*(b*x*e + a*e)*B*b^4*c^3*d*g*e^2*log(-b*e + (b*x*e + a*e)*d/(d*x
 + c))/(d*x + c) - 2*(b*x*e + a*e)*B*a*b^3*c^2*d^2*g*e^2*log(-b*e + (b*x*e
+ a*e)*d/(d*x + c))/(d*x + c) - 2*(b*x*e + a*e)*B*a^2*b^2*c*d^3*g*e^2*log(-
b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c) + 2*(b*x*e + a*e)*B*a^3*b*d^4*g*
e^2*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c) + 2*(b*x*e + a*e)^2*B*b
^3*c^2*d^3*f*e*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^2 - 4*(b*x*e
 + a*e)^2*B*a*b^2*c*d^4*f*e*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)
^2 + 2*(b*x*e + a*e)^2*B*a^2*b*d^5*f*e*log(-b*e + (b*x*e + a*e)*d/(d*x + c)
)/(d*x + c)^2 - (b*x*e + a*e)^2*B*b^3*c^3*d^2*g*e*log(-b*e + (b*x*e + a*e)*
d/(d*x + c))/(d*x + c)^2 + (b*x*e + a*e)^2*B*a*b^2*c^2*d^3*g*e*log(-b*e + (
b*x*e + a*e)*d/(d*x + c))/(d*x + c)^2 + (b*x*e + a*e)^2*B*a^2*b*c*d^4*g*e*l
og(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^2 - (b*x*e + a*e)^2*B*a^3*d^
5*g*e*log(-b*e + (b*x*e + a*e)*d/(d*x + c))/(d*x + c)^2 + 2*(b*x*e + a*e)*B
*b^4*c^2*d^2*f*e^2*log((b*x*e + a*e)/(d*x + c))/(d*x + c) - 4*(b*x*e + a*e)
*B*a*b^3*c*d^3*f*e^2*log((b*x*e + a*e)/(d*x + c))/(d*x + c) + 2*(b*x*e + a*
e)*B*a^2*b^2*d^4*f*e^2*log((b*x*e + a*e)/(d*x + c))/(d*x + c) - 2*(b*x*e +
a*e)*B*a*b^3*c^2*d^2*g*e^2*log((b*x*e + a*e)/(d*x + c))/(d*x + c) + 4*(b*x*
e + a*e)*B*a^2*b^2*c*d^3*g*e^2*log((b*x*e + a*e)/(d*x + c))/(d*x + c) - 2*(
b*x*e + a*e)*B*a^3*b*d^4*g*e^2*log((b*x*e + a*e)/(d*x + c))/(d*x + c) - 2*(
b*x*e + a*e)^2*B*b^3*c^2*d^3*f*e*log((b*x*e + a*e)/(d*x + c))/(d*x + c)^2 +
 4*(b*x*e + a*e)^2*B*a*b^2*c*d^4*f*e*log((b*x*e + a*e)/(d*x + c))/(d*x + c)
^2 - 2*(b*x*e + a*e)^2*B*a^2*b*d^5*f*e*log((b*x*e + a*e)/(d*x + c))/(d*x +
c)^2 + (b*x*e + a*e)^2*B*b^3*c^3*d^2*g*e*log((b*x*e + a*e)/(d*x + c))/(d*x
+ c)^2 - (b*x*e + a*e)^2*B*a*b^2*c^2*d^3*g*e*log((b*x*e + a*e)/(d*x + c))/(
d*x + c)^2 - (b*x*e + a*e)^2*B*a^2*b*c*d^4*g*e*log((b*x*e + a*e)/(d*x + c))
/(d*x + c)^2 + (b*x*e + a*e)^2*B*a^3*d^5*g*e*log((b*x*e + a*e)/(d*x + c))/(
d*x + c)^2 + 2*A*b^5*c^2*d*f*e^3 - 4*A*a*b^4*c*d^2*f*e^3 + 2*A*a^2*b^3*d^3*
```

```
f*e^3 - A*b^5*c^3*g*e^3 - B*b^5*c^3*g*e^3 + A*a*b^4*c^2*d*g*e^3 + 3*B*a*b^4
*c^2*d*g*e^3 + A*a^2*b^3*c*d^2*g*e^3 - 3*B*a^2*b^3*c*d^2*g*e^3 - A*a^3*b^2*
d^3*g*e^3 + B*a^3*b^2*d^3*g*e^3 - 2*(b*x*e + a*e)*A*b^4*c^2*d^2*f*e^2/(d*x
+ c) + 4*(b*x*e + a*e)*A*a*b^3*c*d^3*f*e^2/(d*x + c) - 2*(b*x*e + a*e)*A*a^
2*b^2*d^4*f*e^2/(d*x + c) + 2*(b*x*e + a*e)*A*b^4*c^3*d*g*e^2/(d*x + c) + (
b*x*e + a*e)*B*b^4*c^3*d*g*e^2/(d*x + c) - 4*(b*x*e + a*e)*A*a*b^3*c^2*d^2*
g*e^2/(d*x + c) - 3*(b*x*e + a*e)*B*a*b^3*c^2*d^2*g*e^2/(d*x + c) + 2*(b*x*
e + a*e)*A*a^2*b^2*c*d^3*g*e^2/(d*x + c) + 3*(b*x*e + a*e)*B*a^2*b^2*c*d^3*
g*e^2/(d*x + c) - (b*x*e + a*e)*B*a^3*b*d^4*g*e^2/(d*x + c))*(b*c/((b*c*e -
 a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))/(b^4*d^2*e^2 - 2*
(b*x*e + a*e)*b^3*d^3*e/(d*x + c) + (b*x*e + a*e)^2*b^2*d^4/(d*x + c)^2)
```

Mupad [B]

time = 4.24, size = 144, normalized size = 1.32

$$\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\,\left(\frac{B\,g\,x^2}{2}+B\,f\,x\right)+x\,\left(\frac{2\,A\,a\,d\,g+2\,A\,b\,c\,g+2\,A\,b\,d\,f+B\,a\,d\,g-B\,b\,c\,g}{2\,b\,d}-\frac{A\,g\,(2\,a\,d+2\,b\,c)}{2\,b\,d}\right)-\frac{\ln(a+b\,x)\,\left(B\,a^2\,g-2\,B\,a\,b\,f\right)}{2\,b^2}+\frac{\ln(c+d\,x)\,\left(B\,c^2\,g-2\,B\,c\,d\,f\right)}{2\,d^2}+\frac{A\,g\,x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)*(A + B*log((e*(a + b*x))/(c + d*x))),x)
```

```
[Out] log((e*(a + b*x))/(c + d*x))*(B*f*x + (B*g*x^2)/2) + x*((2*A*a*d*g + 2*A*b*
c*g + 2*A*b*d*f + B*a*d*g - B*b*c*g)/(2*b*d) - (A*g*(2*a*d + 2*b*c))/(2*b*d
)) - (log(a + b*x)*(B*a^2*g - 2*B*a*b*f))/(2*b^2) + (log(c + d*x)*(B*c^2*g
- 2*B*c*d*f))/(2*d^2) + (A*g*x^2)/2
```

3.234 $\int \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$

Optimal. Leaf size=52

$$Ax+\frac{B(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{b}-\frac{B(bc-ad)\log(c+dx)}{bd}$$

```
[Out] A*x+B*(b*x+a)*ln(e*(b*x+a)/(d*x+c))/b-B*(-a*d+b*c)*ln(d*x+c)/b/d
```

Rubi [A]
time = 0.02, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2536, 31}

$$\frac{B(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{b}-\frac{B(bc-ad)\log(c+dx)}{bd}+Ax$$

Antiderivative was successfully verified.

```
[In] Int[A + B*Log[(e*(a + b*x))/(c + d*x)],x]
```

```
[Out] A*x + (B*(a + b*x)*Log[(e*(a + b*x))/(c + d*x)])/b - (B*(b*c - a*d)*Log[c +
 d*x])/(b*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2536

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.), x_Symbol] :> Simp[(a + b*x)*((A + B*Log[e*((a + b*x)^n/(c
+ d*x)^n)])^p/b), x] - Dist[B*n*p*((b*c - a*d)/b), Int[(A + B*Log[e*((a + b
*x)^n/(c + d*x)^n)])^(p - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, A,
B, n}, x] && EqQ[n + mn, 0] && NeQ[b*c - a*d, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx &= Ax+B\int \log\left(\frac{e(a+bx)}{c+dx}\right)\,dx\\ &= Ax+\frac{B(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{b}-\frac{(B(bc-ad))\int \frac{1}{c+dx}\,dx}{b}\\ &= Ax+\frac{B(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{b}-\frac{B(bc-ad)\log(c+dx)}{bd}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 52, normalized size = 1.00

$$Ax+\frac{B(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{b}-\frac{B(bc-ad)\log(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[A + B*Log[(e*(a + b*x))/(c + d*x)],x]
```

```
[Out] A*x + (B*(a + b*x)*Log[(e*(a + b*x))/(c + d*x)])/b - (B*(b*c - a*d)*Log[c +
 d*x])/(b*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 417 vs. 2(52) = 104.
time = 0.37, size = 418, normalized size = 8.04

method	result
risch	$Ax+Bx\ln\left(\frac{e(bx+a)}{dx+c}\right)-\frac{Bc\ln(dx+c)}{d}+\frac{Ba\ln(-bx-a)}{b}$
derivativedivides	$-\frac{e(ad-cb)\left(\frac{dA}{be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d}+\frac{dB\ln\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)}{be}+\frac{d^2B\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{be\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)}\right)}{d^2}$
default	$Ax-\frac{B\ln\left(-be+\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)a}{b}+\frac{B\ln\left(-be+\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)c}{d}+\frac{eB\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)a}{\frac{eda}{dx+c}-\frac{ecb}{dx+c}}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(A+B*ln(e*(b*x+a)/(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] A*x-B/b*ln(-b*e+(b*e/d+(a*d-b*c)*e/d/(d*x+c))*d)*a+B*ln(-b*e+(b*e/d+(a*d-b*
c)*e/d/(d*x+c))*d)/d*c+e*B*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))/(e/(d*x+c)*d*a-e
/(d*x+c)*c*b)*a-e*B*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))/(e/(d*x+c)*d*a-e/(d*x+c
)*c*b)/d*c*b+e*B*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))/b/(e/(d*x+c)*d*a-e/(d*x+c)
*c*b)/(d*x+c)*a^2*d-2*e*B*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))/(e/(d*x+c)*d*a-e/
(d*x+c)*c*b)/(d*x+c)*a*c+e*B*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))/(e/(d*x+c)*d*a
-e/(d*x+c)*c*b)/d/(d*x+c)*c^2*b
```

Maxima [A]
time = 0.28, size = 56, normalized size = 1.08

$$\left(\left(\frac{ae\log\left(bx+a\right)}{b}-\frac{ce\log\left(dx+c\right)}{d}\right)e^{(-1)}+x\log\left(\frac{(bx+a)e}{dx+c}\right)\right)B+Ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(A+B*log(e*(b*x+a)/(d*x+c)),x, algorithm="maxima")
```

[Out] ((a*e*log(b*x + a)/b - c*e*log(d*x + c)/d)*e^(-1) + x*log((b*x + a)*e/(d*x + c)))*B + A*x

Fricas [A]
time = 0.35, size = 55, normalized size = 1.06

$$\frac{Bbdx\log\left(\frac{(bx+a)e}{dx+c}\right)+Abdx+Bad\log\left(bx+a\right)-Bbc\log\left(dx+c\right)}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(A+B*log(e*(b*x+a)/(d*x+c)),x, algorithm="fricas")

[Out] (B*b*d*x*log((b*x + a)*e/(d*x + c)) + A*b*d*x + B*a*d*log(b*x + a) - B*b*c*log(d*x + c))/(b*d)

Sympy [A]
time = 0.87, size = 83, normalized size = 1.60

$$Ax+\frac{Ba\log\left(x+\frac{\frac{Ba^2d}{b}+Bac}{Bad+Bbc}\right)}{b}-\frac{Bc\log\left(x+\frac{Bac+\frac{Bbc^2}{d}}{Bad+Bbc}\right)}{d}+Bx\log\left(\frac{e(a+bx)}{c+dx}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(A+B*ln(e*(b*x+a)/(d*x+c)),x)

[Out] A*x + B*a*log(x + (B*a**2*d/b + B*a*c)/(B*a*d + B*b*c))/b - B*c*log(x + (B*a*c + B*b*c**2/d)/(B*a*d + B*b*c))/d + B*x*log(e*(a + b*x)/(c + d*x))

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 427 vs. 2(53) = 106.
time = 3.27, size = 427, normalized size = 8.21

$$-\left((b^2c^2e^2-2\,abcde^2+a^2d^2e^2)\left(\frac{e^{(-1)}\log\left(\frac{|bxe+ae|}{|dx+c|}\right)}{bd}-\frac{e^{(-1)}\log\left(\left|-be+\frac{(bxe+ae)d}{dx+c}\right|\right)}{bd}\right)-\frac{(b^2c^2e^2-2\,abcde^2+a^2d^2e^2)\log\left(\frac{\left(a-\frac{b\left(\frac{a}{bc-ad}-\frac{(bxe+ae)c}{(bce-ade)(dx+c)}\right)}{\frac{b}{bc-ad}-\frac{(bxe+ae)d}{(bce-ade)(dx+c)}}\right)e}{c-\frac{d\left(\frac{a}{bc-ad}-\frac{(bxe+ae)c}{(bce-ade)(dx+c)}\right)}{\frac{b}{bc-ad}-\frac{(bxe+ae)d}{(bce-ade)(dx+c)}}}\right)}{\left(be-\frac{(bxe+ae)d}{dx+c}\right)d}\right)B\left(\frac{bc}{(bce-ade)(bc-ad)}-\frac{ad}{(bce-ade)(bc-ad)}\right)+Ax$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(A+B*log(e*(b*x+a)/(d*x+c)),x, algorithm="giac")

[Out] -((b^2*c^2*e^2 - 2*a*b*c*d*e^2 + a^2*d^2*e^2)*(e^(-1)*log(abs(b*x*e + a*e)/abs(d*x + c))/(b*d) - e^(-1)*log(abs(-b*e + (b*x*e + a*e)*d/(d*x + c)))/(b*d)) - (b^2*c^2*e^2 - 2*a*b*c*d*e^2 + a^2*d^2*e^2)*log((a - b*(a/(b*c - a*d) - (b*x*e + a*e)*c/((b*c*e - a*d*e)*(d*x + c)))/(b/(b*c - a*d) - (b*x*e + a*e)*d/((b*c*e - a*d*e)*(d*x + c))))*e/(c - d*(a/(b*c - a*d) - (b*x*e + a*e)*c/((b*c*e - a*d*e)*(d*x + c)))/(b/(b*c - a*d) - (b*x*e + a*e)*d/((b*c*e -

```
a*d*e)*(d*x + c)))))/((b*e - (b*x*e + a*e)*d/(d*x + c))*d))*B*(b*c/((b*c*e
- a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d))) + A*x
```

Mupad [B]

time = 4.12, size = 47, normalized size = 0.90

$$A\,x+B\,x\,\ln\left(\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right)+\frac{B\,a\,\ln\left(a+b\,x\right)}{b}-\frac{B\,c\,\ln\left(c+d\,x\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(A + B*log((e*(a + b*x))/(c + d*x)),x)
```

```
[Out] A*x + B*x*log((e*(a + b*x))/(c + d*x)) + (B*a*log(a + b*x))/b - (B*c*log(c
+ d*x))/d
```

3.235 $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{f+gx} dx$

Optimal. Leaf size=140

$$-\frac{B\log\left(-\frac{g(a+bx)}{bf-ag}\right)\log(f+gx)}{g}+\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log(f+gx)}{g}+\frac{B\log\left(-\frac{g(c+dx)}{df-cg}\right)\log(f+gx)}{g}-\frac{B\mathrm{Li}_2}{}$$

```
[Out] -B*ln(-g*(b*x+a)/(-a*g+b*f))*ln(g*x+f)/g+(A+B*ln(e*(b*x+a)/(d*x+c)))*ln(g*x
+f)/g+B*ln(-g*(d*x+c)/(-c*g+d*f))*ln(g*x+f)/g-B*polylog(2,b*(g*x+f)/(-a*g+b
*f))/g+B*polylog(2,d*(g*x+f)/(-c*g+d*f))/g
```

Rubi [A]

time = 0.09, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {2546, 2441, 2440, 2438}

$$-\frac{B\text{PolyLog}\left(2,\frac{b(f+gx)}{bf-ag}\right)}{g}+\frac{B\text{PolyLog}\left(2,\frac{d(f+gx)}{df-cg}\right)}{g}+\frac{\log(f+gx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{g}-\frac{B\log(f+gx)\log\left(-\frac{g(a+bx)}{bf-ag}\right)}{g}+\frac{B\log(f+gx)\log\left(-\frac{g(c+dx)}{df-cg}\right)}{g}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])/(f + g*x),x]
```

```
[Out] -((B*Log[-((g*(a + b*x))/(b*f - a*g))]*Log[f + g*x])/g) + ((A + B*Log[(e*(a
 + b*x))/(c + d*x)])*Log[f + g*x])/g + (B*Log[-((g*(c + d*x))/(d*f - c*g))]
*Log[f + g*x])/g - (B*PolyLog[2, (b*(f + g*x))/(b*f - a*g)])/g + (B*PolyLog
[2, (d*(f + g*x))/(d*f - c*g)])/g
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2546

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))/((f_.) + (g_.)*(x_)), x_Symbol] :> Simp[Log[f + g*x]*((A + B*Log[
e*((a + b*x)^n/(c + d*x)^n)])/g), x] + (-Dist[b*B*(n/g), Int[Log[f + g*x]/(
a + b*x), x], x] + Dist[B*d*(n/g), Int[Log[f + g*x]/(c + d*x), x], x]) /; F
reeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[n + mn, 0] && NeQ[b*c - a*d,
 0]
```

Rubi steps

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{f+gx}\,dx = \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log(f+gx)}{g} - \frac{B\int \frac{(c+dx)\left(-\frac{de(a+bx)}{(c+dx)^2}+\frac{be}{c+dx}\right)\log(f+gx)}{e(a+bx)}\,dx}{g}$$

$$= \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log(f+gx)}{g} - \frac{B\int \frac{(c+dx)\left(-\frac{de(a+bx)}{(c+dx)^2}+\frac{be}{c+dx}\right)\log(f+gx)}{a+bx}\,dx}{eg}$$

$$= \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log(f+gx)}{g} - \frac{B\int \left(\frac{be\log(f+gx)}{a+bx}-\frac{de\log(f+gx)}{c+dx}\right)dx}{eg}$$

$$= \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log(f+gx)}{g} - \frac{(bB)\int \frac{\log(f+gx)}{a+bx}\,dx}{g} + \frac{(Bd)\int \frac{\log(f+gx)}{c+dx}\,dx}{g}$$

$$= -\frac{B\log\left(-\frac{g(a+bx)}{bf-ag}\right)\log(f+gx)}{g} + \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log(f+gx)}{g} + \frac{B\log\left(\right.}{}$$

$$= -\frac{B\log\left(-\frac{g(a+bx)}{bf-ag}\right)\log(f+gx)}{g} + \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log(f+gx)}{g} + \frac{B\log\left(\right.}{}$$

$$= -\frac{B\log\left(-\frac{g(a+bx)}{bf-ag}\right)\log(f+gx)}{g} + \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log(f+gx)}{g} + \frac{B\log\left(\right.}{}$$

Mathematica [**A**]
time = 0.04, size = 115, normalized size = 0.82

$$\frac{\left(A-B\log\left(\frac{g(a+bx)}{-bf+ag}\right)+B\log\left(\frac{e(a+bx)}{c+dx}\right)+B\log\left(\frac{g(c+dx)}{-df+cg}\right)\right)\log(f+gx)-B\text{Li}_2\left(\frac{b(f+gx)}{bf-ag}\right)+B\text{Li}_2\left(\frac{d(f+gx)}{df-cg}\right)}{g}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])/(f + g*x),x]
[Out] ((A - B*Log[(g*(a + b*x))/(-(b*f) + a*g)] + B*Log[(e*(a + b*x))/(c + d*x)]
+ B*Log[(g*(c + d*x))/(-(d*f) + c*g)])*Log[f + g*x] - B*PolyLog[2, (b*(f +
g*x))/(b*f - a*g)] + B*PolyLog[2, (d*(f + g*x))/(d*f - c*g)])/g
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 853 vs. $2(140) = 280$.
time = 2.15, size = 854, normalized size = 6.10 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*(b*x+a)/(d*x+c)))/(g*x+f),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d^2*e*(a*d-b*c)*(d^2*A/e/(a*d-b*c)/(-c*g+d*f)*ln(a*e*g-b*e*f-c*g*(b*e/d+
(a*d-b*c)*e/d/(d*x+c))+d*f*(b*e/d+(a*d-b*c)*e/d/(d*x+c)))*c-d^3*A/e/g/(a*d-
b*c)/(-c*g+d*f)*ln(a*e*g-b*e*f-c*g*(b*e/d+(a*d-b*c)*e/d/(d*x+c))+d*f*(b*e/d
+(a*d-b*c)*e/d/(d*x+c)))*f+d^2*A/e/g/(a*d-b*c)*ln(b*e-(b*e/d+(a*d-b*c)*e/d/
(d*x+c))*d)-d^2*B/e/(a*d-b*c)*dilog(((c*g-d*f)*(b*e/d+(a*d-b*c)*e/d/(d*x+c)
)-a*e*g+b*e*f)/(-a*e*g+b*e*f))/(c*g-d*f)*c+d^3*B/e/g/(a*d-b*c)*dilog(((c*g-
d*f)*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-a*e*g+b*e*f)/(-a*e*g+b*e*f))/(c*g-d*f)*f
-d^2*B/e/(a*d-b*c)*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(((c*g-d*f)*(b*e/d+(a*
d-b*c)*e/d/(d*x+c))-a*e*g+b*e*f)/(-a*e*g+b*e*f))/(c*g-d*f)*c+d^3*B/e/g/(a*d
-b*c)*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(((c*g-d*f)*(b*e/d+(a*d-b*c)*e/d/(d
*x+c))-a*e*g+b*e*f)/(-a*e*g+b*e*f))/(c*g-d*f)*f+d^2*B/e/g/(a*d-b*c)*dilog(-
(-b*e+(b*e/d+(a*d-b*c)*e/d/(d*x+c))*d)/b/e)+d^2*B/e/g/(a*d-b*c)*ln(b*e/d+(a
*d-b*c)*e/d/(d*x+c))*ln(-(-b*e+(b*e/d+(a*d-b*c)*e/d/(d*x+c))*d)/b/e))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(g*x+f),x, algorithm="maxima")
```

```
[Out] -B*integrate(-(log(b*x + a) - log(d*x + c) + 1)/(g*x + f), x) + A*log(g*x +
 f)/g
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(g*x+f),x, algorithm="fricas")
```

```
[Out] integral((B*log((b*x + a)*e/(d*x + c)) + A)/(g*x + f), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(b*x+a)/(d*x+c)))/(g*x+f),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(g*x+f),x, algorithm="giac")
```

```
[Out] integrate((B*log((b*x + a)*e/(d*x + c)) + A)/(g*x + f), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)}{f+g\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x))/(c + d*x)))/(f + g*x),x)
```

```
[Out] int((A + B*log((e*(a + b*x))/(c + d*x)))/(f + g*x), x)
```

3.236 $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(f+gx)^2}\,dx$

Optimal. Leaf size=87

$$\frac{(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bf-ag)(f+gx)}+\frac{B(bc-ad)\log\left(\frac{f+gx}{c+dx}\right)}{(bf-ag)(df-cg)}$$

```
[Out] (b*x+a)*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*g+b*f)/(g*x+f)+B*(-a*d+b*c)*ln((g*x
+f)/(d*x+c))/(-a*g+b*f)/(-c*g+d*f)
```

Rubi [A]
time = 0.06, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2554, 2351, 31}

$$\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{(f+gx)(bf-ag)}+\frac{B(bc-ad)\log\left(\frac{f+gx}{c+dx}\right)}{(bf-ag)(df-cg)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])/(f + g*x)^2,x]
```

```
[Out] ((a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/((b*f - a*g)*(f + g*x)) +
(B*(b*c - a*d)*Log[(f + g*x)/(c + d*x)])/((b*f - a*g)*(d*f - c*g))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2554

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[b*c - a*d, Su
bst[Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m +
2)), x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n
}, x] && EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegerQ[m] &
& IGtQ[p, 0]
```

Rubi steps

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(f+gx)^2}\,dx = -\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{g(f+gx)} + \frac{B\int \frac{bc-ad}{(a+bx)(c+dx)(f+gx)}\,dx}{g}$$

$$= -\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{g(f+gx)} + \frac{(B(bc-ad))\int \frac{1}{(a+bx)(c+dx)(f+gx)}\,dx}{g}$$

$$= -\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{g(f+gx)} + \frac{(B(bc-ad))\int \left(\frac{b^2}{(bc-ad)(bf-ag)(a+bx)} + \frac{d^2}{(bc-ad)(-df+cg)(c+d}\right.}{g}$$

$$= \frac{bB\log(a+bx)}{g(bf-ag)} - \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{g(f+gx)} - \frac{Bd\log(c+dx)}{g(df-cg)} + \frac{B(bc-ad)\log(f+}{(bf-ag)(df-cg}$$

Mathematica [A]
time = 0.09, size = 105, normalized size = 1.21

$$\frac{-\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{f+gx} + \frac{B(b(df-cg)\log(a+bx)+(-bdf+adg)\log(c+dx)+(bc-ad)g\log(f+gx))}{(bf-ag)(df-cg)}}{g}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])/(f + g*x)^2,x]
```

```
[Out] (-((A + B*Log[(e*(a + b*x))/(c + d*x)])/(f + g*x)) + (B*(b*(d*f - c*g)*Log[
a + b*x] + (-(b*d*f) + a*d*g)*Log[c + d*x] + (b*c - a*d)*g*Log[f + g*x]))/(
(b*f - a*g)*(d*f - c*g)))/g
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 301 vs. $2(87) = 174$.
time = 0.43, size = 302, normalized size = 3.47

method	result
risch	$-\frac{B\ln\left(\frac{e(bx+a)}{dx+c}\right)}{g(gx+f)} - \frac{B\ln(gx+f)ad\,g^2x - B\ln(gx+f)bc\,g^2x + B\ln(-bx-a)bc\,g^2x - B\ln(-bx-a)bdfgx - B\ln(-dx-c)ad\,g}{}$
derivativedivides	$-\frac{e(ad-cb)\left(-\frac{d^2A}{\left((-cg+df)\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)+aeg-bef\right)(-cg+df)} - \frac{d^2B\ln\left((-cg+df)\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)+aeg-bef\right)}{e(ag-bf)(-cg+df)} + \frac{d^2}{e(ag-bf)\left(ae\right.}\right.}{d^2}$
default	$-\frac{e(ad-cb)\left(-\frac{d^2A}{\left((-cg+df)\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)+aeg-bef\right)(-cg+df)} - \frac{d^2B\ln\left((-cg+df)\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)+aeg-bef\right)}{e(ag-bf)(-cg+df)} + \frac{d^2}{e(ag-bf)\left(ae\right.}\right.}{d^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*(b*x+a)/(d*x+c)))/(g*x+f)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/d^2*e*(a*d-b*c)*(-d^2*A/((-c*g+d*f)*(b*e/d+(a*d-b*c)*e/d/(d*x+c))+a*e*g-
b*e*f)/(-c*g+d*f)-d^2*B/e/(a*g-b*f)*ln((-c*g+d*f)*(b*e/d+(a*d-b*c)*e/d/(d*x
+c))+a*e*g-b*e*f)/(-c*g+d*f)+d^2*B*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*(b*e/d+(
a*d-b*c)*e/d/(d*x+c))/e/(a*g-b*f)/(a*e*g-b*e*f-c*g*(b*e/d+(a*d-b*c)*e/d/(d*
x+c))+d*f*(b*e/d+(a*d-b*c)*e/d/(d*x+c))))
```

Maxima [A]
time = 0.28, size = 140, normalized size = 1.61

$$B\left(\frac{b\log\left(bx+a\right)}{bfg-ag^2}-\frac{d\log\left(dx+c\right)}{dfg-cg^2}+\frac{(bc-ad)\log\left(gx+f\right)}{bdf^2+acg^2-(bc+ad)fg}-\frac{\log\left(\frac{bxe}{dx+c}+\frac{ae}{dx+c}\right)}{g^2x+fg}\right)-\frac{A}{g^2x+fg}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(g*x+f)^2,x, algorithm="maxima")
```

```
[Out] B*(b*log(b*x + a)/(b*f*g - a*g^2) - d*log(d*x + c)/(d*f*g - c*g^2) + (b*c -
 a*d)*log(g*x + f)/(b*d*f^2 + a*c*g^2 - (b*c + a*d)*f*g) - log(b*x*e/(d*x +
 c) + a*e/(d*x + c))/(g^2*x + f*g)) - A/(g^2*x + f*g)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 254 vs. $2(88) = 176$.
time = 3.51, size = 254, normalized size = 2.92

$$-\frac{Abdf^2+Aacg^2-(Abc+Aad)fg-(Bbdf^2-Bbcfg+(Bbdfg-Bbcg^2)x)\log\left(bx+a\right)+(Bbdf^2-Badfg+(Bbdfg-Badg^2)x)\log\left(dx+c\right)-((Bbc-Bad)g^2x+(Bbc-Bad)fg)\log\left(gx+f\right)+(Bbdf^2+Bacg^2-(Bbc+Bad)fg)\log\left(\frac{(bx+a)e}{dx+c}\right)}{bdf^3g+acfg^3-(bc+ad)f^2g^2+(bdf^2g^2+acg^4-(bc+ad)fg^3)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(g*x+f)^2,x, algorithm="fricas")
```

```
[Out] -(A*b*d*f^2 + A*a*c*g^2 - (A*b*c + A*a*d)*f*g - (B*b*d*f^2 - B*b*c*f*g + (B
*b*d*f*g - B*b*c*g^2)*x)*log(b*x + a) + (B*b*d*f^2 - B*a*d*f*g + (B*b*d*f*g
 - B*a*d*g^2)*x)*log(d*x + c) - ((B*b*c - B*a*d)*g^2*x + (B*b*c - B*a*d)*f*
g)*log(g*x + f) + (B*b*d*f^2 + B*a*c*g^2 - (B*b*c + B*a*d)*f*g)*log((b*x +
a)*e/(d*x + c)))/(b*d*f^3*g + a*c*f*g^3 - (b*c + a*d)*f^2*g^2 + (b*d*f^2*g^
2 + a*c*g^4 - (b*c + a*d)*f*g^3)*x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(b*x+a)/(d*x+c)))/(g*x+f)**2,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1537 vs. $2(88) = 176$.

time = 5.06, size = 1537, normalized size = 17.67

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(g*x+f)^2,x, algorithm="giac")
```

```
[Out] (B*b^3*c^2*f*e^2*log(-b*f*e + a*g*e + (b*x*e + a*e)*d*f/(d*x + c) - (b*x*e
+ a*e)*c*g/(d*x + c)) - 2*B*a*b^2*c*d*f*e^2*log(-b*f*e + a*g*e + (b*x*e + a
*e)*d*f/(d*x + c) - (b*x*e + a*e)*c*g/(d*x + c)) + B*a^2*b*d^2*f*e^2*log(-b
*f*e + a*g*e + (b*x*e + a*e)*d*f/(d*x + c) - (b*x*e + a*e)*c*g/(d*x + c)) -
 B*a*b^2*c^2*g*e^2*log(-b*f*e + a*g*e + (b*x*e + a*e)*d*f/(d*x + c) - (b*x*
e + a*e)*c*g/(d*x + c)) + 2*B*a^2*b*c*d*g*e^2*log(-b*f*e + a*g*e + (b*x*e +
 a*e)*d*f/(d*x + c) - (b*x*e + a*e)*c*g/(d*x + c)) - B*a^3*d^2*g*e^2*log(-b
*f*e + a*g*e + (b*x*e + a*e)*d*f/(d*x + c) - (b*x*e + a*e)*c*g/(d*x + c)) -
 (b*x*e + a*e)*B*b^2*c^2*d*f*e*log(-b*f*e + a*g*e + (b*x*e + a*e)*d*f/(d*x
+ c) - (b*x*e + a*e)*c*g/(d*x + c))/(d*x + c) + 2*(b*x*e + a*e)*B*a*b*c*d^2
*f*e*log(-b*f*e + a*g*e + (b*x*e + a*e)*d*f/(d*x + c) - (b*x*e + a*e)*c*g/(
d*x + c))/(d*x + c) - (b*x*e + a*e)*B*a^2*d^3*f*e*log(-b*f*e + a*g*e + (b*x
*e + a*e)*d*f/(d*x + c) - (b*x*e + a*e)*c*g/(d*x + c))/(d*x + c) + (b*x*e +
 a*e)*B*b^2*c^3*g*e*log(-b*f*e + a*g*e + (b*x*e + a*e)*d*f/(d*x + c) - (b*x
*e + a*e)*c*g/(d*x + c))/(d*x + c) - 2*(b*x*e + a*e)*B*a*b*c^2*d*g*e*log(-b
*f*e + a*g*e + (b*x*e + a*e)*d*f/(d*x + c) - (b*x*e + a*e)*c*g/(d*x + c))/(
d*x + c) + (b*x*e + a*e)*B*a^2*c*d^2*g*e*log(-b*f*e + a*g*e + (b*x*e + a*e)
*d*f/(d*x + c) - (b*x*e + a*e)*c*g/(d*x + c))/(d*x + c) + (b*x*e + a*e)*B*b
^2*c^2*d*f*e*log((b*x*e + a*e)/(d*x + c))/(d*x + c) - 2*(b*x*e + a*e)*B*a*b
*c*d^2*f*e*log((b*x*e + a*e)/(d*x + c))/(d*x + c) + (b*x*e + a*e)*B*a^2*d^3
*f*e*log((b*x*e + a*e)/(d*x + c))/(d*x + c) - (b*x*e + a*e)*B*b^2*c^3*g*e*l
og((b*x*e + a*e)/(d*x + c))/(d*x + c) + 2*(b*x*e + a*e)*B*a*b*c^2*d*g*e*log
((b*x*e + a*e)/(d*x + c))/(d*x + c) - (b*x*e + a*e)*B*a^2*c*d^2*g*e*log((b*
x*e + a*e)/(d*x + c))/(d*x + c) + A*b^3*c^2*f*e^2 - 2*A*a*b^2*c*d*f*e^2 + A
*a^2*b*d^2*f*e^2 - A*a*b^2*c^2*g*e^2 + 2*A*a^2*b*c*d*g*e^2 - A*a^3*d^2*g*e^
2)*(b*c/((b*c*e - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))/
(b^2*d*f^3*e - b^2*c*f^2*g*e - 2*a*b*d*f^2*g*e + 2*a*b*c*f*g^2*e + a^2*d*f*
g^2*e - a^2*c*g^3*e - (b*x*e + a*e)*b*d^2*f^3/(d*x + c) + 2*(b*x*e + a*e)*b
*c*d*f^2*g/(d*x + c) + (b*x*e + a*e)*a*d^2*f^2*g/(d*x + c) - (b*x*e + a*e)*
b*c^2*f*g^2/(d*x + c) - 2*(b*x*e + a*e)*a*c*d*f*g^2/(d*x + c) + (b*x*e + a*
e)*a*c^2*g^3/(d*x + c))
```

Mupad [B]

time = 5.17, size = 166, normalized size = 1.91

$$\frac{B\,d\,\ln\left(c+d\,x\right)}{c\,g^2-d\,f\,g}-\frac{B\,\ln\left(\frac{a\,e+b\,e\,x}{c+d\,x}\right)}{x\,g^2+f\,g}-\frac{B\,b\,\ln\left(a+b\,x\right)}{a\,g^2-b\,f\,g}-\frac{A}{x\,g^2+f\,g}-\frac{B\,a\,d\,\ln\left(f+g\,x\right)}{a\,c\,g^2+b\,d\,f^2-a\,d\,f\,g-b\,c\,f\,g}+\frac{B\,b\,c\,\ln\left(f+g\,x\right)}{a\,c\,g^2+b\,d\,f^2-a\,d\,f\,g-b\,c\,f\,g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x))/(c + d*x)))/(f + g*x)^2,x)
```

```
[Out] (B*d*log(c + d*x))/(c*g^2 - d*f*g) - (B*log((a*e + b*e*x)/(c + d*x)))/(f*g
+ g^2*x) - (B*b*log(a + b*x))/(a*g^2 - b*f*g) - A/(f*g + g^2*x) - (B*a*d*lo
g(f + g*x))/(a*c*g^2 + b*d*f^2 - a*d*f*g - b*c*f*g) + (B*b*c*log(f + g*x))/
(a*c*g^2 + b*d*f^2 - a*d*f*g - b*c*f*g)
```

3.237 $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(f+gx)^3}\,dx$

Optimal. Leaf size=183

$$-\frac{B(bc-ad)}{2(bf-ag)(df-cg)(f+gx)}+\frac{b^2B\log(a+bx)}{2g(bf-ag)^2}-\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{2g(f+gx)^2}-\frac{Bd^2\log(c+dx)}{2g(df-cg)^2}+\frac{B(bc-ad)(2bdf}{2(bf-}$$

```
[Out] -1/2*B*(-a*d+b*c)/(-a*g+b*f)/(-c*g+d*f)/(g*x+f)+1/2*b^2*B*ln(b*x+a)/g/(-a*g
+b*f)^2+1/2*(-A-B*ln(e*(b*x+a)/(d*x+c)))/g/(g*x+f)^2-1/2*B*d^2*ln(d*x+c)/g/
(-c*g+d*f)^2+1/2*B*(-a*d+b*c)*(-a*d*g-b*c*g+2*b*d*f)*ln(g*x+f)/(-a*g+b*f)^2
/(-c*g+d*f)^2
```

Rubi [A]
time = 0.14, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {2548, 84}

$$-\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{2g(f+gx)^2}+\frac{b^2B\log(a+bx)}{2g(bf-ag)^2}-\frac{B(bc-ad)}{2(f+gx)(bf-ag)(df-cg)}+\frac{B(bc-ad)\log(f+gx)(-adg-bcg+2bdf)}{2(bf-ag)^2(df-cg)^2}-\frac{Bd^2\log(c+dx)}{2g(df-cg)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])/(f + g*x)^3,x]
```

```
[Out] -1/2*(B*(b*c - a*d))/((b*f - a*g)*(d*f - c*g)*(f + g*x)) + (b^2*B*Log[a + b
*x])/(2*g*(b*f - a*g)^2) - (A + B*Log[(e*(a + b*x))/(c + d*x)])/(2*g*(f + g
*x)^2) - (B*d^2*Log[c + d*x])/(2*g*(d*f - c*g)^2) + (B*(b*c - a*d)*(2*b*d*f
 - b*c*g - a*d*g)*Log[f + g*x])/(2*(b*f - a*g)^2*(d*f - c*g)^2)
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 2548

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Dist[B*n*((b*c
- a*d)/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /;
 FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c -
 a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Rubi steps

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(f+gx)^3}\,dx = -\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{2g(f+gx)^2} + \frac{B\int \frac{bc-ad}{(a+bx)(c+dx)(f+gx)^2}\,dx}{2g}$$

$$= -\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{2g(f+gx)^2} + \frac{(B(bc-ad))\int \frac{1}{(a+bx)(c+dx)(f+gx)^2}\,dx}{2g}$$

$$= -\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{2g(f+gx)^2} + \frac{(B(bc-ad))\int \left(\frac{b^3}{(bc-ad)(bf-ag)^2(a+bx)} - \frac{d^3}{(bc-ad)(-df+cg)^2}\right.}{2}$$

$$= -\frac{B(bc-ad)}{2(bf-ag)(df-cg)(f+gx)} + \frac{b^2B\log(a+bx)}{2g(bf-ag)^2} - \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{2g(f+gx)^2} - \frac{B}{}$$

Mathematica [A]
time = 0.34, size = 169, normalized size = 0.92

$$\frac{-\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(f+gx)^2} + B(bc-ad)\left(\frac{b^2\log(a+bx)}{(bc-ad)(bf-ag)^2} + \frac{\frac{g(-df+cg)}{(bf-ag)(f+gx)} + \frac{d^2\log(c+dx)}{-bc+ad} - \frac{g(-2bdf+bcg+adg)\log(f+gx)}{(bf-ag)^2}}{(df-cg)^2}\right)}{2g}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])/(f + g*x)^3,x]
```

```
[Out] (-((A + B*Log[(e*(a + b*x))/(c + d*x)])/(f + g*x)^2) + B*(b*c - a*d)*((b^2*
Log[a + b*x])/((b*c - a*d)*(b*f - a*g)^2) + ((g*(-(d*f) + c*g))/((b*f - a*g
)*(f + g*x)) + (d^2*Log[c + d*x])/(-(b*c) + a*d) - (g*(-2*b*d*f + b*c*g + a
*d*g)*Log[f + g*x])/(b*f - a*g)^2)/(d*f - c*g)^2))/(2*g)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2441 vs. $2(176) = 352$.
time = 0.50, size = 2442, normalized size = 13.34

method	result
risch	$-\frac{B\ln\left(\frac{e(bx+a)}{dx+c}\right)}{2g(gx+f)^2} - \frac{-Bab\,d^2f^2g^2x - B\ln(-gx-f)a^2d^2g^4x^2 + B\ln(-gx-f)b^2c^2g^4x^2 + B\ln(-dx-c)a^2d^2g^4x^2 - B\ln(}{}$
derivativedivides	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*(b*x+a)/(d*x+c)))/(g*x+f)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/d^2*e*(a*d-b*c)*(-A*d^2*(-d/(c*g-d*f)/(-c*g+d*f)/(a*e*g-b*e*f-c*g*(b*e/d
+(a*d-b*c)*e/d/(d*x+c))+d*f*(b*e/d+(a*d-b*c)*e/d/(d*x+c)))+1/2*e*g*(a*d-b*c
```

```
)/(c*g-d*f)/(-c*g+d*f)/(a*e*g-b*e*f-c*g*(b*e/d+(a*d-b*c)*e/d/(d*x+c))+d*f*(
b*e/d+(a*d-b*c)*e/d/(d*x+c)))^2)+B*d^3/(c*g-d*f)/e/(a*g-b*f)*ln((-c*g+d*f)*
(b*e/d+(a*d-b*c)*e/d/(d*x+c))+a*e*g-b*e*f)/(-c*g+d*f)-B*d^3/(c*g-d*f)*ln(b*
e/d+(a*d-b*c)*e/d/(d*x+c))*(b*e/d+(a*d-b*c)*e/d/(d*x+c))/e/(a*g-b*f)/(a*e*g
-b*e*f-c*g*(b*e/d+(a*d-b*c)*e/d/(d*x+c))+d*f*(b*e/d+(a*d-b*c)*e/d/(d*x+c)))
+1/2*B*d^3*g^2/(c*g-d*f)/(a*g-b*f)^2/(-c*g+d*f)/(a*e*g-b*e*f-c*g*(b*e/d+(a*
d-b*c)*e/d/(d*x+c))+d*f*(b*e/d+(a*d-b*c)*e/d/(d*x+c)))*a^2-1/2*B*d^2*g^2/(c
*g-d*f)/(a*g-b*f)^2/(-c*g+d*f)/(a*e*g-b*e*f-c*g*(b*e/d+(a*d-b*c)*e/d/(d*x+c
))+d*f*(b*e/d+(a*d-b*c)*e/d/(d*x+c)))*a*b*c-1/2*B*d^3*g/(c*g-d*f)/(a*g-b*f)
^2/(-c*g+d*f)/(a*e*g-b*e*f-c*g*(b*e/d+(a*d-b*c)*e/d/(d*x+c))+d*f*(b*e/d+(a*
d-b*c)*e/d/(d*x+c)))*b*f*a+1/2*B*d^2*g/(c*g-d*f)/(a*g-b*f)^2/(-c*g+d*f)/(a*
e*g-b*e*f-c*g*(b*e/d+(a*d-b*c)*e/d/(d*x+c))+d*f*(b*e/d+(a*d-b*c)*e/d/(d*x+c
)))*b^2*f*c-1/2*B*d^3/e*g/(c*g-d*f)/(a*g-b*f)^2/(-c*g+d*f)*ln(a*e*g-b*e*f-c
*g*(b*e/d+(a*d-b*c)*e/d/(d*x+c))+d*f*(b*e/d+(a*d-b*c)*e/d/(d*x+c)))*a+1/2*B
*d^2/e*g/(c*g-d*f)/(a*g-b*f)^2/(-c*g+d*f)*ln(a*e*g-b*e*f-c*g*(b*e/d+(a*d-b*
c)*e/d/(d*x+c))+d*f*(b*e/d+(a*d-b*c)*e/d/(d*x+c)))*b*c+B*d^3*g^2/(c*g-d*f)*
ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*(b*e/d+(a*d-b*c)*e/d/(d*x+c))/(a*e*g-b*e*f-
c*g*(b*e/d+(a*d-b*c)*e/d/(d*x+c))+d*f*(b*e/d+(a*d-b*c)*e/d/(d*x+c)))^2/(a*g
-b*f)^2*a^2-B*d^2*g^2/(c*g-d*f)*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*(b*e/d+(a*d
-b*c)*e/d/(d*x+c))/(a*e*g-b*e*f-c*g*(b*e/d+(a*d-b*c)*e/d/(d*x+c))+d*f*(b*e/
d+(a*d-b*c)*e/d/(d*x+c)))^2/(a*g-b*f)^2*a*b*c-B*d^3*g/(c*g-d*f)*ln(b*e/d+(a
*d-b*c)*e/d/(d*x+c))*(b*e/d+(a*d-b*c)*e/d/(d*x+c))/(a*e*g-b*e*f-c*g*(b*e/d+
(a*d-b*c)*e/d/(d*x+c))+d*f*(b*e/d+(a*d-b*c)*e/d/(d*x+c)))^2/(a*g-b*f)^2*b*f
*a+B*d^2*g/(c*g-d*f)*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*(b*e/d+(a*d-b*c)*e/d/(
d*x+c))/(a*e*g-b*e*f-c*g*(b*e/d+(a*d-b*c)*e/d/(d*x+c))+d*f*(b*e/d+(a*d-b*c)
*e/d/(d*x+c)))^2/(a*g-b*f)^2*b^2*f*c-1/2*B*d^3/e*g^2/(c*g-d*f)*ln(b*e/d+(a*
d-b*c)*e/d/(d*x+c))*(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2/(a*e*g-b*e*f-c*g*(b*e/d
+(a*d-b*c)*e/d/(d*x+c))+d*f*(b*e/d+(a*d-b*c)*e/d/(d*x+c)))^2/(a*g-b*f)^2*c*
a+1/2*B*d^2/e*g^2/(c*g-d*f)*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*(b*e/d+(a*d-b*c
)*e/d/(d*x+c))^2/(a*e*g-b*e*f-c*g*(b*e/d+(a*d-b*c)*e/d/(d*x+c))+d*f*(b*e/d+
(a*d-b*c)*e/d/(d*x+c)))^2/(a*g-b*f)^2*c^2*b+1/2*B*d^4/e*g/(c*g-d*f)*ln(b*e/
d+(a*d-b*c)*e/d/(d*x+c))*(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2/(a*e*g-b*e*f-c*g*(
b*e/d+(a*d-b*c)*e/d/(d*x+c))+d*f*(b*e/d+(a*d-b*c)*e/d/(d*x+c)))^2/(a*g-b*f)
^2*f*a-1/2*B*d^3/e*g/(c*g-d*f)*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*(b*e/d+(a*d-
b*c)*e/d/(d*x+c))^2/(a*e*g-b*e*f-c*g*(b*e/d+(a*d-b*c)*e/d/(d*x+c))+d*f*(b*e
/d+(a*d-b*c)*e/d/(d*x+c)))^2/(a*g-b*f)^2*f*b*c)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 353 vs. $2(174) = 348$.
time = 0.31, size = 353, normalized size = 1.93

$$\frac{1}{2}\left(\frac{b^2\log\left(bx+a\right)}{b^2f^2g-2\,abfg^2+a^2g^3}-\frac{d^2\log\left(dx+c\right)}{d^2f^2g-2\,cdfg^2+c^2g^3}+\frac{\left(2\left(b^2cd-abd^2\right)f-\left(b^2c^2-a^2d^2\right)g\right)\log\left(gx+f\right)}{b^2d^2f^4+a^2c^2g^4-2\left(b^2cd+abd^2\right)f^3g+\left(b^2c^2+4\,abcd+a^2d^2\right)f^2g^2-2\left(abc^2+a^2cd\right)fg^3}-\frac{bc-ad}{bdf^3+acfg^2-\left(bc+ad\right)f^2g+\left(bdf^2g+acg^3-\left(bc+ad\right)fg^2\right)x}-\frac{\log\left(\frac{bxe}{dx+c}+\frac{ae}{dx+c}\right)}{g^3x^2+2\,fg^2x+f^2g}\right)B-\frac{A}{2\left(g^3x^2+2\,fg^2x+f^2g\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(g*x+f)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(b^2*log(b*x + a)/(b^2*f^2*g - 2*a*b*f*g^2 + a^2*g^3) - d^2*log(d*x + c
)/(d^2*f^2*g - 2*c*d*f*g^2 + c^2*g^3) + (2*(b^2*c*d - a*b*d^2)*f - (b^2*c^2
 - a^2*d^2)*g)*log(g*x + f)/(b^2*d^2*f^4 + a^2*c^2*g^4 - 2*(b^2*c*d + a*b*d
^2)*f^3*g + (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*f^2*g^2 - 2*(a*b*c^2 + a^2*c*d)
*f*g^3) - (b*c - a*d)/(b*d*f^3 + a*c*f*g^2 - (b*c + a*d)*f^2*g + (b*d*f^2*g
 + a*c*g^3 - (b*c + a*d)*f*g^2)*x) - log(b*x*e/(d*x + c) + a*e/(d*x + c))/(
g^3*x^2 + 2*f*g^2*x + f^2*g))*B - 1/2*A/(g^3*x^2 + 2*f*g^2*x + f^2*g)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1016 vs. $2(174) = 348$.
time = 44.83, size = 1016, normalized size = 5.55

$$-\frac{Ab^2d^2f^4 + Aa^2c^2g^4 - ((2A - B)b^2cd + (2A + B)abd^2)f^3g + ((A - B)b^2c^2 + 4Aabcd + (A + B)a^2d^2)f^2g^2 - ((2A - B)abc^2 + (2A + B)a^2cd)fg^3 + ((Bb^2cd - Babd^2)f^2g^2 - (Bb^2c^2 - Ba^2d^2)fg^3 + (Babc^2 - Ba^2cd)g^4)x - (Bb^2d^2f^4 - 2Bb^2cdf^3g + Bb^2c^2f^2g^2 + (Bb^2d^2f^2g^2 - 2Bb^2cdfg^3 + Bb^2c^2g^4)x^2 + 2(Bb^2d^2f^3g - 2Bb^2cdf^2g^2 + Bb^2c^2fg^3)x)\log(bx + a) + (Bb^2d^2f^4 - 2Babd^2f^3g + Ba^2d^2f^2g^2 + (Bb^2d^2f^2g^2 - 2Babd^2fg^3 + Ba^2d^2g^4)x^2 + 2(Bb^2d^2f^3g - 2Babd^2f^2g^2 + Ba^2d^2fg^3)x)\log(dx + c) - (2(Bb^2cd - Babd^2)f^3g - (Bb^2c^2 - Ba^2d^2)f^2g^2 + (2(Bb^2cd - Babd^2)fg^3 - (Bb^2c^2 - Ba^2d^2)g^4)x^2 + 2(2(Bb^2cd - Babd^2)f^2g^2 - (Bb^2c^2 - Ba^2d^2)fg^3)x)\log(gx + f) + (Bb^2d^2f^4 + Ba^2c^2g^4 - 2(Bb^2cd + Babd^2)f^3g + (Bb^2c^2 + 4Babcd + Ba^2d^2)f^2g^2 - 2(Babc^2 + Ba^2cd)fg^3)\log\left(\frac{(bx + a)e}{dx + c}\right)}{2(b^2d^2f^6g + a^2c^2f^2g^5 - 2(b^2cd + abd^2)f^5g^2 + (b^2c^2 + 4abcd + a^2d^2)f^4g^3 - 2(abc^2 + a^2cd)f^3g^4 + (b^2d^2f^4g^3 + a^2c^2g^7 - 2(b^2cd + abd^2)f^3g^4 + (b^2c^2 + 4abcd + a^2d^2)f^2g^5 - 2(abc^2 + a^2cd)fg^6)x^2 + 2(b^2d^2f^5g^2 + a^2c^2fg^6 - 2(b^2cd + abd^2)f^4g^3 + (b^2c^2 + 4abcd + a^2d^2)f^3g^4 - 2(abc^2 + a^2cd)f^2g^5)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(g*x+f)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(A*b^2*d^2*f^4 + A*a^2*c^2*g^4 - ((2*A - B)*b^2*c*d + (2*A + B)*a*b*d^
2)*f^3*g + ((A - B)*b^2*c^2 + 4*A*a*b*c*d + (A + B)*a^2*d^2)*f^2*g^2 - ((2*
A - B)*a*b*c^2 + (2*A + B)*a^2*c*d)*f*g^3 + ((B*b^2*c*d - B*a*b*d^2)*f^2*g^
2 - (B*b^2*c^2 - B*a^2*d^2)*f*g^3 + (B*a*b*c^2 - B*a^2*c*d)*g^4)*x - (B*b^2
*d^2*f^4 - 2*B*b^2*c*d*f^3*g + B*b^2*c^2*f^2*g^2 + (B*b^2*d^2*f^2*g^2 - 2*B
*b^2*c*d*f*g^3 + B*b^2*c^2*g^4)*x^2 + 2*(B*b^2*d^2*f^3*g - 2*B*b^2*c*d*f^2*
g^2 + B*b^2*c^2*f*g^3)*x)*log(b*x + a) + (B*b^2*d^2*f^4 - 2*B*a*b*d^2*f^3*g
 + B*a^2*d^2*f^2*g^2 + (B*b^2*d^2*f^2*g^2 - 2*B*a*b*d^2*f*g^3 + B*a^2*d^2*g
^4)*x^2 + 2*(B*b^2*d^2*f^3*g - 2*B*a*b*d^2*f^2*g^2 + B*a^2*d^2*f*g^3)*x)*lo
g(d*x + c) - (2*(B*b^2*c*d - B*a*b*d^2)*f^3*g - (B*b^2*c^2 - B*a^2*d^2)*f^2
*g^2 + (2*(B*b^2*c*d - B*a*b*d^2)*f*g^3 - (B*b^2*c^2 - B*a^2*d^2)*g^4)*x^2
+ 2*(2*(B*b^2*c*d - B*a*b*d^2)*f^2*g^2 - (B*b^2*c^2 - B*a^2*d^2)*f*g^3)*x)*
log(g*x + f) + (B*b^2*d^2*f^4 + B*a^2*c^2*g^4 - 2*(B*b^2*c*d + B*a*b*d^2)*f
^3*g + (B*b^2*c^2 + 4*B*a*b*c*d + B*a^2*d^2)*f^2*g^2 - 2*(B*a*b*c^2 + B*a^2
*c*d)*f*g^3)*log((b*x + a)*e/(d*x + c)))/(b^2*d^2*f^6*g + a^2*c^2*f^2*g^5 -
 2*(b^2*c*d + a*b*d^2)*f^5*g^2 + (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*f^4*g^3 -
2*(a*b*c^2 + a^2*c*d)*f^3*g^4 + (b^2*d^2*f^4*g^3 + a^2*c^2*g^7 - 2*(b^2*c*d
 + a*b*d^2)*f^3*g^4 + (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*f^2*g^5 - 2*(a*b*c^2
+ a^2*c*d)*f*g^6)*x^2 + 2*(b^2*d^2*f^5*g^2 + a^2*c^2*f*g^6 - 2*(b^2*c*d + a
*b*d^2)*f^4*g^3 + (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*f^3*g^4 - 2*(a*b*c^2 + a^
2*c*d)*f^2*g^5)*x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(b*x+a)/(d*x+c)))/(g*x+f)**3,x)
```

[Out] Timed out

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 7600 vs. $2(174) = 348$.
time = 5.49, size = 7600, normalized size = 41.53

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(g*x+f)^3,x, algorithm="giac")
```

```
[Out] 1/2*(2*B*b^5*c^2*d*f^3*e^3*log(-b*f*e + a*g*e + (b*x*e + a*e)*d*f/(d*x + c)
 - (b*x*e + a*e)*c*g/(d*x + c)) - 4*B*a*b^4*c*d^2*f^3*e^3*log(-b*f*e + a*g*
e + (b*x*e + a*e)*d*f/(d*x + c) - (b*x*e + a*e)*c*g/(d*x + c)) + 2*B*a^2*b^
3*d^3*f^3*e^3*log(-b*f*e + a*g*e + (b*x*e + a*e)*d*f/(d*x + c) - (b*x*e + a
*e)*c*g/(d*x + c)) - B*b^5*c^3*f^2*g*e^3*log(-b*f*e + a*g*e + (b*x*e + a*e)
*d*f/(d*x + c) - (b*x*e + a*e)*c*g/(d*x + c)) - 3*B*a*b^4*c^2*d*f^2*g*e^3*l
og(-b*f*e + a*g*e + (b*x*e + a*e)*d*f/(d*x + c) - (b*x*e + a*e)*c*g/(d*x +
c)) + 9*B*a^2*b^3*c*d^2*f^2*g*e^3*log(-b*f*e + a*g*e + (b*x*e + a*e)*d*f/(d
*x + c) - (b*x*e + a*e)*c*g/(d*x + c)) - 5*B*a^3*b^2*d^3*f^2*g*e^3*log(-b*f
*e + a*g*e + (b*x*e + a*e)*d*f/(d*x + c) - (b*x*e + a*e)*c*g/(d*x + c)) + 2
*B*a*b^4*c^3*f*g^2*e^3*log(-b*f*e + a*g*e + (b*x*e + a*e)*d*f/(d*x + c) - (
b*x*e + a*e)*c*g/(d*x + c)) - 6*B*a^3*b^2*c*d^2*f*g^2*e^3*log(-b*f*e + a*g*
e + (b*x*e + a*e)*d*f/(d*x + c) - (b*x*e + a*e)*c*g/(d*x + c)) + 4*B*a^4*b*
d^3*f*g^2*e^3*log(-b*f*e + a*g*e + (b*x*e + a*e)*d*f/(d*x + c) - (b*x*e + a
*e)*c*g/(d*x + c)) - B*a^2*b^3*c^3*g^3*e^3*log(-b*f*e + a*g*e + (b*x*e + a*
e)*d*f/(d*x + c) - (b*x*e + a*e)*c*g/(d*x + c)) + B*a^3*b^2*c^2*d*g^3*e^3*l
og(-b*f*e + a*g*e + (b*x*e + a*e)*d*f/(d*x + c) - (b*x*e + a*e)*c*g/(d*x +
c)) + B*a^4*b*c*d^2*g^3*e^3*log(-b*f*e + a*g*e + (b*x*e + a*e)*d*f/(d*x + c
) - (b*x*e + a*e)*c*g/(d*x + c)) - B*a^5*d^3*g^3*e^3*log(-b*f*e + a*g*e + (
b*x*e + a*e)*d*f/(d*x + c) - (b*x*e + a*e)*c*g/(d*x + c)) - 4*(b*x*e + a*e)
*B*b^4*c^2*d^2*f^3*e^2*log(-b*f*e + a*g*e + (b*x*e + a*e)*d*f/(d*x + c) - (
b*x*e + a*e)*c*g/(d*x + c))/(d*x + c) + 8*(b*x*e + a*e)*B*a*b^3*c*d^3*f^3*e
^2*log(-b*f*e + a*g*e + (b*x*e + a*e)*d*f/(d*x + c) - (b*x*e + a*e)*c*g/(d*
x + c))/(d*x + c) - 4*(b*x*e + a*e)*B*a^2*b^2*d^4*f^3*e^2*log(-b*f*e + a*g*
e + (b*x*e + a*e)*d*f/(d*x + c) - (b*x*e + a*e)*c*g/(d*x + c))/(d*x + c) +
6*(b*x*e + a*e)*B*b^4*c^3*d*f^2*g*e^2*log(-b*f*e + a*g*e + (b*x*e + a*e)*d*
f/(d*x + c) - (b*x*e + a*e)*c*g/(d*x + c))/(d*x + c) - 6*(b*x*e + a*e)*B*a*
b^3*c^2*d^2*f^2*g*e^2*log(-b*f*e + a*g*e + (b*x*e + a*e)*d*f/(d*x + c) - (b
*x*e + a*e)*c*g/(d*x + c))/(d*x + c) - 6*(b*x*e + a*e)*B*a^2*b^2*c*d^3*f^2*
g*e^2*log(-b*f*e + a*g*e + (b*x*e + a*e)*d*f/(d*x + c) - (b*x*e + a*e)*c*g/
(d*x + c))/(d*x + c) + 6*(b*x*e + a*e)*B*a^3*b*d^4*f^2*g*e^2*log(-b*f*e + a
*g*e + (b*x*e + a*e)*d*f/(d*x + c) - (b*x*e + a*e)*c*g/(d*x + c))/(d*x + c)
 - 2*(b*x*e + a*e)*B*b^4*c^4*f*g^2*e^2*log(-b*f*e + a*g*e + (b*x*e + a*e)*d
*f/(d*x + c) - (b*x*e + a*e)*c*g/(d*x + c))/(d*x + c) - 4*(b*x*e + a*e)*B*a
```

```
*b^3*c^3*d*f*g^2*e^2*log(-b*f*e + a*g*e + (b*x*e + a*e)*d*f/(d*x + c) - (b*
x*e + a*e)*c*g/(d*x + c))/(d*x + c) + 12*(b*x*e + a*e)*B*a^2*b^2*c^2*d^2*f*
g^2*e^2*log(-b*f*e + a*g*e + (b*x*e + a*e)*d*f/(d*x + c) - (b*x*e + a*e)*c*
g/(d*x + c))/(d*x + c) - 4*(b*x*e + a*e)*B*a^3*b*c*d^3*f*g^2*e^2*log(-b*f*e
 + a*g*e + (b*x*e + a*e)*d*f/(d*x + c) - (b*x*e + a*e)*c*g/(d*x + c))/(d*x
+ c) - 2*(b*x*e + a*e)*B*a^4*d^4*f*g^2*e^2*log(-b*f*e + a*g*e + (b*x*e + a*
e)*d*f/(d*x + c) - (b*x*e + a*e)*c*g/(d*x + c))/(d*x + c) + 2*(b*x*e + a*e)
*B*a*b^3*c^4*g^3*e^2*log(-b*f*e + a*g*e + (b*x*e + a*e)*d*f/(d*x + c) - (b*
x*e + a*e)*c*g/(d*x + c))/(d*x + c) - 2*(b*x*e + a*e)*B*a^2*b^2*c^3*d*g^3*e
^2*log(-b*f*e + a*g*e + (b*x*e + a*e)*d*f/(d*x + c) - (b*x*e + a*e)*c*g/(d*
x + c))/(d*x + c) - 2*(b*x*e + a*e)*B*a^3*b*c^2*d^2*g^3*e^2*log(-b*f*e + a*
g*e + (b*x*e + a*e)*d*f/(d*x + c) - (b*x*e + a*e)*c*g/(d*x + c))/(d*x + c)
+ 2*(b*x*e + a*e)*B*a^4*c*d^3*g^3*e^2*log(-b*f*e + a*g*e + (b*x*e + a*e)*d*
f/(d*x + c) - (b*x*e + a*e)*c*g/(d*x + c))/(d*x + c) + 2*(b*x*e + a*e)^2*B*
b^3*c^2*d^3*f^3*e*log(-b*f*e + a*g*e + (b*x*e + a*e)*d*f/(d*x + c) - (b*x*e
 + a*e)*c*g/(d*x + c))/(d*x + c)^2 - 4*(b*x*e + a*e)^2*B*a*b^2*c*d^4*f^3*e*
log(-b*f*e + a*g*e + (b*x*e + a*e)*d*f/(d*x + c) - (b*x*e + a*e)*c*g/(d*x +
 c))/(d*x + c)^2 + 2*(b*x*e + a*e)^2*B*a^2*b*d^5*f^3*e*log(-b*f*e + a*g*e +
 (b*x*e + a*e)*d*f/(d*x + c) - (b*x*e + a*e)*c*g/(d*x + c))/(d*x + c)^2 - 5
*(b*x*e + a*e)^2*B*b^3*c^3*d^2*f^2*g*e*log(-b*f*e + a*g*e + (b*x*e + a*e)*d
*f/(d*x + c) - (b*x*e + a*e)*c*g/(d*x + c))/(d*x + c)^2 + 9*(b*x*e + a*e)^2
*B*a*b^2*c^2*d^3*f^2*g*e*log(-b*f*e + a*g*e + (b*x*e + a*e)*d*f/(d*x + c) -
 (b*x*e + a*e)*c*g/(d*x + c))/(d*x + c)^2 - 3*(b*x*e + a*e)^2*B*a^2*b*c*d^4
*f^2*g*e*log(-b*f*e + a*g*e + (b*x*e + a*e)*d*f/(d*x + c) - (b*x*e + a*e)*c
*g/(d*x + c))/(d*x + c)^2 - (b*x*e + a*e)^2*B*a^3*d^5*f^2*g*e*log(-b*f*e +
a*g*e + (b*x*e + a*e)*d*f/(d*x + c) - (b*x*e + a*e)*c*g/(d*x + c))/(d*x + c
)^2 + 4*(b*x*e + a*e)^2*B*b^3*c^4*d*f*g^2*e*log(-b*f*e + a*g*e + (b*x*e + a
*e)*d*f/(d*x + c) - (b*x*e + a*e)*c*g/(d*x + c))/(d*x + c)^2 - 6*(b*x*e + a
*e)^2*B*a*b^2*c^3*d^2*f*g^2*e*log(-b*f*e + a*g*e + (b*x*e + a*e)*d*f/(d*x +
 c) - (b*x*e + a*e)*c*g/(d*x + c))/(d*x + c)^2 + 2*(b*x*e + a*e)^2*B*a^3*c*
d^4*f*g^2*e*log(-b*f*e + a*g*e + (b*x*e + a*e)*d*f/(d*x + c) - (b*x*e + a*e
)*c*g/(d*x + c))/(d*x + c)^2 - (b*x*e + a*e)^2*B*b^3*c^5*g^3*e*log(-b*f*e +
 a*g*e + (b*x*e + a*e)*d*f/(d*x + c) - (b*x*e +...
```

Mupad [B]

time = 7.21, size = 417, normalized size = 2.28

$$\frac{\ln\left(f+g\,x\right)\,\left(g\,\left(B\,a^2\,d^2-B\,b^2\,c^2\right)-2\,B\,a\,b\,d^2\,f+2\,B\,b^2\,c\,d\,f\right)}{2\,a^2\,c^2\,g^4-4\,a^2\,c\,d\,f\,g^3+2\,a^2\,d^2\,f^2\,g^2-4\,a\,b\,c^2\,f\,g^3+8\,a\,b\,c\,d\,f^2\,g^2-4\,a\,b\,d^2\,f^3\,g+2\,b^2\,c^2\,f^2\,g^2-4\,b^2\,c\,d\,f^3\,g+2\,b^2\,d^2\,f^4}-\frac{\frac{A\,a\,c\,g^2+A\,b\,d\,f^2-A\,a\,d\,f\,g-A\,b\,c\,f\,g-B\,a\,d\,f\,g+B\,b\,c\,f\,g}{a\,c\,g^2+b\,d\,f^2-a\,d\,f\,g-b\,c\,f\,g}-\frac{x\,\left(B\,a\,d\,g^2-B\,b\,c\,g^2\right)}{a\,c\,g^2+b\,d\,f^2-a\,d\,f\,g-b\,c\,f\,g}}{2\,f^2\,g+4\,f\,g^2\,x+2\,g^3\,x^2}+\frac{B\,b^2\,\ln\left(a+b\,x\right)}{2\,a^2\,g^3-4\,a\,b\,f\,g^2+2\,b^2\,f^2\,g}-\frac{B\,\ln\left(\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right)}{2\,g\,\left(f^2+2\,f\,g\,x+g^2\,x^2\right)}-\frac{B\,d^2\,\ln\left(c+d\,x\right)}{2\,c^2\,g^3-4\,c\,d\,f\,g^2+2\,d^2\,f^2\,g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x))/(c + d*x)))/(f + g*x)^3,x)
```

```
[Out] (log(f + g*x)*(g*(B*a^2*d^2 - B*b^2*c^2) - 2*B*a*b*d^2*f + 2*B*b^2*c*d*f))/
(2*a^2*c^2*g^4 + 2*b^2*d^2*f^4 + 2*a^2*d^2*f^2*g^2 + 2*b^2*c^2*f^2*g^2 - 4*
a*b*c^2*f*g^3 - 4*a*b*d^2*f^3*g - 4*a^2*c*d*f*g^3 - 4*b^2*c*d*f^3*g + 8*a*b
```

```
*c*d*f^2*g^2) - ((A*a*c*g^2 + A*b*d*f^2 - A*a*d*f*g - A*b*c*f*g - B*a*d*f*g
 + B*b*c*f*g)/(a*c*g^2 + b*d*f^2 - a*d*f*g - b*c*f*g) - (x*(B*a*d*g^2 - B*b
*c*g^2))/(a*c*g^2 + b*d*f^2 - a*d*f*g - b*c*f*g))/(2*f^2*g + 2*g^3*x^2 + 4*
f*g^2*x) + (B*b^2*log(a + b*x))/(2*a^2*g^3 + 2*b^2*f^2*g - 4*a*b*f*g^2) - (
B*log((e*(a + b*x))/(c + d*x)))/(2*g*(f^2 + g^2*x^2 + 2*f*g*x)) - (B*d^2*lo
g(c + d*x))/(2*c^2*g^3 + 2*d^2*f^2*g - 4*c*d*f*g^2)
```

3.238 $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(f+gx)^4}\,dx$

Optimal. Leaf size=275

$$-\frac{B(bc-ad)}{6(bf-ag)(df-cg)(f+gx)^2}-\frac{B(bc-ad)(2bdf-bcg-adg)}{3(bf-ag)^2(df-cg)^2(f+gx)}+\frac{b^3B\log(a+bx)}{3g(bf-ag)^3}-\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{3g(f+gx)^3}-$$

```
[Out] -1/6*B*(-a*d+b*c)/(-a*g+b*f)/(-c*g+d*f)/(g*x+f)^2-1/3*B*(-a*d+b*c)*(-a*d*g-
b*c*g+2*b*d*f)/(-a*g+b*f)^2/(-c*g+d*f)^2/(g*x+f)+1/3*b^3*B*ln(b*x+a)/g/(-a*
g+b*f)^3+1/3*(-A-B*ln(e*(b*x+a)/(d*x+c)))/g/(g*x+f)^3-1/3*B*d^3*ln(d*x+c)/g
/(-c*g+d*f)^3+1/3*B*(-a*d+b*c)*(a^2*d^2*g^2-a*b*d*g*(-c*g+3*d*f)+b^2*(c^2*g
^2-3*c*d*f*g+3*d^2*f^2))*ln(g*x+f)/(-a*g+b*f)^3/(-c*g+d*f)^3
```

Rubi [A]
time = 0.25, antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {2548, 84}

$$\frac{B(bc-ad)\log(f+gx)\left(a^2d^2g^2-abdg(3df-cg)+b^2(c^2g^2-3cdfg+3d^2f^2)\right)}{3(bf-ag)^3(df-cg)^3}-\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{3g(f+gx)^3}+\frac{b^3B\log(a+bx)}{3g(bf-ag)^3}-\frac{B(bc-ad)(-adg-bcg+2bdf)}{3(f+gx)(bf-ag)^2(df-cg)^2}-\frac{B(bc-ad)}{6(f+gx)^2(bf-ag)(df-cg)}-\frac{Bd^3\log(c+dx)}{3g(df-cg)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])/(f + g*x)^4,x]
```

```
[Out] -1/6*(B*(b*c - a*d))/((b*f - a*g)*(d*f - c*g)*(f + g*x)^2) - (B*(b*c - a*d)
*(2*b*d*f - b*c*g - a*d*g))/(3*(b*f - a*g)^2*(d*f - c*g)^2*(f + g*x)) + (b^
3*B*Log[a + b*x])/(3*g*(b*f - a*g)^3) - (A + B*Log[(e*(a + b*x))/(c + d*x)]
)/(3*g*(f + g*x)^3) - (B*d^3*Log[c + d*x])/(3*g*(d*f - c*g)^3) + (B*(b*c -
a*d)*(a^2*d^2*g^2 - a*b*d*g*(3*d*f - c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c^
2*g^2))*Log[f + g*x])/(3*(b*f - a*g)^3*(d*f - c*g)^3)
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 2548

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Dist[B*n*((b*c
- a*d)/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /;
 FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c -
 a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Rubi steps

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(f+gx)^4}\,dx = -\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{3g(f+gx)^3}+\frac{B\int\frac{bc-ad}{(a+bx)(c+dx)(f+gx)^3}\,dx}{3g}$$
$$= -\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{3g(f+gx)^3}+\frac{(B(bc-ad))\int\frac{1}{(a+bx)(c+dx)(f+gx)^3}\,dx}{3g}$$
$$= -\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{3g(f+gx)^3}+\frac{(B(bc-ad))\int\left(\frac{b^4}{(bc-ad)(bf-ag)^3(a+bx)}+\frac{d^4}{(bc-ad)(-df+cg)^3(c\cdot}\right.}{}$$
$$= -\frac{B(bc-ad)}{6(bf-ag)(df-cg)(f+gx)^2}-\frac{B(bc-ad)(2bdf-bcg-adg)}{3(bf-ag)^2(df-cg)^2(f+gx)}+\frac{b^3B\log(a}{3g(bf-}$$

Mathematica [A]
time = 0.48, size = 260, normalized size = 0.95

$$\frac{-\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(f+gx)^3}+B(bc-ad)\left(-\frac{g}{2(bf-ag)(df-cg)(f+gx)^2}+\frac{g(-2bdf+bcg+adg)}{(bf-ag)^2(df-cg)^2(f+gx)}+\frac{b^3\log(a+bx)}{(bc-ad)(bf-ag)^3}+\frac{d^3\log(c+dx)}{(bc-ad)(-df+cg)^3}+\frac{g\left(a^2d^2g^2+abdg(-3df+cg)+b^2\left(3d^2f^2-3cdfg+c^2g^2\right)\right)\log(f+gx)}{(bf-ag)^3(df-cg)^3}\right)}{3g}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])/(f + g*x)^4,x]
```

```
[Out] (-((A + B*Log[(e*(a + b*x))/(c + d*x)])/(f + g*x)^3) + B*(b*c - a*d)*(-1/2*
g/((b*f - a*g)*(d*f - c*g)*(f + g*x)^2) + (g*(-2*b*d*f + b*c*g + a*d*g))/((
b*f - a*g)^2*(d*f - c*g)^2*(f + g*x)) + (b^3*Log[a + b*x])/((b*c - a*d)*(b*
f - a*g)^3) + (d^3*Log[c + d*x])/((b*c - a*d)*(-(d*f) + c*g)^3) + (g*(a^2*d
^2*g^2 + a*b*d*g*(-3*d*f + c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^2))*Lo
g[f + g*x])/((b*f - a*g)^3*(d*f - c*g)^3)))/(3*g)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 10410 vs. 2(266) = 532.
time = 0.55, size = 10411, normalized size = 37.86

method	result	size
risch	Expression too large to display	2444
derivativedivides	Expression too large to display	10411
default	Expression too large to display	10411

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*(b*x+a)/(d*x+c)))/(g*x+f)^4,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 850 vs. $2(264) = 528$.
time = 0.36, size = 850, normalized size = 3.09

$$\frac{1}{6}\left(\frac{2\,b^3\log\left(bx+a\right)}{b^3f^3g-3\,ab^2f^2g^2+3\,a^2bfg^3-a^3g^4}-\frac{2\,d^3\log\left(dx+c\right)}{d^3f^3g-3\,cd^2f^2g^2+3\,c^2dfg^3-c^3g^4}+\frac{2\left(3\left(b^3cd^2-ab^2d^3\right)f^2-3\left(b^3c^2d-a^2bd^3\right)fg+\left(b^3c^3-a^3d^3\right)g^2\right)\log\left(gx+f\right)}{b^3d^3f^6+a^3c^3g^6-3\left(b^3cd^2+ab^2d^3\right)f^5g+3\left(b^3c^2d+3\,ab^2cd^2+a^2bd^3\right)f^4g^2-\left(b^3c^3+9\,ab^2c^2d+9\,a^2bcd^2+a^3d^3\right)f^3g^3+3\left(ab^2c^3+3\,a^2bc^2d+a^3cd^2\right)f^2g^4-3\left(a^2bc^3+a^3c^2d\right)fg^5}-\frac{5\left(b^2cd-abd^2\right)f^2-3\left(b^2c^2-a^2d^2\right)fg+\left(abc^2-a^2cd\right)g^2+2\left(2\left(b^2cd-abd^2\right)fg-\left(b^2c^2-a^2d^2\right)g^2\right)x}{b^2d^2f^6+a^2c^2f^2g^4-2\left(b^2cd+abd^2\right)f^5g+\left(b^2c^2+4\,abcd+a^2d^2\right)f^4g^2-2\left(abc^2+a^2cd\right)f^3g^3+\left(b^2d^2f^4g^2+a^2c^2g^6-2\left(b^2cd+abd^2\right)f^3g^3+\left(b^2c^2+4\,abcd+a^2d^2\right)f^2g^4-2\left(abc^2+a^2cd\right)fg^5\right)x^2+2\left(b^2d^2f^5g+a^2c^2fg^5-2\left(b^2cd+abd^2\right)f^4g^2+\left(b^2c^2+4\,abcd+a^2d^2\right)f^3g^3-2\left(abc^2+a^2cd\right)f^2g^4\right)x}-\frac{2\log\left(\frac{bxe}{dx+c}+\frac{ae}{dx+c}\right)}{g^4x^3+3\,fg^3x^2+3\,f^2g^2x+f^3g}\right)B-\frac{A}{3\left(g^4x^3+3\,fg^3x^2+3\,f^2g^2x+f^3g\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(g*x+f)^4,x, algorithm="maxima")
```

```
[Out] 1/6*(2*b^3*log(b*x + a)/(b^3*f^3*g - 3*a*b^2*f^2*g^2 + 3*a^2*b*f*g^3 - a^3*
g^4) - 2*d^3*log(d*x + c)/(d^3*f^3*g - 3*c*d^2*f^2*g^2 + 3*c^2*d*f*g^3 - c^
3*g^4) + 2*(3*(b^3*c*d^2 - a*b^2*d^3)*f^2 - 3*(b^3*c^2*d - a^2*b*d^3)*f*g +
 (b^3*c^3 - a^3*d^3)*g^2)*log(g*x + f)/(b^3*d^3*f^6 + a^3*c^3*g^6 - 3*(b^3*
c*d^2 + a*b^2*d^3)*f^5*g + 3*(b^3*c^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3)*f^4*g^
2 - (b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*f^3*g^3 + 3*(a*b^2*
c^3 + 3*a^2*b*c^2*d + a^3*c*d^2)*f^2*g^4 - 3*(a^2*b*c^3 + a^3*c^2*d)*f*g^5)
 - (5*(b^2*c*d - a*b*d^2)*f^2 - 3*(b^2*c^2 - a^2*d^2)*f*g + (a*b*c^2 - a^2*
c*d)*g^2 + 2*(2*(b^2*c*d - a*b*d^2)*f*g - (b^2*c^2 - a^2*d^2)*g^2)*x)/(b^2*
d^2*f^6 + a^2*c^2*f^2*g^4 - 2*(b^2*c*d + a*b*d^2)*f^5*g + (b^2*c^2 + 4*a*b*
c*d + a^2*d^2)*f^4*g^2 - 2*(a*b*c^2 + a^2*c*d)*f^3*g^3 + (b^2*d^2*f^4*g^2 +
 a^2*c^2*g^6 - 2*(b^2*c*d + a*b*d^2)*f^3*g^3 + (b^2*c^2 + 4*a*b*c*d + a^2*d
^2)*f^2*g^4 - 2*(a*b*c^2 + a^2*c*d)*f*g^5)*x^2 + 2*(b^2*d^2*f^5*g + a^2*c^2
*f*g^5 - 2*(b^2*c*d + a*b*d^2)*f^4*g^2 + (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*f^
3*g^3 - 2*(a*b*c^2 + a^2*c*d)*f^2*g^4)*x) - 2*log(b*x*e/(d*x + c) + a*e/(d*
x + c))/(g^4*x^3 + 3*f*g^3*x^2 + 3*f^2*g^2*x + f^3*g))*B - 1/3*A/(g^4*x^3 +
 3*f*g^3*x^2 + 3*f^2*g^2*x + f^3*g)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(g*x+f)^4,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(b*x+a)/(d*x+c)))/(g*x+f)**4,x)
```

```
[Out] Timed out
```

<u>Giac</u> [**B**] Leaf count of result is larger than twice the leaf count of optimal. 21182 vs. $2(264) = 528$.
time = 6.14, size = 21182, normalized size = 77.03

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(g*x+f)^4,x, algorithm="giac")
```

```
[Out] 1/6*(6*B*b^7*c^2*d^2*f^5*e^4*log(-b*f*e + a*g*e + (b*x*e + a*e)*d*f/(d*x +
c) - (b*x*e + a*e)*c*g/(d*x + c)) - 12*B*a*b^6*c*d^3*f^5*e^4*log(-b*f*e + a
*g*e + (b*x*e + a*e)*d*f/(d*x + c) - (b*x*e + a*e)*c*g/(d*x + c)) + 6*B*a^2
*b^5*d^4*f^5*e^4*log(-b*f*e + a*g*e + (b*x*e + a*e)*d*f/(d*x + c) - (b*x*e
+ a*e)*c*g/(d*x + c)) - 6*B*b^7*c^3*d*f^4*g*e^4*log(-b*f*e + a*g*e + (b*x*e
 + a*e)*d*f/(d*x + c) - (b*x*e + a*e)*c*g/(d*x + c)) - 12*B*a*b^6*c^2*d^2*f
^4*g*e^4*log(-b*f*e + a*g*e + (b*x*e + a*e)*d*f/(d*x + c) - (b*x*e + a*e)*c
*g/(d*x + c)) + 42*B*a^2*b^5*c*d^3*f^4*g*e^4*log(-b*f*e + a*g*e + (b*x*e +
a*e)*d*f/(d*x + c) - (b*x*e + a*e)*c*g/(d*x + c)) - 24*B*a^3*b^4*d^4*f^4*g*
e^4*log(-b*f*e + a*g*e + (b*x*e + a*e)*d*f/(d*x + c) - (b*x*e + a*e)*c*g/(d
*x + c)) + 2*B*b^7*c^4*f^3*g^2*e^4*log(-b*f*e + a*g*e + (b*x*e + a*e)*d*f/(
d*x + c) - (b*x*e + a*e)*c*g/(d*x + c)) + 16*B*a*b^6*c^3*d*f^3*g^2*e^4*log(
-b*f*e + a*g*e + (b*x*e + a*e)*d*f/(d*x + c) - (b*x*e + a*e)*c*g/(d*x + c))
 - 56*B*a^3*b^4*c*d^3*f^3*g^2*e^4*log(-b*f*e + a*g*e + (b*x*e + a*e)*d*f/(d
*x + c) - (b*x*e + a*e)*c*g/(d*x + c)) + 38*B*a^4*b^3*d^4*f^3*g^2*e^4*log(-
b*f*e + a*g*e + (b*x*e + a*e)*d*f/(d*x + c) - (b*x*e + a*e)*c*g/(d*x + c))
- 6*B*a*b^6*c^4*f^2*g^3*e^4*log(-b*f*e + a*g*e + (b*x*e + a*e)*d*f/(d*x + c
) - (b*x*e + a*e)*c*g/(d*x + c)) - 12*B*a^2*b^5*c^3*d*f^2*g^3*e^4*log(-b*f*
e + a*g*e + (b*x*e + a*e)*d*f/(d*x + c) - (b*x*e + a*e)*c*g/(d*x + c)) + 12
*B*a^3*b^4*c^2*d^2*f^2*g^3*e^4*log(-b*f*e + a*g*e + (b*x*e + a*e)*d*f/(d*x
+ c) - (b*x*e + a*e)*c*g/(d*x + c)) + 36*B*a^4*b^3*c*d^3*f^2*g^3*e^4*log(-b
*f*e + a*g*e + (b*x*e + a*e)*d*f/(d*x + c) - (b*x*e + a*e)*c*g/(d*x + c)) -
 30*B*a^5*b^2*d^4*f^2*g^3*e^4*log(-b*f*e + a*g*e + (b*x*e + a*e)*d*f/(d*x +
 c) - (b*x*e + a*e)*c*g/(d*x + c)) + 6*B*a^2*b^5*c^4*f*g^4*e^4*log(-b*f*e +
 a*g*e + (b*x*e + a*e)*d*f/(d*x + c) - (b*x*e + a*e)*c*g/(d*x + c)) - 6*B*a
^4*b^3*c^2*d^2*f*g^4*e^4*log(-b*f*e + a*g*e + (b*x*e + a*e)*d*f/(d*x + c) -
 (b*x*e + a*e)*c*g/(d*x + c)) - 12*B*a^5*b^2*c*d^3*f*g^4*e^4*log(-b*f*e + a
*g*e + (b*x*e + a*e)*d*f/(d*x + c) - (b*x*e + a*e)*c*g/(d*x + c)) + 12*B*a^
6*b*d^4*f*g^4*e^4*log(-b*f*e + a*g*e + (b*x*e + a*e)*d*f/(d*x + c) - (b*x*e
 + a*e)*c*g/(d*x + c)) - 2*B*a^3*b^4*c^4*g^5*e^4*log(-b*f*e + a*g*e + (b*x*
e + a*e)*d*f/(d*x + c) - (b*x*e + a*e)*c*g/(d*x + c)) + 2*B*a^4*b^3*c^3*d*g
^5*e^4*log(-b*f*e + a*g*e + (b*x*e + a*e)*d*f/(d*x + c) - (b*x*e + a*e)*c*g
/(d*x + c)) + 2*B*a^6*b*c*d^3*g^5*e^4*log(-b*f*e + a*g*e + (b*x*e + a*e)*d*
f/(d*x + c) - (b*x*e + a*e)*c*g/(d*x + c)) - 2*B*a^7*d^4*g^5*e^4*log(-b*f*e
 + a*g*e + (b*x*e + a*e)*d*f/(d*x + c) - (b*x*e + a*e)*c*g/(d*x + c)) - 18*
(b*x*e + a*e)*B*b^6*c^2*d^3*f^5*e^3*log(-b*f*e + a*g*e + (b*x*e + a*e)*d*f/
```

```
(d*x + c) - (b*x*e + a*e)*c*g/(d*x + c))/(d*x + c) + 36*(b*x*e + a*e)*B*a*b
^5*c*d^4*f^5*e^3*log(-b*f*e + a*g*e + (b*x*e + a*e)*d*f/(d*x + c) - (b*x*e
+ a*e)*c*g/(d*x + c))/(d*x + c) - 18*(b*x*e + a*e)*B*a^2*b^4*d^5*f^5*e^3*lo
g(-b*f*e + a*g*e + (b*x*e + a*e)*d*f/(d*x + c) - (b*x*e + a*e)*c*g/(d*x + c
))/(d*x + c) + 36*(b*x*e + a*e)*B*b^6*c^3*d^2*f^4*g*e^3*log(-b*f*e + a*g*e
+ (b*x*e + a*e)*d*f/(d*x + c) - (b*x*e + a*e)*c*g/(d*x + c))/(d*x + c) - 18
*(b*x*e + a*e)*B*a*b^5*c^2*d^3*f^4*g*e^3*log(-b*f*e + a*g*e + (b*x*e + a*e)
*d*f/(d*x + c) - (b*x*e + a*e)*c*g/(d*x + c))/(d*x + c) - 72*(b*x*e + a*e)*
B*a^2*b^4*c*d^4*f^4*g*e^3*log(-b*f*e + a*g*e + (b*x*e + a*e)*d*f/(d*x + c)
- (b*x*e + a*e)*c*g/(d*x + c))/(d*x + c) + 54*(b*x*e + a*e)*B*a^3*b^3*d^5*f
^4*g*e^3*log(-b*f*e + a*g*e + (b*x*e + a*e)*d*f/(d*x + c) - (b*x*e + a*e)*c
*g/(d*x + c))/(d*x + c) - 24*(b*x*e + a*e)*B*b^6*c^4*d*f^3*g^2*e^3*log(-b*f
*e + a*g*e + (b*x*e + a*e)*d*f/(d*x + c) - (b*x*e + a*e)*c*g/(d*x + c))/(d*
x + c) - 48*(b*x*e + a*e)*B*a*b^5*c^3*d^2*f^3*g^2*e^3*log(-b*f*e + a*g*e +
(b*x*e + a*e)*d*f/(d*x + c) - (b*x*e + a*e)*c*g/(d*x + c))/(d*x + c) + 108*
(b*x*e + a*e)*B*a^2*b^4*c^2*d^3*f^3*g^2*e^3*log(-b*f*e + a*g*e + (b*x*e + a
*e)*d*f/(d*x + c) - (b*x*e + a*e)*c*g/(d*x + c))/(d*x + c) + 24*(b*x*e + a*
e)*B*a^3*b^3*c*d^4*f^3*g^2*e^3*log(-b*f*e + a*g*e + (b*x*e + a*e)*d*f/(d*x
+ c) - (b*x*e + a*e)*c*g/(d*x + c))/(d*x + c) - 60*(b*x*e + a*e)*B*a^4*b^2*
d^5*f^3*g^2*e^3*log(-b*f*e + a*g*e + (b*x*e + a*e)*d*f/(d*x + c) - (b*x*e +
 a*e)*c*g/(d*x + c))/(d*x + c) + 6*(b*x*e + a*e)*B*b^6*c^5*f^2*g^3*e^3*log(
-b*f*e + a*g*e + (b*x*e + a*e)*d*f/(d*x + c) - (b*x*e + a*e)*c*g/(d*x + c))
/(d*x + c) + 42*(b*x*e + a*e)*B*a*b^5*c^4*d*f^2*g^3*e^3*log(-b*f*e + a*g*e
+ (b*x*e + a*e)*d*f/(d*x + c) - (b*x*e + a*e)*c*g/(d*x + c))/(d*x + c) - 12
*(b*x*e + a*e)*B*a^2*b^4*c^3*d^2*f^2*g^3*e^3*log(-b*f*e + a*g*e + (b*x*e +
a*e)*d*f/(d*x + c) - (b*x*e + a*e)*c*g/(d*x + c))/(d*x + c) - 96*(b*x*e + a
*e)*B*a^3*b^3*c^2*d^3*f^2*g^3*e^3*log(-b*f*e + a*g*e + (b*x*e + a*e)*d*f/(d
*x + c) - (b*x*e + a*e)*c*g/(d*x + c))/(d*x + c) + 30*(b*x*e + a*e)*B*a^4*b
^2*c*d^4*f^2*g^3*e^3*log(-b*f*e + a*g*e + (b*x*e + a*e)*d*f/(d*x + c) - (b*
x*e + a*e)*c*g/(d*x + c))/(d*x + c) + 30*(b*x*e + a*e)*B*a^5*b*d^5*f^2*g^3*
e^3*log(-b*f*e + a*g*e + (b*x*e + a*e)*d*f/(d*x...
```

Mupad [B]
time = 10.67, size = 1154, normalized size = 4.20

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x))/(c + d*x)))/(f + g*x)^4,x)
```

```
[Out] (log(f + g*x)*(g*(3*B*a^2*b*d^3*f - 3*B*b^3*c^2*d*f) - g^2*(B*a^3*d^3 - B*b
^3*c^3) - 3*B*a*b^2*d^3*f^2 + 3*B*b^3*c*d^2*f^2))/(3*a^3*c^3*g^6 + 3*b^3*d^
3*f^6 - 3*a^3*d^3*f^3*g^3 - 3*b^3*c^3*f^3*g^3 - 9*a^2*b*c^3*f*g^5 - 9*a*b^2
*d^3*f^5*g - 9*a^3*c^2*d*f*g^5 - 9*b^3*c*d^2*f^5*g + 9*a*b^2*c^3*f^2*g^4 +
9*a^2*b*d^3*f^4*g^2 + 9*a^3*c*d^2*f^2*g^4 + 9*b^3*c^2*d*f^4*g^2 + 27*a*b^2*
```

```
c*d^2*f^4*g^2 - 27*a*b^2*c^2*d*f^3*g^3 - 27*a^2*b*c*d^2*f^3*g^3 + 27*a^2*b*
c^2*d*f^2*g^4) - ((2*A*a^2*c^2*g^4 + 2*A*b^2*d^2*f^4 + 2*A*a^2*d^2*f^2*g^2
+ 2*A*b^2*c^2*f^2*g^2 + 3*B*a^2*d^2*f^2*g^2 - 3*B*b^2*c^2*f^2*g^2 - 4*A*a*b
*c^2*f*g^3 - 4*A*a*b*d^2*f^3*g + B*a*b*c^2*f*g^3 - 4*A*a^2*c*d*f*g^3 - 5*B*
a*b*d^2*f^3*g - 4*A*b^2*c*d*f^3*g - B*a^2*c*d*f*g^3 + 5*B*b^2*c*d*f^3*g + 8
*A*a*b*c*d*f^2*g^2)/(2*(a^2*c^2*g^4 + b^2*d^2*f^4 + a^2*d^2*f^2*g^2 + b^2*c
^2*f^2*g^2 - 2*a*b*c^2*f*g^3 - 2*a*b*d^2*f^3*g - 2*a^2*c*d*f*g^3 - 2*b^2*c*
d*f^3*g + 4*a*b*c*d*f^2*g^2)) + (x^2*(B*a^2*d^2*g^4 - B*b^2*c^2*g^4 - 2*B*a
*b*d^2*f*g^3 + 2*B*b^2*c*d*f*g^3))/(a^2*c^2*g^4 + b^2*d^2*f^4 + a^2*d^2*f^2
*g^2 + b^2*c^2*f^2*g^2 - 2*a*b*c^2*f*g^3 - 2*a*b*d^2*f^3*g - 2*a^2*c*d*f*g^
3 - 2*b^2*c*d*f^3*g + 4*a*b*c*d*f^2*g^2) + (x*(5*B*a^2*d^2*f*g^3 - 5*B*b^2*
c^2*f*g^3 + B*a*b*c^2*g^4 - B*a^2*c*d*g^4 - 9*B*a*b*d^2*f^2*g^2 + 9*B*b^2*c
*d*f^2*g^2))/(2*(a^2*c^2*g^4 + b^2*d^2*f^4 + a^2*d^2*f^2*g^2 + b^2*c^2*f^2*
g^2 - 2*a*b*c^2*f*g^3 - 2*a*b*d^2*f^3*g - 2*a^2*c*d*f*g^3 - 2*b^2*c*d*f^3*g
 + 4*a*b*c*d*f^2*g^2)))/(3*f^3*g + 3*g^4*x^3 + 9*f^2*g^2*x + 9*f*g^3*x^2) -
 (B*b^3*log(a + b*x))/(3*a^3*g^4 - 3*b^3*f^3*g + 9*a*b^2*f^2*g^2 - 9*a^2*b*
f*g^3) + (B*d^3*log(c + d*x))/(3*c^3*g^4 - 3*d^3*f^3*g + 9*c*d^2*f^2*g^2 -
9*c^2*d*f*g^3) - (B*log((e*(a + b*x))/(c + d*x)))/(3*g*(f^3 + g^3*x^3 + 3*f
^2*g*x + 3*f*g^2*x^2))
```

3.239 $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(f+gx)^5}\,dx$

Optimal. Leaf size=379

$$-\frac{B(bc-ad)}{12(bf-ag)(df-cg)(f+gx)^3}-\frac{B(bc-ad)(2bdf-bcg-adg)}{8(bf-ag)^2(df-cg)^2(f+gx)^2}-\frac{B(bc-ad)\left(a^2d^2g^2-abdg(3df-cg)\right.}{4(bf-ag)^3(df-cg}$$

```
[Out] -1/12*B*(-a*d+b*c)/(-a*g+b*f)/(-c*g+d*f)/(g*x+f)^3-1/8*B*(-a*d+b*c)*(-a*d*g
-b*c*g+2*b*d*f)/(-a*g+b*f)^2/(-c*g+d*f)^2/(g*x+f)^2-1/4*B*(-a*d+b*c)*(a^2*d
^2*g^2-a*b*d*g*(-c*g+3*d*f)+b^2*(c^2*g^2-3*c*d*f*g+3*d^2*f^2))/(-a*g+b*f)^3
/(-c*g+d*f)^3/(g*x+f)+1/4*b^4*B*ln(b*x+a)/g/(-a*g+b*f)^4+1/4*(-A-B*ln(e*(b*
x+a)/(d*x+c)))/g/(g*x+f)^4-1/4*B*d^4*ln(d*x+c)/g/(-c*g+d*f)^4-1/4*B*(-a*d+b
*c)*(-a*d*g-b*c*g+2*b*d*f)*(2*a*b*d^2*f*g-a^2*d^2*g^2-b^2*(c^2*g^2-2*c*d*f*
g+2*d^2*f^2))*ln(g*x+f)/(-a*g+b*f)^4/(-c*g+d*f)^4
```

Rubi [A]
time = 0.40, antiderivative size = 379, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {2548, 84}

$$-\frac{B(bc-ad)\left(a^2d^2g^2-abdg(3df-cg)+b^2(c^2g^2-3cdfg+3d^2f^2)\right)}{4(f+gx)(bf-ag)^3(df-cg)^3}-\frac{B(bc-ad)\log(f+gx)(-adg-bcg+2bdf)\left(-a^2d^2g^2+2abd^2fg-(b^2(c^2g^2-2cdfg+2d^2f^2))\right)}{4(bf-ag)^4(df-cg)^4}-\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{4g(f+gx)^4}+\frac{b^4B\log(a+bx)}{4g(bf-ag)^4}-\frac{B(bc-ad)(-adg-bcg+2bdf)}{8(f+gx)^2(bf-ag)^2(df-cg)^2}-\frac{B(bc-ad)}{12(f+gx)^3(bf-ag)(df-cg)}-\frac{Bd^4\log(c+dx)}{4g(df-cg)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])/(f + g*x)^5,x]
```

```
[Out] -1/12*(B*(b*c - a*d))/((b*f - a*g)*(d*f - c*g)*(f + g*x)^3) - (B*(b*c - a*d
)*(2*b*d*f - b*c*g - a*d*g))/(8*(b*f - a*g)^2*(d*f - c*g)^2*(f + g*x)^2) -
(B*(b*c - a*d)*(a^2*d^2*g^2 - a*b*d*g*(3*d*f - c*g) + b^2*(3*d^2*f^2 - 3*c*
d*f*g + c^2*g^2)))/(4*(b*f - a*g)^3*(d*f - c*g)^3*(f + g*x)) + (b^4*B*Log[a
 + b*x])/(4*g*(b*f - a*g)^4) - (A + B*Log[(e*(a + b*x))/(c + d*x)])/(4*g*(f
 + g*x)^4) - (B*d^4*Log[c + d*x])/(4*g*(d*f - c*g)^4) - (B*(b*c - a*d)*(2*b
*d*f - b*c*g - a*d*g)*(2*a*b*d^2*f*g - a^2*d^2*g^2 - b^2*(2*d^2*f^2 - 2*c*d
*f*g + c^2*g^2))*Log[f + g*x])/(4*(b*f - a*g)^4*(d*f - c*g)^4)
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 2548

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
```

```
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Dist[B*n*((b*c
- a*d)/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /;
 FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c -
 a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(f+gx)^5}\,dx &= -\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{4g(f+gx)^4}+\frac{B\int \frac{bc-ad}{(a+bx)(c+dx)(f+gx)^4}\,dx}{4g}\\ &= -\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{4g(f+gx)^4}+\frac{(B(bc-ad))\int \frac{1}{(a+bx)(c+dx)(f+gx)^4}\,dx}{4g}\\ &= -\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{4g(f+gx)^4}+\frac{(B(bc-ad))\int \left(\frac{b^5}{(bc-ad)(bf-ag)^4(a+bx)}-\frac{d^5}{(bc-ad)(-df+cg)^4(c}\right.}{}\\ &= -\frac{B(bc-ad)}{12(bf-ag)(df-cg)(f+gx)^3}-\frac{B(bc-ad)(2bdf-bcg-adg)}{8(bf-ag)^2(df-cg)^2(f+gx)^2}-\frac{B(bc-}{}\end{aligned}$$

Mathematica [**A**]
time = 0.62, size = 355, normalized size = 0.94

$$\frac{-\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(f+gx)^4}+B(bc-ad)\left(-\frac{g}{3(bf-ag)(df-cg)(f+gx)^3}+\frac{g(-2bdf+bcg+adg)}{2(bf-ag)^2(df-cg)^2(f+gx)^2}-\frac{g\left(a^2d^2g^2+abdg(-3df+cg)+b^2\left(3d^2f^2-3cdfg+c^2g^2\right)\right)}{(bf-ag)^3(df-cg)^3(f+gx)}+\frac{b^4\log(a+bx)}{(bc-ad)(bf-ag)^4}-\frac{d^4\log(c+dx)}{(bc-ad)(df-cg)^4}-\frac{g(-2bdf+bcg+adg)\left(-2abd^2fg+a^2d^2g^2+b^2\left(2d^2f^2-2cdfg+c^2g^2\right)\right)\log(f+gx)}{(bf-ag)^4(df-cg)^4}\right)}{4g}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])/(f + g*x)^5,x]
```

```
[Out] (-((A + B*Log[(e*(a + b*x))/(c + d*x)])/(f + g*x)^4) + B*(b*c - a*d)*(-1/3*
g/((b*f - a*g)*(d*f - c*g)*(f + g*x)^3) + (g*(-2*b*d*f + b*c*g + a*d*g))/(2
*(b*f - a*g)^2*(d*f - c*g)^2*(f + g*x)^2) - (g*(a^2*d^2*g^2 + a*b*d*g*(-3*d
*f + c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^2)))/((b*f - a*g)^3*(d*f - c
*g)^3*(f + g*x)) + (b^4*Log[a + b*x])/((b*c - a*d)*(b*f - a*g)^4) - (d^4*Lo
g[c + d*x])/((b*c - a*d)*(d*f - c*g)^4) - (g*(-2*b*d*f + b*c*g + a*d*g)*(-2
*a*b*d^2*f*g + a^2*d^2*g^2 + b^2*(2*d^2*f^2 - 2*c*d*f*g + c^2*g^2))*Log[f +
 g*x])/((b*f - a*g)^4*(d*f - c*g)^4)))/(4*g)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 29345 vs. 2(368) = 736.
time = 0.88, size = 29346, normalized size = 77.43

method	result	size

risch	Expression too large to display	4167
derivativedivides	Expression too large to display	29346
default	Expression too large to display	29346

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*(b*x+a)/(d*x+c)))/(g*x+f)^5,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1759 vs. $2(366) = 732$.
time = 0.52, size = 1759, normalized size = 4.64

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(g*x+f)^5,x, algorithm="maxima")
```

```
[Out] 1/24*(6*b^4*log(b*x + a)/(b^4*f^4*g - 4*a*b^3*f^3*g^2 + 6*a^2*b^2*f^2*g^3 -
 4*a^3*b*f*g^4 + a^4*g^5) - 6*d^4*log(d*x + c)/(d^4*f^4*g - 4*c*d^3*f^3*g^2
 + 6*c^2*d^2*f^2*g^3 - 4*c^3*d*f*g^4 + c^4*g^5) + 6*(4*(b^4*c*d^3 - a*b^3*d
^4)*f^3 - 6*(b^4*c^2*d^2 - a^2*b^2*d^4)*f^2*g + 4*(b^4*c^3*d - a^3*b*d^4)*f
*g^2 - (b^4*c^4 - a^4*d^4)*g^3)*log(g*x + f)/(b^4*d^4*f^8 + a^4*c^4*g^8 - 4
*(b^4*c*d^3 + a*b^3*d^4)*f^7*g + 2*(3*b^4*c^2*d^2 + 8*a*b^3*c*d^3 + 3*a^2*b
^2*d^4)*f^6*g^2 - 4*(b^4*c^3*d + 6*a*b^3*c^2*d^2 + 6*a^2*b^2*c*d^3 + a^3*b*
d^4)*f^5*g^3 + (b^4*c^4 + 16*a*b^3*c^3*d + 36*a^2*b^2*c^2*d^2 + 16*a^3*b*c*
d^3 + a^4*d^4)*f^4*g^4 - 4*(a*b^3*c^4 + 6*a^2*b^2*c^3*d + 6*a^3*b*c^2*d^2 +
 a^4*c*d^3)*f^3*g^5 + 2*(3*a^2*b^2*c^4 + 8*a^3*b*c^3*d + 3*a^4*c^2*d^2)*f^2
*g^6 - 4*(a^3*b*c^4 + a^4*c^3*d)*f*g^7) - (26*(b^3*c*d^2 - a*b^2*d^3)*f^4 -
 31*(b^3*c^2*d - a^2*b*d^3)*f^3*g + (11*b^3*c^3 + 15*a*b^2*c^2*d - 15*a^2*b
*c*d^2 - 11*a^3*d^3)*f^2*g^2 - 7*(a*b^2*c^3 - a^3*c*d^2)*f*g^3 + 2*(a^2*b*c
^3 - a^3*c^2*d)*g^4 + 6*(3*(b^3*c*d^2 - a*b^2*d^3)*f^2*g^2 - 3*(b^3*c^2*d -
 a^2*b*d^3)*f*g^3 + (b^3*c^3 - a^3*d^3)*g^4)*x^2 + 3*(14*(b^3*c*d^2 - a*b^2
*d^3)*f^3*g - 15*(b^3*c^2*d - a^2*b*d^3)*f^2*g^2 + (5*b^3*c^3 + 3*a*b^2*c^2
*d - 3*a^2*b*c*d^2 - 5*a^3*d^3)*f*g^3 - (a*b^2*c^3 - a^3*c*d^2)*g^4)*x)/(b^
3*d^3*f^9 + a^3*c^3*f^3*g^6 - 3*(b^3*c*d^2 + a*b^2*d^3)*f^8*g + 3*(b^3*c^2*
d + 3*a*b^2*c*d^2 + a^2*b*d^3)*f^7*g^2 - (b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2*b
*c*d^2 + a^3*d^3)*f^6*g^3 + 3*(a*b^2*c^3 + 3*a^2*b*c^2*d + a^3*c*d^2)*f^5*g
^4 - 3*(a^2*b*c^3 + a^3*c^2*d)*f^4*g^5 + (b^3*d^3*f^6*g^3 + a^3*c^3*g^9 - 3
*(b^3*c*d^2 + a*b^2*d^3)*f^5*g^4 + 3*(b^3*c^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3
)*f^4*g^5 - (b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*f^3*g^6 + 3
*(a*b^2*c^3 + 3*a^2*b*c^2*d + a^3*c*d^2)*f^2*g^7 - 3*(a^2*b*c^3 + a^3*c^2*d
)*f*g^8)*x^3 + 3*(b^3*d^3*f^7*g^2 + a^3*c^3*f*g^8 - 3*(b^3*c*d^2 + a*b^2*d^
```

```
3)*f^6*g^3 + 3*(b^3*c^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3)*f^5*g^4 - (b^3*c^3 +
 9*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*f^4*g^5 + 3*(a*b^2*c^3 + 3*a^2*b*
c^2*d + a^3*c*d^2)*f^3*g^6 - 3*(a^2*b*c^3 + a^3*c^2*d)*f^2*g^7)*x^2 + 3*(b^
3*d^3*f^8*g + a^3*c^3*f^2*g^7 - 3*(b^3*c*d^2 + a*b^2*d^3)*f^7*g^2 + 3*(b^3*
c^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3)*f^6*g^3 - (b^3*c^3 + 9*a*b^2*c^2*d + 9*a
^2*b*c*d^2 + a^3*d^3)*f^5*g^4 + 3*(a*b^2*c^3 + 3*a^2*b*c^2*d + a^3*c*d^2)*f
^4*g^5 - 3*(a^2*b*c^3 + a^3*c^2*d)*f^3*g^6)*x) - 6*log(b*x*e/(d*x + c) + a*
e/(d*x + c))/(g^5*x^4 + 4*f*g^4*x^3 + 6*f^2*g^3*x^2 + 4*f^3*g^2*x + f^4*g))
*B - 1/4*A/(g^5*x^4 + 4*f*g^4*x^3 + 6*f^2*g^3*x^2 + 4*f^3*g^2*x + f^4*g)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(g*x+f)^5,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(b*x+a)/(d*x+c)))/(g*x+f)**5,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 44231 vs. 2(366) = 732.
time = 4.71, size = 44231, normalized size = 116.70

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(g*x+f)^5,x, algorithm="giac")
```

```
[Out] 1/24*(24*B*b^9*c^2*d^3*f^7*e^5*log(-b*f*e + a*g*e + (b*x*e + a*e)*d*f/(d*x
+ c) - (b*x*e + a*e)*c*g/(d*x + c)) - 48*B*a*b^8*c*d^4*f^7*e^5*log(-b*f*e +
 a*g*e + (b*x*e + a*e)*d*f/(d*x + c) - (b*x*e + a*e)*c*g/(d*x + c)) + 24*B*
a^2*b^7*d^5*f^7*e^5*log(-b*f*e + a*g*e + (b*x*e + a*e)*d*f/(d*x + c) - (b*x
*e + a*e)*c*g/(d*x + c)) - 36*B*b^9*c^3*d^2*f^6*g*e^5*log(-b*f*e + a*g*e +
(b*x*e + a*e)*d*f/(d*x + c) - (b*x*e + a*e)*c*g/(d*x + c)) - 60*B*a*b^8*c^2
*d^3*f^6*g*e^5*log(-b*f*e + a*g*e + (b*x*e + a*e)*d*f/(d*x + c) - (b*x*e +
```

a*e)*c*g/(d*x + c)) + 228*B*a^2*b^7*c*d^4*f^6*g*e^5*log(-b*f*e + a*g*e + (b*x*e + a*e)*d*f/(d*x + c) - (b*x*e + a*e)*c*g/(d*x + c)) - 132*B*a^3*b^6*d^5*f^6*g*e^5*log(-b*f*e + a*g*e + (b*x*e + a*e)*d*f/(d*x + c) - (b*x*e + a*e)*c*g/(d*x + c)) + 24*B*b^9*c^4*d*f^5*g^2*e^5*log(-b*f*e + a*g*e + (b*x*e + a*e)*d*f/(d*x + c) - (b*x*e + a*e)*c*g/(d*x + c)) + 120*B*a*b^8*c^3*d^2*f^5*g^2*e^5*log(-b*f*e + a*g*e + (b*x*e + a*e)*d*f/(d*x + c) - (b*x*e + a*e)*c*g/(d*x + c)) - 456*B*a^3*b^6*c*d^4*f^5*g^2*e^5*log(-b*f*e + a*g*e + (b*x*e + a*e)*d*f/(d*x + c) - (b*x*e + a*e)*c*g/(d*x + c)) + 312*B*a^4*b^5*d^5*f^5*g^2*e^5*log(-b*f*e + a*g*e + (b*x*e + a*e)*d*f/(d*x + c) - (b*x*e + a*e)*c*g/(d*x + c)) - 6*B*b^9*c^5*f^4*g^3*e^5*log(-b*f*e + a*g*e + (b*x*e + a*e)*d*f/(d*x + c) - (b*x*e + a*e)*c*g/(d*x + c)) - 90*B*a*b^8*c^4*d*f^4*g^3*e^5*log(-b*f*e + a*g*e + (b*x*e + a*e)*d*f/(d*x + c) - (b*x*e + a*e)*c*g/(d*x + c)) - 120*B*a^2*b^7*c^3*d^2*f^4*g^3*e^5*log(-b*f*e + a*g*e + (b*x*e + a*e)*d*f/(d*x + c) - (b*x*e + a*e)*c*g/(d*x + c)) + 120*B*a^3*b^6*c^2*d^3*f^4*g^3*e^5*log(-b*f*e + a*g*e + (b*x*e + a*e)*d*f/(d*x + c) - (b*x*e + a*e)*c*g/(d*x + c)) + 510*B*a^4*b^5*c*d^4*f^4*g^3*e^5*log(-b*f*e + a*g*e + (b*x*e + a*e)*d*f/(d*x + c) - (b*x*e + a*e)*c*g/(d*x + c)) - 414*B*a^5*b^4*d^5*f^4*g^3*e^5*log(-b*f*e + a*g*e + (b*x*e + a*e)*d*f/(d*x + c) - (b*x*e + a*e)*c*g/(d*x + c)) + 24*B*a*b^8*c^5*f^3*g^4*e^5*log(-b*f*e + a*g*e + (b*x*e + a*e)*d*f/(d*x + c) - (b*x*e + a*e)*c*g/(d*x + c)) + 120*B*a^2*b^7*c^4*d*f^3*g^4*e^5*log(-b*f*e + a*g*e + (b*x*e + a*e)*d*f/(d*x + c) - (b*x*e + a*e)*c*g/(d*x + c)) - 120*B*a^4*b^5*c^2*d^3*f^3*g^4*e^5*log(-b*f*e + a*g*e + (b*x*e + a*e)*d*f/(d*x + c) - (b*x*e + a*e)*c*g/(d*x + c)) - 360*B*a^5*b^4*c*d^4*f^3*g^4*e^5*log(-b*f*e + a*g*e + (b*x*e + a*e)*d*f/(d*x + c) - (b*x*e + a*e)*c*g/(d*x + c)) + 336*B*a^6*b^3*d^5*f^3*g^4*e^5*log(-b*f*e + a*g*e + (b*x*e + a*e)*d*f/(d*x + c) - (b*x*e + a*e)*c*g/(d*x + c)) - 36*B*a^2*b^7*c^5*f^2*g^5*e^5*log(-b*f*e + a*g*e + (b*x*e + a*e)*d*f/(d*x + c) - (b*x*e + a*e)*c*g/(d*x + c)) - 60*B*a^3*b^6*c^4*d*f^2*g^5*e^5*log(-b*f*e + a*g*e + (b*x*e + a*e)*d*f/(d*x + c) - (b*x*e + a*e)*c*g/(d*x + c)) + 60*B*a^4*b^5*c^3*d^2*f^2*g^5*e^5*log(-b*f*e + a*g*e + (b*x*e + a*e)*d*f/(d*x + c) - (b*x*e + a*e)*c*g/(d*x + c)) + 36*B*a^5*b^4*c^2*d^3*f^2*g^5*e^5*log(-b*f*e + a*g*e + (b*x*e + a*e)*d*f/(d*x + c) - (b*x*e + a*e)*c*g/(d*x + c)) + 168*B*a^6*b^3*c*d^4*f^2*g^5*e^5*log(-b*f*e + a*g*e + (b*x*e + a*e)*d*f/(d*x + c) - (b*x*e + a*e)*c*g/(d*x + c)) - 168*B*a^7*b^2*d^5*f^2*g^5*e^5*log(-b*f*e + a*g*e + (b*x*e + a*e)*d*f/(d*x + c) - (b*x*e + a*e)*c*g/(d*x + c)) + 24*B*a^3*b^6*c^5*f*g^6*e^5*log(-b*f*e + a*g*e + (b*x*e + a*e)*d*f/(d*x + c) - (b*x*e + a*e)*c*g/(d*x + c)) - 24*B*a^5*b^4*c^3*d^2*f*g^6*e^5*log(-b*f*e + a*g*e + (b*x*e + a*e)*d*f/(d*x + c) - (b*x*e + a*e)*c*g/(d*x + c)) - 48*B*a^7*b^2*c*d^4*f*g^6*e^5*log(-b*f*e + a*g*e + (b*x*e + a*e)*d*f/(d*x + c) - (b*x*e + a*e)*c*g/(d*x + c)) + 48*B*a^8*b*d^5*f*g^6*e^5*log(-b*f*e + a*g*e + (b*x*e + a*e)*d*f/(d*x + c) - (b*x*e + a*e)*c*g/(d*x + c)) - 6*B*a^4*b^5*c^5*g^7*e^5*log(-b*f*e + a*g*e + (b*x*e + a*e)*d*f/(d*x + c) - (b*x*e + a*e)*c*g/(d*x + c)) + 6*B*a^5*b^4*c^4*d*g^7*e^5*log(-b*f*e + a*g*e + (b*x*e + a*e)*d*f/(d*x + c) - (b*x*e + a*e)*c*g/(d*x + c)) + 6*B*a^8*b*c*d^4*g^7*e^5*log(-b*f*e + a*g*e + (b*x*e + a*e)*d*f/(d*x + c) - (b*x*e + a*e)*c*g/(d*x + c)) - 6*

```
B*a^9*d^5*g^7*e^5*log(-b*f*e + a*g*e + (b*x*e + a*e)*d*f/(d*x + c) - (b*x*e
 + a*e)*c*g/(d*x + c)) - 96*(b*x*e + a*e)*B*b^8*c^2*d^4*f^7*e^4*log(-b*f*e
+ a*g*e + (b*x*e + a*e)*d*f/(d*x + c) - (b*x*e + a*e)*c*g/(d*x + c))/(d*x +
 c) + 192*(b*x*e + a*e)*B*a*b^7*c*d^5*f^7*e^4*log(-b*f*e + a*g*e + (b*x*e +
 a*e)*d*f/(d*x + c) - (b*x*e + a*e)*c*g/(d*x + c))/(d*x + c) - 96*(b*x*e +
a*e)*B*a^2*b^6*d^6*f^7*e^4*log(-b*f*e + a*g*e + (b*x*e + a*e)*d*f/(d*x + c)
 - (b*x*e + a*e)*c*g/(d*x + c))/(d*x + c) + 240*(b*x*e + a*e)*B*b^8*c^3*d^3
*f^6*g*e^4*log(-b*f*e + a*g*e + (b*x*e + a*e)*d*f/(d*x + c) - (b*x*e + a*e)
*c*g/(d*x + c))/(d*x + c) - 48*(b*x*e + a*e)*B*a*b^7*c^2*d^4*f^6*g*e^4*log(
-b*f*e + a*g*e + (b*x*e + a*e)*d*f/(d*x + c) - (b*x*e + a*e)*c*g/(d*x + c))
/(d*x + c) - 624*(b*x*e + a*e)*B*a^2*b^6*c*d^5*f^6*g*e^4*log(-b*f*e + a*g*e
 + (b*x*e + a*e)*d*f/(d*x + c) - (b*x*e + a*e)*c*g/(d*x + c))/(d*x + c) + 4
32*(b*x*e + a*e)*B*a^3*b^5*d^6*f^6*g*e^4*log(-b*f*e + a*g*e + (b*x*e + a*e)
*d*f/(d*x + c) - (b*x*e + a*e)*c*g/(d*x + c))/(d*x + c) - 240*(b*x*e + a*e)
*B*b^8*c^4*d^2*f^5*g^2*e^4*log(-b*f*e + a*g*e +...
```

Mupad [B]
time = 16.22, size = 2518, normalized size = 6.64

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x))/(c + d*x)))/(f + g*x)^5,x)
```

```
[Out] (log(f + g*x)*(g*(6*B*a^2*b^2*d^4*f^2 - 6*B*b^4*c^2*d^2*f^2) - g^2*(4*B*a^3
*b*d^4*f - 4*B*b^4*c^3*d*f) + g^3*(B*a^4*d^4 - B*b^4*c^4) - 4*B*a*b^3*d^4*f
^3 + 4*B*b^4*c*d^3*f^3))/(4*a^4*c^4*g^8 + 4*b^4*d^4*f^8 + 4*a^4*d^4*f^4*g^4
 + 4*b^4*c^4*f^4*g^4 + 24*a^2*b^2*c^4*f^2*g^6 + 24*a^2*b^2*d^4*f^6*g^2 + 24
*a^4*c^2*d^2*f^2*g^6 + 24*b^4*c^2*d^2*f^6*g^2 - 16*a^3*b*c^4*f*g^7 - 16*a*b
^3*d^4*f^7*g - 16*a^4*c^3*d*f*g^7 - 16*b^4*c*d^3*f^7*g - 16*a*b^3*c^4*f^3*g
^5 - 16*a^3*b*d^4*f^5*g^3 - 16*a^4*c*d^3*f^3*g^5 - 16*b^4*c^3*d*f^5*g^3 + 6
4*a*b^3*c*d^3*f^6*g^2 + 64*a*b^3*c^3*d*f^4*g^4 + 64*a^3*b*c*d^3*f^4*g^4 + 6
4*a^3*b*c^3*d*f^2*g^6 - 96*a*b^3*c^2*d^2*f^5*g^3 - 96*a^2*b^2*c*d^3*f^5*g^3
 - 96*a^2*b^2*c^3*d*f^3*g^5 - 96*a^3*b*c^2*d^2*f^3*g^5 + 144*a^2*b^2*c^2*d^
2*f^4*g^4) - ((6*A*a^3*c^3*g^6 + 6*A*b^3*d^3*f^6 - 6*A*a^3*d^3*f^3*g^3 - 6*
A*b^3*c^3*f^3*g^3 - 11*B*a^3*d^3*f^3*g^3 + 11*B*b^3*c^3*f^3*g^3 + 18*A*a*b^
2*c^3*f^2*g^4 + 18*A*a^2*b*d^3*f^4*g^2 - 7*B*a*b^2*c^3*f^2*g^4 + 18*A*a^3*c
*d^2*f^2*g^4 + 31*B*a^2*b*d^3*f^4*g^2 + 18*A*b^3*c^2*d*f^4*g^2 + 7*B*a^3*c*
d^2*f^2*g^4 - 31*B*b^3*c^2*d*f^4*g^2 - 18*A*a^2*b*c^3*f*g^5 - 18*A*a*b^2*d^
3*f^5*g + 2*B*a^2*b*c^3*f*g^5 - 18*A*a^3*c^2*d*f*g^5 - 26*B*a*b^2*d^3*f^5*g
 - 18*A*b^3*c*d^2*f^5*g - 2*B*a^3*c^2*d*f*g^5 + 26*B*b^3*c*d^2*f^5*g + 54*A
*a*b^2*c*d^2*f^4*g^2 - 54*A*a*b^2*c^2*d*f^3*g^3 - 54*A*a^2*b*c*d^2*f^3*g^3
+ 54*A*a^2*b*c^2*d*f^2*g^4 + 15*B*a*b^2*c^2*d*f^3*g^3 - 15*B*a^2*b*c*d^2*f^
3*g^3)/(6*(a^3*c^3*g^6 + b^3*d^3*f^6 - a^3*d^3*f^3*g^3 - b^3*c^3*f^3*g^3 -
3*a^2*b*c^3*f*g^5 - 3*a*b^2*d^3*f^5*g - 3*a^3*c^2*d*f*g^5 - 3*b^3*c*d^2*f^5
```

*g + 3*a*b^2*c^3*f^2*g^4 + 3*a^2*b*d^3*f^4*g^2 + 3*a^3*c*d^2*f^2*g^4 + 3*b^3*c^2*d*f^4*g^2 + 9*a*b^2*c*d^2*f^4*g^2 - 9*a*b^2*c^2*d*f^3*g^3 - 9*a^2*b*c*d^2*f^3*g^3 + 9*a^2*b*c^2*d*f^2*g^4)) - (x^2*(B*a*b^2*c^3*g^6 - B*a^3*c*d^2*g^6 + 7*B*a^3*d^3*f*g^5 - 7*B*b^3*c^3*f*g^5 + 20*B*a*b^2*d^3*f^3*g^3 - 21*B*a^2*b*d^3*f^2*g^4 - 20*B*b^3*c*d^2*f^3*g^3 + 21*B*b^3*c^2*d*f^2*g^4 - 3*B*a*b^2*c^2*d*f*g^5 + 3*B*a^2*b*c*d^2*f*g^5))/(2*(a^3*c^3*g^6 + b^3*d^3*f^6 - a^3*d^3*f^3*g^3 - b^3*c^3*f^3*g^3 - 3*a^2*b*c^3*f*g^5 - 3*a*b^2*d^3*f^5*g - 3*a^3*c^2*d*f*g^5 - 3*b^3*c*d^2*f^5*g + 3*a*b^2*c^3*f^2*g^4 + 3*a^2*b*d^3*f^4*g^2 + 3*a^3*c*d^2*f^2*g^4 + 3*b^3*c^2*d*f^4*g^2 + 9*a*b^2*c*d^2*f^4*g^2 - 9*a*b^2*c^2*d*f^3*g^3 - 9*a^2*b*c*d^2*f^3*g^3 + 9*a^2*b*c^2*d*f^2*g^4)) + (x*(B*a^2*b*c^3*g^6 - B*a^3*c^2*d*g^6 - 13*B*a^3*d^3*f^2*g^4 + 13*B*b^3*c^3*f^2*g^4 - 34*B*a*b^2*d^3*f^4*g^2 + 38*B*a^2*b*d^3*f^3*g^3 + 34*B*b^3*c*d^2*f^4*g^2 - 38*B*b^3*c^2*d*f^3*g^3 - 5*B*a*b^2*c^3*f*g^5 + 5*B*a^3*c*d^2*f*g^5 + 12*B*a*b^2*c^2*d*f^2*g^4 - 12*B*a^2*b*c*d^2*f^2*g^4))/(3*(a^3*c^3*g^6 + b^3*d^3*f^6 - a^3*d^3*f^3*g^3 - b^3*c^3*f^3*g^3 - 3*a^2*b*c^3*f*g^5 - 3*a*b^2*d^3*f^5*g - 3*a^3*c^2*d*f*g^5 - 3*b^3*c*d^2*f^5*g + 3*a*b^2*c^3*f^2*g^4 + 3*a^2*b*d^3*f^4*g^2 + 3*a^3*c*d^2*f^2*g^4 + 3*b^3*c^2*d*f^4*g^2 + 9*a*b^2*c*d^2*f^4*g^2 - 9*a*b^2*c^2*d*f^3*g^3 - 9*a^2*b*c*d^2*f^3*g^3 + 9*a^2*b*c^2*d*f^2*g^4)) - (x^3*(B*a^3*d^3*g^6 - B*b^3*c^3*g^6 + 3*B*a*b^2*d^3*f^2*g^4 - 3*B*b^3*c*d^2*f^2*g^4 - 3*B*a^2*b*d^3*f*g^5 + 3*B*b^3*c^2*d*f*g^5))/(a^3*c^3*g^6 + b^3*d^3*f^6 - a^3*d^3*f^3*g^3 - b^3*c^3*f^3*g^3 - 3*a^2*b*c^3*f*g^5 - 3*a*b^2*d^3*f^5*g - 3*a^3*c^2*d*f*g^5 - 3*b^3*c*d^2*f^5*g + 3*a*b^2*c^3*f^2*g^4 + 3*a^2*b*d^3*f^4*g^2 + 3*a^3*c*d^2*f^2*g^4 + 3*b^3*c^2*d*f^4*g^2 + 9*a*b^2*c*d^2*f^4*g^2 - 9*a*b^2*c^2*d*f^3*g^3 - 9*a^2*b*c*d^2*f^3*g^3 + 9*a^2*b*c^2*d*f^2*g^4))/(4*f^4*g + 4*g^5*x^4 + 16*f^3*g^2*x + 16*f*g^4*x^3 + 24*f^2*g^3*x^2) - (B*log((e*(a + b*x))/(c + d*x)))/(4*g*(f^4 + g^4*x^4 + 4*f^3*g*x + 4*f*g^3*x^3 + 6*f^2*g^2*x^2)) + (B*b^4*log(a + b*x))/(4*a^4*g^5 + 4*b^4*f^4*g - 16*a*b^3*f^3*g^2 + 24*a^2*b^2*f^2*g^3 - 16*a^3*b*f*g^4) - (B*d^4*log(c + d*x))/(4*c^4*g^5 + 4*d^4*f^4*g - 16*c*d^3*f^3*g^2 + 24*c^2*d^2*f^2*g^3 - 16*c^3*d*f*g^4)

3.240 $\int (f+gx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$

Optimal. Leaf size=874

$$\frac{B^2(bc-ad)^3g^3x}{6b^3d^3}+\frac{B^2(bc-ad)^2g^2(4bdf-3bcg-adg)x}{4b^3d^3}+\frac{B^2(bc-ad)^2g^3(c+dx)^2}{12b^2d^4}+\frac{B^2(bc-ad)^4g^3\log\left(\frac{a+bx}{c+dx}\right)}{6b^4d^4}$$

```
[Out] 1/6*B^2*(-a*d+b*c)^3*g^3*x/b^3/d^3+1/4*B^2*(-a*d+b*c)^2*g^2*(-a*d*g-3*b*c*g
+4*b*d*f)*x/b^3/d^3+1/12*B^2*(-a*d+b*c)^2*g^3*(d*x+c)^2/b^2/d^4+1/6*B^2*(-a
*d+b*c)^4*g^3*ln((b*x+a)/(d*x+c))/b^4/d^4+1/4*B^2*(-a*d+b*c)^3*g^2*(-a*d*g-
3*b*c*g+4*b*d*f)*ln((b*x+a)/(d*x+c))/b^4/d^4-1/2*B*(-a*d+b*c)*g*(a^2*d^2*g^
2-2*a*b*d*g*(-c*g+2*d*f)+b^2*(3*c^2*g^2-8*c*d*f*g+6*d^2*f^2))*(b*x+a)*(A+B*
ln(e*(b*x+a)/(d*x+c)))/b^4/d^3-1/4*B*(-a*d+b*c)*g^2*(-a*d*g-3*b*c*g+4*b*d*f
)*(d*x+c)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/b^2/d^4-1/6*B*(-a*d+b*c)*g^3*(d*x+c
)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))/b/d^4-1/2*B*(-a*d+b*c)*(-a*d*g-b*c*g+2*b*d*
f)*(2*a*b*d^2*f*g-a^2*d^2*g^2-b^2*(c^2*g^2-2*c*d*f*g+2*d^2*f^2))*ln((-a*d+b
*c)/b/(d*x+c))*(A+B*ln(e*(b*x+a)/(d*x+c)))/b^4/d^4-1/4*(-a*g+b*f)^4*(A+B*ln
(e*(b*x+a)/(d*x+c)))^2/b^4/g+1/4*(g*x+f)^4*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/g+
1/6*B^2*(-a*d+b*c)^4*g^3*ln(d*x+c)/b^4/d^4+1/4*B^2*(-a*d+b*c)^3*g^2*(-a*d*g
-3*b*c*g+4*b*d*f)*ln(d*x+c)/b^4/d^4+1/2*B^2*(-a*d+b*c)^2*g*(a^2*d^2*g^2-2*a
*b*d*g*(-c*g+2*d*f)+b^2*(3*c^2*g^2-8*c*d*f*g+6*d^2*f^2))*ln(d*x+c)/b^4/d^4-
1/2*B^2*(-a*d+b*c)*(-a*d*g-b*c*g+2*b*d*f)*(2*a*b*d^2*f*g-a^2*d^2*g^2-b^2*(c
^2*g^2-2*c*d*f*g+2*d^2*f^2))*polylog(2,d*(b*x+a)/b/(d*x+c))/b^4/d^4
```

Rubi [A]
time = 1.10, antiderivative size = 874, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.345$, Rules used = {2554, 2398, 2404, 2338, 2356, 46, 2351, 31, 2354, 2438}

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]
```

```
[Out] (B^2*(b*c - a*d)^3*g^3*x)/(6*b^3*d^3) + (B^2*(b*c - a*d)^2*g^2*(4*b*d*f - 3
*b*c*g - a*d*g)*x)/(4*b^3*d^3) + (B^2*(b*c - a*d)^2*g^3*(c + d*x)^2)/(12*b^
2*d^4) + (B^2*(b*c - a*d)^4*g^3*Log[(a + b*x)/(c + d*x)])/(6*b^4*d^4) + (B^
2*(b*c - a*d)^3*g^2*(4*b*d*f - 3*b*c*g - a*d*g)*Log[(a + b*x)/(c + d*x)])/(
4*b^4*d^4) - (B*(b*c - a*d)*g*(a^2*d^2*g^2 - 2*a*b*d*g*(2*d*f - c*g) + b^2*
(6*d^2*f^2 - 8*c*d*f*g + 3*c^2*g^2))*(a + b*x)*(A + B*Log[(e*(a + b*x))/(c
+ d*x)]))/(2*b^4*d^3) - (B*(b*c - a*d)*g^2*(4*b*d*f - 3*b*c*g - a*d*g)*(c +
 d*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(4*b^2*d^4) - (B*(b*c - a*d)*
g^3*(c + d*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(6*b*d^4) - (B*(b*c -
```

```
a*d)*(2*b*d*f - b*c*g - a*d*g)*(2*a*b*d^2*f*g - a^2*d^2*g^2 - b^2*(2*d^2*f
^2 - 2*c*d*f*g + c^2*g^2))*Log[(b*c - a*d)/(b*(c + d*x))]*(A + B*Log[(e*(a
+ b*x))/(c + d*x)]))/(2*b^4*d^4) - ((b*f - a*g)^4*(A + B*Log[(e*(a + b*x))/
(c + d*x)])^2)/(4*b^4*g) + ((f + g*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)]
)^2)/(4*g) + (B^2*(b*c - a*d)^4*g^3*Log[c + d*x])/(6*b^4*d^4) + (B^2*(b*c -
 a*d)^3*g^2*(4*b*d*f - 3*b*c*g - a*d*g)*Log[c + d*x])/(4*b^4*d^4) + (B^2*(b
*c - a*d)^2*g*(a^2*d^2*g^2 - 2*a*b*d*g*(2*d*f - c*g) + b^2*(6*d^2*f^2 - 8*c
*d*f*g + 3*c^2*g^2))*Log[c + d*x])/(2*b^4*d^4) - (B^2*(b*c - a*d)*(2*b*d*f
- b*c*g - a*d*g)*(2*a*b*d^2*f*g - a^2*d^2*g^2 - b^2*(2*d^2*f^2 - 2*c*d*f*g
+ c^2*g^2))*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/(2*b^4*d^4)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
```

```
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_)*((
f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(d + e*x)^(q +
 1)*((a + b*Log[c*x^n])^p/((q + 1)*(e*f - d*g))), x] - Dist[b*n*(p/((q + 1)
*(e*f - d*g))), Int[(f + g*x)^(m + 1)*(d + e*x)^(q + 1)*((a + b*Log[c*x^n])
^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, q}, x] && NeQ[e*f
- d*g, 0] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 2404

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2554

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[b*c - a*d, Su
bst[Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m +
2)), x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n
}, x] && EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegerQ[m] &
& IGtQ[p, 0]
```

Rubi steps

$$\int (f+gx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \frac{(f+gx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{4g} - \frac{B\int \frac{(bc-ad)(f+gx)^4\left(A+B\,\mathrm{l}\right.}{(a+bx)(c+d}}{2g}$$

$$= \frac{(f+gx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{4g} - \frac{(B(bc-ad))\int \frac{(f+gx)^4(}{(}}{2g}$$

$$= \frac{(f+gx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{4g} - \frac{(B(bc-ad))\int\left(\frac{g^2(a^2d}{}\right.}{}$$

$$= \frac{(f+gx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{4g} - \frac{(B(bc-ad)g^3)\int x^2\left(A\right.}{2}$$

$$= -\frac{AB(bc-ad)g(a^2d^2g^2-abdg(4df-cg)+b^2(6d^2f^2-4cd}{2b^3d^3}$$

$$= -\frac{AB(bc-ad)g(a^2d^2g^2-abdg(4df-cg)+b^2(6d^2f^2-4cd}{2b^3d^3}$$

$$= -\frac{AB(bc-ad)g(a^2d^2g^2-abdg(4df-cg)+b^2(6d^2f^2-4cd}{2b^3d^3}$$

$$= -\frac{B^2(bc-ad)^2(bc+ad)g^3x}{6b^3d^3} + \frac{B^2(bc-ad)^2g^2(4bdf-bcg-}{4b^3d^3}$$

$$= -\frac{B^2(bc-ad)^2(bc+ad)g^3x}{6b^3d^3} + \frac{B^2(bc-ad)^2g^2(4bdf-bcg-}{4b^3d^3}$$

$$= -\frac{B^2(bc-ad)^2(bc+ad)g^3x}{6b^3d^3} + \frac{B^2(bc-ad)^2g^2(4bdf-bcg-}{4b^3d^3}$$

$$= -\frac{B^2(bc-ad)^2(bc+ad)g^3x}{6b^3d^3} + \frac{B^2(bc-ad)^2g^2(4bdf-bcg-}{4b^3d^3}$$

Mathematica [A]
time = 0.64, size = 733, normalized size = 0.84

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]
```

```
[Out] ((f + g*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 - (B*(6*A*b*d*(b*c - a*
d)*g^2*(a^2*d^2*g^2 + a*b*d*g*(-4*d*f + c*g) + b^2*(6*d^2*f^2 - 4*c*d*f*g +
 c^2*g^2))*x + 6*B*d*(b*c - a*d)*g^2*(a^2*d^2*g^2 + a*b*d*g*(-4*d*f + c*g)
+ b^2*(6*d^2*f^2 - 4*c*d*f*g + c^2*g^2))*(a + b*x)*Log[(e*(a + b*x))/(c + d
*x)] + 3*b^2*d^2*(b*c - a*d)*g^3*(4*b*d*f - b*c*g - a*d*g)*x^2*(A + B*Log[(
e*(a + b*x))/(c + d*x)]) + 2*b^3*d^3*(b*c - a*d)*g^4*x^3*(A + B*Log[(e*(a +
 b*x))/(c + d*x)]) + 6*d^4*(b*f - a*g)^4*Log[a + b*x]*(A + B*Log[(e*(a + b*
x))/(c + d*x)]) - 6*B*(b*c - a*d)^2*g^2*(a^2*d^2*g^2 + a*b*d*g*(-4*d*f + c*
g) + b^2*(6*d^2*f^2 - 4*c*d*f*g + c^2*g^2))*Log[c + d*x] - 6*b^4*(d*f - c*g
)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)])*Log[c + d*x] + B*(b*c - a*d)*g^4*(
b*d*(b*c - a*d)*x*(2*b*c + 2*a*d - b*d*x) + 2*a^3*d^3*Log[a + b*x] - 2*b^3*
c^3*Log[c + d*x]) - 3*B*(b*c - a*d)*g^3*(-4*b*d*f + b*c*g + a*d*g)*(-(a^2*d
^2*Log[a + b*x]) + b*(d*(-(b*c) + a*d)*x + b*c^2*Log[c + d*x])) - 3*B*d^4*(
b*f - a*g)^4*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]
) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) + 3*b^4*B*(d*f - c*g)^4*((2
*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog
[2, (b*(c + d*x))/(b*c - a*d)])))/(3*b^4*d^4))/(4*g)
```

Maple [F]
time = 0.35, size = 0, normalized size = 0.00

$$\int (gx+f)^3 \left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

```
[Out] int((g*x+f)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1973 vs. 2(849) = 1698.
time = 0.40, size = 1973, normalized size = 2.26

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="maxima")
```

```
[Out] 1/4*A^2*g^3*x^4 + A^2*f*g^2*x^3 + 3/2*A^2*f^2*g*x^2 + 2*(x*log(b*x*e/(d*x +
 c) + a*e/(d*x + c)) + a*log(b*x + a)/b - c*log(d*x + c)/d)*A*B*f^3 + 3*(x^
2*log(b*x*e/(d*x + c) + a*e/(d*x + c)) - a^2*log(b*x + a)/b^2 + c^2*log(d*x
 + c)/d^2 - (b*c - a*d)*x/(b*d))*A*B*f^2*g + (2*x^3*log(b*x*e/(d*x + c) + a
*e/(d*x + c)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d
 - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*A*B*f*g^2 + 1/12*(6*x
^4*log(b*x*e/(d*x + c) + a*e/(d*x + c)) - 6*a^4*log(b*x + a)/b^4 + 6*c^4*lo
g(d*x + c)/d^4 - (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)
```

```
*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3))*A*B*g^3 + A^2*f^3*x - 1/12*(6*a^
3*c*d^3*g^3 - 3*(8*c*d^3*f*g^2 - c^2*d^2*g^3)*a^2*b + 2*(18*c*d^3*f^2*g - 6
*c^2*d^2*f*g^2 + c^3*d*g^3)*a*b^2 + (24*c*d^3*f^3 - 72*c^2*d^2*f^2*g + 60*c
^3*d*f*g^2 - 17*c^4*g^3)*b^3)*B^2*log(d*x + c)/(b^3*d^4) + 1/2*(4*a*b^3*d^4
*f^3 - 6*a^2*b^2*d^4*f^2*g + 4*a^3*b*d^4*f*g^2 - a^4*d^4*g^3 - (4*c*d^3*f^3
 - 6*c^2*d^2*f^2*g + 4*c^3*d*f*g^2 - c^4*g^3)*b^4)*(log(b*x + a)*log((b*d*x
 + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B^2/(b^4*d^4)
 + 1/12*(3*B^2*b^4*d^4*g^3*x^4 + 2*(a*b^3*d^4*g^3 + (6*d^4*f*g^2 - c*d^3*g^
3)*b^4)*B^2*x^3 - 2*(a^2*b^2*d^4*g^3 - (6*d^4*f*g^2 - c*d^3*g^3)*a*b^3 - (9
*d^4*f^2*g - 6*c*d^3*f*g^2 + 2*c^2*d^2*g^3)*b^4)*B^2*x^2 + (a^3*b*d^4*g^3 -
 (12*d^4*f*g^2 - 5*c*d^3*g^3)*a^2*b^2 + (36*d^4*f^2*g - 24*c*d^3*f*g^2 + 5*
c^2*d^2*g^3)*a*b^3 + (12*d^4*f^3 - 36*c*d^3*f^2*g + 36*c^2*d^2*f*g^2 - 11*c
^3*d*g^3)*b^4)*B^2*x + 3*(B^2*b^4*d^4*g^3*x^4 + 4*B^2*b^4*d^4*f*g^2*x^3 + 6
*B^2*b^4*d^4*f^2*g*x^2 + 4*B^2*b^4*d^4*f^3*x + (4*a*b^3*d^4*f^3 - 6*a^2*b^2
*d^4*f^2*g + 4*a^3*b*d^4*f*g^2 - a^4*d^4*g^3)*B^2)*log(b*x + a)^2 + 3*(B^2*
b^4*d^4*g^3*x^4 + 4*B^2*b^4*d^4*f*g^2*x^3 + 6*B^2*b^4*d^4*f^2*g*x^2 + 4*B^2
*b^4*d^4*f^3*x + (4*c*d^3*f^3 - 6*c^2*d^2*f^2*g + 4*c^3*d*f*g^2 - c^4*g^3)*
B^2*b^4)*log(d*x + c)^2 + (6*B^2*b^4*d^4*g^3*x^4 + 2*(a*b^3*d^4*g^3 + (12*d
^4*f*g^2 - c*d^3*g^3)*b^4)*B^2*x^3 + 3*(4*a*b^3*d^4*f*g^2 - a^2*b^2*d^4*g^3
 + (12*d^4*f^2*g - 4*c*d^3*f*g^2 + c^2*d^2*g^3)*b^4)*B^2*x^2 + 6*(6*a*b^3*d
^4*f^2*g - 4*a^2*b^2*d^4*f*g^2 + a^3*b*d^4*g^3 + (4*d^4*f^3 - 6*c*d^3*f^2*g
 + 4*c^2*d^2*f*g^2 - c^3*d*g^3)*b^4)*B^2*x + (5*a^4*d^4*g^3 - 2*(6*d^4*f*g^
2 + c*d^3*g^3)*a^3*b + 3*(4*c*d^3*f*g^2 - c^2*d^2*g^3)*a^2*b^2 + 6*(4*d^4*f
^3 - 6*c*d^3*f^2*g + 4*c^2*d^2*f*g^2 - c^3*d*g^3)*a*b^3)*B^2)*log(b*x + a)
- (6*B^2*b^4*d^4*g^3*x^4 + 2*(a*b^3*d^4*g^3 + (12*d^4*f*g^2 - c*d^3*g^3)*b^
4)*B^2*x^3 + 3*(4*a*b^3*d^4*f*g^2 - a^2*b^2*d^4*g^3 + (12*d^4*f^2*g - 4*c*d
^3*f*g^2 + c^2*d^2*g^3)*b^4)*B^2*x^2 + 6*(6*a*b^3*d^4*f^2*g - 4*a^2*b^2*d^4
*f*g^2 + a^3*b*d^4*g^3 + (4*d^4*f^3 - 6*c*d^3*f^2*g + 4*c^2*d^2*f*g^2 - c^3
*d*g^3)*b^4)*B^2*x + 6*(B^2*b^4*d^4*g^3*x^4 + 4*B^2*b^4*d^4*f*g^2*x^3 + 6*B
^2*b^4*d^4*f^2*g*x^2 + 4*B^2*b^4*d^4*f^3*x + (4*a*b^3*d^4*f^3 - 6*a^2*b^2*d
^4*f^2*g + 4*a^3*b*d^4*f*g^2 - a^4*d^4*g^3)*B^2)*log(b*x + a))*log(d*x + c)
)/(b^4*d^4)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="fricas")
```

```
[Out] integral(A^2*g^3*x^3 + 3*A^2*f*g^2*x^2 + 3*A^2*f^2*g*x + A^2*f^3 + (B^2*g^3
*x^3 + 3*B^2*f*g^2*x^2 + 3*B^2*f^2*g*x + B^2*f^3)*log((b*x + a)*e/(d*x + c)
)^2 + 2*(A*B*g^3*x^3 + 3*A*B*f*g^2*x^2 + 3*A*B*f^2*g*x + A*B*f^3)*log((b*x
+ a)*e/(d*x + c)), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**3*(A+B*ln(e*(b*x+a)/(d*x+c)))**2,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="giac")
```

```
[Out] integrate((g*x + f)^3*(B*log((b*x + a)*e/(d*x + c)) + A)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (f+g\,x)^3\,\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^3*(A + B*log((e*(a + b*x))/(c + d*x)))^2,x)
```

```
[Out] int((f + g*x)^3*(A + B*log((e*(a + b*x))/(c + d*x)))^2, x)
```

3.241 $\int (f+gx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$

Optimal. Leaf size=532

$$\frac{B^2(bc-ad)^2g^2x}{3b^2d^2}+\frac{B^2(bc-ad)^3g^2\log\left(\frac{a+bx}{c+dx}\right)}{3b^3d^3}-\frac{2B(bc-ad)g(3bdf-2bcg-adg)(a+bx)\left(A+B\log\left(\frac{e(a+b}{c+d}\right.\right.}{3b^3d^2}$$

```
[Out] 1/3*B^2*(-a*d+b*c)^2*g^2*x/b^2/d^2+1/3*B^2*(-a*d+b*c)^3*g^2*ln((b*x+a)/(d*x
+c))/b^3/d^3-2/3*B*(-a*d+b*c)*g*(-a*d*g-2*b*c*g+3*b*d*f)*(b*x+a)*(A+B*ln(e*
(b*x+a)/(d*x+c)))/b^3/d^2-1/3*B*(-a*d+b*c)*g^2*(d*x+c)^2*(A+B*ln(e*(b*x+a)/
(d*x+c)))/b/d^3+2/3*B*(-a*d+b*c)*(a^2*d^2*g^2-a*b*d*g*(-c*g+3*d*f)+b^2*(c^2
*g^2-3*c*d*f*g+3*d^2*f^2))*ln((-a*d+b*c)/b/(d*x+c))*(A+B*ln(e*(b*x+a)/(d*x+
c)))/b^3/d^3-1/3*(-a*g+b*f)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/b^3/g+1/3*(g*x+
f)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/g+1/3*B^2*(-a*d+b*c)^3*g^2*ln(d*x+c)/b^3
/d^3+2/3*B^2*(-a*d+b*c)^2*g*(-a*d*g-2*b*c*g+3*b*d*f)*ln(d*x+c)/b^3/d^3+2/3*
B^2*(-a*d+b*c)*(a^2*d^2*g^2-a*b*d*g*(-c*g+3*d*f)+b^2*(c^2*g^2-3*c*d*f*g+3*d
^2*f^2))*polylog(2,d*(b*x+a)/b/(d*x+c))/b^3/d^3
```

Rubi [A]

time = 0.72, antiderivative size = 532, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.345$, Rules used = {2554, 2398, 2404, 2338, 2356, 46, 2351, 31, 2354, 2438}

$$\frac{2B^2(bc-ad)\left(a^2d^2g^2-abdg(3df-cg)+b^2(c^2g^2-3cdfg+3d^2f^2)\right)\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{3b^3d^3}+\frac{2B(bc-ad)\left(a^2d^2g^2-abdg(3df-cg)+b^2(c^2g^2-3cdfg+3d^2f^2)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3b^3d^3}-\frac{2Bg(a+bx)(bc-ad)(-adg-2bcg+3bdf)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3b^3d^2}-\frac{(bf-ag)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{3b^3g}-\frac{Bg^2(c+dx)^2(bc-ad)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3bd^3}+\frac{(f+gx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{3g}+\frac{2B^2g(bc-ad)^2\log(c+dx)(-adg-2bcg+3bdf)}{3b^3d^3}+\frac{B^2g^2(bc-ad)^3\log\left(\frac{a+bx}{c+dx}\right)}{3b^3d^3}+\frac{B^2g^2(bc-ad)^3\log(c+dx)}{3b^3d^3}+\frac{B^2g^2x(bc-ad)^2}{3b^2d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]
```

```
[Out] (B^2*(b*c - a*d)^2*g^2*x)/(3*b^2*d^2) + (B^2*(b*c - a*d)^3*g^2*Log[(a + b*x
)/(c + d*x)])/(3*b^3*d^3) - (2*B*(b*c - a*d)*g*(3*b*d*f - 2*b*c*g - a*d*g)*
(a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(3*b^3*d^2) - (B*(b*c - a*d
)*g^2*(c + d*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(3*b*d^3) + (2*B*(b
*c - a*d)*(a^2*d^2*g^2 - a*b*d*g*(3*d*f - c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g
 + c^2*g^2))*Log[(b*c - a*d)/(b*(c + d*x))]*(A + B*Log[(e*(a + b*x))/(c + d
*x)]))/(3*b^3*d^3) - ((b*f - a*g)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)
/(3*b^3*g) + ((f + g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(3*g) + (
B^2*(b*c - a*d)^3*g^2*Log[c + d*x])/(3*b^3*d^3) + (2*B^2*(b*c - a*d)^2*g*(3
*b*d*f - 2*b*c*g - a*d*g)*Log[c + d*x])/(3*b^3*d^3) + (2*B^2*(b*c - a*d)*(a
^2*d^2*g^2 - a*b*d*g*(3*d*f - c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^2))
*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/(3*b^3*d^3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_)*((
f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(d + e*x)^(q +
 1)*((a + b*Log[c*x^n])^p/((q + 1)*(e*f - d*g))), x] - Dist[b*n*(p/((q + 1)
*(e*f - d*g))), Int[(f + g*x)^(m + 1)*(d + e*x)^(q + 1)*((a + b*Log[c*x^n])
^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, q}, x] && NeQ[e*f
- d*g, 0] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 2404

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2554

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[b*c - a*d, Su
bst[Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m +
2)), x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n
}, x] && EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegerQ[m] &
& IGtQ[p, 0]
```

Rubi steps

$$\int (f+gx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \frac{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3g} - \frac{(2B)\int \frac{(bc-ad)(f+gx)^3\left(A+B\right.}{(a+bx)(c+}}{3g}$$

$$= \frac{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3g} - \frac{(2B(bc-ad))\int \frac{(f+gx)^3\left(\right.}{(a}}{3g}$$

$$= \frac{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3g} - \frac{(2B(bc-ad))\int \left(\frac{g^2(3bdf}{}\right.}{}$$

$$= \frac{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3g} - \frac{(2B(bc-ad)g^2)\int x\left(A\right.}{3bc}$$

$$= -\frac{2AB(bc-ad)g(3bdf-bcg-adg)x}{3b^2d^2} - \frac{B(bc-ad)g^2x^2\left(A\right.}{3bc}$$

$$= -\frac{2AB(bc-ad)g(3bdf-bcg-adg)x}{3b^2d^2} - \frac{2B^2(bc-ad)g(3bdf}{}$$

$$= -\frac{2AB(bc-ad)g(3bdf-bcg-adg)x}{3b^2d^2} - \frac{2B^2(bc-ad)g(3bdf}{}$$

$$= \frac{B^2(bc-ad)^2g^2x}{3b^2d^2} - \frac{2AB(bc-ad)g(3bdf-bcg-adg)x}{3b^2d^2} + \frac{a^2}{}$$

$$= \frac{B^2(bc-ad)^2g^2x}{3b^2d^2} - \frac{2AB(bc-ad)g(3bdf-bcg-adg)x}{3b^2d^2} + \frac{a^2}{}$$

$$= \frac{B^2(bc-ad)^2g^2x}{3b^2d^2} - \frac{2AB(bc-ad)g(3bdf-bcg-adg)x}{3b^2d^2} + \frac{a^2}{}$$

$$= \frac{B^2(bc-ad)^2g^2x}{3b^2d^2} - \frac{2AB(bc-ad)g(3bdf-bcg-adg)x}{3b^2d^2} + \frac{a^2}{}$$

Mathematica [A]
time = 0.35, size = 486, normalized size = 0.91

$$\frac{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 - \frac{B\left(2Abd(bc-ad)g^2(3bdf-bcg-adg)x+2Bd(bc-ad)g^2(3bdf-bcg-adg)(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)+b^2d^2(bc-ad)g^3x^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)+2d^3(bf-ag)^3\log(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)+2B(bc-ad)^2g^2(-3bdf+bcg+adg)\log(c+dx)-2b^3(df-cg)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log(c+dx)-B(bc-ad)g^3\left(a^2d^2\log(a+bx)-b\left(d(-bc+ad)x+bc^2\log(c+dx)\right)\right)-Bd^3(bf-ag)^3\left(\log(a+bx)\left(\log(a+bx)-2\log\left(\frac{b(c+dx)}{bc-ad}\right)\right)-2\text{Li}_2\left(\frac{d(a+bx)}{-bc+ad}\right)\right)+b^3B(df-cg)^3\left(\left(2\log\left(\frac{d(a+bx)}{-bc+ad}\right)-\log(c+dx)\right)\log(c+dx)+2\text{Li}_2\left(\frac{b(c+dx)}{bc-ad}\right)\right)\right)}{b^3d^3}}{3g}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]
```

```
[Out] ((f + g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 - (B*(2*A*b*d*(b*c - a*
d)*g^2*(3*b*d*f - b*c*g - a*d*g)*x + 2*B*d*(b*c - a*d)*g^2*(3*b*d*f - b*c*g
 - a*d*g)*(a + b*x)*Log[(e*(a + b*x))/(c + d*x)] + b^2*d^2*(b*c - a*d)*g^3*
x^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 2*d^3*(b*f - a*g)^3*Log[a + b*x]
*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 2*B*(b*c - a*d)^2*g^2*(-3*b*d*f + b
*c*g + a*d*g)*Log[c + d*x] - 2*b^3*(d*f - c*g)^3*(A + B*Log[(e*(a + b*x))/(
c + d*x)])*Log[c + d*x] - B*(b*c - a*d)*g^3*(a^2*d^2*Log[a + b*x] - b*(d*(-
(b*c) + a*d)*x + b*c^2*Log[c + d*x])) - B*d^3*(b*f - a*g)^3*(Log[a + b*x]*(
Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x
))/(-(b*c) + a*d)]) + b^3*B*(d*f - c*g)^3*((2*Log[(d*(a + b*x))/(-(b*c) + a
*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]
)))/(b^3*d^3))/(3*g)
```

Maple [F]
time = 0.24, size = 0, normalized size = 0.00

$$\int (gx + f)^2 \left(A + B \ln\left(\frac{e(bx + a)}{dx + c}\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

```
[Out] int((g*x+f)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1209 vs. 2(516) = 1032.
time = 0.36, size = 1209, normalized size = 2.27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="maxima")
```

```
[Out] 1/3*A^2*g^2*x^3 + A^2*f*g*x^2 + 2*(x*log(b*x*e/(d*x + c) + a*e/(d*x + c)) +
 a*log(b*x + a)/b - c*log(d*x + c)/d)*A*B*f^2 + 2*(x^2*log(b*x*e/(d*x + c)
+ a*e/(d*x + c)) - a^2*log(b*x + a)/b^2 + c^2*log(d*x + c)/d^2 - (b*c - a*d
)*x/(b*d))*A*B*f*g + 1/3*(2*x^3*log(b*x*e/(d*x + c) + a*e/(d*x + c)) + 2*a^
3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*
(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*A*B*g^2 + A^2*f^2*x + 1/3*(2*a^2*c*d^2*g^
2 - (6*c*d^2*f*g - c^2*d*g^2)*a*b - (6*c*d^2*f^2 - 12*c^2*d*f*g + 5*c^3*g^2
)*b^2)*B^2*log(d*x + c)/(b^2*d^3) + 2/3*(3*a*b^2*d^3*f^2 - 3*a^2*b*d^3*f*g
+ a^3*d^3*g^2 - (3*c*d^2*f^2 - 3*c^2*d*f*g + c^3*g^2)*b^3)*(log(b*x + a)*lo
g((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B^2/(
b^3*d^3) + 1/3*(B^2*b^3*d^3*g^2*x^3 + (a*b^2*d^3*g^2 + (3*d^3*f*g - c*d^2*g
^2)*b^3)*B^2*x^2 - (a^2*b*d^3*g^2 - 2*(3*d^3*f*g - c*d^2*g^2)*a*b^2 - 3*(d^
3*f^2 - 2*c*d^2*f*g + c^2*d*g^2)*b^3)*B^2*x + (B^2*b^3*d^3*g^2*x^3 + 3*B^2*
```

```
b^3*d^3*f*g*x^2 + 3*B^2*b^3*d^3*f^2*x + (3*a*b^2*d^3*f^2 - 3*a^2*b*d^3*f*g
+ a^3*d^3*g^2)*B^2)*log(b*x + a)^2 + (B^2*b^3*d^3*g^2*x^3 + 3*B^2*b^3*d^3*f
*g*x^2 + 3*B^2*b^3*d^3*f^2*x + (3*c*d^2*f^2 - 3*c^2*d*f*g + c^3*g^2)*B^2*b^
3)*log(d*x + c)^2 + (2*B^2*b^3*d^3*g^2*x^3 + (a*b^2*d^3*g^2 + (6*d^3*f*g -
c*d^2*g^2)*b^3)*B^2*x^2 + 2*(3*a*b^2*d^3*f*g - a^2*b*d^3*g^2 + (3*d^3*f^2 -
 3*c*d^2*f*g + c^2*d*g^2)*b^3)*B^2*x + (a^2*b*c*d^2*g^2 - a^3*d^3*g^2 + 2*(
3*d^3*f^2 - 3*c*d^2*f*g + c^2*d*g^2)*a*b^2)*B^2)*log(b*x + a) - (2*B^2*b^3*
d^3*g^2*x^3 + (a*b^2*d^3*g^2 + (6*d^3*f*g - c*d^2*g^2)*b^3)*B^2*x^2 + 2*(3*
a*b^2*d^3*f*g - a^2*b*d^3*g^2 + (3*d^3*f^2 - 3*c*d^2*f*g + c^2*d*g^2)*b^3)*
B^2*x + 2*(B^2*b^3*d^3*g^2*x^3 + 3*B^2*b^3*d^3*f*g*x^2 + 3*B^2*b^3*d^3*f^2*
x + (3*a*b^2*d^3*f^2 - 3*a^2*b*d^3*f*g + a^3*d^3*g^2)*B^2)*log(b*x + a))*lo
g(d*x + c))/(b^3*d^3)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="fricas")
```

```
[Out] integral(A^2*g^2*x^2 + 2*A^2*f*g*x + A^2*f^2 + (B^2*g^2*x^2 + 2*B^2*f*g*x +
 B^2*f^2)*log((b*x + a)*e/(d*x + c))^2 + 2*(A*B*g^2*x^2 + 2*A*B*f*g*x + A*B
*f^2)*log((b*x + a)*e/(d*x + c)), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2*(A+B*ln(e*(b*x+a)/(d*x+c)))**2,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="giac")
```

```
[Out] integrate((g*x + f)^2*(B*log((b*x + a)*e/(d*x + c)) + A)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (f+g\,x)^2\,\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^2*(A + B*log((e*(a + b*x))/(c + d*x)))^2,x)
[Out] int((f + g*x)^2*(A + B*log((e*(a + b*x))/(c + d*x)))^2, x)
```

3.242 $\int (f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx$

Optimal. Leaf size=270

$$-\frac{B(bc-ad)g(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b^2d}+\frac{B(bc-ad)(2bdf-bcg-adg)\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right.}{b^2d^2}$$

```
[Out] -B*(-a*d+b*c)*g*(b*x+a)*(A+B*ln(e*(b*x+a)/(d*x+c)))/b^2/d+B*(-a*d+b*c)*(-a*
d*g-b*c*g+2*b*d*f)*ln((-a*d+b*c)/b/(d*x+c))*(A+B*ln(e*(b*x+a)/(d*x+c)))/b^2
/d^2-1/2*(-a*g+b*f)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/b^2/g+1/2*(g*x+f)^2*(A+
B*ln(e*(b*x+a)/(d*x+c)))^2/g+B^2*(-a*d+b*c)^2*g*ln(d*x+c)/b^2/d^2+B^2*(-a*d
+b*c)*(-a*d*g-b*c*g+2*b*d*f)*polylog(2,d*(b*x+a)/b/(d*x+c))/b^2/d^2
```

Rubi [A]
time = 0.36, antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {2554, 2398, 2404, 2338, 2351, 31, 2354, 2438}

$$\frac{B^2(bc-ad)(-adg-bcg+2bdf)\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{b^2d^2}+\frac{B(bc-ad)(-adg-bcg+2bdf)\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{b^2d^2}-\frac{(bf-ag)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{2b^2g}-\frac{Bg(a+bx)(bc-ad)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{b^2d}+\frac{(f+gx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{2g}+\frac{B^2g(bc-ad)^2\log(c+dx)}{b^2d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]
```

```
[Out] -((B*(b*c - a*d)*g*(a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(b^2*d))
 + (B*(b*c - a*d)*(2*b*d*f - b*c*g - a*d*g)*Log[(b*c - a*d)/(b*(c + d*x))]*
(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(b^2*d^2) - ((b*f - a*g)^2*(A + B*Log
[(e*(a + b*x))/(c + d*x)])^2)/(2*b^2*g) + ((f + g*x)^2*(A + B*Log[(e*(a + b
*x))/(c + d*x)])^2)/(2*g) + (B^2*(b*c - a*d)^2*g*Log[c + d*x])/(b^2*d^2) +
(B^2*(b*c - a*d)*(2*b*d*f - b*c*g - a*d*g)*PolyLog[2, (d*(a + b*x))/(b*(c +
 d*x))])/(b^2*d^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
```

```
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_)*((
f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(d + e*x)^(q +
 1)*((a + b*Log[c*x^n])^p/((q + 1)*(e*f - d*g))), x] - Dist[b*n*(p/((q + 1)
*(e*f - d*g))), Int[(f + g*x)^(m + 1)*(d + e*x)^(q + 1)*((a + b*Log[c*x^n])
^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, q}, x] && NeQ[e*f
- d*g, 0] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 2404

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2554

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[b*c - a*d, Su
bst[Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m +
2)), x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n
}, x] && EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegerQ[m] &
& IGtQ[p, 0]
```

Rubi steps

$$\int (f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \frac{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2g} - \frac{B\int \frac{(bc-ad)(f+gx)^2\left(A+B\log\left(\right.\right.}{(a+bx)(c+dx)}}{g}$$

$$= \frac{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2g} - \frac{(B(bc-ad))\int \frac{(f+gx)^2\left(A+\right.}{(a+b}}{g}$$

$$= \frac{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2g} - \frac{(B(bc-ad))\int \left(\frac{g^2\left(A+B\,\mathrm{l}\right.}{}\right.}{}$$

$$= \frac{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2g} - \frac{(B(bc-ad)g)\int \left(A+B\right.}{bd}$$

$$= -\frac{AB(bc-ad)gx}{bd} - \frac{B(bf-ag)^2\log(a+bx)\left(A+B\log\left(\frac{e(a+}{c+}\right.\right.}{b^2g}$$

$$= -\frac{AB(bc-ad)gx}{bd} - \frac{B^2(bc-ad)g(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{b^2d} - \frac{B(}{}$$

$$= -\frac{AB(bc-ad)gx}{bd} - \frac{B^2(bc-ad)g(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{b^2d} - \frac{B(}{}$$

$$= -\frac{AB(bc-ad)gx}{bd} - \frac{B^2(bc-ad)g(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{b^2d} - \frac{B(}{}$$

$$= -\frac{AB(bc-ad)gx}{bd} - \frac{B^2(bc-ad)g(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{b^2d} - \frac{B(}{}$$

$$= -\frac{AB(bc-ad)gx}{bd} + \frac{B^2(bf-ag)^2\log^2(a+bx)}{2b^2g} - \frac{B^2(bc-ad)}{}$$

$$= -\frac{AB(bc-ad)gx}{bd} + \frac{B^2(bf-ag)^2\log^2(a+bx)}{2b^2g} - \frac{B^2(bc-ad)}{}$$

<u>Mathematica</u> [A]

time = 0.20, size = 346, normalized size = 1.28

$$\frac{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 - \frac{B\left(2Abd(bc-ad)g^2x+2Bd(bc-ad)g^2(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)+2d^2(bf-ag)^2\log(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)-2B(bc-ad)^2g^2\log(c+dx)-2b^2(df-cg)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log(c+dx)-Bd^2(bf-ag)^2\left(\log(a+bx)\left(\log(a+bx)-2\log\left(\frac{b(c+dx)}{bc-ad}\right)\right)-2\mathrm{Li}_2\left(\frac{d(a+bx)}{-bc+ad}\right)\right)+b^2B(df-cg)^2\left(\left(2\log\left(\frac{d(a+bx)}{-bc+ad}\right)-\log(c+dx)\right)\log(c+dx)+2\mathrm{Li}_2\left(\frac{b(c+dx)}{bc-ad}\right)\right)\right)}{b^2d^2}}{2g}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]
```

```
[Out] ((f + g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 - (B*(2*A*b*d*(b*c - a*
d)*g^2*x + 2*B*d*(b*c - a*d)*g^2*(a + b*x)*Log[(e*(a + b*x))/(c + d*x)] + 2
*d^2*(b*f - a*g)^2*Log[a + b*x]*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - 2*B*
(b*c - a*d)^2*g^2*Log[c + d*x] - 2*b^2*(d*f - c*g)^2*(A + B*Log[(e*(a + b*x
))/(c + d*x)])*Log[c + d*x] - B*d^2*(b*f - a*g)^2*(Log[a + b*x]*(Log[a + b*
x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c)
 + a*d)]) + b^2*B*(d*f - c*g)^2*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log
[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(b^2*d
^2))/(2*g)
```

Maple [F]
time = 0.13, size = 0, normalized size = 0.00

$$\int (gx+f)\left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

```
[Out] int((g*x+f)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 629 vs. $2(269) = 538$.
time = 0.36, size = 629, normalized size = 2.33

$$\frac{1}{2}A^2gx^2 + 2\left(x\log\left(\frac{bxe}{dx+c}+\frac{ae}{dx+c}\right)+\frac{a\log(bx+a)}{b}-\frac{c\log(dx+c)}{d}\right)ABf + \left(x^2\log\left(\frac{bxe}{dx+c}+\frac{ae}{dx+c}\right)-\frac{a^2\log(bx+a)}{b^2}+\frac{c^2\log(dx+c)}{d^2}-\frac{(bc-ad)x}{bd}\right)ABg + A^2fx - \frac{(acdg+2(cdf-c^2g)b)B^2\log(dx+c)}{bd^2} + \frac{(2abd^2f-a^2d^2g-(2cdf-c^2g)b^2)\left(\log(bx+a)\log\left(\frac{bdx+ad}{bc-ad}+1\right)+\mathrm{Li}_2\left(-\frac{bdx+ad}{bc-ad}\right)\right)B^2}{b^2d^2} + \frac{B^2b^2d^2gx^2+2(abd^2g+(d^2f-cdg)b^2)B^2x+(B^2b^2d^2gx^2+2B^2b^2d^2fx+(2abd^2f-a^2d^2g)B^2)\log(bx+a)^2+(B^2b^2d^2gx^2+2B^2b^2d^2fx+(2cdf-c^2g)B^2b^2)\log(dx+c)^2+2(B^2b^2d^2gx^2+(2d^2f-cdg)B^2ab+(abd^2g+(2d^2f-cdg)b^2)B^2x)\log(bx+a)-2(B^2b^2d^2gx^2+(abd^2g+(2d^2f-cdg)b^2)B^2x+(B^2b^2d^2gx^2+2B^2b^2d^2fx+(2abd^2f-a^2d^2g)B^2)\log(bx+a))\log(dx+c)}{2b^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="maxima")
```

```
[Out] 1/2*A^2*g*x^2 + 2*(x*log(b*x*e/(d*x + c) + a*e/(d*x + c)) + a*log(b*x + a)/
b - c*log(d*x + c)/d)*A*B*f + (x^2*log(b*x*e/(d*x + c) + a*e/(d*x + c)) - a
^2*log(b*x + a)/b^2 + c^2*log(d*x + c)/d^2 - (b*c - a*d)*x/(b*d))*A*B*g + A
^2*f*x - (a*c*d*g + 2*(c*d*f - c^2*g)*b)*B^2*log(d*x + c)/(b*d^2) + (2*a*b*
d^2*f - a^2*d^2*g - (2*c*d*f - c^2*g)*b^2)*(log(b*x + a)*log((b*d*x + a*d)/
(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B^2/(b^2*d^2) + 1/2*(
B^2*b^2*d^2*g*x^2 + 2*(a*b*d^2*g + (d^2*f - c*d*g)*b^2)*B^2*x + (B^2*b^2*d^
2*g*x^2 + 2*B^2*b^2*d^2*f*x + (2*a*b*d^2*f - a^2*d^2*g)*B^2)*log(b*x + a)^2
 + (B^2*b^2*d^2*g*x^2 + 2*B^2*b^2*d^2*f*x + (2*c*d*f - c^2*g)*B^2*b^2)*log(
d*x + c)^2 + 2*(B^2*b^2*d^2*g*x^2 + (2*d^2*f - c*d*g)*B^2*a*b + (a*b*d^2*g
+ (2*d^2*f - c*d*g)*b^2)*B^2*x)*log(b*x + a) - 2*(B^2*b^2*d^2*g*x^2 + (a*b*
d^2*g + (2*d^2*f - c*d*g)*b^2)*B^2*x + (B^2*b^2*d^2*g*x^2 + 2*B^2*b^2*d^2*f
*x + (2*a*b*d^2*f - a^2*d^2*g)*B^2)*log(b*x + a))*log(d*x + c))/(b^2*d^2)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="fricas")
```

```
[Out] integral(A^2*g*x + A^2*f + (B^2*g*x + B^2*f)*log((b*x + a)*e/(d*x + c))^2 +
 2*(A*B*g*x + A*B*f)*log((b*x + a)*e/(d*x + c)), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(A+B*ln(e*(b*x+a)/(d*x+c)))**2,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="giac")
```

```
[Out] integrate((g*x + f)*(B*log((b*x + a)*e/(d*x + c)) + A)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int (f+g\,x)\,\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)*(A + B*log((e*(a + b*x))/(c + d*x)))^2,x)
```

```
[Out] int((f + g*x)*(A + B*log((e*(a + b*x))/(c + d*x)))^2, x)
```

3.243 $\int \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$

Optimal. Leaf size=125

$$\frac{2B(bc-ad)\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{bd}+\frac{(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{b}+\frac{2B^2(bc-ad)\text{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{bd}$$

[Out] 2*B*(-a*d+b*c)*ln((-a*d+b*c)/b/(d*x+c))*(A+B*ln(e*(b*x+a)/(d*x+c)))/b/d+(b*x+a)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/b+2*B^2*(-a*d+b*c)*polylog(2,d*(b*x+a)/b/(d*x+c))/b/d

Rubi [A]
time = 0.16, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2536, 2544, 2458, 2378, 2370, 2352}

$$\frac{2B^2(bc-ad)\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{bd}+\frac{2B(bc-ad)\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{bd}+\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{b}$$

Antiderivative was successfully verified.

[In] Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]

[Out] (2*B*(b*c - a*d)*Log[(b*c - a*d)/(b*(c + d*x))]*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(b*d) + ((a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/b + (2*B^2*(b*c - a*d)*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/(b*d)

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLog[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2370

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)/(x_))^(q_.)*(x_)^(m_.), x_Symbol] :> Int[(e + d*x)^q*(a + b*Log[c*x^n])^p, x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && EqQ[m, q] && IntegerQ[q]
```

Rule 2378

```
Int[((a_.) + Log[(c_.)*(x_)^(n_)]*(b_.))/((x_)*((d_) + (e_.)*(x_)^(r_.))), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*Log[c*x])/(x*(d + e*x^(r/n))), x], x, x^n], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IntegerQ[r/n]
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2536

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.), x_Symbol] :> Simp[(a + b*x)*((A + B*Log[e*((a + b*x)^n/(c
+ d*x)^n)])^p/b), x] - Dist[B*n*p*((b*c - a*d)/b), Int[(A + B*Log[e*((a + b
*x)^n/(c + d*x)^n)])^(p - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, A,
B, n}, x] && EqQ[n + mn, 0] && NeQ[b*c - a*d, 0] && IGtQ[p, 0]
```

Rule 2544

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))/((f_.) + (g_.)*(x_)), x_Symbol] :> Simp[(-Log[(b*c - a*d)/(b*(c +
 d*x))])*((A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/g), x] + Dist[B*n*((b*c
- a*d)/g), Int[Log[(b*c - a*d)/(b*(c + d*x))]/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[n + mn, 0] && NeQ[b*c -
 a*d, 0] && EqQ[d*f - c*g, 0]
```

Rubi steps

$$\int \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = x\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 - (2B)\int \frac{(bc-ad)x\left(A+B\log\left(\frac{e(a+b}{c+dx}\right.\right.}{(a+bx)(c+dx)}$$

$$= x\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 - (2B(bc-ad))\int \frac{x\left(A+B\log\left(\frac{e(a+b}{c+dx}\right.\right.}{(a+bx)(c+dx}$$

$$= x\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 - (2B(bc-ad))\int \left(-\frac{a\left(A+B\log\left(\frac{e}{}\right.\right.}{(bc-ad)(a+}\right.$$

$$= x\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 + (2aB)\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{a+bx}dx - (2$$

$$= \frac{2aB\log(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b} + x\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right.$$

$$= \frac{2aB\log(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b} + x\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right.$$

$$= \frac{2aB\log(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b} + x\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right.$$

$$= \frac{2aB\log(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b} + x\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right.$$

$$= \frac{2aB\log(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b} + x\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right.$$

$$= -\frac{aB^2\log^2(a+bx)}{b} + \frac{2aB\log(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b} + x\left(A\right.$$

$$= -\frac{aB^2\log^2(a+bx)}{b} + \frac{2aB\log(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b} + x\left(A\right.$$

Mathematica [A]
time = 0.12, size = 214, normalized size = 1.71

$$x\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 + \frac{B\left(2ad\log(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right) - 2bc\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log(c+dx) - aBd\left(\log(a+bx)\left(\log(a+bx) - 2\log\left(\frac{b(c+dx)}{bc-ad}\right)\right) - 2\text{Li}_2\left(\frac{d(a+bx)}{-bc+ad}\right)\right) + bBc\left(\left(2\log\left(\frac{d(a+bx)}{-bc+ad}\right) - \log(c+dx)\right)\log(c+dx) + 2\text{Li}_2\left(\frac{b(c+dx)}{bc-ad}\right)\right)\right)}{bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]
```

```
[Out] x*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 + (B*(2*a*d*Log[a + b*x]*(A + B*Lo
g[(e*(a + b*x))/(c + d*x)]) - 2*b*c*(A + B*Log[(e*(a + b*x))/(c + d*x)])*Lo
g[c + d*x] - a*B*d*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c -
 a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) + b*B*c*((2*Log[(d*(a
 + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c
+ d*x))/(b*c - a*d)])))/(b*d)
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \left(A + B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

```
[Out] int((A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="maxima")
```

```
[Out] 2*((a*e*log(b*x + a)/b - c*e*log(d*x + c)/d)*e^(-1) + x*log((b*x + a)*e/(d*
x + c)))*A*B + A^2*x + B^2*((b*d*x*log(b*x + a)^2 + (b*d*x + b*c)*log(d*x +
 c)^2 - 2*(b*d*x + (b*d*x + a*d)*log(b*x + a))*log(d*x + c))/(b*d) + integr
ate((3*b^2*d*x^2 + a*b*c + (b^2*c + 3*a*b*d)*x + 2*(b^2*d*x^2 + 3*a*b*d*x +
 a*b*c + a^2*d)*log(b*x + a))/(b^2*d*x^2 + a*b*c + (b^2*c + a*b*d)*x), x))
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="fricas")
```

```
[Out] integral(B^2*log((b*x + a)*e/(d*x + c))^2 + 2*A*B*log((b*x + a)*e/(d*x + c)
) + A^2, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(b*x+a)/(d*x+c)))**2,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="giac")
```

```
[Out] integrate((B*log((b*x + a)*e/(d*x + c)) + A)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(A+B\,\ln\left(\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right)\right)}^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x))/(c + d*x)))^2,x)
```

```
[Out] int((A + B*log((e*(a + b*x))/(c + d*x)))^2, x)
```

3.244 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{f+gx}\,dx$

Optimal. Leaf size=277

$$-\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{g}+\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\log\left(1-\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{g}-\frac{2B\left(A+B\log\left(\frac{e(a\right.\right.}{c-}$$

```
[Out] -ln((-a*d+b*c)/b/(d*x+c))*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/g+(A+B*ln(e*(b*x+a)
/(d*x+c)))^2*ln(1-(-c*g+d*f)*(b*x+a)/(-a*g+b*f)/(d*x+c))/g-2*B*(A+B*ln(e*(b
*x+a)/(d*x+c)))*polylog(2,d*(b*x+a)/b/(d*x+c))/g+2*B*(A+B*ln(e*(b*x+a)/(d*x
+c)))*polylog(2,(-c*g+d*f)*(b*x+a)/(-a*g+b*f)/(d*x+c))/g+2*B^2*polylog(3,d*
(b*x+a)/b/(d*x+c))/g-2*B^2*polylog(3,(-c*g+d*f)*(b*x+a)/(-a*g+b*f)/(d*x+c))
/g
```

Rubi [A]
time = 0.34, antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {2554, 2404, 2354, 2421, 6724}

$$\frac{2B\,\text{PolyLog}\left(2,\frac{(a+bx)(df-cg)}{(c+dx)(bf-ag)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{g}-\frac{2B\,\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{g}-\frac{2B^2\,\text{PolyLog}\left(3,\frac{(a+bx)(df-cg)}{(c+dx)(bf-ag)}\right)}{g}+\frac{2B^2\,\text{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)}{g}+\frac{\log\left(1-\frac{(a+bx)(df-cg)}{(c+dx)(bf-ag)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{g}-\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{g}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(f + g*x),x]
```

```
[Out] -((Log[(b*c - a*d)/(b*(c + d*x))]*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/g
) + ((A + B*Log[(e*(a + b*x))/(c + d*x)])^2*Log[1 - ((d*f - c*g)*(a + b*x))
/((b*f - a*g)*(c + d*x))])/g - (2*B*(A + B*Log[(e*(a + b*x))/(c + d*x)])*Po
lyLog[2, (d*(a + b*x))/(b*(c + d*x))])/g + (2*B*(A + B*Log[(e*(a + b*x))/(c
 + d*x)])*PolyLog[2, ((d*f - c*g)*(a + b*x))/((b*f - a*g)*(c + d*x))])/g +
(2*B^2*PolyLog[3, (d*(a + b*x))/(b*(c + d*x))])/g - (2*B^2*PolyLog[3, ((d*f
 - c*g)*(a + b*x))/((b*f - a*g)*(c + d*x))])/g
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2404

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2554

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[b*c - a*d, Su
bst[Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m +
2)), x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n
}, x] && EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegerQ[m] &
& IGtQ[p, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{f+gx}\,dx = \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\log(f+gx)}{g} - \frac{(2B)\int \frac{(c+dx)\left(-\frac{de(a+bx)}{(c+dx)^2}+\frac{be}{c+dx}\right)\left(A+B\log\left(\right.\right.}{e(a+bx)}}{g}$$

$$= \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\log(f+gx)}{g} - \frac{(2B)\int \frac{(c+dx)\left(-\frac{de(a+bx)}{(c+dx)^2}+\frac{be}{c+dx}\right)\left(A+B\log\left(\right.\right.}{a+bx}}{eg}$$

$$= \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\log(f+gx)}{g} - \frac{(2B)\int \frac{(bc-ad)e\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log(f+}{(a+bx)(c+dx)}}{eg}$$

$$= \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\log(f+gx)}{g} - \frac{(2B(bc-ad))\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log(}{(a+bx)(c+dx)}}{g}$$

$$= \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\log(f+gx)}{g} - \frac{(2B(bc-ad))\int \left(\frac{b\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)(a+bx)}\right.}{}$$

$$= \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\log(f+gx)}{g} - \frac{(2bB)\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log(f+gx)}{a+bx}\,dx}{g}$$

$$= \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\log(f+gx)}{g} - \frac{(2bB)\int \left(\frac{A\log(f+gx)}{a+bx}+\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)\log}{a+bx}\right.}{g}$$

$$= \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\log(f+gx)}{g} - \frac{(2AbB)\int \frac{\log(f+gx)}{a+bx}\,dx}{g} - \frac{(2bB^2)\int \frac{\log}{}}{}$$

$$= -\frac{2AB\log\left(-\frac{g(a+bx)}{bf-ag}\right)\log(f+gx)}{g} + \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\log(f+gx)}{g} +$$

$$= -\frac{2AB\log\left(-\frac{g(a+bx)}{bf-ag}\right)\log(f+gx)}{g} + \frac{2B^2\log\left(-\frac{g(a+bx)}{bf-ag}\right)\left(\log(a+bx)+\log\right.}{g}$$

$$= -\frac{B^2\log^2(a+bx)\log(f+gx)}{g} - \frac{2AB\log\left(-\frac{g(a+bx)}{bf-ag}\right)\log(f+gx)}{g} - \frac{B^2\log^2}{}$$

$$= -\frac{B^2\log^2(a+bx)\log(f+gx)}{g} - \frac{2AB\log\left(-\frac{g(a+bx)}{bf-ag}\right)\log(f+gx)}{g} - \frac{B^2\log^2}{}$$

$$= -\frac{B^2\log^2(a+bx)\log(f+gx)}{g} - \frac{2AB\log\left(-\frac{g(a+bx)}{bf-ag}\right)\log(f+gx)}{g} - \frac{B^2\log^2}{}$$

$$= -\frac{B^2\log^2(a+bx)\log(f+gx)}{g} - \frac{2AB\log\left(-\frac{g(a+bx)}{bf-ag}\right)\log(f+gx)}{g} - \frac{B^2\log^2}{}$$

Mathematica [A]
time = 0.48, size = 431, normalized size = 1.56

$$\frac{-B^2\log^2\left(\frac{e(a+bx)}{c+dx}\right)\log\left(\frac{bc-ad}{bc+bdx}\right)+A^2\log(f+gx)-2AB\log\left(\frac{a}{b}+x\right)\log(f+gx)+2AB\log\left(\frac{c}{d}+x\right)\log(f+gx)+2AB\log\left(\frac{e(a+bx)}{c+dx}\right)\log(f+gx)+2AB\log\left(\frac{a}{b}+x\right)\log\left(\frac{b(f+gx)}{bf-ag}\right)-2AB\log\left(\frac{c}{d}+x\right)\log\left(\frac{d(f+gx)}{df-cg}\right)+B^2\log^2\left(\frac{e(a+bx)}{c+dx}\right)\log\left(\frac{(bc-ad)(f+gx)}{(bf-ag)(c+dx)}\right)+2AB\text{Li}_2\left(\frac{g(a+bx)}{-bf+ag}\right)-2B^2\log\left(\frac{e(a+bx)}{c+dx}\right)\text{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)+2B^2\log\left(\frac{e(a+bx)}{c+dx}\right)\text{Li}_2\left(\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)-2AB\text{Li}_2\left(\frac{g(c+dx)}{-df+cg}\right)+2B^2\text{Li}_3\left(\frac{d(a+bx)}{b(c+dx)}\right)-2B^2\text{Li}_3\left(\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{g}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(f + g*x),x]
```

```
[Out] (-(B^2*Log[(e*(a + b*x))/(c + d*x)]^2*Log[(b*c - a*d)/(b*c + b*d*x)]) + A^2
*Log[f + g*x] - 2*A*B*Log[a/b + x]*Log[f + g*x] + 2*A*B*Log[c/d + x]*Log[f
+ g*x] + 2*A*B*Log[(e*(a + b*x))/(c + d*x)]*Log[f + g*x] + 2*A*B*Log[a/b +
x]*Log[(b*(f + g*x))/(b*f - a*g)] - 2*A*B*Log[c/d + x]*Log[(d*(f + g*x))/(d
*f - c*g)] + B^2*Log[(e*(a + b*x))/(c + d*x)]^2*Log[((b*c - a*d)*(f + g*x))
/((b*f - a*g)*(c + d*x))] + 2*A*B*PolyLog[2, (g*(a + b*x))/(-(b*f) + a*g)]
- 2*B^2*Log[(e*(a + b*x))/(c + d*x)]*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))
] + 2*B^2*Log[(e*(a + b*x))/(c + d*x)]*PolyLog[2, ((d*f - c*g)*(a + b*x))/(
(b*f - a*g)*(c + d*x))] - 2*A*B*PolyLog[2, (g*(c + d*x))/(-(d*f) + c*g)] +
2*B^2*PolyLog[3, (d*(a + b*x))/(b*(c + d*x))] - 2*B^2*PolyLog[3, ((d*f - c*
g)*(a + b*x))/((b*f - a*g)*(c + d*x))])/g
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1394 vs. $2(277) = 554$.
time = 2.15, size = 1395, normalized size = 5.04

method	result	size
derivativedivides	Expression too large to display	1395
default	Expression too large to display	1395
risch	Expression too large to display	2149

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*(b*x+a)/(d*x+c)))^2/(g*x+f),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d^2*e*(a*d-b*c)*(d^2*A^2/e/(a*d-b*c)/(-c*g+d*f)*ln(a*e*g-b*e*f-c*g*(b*e/
d+(a*d-b*c)*e/d/(d*x+c))+d*f*(b*e/d+(a*d-b*c)*e/d/(d*x+c)))*c-d^3*A^2/e/g/(
a*d-b*c)/(-c*g+d*f)*ln(a*e*g-b*e*f-c*g*(b*e/d+(a*d-b*c)*e/d/(d*x+c))+d*f*(b
*e/d+(a*d-b*c)*e/d/(d*x+c)))*f+d^2*A^2/e/g/(a*d-b*c)*ln(b*e-(b*e/d+(a*d-b*c
)*e/d/(d*x+c))*d)-d^2*B^2/e/g/(a*d-b*c)*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*l
n(1+(c*g-d*f)/(-a*e*g+b*e*f)*(b*e/d+(a*d-b*c)*e/d/(d*x+c)))-2*d^2*B^2/e/g/(
a*d-b*c)*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*polylog(2,-(c*g-d*f)/(-a*e*g+b*e*f
)*(b*e/d+(a*d-b*c)*e/d/(d*x+c)))+2*d^2*B^2/e/g/(a*d-b*c)*polylog(3,-(c*g-d*
f)/(-a*e*g+b*e*f)*(b*e/d+(a*d-b*c)*e/d/(d*x+c)))+d^2*B^2/e/g/(a*d-b*c)*ln(b
*e/d+(a*d-b*c)*e/d/(d*x+c))^2*ln(1-1/b/e*d*(b*e/d+(a*d-b*c)*e/d/(d*x+c)))+2
*d^2*B^2/e/g/(a*d-b*c)*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*polylog(2,1/b/e*d*(b
*e/d+(a*d-b*c)*e/d/(d*x+c)))-2*d^2*B^2/e/g/(a*d-b*c)*polylog(3,1/b/e*d*(b*e
```

```
/d+(a*d-b*c)*e/d/(d*x+c))-2*d^2*A*B/e/(a*d-b*c)*dilog(((c*g-d*f)*(b*e/d+(a
*d-b*c)*e/d/(d*x+c))-a*e*g+b*e*f)/(-a*e*g+b*e*f))/(c*g-d*f)*c+2*d^3*A*B/e/g
/(a*d-b*c)*dilog(((c*g-d*f)*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-a*e*g+b*e*f)/(-a*
e*g+b*e*f))/(c*g-d*f)*f-2*d^2*A*B/e/(a*d-b*c)*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c
))*ln(((c*g-d*f)*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-a*e*g+b*e*f)/(-a*e*g+b*e*f))
/(c*g-d*f)*c+2*d^3*A*B/e/g/(a*d-b*c)*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(((c
*g-d*f)*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-a*e*g+b*e*f)/(-a*e*g+b*e*f))/(c*g-d*f
)*f+2*d^2*A*B/e/g/(a*d-b*c)*dilog(-(-b*e+(b*e/d+(a*d-b*c)*e/d/(d*x+c))*d)/b
/e)+2*d^2*A*B/e/g/(a*d-b*c)*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(-(-b*e+(b*e/
d+(a*d-b*c)*e/d/(d*x+c))*d)/b/e))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(g*x+f),x, algorithm="maxima")
```

```
[Out] A^2*log(g*x + f)/g - integrate(-(B^2*log(b*x + a)^2 + 2*A*B + B^2 + 2*(A*B
+ B^2)*log(b*x + a) - 2*(B^2*log(b*x + a) + A*B + B^2)*log(d*x + c))/(g*x +
 f), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(g*x+f),x, algorithm="fricas")
```

```
[Out] integral((B^2*log((b*x + a)*e/(d*x + c))^2 + 2*A*B*log((b*x + a)*e/(d*x + c
)) + A^2)/(g*x + f), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(b*x+a)/(d*x+c)))**2/(g*x+f),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(g*x+f),x, algorithm="giac")
```

```
[Out] integrate((B*log((b*x + a)*e/(d*x + c)) + A)^2/(g*x + f), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2}{f+g\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x))/(c + d*x)))^2/(f + g*x),x)
```

```
[Out] int((A + B*log((e*(a + b*x))/(c + d*x)))^2/(f + g*x), x)
```

3.245 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(f+gx)^2}\,dx$

Optimal. Leaf size=196

$$\frac{(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bf-ag)(f+gx)}+\frac{2B(bc-ad)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log\left(1-\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{(bf-ag)(df-cg)}+\frac{2B^2(bc-ad)}{(bf-a}$$

```
[Out] (b*x+a)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(-a*g+b*f)/(g*x+f)+2*B*(-a*d+b*c)*(A+
B*ln(e*(b*x+a)/(d*x+c)))*ln(1-(-c*g+d*f)*(b*x+a)/(-a*g+b*f)/(d*x+c))/(-a*g+
b*f)/(-c*g+d*f)+2*B^2*(-a*d+b*c)*polylog(2,(-c*g+d*f)*(b*x+a)/(-a*g+b*f)/(d
*x+c))/(-a*g+b*f)/(-c*g+d*f)
```

Rubi [A]
time = 0.12, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {2554, 2355, 2354, 2438}

$$\frac{2B^2(bc-ad)\text{PolyLog}\left(2,\frac{(a+bx)(df-cg)}{(c+dx)(bf-ag)}\right)}{(bf-ag)(df-cg)}+\frac{2B(bc-ad)\log\left(1-\frac{(a+bx)(df-cg)}{(c+dx)(bf-ag)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{(bf-ag)(df-cg)}+\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{(f+gx)(bf-ag)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(f + g*x)^2,x]
```

```
[Out] ((a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/((b*f - a*g)*(f + g*x))
+ (2*B*(b*c - a*d)*(A + B*Log[(e*(a + b*x))/(c + d*x)])*Log[1 - ((d*f - c*g
)*(a + b*x))/((b*f - a*g)*(c + d*x))])/((b*f - a*g)*(d*f - c*g)) + (2*B^2*(
b*c - a*d)*PolyLog[2, ((d*f - c*g)*(a + b*x))/((b*f - a*g)*(c + d*x))])/((b
*f - a*g)*(d*f - c*g))
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2355

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[x*((a + b*Log[c*x^n])^p/(d*(d + e*x))), x] - Dist[b*n*(p/d),
Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n
, p}, x] && GtQ[p, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2554

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[b*c - a*d, Su
bst[Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m +
2)), x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n
}, x] && EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegerQ[m] &
& IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(f+gx)^2}\,dx = -\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{g(f+gx)} + \frac{(2B)\int \frac{(bc-ad)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)(c+dx)(f+gx)}\,dx}{g}$$

$$= -\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{g(f+gx)} + \frac{(2B(bc-ad))\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(a+bx)(c+dx)(f+gx)}\,dx}{g}$$

$$= -\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{g(f+gx)} + \frac{(2B(bc-ad))\int \left(\frac{b^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)(bf-ag)(a+bx)} + \frac{d^2\left(A+B\right.}{(bc-ad)(}\right.}{g}$$

$$= -\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{g(f+gx)} + \frac{(2b^2B)\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{a+bx}\,dx}{g(bf-ag)} - \frac{(2Bd^2)\int \frac{A+B\log}{c}}{g(df-}$$

$$= \frac{2bB\log(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{g(bf-ag)} - \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{g(f+gx)} - \frac{2Bd(}{}$$

$$= \frac{2bB\log(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{g(bf-ag)} - \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{g(f+gx)} - \frac{2Bd(}{}$$

$$= \frac{2bB\log(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{g(bf-ag)} - \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{g(f+gx)} - \frac{2Bd(}{}$$

$$= \frac{2bB\log(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{g(bf-ag)} - \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{g(f+gx)} - \frac{2Bd(}{}$$

$$= \frac{2bB\log(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{g(bf-ag)} - \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{g(f+gx)} + \frac{2B^2d}{}$$

$$= -\frac{bB^2\log^2(a+bx)}{g(bf-ag)} + \frac{2bB\log(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{g(bf-ag)} - \frac{\left(A+B\log\right.}{g(f}$$

$$= -\frac{bB^2\log^2(a+bx)}{g(bf-ag)} + \frac{2bB\log(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{g(bf-ag)} - \frac{\left(A+B\log\right.}{g(f}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 402 vs. 2(196) = 392.

time = 0.37, size = 402, normalized size = 2.05

$$\frac{-\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{f+gx}+\frac{B\left(2b(df-cg)\log(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)-2d(bf-ag)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log(c+dx)+2(bc-ad)g\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log(f+gx)-bB(df-cg)\left(\log(a+bx)\left(\log(a+bx)-2\log\left(\frac{b(c+dx)}{bc-ad}\right)\right)-2\text{Li}_2\left(\frac{d(a+bx)}{-bc+ad}\right)\right)+Bd(bf-ag)\left(\left(2\log\left(\frac{d(a+bx)}{-bc+ad}\right)-\log(c+dx)\right)\log(c+dx)+2\text{Li}_2\left(\frac{b(c+dx)}{bc-ad}\right)\right)-2B(bc-ad)g\left(\left(\log\left(\frac{g(a+bx)}{-bf+ag}\right)-\log\left(\frac{g(c+dx)}{-df+cg}\right)\right)\log(f+gx)+\text{Li}_2\left(\frac{b(f+gx)}{bf-ag}\right)-\text{Li}_2\left(\frac{d(f+gx)}{df-cg}\right)\right)\right)}{(bf-ag)(df-cg)}}{g}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(f + g*x)^2,x]
```

```
[Out] (-((A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(f + g*x)) + (B*(2*b*(d*f - c*g)*
Log[a + b*x]*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - 2*d*(b*f - a*g)*(A + B*
Log[(e*(a + b*x))/(c + d*x)])*Log[c + d*x] + 2*(b*c - a*d)*g*(A + B*Log[(e*
(a + b*x))/(c + d*x)])*Log[f + g*x] - b*B*(d*f - c*g)*(Log[a + b*x]*(Log[a
+ b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(
b*c) + a*d)]) + B*d*(b*f - a*g)*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log
[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]) - 2*B*(b
*c - a*d)*g*((Log[(g*(a + b*x))/(-(b*f) + a*g)] - Log[(g*(c + d*x))/(-(d*f)
 + c*g)])*Log[f + g*x] + PolyLog[2, (b*(f + g*x))/(b*f - a*g)] - PolyLog[2,
 (d*(f + g*x))/(d*f - c*g)])))/((b*f - a*g)*(d*f - c*g)))/g
```

Maple [F]
time = 0.19, size = 0, normalized size = 0.00

$$\int \frac{\left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2}{(gx+f)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*(b*x+a)/(d*x+c)))^2/(g*x+f)^2,x)
```

```
[Out] int((A+B*ln(e*(b*x+a)/(d*x+c)))^2/(g*x+f)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(g*x+f)^2,x, algorithm="maxima")
```

```
[Out] 2*A*B*(b*log(b*x + a)/(b*f*g - a*g^2) - d*log(d*x + c)/(d*f*g - c*g^2) + (b
*c - a*d)*log(g*x + f)/(b*d*f^2 + a*c*g^2 - (b*c + a*d)*f*g) - log(b*x*e/(d
*x + c) + a*e/(d*x + c))/(g^2*x + f*g)) - B^2*(log(d*x + c)^2/(g^2*x + f*g)
 + integrate(-(d*g*x + (d*g*x + c*g)*log(b*x + a)^2 + c*g + 2*(d*g*x + c*g)
*log(b*x + a) + 2*(d*f - c*g - (d*g*x + c*g)*log(b*x + a))*log(d*x + c))/(d
*g^3*x^3 + c*f^2*g + (2*d*f*g^2 + c*g^3)*x^2 + (d*f^2*g + 2*c*f*g^2)*x), x)
) - A^2/(g^2*x + f*g)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(g*x+f)^2,x, algorithm="fricas")

[Out] integral((B^2*log((b*x + a)*e/(d*x + c))^2 + 2*A*B*log((b*x + a)*e/(d*x + c
)) + A^2)/(g^2*x^2 + 2*f*g*x + f^2), x)

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*ln(e*(b*x+a)/(d*x+c)))**2/(g*x+f)**2,x)

[Out] Timed out

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(g*x+f)^2,x, algorithm="giac")

[Out] integrate((B*log((b*x + a)*e/(d*x + c)) + A)^2/(g*x + f)^2, x)

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2}{(f+g\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*log((e*(a + b*x))/(c + d*x)))^2/(f + g*x)^2,x)

[Out] int((A + B*log((e*(a + b*x))/(c + d*x)))^2/(f + g*x)^2, x)

3.246 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(f+gx)^3}\,dx$

Optimal. Leaf size=369

$$\frac{B(bc-ad)g(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bf-ag)^2(df-cg)(f+gx)}+\frac{b^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2g(bf-ag)^2}-\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2g(f+gx)^2}+\frac{B^2(bc-}{(bf-}$$

```
[Out] B*(-a*d+b*c)*g*(b*x+a)*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*g+b*f)^2/(-c*g+d*f)/
(g*x+f)+1/2*b^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/g/(-a*g+b*f)^2-1/2*(A+B*ln(e*
(b*x+a)/(d*x+c)))^2/g/(g*x+f)^2+B^2*(-a*d+b*c)^2*g*ln((g*x+f)/(d*x+c))/(-a*
g+b*f)^2/(-c*g+d*f)^2+B*(-a*d+b*c)*(-a*d*g-b*c*g+2*b*d*f)*(A+B*ln(e*(b*x+a)
/(d*x+c)))*ln(1-(-c*g+d*f)*(b*x+a)/(-a*g+b*f)/(d*x+c))/(-a*g+b*f)^2/(-c*g+d
*f)^2+B^2*(-a*d+b*c)*(-a*d*g-b*c*g+2*b*d*f)*polylog(2,(-c*g+d*f)*(b*x+a)/(-
a*g+b*f)/(d*x+c))/(-a*g+b*f)^2/(-c*g+d*f)^2
```

Rubi [A]
time = 0.52, antiderivative size = 369, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {2554, 2398, 2404, 2338, 2351, 31, 2354, 2438}

$$\frac{B^2(bc-ad)(-adg-bcg+2bdf)\text{PolyLog}\left(2,\frac{(a+bx)(df-cg)}{(c+dx)(bf-ag)}\right)}{(bf-ag)^2(df-cg)^2}+\frac{b^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{2g(bf-ag)^2}+\frac{Bg(a+bx)(bc-ad)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{(f+gx)(bf-ag)^2(df-cg)}+\frac{B(bc-ad)(-adg-bcg+2bdf)\log\left(1-\frac{(a+bx)(df-cg)}{(c+dx)(bf-ag)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{(bf-ag)^2(df-cg)^2}-\frac{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{2g(f+gx)^2}+\frac{B^2g(bc-ad)^2\log\left(\frac{f+gx}{c+dx}\right)}{(bf-ag)^2(df-cg)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(f + g*x)^3,x]
```

```
[Out] (B*(b*c - a*d)*g*(a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/((b*f - a*
g)^2*(d*f - c*g)*(f + g*x)) + (b^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/
(2*g*(b*f - a*g)^2) - (A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(2*g*(f + g*x)
^2) + (B^2*(b*c - a*d)^2*g*Log[(f + g*x)/(c + d*x)])/((b*f - a*g)^2*(d*f -
c*g)^2) + (B*(b*c - a*d)*(2*b*d*f - b*c*g - a*d*g)*(A + B*Log[(e*(a + b*x))
/(c + d*x)])*Log[1 - ((d*f - c*g)*(a + b*x))/((b*f - a*g)*(c + d*x))])/((b*
f - a*g)^2*(d*f - c*g)^2) + (B^2*(b*c - a*d)*(2*b*d*f - b*c*g - a*d*g)*Poly
Log[2, ((d*f - c*g)*(a + b*x))/((b*f - a*g)*(c + d*x))])/((b*f - a*g)^2*(d*
f - c*g)^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_)*((
f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(d + e*x)^(q +
 1)*((a + b*Log[c*x^n])^p/((q + 1)*(e*f - d*g))), x] - Dist[b*n*(p/((q + 1)
*(e*f - d*g))), Int[(f + g*x)^(m + 1)*(d + e*x)^(q + 1)*((a + b*Log[c*x^n])
^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, q}, x] && NeQ[e*f
- d*g, 0] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 2404

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2554

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[b*c - a*d, Su
bst[Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m +
2)), x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n
}, x] && EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegerQ[m] &
& IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(f+gx)^3}\,dx = -\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2g(f+gx)^2} + \frac{B\int \frac{(bc-ad)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)(c+dx)(f+gx)^2}\,dx}{g}$$

$$= -\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2g(f+gx)^2} + \frac{(B(bc-ad))\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(a+bx)(c+dx)(f+gx)^2}\,dx}{g}$$

$$= -\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2g(f+gx)^2} + \frac{(B(bc-ad))\int \left(\frac{b^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)(bf-ag)^2(a+bx)} - \frac{d^3\left(A+\right.}{(bc-ad)}\right.}{}$$

$$= -\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2g(f+gx)^2} + \frac{(b^3B)\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{a+bx}\,dx}{g(bf-ag)^2} - \frac{(Bd^3)\int \frac{A+B\log(}{c+}}{g(df-c}$$

$$= -\frac{B(bc-ad)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bf-ag)(df-cg)(f+gx)} + \frac{b^2B\log(a+bx)\left(A+B\log\left(\frac{e(a+bx}{c+dx}\right.\right.}{g(bf-ag)^2}$$

$$= -\frac{B(bc-ad)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bf-ag)(df-cg)(f+gx)} + \frac{b^2B\log(a+bx)\left(A+B\log\left(\frac{e(a+bx}{c+dx}\right.\right.}{g(bf-ag)^2}$$

$$= -\frac{B(bc-ad)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bf-ag)(df-cg)(f+gx)} + \frac{b^2B\log(a+bx)\left(A+B\log\left(\frac{e(a+bx}{c+dx}\right.\right.}{g(bf-ag)^2}$$

$$= \frac{bB^2(bc-ad)\log(a+bx)}{(bf-ag)^2(df-cg)} - \frac{B(bc-ad)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bf-ag)(df-cg)(f+gx)} + \frac{b^2B\log(}{}$$

$$= \frac{bB^2(bc-ad)\log(a+bx)}{(bf-ag)^2(df-cg)} - \frac{B(bc-ad)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bf-ag)(df-cg)(f+gx)} + \frac{b^2B\log(}{}$$

$$= \frac{bB^2(bc-ad)\log(a+bx)}{(bf-ag)^2(df-cg)} - \frac{b^2B^2\log^2(a+bx)}{2g(bf-ag)^2} - \frac{B(bc-ad)\left(A+B\log\left(\right.\right.}{(bf-ag)(df-cg)(f}$$

$$= \frac{bB^2(bc-ad)\log(a+bx)}{(bf-ag)^2(df-cg)} - \frac{b^2B^2\log^2(a+bx)}{2g(bf-ag)^2} - \frac{B(bc-ad)\left(A+B\log\left(\right.\right.}{(bf-ag)(df-cg)(f}$$

Mathematica [A]
time = 0.95, size = 595, normalized size = 1.61

$$-\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2+\frac{B(f+gx)\left(2(bc-ad)g(bf-ag)(df-cg)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)-2b^2(df-cg)^2(f+gx)\log(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)+2d^2(bf-ag)^2(f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log(c+dx)+2(bc-ad)g(-2bdf+bcg+adg)(f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log(f+gx)-2B(bc-ad)g(f+gx)(b(df-cg)\log(a+bx)+(-bdf+adg)\log(c+dx)+(bc-ad)g\log(f+gx))+b^2B(df-cg)^2(f+gx)\left(\log(a+bx)\left(\log(a+bx)-2\log\left(\frac{b(c+dx)}{bc-ad}\right)\right)-2\mathrm{Li}_2\left(\frac{d(a+bx)}{-bc+ad}\right)\right)-Bd^2(bf-ag)^2(f+gx)\left(\left(2\log\left(\frac{d(a+bx)}{-bc+ad}\right)-\log(c+dx)\right)\log(c+dx)+2\mathrm{Li}_2\left(\frac{b(c+dx)}{bc-ad}\right)\right)-2B(bc-ad)g(-2bdf+bcg+adg)(f+gx)\left(\left(\log\left(\frac{g(a+bx)}{-bf+ag}\right)-\log\left(\frac{g(c+dx)}{-df+cg}\right)\right)\log(f+gx)+\mathrm{Li}_2\left(\frac{b(f+gx)}{bf-ag}\right)-\mathrm{Li}_2\left(\frac{d(f+gx)}{df-cg}\right)\right)\right)}{(bf-ag)^2(df-cg)^2}}{2g(f+gx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(f + g*x)^3,x]
```

```
[Out] -1/2*((A + B*Log[(e*(a + b*x))/(c + d*x)])^2 + (B*(f + g*x)*(2*(b*c - a*d)*
g*(b*f - a*g)*(d*f - c*g)*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - 2*b^2*(d*f
 - c*g)^2*(f + g*x)*Log[a + b*x]*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 2*d
^2*(b*f - a*g)^2*(f + g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])*Log[c + d*x
] + 2*(b*c - a*d)*g*(-2*b*d*f + b*c*g + a*d*g)*(f + g*x)*(A + B*Log[(e*(a +
 b*x))/(c + d*x)])*Log[f + g*x] - 2*B*(b*c - a*d)*g*(f + g*x)*(b*(d*f - c*g
)*Log[a + b*x] + (-(b*d*f) + a*d*g)*Log[c + d*x] + (b*c - a*d)*g*Log[f + g*
x]) + b^2*B*(d*f - c*g)^2*(f + g*x)*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*
(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) - B*
d^2*(b*f - a*g)^2*(f + g*x)*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c +
 d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]) - 2*B*(b*c -
 a*d)*g*(-2*b*d*f + b*c*g + a*d*g)*(f + g*x)*((Log[(g*(a + b*x))/(-(b*f) +
a*g)] - Log[(g*(c + d*x))/(-(d*f) + c*g)])*Log[f + g*x] + PolyLog[2, (b*(f
+ g*x))/(b*f - a*g)] - PolyLog[2, (d*(f + g*x))/(d*f - c*g)])))/((b*f - a*g
)^2*(d*f - c*g)^2))/(g*(f + g*x)^2)
```

Maple [F]
time = 0.45, size = 0, normalized size = 0.00

$$\int \frac{\left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2}{(gx+f)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*(b*x+a)/(d*x+c)))^2/(g*x+f)^3,x)
```

```
[Out] int((A+B*ln(e*(b*x+a)/(d*x+c)))^2/(g*x+f)^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(g*x+f)^3,x, algorithm="maxima")
```

```
[Out] (b^2*log(b*x + a)/(b^2*f^2*g - 2*a*b*f*g^2 + a^2*g^3) - d^2*log(d*x + c)/(d
^2*f^2*g - 2*c*d*f*g^2 + c^2*g^3) + (2*(b^2*c*d - a*b*d^2)*f - (b^2*c^2 - a
^2*d^2)*g)*log(g*x + f)/(b^2*d^2*f^4 + a^2*c^2*g^4 - 2*(b^2*c*d + a*b*d^2)*
f^3*g + (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*f^2*g^2 - 2*(a*b*c^2 + a^2*c*d)*f*g
^3) - (b*c - a*d)/(b*d*f^3 + a*c*f*g^2 - (b*c + a*d)*f^2*g + (b*d*f^2*g + a
*c*g^3 - (b*c + a*d)*f*g^2)*x) - log(b*x*e/(d*x + c) + a*e/(d*x + c))/(g^3*
x^2 + 2*f*g^2*x + f^2*g))*A*B - 1/2*B^2*(log(d*x + c)^2/(g^3*x^2 + 2*f*g^2*
x + f^2*g) + 2*integrate(-(d*g*x + (d*g*x + c*g)*log(b*x + a)^2 + c*g + 2*(
d*g*x + c*g)*log(b*x + a) - (d*g*x - d*f + 2*c*g + 2*(d*g*x + c*g)*log(b*x
+ a))*log(d*x + c))/(d*g^4*x^4 + c*f^3*g + (3*d*f*g^3 + c*g^4)*x^3 + 3*(d*f
```

```
^2*g^2 + c*f*g^3)*x^2 + (d*f^3*g + 3*c*f^2*g^2)*x), x)) - 1/2*A^2/(g^3*x^2
+ 2*f*g^2*x + f^2*g)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(g*x+f)^3,x, algorithm="fricas")
[Out] integral((B^2*log((b*x + a)*e/(d*x + c))^2 + 2*A*B*log((b*x + a)*e/(d*x + c
)) + A^2)/(g^3*x^3 + 3*f*g^2*x^2 + 3*f^2*g*x + f^3), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(b*x+a)/(d*x+c)))**2/(g*x+f)**3,x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(g*x+f)^3,x, algorithm="giac")
[Out] integrate((B*log((b*x + a)*e/(d*x + c)) + A)^2/(g*x + f)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2}{{(f+g\,x)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x))/(c + d*x)))^2/(f + g*x)^3,x)
[Out] int((A + B*log((e*(a + b*x))/(c + d*x)))^2/(f + g*x)^3, x)
```

3.247 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(f+gx)^4}\,dx$

Optimal. Leaf size=714

$$\frac{B^2(bc-ad)^2g^2(c+dx)}{3(bf-ag)^2(df-cg)^3(f+gx)}+\frac{B^2(bc-ad)^3g^2\log\left(\frac{a+bx}{c+dx}\right)}{3(bf-ag)^3(df-cg)^3}-\frac{B(bc-ad)g^2(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3(bf-ag)(df-cg)^3(f+gx)^2}+$$

```
[Out] 1/3*B^2*(-a*d+b*c)^2*g^2*(d*x+c)/(-a*g+b*f)^2/(-c*g+d*f)^3/(g*x+f)+1/3*B^2*
(-a*d+b*c)^3*g^2*ln((b*x+a)/(d*x+c))/(-a*g+b*f)^3/(-c*g+d*f)^3-1/3*B*(-a*d+
b*c)*g^2*(d*x+c)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*g+b*f)/(-c*g+d*f)^3/(g*x
+f)^2+2/3*B*(-a*d+b*c)*g*(-2*a*d*g-b*c*g+3*b*d*f)*(b*x+a)*(A+B*ln(e*(b*x+a)
/(d*x+c)))/(-a*g+b*f)^3/(-c*g+d*f)^2/(g*x+f)+1/3*b^3*(A+B*ln(e*(b*x+a)/(d*x
+c)))^2/g/(-a*g+b*f)^3-1/3*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/g/(g*x+f)^3-1/3*B^
2*(-a*d+b*c)^3*g^2*ln((g*x+f)/(d*x+c))/(-a*g+b*f)^3/(-c*g+d*f)^3+2/3*B^2*(-
a*d+b*c)^2*g*(-2*a*d*g-b*c*g+3*b*d*f)*ln((g*x+f)/(d*x+c))/(-a*g+b*f)^3/(-c*
g+d*f)^3+2/3*B*(-a*d+b*c)*(a^2*d^2*g^2-a*b*d*g*(-c*g+3*d*f)+b^2*(c^2*g^2-3*
c*d*f*g+3*d^2*f^2))*(A+B*ln(e*(b*x+a)/(d*x+c)))*ln(1-(-c*g+d*f)*(b*x+a)/(-a
*g+b*f)/(d*x+c))/(-a*g+b*f)^3/(-c*g+d*f)^3+2/3*B^2*(-a*d+b*c)*(a^2*d^2*g^2-
a*b*d*g*(-c*g+3*d*f)+b^2*(c^2*g^2-3*c*d*f*g+3*d^2*f^2))*polylog(2,(-c*g+d*f
)*(b*x+a)/(-a*g+b*f)/(d*x+c))/(-a*g+b*f)^3/(-c*g+d*f)^3
```

Rubi [A]
time = 1.06, antiderivative size = 714, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.345$, Rules used = {2554, 2398, 2404, 2338, 2356, 46, 2351, 31, 2354, 2438}

$$\frac{2B^2(bc-ad)\left(a^2d^2g^2-abdg(3df-cg)+b^2(c^2g^2-3cdfg+3d^2f^2)\right)\text{PolyLog}\left(2,\frac{(a+bx)(df-cg)}{(c+dx)(bf-ag)}\right)}{3(bf-ag)^3(df-cg)^3}+\frac{2B(bc-ad)\left(a^2d^2g^2-abdg(3df-cg)+b^2(c^2g^2-3cdfg+3d^2f^2)\right)\log\left(1-\frac{(a+bx)(df-cg)}{(c+dx)(bf-ag)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3(bf-ag)^3(df-cg)^3}+\frac{b^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{3g(bf-ag)^3}-\frac{Bg^2(c+dx)^2(bc-ad)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3(f+gx)^2(bf-ag)(df-cg)^3}-\frac{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{3g(f+gx)^3}+\frac{2Bg(a+bx)(bc-ad)(-2adg-bcg+3bdf)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3(f+gx)(bf-ag)^3(df-cg)^2}+\frac{B^2g^2(c+dx)(bc-ad)^2}{3(f+gx)(bf-ag)^2(df-cg)^3}+\frac{B^2g^2(bc-ad)^3\log\left(\frac{a+bx}{c+dx}\right)}{3(bf-ag)^3(df-cg)^3}-\frac{B^2g^2(bc-ad)^3\log\left(\frac{f+gx}{c+dx}\right)}{3(bf-ag)^3(df-cg)^3}+\frac{2B^2g(bc-ad)^2(-2adg-bcg+3bdf)\log\left(\frac{f+gx}{c+dx}\right)}{3(bf-ag)^3(df-cg)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(f + g*x)^4,x]
```

```
[Out] (B^2*(b*c - a*d)^2*g^2*(c + d*x))/(3*(b*f - a*g)^2*(d*f - c*g)^3*(f + g*x))
 + (B^2*(b*c - a*d)^3*g^2*Log[(a + b*x)/(c + d*x)])/(3*(b*f - a*g)^3*(d*f -
 c*g)^3) - (B*(b*c - a*d)*g^2*(c + d*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x
)]))/(3*(b*f - a*g)*(d*f - c*g)^3*(f + g*x)^2) + (2*B*(b*c - a*d)*g*(3*b*d*
f - b*c*g - 2*a*d*g)*(a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(3*(b*
f - a*g)^3*(d*f - c*g)^2*(f + g*x)) + (b^3*(A + B*Log[(e*(a + b*x))/(c + d*
x)])^2)/(3*g*(b*f - a*g)^3) - (A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(3*g*(
f + g*x)^3) - (B^2*(b*c - a*d)^3*g^2*Log[(f + g*x)/(c + d*x)])/(3*(b*f - a*
g)^3*(d*f - c*g)^3) + (2*B^2*(b*c - a*d)^2*g*(3*b*d*f - b*c*g - 2*a*d*g)*Lo
g[(f + g*x)/(c + d*x)])/(3*(b*f - a*g)^3*(d*f - c*g)^3) + (2*B*(b*c - a*d)*
(a^2*d^2*g^2 - a*b*d*g*(3*d*f - c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^2
))*(A + B*Log[(e*(a + b*x))/(c + d*x)])*Log[1 - ((d*f - c*g)*(a + b*x))/((b
```

```
*f - a*g)*(c + d*x))])/(3*(b*f - a*g)^3*(d*f - c*g)^3) + (2*B^2*(b*c - a*d)
*(a^2*d^2*g^2 - a*b*d*g*(3*d*f - c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^
2))*PolyLog[2, ((d*f - c*g)*(a + b*x))/((b*f - a*g)*(c + d*x))])/(3*(b*f -
a*g)^3*(d*f - c*g)^3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_)*((
f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(d + e*x)^(q +
 1)*((a + b*Log[c*x^n])^p/((q + 1)*(e*f - d*g))), x] - Dist[b*n*(p/((q + 1)
*(e*f - d*g))), Int[(f + g*x)^(m + 1)*(d + e*x)^(q + 1)*((a + b*Log[c*x^n])
^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, q}, x] && NeQ[e*f
- d*g, 0] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 2404

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2554

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[b*c - a*d, Su
bst[Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m +
2)), x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n
}, x] && EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegerQ[m] &
& IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(f+gx)^4}\,dx = -\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3g(f+gx)^3}+\frac{(2B)\int\frac{(bc-ad)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)(c+dx)(f+gx)^3}\,dx}{3g}$$

$$= -\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3g(f+gx)^3}+\frac{(2B(bc-ad))\int\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(a+bx)(c+dx)(f+gx)^3}\,dx}{3g}$$

$$= -\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3g(f+gx)^3}+\frac{(2B(bc-ad))\int\left(\frac{b^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)(bf-ag)^3(a+bx)}+\frac{d^4\left(A\cdots\right.}{(bc-ad\cdots}\right.}{\cdots}$$

$$= -\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3g(f+gx)^3}+\frac{(2b^4B)\int\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{a+bx}\,dx}{3g(bf-ag)^3}-\frac{(2Bd^4)\int\frac{A+B\cdots}{\cdots}}{3g(df-\cdots}$$

$$= -\frac{B(bc-ad)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3(bf-ag)(df-cg)(f+gx)^2}-\frac{2B(bc-ad)(2bdf-bcg-adg)\left(A\cdots\right.}{3(bf-ag)^2(df-cg)^2\cdots}$$

$$= -\frac{B(bc-ad)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3(bf-ag)(df-cg)(f+gx)^2}-\frac{2B(bc-ad)(2bdf-bcg-adg)\left(A\cdots\right.}{3(bf-ag)^2(df-cg)^2\cdots}$$

$$= -\frac{B(bc-ad)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3(bf-ag)(df-cg)(f+gx)^2}-\frac{2B(bc-ad)(2bdf-bcg-adg)\left(A\cdots\right.}{3(bf-ag)^2(df-cg)^2\cdots}$$

$$= -\frac{B^2(bc-ad)^2g}{3(bf-ag)^2(df-cg)^2(f+gx)}+\frac{b^2B^2(bc-ad)\log(a+bx)}{3(bf-ag)^3(df-cg)}+\frac{2bB^2(bc\cdots}{\cdots}$$

$$= -\frac{B^2(bc-ad)^2g}{3(bf-ag)^2(df-cg)^2(f+gx)}+\frac{b^2B^2(bc-ad)\log(a+bx)}{3(bf-ag)^3(df-cg)}+\frac{2bB^2(bc\cdots}{\cdots}$$

$$= -\frac{B^2(bc-ad)^2g}{3(bf-ag)^2(df-cg)^2(f+gx)}+\frac{b^2B^2(bc-ad)\log(a+bx)}{3(bf-ag)^3(df-cg)}+\frac{2bB^2(bc\cdots}{\cdots}$$

$$= -\frac{B^2(bc-ad)^2g}{3(bf-ag)^2(df-cg)^2(f+gx)}+\frac{b^2B^2(bc-ad)\log(a+bx)}{3(bf-ag)^3(df-cg)}+\frac{2bB^2(bc\cdots}{\cdots}$$

Mathematica [A]
time = 1.98, size = 894, normalized size = 1.25

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(f + g*x)^4,x]
```

```
[Out] -1/3*((A + B*Log[(e*(a + b*x))/(c + d*x)])^2 + (B*(f + g*x)*((b*c - a*d)*g*
(b*f - a*g)^2*(d*f - c*g)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 2*(b*c -
 a*d)*g*(b*f - a*g)*(-(d*f) + c*g)*(-2*b*d*f + b*c*g + a*d*g)*(f + g*x)*(A
+ B*Log[(e*(a + b*x))/(c + d*x)]) - 2*b^3*(d*f - c*g)^3*(f + g*x)^2*Log[a +
 b*x]*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 2*d^3*(b*f - a*g)^3*(f + g*x)^
2*(A + B*Log[(e*(a + b*x))/(c + d*x)])*Log[c + d*x] - 2*(b*c - a*d)*g*(a^2*
d^2*g^2 + a*b*d*g*(-3*d*f + c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^2))*(
f + g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])*Log[f + g*x] - 2*B*(b*c - a
*d)*g*(2*b*d*f - b*c*g - a*d*g)*(f + g*x)^2*(b*(d*f - c*g)*Log[a + b*x] + (
-(b*d*f) + a*d*g)*Log[c + d*x] + (b*c - a*d)*g*Log[f + g*x]) + B*(b*c - a*d
)*g*(f + g*x)*((b*c - a*d)*g*(b*f - a*g)*(d*f - c*g) - b^2*(d*f - c*g)^2*(f
 + g*x)*Log[a + b*x] + d^2*(b*f - a*g)^2*(f + g*x)*Log[c + d*x] + (b*c - a*
d)*g*(-2*b*d*f + b*c*g + a*d*g)*(f + g*x)*Log[f + g*x]) + b^3*B*(d*f - c*g)
^3*(f + g*x)^2*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d
)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) - B*d^3*(b*f - a*g)^3*(f
+ g*x)^2*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x]
 + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]) + 2*B*(b*c - a*d)*g*(a^2*d^2*g^
2 + a*b*d*g*(-3*d*f + c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^2))*(f + g*
x)^2*((Log[(g*(a + b*x))/(-(b*f) + a*g)] - Log[(g*(c + d*x))/(-(d*f) + c*g)
])*Log[f + g*x] + PolyLog[2, (b*(f + g*x))/(b*f - a*g)] - PolyLog[2, (d*(f
+ g*x))/(d*f - c*g)])))/((b*f - a*g)^3*(d*f - c*g)^3))/(g*(f + g*x)^3)
```

Maple [F]
time = 0.81, size = 0, normalized size = 0.00

$$\int \frac{\left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^{2}}{\left(gx+f\right)^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*(b*x+a)/(d*x+c)))^2/(g*x+f)^4,x)
```

```
[Out] int((A+B*ln(e*(b*x+a)/(d*x+c)))^2/(g*x+f)^4,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(g*x+f)^4,x, algorithm="maxima")
```

```
[Out] 1/3*(2*b^3*log(b*x + a)/(b^3*f^3*g - 3*a*b^2*f^2*g^2 + 3*a^2*b*f*g^3 - a^3*
g^4) - 2*d^3*log(d*x + c)/(d^3*f^3*g - 3*c*d^2*f^2*g^2 + 3*c^2*d*f*g^3 - c^
3*g^4) + 2*(3*(b^3*c*d^2 - a*b^2*d^3)*f^2 - 3*(b^3*c^2*d - a^2*b*d^3)*f*g +
 (b^3*c^3 - a^3*d^3)*g^2)*log(g*x + f)/(b^3*d^3*f^6 + a^3*c^3*g^6 - 3*(b^3*
```

```
c*d^2 + a*b^2*d^3)*f^5*g + 3*(b^3*c^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3)*f^4*g^
2 - (b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*f^3*g^3 + 3*(a*b^2*
c^3 + 3*a^2*b*c^2*d + a^3*c*d^2)*f^2*g^4 - 3*(a^2*b*c^3 + a^3*c^2*d)*f*g^5)
 - (5*(b^2*c*d - a*b*d^2)*f^2 - 3*(b^2*c^2 - a^2*d^2)*f*g + (a*b*c^2 - a^2*
c*d)*g^2 + 2*(2*(b^2*c*d - a*b*d^2)*f*g - (b^2*c^2 - a^2*d^2)*g^2)*x)/(b^2*
d^2*f^6 + a^2*c^2*f^2*g^4 - 2*(b^2*c*d + a*b*d^2)*f^5*g + (b^2*c^2 + 4*a*b*
c*d + a^2*d^2)*f^4*g^2 - 2*(a*b*c^2 + a^2*c*d)*f^3*g^3 + (b^2*d^2*f^4*g^2 +
 a^2*c^2*g^6 - 2*(b^2*c*d + a*b*d^2)*f^3*g^3 + (b^2*c^2 + 4*a*b*c*d + a^2*d
^2)*f^2*g^4 - 2*(a*b*c^2 + a^2*c*d)*f*g^5)*x^2 + 2*(b^2*d^2*f^5*g + a^2*c^2
*f*g^5 - 2*(b^2*c*d + a*b*d^2)*f^4*g^2 + (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*f^
3*g^3 - 2*(a*b*c^2 + a^2*c*d)*f^2*g^4)*x) - 2*log(b*x*e/(d*x + c) + a*e/(d*
x + c))/(g^4*x^3 + 3*f*g^3*x^2 + 3*f^2*g^2*x + f^3*g))*A*B - 1/3*B^2*(log(d
*x + c)^2/(g^4*x^3 + 3*f*g^3*x^2 + 3*f^2*g^2*x + f^3*g) + 3*integrate(-1/3*
(3*d*g*x + 3*(d*g*x + c*g)*log(b*x + a)^2 + 3*c*g + 6*(d*g*x + c*g)*log(b*x
 + a) - 2*(2*d*g*x - d*f + 3*c*g + 3*(d*g*x + c*g)*log(b*x + a))*log(d*x +
c))/(d*g^5*x^5 + c*f^4*g + (4*d*f*g^4 + c*g^5)*x^4 + 2*(3*d*f^2*g^3 + 2*c*f
*g^4)*x^3 + 2*(2*d*f^3*g^2 + 3*c*f^2*g^3)*x^2 + (d*f^4*g + 4*c*f^3*g^2)*x),
 x)) - 1/3*A^2/(g^4*x^3 + 3*f*g^3*x^2 + 3*f^2*g^2*x + f^3*g)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(g*x+f)^4,x, algorithm="fricas")
```

```
[Out] integral((B^2*log((b*x + a)*e/(d*x + c))^2 + 2*A*B*log((b*x + a)*e/(d*x + c
)) + A^2)/(g^4*x^4 + 4*f*g^3*x^3 + 6*f^2*g^2*x^2 + 4*f^3*g*x + f^4), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(b*x+a)/(d*x+c)))**2/(g*x+f)**4,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(g*x+f)^4,x, algorithm="giac")
```

```
[Out] integrate((B*log((b*x + a)*e/(d*x + c)) + A)^2/(g*x + f)^4, x)
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2}{(f+g\,x)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x))/(c + d*x)))^2/(f + g*x)^4,x)
```

```
[Out] int((A + B*log((e*(a + b*x))/(c + d*x)))^2/(f + g*x)^4, x)
```

3.248 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(f+gx)^5}\,dx$

Optimal. Leaf size=1159

$$-\frac{B^2(bc-ad)^2g^3(c+dx)^2}{12(bf-ag)^2(df-cg)^4(f+gx)^2}-\frac{B^2(bc-ad)^3g^3(c+dx)}{6(bf-ag)^3(df-cg)^4(f+gx)}+\frac{B^2(bc-ad)^2g^2(4bdf-bcg-3adg)(c}{4(bf-ag)^3(df-cg)^4(f+gx)}$$

[Out] -1/12*B^2*(-a*d+b*c)^2*g^3*(d*x+c)^2/(-a*g+b*f)^2/(-c*g+d*f)^4/(g*x+f)^2-1/6*B^2*(-a*d+b*c)^3*g^3*(d*x+c)/(-a*g+b*f)^3/(-c*g+d*f)^4/(g*x+f)+1/4*B^2*(-a*d+b*c)^2*g^2*(-3*a*d*g-b*c*g+4*b*d*f)*(d*x+c)/(-a*g+b*f)^3/(-c*g+d*f)^4/(g*x+f)-1/6*B^2*(-a*d+b*c)^4*g^3*ln((b*x+a)/(d*x+c))/(-a*g+b*f)^4/(-c*g+d*f)^4+1/4*B^2*(-a*d+b*c)^3*g^2*(-3*a*d*g-b*c*g+4*b*d*f)*ln((b*x+a)/(d*x+c))/(-a*g+b*f)^4/(-c*g+d*f)^4+1/6*B*(-a*d+b*c)*g^3*(d*x+c)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*g+b*f)/(-c*g+d*f)^4/(g*x+f)^3-1/4*B*(-a*d+b*c)*g^2*(-3*a*d*g-b*c*g+4*b*d*f)*(d*x+c)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*g+b*f)^2/(-c*g+d*f)^4/(g*x+f)^2+1/2*B*(-a*d+b*c)*g*(3*a^2*d^2*g^2-2*a*b*d*g*(-c*g+4*d*f)+b^2*(c^2*g^2-4*c*d*f*g+6*d^2*f^2))*(b*x+a)*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*g+b*f)^4/(-c*g+d*f)^3/(g*x+f)+1/4*b^4*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/g/(-a*g+b*f)^4-1/4*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/g/(g*x+f)^4+1/6*B^2*(-a*d+b*c)^4*g^3*ln((g*x+f)/(d*x+c))/(-a*g+b*f)^4/(-c*g+d*f)^4-1/4*B^2*(-a*d+b*c)^3*g^2*(-3*a*d*g-b*c*g+4*b*d*f)*ln((g*x+f)/(d*x+c))/(-a*g+b*f)^4/(-c*g+d*f)^4+1/2*B^2*(-a*d+b*c)^2*g*(3*a^2*d^2*g^2-2*a*b*d*g*(-c*g+4*d*f)+b^2*(c^2*g^2-4*c*d*f*g+6*d^2*f^2))*ln((g*x+f)/(d*x+c))/(-a*g+b*f)^4/(-c*g+d*f)^4-1/2*B*(-a*d+b*c)*(-a*d*g-b*c*g+2*b*d*f)*(2*a*b*d^2*f*g-a^2*d^2*g^2-b^2*(c^2*g^2-2*c*d*f*g+2*d^2*f^2))*(A+B*ln(e*(b*x+a)/(d*x+c)))*ln(1-(-c*g+d*f)*(b*x+a)/(-a*g+b*f)/(d*x+c))/(-a*g+b*f)^4/(-c*g+d*f)^4-1/2*B^2*(-a*d+b*c)*(-a*d*g-b*c*g+2*b*d*f)*(2*a*b*d^2*f*g-a^2*d^2*g^2-b^2*(c^2*g^2-2*c*d*f*g+2*d^2*f^2))*polylog(2,(-c*g+d*f)*(b*x+a)/(-a*g+b*f)/(d*x+c))/(-a*g+b*f)^4/(-c*g+d*f)^4

Rubi [A]

time = 1.67, antiderivative size = 1159, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.345$, Rules used = {2554, 2398, 2404, 2338, 2356, 46, 2351, 31, 2354, 2438}

Antiderivative was successfully verified.

[In] Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(f + g*x)^5,x]

[Out] -1/12*(B^2*(b*c - a*d)^2*g^3*(c + d*x)^2)/((b*f - a*g)^2*(d*f - c*g)^4*(f + g*x)^2) - (B^2*(b*c - a*d)^3*g^3*(c + d*x))/(6*(b*f - a*g)^3*(d*f - c*g)^4*(f + g*x)) + (B^2*(b*c - a*d)^2*g^2*(4*b*d*f - b*c*g - 3*a*d*g)*(c + d*x))/(4*(b*f - a*g)^3*(d*f - c*g)^4*(f + g*x)) - (B^2*(b*c - a*d)^4*g^3*Log[(a

```
+ b*x)/(c + d*x)])/(6*(b*f - a*g)^4*(d*f - c*g)^4) + (B^2*(b*c - a*d)^3*g^2
*(4*b*d*f - b*c*g - 3*a*d*g)*Log[(a + b*x)/(c + d*x)])/(4*(b*f - a*g)^4*(d*
f - c*g)^4) + (B*(b*c - a*d)*g^3*(c + d*x)^3*(A + B*Log[(e*(a + b*x))/(c +
d*x)]))/(6*(b*f - a*g)*(d*f - c*g)^4*(f + g*x)^3) - (B*(b*c - a*d)*g^2*(4*b
*d*f - b*c*g - 3*a*d*g)*(c + d*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(
4*(b*f - a*g)^2*(d*f - c*g)^4*(f + g*x)^2) + (B*(b*c - a*d)*g*(3*a^2*d^2*g^
2 - 2*a*b*d*g*(4*d*f - c*g) + b^2*(6*d^2*f^2 - 4*c*d*f*g + c^2*g^2))*(a + b
*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(2*(b*f - a*g)^4*(d*f - c*g)^3*(f
 + g*x)) + (b^4*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(4*g*(b*f - a*g)^4)
 - (A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(4*g*(f + g*x)^4) + (B^2*(b*c - a
*d)^4*g^3*Log[(f + g*x)/(c + d*x)])/(6*(b*f - a*g)^4*(d*f - c*g)^4) - (B^2*
(b*c - a*d)^3*g^2*(4*b*d*f - b*c*g - 3*a*d*g)*Log[(f + g*x)/(c + d*x)])/(4*
(b*f - a*g)^4*(d*f - c*g)^4) + (B^2*(b*c - a*d)^2*g*(3*a^2*d^2*g^2 - 2*a*b*
d*g*(4*d*f - c*g) + b^2*(6*d^2*f^2 - 4*c*d*f*g + c^2*g^2))*Log[(f + g*x)/(c
 + d*x)])/(2*(b*f - a*g)^4*(d*f - c*g)^4) - (B*(b*c - a*d)*(2*b*d*f - b*c*g
 - a*d*g)*(2*a*b*d^2*f*g - a^2*d^2*g^2 - b^2*(2*d^2*f^2 - 2*c*d*f*g + c^2*g
^2))*(A + B*Log[(e*(a + b*x))/(c + d*x)])*Log[1 - ((d*f - c*g)*(a + b*x))/(
(b*f - a*g)*(c + d*x))])/(2*(b*f - a*g)^4*(d*f - c*g)^4) - (B^2*(b*c - a*d)
*(2*b*d*f - b*c*g - a*d*g)*(2*a*b*d^2*f*g - a^2*d^2*g^2 - b^2*(2*d^2*f^2 -
2*c*d*f*g + c^2*g^2))*PolyLog[2, ((d*f - c*g)*(a + b*x))/((b*f - a*g)*(c +
d*x))])/(2*(b*f - a*g)^4*(d*f - c*g)^4)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_)*((
f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(d + e*x)^(q +
 1)*((a + b*Log[c*x^n])^p/((q + 1)*(e*f - d*g))), x] - Dist[b*n*(p/((q + 1)
*(e*f - d*g))), Int[(f + g*x)^(m + 1)*(d + e*x)^(q + 1)*((a + b*Log[c*x^n])
^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, q}, x] && NeQ[e*f
- d*g, 0] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 2404

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2554

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[b*c - a*d, Su
bst[Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m +
2)), x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n
}, x] && EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegerQ[m] &
& IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(f+gx)^5}\,dx = -\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{4g(f+gx)^4}+\frac{B\int\frac{(bc-ad)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)(c+dx)(f+gx)^4}\,dx}{2g}$$

$$= -\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{4g(f+gx)^4}+\frac{(B(bc-ad))\int\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(a+bx)(c+dx)(f+gx)^4}\,dx}{2g}$$

$$= -\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{4g(f+gx)^4}+\frac{(B(bc-ad))\int\left(\frac{b^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)(bf-ag)^4(a+bx)}-\frac{d^5\left(A+B\right.}{(bc-ad)(-}\right.}{}$$

$$= -\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{4g(f+gx)^4}+\frac{(b^5B)\int\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{a+bx}\,dx}{2g(bf-ag)^4}-\frac{(Bd^5)\int\frac{A+B\log\left(\frac{e}{}\right.}{c+dx}}{2g(df-cg}$$

$$= -\frac{B(bc-ad)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{6(bf-ag)(df-cg)(f+gx)^3}-\frac{B(bc-ad)(2bdf-bcg-adg)\left(A+\right.}{4(bf-ag)^2(df-cg)^2(f}$$

$$= -\frac{B(bc-ad)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{6(bf-ag)(df-cg)(f+gx)^3}-\frac{B(bc-ad)(2bdf-bcg-adg)\left(A+\right.}{4(bf-ag)^2(df-cg)^2(f}$$

$$= -\frac{B(bc-ad)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{6(bf-ag)(df-cg)(f+gx)^3}-\frac{B(bc-ad)(2bdf-bcg-adg)\left(A+\right.}{4(bf-ag)^2(df-cg)^2(f}$$

$$= -\frac{B^2(bc-ad)^2g}{12(bf-ag)^2(df-cg)^2(f+gx)^2}-\frac{5B^2(bc-ad)^2g(2bdf-bcg-adg)}{12(bf-ag)^3(df-cg)^3(f+gx)}+$$

$$= -\frac{B^2(bc-ad)^2g}{12(bf-ag)^2(df-cg)^2(f+gx)^2}-\frac{5B^2(bc-ad)^2g(2bdf-bcg-adg)}{12(bf-ag)^3(df-cg)^3(f+gx)}+$$

$$= -\frac{B^2(bc-ad)^2g}{12(bf-ag)^2(df-cg)^2(f+gx)^2}-\frac{5B^2(bc-ad)^2g(2bdf-bcg-adg)}{12(bf-ag)^3(df-cg)^3(f+gx)}+$$

$$= -\frac{B^2(bc-ad)^2g}{12(bf-ag)^2(df-cg)^2(f+gx)^2}-\frac{5B^2(bc-ad)^2g(2bdf-bcg-adg)}{12(bf-ag)^3(df-cg)^3(f+gx)}+$$

__Mathematica__ [A]
time = 4.82, size = 1301, normalized size = 1.12

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(f + g*x)^5,x]
```

```
[Out] -1/12*(3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 + (B*(f + g*x)*(2*(b*c - a*
d)*g*(b*f - a*g)^3*(d*f - c*g)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - 3*(
b*c - a*d)*g*(b*f - a*g)^2*(d*f - c*g)^2*(-2*b*d*f + b*c*g + a*d*g)*(f + g*
x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 6*(b*c - a*d)*g*(b*f - a*g)*(d*f
- c*g)*(a^2*d^2*g^2 + a*b*d*g*(-3*d*f + c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g +
 c^2*g^2))*(f + g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - 6*b^4*(d*f -
c*g)^4*(f + g*x)^3*Log[a + b*x]*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 6*d^
4*(b*f - a*g)^4*(f + g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])*Log[c + d*
x] + 6*(b*c - a*d)*g*(-2*b*d*f + b*c*g + a*d*g)*(-2*a*b*d^2*f*g + a^2*d^2*g
^2 + b^2*(2*d^2*f^2 - 2*c*d*f*g + c^2*g^2))*(f + g*x)^3*(A + B*Log[(e*(a +
b*x))/(c + d*x)])*Log[f + g*x] - 6*B*(b*c - a*d)*g*(a^2*d^2*g^2 + a*b*d*g*(
-3*d*f + c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^2))*(f + g*x)^3*(b*(d*f
- c*g)*Log[a + b*x] + (-(b*d*f) + a*d*g)*Log[c + d*x] + (b*c - a*d)*g*Log[f
 + g*x]) + 3*B*(b*c - a*d)*g*(2*b*d*f - b*c*g - a*d*g)*(f + g*x)^2*((b*c -
a*d)*g*(b*f - a*g)*(d*f - c*g) - b^2*(d*f - c*g)^2*(f + g*x)*Log[a + b*x] +
 d^2*(b*f - a*g)^2*(f + g*x)*Log[c + d*x] + (b*c - a*d)*g*(-2*b*d*f + b*c*g
 + a*d*g)*(f + g*x)*Log[f + g*x]) + B*(b*c - a*d)*g*(f + g*x)*((b*c - a*d)*
g*(b*f - a*g)^2*(d*f - c*g)^2 + 2*(b*c - a*d)*g*(b*f - a*g)*(-(d*f) + c*g)*
(-2*b*d*f + b*c*g + a*d*g)*(f + g*x) - 2*b^3*(d*f - c*g)^3*(f + g*x)^2*Log[
a + b*x] + 2*d^3*(b*f - a*g)^3*(f + g*x)^2*Log[c + d*x] - 2*(b*c - a*d)*g*(
a^2*d^2*g^2 + a*b*d*g*(-3*d*f + c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^2
))*(f + g*x)^2*Log[f + g*x]) + 3*b^4*B*(d*f - c*g)^4*(f + g*x)^3*(Log[a + b
*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a
+ b*x))/(-(b*c) + a*d)]) - 3*B*d^4*(b*f - a*g)^4*(f + g*x)^3*((2*Log[(d*(a
+ b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c +
 d*x))/(b*c - a*d)]) - 6*B*(b*c - a*d)*g*(-2*b*d*f + b*c*g + a*d*g)*(-2*a*b
*d^2*f*g + a^2*d^2*g^2 + b^2*(2*d^2*f^2 - 2*c*d*f*g + c^2*g^2))*(f + g*x)^3
*((Log[(g*(a + b*x))/(-(b*f) + a*g)] - Log[(g*(c + d*x))/(-(d*f) + c*g)])*L
og[f + g*x] + PolyLog[2, (b*(f + g*x))/(b*f - a*g)] - PolyLog[2, (d*(f + g*
x))/(d*f - c*g)])))/((b*f - a*g)^4*(d*f - c*g)^4))/(g*(f + g*x)^4)
```

Maple [F]
time = 1.45, size = 0, normalized size = 0.00

$$\int \frac{\left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^{2}}{(gx+f)^{5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*(b*x+a)/(d*x+c)))^2/(g*x+f)^5,x)
```

```
[Out] int((A+B*ln(e*(b*x+a)/(d*x+c)))^2/(g*x+f)^5,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(g*x+f)^5,x, algorithm="maxima")
```

```
[Out] 1/12*(6*b^4*log(b*x + a)/(b^4*f^4*g - 4*a*b^3*f^3*g^2 + 6*a^2*b^2*f^2*g^3 -
 4*a^3*b*f*g^4 + a^4*g^5) - 6*d^4*log(d*x + c)/(d^4*f^4*g - 4*c*d^3*f^3*g^2
 + 6*c^2*d^2*f^2*g^3 - 4*c^3*d*f*g^4 + c^4*g^5) + 6*(4*(b^4*c*d^3 - a*b^3*d
^4)*f^3 - 6*(b^4*c^2*d^2 - a^2*b^2*d^4)*f^2*g + 4*(b^4*c^3*d - a^3*b*d^4)*f
*g^2 - (b^4*c^4 - a^4*d^4)*g^3)*log(g*x + f)/(b^4*d^4*f^8 + a^4*c^4*g^8 - 4
*(b^4*c*d^3 + a*b^3*d^4)*f^7*g + 2*(3*b^4*c^2*d^2 + 8*a*b^3*c*d^3 + 3*a^2*b
^2*d^4)*f^6*g^2 - 4*(b^4*c^3*d + 6*a*b^3*c^2*d^2 + 6*a^2*b^2*c*d^3 + a^3*b*
d^4)*f^5*g^3 + (b^4*c^4 + 16*a*b^3*c^3*d + 36*a^2*b^2*c^2*d^2 + 16*a^3*b*c*
d^3 + a^4*d^4)*f^4*g^4 - 4*(a*b^3*c^4 + 6*a^2*b^2*c^3*d + 6*a^3*b*c^2*d^2 +
 a^4*c*d^3)*f^3*g^5 + 2*(3*a^2*b^2*c^4 + 8*a^3*b*c^3*d + 3*a^4*c^2*d^2)*f^2
*g^6 - 4*(a^3*b*c^4 + a^4*c^3*d)*f*g^7) - (26*(b^3*c*d^2 - a*b^2*d^3)*f^4 -
 31*(b^3*c^2*d - a^2*b*d^3)*f^3*g + (11*b^3*c^3 + 15*a*b^2*c^2*d - 15*a^2*b
*c*d^2 - 11*a^3*d^3)*f^2*g^2 - 7*(a*b^2*c^3 - a^3*c*d^2)*f*g^3 + 2*(a^2*b*c
^3 - a^3*c^2*d)*g^4 + 6*(3*(b^3*c*d^2 - a*b^2*d^3)*f^2*g^2 - 3*(b^3*c^2*d -
 a^2*b*d^3)*f*g^3 + (b^3*c^3 - a^3*d^3)*g^4)*x^2 + 3*(14*(b^3*c*d^2 - a*b^2
*d^3)*f^3*g - 15*(b^3*c^2*d - a^2*b*d^3)*f^2*g^2 + (5*b^3*c^3 + 3*a*b^2*c^2
*d - 3*a^2*b*c*d^2 - 5*a^3*d^3)*f*g^3 - (a*b^2*c^3 - a^3*c*d^2)*g^4)*x)/(b^
3*d^3*f^9 + a^3*c^3*f^3*g^6 - 3*(b^3*c*d^2 + a*b^2*d^3)*f^8*g + 3*(b^3*c^2*
d + 3*a*b^2*c*d^2 + a^2*b*d^3)*f^7*g^2 - (b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2*b
*c*d^2 + a^3*d^3)*f^6*g^3 + 3*(a*b^2*c^3 + 3*a^2*b*c^2*d + a^3*c*d^2)*f^5*g
^4 - 3*(a^2*b*c^3 + a^3*c^2*d)*f^4*g^5 + (b^3*d^3*f^6*g^3 + a^3*c^3*g^9 - 3
*(b^3*c*d^2 + a*b^2*d^3)*f^5*g^4 + 3*(b^3*c^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3
)*f^4*g^5 - (b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*f^3*g^6 + 3
*(a*b^2*c^3 + 3*a^2*b*c^2*d + a^3*c*d^2)*f^2*g^7 - 3*(a^2*b*c^3 + a^3*c^2*d
)*f*g^8)*x^3 + 3*(b^3*d^3*f^7*g^2 + a^3*c^3*f*g^8 - 3*(b^3*c*d^2 + a*b^2*d^
3)*f^6*g^3 + 3*(b^3*c^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3)*f^5*g^4 - (b^3*c^3 +
 9*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*f^4*g^5 + 3*(a*b^2*c^3 + 3*a^2*b*
c^2*d + a^3*c*d^2)*f^3*g^6 - 3*(a^2*b*c^3 + a^3*c^2*d)*f^2*g^7)*x^2 + 3*(b^
3*d^3*f^8*g + a^3*c^3*f^2*g^7 - 3*(b^3*c*d^2 + a*b^2*d^3)*f^7*g^2 + 3*(b^3*
c^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3)*f^6*g^3 - (b^3*c^3 + 9*a*b^2*c^2*d + 9*a
^2*b*c*d^2 + a^3*d^3)*f^5*g^4 + 3*(a*b^2*c^3 + 3*a^2*b*c^2*d + a^3*c*d^2)*f
^4*g^5 - 3*(a^2*b*c^3 + a^3*c^2*d)*f^3*g^6)*x) - 6*log(b*x*e/(d*x + c) + a*
e/(d*x + c))/(g^5*x^4 + 4*f*g^4*x^3 + 6*f^2*g^3*x^2 + 4*f^3*g^2*x + f^4*g))
*A*B - 1/4*B^2*(log(d*x + c)^2/(g^5*x^4 + 4*f*g^4*x^3 + 6*f^2*g^3*x^2 + 4*f
^3*g^2*x + f^4*g) + 4*integrate(-1/2*(2*d*g*x + 2*(d*g*x + c*g)*log(b*x + a
)^2 + 2*c*g + 4*(d*g*x + c*g)*log(b*x + a) - (3*d*g*x - d*f + 4*c*g + 4*(d*
g*x + c*g)*log(b*x + a))*log(d*x + c))/(d*g^6*x^6 + c*f^5*g + (5*d*f*g^5 +
c*g^6)*x^5 + 5*(2*d*f^2*g^4 + c*f*g^5)*x^4 + 10*(d*f^3*g^3 + c*f^2*g^4)*x^3
 + 5*(d*f^4*g^2 + 2*c*f^3*g^3)*x^2 + (d*f^5*g + 5*c*f^4*g^2)*x), x)) - 1/4*
A^2/(g^5*x^4 + 4*f*g^4*x^3 + 6*f^2*g^3*x^2 + 4*f^3*g^2*x + f^4*g)
```

Fricas [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(g*x+f)^5,x, algorithm="fricas")
```

```
[Out] integral((B^2*log((b*x + a)*e/(d*x + c))^2 + 2*A*B*log((b*x + a)*e/(d*x + c
)) + A^2)/(g^5*x^5 + 5*f*g^4*x^4 + 10*f^2*g^3*x^3 + 10*f^3*g^2*x^2 + 5*f^4*
g*x + f^5), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(b*x+a)/(d*x+c)))**2/(g*x+f)**5,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(g*x+f)^5,x, algorithm="giac")
```

```
[Out] integrate((B*log((b*x + a)*e/(d*x + c)) + A)^2/(g*x + f)^5, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2}{(f+g\,x)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x))/(c + d*x)))^2/(f + g*x)^5,x)
```

```
[Out] int((A + B*log((e*(a + b*x))/(c + d*x)))^2/(f + g*x)^5, x)
```

3.249 $\int \frac{\log\left(\frac{1+x}{-1+x}\right)}{x^2} dx$

Optimal. Leaf size=35

$$2\log\left(-\frac{x}{1-x}\right) - \frac{(1+x)\log\left(-\frac{1+x}{1-x}\right)}{x}$$

[Out] 2*ln(-x/(1-x))-(1+x)*ln((-1-x)/(1-x))/x

Rubi [A]
time = 0.02, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {2553, 2351, 31}

$$2\log\left(-\frac{x}{1-x}\right) - \frac{(x+1)\log\left(-\frac{x+1}{1-x}\right)}{x}$$

Antiderivative was successfully verified.

[In] Int[Log[(1 + x)/(-1 + x)]/x^2,x]

[Out] 2*Log[-(x/(1 - x))] - ((1 + x)*Log[-((1 + x)/(1 - x))])/x

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2553

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[b*c - a*d, Subst[
Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)),
 x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x
] && NeQ[b*c - a*d, 0] && IntegerQ[m] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\log\left(\frac{1+x}{-1+x}\right)}{x^2}\,dx &= -\frac{(1-x)\log\left(-\frac{1+x}{1-x}\right)}{x} + 2\int \frac{1}{x(1+x)}\,dx \\ &= -\frac{(1-x)\log\left(-\frac{1+x}{1-x}\right)}{x} + 2\int \frac{1}{x}\,dx - 2\int \frac{1}{1+x}\,dx \\ &= 2\log(x) - 2\log(1+x) - \frac{(1-x)\log\left(-\frac{1+x}{1-x}\right)}{x}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 30, normalized size = 0.86

$$2\log(x) - \frac{\log\left(\frac{1+x}{-1+x}\right)}{x} - \log\left(1-x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[(1 + x)/(-1 + x)]/x^2,x]
```

```
[Out] 2*Log[x] - Log[(1 + x)/(-1 + x)]/x - Log[1 - x^2]
```

Maple [A]
time = 0.15, size = 46, normalized size = 1.31

method	result	size
risch	$-\frac{\ln\left(\frac{x+1}{-1+x}\right)}{x} + 2\ln(x) - \ln\left(x^2-1\right)$	29
derivativedivides	$2\ln\left(2+\frac{2}{-1+x}\right) - \frac{2\ln\left(1+\frac{2}{-1+x}\right)\left(1+\frac{2}{-1+x}\right)}{2+\frac{2}{-1+x}}$	46
default	$2\ln\left(2+\frac{2}{-1+x}\right) - \frac{2\ln\left(1+\frac{2}{-1+x}\right)\left(1+\frac{2}{-1+x}\right)}{2+\frac{2}{-1+x}}$	46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln((x+1)/(-1+x))/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2*ln(2+2/(-1+x))-2*ln(1+2/(-1+x))*(1+2/(-1+x))/(2+2/(-1+x))
```

Maxima [A]
time = 0.27, size = 32, normalized size = 0.91

$$-\frac{\log\left(\frac{x+1}{x-1}\right)}{x} - \log(x+1) - \log(x-1) + 2\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log((1+x)/(-1+x))/x^2,x, algorithm="maxima")

[Out] -log((x + 1)/(x - 1))/x - log(x + 1) - log(x - 1) + 2*log(x)

Fricas [A]
time = 0.35, size = 29, normalized size = 0.83

$$-\frac{x\log\left(x^2-1\right)-2\,x\log\left(x\right)+\log\left(\frac{x+1}{x-1}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log((1+x)/(-1+x))/x^2,x, algorithm="fricas")

[Out] -(x*log(x^2 - 1) - 2*x*log(x) + log((x + 1)/(x - 1)))/x

Sympy [A]
time = 0.05, size = 20, normalized size = 0.57

$$2\log\left(x\right)-\log\left(x^2-1\right)-\frac{\log\left(\frac{x+1}{x-1}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(ln((1+x)/(-1+x))/x**2,x)

[Out] 2*log(x) - log(x**2 - 1) - log((x + 1)/(x - 1))/x

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 103 vs. 2(29) = 58.
time = 5.51, size = 103, normalized size = 2.94

$$\frac{2\,\log\left(\frac{\frac{\frac{x+1}{x-1}+1}{\frac{x+1}{x-1}-1}+1}{\frac{\frac{x+1}{x-1}+1}{\frac{x+1}{x-1}-1}-1}\right)}{\frac{x+1}{x-1}+1}-2\,\log\left(\frac{|x+1|}{|x-1|}\right)+2\,\log\left(\left|\frac{x+1}{x-1}+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log((1+x)/(-1+x))/x^2,x, algorithm="giac")

[Out] 2*log((((x + 1)/(x - 1) + 1)/((x + 1)/(x - 1) - 1) + 1)/(((x + 1)/(x - 1) + 1)/((x + 1)/(x - 1) - 1) - 1))/((x + 1)/(x - 1) + 1) - 2*log(abs(x + 1)/abs(x - 1)) + 2*log(abs((x + 1)/(x - 1) + 1))

Mupad [B]
time = 0.19, size = 28, normalized size = 0.80

$$2\,\ln\left(x\right)-\ln\left(x^2-1\right)-\frac{\ln\left(\frac{x+1}{x-1}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(log((x + 1)/(x - 1))/x^2,x)

[Out] 2*log(x) - log(x^2 - 1) - log((x + 1)/(x - 1))/x

3.250 $\int \frac{(f+gx)^2}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx$

Optimal. Leaf size=32

$$\text{Int}\left(\frac{(f+gx)^2}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)},x\right)$$

```
[Out] Unintegrable((g*x+f)^2/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(f+gx)^2}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(f + g*x)^2/(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
[Out] Defer[Int][(f + g*x)^2/(A + B*Log[(e*(a + b*x))/(c + d*x)]), x]
```

Rubi steps

$$\begin{aligned}\int \frac{(f+gx)^2}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx &= \int \left(\frac{f^2}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}+\frac{2fgx}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}+\frac{g^2x^2}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\right)dx\\ &= f^2\int \frac{1}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx+(2fg)\int \frac{x}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx+g^2\int \frac{}{A+B}\end{aligned}$$

Mathematica [A]
time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{(f+gx)^2}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(f + g*x)^2/(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
[Out] Integrate[(f + g*x)^2/(A + B*Log[(e*(a + b*x))/(c + d*x)]), x]
```

Maple [A]
time = 0.92, size = 0, normalized size = 0.00

$$\int \frac{(gx+f)^2}{A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
[Out] int((g*x+f)^2/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="maxima")
[Out] integrate((g*x + f)^2/(B*log((b*x + a)*e/(d*x + c)) + A), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="fricas")
[Out] integral((g^2*x^2 + 2*f*g*x + f^2)/(B*log((b*x + a)*e/(d*x + c)) + A), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(f+gx)^2}{A+B\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
[Out] Integral((f + g*x)**2/(A + B*log(a*e/(c + d*x) + b*e*x/(c + d*x))), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="giac")
[Out] integrate((g*x + f)^2/(B*log((b*x + a)*e/(d*x + c)) + A), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{(f+g\,x)^2}{A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^2/(A + B*log((e*(a + b*x))/(c + d*x))),x)
[Out] int((f + g*x)^2/(A + B*log((e*(a + b*x))/(c + d*x))), x)
```

3.251 $\int \frac{f+gx}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)} dx$

Optimal. Leaf size=30

$$\text{Int}\left(\frac{f+gx}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)},x\right)$$

```
[Out] Unintegrable((g*x+f)/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{f+gx}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)} dx$$

Verification is not applicable to the result.

```
[In] Int[(f + g*x)/(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
[Out] Defer[Int][(f + g*x)/(A + B*Log[(e*(a + b*x))/(c + d*x)]), x]
```

Rubi steps

$$\begin{aligned}\int \frac{f+gx}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)} dx &= \int \left(\frac{f}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}+\frac{gx}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\right) dx\\ &= f\int \frac{1}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)} dx+g\int \frac{x}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)} dx\end{aligned}$$

Mathematica [A]
time = 0.17, size = 0, normalized size = 0.00

$$\int \frac{f+gx}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)} dx$$

Verification is not applicable to the result.

```
[In] Integrate[(f + g*x)/(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
[Out] Integrate[(f + g*x)/(A + B*Log[(e*(a + b*x))/(c + d*x)]), x]
```

Maple [A]
time = 0.68, size = 0, normalized size = 0.00

$$\int \frac{gx+f}{A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
[Out] int((g*x+f)/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="maxima")
[Out] integrate((g*x + f)/(B*log((b*x + a)*e/(d*x + c)) + A), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="fricas")
[Out] integral((g*x + f)/(B*log((b*x + a)*e/(d*x + c)) + A), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{f+gx}{A+B\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
[Out] Integral((f + g*x)/(A + B*log(a*e/(c + d*x) + b*e*x/(c + d*x))), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="giac")
```

```
[Out] integrate((g*x + f)/(B*log((b*x + a)*e/(d*x + c)) + A), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{f+g\,x}{A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)/(A + B*log((e*(a + b*x))/(c + d*x))),x)
```

```
[Out] int((f + g*x)/(A + B*log((e*(a + b*x))/(c + d*x))), x)
```

3.252 $\int \frac{1}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx$

Optimal. Leaf size=24

$$\text{Int}\left(\frac{1}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)},x\right)$$

```
[Out] Unintegrable(1/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

Rubi [A]
time = 0.00, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])^(-1),x]
[Out] Defer[Int][(A + B*Log[(e*(a + b*x))/(c + d*x)])^(-1), x]
```

Rubi steps

$$\int \frac{1}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx = \int \frac{1}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx$$

Mathematica [A]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])^(-1),x]
[Out] Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])^(-1), x]
```

Maple [A]
time = 0.63, size = 0, normalized size = 0.00

$$\int \frac{1}{A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
[Out] int(1/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="maxima")
[Out] integrate(1/(B*log((b*x + a)*e/(d*x + c)) + A), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="fricas")
[Out] integral(1/(B*log((b*x + a)*e/(d*x + c)) + A), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{A + B \log\left(\frac{e(a+bx)}{c+dx}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
[Out] Integral(1/(A + B*log(e*(a + b*x)/(c + d*x))), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="giac")
[Out] integrate(1/(B*log((b*x + a)*e/(d*x + c)) + A), x)
```

Mupad [A]

time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{A + B\ln\left(\frac{e(a+bx)}{c+dx}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(A + B*log((e*(a + b*x))/(c + d*x))),x)
[Out] int(1/(A + B*log((e*(a + b*x))/(c + d*x))), x)
```

3.253 $\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx$

Optimal. Leaf size=32

$$\operatorname{Int}\left(\frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)},x\right)$$

[Out] Unintegrable(1/(g*x+f)/(A+B*ln(e*(b*x+a)/(d*x+c))),x)

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx$$

Verification is not applicable to the result.

[In] Int[1/((f + g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])),x]

[Out] Defer[Int][1/((f + g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])), x]

Rubi steps

$$\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx = \int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx$$

Mathematica [A]
time = 0.52, size = 0, normalized size = 0.00

$$\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx$$

Verification is not applicable to the result.

[In] Integrate[1/((f + g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])),x]

[Out] Integrate[1/((f + g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])), x]

Maple [A]
time = 0.72, size = 0, normalized size = 0.00

$$\int \frac{1}{(gx+f)\left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(g*x+f)/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

```
[Out] int(1/(g*x+f)/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="maxima")
```

```
[Out] integrate(1/((g*x + f)*(B*log((b*x + a)*e/(d*x + c)) + A)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="fricas")
```

```
[Out] integral(1/(A*g*x + A*f + (B*g*x + B*f)*log((b*x + a)*e/(d*x + c))), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(A + B\log\left(\frac{ae}{c+dx} + \frac{bex}{c+dx}\right)\right)(f + gx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

```
[Out] Integral(1/((A + B*log(a*e/(c + d*x) + b*e*x/(c + d*x)))*(f + g*x)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="giac")
```

```
[Out] integrate(1/((g*x + f)*(B*log((b*x + a)*e/(d*x + c)) + A)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{(f+g\,x)\,\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((f + g*x)*(A + B*log((e*(a + b*x))/(c + d*x)))),x)
[Out] int(1/((f + g*x)*(A + B*log((e*(a + b*x))/(c + d*x)))), x)
```

3.254 $\int \frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx$

Optimal. Leaf size=32

$$\text{Int}\left(\frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)},x\right)$$

```
[Out] Unintegrable(1/(g*x+f)^2/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((f + g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])),x]
[Out] Defer[Int][1/((f + g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])), x]
```

Rubi steps

$$\int \frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx=\int \frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx$$

Mathematica [A]
time = 1.40, size = 0, normalized size = 0.00

$$\int \frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((f + g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])),x]
[Out] Integrate[1/((f + g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])), x]
```

Maple [A]
time = 0.68, size = 0, normalized size = 0.00

$$\int \frac{1}{(gx+f)^2\left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(g*x+f)^2/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

```
[Out] int(1/(g*x+f)^2/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)^2/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="maxima")
```

```
[Out] integrate(1/((g*x + f)^2*(B*log((b*x + a)*e/(d*x + c)) + A)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)^2/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="fricas")
```

```
[Out] integral(1/(A*g^2*x^2 + 2*A*f*g*x + A*f^2 + (B*g^2*x^2 + 2*B*f*g*x + B*f^2)
*log((b*x + a)*e/(d*x + c))), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(A + B\log\left(\frac{ae}{c+dx} + \frac{bex}{c+dx}\right)\right)(f+gx)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)**2/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

```
[Out] Integral(1/((A + B*log(a*e/(c + d*x) + b*e*x/(c + d*x)))*(f + g*x)**2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)^2/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="giac")
```

```
[Out] integrate(1/((g*x + f)^2*(B*log((b*x + a)*e/(d*x + c)) + A)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{(f+g\,x)^2\,\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((f + g*x)^2*(A + B*log((e*(a + b*x))/(c + d*x)))),x)
[Out] int(1/((f + g*x)^2*(A + B*log((e*(a + b*x))/(c + d*x)))), x)
```

3.255 $\int \frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx$

Optimal. Leaf size=32

$$\text{Int}\left(\frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)},x\right)$$

```
[Out] Unintegrable(1/(g*x+f)^3/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((f + g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])),x]
[Out] Defer[Int][1/((f + g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])), x]
```

Rubi steps

$$\int \frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx = \int \frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx$$

Mathematica [A]
time = 30.38, size = 0, normalized size = 0.00

$$\int \frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((f + g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])),x]
[Out] Integrate[1/((f + g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])), x]
```

Maple [A]
time = 1.26, size = 0, normalized size = 0.00

$$\int \frac{1}{(gx+f)^3\left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(g*x+f)^3/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

```
[Out] int(1/(g*x+f)^3/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

Maxima [**A**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)^3/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="maxima")
```

```
[Out] integrate(1/((g*x + f)^3*(B*log((b*x + a)*e/(d*x + c)) + A)), x)
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)^3/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="fricas")
```

```
[Out] integral(1/(A*g^3*x^3 + 3*A*f*g^2*x^2 + 3*A*f^2*g*x + A*f^3 + (B*g^3*x^3 +
3*B*f*g^2*x^2 + 3*B*f^2*g*x + B*f^3)*log((b*x + a)*e/(d*x + c))), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)**3/(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)^3/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="giac")
```

```
[Out] integrate(1/((g*x + f)^3*(B*log((b*x + a)*e/(d*x + c)) + A)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{(f+gx)^3\left(A+B\ln\left(\frac{e(a+bx)}{c+dx}\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((f + g*x)^3*(A + B*log((e*(a + b*x))/(c + d*x)))),x)
[Out] int(1/((f + g*x)^3*(A + B*log((e*(a + b*x))/(c + d*x)))), x)
```

3.256 $\int \frac{(f+gx)^2}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$

Optimal. Leaf size=32

$$\text{Int}\left(\frac{(f+gx)^2}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2},x\right)$$

```
[Out] Unintegrable((g*x+f)^2/(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(f+gx)^2}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(f + g*x)^2/(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]
[Out] Defer[Int][(f + g*x)^2/(A + B*Log[(e*(a + b*x))/(c + d*x)])^2, x]
```

Rubi steps

$$\int \frac{(f+gx)^2}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx = \int \left(\frac{f^2}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}+\frac{2fgx}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}+\frac{g^2x^2}{\left(A+B\log\left(\frac{e(}{c}\right.\right.}\right.$$

$$= f^2\int \frac{1}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx+(2fg)\int \frac{x}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx+$$

Mathematica [A]
time = 0.78, size = 0, normalized size = 0.00

$$\int \frac{(f+gx)^2}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(f + g*x)^2/(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]
```

```
[Out] Integrate[(f + g*x)^2/(A + B*Log[(e*(a + b*x))/(c + d*x)])^2, x]
```

Maple [A]
time = 1.07, size = 0, normalized size = 0.00

$$\int \frac{(gx+f)^2}{\left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2/(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

```
[Out] int((g*x+f)^2/(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="maxima")
```

```
[Out] -(b*d*g^2*x^4 + a*c*f^2 + (a*d*g^2 + (2*d*f*g + c*g^2)*b)*x^3 + ((2*d*f*g +
 c*g^2)*a + (d*f^2 + 2*c*f*g)*b)*x^2 + (b*c*f^2 + (d*f^2 + 2*c*f*g)*a)*x)/(
(b*c - a*d)*B^2*log(b*x + a) - (b*c - a*d)*B^2*log(d*x + c) + (b*c - a*d)*A
*B + (b*c - a*d)*B^2) + integrate((4*b*d*g^2*x^3 + b*c*f^2 + 3*(a*d*g^2 + (
2*d*f*g + c*g^2)*b)*x^2 + (d*f^2 + 2*c*f*g)*a + 2*((2*d*f*g + c*g^2)*a + (d
*f^2 + 2*c*f*g)*b)*x)/((b*c - a*d)*B^2*log(b*x + a) - (b*c - a*d)*B^2*log(d
*x + c) + (b*c - a*d)*A*B + (b*c - a*d)*B^2), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="fricas")
```

```
[Out] integral((g^2*x^2 + 2*f*g*x + f^2)/(B^2*log((b*x + a)*e/(d*x + c))^2 + 2*A*
B*log((b*x + a)*e/(d*x + c)) + A^2), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2/(A+B*ln(e*(b*x+a)/(d*x+c)))**2,x)
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="giac")
[Out] integrate((g*x + f)^2/(B*log((b*x + a)*e/(d*x + c)) + A)^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{(f+g\,x)^2}{\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^2/(A + B*log((e*(a + b*x))/(c + d*x)))^2,x)
[Out] int((f + g*x)^2/(A + B*log((e*(a + b*x))/(c + d*x)))^2, x)
```

3.257 $\int \frac{f+gx}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$

Optimal. Leaf size=30

$$\mathrm{Int}\left(\frac{f+gx}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2},x\right)$$

```
[Out] Unintegrable((g*x+f)/(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{f+gx}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(f + g*x)/(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]
```

```
[Out] Defer[Int][(f + g*x)/(A + B*Log[(e*(a + b*x))/(c + d*x)])^2, x]
```

Rubi steps

$$\begin{aligned}\int \frac{f+gx}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx &= \int \left(\frac{f}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}+\frac{gx}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\right)dx\\ &= f\int \frac{1}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx+g\int \frac{x}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx\end{aligned}$$

Mathematica [A]
time = 0.58, size = 0, normalized size = 0.00

$$\int \frac{f+gx}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(f + g*x)/(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]
```

```
[Out] Integrate[(f + g*x)/(A + B*Log[(e*(a + b*x))/(c + d*x)])^2, x]
```

Maple [A]
time = 0.81, size = 0, normalized size = 0.00

$$\int \frac{gx+f}{\left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)/(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

```
[Out] int((g*x+f)/(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="maxima")
```

```
[Out] -(b*d*g*x^3 + a*c*f + (a*d*g + (d*f + c*g)*b)*x^2 + (b*c*f + (d*f + c*g)*a)
*x)/((b*c - a*d)*B^2*log(b*x + a) - (b*c - a*d)*B^2*log(d*x + c) + (b*c - a
*d)*A*B + (b*c - a*d)*B^2) + integrate((3*b*d*g*x^2 + b*c*f + (d*f + c*g)*a
 + 2*(a*d*g + (d*f + c*g)*b)*x)/((b*c - a*d)*B^2*log(b*x + a) - (b*c - a*d)
*B^2*log(d*x + c) + (b*c - a*d)*A*B + (b*c - a*d)*B^2), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="fricas")
```

```
[Out] integral((g*x + f)/(B^2*log((b*x + a)*e/(d*x + c))^2 + 2*A*B*log((b*x + a)*
e/(d*x + c)) + A^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{acf+acgx+adfx+adgx^{2}+bcfx+bcgx^{2}+bdfx^{2}+bdgx^{3}}{ABad-ABbc+\left(B^{2}ad-B^{2}bc\right)\log\left(\frac{e(a+bx)}{c+dx}\right)}-\frac{\int\frac{acg}{A+B\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}\,dx+\int\frac{adf}{A+B\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}\,dx+\int\frac{bcf}{A+B\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}\,dx+\int\frac{2adgx}{A+B\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}\,dx+\int\frac{2bcgx}{A+B\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}\,dx+\int\frac{2bdfx}{A+B\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}\,dx+\int\frac{3bdgx^{2}}{A+B\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}\,dx}{B\left(ad-bc\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)/(A+B*ln(e*(b*x+a)/(d*x+c)))**2,x)
```

```
[Out] (a*c*f + a*c*g*x + a*d*f*x + a*d*g*x**2 + b*c*f*x + b*c*g*x**2 + b*d*f*x**2
 + b*d*g*x**3)/(A*B*a*d - A*B*b*c + (B**2*a*d - B**2*b*c)*log(e*(a + b*x)/(
c + d*x))) - (Integral(a*c*g/(A + B*log(a*e/(c + d*x) + b*e*x/(c + d*x))),
x) + Integral(a*d*f/(A + B*log(a*e/(c + d*x) + b*e*x/(c + d*x))), x) + Inte
gral(b*c*f/(A + B*log(a*e/(c + d*x) + b*e*x/(c + d*x))), x) + Integral(2*a*
d*g*x/(A + B*log(a*e/(c + d*x) + b*e*x/(c + d*x))), x) + Integral(2*b*c*g*x
/(A + B*log(a*e/(c + d*x) + b*e*x/(c + d*x))), x) + Integral(2*b*d*f*x/(A +
 B*log(a*e/(c + d*x) + b*e*x/(c + d*x))), x) + Integral(3*b*d*g*x**2/(A + B
*log(a*e/(c + d*x) + b*e*x/(c + d*x))), x))/(B*(a*d - b*c))
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="giac")
```

```
[Out] integrate((g*x + f)/(B*log((b*x + a)*e/(d*x + c)) + A)^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{f+g\,x}{{\left(A+B\,\ln\left(\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)/(A + B*log((e*(a + b*x))/(c + d*x)))^2,x)
```

```
[Out] int((f + g*x)/(A + B*log((e*(a + b*x))/(c + d*x)))^2, x)
```

3.258 $\int \frac{1}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$

Optimal. Leaf size=24

$$\text{Int}\left(\frac{1}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2},x\right)$$

```
[Out] Unintegrable(1/(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

Rubi [A]
time = 0.00, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])^(-2),x]
[Out] Defer[Int][(A + B*Log[(e*(a + b*x))/(c + d*x)])^(-2), x]
```

Rubi steps

$$\int \frac{1}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx=\int \frac{1}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$$

Mathematica [A]
time = 0.34, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])^(-2),x]
[Out] Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])^(-2), x]
```

Maple [A]
time = 0.68, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

```
[Out] int(1/(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="maxima")
```

```
[Out] -(b*d*x^2 + a*c + (b*c + a*d)*x)/((b*c - a*d)*B^2*log(b*x + a) - (b*c - a*d
)*B^2*log(d*x + c) + (b*c - a*d)*A*B + (b*c - a*d)*B^2) + integrate((2*b*d*
x + b*c + a*d)/((b*c - a*d)*B^2*log(b*x + a) - (b*c - a*d)*B^2*log(d*x + c)
 + (b*c - a*d)*A*B + (b*c - a*d)*B^2), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="fricas")
```

```
[Out] integral(1/(B^2*log((b*x + a)*e/(d*x + c))^2 + 2*A*B*log((b*x + a)*e/(d*x +
 c)) + A^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{ac+adx+bcx+bdx^2}{ABad-ABbc+(B^2ad-B^2bc)\log\left(\frac{e(a+bx)}{c+dx}\right)}-\frac{\int\frac{ad}{A+B\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}\,dx+\int\frac{bc}{A+B\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}\,dx+\int\frac{2bdx}{A+B\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}\,dx}{B\,(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(A+B*ln(e*(b*x+a)/(d*x+c)))**2,x)
```

```
[Out] (a*c + a*d*x + b*c*x + b*d*x**2)/(A*B*a*d - A*B*b*c + (B**2*a*d - B**2*b*c)
*log(e*(a + b*x)/(c + d*x))) - (Integral(a*d/(A + B*log(a*e/(c + d*x) + b*e
*x/(c + d*x))), x) + Integral(b*c/(A + B*log(a*e/(c + d*x) + b*e*x/(c + d*x
))), x) + Integral(2*b*d*x/(A + B*log(a*e/(c + d*x) + b*e*x/(c + d*x))), x)
)/(B*(a*d - b*c))
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="giac")
[Out] integrate((B*log((b*x + a)*e/(d*x + c)) + A)^(-2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(A + B*log((e*(a + b*x))/(c + d*x)))^2,x)
[Out] int(1/(A + B*log((e*(a + b*x))/(c + d*x)))^2, x)
```

3.259 $\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$

Optimal. Leaf size=32

$$\text{Int}\left(\frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2},x\right)$$

```
[Out] Unintegrable(1/(g*x+f)/(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((f + g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2),x]
[Out] Defer[Int][1/((f + g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2), x]
```

Rubi steps

$$\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx = \int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$$

Mathematica [A]
time = 1.52, size = 0, normalized size = 0.00

$$\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((f + g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2),x]
[Out] Integrate[1/((f + g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2), x]
```

Maple [A]
time = 0.74, size = 0, normalized size = 0.00

$$\int \frac{1}{(gx+f)\left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(g*x+f)/(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

```
[Out] int(1/(g*x+f)/(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="maxima")
```

```
[Out] -(b*d*x^2 + a*c + (b*c + a*d)*x)/((b*c*f - a*d*f)*A*B + (b*c*f - a*d*f)*B^2
 + ((b*c*g - a*d*g)*A*B + (b*c*g - a*d*g)*B^2)*x + ((b*c*g - a*d*g)*B^2*x +
 (b*c*f - a*d*f)*B^2)*log(b*x + a) - ((b*c*g - a*d*g)*B^2*x + (b*c*f - a*d*
f)*B^2)*log(d*x + c)) + integrate((b*d*g*x^2 + 2*b*d*f*x + b*c*f + (d*f - c
*g)*a)/((b*c*f^2 - a*d*f^2)*A*B + (b*c*f^2 - a*d*f^2)*B^2 + ((b*c*g^2 - a*d
*g^2)*A*B + (b*c*g^2 - a*d*g^2)*B^2)*x^2 + 2*((b*c*f*g - a*d*f*g)*A*B + (b*
c*f*g - a*d*f*g)*B^2)*x + ((b*c*g^2 - a*d*g^2)*B^2*x^2 + 2*(b*c*f*g - a*d*f
*g)*B^2*x + (b*c*f^2 - a*d*f^2)*B^2)*log(b*x + a) - ((b*c*g^2 - a*d*g^2)*B^
2*x^2 + 2*(b*c*f*g - a*d*f*g)*B^2*x + (b*c*f^2 - a*d*f^2)*B^2)*log(d*x + c)
), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="fricas")
```

```
[Out] integral(1/(A^2*g*x + A^2*f + (B^2*g*x + B^2*f)*log((b*x + a)*e/(d*x + c))^
2 + 2*(A*B*g*x + A*B*f)*log((b*x + a)*e/(d*x + c))), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)/(A+B*ln(e*(b*x+a)/(d*x+c)))**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((g*x + f)*(B*log((b*x + a)*e/(d*x + c)) + A)^2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{(f+g\,x)\,\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((f + g*x)*(A + B*log((e*(a + b*x))/(c + d*x)))^2),x)
```

```
[Out] int(1/((f + g*x)*(A + B*log((e*(a + b*x))/(c + d*x)))^2), x)
```

3.260 $\int \frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$

Optimal. Leaf size=32

$$\mathrm{Int}\left(\frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2},x\right)$$

```
[Out] Unintegrable(1/(g*x+f)^2/(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((f + g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2),x]
[Out] Defer[Int][1/((f + g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2), x]
```

Rubi steps

$$\int \frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx=\int \frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$$

Mathematica [A]
time = 2.05, size = 0, normalized size = 0.00

$$\int \frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((f + g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2),x]
[Out] Integrate[1/((f + g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2), x]
```

Maple [A]
time = 0.77, size = 0, normalized size = 0.00

$$\int \frac{1}{(gx+f)^2 \left(A + B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(g*x+f)^2/(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

```
[Out] int(1/(g*x+f)^2/(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)^2/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="maxima")
```

```
[Out] -(b*d*x^2 + a*c + (b*c + a*d)*x)/((b*c*f^2 - a*d*f^2)*A*B + (b*c*f^2 - a*d*
f^2)*B^2 + ((b*c*g^2 - a*d*g^2)*A*B + (b*c*g^2 - a*d*g^2)*B^2)*x^2 + 2*((b*
c*f*g - a*d*f*g)*A*B + (b*c*f*g - a*d*f*g)*B^2)*x + ((b*c*g^2 - a*d*g^2)*B^
2*x^2 + 2*(b*c*f*g - a*d*f*g)*B^2*x + (b*c*f^2 - a*d*f^2)*B^2)*log(b*x + a)
 - ((b*c*g^2 - a*d*g^2)*B^2*x^2 + 2*(b*c*f*g - a*d*f*g)*B^2*x + (b*c*f^2 -
a*d*f^2)*B^2)*log(d*x + c)) - integrate(-(b*c*f + (d*f - 2*c*g)*a - (a*d*g
- (2*d*f - c*g)*b)*x)/(((b*c*g^3 - a*d*g^3)*A*B + (b*c*g^3 - a*d*g^3)*B^2)*
x^3 + (b*c*f^3 - a*d*f^3)*A*B + (b*c*f^3 - a*d*f^3)*B^2 + 3*((b*c*f*g^2 - a
*d*f*g^2)*A*B + (b*c*f*g^2 - a*d*f*g^2)*B^2)*x^2 + 3*((b*c*f^2*g - a*d*f^2*
g)*A*B + (b*c*f^2*g - a*d*f^2*g)*B^2)*x + ((b*c*g^3 - a*d*g^3)*B^2*x^3 + 3*
(b*c*f*g^2 - a*d*f*g^2)*B^2*x^2 + 3*(b*c*f^2*g - a*d*f^2*g)*B^2*x + (b*c*f^
3 - a*d*f^3)*B^2)*log(b*x + a) - ((b*c*g^3 - a*d*g^3)*B^2*x^3 + 3*(b*c*f*g^
2 - a*d*f*g^2)*B^2*x^2 + 3*(b*c*f^2*g - a*d*f^2*g)*B^2*x + (b*c*f^3 - a*d*f
^3)*B^2)*log(d*x + c)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)^2/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="fricas")
```

```
[Out] integral(1/(A^2*g^2*x^2 + 2*A^2*f*g*x + A^2*f^2 + (B^2*g^2*x^2 + 2*B^2*f*g*
x + B^2*f^2)*log((b*x + a)*e/(d*x + c))^2 + 2*(A*B*g^2*x^2 + 2*A*B*f*g*x +
A*B*f^2)*log((b*x + a)*e/(d*x + c))), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)**2/(A+B*ln(e*(b*x+a)/(d*x+c)))**2,x)
[Out] Timed out
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)^2/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="giac")
[Out] integrate(1/((g*x + f)^2*(B*log((b*x + a)*e/(d*x + c)) + A)^2), x)
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{(f+g\,x)^2\,\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((f + g*x)^2*(A + B*log((e*(a + b*x))/(c + d*x)))^2),x)
[Out] int(1/((f + g*x)^2*(A + B*log((e*(a + b*x))/(c + d*x)))^2), x)
```

3.261 $\int \frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$

Optimal. Leaf size=32

$$\text{Int}\left(\frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2},x\right)$$

```
[Out] Unintegrable(1/(g*x+f)^3/(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((f + g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2),x]
[Out] Defer[Int][1/((f + g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2), x]
```

Rubi steps

$$\int \frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx = \int \frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$$

Mathematica [A]
time = 112.18, size = 0, normalized size = 0.00

$$\int \frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((f + g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2),x]
[Out] Integrate[1/((f + g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2), x]
```

Maple [A]
time = 1.59, size = 0, normalized size = 0.00

$$\int \frac{1}{(gx+f)^3 \left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(g*x+f)^3/(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
[Out] int(1/(g*x+f)^3/(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)^3/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="maxima")
[Out] -(b*d*x^2 + a*c + (b*c + a*d)*x)/(((b*c*g^3 - a*d*g^3)*A*B + (b*c*g^3 - a*d
*g^3)*B^2)*x^3 + (b*c*f^3 - a*d*f^3)*A*B + (b*c*f^3 - a*d*f^3)*B^2 + 3*((b*
c*f*g^2 - a*d*f*g^2)*A*B + (b*c*f*g^2 - a*d*f*g^2)*B^2)*x^2 + 3*((b*c*f^2*g
 - a*d*f^2*g)*A*B + (b*c*f^2*g - a*d*f^2*g)*B^2)*x + ((b*c*g^3 - a*d*g^3)*B
^2*x^3 + 3*(b*c*f*g^2 - a*d*f*g^2)*B^2*x^2 + 3*(b*c*f^2*g - a*d*f^2*g)*B^2*
x + (b*c*f^3 - a*d*f^3)*B^2)*log(b*x + a) - ((b*c*g^3 - a*d*g^3)*B^2*x^3 +
3*(b*c*f*g^2 - a*d*f*g^2)*B^2*x^2 + 3*(b*c*f^2*g - a*d*f^2*g)*B^2*x + (b*c*
f^3 - a*d*f^3)*B^2)*log(d*x + c)) - integrate((b*d*g*x^2 - b*c*f - (d*f - 3
*c*g)*a + 2*(a*d*g - (d*f - c*g)*b)*x)/(((b*c*g^4 - a*d*g^4)*A*B + (b*c*g^4
 - a*d*g^4)*B^2)*x^4 + 4*((b*c*f*g^3 - a*d*f*g^3)*A*B + (b*c*f*g^3 - a*d*f*
g^3)*B^2)*x^3 + (b*c*f^4 - a*d*f^4)*A*B + (b*c*f^4 - a*d*f^4)*B^2 + 6*((b*c
*f^2*g^2 - a*d*f^2*g^2)*A*B + (b*c*f^2*g^2 - a*d*f^2*g^2)*B^2)*x^2 + 4*((b*
c*f^3*g - a*d*f^3*g)*A*B + (b*c*f^3*g - a*d*f^3*g)*B^2)*x + ((b*c*g^4 - a*d
*g^4)*B^2*x^4 + 4*(b*c*f*g^3 - a*d*f*g^3)*B^2*x^3 + 6*(b*c*f^2*g^2 - a*d*f^
2*g^2)*B^2*x^2 + 4*(b*c*f^3*g - a*d*f^3*g)*B^2*x + (b*c*f^4 - a*d*f^4)*B^2)
*log(b*x + a) - ((b*c*g^4 - a*d*g^4)*B^2*x^4 + 4*(b*c*f*g^3 - a*d*f*g^3)*B^
2*x^3 + 6*(b*c*f^2*g^2 - a*d*f^2*g^2)*B^2*x^2 + 4*(b*c*f^3*g - a*d*f^3*g)*B
^2*x + (b*c*f^4 - a*d*f^4)*B^2)*log(d*x + c)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)^3/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="fricas")
```

```
[Out] integral(1/(A^2*g^3*x^3 + 3*A^2*f*g^2*x^2 + 3*A^2*f^2*g*x + A^2*f^3 + (B^2*
g^3*x^3 + 3*B^2*f*g^2*x^2 + 3*B^2*f^2*g*x + B^2*f^3)*log((b*x + a)*e/(d*x +
 c))^2 + 2*(A*B*g^3*x^3 + 3*A*B*f*g^2*x^2 + 3*A*B*f^2*g*x + A*B*f^3)*log((b
*x + a)*e/(d*x + c))), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)**3/(A+B*ln(e*(b*x+a)/(d*x+c)))**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)^3/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((g*x + f)^3*(B*log((b*x + a)*e/(d*x + c)) + A)^2), x)
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{(f+g\,x)^3\,\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((f + g*x)^3*(A + B*log((e*(a + b*x))/(c + d*x)))^2),x)
```

```
[Out] int(1/((f + g*x)^3*(A + B*log((e*(a + b*x))/(c + d*x)))^2), x)
```

3.262 $\int (f+gx)^4 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx$

Optimal. Leaf size=357

$$\frac{2B(bc-ad)g(a^3d^3g^3-a^2bd^2g^2(5df-cg)+ab^2dg(10d^2f^2-5cdfg+c^2g^2)-b^3(10d^3f^3-10cd^2f^2g+5c^2d}{5b^4d^4}$$

```
[Out] 2/5*B*(-a*d+b*c)*g*(a^3*d^3*g^3-a^2*b*d^2*g^2*(-c*g+5*d*f)+a*b^2*d*g*(c^2*g
^2-5*c*d*f*g+10*d^2*f^2)-b^3*(-c^3*g^3+5*c^2*d*f*g^2-10*c*d^2*f^2*g+10*d^3*
f^3))*x/b^4/d^4-1/5*B*(-a*d+b*c)*g^2*(a^2*d^2*g^2-a*b*d*g*(-c*g+5*d*f)+b^2*
(c^2*g^2-5*c*d*f*g+10*d^2*f^2))*x^2/b^3/d^3-2/15*B*(-a*d+b*c)*g^3*(-a*d*g-b
*c*g+5*b*d*f)*x^3/b^2/d^2-1/10*B*(-a*d+b*c)*g^4*x^4/b/d-2/5*B*(-a*g+b*f)^5*
ln(b*x+a)/b^5/g+1/5*(g*x+f)^5*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/g+2/5*B*(-c*g
+d*f)^5*ln(d*x+c)/d^5/g
```

Rubi [A]

time = 0.32, antiderivative size = 357, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {2548, 84}

$$-\frac{Bg^2x^2(bc-ad)\left(a^2d^2g^2-abdg(5df-cg)+b^2(c^2g^2-5cdfg+10d^2f^2)\right)}{5b^3d^3}+\frac{2Bgx(bc-ad)\left(a^3d^3g^3-a^2bd^2g^2(5df-cg)+ab^2dg(c^2g^2-5cdfg+10d^2f^2)-(b^3(-c^3g^3+5c^2dfg^2-10cd^2f^2g+10d^3f^3))\right)}{5b^4d^4}+\frac{(f+gx)^5\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{5g}-\frac{2B(bf-ag)^5\log(a+bx)}{5b^5g}-\frac{2Bg^3x^3(bc-ad)(-adg-bcg+5bdf)}{15b^2d^2}-\frac{Bg^4x^4(bc-ad)}{10bd}+\frac{2B(df-cg)^5\log(c+dx)}{5d^5g}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^4*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]),x]
```

```
[Out] (2*B*(b*c - a*d)*g*(a^3*d^3*g^3 - a^2*b*d^2*g^2*(5*d*f - c*g) + a*b^2*d*g*(
10*d^2*f^2 - 5*c*d*f*g + c^2*g^2) - b^3*(10*d^3*f^3 - 10*c*d^2*f^2*g + 5*c^
2*d*f*g^2 - c^3*g^3))*x)/(5*b^4*d^4) - (B*(b*c - a*d)*g^2*(a^2*d^2*g^2 - a*
b*d*g*(5*d*f - c*g) + b^2*(10*d^2*f^2 - 5*c*d*f*g + c^2*g^2))*x^2)/(5*b^3*d
^3) - (2*B*(b*c - a*d)*g^3*(5*b*d*f - b*c*g - a*d*g)*x^3)/(15*b^2*d^2) - (B
*(b*c - a*d)*g^4*x^4)/(10*b*d) - (2*B*(b*f - a*g)^5*Log[a + b*x])/(5*b^5*g)
 + ((f + g*x)^5*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]))/(5*g) + (2*B*(d*f
 - c*g)^5*Log[c + d*x])/(5*d^5*g)
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 2548

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Dist[B*n*((b*c
```

```
- a*d)/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /;
 FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c -
 a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Rubi steps

$$\int (f+gx)^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)dx = \frac{(f+gx)^5\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{5g}-\frac{B\int\frac{2(bc-ad)(f+gx)^5}{(a+bx)(c+dx)}\,dx}{5g}$$

$$=\frac{(f+gx)^5\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{5g}-\frac{(2B(bc-ad))\int\frac{(f+gx)}{(a+bx)(c}}{5g}$$

$$=\frac{(f+gx)^5\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{5g}-\frac{(2B(bc-ad))\int\left(\frac{g^2(-a^3}{}\right.}{}$$

$$=\frac{2B(bc-ad)g(a^3d^3g^3-a^2bd^2g^2(5df-cg)+ab^2dg(10d^2f^2-}{5b^4}$$

Mathematica [A]
time = 0.39, size = 282, normalized size = 0.79

$$\frac{\frac{B(-bc+ad)g^2x(-12a^3d^3g^3+6a^2bd^2g^2(10df-2cg+dgx)-2ab^2dg(6c^2g^2-3cdg(10f+gx)+d^2(60f^2+15fgx+2g^2x^2))+b^3(-12c^3g^3+6c^2dg^2(10f+gx)-2cd^2g(60f^2+15fgx+2g^2x^2)+d^3(120f^3+60f^2gx+20fg^2x^2+3g^3x^3)))}{6b^4d^4}-\frac{2B(bf-ag)^5\log(a+bx)}{b^5}+(f+gx)^5\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)+\frac{2B(df-cg)^5\log(c+dx)}{d^5}}{5g}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^4*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]),x]
```

```
[Out] ((B*(-(b*c) + a*d)*g^2*x*(-12*a^3*d^3*g^3 + 6*a^2*b*d^2*g^2*(10*d*f - 2*c*g
 + d*g*x) - 2*a*b^2*d*g*(6*c^2*g^2 - 3*c*d*g*(10*f + g*x) + d^2*(60*f^2 + 1
5*f*g*x + 2*g^2*x^2)) + b^3*(-12*c^3*g^3 + 6*c^2*d*g^2*(10*f + g*x) - 2*c*d
^2*g*(60*f^2 + 15*f*g*x + 2*g^2*x^2) + d^3*(120*f^3 + 60*f^2*g*x + 20*f*g^2
*x^2 + 3*g^3*x^3))))/(6*b^4*d^4) - (2*B*(b*f - a*g)^5*Log[a + b*x])/b^5 + (
f + g*x)^5*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]) + (2*B*(d*f - c*g)^5*Lo
g[c + d*x])/d^5)/(5*g)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2906 vs. $2(343) = 686$.
time = 0.49, size = 2907, normalized size = 8.14

method	result
risch	$g^3Afx^4+\frac{g^4Bax^4}{10b}-\frac{g^4Bcx^4}{10d}+2g^2Af^2x^3-\frac{2g^4Ba^2x^3}{15b^2}+\frac{2g^4Bc^2x^3}{15d^2}+2gAf^3x^2+\frac{g^4Ba^3x^2}{5b^3}-$

derivativedivides	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^4*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d*(A/d^4*(-2*g^2*(c^2*g^2-2*c*d*f*g+d^2*f^2)*(d*x+c)^3-1/5*g^4*(d*x+c)^5
+2*g*(c^3*g^3-3*c^2*d*f*g^2+3*c*d^2*f^2*g-d^3*f^3)*(d*x+c)^2-(c^4*g^4-4*c^3
*d*f*g^3+6*c^2*d^2*f^2*g^2-4*c*d^3*f^3*g+d^4*f^4)*(d*x+c)+g^3*(c*g-d*f)*(d*
x+c)^4)-B/d^4*(d*x+c)*ln(e*(a/(d*x+c)*d-b*c/(d*x+c)+b)^2/d^2)*g^4*c^4-8*B/d
^2*g^3*a/b*ln(a/(d*x+c)*d-b*c/(d*x+c)+b)*c^3*f-8*B*g/b*ln(a/(d*x+c)*d-b*c/(
d*x+c)+b)*a*c*f^3-2*B*d*g^3*a^4/b^4*ln(1/(d*x+c))*f+8*B/d/b/(a*d-b*c)*ln(a/
(d*x+c)*d-b*c/(d*x+c)+b)*a^2*f*g^3*c^3+8*B*d/b/(a*d-b*c)*ln(a/(d*x+c)*d-b*c
/(d*x+c)+b)*a^2*f^3*g*c+12*B/d*g^2*a/b*ln(a/(d*x+c)*d-b*c/(d*x+c)+b)*c^2*f^
2-12*B/d^2/(a*d-b*c)*ln(a/(d*x+c)*d-b*c/(d*x+c)+b)*c^4*b*f^2*g^2+8*B/d^3/(a
*d-b*c)*ln(a/(d*x+c)*d-b*c/(d*x+c)+b)*c^5*b*f*g^3-2*B/d^2/b/(a*d-b*c)*ln(a/
(d*x+c)*d-b*c/(d*x+c)+b)*a^2*g^4*c^4-16*B/d^2/(a*d-b*c)*ln(a/(d*x+c)*d-b*c/
(d*x+c)+b)*a*c^4*f*g^3+24*B/d/(a*d-b*c)*ln(a/(d*x+c)*d-b*c/(d*x+c)+b)*a*c^3
*f^2*g^2+4*B/d*g^2*a/b*(d*x+c)*c*f^2+2*B/d^2*g^3*a/b*(d*x+c)^2*c*f-12*B/b/(
a*d-b*c)*ln(a/(d*x+c)*d-b*c/(d*x+c)+b)*a^2*f^2*g^2*c^2+8*B/d/(a*d-b*c)*ln(a
/(d*x+c)*d-b*c/(d*x+c)+b)*c^3*b*f^3*g-2*B/d*g^3*a^2/b^2*(d*x+c)*c*f-2*B/d^2
*g^3*a/b*(d*x+c)*c^2*f-2/5*B*d*g^4*a^5/b^5*ln(a/(d*x+c)*d-b*c/(d*x+c)+b)-2*
B/(a*d-b*c)*ln(a/(d*x+c)*d-b*c/(d*x+c)+b)*c^2*b*f^4+2*B/d^3*g^3*c^4*ln(1/(d
*x+c))*f+6*B/d^3*g^3*c^4*ln(a/(d*x+c)*d-b*c/(d*x+c)+b)*f-B*(d*x+c)*ln(e*(a/
(d*x+c)*d-b*c/(d*x+c)+b)^2/d^2)*f^4-2*B*ln(1/(d*x+c))*c*f^4+6/5*B/d^4*g^4*c
^3*(d*x+c)^2-8/5*B/d^4*g^4*c^4*(d*x+c)-8/15*B/d^4*g^4*c^2*(d*x+c)^3+1/10*B/
d^4*g^4*c*(d*x+c)^4-8/5*B/d^4*g^4*c^5*ln(a/(d*x+c)*d-b*c/(d*x+c)+b)-2/5*B/d
^4*g^4*c^5*ln(1/(d*x+c))+4*B*g^2*a^2/b^2*(d*x+c)*f^2-3*B/d^3*g^3*c^2*(d*x+c
)^2*f+6*B/d^3*g^3*c^3*(d*x+c)*f+2/15*B/d^2*g^4*a^2/b^2*(d*x+c)^3-1/5*B/d*g^
4*a^3/b^3*(d*x+c)^2-4*B*d*g^2*a^3/b^3*ln(a/(d*x+c)*d-b*c/(d*x+c)+b)*f^2+2*B
/d^4*(d*x+c)^2*ln(e*(a/(d*x+c)*d-b*c/(d*x+c)+b)^2/d^2)*c^3*g^4+2/5*B/d*g^4*
a^3/b^3*(d*x+c)*c-16*B/(a*d-b*c)*ln(a/(d*x+c)*d-b*c/(d*x+c)+b)*a*c^2*f^3*g+
B/d^4*g^4*(d*x+c)^4*ln(e*(a/(d*x+c)*d-b*c/(d*x+c)+b)^2/d^2)*c-2*B/d*g*(d*x+
c)^2*ln(e*(a/(d*x+c)*d-b*c/(d*x+c)+b)^2/d^2)*f^3+4*B/d*g*ln(a/(d*x+c)*d-b*c
/(d*x+c)+b)*c^2*f^3-B/d^3*g^3*(d*x+c)^4*ln(e*(a/(d*x+c)*d-b*c/(d*x+c)+b)^2/
d^2)*f-1/5*B/d^4*g^4*(d*x+c)^5*ln(e*(a/(d*x+c)*d-b*c/(d*x+c)+b)^2/d^2)+2/5*
B*g^4*a^4/b^4*(d*x+c)+B/d*g^3*a^2*(d*x+c)^2/b^2*f-2*B/d*g^2*a/b*(d*x+c)^2*f
^2+2/5*B/d^3*g^4*a/b*(d*x+c)^3*c-2/5*B/d^2*g^4*a^2/b^2*(d*x+c)^2*c-2*B*d^2/
b/(a*d-b*c)*ln(a/(d*x+c)*d-b*c/(d*x+c)+b)*a^2*f^4+4*B/d^3/(a*d-b*c)*ln(a/(d
*x+c)*d-b*c/(d*x+c)+b)*a*c^5*g^4+4*B*d/(a*d-b*c)*ln(a/(d*x+c)*d-b*c/(d*x+c)
+b)*a*c*f^4-4*B*d*g/b^2*ln(1/(d*x+c))*a^2*f^3+4*B/d^3*(d*x+c)*ln(e*(a/(d*x+
c)*d-b*c/(d*x+c)+b)^2/d^2)*f*g^3*c^3-6*B/d^2*(d*x+c)*ln(e*(a/(d*x+c)*d-b*c/
(d*x+c)+b)^2/d^2)*f^2*g^2*c^2+4*B/d*(d*x+c)*ln(e*(a/(d*x+c)*d-b*c/(d*x+c)+b
)^2/d^2)*f^3*g*c-3/5*B/d^3*g^4*a/b*(d*x+c)^2*c^2+2/5*B/d^2*g^4*a^2/b^2*(d*x
```

```
+c)*c^2+2/5*B/d^3*g^4*a/b*(d*x+c)*c^3-2/3*B/d^2*g^3*a/b*(d*x+c)^3*f+4*B/d*g
*ln(1/(d*x+c))*c^2*f^3-4*B*g/b*(d*x+c)*a*f^3+6*B/d^2*(d*x+c)^2*ln(e*(a/(d*x
+c)*d-b*c/(d*x+c)+b)^2/d^2)*c*f^2*g^2+2*B*d*g^3*a^4/b^4*ln(a/(d*x+c)*d-b*c/
(d*x+c)+b)*f+2*B/d^3*g^4*a/b*ln(a/(d*x+c)*d-b*c/(d*x+c)+b)*c^4+4*B/d^3*g^3*
(d*x+c)^3*ln(e*(a/(d*x+c)*d-b*c/(d*x+c)+b)^2/d^2)*c*f+4*B*d*g^2*a^3/b^3*ln(
1/(d*x+c))*f^2+4*B*d*g/b^2*ln(a/(d*x+c)*d-b*c/(d*x+c)+b)*a^2*f^3-6*B/d^3*(d
*x+c)^2*ln(e*(a/(d*x+c)*d-b*c/(d*x+c)+b)^2/d^2)*c^2*f*g^3-2*B/d^4/(a*d-b*c)
*ln(a/(d*x+c)*d-b*c/(d*x+c)+b)*c^6*b*g^4-1/10*B/d^3*g^4*a/b*(d*x+c)^4+2/3*B
/d^3*g^3*c*(d*x+c)^3*f+4*B/d*g*(d*x+c)*c*f^3-8*B/d^2*g^2*c^3*ln(a/(d*x+c)*d
-b*c/(d*x+c)+b)*f^2-4*B/d^2*g^2*c^3*ln(1/(d*x+c))*f^2+2*B/d^2*g^2*c*(d*x+c)
^2*f^2-8*B/d^2*g^2*c^2*(d*x+c)*f^2-2*B*g^3*a^3*(d*x+c)/b^3*f-2*B/d^2*g^2*(d
*x+c)^3*ln(e*(a/(d*x+c)*d-b*c/(d*x+c)+b)^2/d^2)*f^2+2/5*B*d*g^4*a^5/b^5*ln(
1/(d*x+c))-2*B/d^4*g^4*(d*x+c)^3*ln(e*(a/(d*x+c)*d-b*c/(d*x+c)+b)^2/d^2)*c^
2+2*B*d/b*ln(1/(d*x+c))*a*f^4)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 870 vs. $2(344) = 688$.
time = 0.32, size = 870, normalized size = 2.44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^4*(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="maxima")
```

```
[Out] 1/5*A*g^4*x^5 + A*f*g^3*x^4 + 2*A*f^2*g^2*x^3 + 2*A*f^3*g*x^2 + (x*log(b^2*
x^2*e/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*x*e/(d^2*x^2 + 2*c*d*x + c^2) + a^2
*e/(d^2*x^2 + 2*c*d*x + c^2)) + 2*a*log(b*x + a)/b - 2*c*log(d*x + c)/d)*B*
f^4 + 2*(x^2*log(b^2*x^2*e/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*x*e/(d^2*x^2 +
 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) - 2*a^2*log(b*x + a)/b^2
 + 2*c^2*log(d*x + c)/d^2 - 2*(b*c - a*d)*x/(b*d))*B*f^3*g + 2*(x^3*log(b^2
*x^2*e/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*x*e/(d^2*x^2 + 2*c*d*x + c^2) + a^
2*e/(d^2*x^2 + 2*c*d*x + c^2)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c
)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*B*f^
2*g^2 + 1/3*(3*x^4*log(b^2*x^2*e/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*x*e/(d^2
*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) - 6*a^4*log(b*x +
a)/b^4 + 6*c^4*log(d*x + c)/d^4 - (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c
^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3))*B*f*g^3 + 1/30*
(6*x^5*log(b^2*x^2*e/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*x*e/(d^2*x^2 + 2*c*d
*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) + 12*a^5*log(b*x + a)/b^5 - 12
*c^5*log(d*x + c)/d^5 - (3*(b^4*c*d^3 - a*b^3*d^4)*x^4 - 4*(b^4*c^2*d^2 - a
^2*b^2*d^4)*x^3 + 6*(b^4*c^3*d - a^3*b*d^4)*x^2 - 12*(b^4*c^4 - a^4*d^4)*x)
/(b^4*d^4))*B*g^4 + A*f^4*x
```

Fricas [A]
time = 0.75, size = 658, normalized size = 1.84

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^4*(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="fricas")
```

```
[Out] 1/30*(6*A*b^5*d^5*g^4*x^5 + 3*(10*A*b^5*d^5*f*g^3 - (B*b^5*c*d^4 - B*a*b^4*
d^5)*g^4)*x^4 + 4*(15*A*b^5*d^5*f^2*g^2 - 5*(B*b^5*c*d^4 - B*a*b^4*d^5)*f*g
^3 + (B*b^5*c^2*d^3 - B*a^2*b^3*d^5)*g^4)*x^3 + 6*(10*A*b^5*d^5*f^3*g - 10*
(B*b^5*c*d^4 - B*a*b^4*d^5)*f^2*g^2 + 5*(B*b^5*c^2*d^3 - B*a^2*b^3*d^5)*f*g
^3 - (B*b^5*c^3*d^2 - B*a^3*b^2*d^5)*g^4)*x^2 + 6*(5*A*b^5*d^5*f^4 - 20*(B*
b^5*c*d^4 - B*a*b^4*d^5)*f^3*g + 20*(B*b^5*c^2*d^3 - B*a^2*b^3*d^5)*f^2*g^2
 - 10*(B*b^5*c^3*d^2 - B*a^3*b^2*d^5)*f*g^3 + 2*(B*b^5*c^4*d - B*a^4*b*d^5)
*g^4)*x + 12*(5*B*a*b^4*d^5*f^4 - 10*B*a^2*b^3*d^5*f^3*g + 10*B*a^3*b^2*d^5
*f^2*g^2 - 5*B*a^4*b*d^5*f*g^3 + B*a^5*d^5*g^4)*log(b*x + a) - 12*(5*B*b^5*
c*d^4*f^4 - 10*B*b^5*c^2*d^3*f^3*g + 10*B*b^5*c^3*d^2*f^2*g^2 - 5*B*b^5*c^4
*d*f*g^3 + B*b^5*c^5*g^4)*log(d*x + c) + 6*(B*b^5*d^5*g^4*x^5 + 5*B*b^5*d^5
*f*g^3*x^4 + 10*B*b^5*d^5*f^2*g^2*x^3 + 10*B*b^5*d^5*f^3*g*x^2 + 5*B*b^5*d^
5*f^4*x)*log((b^2*x^2 + 2*a*b*x + a^2)*e/(d^2*x^2 + 2*c*d*x + c^2)))/(b^5*d
^5)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1477 vs. $2(347) = 694$.
time = 58.22, size = 1477, normalized size = 4.14

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**4*(A+B*ln(e*(b*x+a)**2/(d*x+c)**2)),x)
```

```
[Out] A*g**4*x**5/5 + 2*B*a*(a**4*g**4 - 5*a**3*b*f*g**3 + 10*a**2*b**2*f**2*g**2
 - 10*a*b**3*f**3*g + 5*b**4*f**4)*log(x + (2*B*a**5*c*d**4*g**4 - 10*B*a**
4*b*c*d**4*f*g**3 + 20*B*a**3*b**2*c*d**4*f**2*g**2 - 20*B*a**2*b**3*c*d**4
*f**3*g + 2*B*a**2*d**5*(a**4*g**4 - 5*a**3*b*f*g**3 + 10*a**2*b**2*f**2*g*
*2 - 10*a*b**3*f**3*g + 5*b**4*f**4)/b + 2*B*a*b**4*c**5*g**4 - 10*B*a*b**4
*c**4*d*f*g**3 + 20*B*a*b**4*c**3*d**2*f**2*g**2 - 20*B*a*b**4*c**2*d**3*f*
*3*g + 20*B*a*b**4*c*d**4*f**4 - 2*B*a*c*d**4*(a**4*g**4 - 5*a**3*b*f*g**3
+ 10*a**2*b**2*f**2*g**2 - 10*a*b**3*f**3*g + 5*b**4*f**4))/(2*B*a**5*d**5*
g**4 - 10*B*a**4*b*d**5*f*g**3 + 20*B*a**3*b**2*d**5*f**2*g**2 - 20*B*a**2*
b**3*d**5*f**3*g + 10*B*a*b**4*d**5*f**4 + 2*B*b**5*c**5*g**4 - 10*B*b**5*c
**4*d*f*g**3 + 20*B*b**5*c**3*d**2*f**2*g**2 - 20*B*b**5*c**2*d**3*f**3*g +
 10*B*b**5*c*d**4*f**4))/(5*b**5) - 2*B*c*(c**4*g**4 - 5*c**3*d*f*g**3 + 10
*c**2*d**2*f**2*g**2 - 10*c*d**3*f**3*g + 5*d**4*f**4)*log(x + (2*B*a**5*c*
d**4*g**4 - 10*B*a**4*b*c*d**4*f*g**3 + 20*B*a**3*b**2*c*d**4*f**2*g**2 - 2
0*B*a**2*b**3*c*d**4*f**3*g + 2*B*a*b**4*c**5*g**4 - 10*B*a*b**4*c**4*d*f*g
**3 + 20*B*a*b**4*c**3*d**2*f**2*g**2 - 20*B*a*b**4*c**2*d**3*f**3*g + 20*B
*a*b**4*c*d**4*f**4 - 2*B*a*b**4*c*(c**4*g**4 - 5*c**3*d*f*g**3 + 10*c**2*d
**2*f**2*g**2 - 10*c*d**3*f**3*g + 5*d**4*f**4) + 2*B*b**5*c**2*(c**4*g**4
```

```
- 5*c**3*d*f*g**3 + 10*c**2*d**2*f**2*g**2 - 10*c*d**3*f**3*g + 5*d**4*f**4
)/d)/(2*B*a**5*d**5*g**4 - 10*B*a**4*b*d**5*f*g**3 + 20*B*a**3*b**2*d**5*f*
*2*g**2 - 20*B*a**2*b**3*d**5*f**3*g + 10*B*a*b**4*d**5*f**4 + 2*B*b**5*c**
5*g**4 - 10*B*b**5*c**4*d*f*g**3 + 20*B*b**5*c**3*d**2*f**2*g**2 - 20*B*b**
5*c**2*d**3*f**3*g + 10*B*b**5*c*d**4*f**4))/(5*d**5) + x**4*(A*f*g**3 + B*
a*g**4/(10*b) - B*c*g**4/(10*d)) + x**3*(2*A*f**2*g**2 - 2*B*a**2*g**4/(15*
b**2) + 2*B*a*f*g**3/(3*b) + 2*B*c**2*g**4/(15*d**2) - 2*B*c*f*g**3/(3*d))
+ x**2*(2*A*f**3*g + B*a**3*g**4/(5*b**3) - B*a**2*f*g**3/b**2 + 2*B*a*f**2
*g**2/b - B*c**3*g**4/(5*d**3) + B*c**2*f*g**3/d**2 - 2*B*c*f**2*g**2/d) +
x*(A*f**4 - 2*B*a**4*g**4/(5*b**4) + 2*B*a**3*f*g**3/b**3 - 4*B*a**2*f**2*g
**2/b**2 + 4*B*a*f**3*g/b + 2*B*c**4*g**4/(5*d**4) - 2*B*c**3*f*g**3/d**3 +
 4*B*c**2*f**2*g**2/d**2 - 4*B*c*f**3*g/d) + (B*f**4*x + 2*B*f**3*g*x**2 +
2*B*f**2*g**2*x**3 + B*f*g**3*x**4 + B*g**4*x**5/5)*log(e*(a + b*x)**2/(c +
 d*x)**2)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^4*(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**B**]
time = 5.33, size = 1403, normalized size = 3.93

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^4*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2)),x)
```

```
[Out] log((e*(a + b*x)^2)/(c + d*x)^2)*((B*g^4*x^5)/5 + B*f^4*x + 2*B*f^2*g^2*x^3
 + 2*B*f^3*g*x^2 + B*f*g^3*x^4) + x^2*((20*A*a*c*f*g^3 + 20*A*b*d*f^3*g + 3
0*A*a*d*f^2*g^2 + 30*A*b*c*f^2*g^2 + 20*B*a*d*f^2*g^2 - 20*B*b*c*f^2*g^2)/(
10*b*d) + ((5*a*d + 5*b*c)*((((5*A*a*d*g^4 + 5*A*b*c*g^4 + 2*B*a*d*g^4 - 2*
B*b*c*g^4 + 20*A*b*d*f*g^3)/(5*b*d) - (A*g^4*(5*a*d + 5*b*c))/(5*b*d))*(5*a
*d + 5*b*c))/(5*b*d) - (5*A*a*c*g^4 + 20*A*a*d*f*g^3 + 20*A*b*c*f*g^3 + 10*
B*a*d*f*g^3 - 10*B*b*c*f*g^3 + 30*A*b*d*f^2*g^2)/(5*b*d) + (A*a*c*g^4)/(b*d
)))/(10*b*d) - (a*c*((5*A*a*d*g^4 + 5*A*b*c*g^4 + 2*B*a*d*g^4 - 2*B*b*c*g^4
 + 20*A*b*d*f*g^3)/(5*b*d) - (A*g^4*(5*a*d + 5*b*c))/(5*b*d)))/(2*b*d)) + x
^4*((5*A*a*d*g^4 + 5*A*b*c*g^4 + 2*B*a*d*g^4 - 2*B*b*c*g^4 + 20*A*b*d*f*g^3
)/(20*b*d) - (A*g^4*(5*a*d + 5*b*c))/(20*b*d)) + x*((5*A*b*d*f^4 + 20*A*a*d
*f^3*g + 20*A*b*c*f^3*g + 20*B*a*d*f^3*g - 20*B*b*c*f^3*g + 30*A*a*c*f^2*g^
2)/(5*b*d) - ((5*a*d + 5*b*c)*((20*A*a*c*f*g^3 + 20*A*b*d*f^3*g + 30*A*a*d*
```

```
f^2*g^2 + 30*A*b*c*f^2*g^2 + 20*B*a*d*f^2*g^2 - 20*B*b*c*f^2*g^2)/(5*b*d) +
 ((5*a*d + 5*b*c)*((((5*A*a*d*g^4 + 5*A*b*c*g^4 + 2*B*a*d*g^4 - 2*B*b*c*g^4
 + 20*A*b*d*f*g^3)/(5*b*d) - (A*g^4*(5*a*d + 5*b*c))/(5*b*d))*(5*a*d + 5*b*
c))/(5*b*d) - (5*A*a*c*g^4 + 20*A*a*d*f*g^3 + 20*A*b*c*f*g^3 + 10*B*a*d*f*g
^3 - 10*B*b*c*f*g^3 + 30*A*b*d*f^2*g^2)/(5*b*d) + (A*a*c*g^4)/(b*d)))/(5*b*
d) - (a*c*((5*A*a*d*g^4 + 5*A*b*c*g^4 + 2*B*a*d*g^4 - 2*B*b*c*g^4 + 20*A*b*
d*f*g^3)/(5*b*d) - (A*g^4*(5*a*d + 5*b*c))/(5*b*d)))/(b*d)))/(5*b*d) + (a*c
*((((5*A*a*d*g^4 + 5*A*b*c*g^4 + 2*B*a*d*g^4 - 2*B*b*c*g^4 + 20*A*b*d*f*g^3
)/(5*b*d) - (A*g^4*(5*a*d + 5*b*c))/(5*b*d))*(5*a*d + 5*b*c))/(5*b*d) - (5*
A*a*c*g^4 + 20*A*a*d*f*g^3 + 20*A*b*c*f*g^3 + 10*B*a*d*f*g^3 - 10*B*b*c*f*g
^3 + 30*A*b*d*f^2*g^2)/(5*b*d) + (A*a*c*g^4)/(b*d)))/(b*d)) - x^3*((((5*A*a
*d*g^4 + 5*A*b*c*g^4 + 2*B*a*d*g^4 - 2*B*b*c*g^4 + 20*A*b*d*f*g^3)/(5*b*d)
- (A*g^4*(5*a*d + 5*b*c))/(5*b*d))*(5*a*d + 5*b*c))/(15*b*d) - (5*A*a*c*g^4
 + 20*A*a*d*f*g^3 + 20*A*b*c*f*g^3 + 10*B*a*d*f*g^3 - 10*B*b*c*f*g^3 + 30*A
*b*d*f^2*g^2)/(15*b*d) + (A*a*c*g^4)/(3*b*d)) + (A*g^4*x^5)/5 + (log(a + b*
x)*((2*B*a^5*g^4)/5 + 2*B*a*b^4*f^4 - 4*B*a^2*b^3*f^3*g + 4*B*a^3*b^2*f^2*g
^2 - 2*B*a^4*b*f*g^3))/b^5 - (log(c + d*x)*(2*B*c^5*g^4 + 10*B*c*d^4*f^4 -
20*B*c^2*d^3*f^3*g + 20*B*c^3*d^2*f^2*g^2 - 10*B*c^4*d*f*g^3))/(5*d^5)
```

3.263 $\int (f+gx)^3 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right) dx$

Optimal. Leaf size=229

$$-\frac{B(bc-ad)g(a^2d^2g^2-abdg(4df-cg)+b^2(6d^2f^2-4cdfg+c^2g^2))\,x}{2b^3d^3}-\frac{B(bc-ad)g^2(4bdf-bcg-adg)x^2}{4b^2d^2}$$

```
[Out] -1/2*B*(-a*d+b*c)*g*(a^2*d^2*g^2-a*b*d*g*(-c*g+4*d*f)+b^2*(c^2*g^2-4*c*d*f*
g+6*d^2*f^2))*x/b^3/d^3-1/4*B*(-a*d+b*c)*g^2*(-a*d*g-b*c*g+4*b*d*f)*x^2/b^2
/d^2-1/6*B*(-a*d+b*c)*g^3*x^3/b/d-1/2*B*(-a*g+b*f)^4*ln(b*x+a)/b^4/g+1/4*(g
*x+f)^4*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/g+1/2*B*(-c*g+d*f)^4*ln(d*x+c)/d^4/
g
```

Rubi [A]

time = 0.20, antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {2548, 84}

$$-\frac{Bgx(bc-ad)\left(a^2d^2g^2-abdg(4df-cg)+b^2(c^2g^2-4cdfg+6d^2f^2)\right)}{2b^3d^3}+\frac{(f+gx)^4\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{4g}-\frac{B(bf-ag)^4\log(a+bx)}{2b^4g}-\frac{Bg^2x^2(bc-ad)(-adg-bcg+4bdf)}{4b^2d^2}-\frac{Bg^3x^3(bc-ad)}{6bd}+\frac{B(df-cg)^4\log(c+dx)}{2d^4g}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]),x]
```

```
[Out] -1/2*(B*(b*c - a*d)*g*(a^2*d^2*g^2 - a*b*d*g*(4*d*f - c*g) + b^2*(6*d^2*f^2
 - 4*c*d*f*g + c^2*g^2))*x)/(b^3*d^3) - (B*(b*c - a*d)*g^2*(4*b*d*f - b*c*g
 - a*d*g)*x^2)/(4*b^2*d^2) - (B*(b*c - a*d)*g^3*x^3)/(6*b*d) - (B*(b*f - a*
g)^4*Log[a + b*x])/(2*b^4*g) + ((f + g*x)^4*(A + B*Log[(e*(a + b*x)^2)/(c +
 d*x)^2]))/(4*g) + (B*(d*f - c*g)^4*Log[c + d*x])/(2*d^4*g)
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 2548

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Dist[B*n*((b*c
- a*d)/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /;
 FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c -
 a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Rubi steps

$$\int (f+gx)^3 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx = \frac{(f+gx)^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{4g} - \frac{B\int \frac{2(bc-ad)(f+gx)^4}{(a+bx)(c+dx)}\,dx}{4g}$$

$$= \frac{(f+gx)^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{4g} - \frac{(B(bc-ad))\int \frac{(f+gx}{(a+bx)(c\cdot}}{2g}$$

$$= \frac{(f+gx)^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{4g} - \frac{(B(bc-ad))\int \Big(\frac{g^2(a^2d^2}{}}{}$$

$$= -\frac{B(bc-ad)g(a^2d^2g^2-abdg(4df-cg)+b^2(6d^2f^2-4cdfg}{2b^3d^3}$$

Mathematica [A]
time = 0.18, size = 217, normalized size = 0.95

$$\frac{(f+gx)^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)-\frac{B\left(6bd(bc-ad)g^2\left(a^2d^2g^2+abdg(-4df+cg)+b^2\left(6d^2f^2-4cdfg+c^2g^2\right)\right)x+3b^2d^2(bc-ad)g^3(4bdf-bcg-adg)x^2+2b^3d^3(bc-ad)g^4x^3+6d^4(bf-ag)^4\log(a+bx)-6b^4(df-cg)^4\log(c+dx)\right)}{3b^4d^4}}{4g}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]),x]
```

```
[Out] ((f + g*x)^4*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]) - (B*(6*b*d*(b*c - a*
d)*g^2*(a^2*d^2*g^2 + a*b*d*g*(-4*d*f + c*g) + b^2*(6*d^2*f^2 - 4*c*d*f*g +
 c^2*g^2))*x + 3*b^2*d^2*(b*c - a*d)*g^3*(4*b*d*f - b*c*g - a*d*g)*x^2 + 2*
b^3*d^3*(b*c - a*d)*g^4*x^3 + 6*d^4*(b*f - a*g)^4*Log[a + b*x] - 6*b^4*(d*f
 - c*g)^4*Log[c + d*x]))/(3*b^4*d^4))/(4*g)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1965 vs. $2(217) = 434$.
time = 0.45, size = 1966, normalized size = 8.59

method	result
risch	$\frac{g^3Ba^3x}{2b^3}-\frac{g^3Bc^3x}{2d^3}+\frac{2g^2B\ln(bx+a)a^3f}{b^3}+\frac{g^3Ax^4}{4}-\frac{g^3B\ln(bx+a)a^4}{2b^4}+\frac{g^3B\ln(-dx-c)c^4}{2d^4}+\frac{2B\ln(bx+a)a}{b}$
derivativedivides	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^3*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d*(-2*B/(a*d-b*c)*ln(a/(d*x+c)*d-b*c/(d*x+c)+b)*c^2*b*f^3+6*B/d/(a*d-b*c
)*ln(a/(d*x+c)*d-b*c/(d*x+c)+b)*c^3*b*f^2*g+2*B/d*g^2*a/b*(d*x+c)*c*f+6*B/d
```

```
*g^2*a/b*ln(a/(d*x+c)*d-b*c/(d*x+c)+b)*c^2*f-6*B/d^2/(a*d-b*c)*ln(a/(d*x+c)
*d-b*c/(d*x+c)+b)*c^4*b*f*g^2+12*B/d/(a*d-b*c)*ln(a/(d*x+c)*d-b*c/(d*x+c)+b
)*a*c^3*f*g^2+2*B/d/b/(a*d-b*c)*ln(a/(d*x+c)*d-b*c/(d*x+c)+b)*a^2*c^3*g^3+6
*B*d/b/(a*d-b*c)*ln(a/(d*x+c)*d-b*c/(d*x+c)+b)*a^2*c*f^2*g-6*B/b/(a*d-b*c)*
ln(a/(d*x+c)*d-b*c/(d*x+c)+b)*a^2*c^2*f*g^2-A/d^3*(1/4*g^3*(d*x+c)^4-(c^3*g
^3-3*c^2*d*f*g^2+3*c*d^2*f^2*g-d^3*f^3)*(d*x+c)-g^2*(c*g-d*f)*(d*x+c)^3+3/2
*g*(c^2*g^2-2*c*d*f*g+d^2*f^2)*(d*x+c)^2)-B*(d*x+c)*ln(e*(a/(d*x+c)*d-b*c/(
d*x+c)+b)^2/d^2)*f^3-2*B*ln(1/(d*x+c))*c*f^3+3*B/d*ln(1/(d*x+c))*c^2*f^2*g+
B/d^3*g^3*(d*x+c)^3*ln(e*(a/(d*x+c)*d-b*c/(d*x+c)+b)^2/d^2)*c-1/2*B*g^3*a^3
/b^3*(d*x+c)-1/4*B/d^3*g^3*(d*x+c)^4*ln(e*(a/(d*x+c)*d-b*c/(d*x+c)+b)^2/d^2
)-3*B*g/b*(d*x+c)*a*f^2+B/d^3*(d*x+c)*ln(e*(a/(d*x+c)*d-b*c/(d*x+c)+b)^2/d^
2)*c^3*g^3-4*B/d^2*g^2*c^3*ln(a/(d*x+c)*d-b*c/(d*x+c)+b)*f-3/2*B/d^3*(d*x+c
)^2*ln(e*(a/(d*x+c)*d-b*c/(d*x+c)+b)^2/d^2)*c^2*g^3+1/4*B/d*g^3*a^2/b^2*(d*
x+c)^2+3*B/d*g*(d*x+c)*c*f^2+2*B*g^2*a^2/b^2*(d*x+c)*f-4*B/d^2*g^2*c^2*(d*x
+c)*f+2*B*d/b*ln(1/(d*x+c))*a*f^3+1/2*B/d^3*g^3*c^4*ln(1/(d*x+c))-3/4*B/d^3
*g^3*c^2*(d*x+c)^2+3/2*B/d^3*g^3*c^3*(d*x+c)+1/6*B/d^3*g^3*c*(d*x+c)^3+3/2*
B/d^3*g^3*c^4*ln(a/(d*x+c)*d-b*c/(d*x+c)+b)+B/d^2*g^2*c*(d*x+c)^2*f+3*B/d*g
*ln(a/(d*x+c)*d-b*c/(d*x+c)+b)*c^2*f^2-1/2*B*d*g^3*a^4/b^4*ln(1/(d*x+c))-2*
B*d^2/b/(a*d-b*c)*ln(a/(d*x+c)*d-b*c/(d*x+c)+b)*a^2*f^3-4*B/d^2/(a*d-b*c)*l
n(a/(d*x+c)*d-b*c/(d*x+c)+b)*a*c^4*g^3+4*B*d/(a*d-b*c)*ln(a/(d*x+c)*d-b*c/(
d*x+c)+b)*a*c*f^3-3*B/d^2*(d*x+c)*ln(e*(a/(d*x+c)*d-b*c/(d*x+c)+b)^2/d^2)*c
^2*f*g^2-1/2*B/d*g^3*a^2/b^2*(d*x+c)*c-1/2*B/d^2*g^3*a/b*(d*x+c)*c^2-2*B*d*
g^2*a^3/b^3*ln(a/(d*x+c)*d-b*c/(d*x+c)+b)*f+2*B/d^3/(a*d-b*c)*ln(a/(d*x+c)*
d-b*c/(d*x+c)+b)*c^5*b*g^3+3*B/d*(d*x+c)*ln(e*(a/(d*x+c)*d-b*c/(d*x+c)+b)^2
/d^2)*c*f^2*g-3*B*d*g/b^2*ln(1/(d*x+c))*a^2*f^2+2*B*d*g^2*a^3/b^3*ln(1/(d*x
+c))*f-6*B*g/b*ln(a/(d*x+c)*d-b*c/(d*x+c)+b)*a*c*f^2-12*B/(a*d-b*c)*ln(a/(d
*x+c)*d-b*c/(d*x+c)+b)*a*c^2*f^2*g-2*B/d^2*g^3*a/b*ln(a/(d*x+c)*d-b*c/(d*x+
c)+b)*c^3+1/2*B/d^2*g^3*a/b*(d*x+c)^2*c-B/d*g^2*a/b*(d*x+c)^2*f+3*B*d*g/b^2
*ln(a/(d*x+c)*d-b*c/(d*x+c)+b)*a^2*f^2+3*B/d^2*(d*x+c)^2*ln(e*(a/(d*x+c)*d-
b*c/(d*x+c)+b)^2/d^2)*c*f*g^2+1/2*B*d*g^3*a^4/b^4*ln(a/(d*x+c)*d-b*c/(d*x+c
)+b)-B/d^2*g^2*(d*x+c)^3*ln(e*(a/(d*x+c)*d-b*c/(d*x+c)+b)^2/d^2)*f-3/2*B/d*
g*(d*x+c)^2*ln(e*(a/(d*x+c)*d-b*c/(d*x+c)+b)^2/d^2)*f^2-2*B/d^2*ln(1/(d*x+c
))*c^3*f*g^2-1/6*B/d^2*g^3*a/b*(d*x+c)^3)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 635 vs. $2(218) = 436$.
time = 0.31, size = 635, normalized size = 2.77

$$\frac{1}{4}Ag^3x^4 + Afg^2x^3 + \frac{3}{2}Af^2gx^2 + \left(x\log\left(\frac{b^2x^2e}{d^2x^2+2cdx+c^2} + \frac{2abxe}{d^2x^2+2cdx+c^2} + \frac{a^2e}{d^2x^2+2cdx+c^2}\right) + \frac{2a\log(bx+a)}{b} - \frac{2c\log(dx+c)}{d}\right)Bf^3 + \frac{3}{2}\left(x^2\log\left(\frac{b^2x^2e}{d^2x^2+2cdx+c^2} + \frac{2abxe}{d^2x^2+2cdx+c^2} + \frac{a^2e}{d^2x^2+2cdx+c^2}\right) - \frac{2a^2\log(bx+a)}{b^2} + \frac{2c^2\log(dx+c)}{d^2} - \frac{2(bc-ad)x}{bd}\right)Bf^2g + \left(x^3\log\left(\frac{b^2x^2e}{d^2x^2+2cdx+c^2} + \frac{2abxe}{d^2x^2+2cdx+c^2} + \frac{a^2e}{d^2x^2+2cdx+c^2}\right) + \frac{2a^3\log(bx+a)}{b^3} - \frac{2c^3\log(dx+c)}{d^3} - \frac{(b^2cd-abd^2)x^2 - 2(b^2c^2-a^2d^2)x}{b^2d^2}\right)Bfg^2 + \frac{1}{12}\left(3x^4\log\left(\frac{b^2x^2e}{d^2x^2+2cdx+c^2} + \frac{2abxe}{d^2x^2+2cdx+c^2} + \frac{a^2e}{d^2x^2+2cdx+c^2}\right) - \frac{6a^4\log(bx+a)}{b^4} + \frac{6c^4\log(dx+c)}{d^4} - \frac{2(b^3cd^2-ab^2d^3)x^3 - 3(b^3c^2d-a^2bd^3)x^2 + 6(b^3c^3-a^3d^3)x}{b^3d^3}\right)Bg^3 + Af^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="maxima")
```

```
[Out] 1/4*A*g^3*x^4 + A*f*g^2*x^3 + 3/2*A*f^2*g*x^2 + (x*log(b^2*x^2*e/(d^2*x^2 +
 2*c*d*x + c^2) + 2*a*b*x*e/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*
```

```
c*d*x + c^2)) + 2*a*log(b*x + a)/b - 2*c*log(d*x + c)/d)*B*f^3 + 3/2*(x^2*l
og(b^2*x^2*e/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*x*e/(d^2*x^2 + 2*c*d*x + c^2
) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) - 2*a^2*log(b*x + a)/b^2 + 2*c^2*log(d
*x + c)/d^2 - 2*(b*c - a*d)*x/(b*d))*B*f^2*g + (x^3*log(b^2*x^2*e/(d^2*x^2
+ 2*c*d*x + c^2) + 2*a*b*x*e/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2
*c*d*x + c^2)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*
d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*B*f*g^2 + 1/12*(3*x^
4*log(b^2*x^2*e/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*x*e/(d^2*x^2 + 2*c*d*x +
c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) - 6*a^4*log(b*x + a)/b^4 + 6*c^4*lo
g(d*x + c)/d^4 - (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)
*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3))*B*g^3 + A*f^3*x
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 466 vs. $2(218) = 436$.
time = 0.48, size = 466, normalized size = 2.03

$$\frac{3\,Ab^4d^4g^3x^4 + 2\,(6\,Ab^4d^4fg^2 - (Bb^4cd^3 - Bab^3d^4)g^3)x^3 + 3\,(6\,Ab^4d^4f^2g - 4\,(Bb^4cd^3 - Bab^3d^4)fg^2 + (Bb^4c^2d^2 - Ba^2b^2d^4)g^3)x^2 + 6\,(2\,Ab^4d^4f^3 - 6\,(Bb^4cd^3 - Bab^3d^4)f^2g + 4\,(Bb^4c^2d^2 - Ba^2b^2d^4)fg^2 - (Bb^4c^3d - Ba^3bd^4)g^3)x + 6\,(4\,Bab^3d^4f^3 - 6\,Ba^2b^2d^4f^2g + 4\,Ba^3bd^4fg^2 - Ba^4d^4g^3)\log\left(bx + a\right) - 6\,(4\,Bb^4cd^3f^3 - 6\,Bb^4c^2d^2f^2g + 4\,Bb^4c^3dfg^2 - Bb^4c^4g^3)\log\left(dx + c\right) + 3\,(Bb^4d^4g^3x^4 + 4\,Bb^4d^4fg^2x^3 + 6\,Bb^4d^4f^2gx^2 + 4\,Bb^4d^4f^3x)\log\left(\frac{(b^2x^2 + 2\,abx + a^2)e}{d^2x^2 + 2\,cdx + c^2}\right)}{12\,b^4d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="fricas")
```

```
[Out] 1/12*(3*A*b^4*d^4*g^3*x^4 + 2*(6*A*b^4*d^4*f*g^2 - (B*b^4*c*d^3 - B*a*b^3*d
^4)*g^3)*x^3 + 3*(6*A*b^4*d^4*f^2*g - 4*(B*b^4*c*d^3 - B*a*b^3*d^4)*f*g^2 +
 (B*b^4*c^2*d^2 - B*a^2*b^2*d^4)*g^3)*x^2 + 6*(2*A*b^4*d^4*f^3 - 6*(B*b^4*c
*d^3 - B*a*b^3*d^4)*f^2*g + 4*(B*b^4*c^2*d^2 - B*a^2*b^2*d^4)*f*g^2 - (B*b^
4*c^3*d - B*a^3*b*d^4)*g^3)*x + 6*(4*B*a*b^3*d^4*f^3 - 6*B*a^2*b^2*d^4*f^2*
g + 4*B*a^3*b*d^4*f*g^2 - B*a^4*d^4*g^3)*log(b*x + a) - 6*(4*B*b^4*c*d^3*f^
3 - 6*B*b^4*c^2*d^2*f^2*g + 4*B*b^4*c^3*d*f*g^2 - B*b^4*c^4*g^3)*log(d*x +
c) + 3*(B*b^4*d^4*g^3*x^4 + 4*B*b^4*d^4*f*g^2*x^3 + 6*B*b^4*d^4*f^2*g*x^2 +
 4*B*b^4*d^4*f^3*x)*log((b^2*x^2 + 2*a*b*x + a^2)*e/(d^2*x^2 + 2*c*d*x + c^
2)))/(b^4*d^4)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 998 vs. $2(211) = 422$.
time = 9.14, size = 998, normalized size = 4.36

$$\frac{Ag^3x^4}{4} - \frac{Ba(ag - 2bf)\left(a^2g^2 - 2abfg + 2b^2f^2\right)\log\left(x + \frac{Ba^4cd^3g^3 - 4Ba^3bcd^3fg^2 + 6Ba^2b^2cd^3f^2g + \frac{Ba^2d^4(ag - 2bf)\left(a^2g^2 - 2abfg + 2b^2f^2\right)}{b} + Bab^3c^4g^3 - 4Bab^3c^3dfg^2 + 6Bab^3c^2d^2f^2g - 8Bab^3cd^3f^3 - Bacd^3(ag - 2bf)\left(a^2g^2 - 2abfg + 2b^2f^2\right)}{Ba^4d^4g^3 - 4Ba^3bd^4fg^2 + 6Ba^2b^2d^4f^2g - 4Bab^3d^4f^3 + Bb^4c^4g^3 - 4Bb^4c^3dfg^2 + 6Bb^4c^2d^2f^2g - 4Bb^4cd^3f^3}\right)}{2b^4} + \frac{Bc(cg - 2df)\left(c^2g^2 - 2cdfg + 2d^2f^2\right)\log\left(x + \frac{Ba^4cd^3g^3 - 4Ba^3bcd^3fg^2 + 6Ba^2b^2cd^3f^2g + Bab^3c^4g^3 - 4Bab^3c^3dfg^2 + 6Bab^3c^2d^2f^2g - 8Bab^3cd^3f^3 - Bab^3c(cg - 2df)\left(c^2g^2 - 2cdfg + 2d^2f^2\right) + \frac{Bb^4c^2(cg - 2df)\left(c^2g^2 - 2cdfg + 2d^2f^2\right)}{d}}{Ba^4d^4g^3 - 4Ba^3bd^4fg^2 + 6Ba^2b^2d^4f^2g - 4Bab^3d^4f^3 + Bb^4c^4g^3 - 4Bb^4c^3dfg^2 + 6Bb^4c^2d^2f^2g - 4Bb^4cd^3f^3}\right)}{2d^4} + x^3\left(Afg^2 + \frac{Bag^3}{6b} - \frac{Bcg^3}{6d}\right) + x^2\cdot\left(\frac{3Af^2g}{2} - \frac{Ba^2g^3}{4b^2} + \frac{Bafg^2}{b} + \frac{Bc^2g^3}{4d^2} - \frac{Bcfg^2}{d}\right) + x\left(Af^3 + \frac{Ba^3g^3}{2b^3} - \frac{2Ba^2fg^2}{b^2} + \frac{3Baf^2g}{b} - \frac{Bc^3g^3}{2d^3} + \frac{2Bc^2fg^2}{d^2} - \frac{3Bcf^2g}{d}\right) + \left(Bf^3x + \frac{3Bf^2gx^2}{2} + Bfg^2x^3 + \frac{Bg^3x^4}{4}\right)\log\left(\frac{e(a + bx)^2}{(c + dx)^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**3*(A+B*ln(e*(b*x+a)**2/(d*x+c)**2)),x)
```

```
[Out] A*g**3*x**4/4 - B*a*(a*g - 2*b*f)*(a**2*g**2 - 2*a*b*f*g + 2*b**2*f**2)*log
(x + (B*a**4*c*d**3*g**3 - 4*B*a**3*b*c*d**3*f*g**2 + 6*B*a**2*b**2*c*d**3*
f**2*g + B*a**2*d**4*(a*g - 2*b*f)*(a**2*g**2 - 2*a*b*f*g + 2*b**2*f**2)/b
+ B*a*b**3*c**4*g**3 - 4*B*a*b**3*c**3*d*f*g**2 + 6*B*a*b**3*c**2*d**2*f**2
*g - 8*B*a*b**3*c*d**3*f**3 - B*a*c*d**3*(a*g - 2*b*f)*(a**2*g**2 - 2*a*b*f
```

```
*g + 2*b**2*f**2))/(B*a**4*d**4*g**3 - 4*B*a**3*b*d**4*f*g**2 + 6*B*a**2*b*
*2*d**4*f**2*g - 4*B*a*b**3*d**4*f**3 + B*b**4*c**4*g**3 - 4*B*b**4*c**3*d*
f*g**2 + 6*B*b**4*c**2*d**2*f**2*g - 4*B*b**4*c*d**3*f**3))/(2*b**4) + B*c*
(c*g - 2*d*f)*(c**2*g**2 - 2*c*d*f*g + 2*d**2*f**2)*log(x + (B*a**4*c*d**3*
g**3 - 4*B*a**3*b*c*d**3*f*g**2 + 6*B*a**2*b**2*c*d**3*f**2*g + B*a*b**3*c*
*4*g**3 - 4*B*a*b**3*c**3*d*f*g**2 + 6*B*a*b**3*c**2*d**2*f**2*g - 8*B*a*b*
*3*c*d**3*f**3 - B*a*b**3*c*(c*g - 2*d*f)*(c**2*g**2 - 2*c*d*f*g + 2*d**2*f
**2) + B*b**4*c**2*(c*g - 2*d*f)*(c**2*g**2 - 2*c*d*f*g + 2*d**2*f**2)/d)/(
B*a**4*d**4*g**3 - 4*B*a**3*b*d**4*f*g**2 + 6*B*a**2*b**2*d**4*f**2*g - 4*B
*a*b**3*d**4*f**3 + B*b**4*c**4*g**3 - 4*B*b**4*c**3*d*f*g**2 + 6*B*b**4*c*
*2*d**2*f**2*g - 4*B*b**4*c*d**3*f**3))/(2*d**4) + x**3*(A*f*g**2 + B*a*g**
3/(6*b) - B*c*g**3/(6*d)) + x**2*(3*A*f**2*g/2 - B*a**2*g**3/(4*b**2) + B*a
*f*g**2/b + B*c**2*g**3/(4*d**2) - B*c*f*g**2/d) + x*(A*f**3 + B*a**3*g**3/
(2*b**3) - 2*B*a**2*f*g**2/b**2 + 3*B*a*f**2*g/b - B*c**3*g**3/(2*d**3) + 2
*B*c**2*f*g**2/d**2 - 3*B*c*f**2*g/d) + (B*f**3*x + 3*B*f**2*g*x**2/2 + B*f
*g**2*x**3 + B*g**3*x**4/4)*log(e*(a + b*x)**2/(c + d*x)**2)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**B**]
time = 5.03, size = 743, normalized size = 3.24

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^3*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2)),x)
```

```
[Out] log((e*(a + b*x)^2)/(c + d*x)^2)*((B*g^3*x^4)/4 + B*f^3*x + (3*B*f^2*g*x^2)
/2 + B*f*g^2*x^3) + x*((2*A*b*d*f^3 + 6*A*a*c*f*g^2 + 6*A*a*d*f^2*g + 6*A*b
*c*f^2*g + 6*B*a*d*f^2*g - 6*B*b*c*f^2*g)/(2*b*d) + ((2*a*d + 2*b*c)*((((2*
A*a*d*g^3 + 2*A*b*c*g^3 + B*a*d*g^3 - B*b*c*g^3 + 6*A*b*d*f*g^2)/(2*b*d) -
(A*g^3*(2*a*d + 2*b*c))/(2*b*d))*(2*a*d + 2*b*c))/(2*b*d) - (2*A*a*c*g^3 +
6*A*a*d*f*g^2 + 6*A*b*c*f*g^2 + 6*A*b*d*f^2*g + 4*B*a*d*f*g^2 - 4*B*b*c*f*g
^2)/(2*b*d) + (A*a*c*g^3)/(b*d)))/(2*b*d) - (a*c*((2*A*a*d*g^3 + 2*A*b*c*g^
3 + B*a*d*g^3 - B*b*c*g^3 + 6*A*b*d*f*g^2)/(2*b*d) - (A*g^3*(2*a*d + 2*b*c)
)/(2*b*d)))/(b*d)) - x^2*((((2*A*a*d*g^3 + 2*A*b*c*g^3 + B*a*d*g^3 - B*b*c*
g^3 + 6*A*b*d*f*g^2)/(2*b*d) - (A*g^3*(2*a*d + 2*b*c))/(2*b*d))*(2*a*d + 2*
b*c))/(4*b*d) - (2*A*a*c*g^3 + 6*A*a*d*f*g^2 + 6*A*b*c*f*g^2 + 6*A*b*d*f^2*
```

```
g + 4*B*a*d*f*g^2 - 4*B*b*c*f*g^2)/(4*b*d) + (A*a*c*g^3)/(2*b*d)) + x^3*((2
*A*a*d*g^3 + 2*A*b*c*g^3 + B*a*d*g^3 - B*b*c*g^3 + 6*A*b*d*f*g^2)/(6*b*d) -
 (A*g^3*(2*a*d + 2*b*c))/(6*b*d)) + (A*g^3*x^4)/4 - (log(a + b*x)*(B*a^4*g^
3 - 4*B*a*b^3*f^3 + 6*B*a^2*b^2*f^2*g - 4*B*a^3*b*f*g^2))/(2*b^4) + (log(c
+ d*x)*(B*c^4*g^3 - 4*B*c*d^3*f^3 + 6*B*c^2*d^2*f^2*g - 4*B*c^3*d*f*g^2))/(
2*d^4)
```

3.264 $\int (f+gx)^2 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx$

Optimal. Leaf size=152

$$-\frac{2B(bc-ad)g(3bdf-bcg-adg)x}{3b^2d^2}-\frac{B(bc-ad)g^2x^2}{3bd}-\frac{2B(bf-ag)^3\log(a+bx)}{3b^3g}+\frac{(f+gx)^3\left(A+B\log\left(\frac{e}{(}\right.\right.}{3g}$$

```
[Out] -2/3*B*(-a*d+b*c)*g*(-a*d*g-b*c*g+3*b*d*f)*x/b^2/d^2-1/3*B*(-a*d+b*c)*g^2*x
^2/b/d-2/3*B*(-a*g+b*f)^3*ln(b*x+a)/b^3/g+1/3*(g*x+f)^3*(A+B*ln(e*(b*x+a)^2
/(d*x+c)^2))/g+2/3*B*(-c*g+d*f)^3*ln(d*x+c)/d^3/g
```

Rubi [A]
time = 0.10, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {2548, 84}

$$\frac{(f+gx)^3\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{3g}-\frac{2B(bf-ag)^3\log(a+bx)}{3b^3g}-\frac{2Bgx(bc-ad)(-adg-bcg+3bdf)}{3b^2d^2}-\frac{Bg^2x^2(bc-ad)}{3bd}+\frac{2B(df-cg)^3\log(c+dx)}{3d^3g}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]),x]
```

```
[Out] (-2*B*(b*c - a*d)*g*(3*b*d*f - b*c*g - a*d*g)*x)/(3*b^2*d^2) - (B*(b*c - a*
d)*g^2*x^2)/(3*b*d) - (2*B*(b*f - a*g)^3*Log[a + b*x])/(3*b^3*g) + ((f + g*
x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]))/(3*g) + (2*B*(d*f - c*g)^3*L
og[c + d*x])/(3*d^3*g)
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 2548

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Dist[B*n*((b*c
- a*d)/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /;
 FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c -
 a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Rubi steps

$$\int (f+gx)^2 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right) dx = \frac{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{3g} - \frac{B\int \frac{2(bc-ad)(f+gx)^3}{(a+bx)(c+dx)}dx}{3g}$$
$$= \frac{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{3g} - \frac{(2B(bc-ad))\int \frac{(f+g}{(a+bx)(}}{3g}$$
$$= \frac{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{3g} - \frac{(2B(bc-ad))\int \left(\frac{g^2(3bc}{}\right.}{}$$
$$= -\frac{2B(bc-ad)g(3bdf-bcg-adg)x}{3b^2d^2} - \frac{B(bc-ad)g^2x^2}{3bd} - \frac{2}{}$$

Mathematica [A]
time = 0.10, size = 142, normalized size = 0.93

$$\frac{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right) - \frac{B\left(2bd(bc-ad)g^2(3bdf-bcg-adg)x+b^2d^2(bc-ad)g^3x^2+2d^3(bf-ag)^3\log(a+bx)-2b^3(df-cg)^3\log(c+dx)\right)}{b^3d^3}}{3g}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]),x]
```

```
[Out] ((f + g*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]) - (B*(2*b*d*(b*c - a*
d)*g^2*(3*b*d*f - b*c*g - a*d*g)*x + b^2*d^2*(b*c - a*d)*g^3*x^2 + 2*d^3*(b
*f - a*g)^3*Log[a + b*x] - 2*b^3*(d*f - c*g)^3*Log[c + d*x]))/(b^3*d^3))/(3
*g)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1209 vs. $2(142) = 284$.
time = 0.38, size = 1210, normalized size = 7.96 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d*(4*B/d/(a*d-b*c)*ln(a/(d*x+c)*d-b*c/(d*x+c)+b)*c^3*b*f*g-2*B*g/b*(d*x+
c)*a*f-1/3*B/d^2*g^2*(d*x+c)^3*ln(e*(a/(d*x+c)*d-b*c/(d*x+c)+b)^2/d^2)-2/3*
B/d^2*g^2*c^3*ln(1/(d*x+c))-4/3*B/d^2*g^2*c^2*(d*x+c)+4*B*d/b/(a*d-b*c)*ln(
a/(d*x+c)*d-b*c/(d*x+c)+b)*a^2*c*f*g+2/3*B*g^2*a^2/b^2*(d*x+c)-4/3*B/d^2*g^
2*c^3*ln(a/(d*x+c)*d-b*c/(d*x+c)+b)-2*B*ln(1/(d*x+c))*c*f^2+A/d^2*(-1/3*g^2
*(d*x+c)^3-(c^2*g^2-2*c*d*f*g+d^2*f^2)*(d*x+c)+g*(c*g-d*f)*(d*x+c)^2)-B*(d*
x+c)*ln(e*(a/(d*x+c)*d-b*c/(d*x+c)+b)^2/d^2)*f^2+1/3*B/d^2*g^2*c*(d*x+c)^2-
2*B*d^2/b/(a*d-b*c)*ln(a/(d*x+c)*d-b*c/(d*x+c)+b)*a^2*f^2+2*B*d*g/b^2*ln(a/
(d*x+c)*d-b*c/(d*x+c)+b)*a^2*f-B/d^2*(d*x+c)*ln(e*(a/(d*x+c)*d-b*c/(d*x+c)+
b)^2/d^2)*c^2*g^2+2*B*d/b*ln(1/(d*x+c))*a*f^2-8*B/(a*d-b*c)*ln(a/(d*x+c)*d-
```

```
b*c/(d*x+c)+b)*a*c^2*f*g+4*B*d/(a*d-b*c)*ln(a/(d*x+c)*d-b*c/(d*x+c)+b)*a*c*
f^2+2/3*B/d*g^2*a/b*(d*x+c)*c-1/3*B/d*g^2*a/b*(d*x+c)^2-B/d*g*(d*x+c)^2*ln(
e*(a/(d*x+c)*d-b*c/(d*x+c)+b)^2/d^2)*f+2*B/d*g*(d*x+c)*c*f-4*B*g/b*ln(a/(d*
x+c)*d-b*c/(d*x+c)+b)*a*c*f-2*B/d^2/(a*d-b*c)*ln(a/(d*x+c)*d-b*c/(d*x+c)+b)
*c^4*b*g^2-2*B*d*g/b^2*ln(1/(d*x+c))*a^2*f+2*B/d*(d*x+c)*ln(e*(a/(d*x+c)*d-
b*c/(d*x+c)+b)^2/d^2)*c*f*g-2*B/b/(a*d-b*c)*ln(a/(d*x+c)*d-b*c/(d*x+c)+b)*a
^2*c^2*g^2+4*B/d/(a*d-b*c)*ln(a/(d*x+c)*d-b*c/(d*x+c)+b)*a*c^3*g^2-2/3*B*d*
g^2*a^3/b^3*ln(a/(d*x+c)*d-b*c/(d*x+c)+b)+B/d^2*(d*x+c)^2*ln(e*(a/(d*x+c)*d
-b*c/(d*x+c)+b)^2/d^2)*c*g^2+2/3*B*d*g^2*a^3/b^3*ln(1/(d*x+c))+2*B/d*g^2*a/
b*ln(a/(d*x+c)*d-b*c/(d*x+c)+b)*c^2+2*B/d*g*ln(a/(d*x+c)*d-b*c/(d*x+c)+b)*c
^2*f-2*B/(a*d-b*c)*ln(a/(d*x+c)*d-b*c/(d*x+c)+b)*c^2*b*f^2+2*B/d*ln(1/(d*x+
c))*c^2*f*g)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 428 vs. $2(143) = 286$.
time = 0.29, size = 428, normalized size = 2.82

$$\frac{1}{3}\,Ag^2x^3 + Afgx^2 + \left(x\log\left(\frac{b^2x^2e}{d^2x^2 + 2\,cdx + c^2} + \frac{2\,abxe}{d^2x^2 + 2\,cdx + c^2} + \frac{a^2e}{d^2x^2 + 2\,cdx + c^2}\right) + \frac{2\,a\log\left(bx + a\right)}{b} - \frac{2\,c\log\left(dx + c\right)}{d}\right)Bf^2 + \left(x^2\log\left(\frac{b^2x^2e}{d^2x^2 + 2\,cdx + c^2} + \frac{2\,abxe}{d^2x^2 + 2\,cdx + c^2} + \frac{a^2e}{d^2x^2 + 2\,cdx + c^2}\right) - \frac{2\,a^2\log\left(bx + a\right)}{b^2} + \frac{2\,c^2\log\left(dx + c\right)}{d^2} - \frac{2\,(bc - ad)x}{bd}\right)Bfg + \frac{1}{3}\left(x^3\log\left(\frac{b^2x^2e}{d^2x^2 + 2\,cdx + c^2} + \frac{2\,abxe}{d^2x^2 + 2\,cdx + c^2} + \frac{a^2e}{d^2x^2 + 2\,cdx + c^2}\right) + \frac{2\,a^3\log\left(bx + a\right)}{b^3} - \frac{2\,c^3\log\left(dx + c\right)}{d^3} - \frac{(b^2cd - abd^2)x^2 - 2\,(b^2c^2 - a^2d^2)x}{b^2d^2}\right)Bg^2 + Af^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="maxima")
```

```
[Out] 1/3*A*g^2*x^3 + A*f*g*x^2 + (x*log(b^2*x^2*e/(d^2*x^2 + 2*c*d*x + c^2) + 2*
a*b*x*e/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) + 2*a*
log(b*x + a)/b - 2*c*log(d*x + c)/d)*B*f^2 + (x^2*log(b^2*x^2*e/(d^2*x^2 +
2*c*d*x + c^2) + 2*a*b*x*e/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c
*d*x + c^2)) - 2*a^2*log(b*x + a)/b^2 + 2*c^2*log(d*x + c)/d^2 - 2*(b*c - a
*d)*x/(b*d))*B*f*g + 1/3*(x^3*log(b^2*x^2*e/(d^2*x^2 + 2*c*d*x + c^2) + 2*a
*b*x*e/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) + 2*a^3
*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(
b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*B*g^2 + A*f^2*x
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 299 vs. $2(143) = 286$.
time = 0.40, size = 299, normalized size = 1.97

$$\frac{Ab^3d^3g^2x^3 + (3\,Ab^3d^3fg - (Bb^3cd^2 - Bab^2d^3)g^2)x^2 + (3\,Ab^3d^3f^2 - 6\,(Bb^3cd^2 - Bab^2d^3)fg + 2\,(Bb^3c^2d - Ba^2bd^3)g^2)x + 2\,(3\,Bab^2d^3f^2 - 3\,Ba^2bd^3fg + Ba^3d^3g^2)\log\left(bx + a\right) - 2\,(3\,Bb^3cd^2f^2 - 3\,Bb^3c^2dfg + Bb^3c^3g^2)\log\left(dx + c\right) + (Bb^3d^3g^2x^3 + 3\,Bb^3d^3fgx^2 + 3\,Bb^3d^3f^2x)\log\left(\frac{(b^2x^2 + 2\,abx + a^2)e}{d^2x^2 + 2\,cdx + c^2}\right)}{3\,b^3d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="fricas")
```

```
[Out] 1/3*(A*b^3*d^3*g^2*x^3 + (3*A*b^3*d^3*f*g - (B*b^3*c*d^2 - B*a*b^2*d^3)*g^2
)*x^2 + (3*A*b^3*d^3*f^2 - 6*(B*b^3*c*d^2 - B*a*b^2*d^3)*f*g + 2*(B*b^3*c^2
*d - B*a^2*b*d^3)*g^2)*x + 2*(3*B*a*b^2*d^3*f^2 - 3*B*a^2*b*d^3*f*g + B*a^3
*d^3*g^2)*log(b*x + a) - 2*(3*B*b^3*c*d^2*f^2 - 3*B*b^3*c^2*d*f*g + B*b^3*c
^3*g^2)*log(d*x + c) + (B*b^3*d^3*g^2*x^3 + 3*B*b^3*d^3*f*g*x^2 + 3*B*b^3*d
```

```
^3*f^2*x)*log((b^2*x^2 + 2*a*b*x + a^2)*e/(d^2*x^2 + 2*c*d*x + c^2)))/(b^3*
d^3)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 692 vs. $2(139) = 278$.

time = 3.89, size = 692, normalized size = 4.55

$$\frac{Ag^2x^3}{3}+\frac{2Ba(a^2g^2-3abfg+3b^2f^2)\log\left(x+\frac{2Ba^3cd^2g^2-6Ba^2bcd^2fg+\frac{2Ba^2d^3(a^2g^2-3abfg+3b^2f^2)}{b}+2Bab^2c^3g^2-6Bab^2c^2dfg+12Bab^2cd^2f^2-2Bacd^2(a^2g^2-3abfg+3b^2f^2)}{2Ba^3d^3g^2-6Ba^2bd^3fg+6Bab^2d^3f^2+2Bb^3c^3g^2-6Bb^3c^2dfg+6Bb^3cd^2f^2}\right)}{3b^3}-\frac{2Bc(c^2g^2-3cdfg+3d^2f^2)\log\left(x+\frac{2Ba^3cd^2g^2-6Ba^2bcd^2fg+2Bab^2c^3g^2-6Bab^2c^2dfg+12Bab^2cd^2f^2-2Bab^2c(c^2g^2-3cdfg+3d^2f^2)+\frac{2Bb^3c^2(c^2g^2-3cdfg+3d^2f^2)}{d}}{2Ba^3d^3g^2-6Ba^2bd^3fg+6Bab^2d^3f^2+2Bb^3c^3g^2-6Bb^3c^2dfg+6Bb^3cd^2f^2}\right)}{3d^3}+x^2\left(Afg+\frac{Bag^2}{3b}-\frac{Bcg^2}{3d}\right)+x\left(Af^2-\frac{2Ba^2g^2}{3b^2}+\frac{2Bafg}{b}+\frac{2Bc^2g^2}{3d^2}-\frac{2Bcfg}{d}\right)+\left(Bf^2x+Bfgx^2+\frac{Bg^2x^3}{3}\right)\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2*(A+B*ln(e*(b*x+a)**2/(d*x+c)**2)),x)
```

```
[Out] A*g**2*x**3/3 + 2*B*a*(a**2*g**2 - 3*a*b*f*g + 3*b**2*f**2)*log(x + (2*B*a*
*3*c*d**2*g**2 - 6*B*a**2*b*c*d**2*f*g + 2*B*a**2*d**3*(a**2*g**2 - 3*a*b*f
*g + 3*b**2*f**2)/b + 2*B*a*b**2*c**3*g**2 - 6*B*a*b**2*c**2*d*f*g + 12*B*a
*b**2*c*d**2*f**2 - 2*B*a*c*d**2*(a**2*g**2 - 3*a*b*f*g + 3*b**2*f**2))/(2*
B*a**3*d**3*g**2 - 6*B*a**2*b*d**3*f*g + 6*B*a*b**2*d**3*f**2 + 2*B*b**3*c*
*3*g**2 - 6*B*b**3*c**2*d*f*g + 6*B*b**3*c*d**2*f**2))/(3*b**3) - 2*B*c*(c*
*2*g**2 - 3*c*d*f*g + 3*d**2*f**2)*log(x + (2*B*a**3*c*d**2*g**2 - 6*B*a**2
*b*c*d**2*f*g + 2*B*a*b**2*c**3*g**2 - 6*B*a*b**2*c**2*d*f*g + 12*B*a*b**2*
c*d**2*f**2 - 2*B*a*b**2*c*(c**2*g**2 - 3*c*d*f*g + 3*d**2*f**2) + 2*B*b**3
*c**2*(c**2*g**2 - 3*c*d*f*g + 3*d**2*f**2)/d)/(2*B*a**3*d**3*g**2 - 6*B*a*
*2*b*d**3*f*g + 6*B*a*b**2*d**3*f**2 + 2*B*b**3*c**3*g**2 - 6*B*b**3*c**2*d
*f*g + 6*B*b**3*c*d**2*f**2))/(3*d**3) + x**2*(A*f*g + B*a*g**2/(3*b) - B*c
*g**2/(3*d)) + x*(A*f**2 - 2*B*a**2*g**2/(3*b**2) + 2*B*a*f*g/b + 2*B*c**2*
g**2/(3*d**2) - 2*B*c*f*g/d) + (B*f**2*x + B*f*g*x**2 + B*g**2*x**3/3)*log(
e*(a + b*x)**2/(c + d*x)**2)
```

Giac [A]

time = 30.38, size = 279, normalized size = 1.84

$$\frac{1}{3}\,(Ag^2+Bg^2)x^3+\frac{1}{3}\,(Bg^2x^3+3\,Bfgx^2+3\,Bf^2x)\log\left(\frac{b^2x^2+2\,abx+a^2}{d^2x^2+2\,cdx+c^2}\right)+\frac{(3\,Abdfg+3\,Bbdfg-Bbcg^2+Badg^2)x^2}{3\,bd}+\frac{2\,(3\,Bab^2f^2-3\,Ba^2bfg+Ba^3g^2)\log(bx+a)}{3\,b^3}-\frac{2\,(3\,Bcd^2f^2-3\,Bc^2dfg+Bc^3g^2)\log(-dx-c)}{3\,d^3}+\frac{(3\,Ab^2d^2f^2+3\,Bb^2d^2f^2-6\,Bb^2cdfg+6\,Babd^2fg+2\,Bb^2c^2g^2-2\,Ba^2d^2g^2)x}{3\,b^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="giac")
```

```
[Out] 1/3*(A*g^2 + B*g^2)*x^3 + 1/3*(B*g^2*x^3 + 3*B*f*g*x^2 + 3*B*f^2*x)*log((b^
2*x^2 + 2*a*b*x + a^2)/(d^2*x^2 + 2*c*d*x + c^2)) + 1/3*(3*A*b*d*f*g + 3*B*
b*d*f*g - B*b*c*g^2 + B*a*d*g^2)*x^2/(b*d) + 2/3*(3*B*a*b^2*f^2 - 3*B*a^2*b
*f*g + B*a^3*g^2)*log(b*x + a)/b^3 - 2/3*(3*B*c*d^2*f^2 - 3*B*c^2*d*f*g + B
*c^3*g^2)*log(-d*x - c)/d^3 + 1/3*(3*A*b^2*d^2*f^2 + 3*B*b^2*d^2*f^2 - 6*B*
b^2*c*d*f*g + 6*B*a*b*d^2*f*g + 2*B*b^2*c^2*g^2 - 2*B*a^2*d^2*g^2)*x/(b^2*d
^2)
```

Mupad [B]

time = 4.79, size = 362, normalized size = 2.38

$$\ln\left(\frac{e\,{\left(a+b\,x\right)}^2}{{\left(c+d\,x\right)}^2}\right)\,\left(B\,f^2\,x+B\,f\,g\,x^2+\frac{B\,g^2\,x^3}{3}\right)+x^2\,\left(\frac{3\,A\,a\,d\,g^2+3\,A\,b\,c\,g^2+2\,B\,a\,d\,g^2-2\,B\,b\,c\,g^2+6\,A\,b\,d\,f\,g}{6\,b\,d}-\frac{A\,g^2\,\left(3\,a\,d+3\,b\,c\right)}{6\,b\,d}\right)-x\,\left(\frac{\left(\frac{3\,A\,a\,d\,g^2+3\,A\,b\,c\,g^2+2\,B\,a\,d\,g^2-2\,B\,b\,c\,g^2+6\,A\,b\,d\,f\,g}{3\,b\,d}-\frac{A\,g^2\,\left(3\,a\,d+3\,b\,c\right)}{3\,b\,d}\right)\,\left(3\,a\,d+3\,b\,c\right)}{3\,b\,d}-\frac{3\,A\,a\,c\,g^2+3\,A\,b\,d\,f^2+6\,A\,a\,d\,f\,g+6\,A\,b\,c\,f\,g+6\,B\,a\,d\,f\,g-6\,B\,b\,c\,f\,g}{3\,b\,d}+\frac{A\,a\,c\,g^2}{b\,d}\right)+\frac{\ln\left(a+b\,x\right)\,\left(2\,B\,a^3\,g^2-6\,B\,a^2\,b\,f\,g+6\,B\,a\,b^2\,f^2\right)}{3\,b^3}-\frac{\ln\left(c+d\,x\right)\,\left(2\,B\,c^3\,g^2-6\,B\,c^2\,d\,f\,g+6\,B\,c\,d^2\,f^2\right)}{3\,d^3}+\frac{A\,g^2\,x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^2*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2)),x)
```

```
[Out] log((e*(a + b*x)^2)/(c + d*x)^2)*((B*g^2*x^3)/3 + B*f^2*x + B*f*g*x^2) + x^
2*((3*A*a*d*g^2 + 3*A*b*c*g^2 + 2*B*a*d*g^2 - 2*B*b*c*g^2 + 6*A*b*d*f*g)/(6
*b*d) - (A*g^2*(3*a*d + 3*b*c))/(6*b*d)) - x*((((3*A*a*d*g^2 + 3*A*b*c*g^2
+ 2*B*a*d*g^2 - 2*B*b*c*g^2 + 6*A*b*d*f*g)/(3*b*d) - (A*g^2*(3*a*d + 3*b*c)
)/(3*b*d))*(3*a*d + 3*b*c))/(3*b*d) - (3*A*a*c*g^2 + 3*A*b*d*f^2 + 6*A*a*d*
f*g + 6*A*b*c*f*g + 6*B*a*d*f*g - 6*B*b*c*f*g)/(3*b*d) + (A*a*c*g^2)/(b*d))
 + (log(a + b*x)*(2*B*a^3*g^2 + 6*B*a*b^2*f^2 - 6*B*a^2*b*f*g))/(3*b^3) - (
log(c + d*x)*(2*B*c^3*g^2 + 6*B*c*d^2*f^2 - 6*B*c^2*d*f*g))/(3*d^3) + (A*g^
2*x^3)/3
```

3.265 $\int (f+gx) \left(A+B \log \left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right) \, dx$

Optimal. Leaf size=104

$$-\frac{B(bc-ad)gx}{bd}-\frac{B(bf-ag)^2\log(a+bx)}{b^2g}+\frac{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{2g}+\frac{B(df-cg)^2\log(c+dx)}{d^2g}$$

```
[Out] -B*(-a*d+b*c)*g*x/b/d-B*(-a*g+b*f)^2*ln(b*x+a)/b^2/g+1/2*(g*x+f)^2*(A+B*ln(
e*(b*x+a)^2/(d*x+c)^2))/g+B*(-c*g+d*f)^2*ln(d*x+c)/d^2/g
```

Rubi [A]
time = 0.05, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {2548, 84}

$$\frac{(f+gx)^2\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{2g}-\frac{B(bf-ag)^2\log(a+bx)}{b^2g}-\frac{Bgx(bc-ad)}{bd}+\frac{B(df-cg)^2\log(c+dx)}{d^2g}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]),x]
```

```
[Out] -((B*(b*c - a*d)*g*x)/(b*d)) - (B*(b*f - a*g)^2*Log[a + b*x])/(b^2*g) + ((f
 + g*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]))/(2*g) + (B*(d*f - c*g)^
2*Log[c + d*x])/(d^2*g)
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 2548

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Dist[B*n*((b*c
- a*d)/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /;
 FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c -
 a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Rubi steps

$$\int (f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)dx = \frac{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{2g} - \frac{B\int \frac{2(bc-ad)(f+gx)^2}{(a+bx)(c+dx)}dx}{2g}$$

$$= \frac{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{2g} - \frac{(B(bc-ad))\int \frac{(f+gx)^2}{(a+bx)(c+dx)}$$

$$= \frac{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{2g} - \frac{(B(bc-ad))\int\left(\frac{g^2}{bd}+\frac{(}{b(bc}\right.}{}$$

$$= -\frac{B(bc-ad)gx}{bd} - \frac{B(bf-ag)^2\log(a+bx)}{b^2g} + \frac{(f+gx)^2\left(A+\right.}{}$$

Mathematica [A]
time = 0.08, size = 118, normalized size = 1.13

$$\frac{-2Bd^2(bf-ag)^2\log(a+bx)+b\left(d(2B(-bc+ad)g^2x+Abd(f+gx)^2)+bBd^2(f+gx)^2\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+2bB(df-cg)^2\log(c+dx)\right)}{2b^2d^2g}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]),x]
```

```
[Out] (-2*B*d^2*(b*f - a*g)^2*Log[a + b*x] + b*(d*(2*B*(-(b*c) + a*d)*g^2*x + A*b
*d*(f + g*x)^2) + b*B*d^2*(f + g*x)^2*Log[(e*(a + b*x)^2)/(c + d*x)^2] + 2*
b*B*(d*f - c*g)^2*Log[c + d*x]))/(2*b^2*d^2*g)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 620 vs. 2(102) = 204.
time = 0.36, size = 621, normalized size = 5.97

method	result
risch	$\frac{Bx(gx+2f)\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)}{2}+\frac{Agx^2}{2}+Afx+\frac{B\ln(-dx-c)c^2g}{d^2}-\frac{2B\ln(-dx-c)cf}{d}-\frac{B\ln(bx+a)a^2g}{b^2}+\frac{2B\ln(}{}$
derivativedivides	$-\frac{-\frac{A\left(\frac{g(dx+c)^2}{2}-(cg-df)(dx+c)\right)}{d}-\frac{Bg(dx+c)^2\ln\left(\frac{e\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^2}{d^2}\right)}{2d}-\frac{Bg(dx+c)a}{b}+\frac{Bg(dx+c)c}{d}-\frac{Bdg\ln\left(\frac{1}{dx+c}\right)a^2}{b^2}+\frac{Bg\,l}{}}{}$
default	$-\frac{-\frac{A\left(\frac{g(dx+c)^2}{2}-(cg-df)(dx+c)\right)}{d}-\frac{Bg(dx+c)^2\ln\left(\frac{e\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^2}{d^2}\right)}{2d}-\frac{Bg(dx+c)a}{b}+\frac{Bg(dx+c)c}{d}-\frac{Bdg\ln\left(\frac{1}{dx+c}\right)a^2}{b^2}+\frac{Bg\,l}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d*(-A/d*(1/2*g*(d*x+c)^2-(c*g-d*f)*(d*x+c))-1/2*B/d*g*(d*x+c)^2*ln(e*(a/
(d*x+c)*d-b*c/(d*x+c)+b)^2/d^2)-B*g/b*(d*x+c)*a+B/d*g*(d*x+c)*c-B*d*g/b^2*l
n(1/(d*x+c))*a^2+B/d*g*ln(1/(d*x+c))*c^2+B*d*g/b^2*ln(a/(d*x+c)*d-b*c/(d*x+
c)+b)*a^2-2*B*g/b*ln(a/(d*x+c)*d-b*c/(d*x+c)+b)*a*c+B/d*g*ln(a/(d*x+c)*d-b*
c/(d*x+c)+b)*c^2+B/d*(d*x+c)*ln(e*(a/(d*x+c)*d-b*c/(d*x+c)+b)^2/d^2)*c*g-B*
(d*x+c)*ln(e*(a/(d*x+c)*d-b*c/(d*x+c)+b)^2/d^2)*f+2*B*d/b*ln(1/(d*x+c))*a*f
-2*B*ln(1/(d*x+c))*c*f+2*B*d/b/(a*d-b*c)*ln(a/(d*x+c)*d-b*c/(d*x+c)+b)*a^2*
c*g-2*B*d^2/b/(a*d-b*c)*ln(a/(d*x+c)*d-b*c/(d*x+c)+b)*a^2*f-4*B/(a*d-b*c)*l
n(a/(d*x+c)*d-b*c/(d*x+c)+b)*a*c^2*g+4*B*d/(a*d-b*c)*ln(a/(d*x+c)*d-b*c/(d*
x+c)+b)*a*c*f+2*B/d/(a*d-b*c)*ln(a/(d*x+c)*d-b*c/(d*x+c)+b)*c^3*b*g-2*B/(a*
d-b*c)*ln(a/(d*x+c)*d-b*c/(d*x+c)+b)*c^2*b*f)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 252 vs. $2(103) = 206$.
time = 0.29, size = 252, normalized size = 2.42

$$\frac{1}{2}\,Agx^2 + \left(x\log\left(\frac{b^2x^2e}{d^2x^2+2\,cdx+c^2} + \frac{2\,abxe}{d^2x^2+2\,cdx+c^2} + \frac{a^2e}{d^2x^2+2\,cdx+c^2}\right) + \frac{2\,a\log\left(bx+a\right)}{b} - \frac{2\,c\log\left(dx+c\right)}{d}\right)Bf + \frac{1}{2}\left(x^2\log\left(\frac{b^2x^2e}{d^2x^2+2\,cdx+c^2} + \frac{2\,abxe}{d^2x^2+2\,cdx+c^2} + \frac{a^2e}{d^2x^2+2\,cdx+c^2}\right) - \frac{2\,a^2\log\left(bx+a\right)}{b^2} + \frac{2\,c^2\log\left(dx+c\right)}{d^2} - \frac{2\,(bc-ad)x}{bd}\right)Bg + Afx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="maxima")
```

```
[Out] 1/2*A*g*x^2 + (x*log(b^2*x^2*e/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*x*e/(d^2*x
^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) + 2*a*log(b*x + a)/b
 - 2*c*log(d*x + c)/d)*B*f + 1/2*(x^2*log(b^2*x^2*e/(d^2*x^2 + 2*c*d*x + c^
2) + 2*a*b*x*e/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2))
 - 2*a^2*log(b*x + a)/b^2 + 2*c^2*log(d*x + c)/d^2 - 2*(b*c - a*d)*x/(b*d))
*B*g + A*f*x
```

Fricas [A]
time = 0.36, size = 172, normalized size = 1.65

$$\frac{Ab^2d^2gx^2 + 2\,(Ab^2d^2f - (Bb^2cd - Babd^2)g)x + 2\,(2\,Babd^2f - Ba^2d^2g)\log\left(bx+a\right) - 2\,(2\,Bb^2cdf - Bb^2c^2g)\log\left(dx+c\right) + (Bb^2d^2gx^2 + 2\,Bb^2d^2fx)\log\left(\frac{(b^2x^2+2\,abx+a^2)e}{d^2x^2+2\,cdx+c^2}\right)}{2\,b^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="fricas")
```

```
[Out] 1/2*(A*b^2*d^2*g*x^2 + 2*(A*b^2*d^2*f - (B*b^2*c*d - B*a*b*d^2)*g)*x + 2*(2
*B*a*b*d^2*f - B*a^2*d^2*g)*log(b*x + a) - 2*(2*B*b^2*c*d*f - B*b^2*c^2*g)*
log(d*x + c) + (B*b^2*d^2*g*x^2 + 2*B*b^2*d^2*f*x)*log((b^2*x^2 + 2*a*b*x +
 a^2)*e/(d^2*x^2 + 2*c*d*x + c^2)))/(b^2*d^2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 314 vs. $2(88) = 176$.
time = 1.61, size = 314, normalized size = 3.02

$$\frac{Agx^2}{2} - \frac{Ba(ag-2bf)\log\left(x + \frac{Ba^2cdg + \frac{Ba^2d^2(ag-2bf)}{b} + Babc^2g - 4Babcdf - Bacd(ag-2bf)}{Ba^2d^2g - 2Babd^2f + Bb^2c^2g - 2Bb^2cdf}\right)}{b^2} + \frac{Bc(cg-2df)\log\left(x + \frac{Ba^2cdg + Babc^2g - 4Babcdf - Babc(cg-2df) + \frac{Bb^2c^2(cg-2df)}{d}}{Ba^2d^2g - 2Babd^2f + Bb^2c^2g - 2Bb^2cdf}\right)}{d^2} + x\left(Af + \frac{Bag}{b} - \frac{Bcg}{d}\right) + \left(Bfx + \frac{Bgx^2}{2}\right)\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(A+B*ln(e*(b*x+a)**2/(d*x+c)**2)),x)
```

```
[Out] A*g*x**2/2 - B*a*(a*g - 2*b*f)*log(x + (B*a**2*c*d*g + B*a**2*d**2*(a*g - 2
*b*f)/b + B*a*b*c**2*g - 4*B*a*b*c*d*f - B*a*c*d*(a*g - 2*b*f))/(B*a**2*d**
2*g - 2*B*a*b*d**2*f + B*b**2*c**2*g - 2*B*b**2*c*d*f))/b**2 + B*c*(c*g - 2
*d*f)*log(x + (B*a**2*c*d*g + B*a*b*c**2*g - 4*B*a*b*c*d*f - B*a*b*c*(c*g -
 2*d*f) + B*b**2*c**2*(c*g - 2*d*f)/d)/(B*a**2*d**2*g - 2*B*a*b*d**2*f + B*
b**2*c**2*g - 2*B*b**2*c*d*f))/d**2 + x*(A*f + B*a*g/b - B*c*g/d) + (B*f*x
+ B*g*x**2/2)*log(e*(a + b*x)**2/(c + d*x)**2)
```

Giac [A]
time = 6.87, size = 145, normalized size = 1.39

$$\frac{1}{2}\,(Ag+Bg)x^2+\frac{1}{2}\,(Bgx^2+2\,Bfx)\log\left(\frac{b^2x^2+2\,abx+a^2}{d^2x^2+2\,cdx+c^2}\right)+\frac{(Abdf+Bbdf-Bbcg+Badg)x}{bd}+\frac{(2\,Babf-Ba^2g)\log(bx+a)}{b^2}-\frac{(2\,Bcdf-Bc^2g)\log(-dx-c)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="giac")
```

```
[Out] 1/2*(A*g + B*g)*x^2 + 1/2*(B*g*x^2 + 2*B*f*x)*log((b^2*x^2 + 2*a*b*x + a^2)
/(d^2*x^2 + 2*c*d*x + c^2)) + (A*b*d*f + B*b*d*f - B*b*c*g + B*a*d*g)*x/(b*
d) + (2*B*a*b*f - B*a^2*g)*log(b*x + a)/b^2 - (2*B*c*d*f - B*c^2*g)*log(-d*
x - c)/d^2
```

Mupad [B]
time = 4.50, size = 133, normalized size = 1.28

$$\ln\left(\frac{e\,{\left(a+b\,x\right)}^2}{{\left(c+d\,x\right)}^2}\right)\,\left(\frac{B\,g\,x^2}{2}+B\,f\,x\right)+x\,\left(\frac{A\,a\,d\,g+A\,b\,c\,g+A\,b\,d\,f+B\,a\,d\,g-B\,b\,c\,g}{b\,d}-\frac{A\,g\,\left(a\,d+b\,c\right)}{b\,d}\right)+\frac{A\,g\,x^2}{2}-\frac{B\,a\,\ln\left(a+b\,x\right)\,\left(a\,g-2\,b\,f\right)}{b^2}+\frac{B\,c\,\ln\left(c+d\,x\right)\,\left(c\,g-2\,d\,f\right)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2)),x)
```

```
[Out] log((e*(a + b*x)^2)/(c + d*x)^2)*(B*f*x + (B*g*x^2)/2) + x*((A*a*d*g + A*b*
c*g + A*b*d*f + B*a*d*g - B*b*c*g)/(b*d) - (A*g*(a*d + b*c))/(b*d)) + (A*g*
x^2)/2 - (B*a*log(a + b*x)*(a*g - 2*b*f))/b^2 + (B*c*log(c + d*x)*(c*g - 2*
d*f))/d^2
```

3.266 $\int \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx$

Optimal. Leaf size=54

$$Ax+\frac{B(a+bx)\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{b}-\frac{2B(bc-ad)\log(c+dx)}{bd}$$

```
[Out] A*x+B*(b*x+a)*ln(e*(b*x+a)^2/(d*x+c)^2)/b-2*B*(-a*d+b*c)*ln(d*x+c)/b/d
```

Rubi [A]

time = 0.02, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2536, 31}

$$\frac{B(a+bx)\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{b}-\frac{2B(bc-ad)\log(c+dx)}{bd}+Ax$$

Antiderivative was successfully verified.

```
[In] Int[A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2],x]
```

```
[Out] A*x + (B*(a + b*x)*Log[(e*(a + b*x)^2)/(c + d*x)^2])/b - (2*B*(b*c - a*d)*L
og[c + d*x])/(b*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2536

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.), x_Symbol] :> Simp[(a + b*x)*((A + B*Log[e*((a + b*x)^n/(c
+ d*x)^n)])^p/b), x] - Dist[B*n*p*((b*c - a*d)/b), Int[(A + B*Log[e*((a + b
*x)^n/(c + d*x)^n)])^(p - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, A,
B, n}, x] && EqQ[n + mn, 0] && NeQ[b*c - a*d, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\,dx &= Ax+B\int \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\,dx\\
&= Ax+\frac{B(a+bx)\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{b}-\frac{(2B(bc-ad))\int \frac{1}{c+dx}\,dx}{b}\\
&= Ax+\frac{B(a+bx)\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{b}-\frac{2B(bc-ad)\log(c+dx)}{bd}
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 54, normalized size = 1.00

$$Ax+\frac{B(a+bx)\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{b}-\frac{2B(bc-ad)\log(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2],x]
```

```
[Out] A*x + (B*(a + b*x)*Log[(e*(a + b*x)^2)/(c + d*x)^2])/b - (2*B*(b*c - a*d)*L
og[c + d*x])/(b*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 232 vs. $2(54) = 108$.
time = 0.30, size = 233, normalized size = 4.31

method	result
risch	$Ax+Bx\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)-\frac{2Bc\ln(dx+c)}{d}+\frac{2Ba\ln(-bx-a)}{b}$
derivativedivides	$-\frac{-A(dx+c)-B(dx+c)\ln\left(\frac{e\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^2}{d^2}\right)-\frac{2B\ln\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)a^2d^2}{b(ad-cb)}+\frac{4B\ln\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)adc}{ad-cb}-}{d}$
default	$Ax+B\ln\left(\frac{e\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^2}{d^2}\right)x+\frac{B\ln\left(\frac{e\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^2}{d^2}\right)c}{d}+\frac{2Bd\ln\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)a^2}{b(ad-cb)}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(A+B*ln(e*(b*x+a)^2/(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] A*x+B*ln(e*(a/(d*x+c)*d-b*c/(d*x+c)+b)^2/d^2)*x+B/d*ln(e*(a/(d*x+c)*d-b*c/(
d*x+c)+b)^2/d^2)*c+2*B*d/b/(a*d-b*c)*ln(a/(d*x+c)*d-b*c/(d*x+c)+b)*a^2-4*B/
(a*d-b*c)*ln(a/(d*x+c)*d-b*c/(d*x+c)+b)*a*c+2*B/d/(a*d-b*c)*ln(a/(d*x+c)*d-
b*c/(d*x+c)+b)*c^2*b-2*B/b*ln(1/(d*x+c))*a+2*B/d*ln(1/(d*x+c))*c
```

Maxima [A]
time = 0.27, size = 59, normalized size = 1.09

$$\left(2\left(\frac{ae\log(bx+a)}{b}-\frac{ce\log(dx+c)}{d}\right)e^{(-1)}+x\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)\right)B+Ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(A+B*log(e*(b*x+a)^2/(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] (2*(a*e*log(b*x + a)/b - c*e*log(d*x + c)/d)*e^(-1) + x*log((b*x + a)^2*e/(
d*x + c)^2))*B + A*x
```

Fricas [A]
time = 0.34, size = 78, normalized size = 1.44

$$\frac{Bbdx\log\left(\frac{(b^2x^2+2\,abx+a^2)e}{d^2x^2+2\,cdx+c^2}\right)+Abdx+2\,Bad\log\left(bx+a\right)-2\,Bbc\log\left(dx+c\right)}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(A+B*log(e*(b*x+a)^2/(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] (B*b*d*x*log((b^2*x^2 + 2*a*b*x + a^2)*e/(d^2*x^2 + 2*c*d*x + c^2)) + A*b*d
*x + 2*B*a*d*log(b*x + a) - 2*B*b*c*log(d*x + c))/(b*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 104 vs. $2(48) = 96$.
time = 0.51, size = 104, normalized size = 1.93

$$Ax+\frac{2Ba\log\left(x+\frac{\frac{2Ba^2d}{b}+2Bac}{2Bad+2Bbc}\right)}{b}-\frac{2Bc\log\left(x+\frac{2Bac+\frac{2Bbc^2}{d}}{2Bad+2Bbc}\right)}{d}+Bx\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(A+B*ln(e*(b*x+a)**2/(d*x+c)**2),x)
```

```
[Out] A*x + 2*B*a*log(x + (2*B*a**2*d/b + 2*B*a*c)/(2*B*a*d + 2*B*b*c))/b - 2*B*c
*log(x + (2*B*a*c + 2*B*b*c**2/d)/(2*B*a*d + 2*B*b*c))/d + B*x*log(e*(a + b
*x)**2/(c + d*x)**2)
```

Giac [A]
time = 3.75, size = 83, normalized size = 1.54

$$\left(2\,(bc-ad)\left(\frac{a\log\left(|bx+a|\right)}{b^2c-abd}-\frac{c\log\left(|dx+c|\right)}{bcd-ad^2}\right)+x\log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right)\right)B+Ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(A+B*log(e*(b*x+a)^2/(d*x+c)^2),x, algorithm="giac")
```

```
[Out] (2*(b*c - a*d)*(a*log(abs(b*x + a))/(b^2*c - a*b*d) - c*log(abs(d*x + c))/(
b*c*d - a*d^2)) + x*log((b*x + a)^2*e/(d*x + c)^2))*B + A*x
```

Mupad [B]
time = 4.29, size = 50, normalized size = 0.93

$$A\,x+B\,x\,\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)+\frac{2\,B\,a\,\ln\left(a+b\,x\right)}{b}-\frac{2\,B\,c\,\ln\left(c+d\,x\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(A + B*log((e*(a + b*x)^2)/(c + d*x)^2),x)
```

```
[Out] A*x + B*x*log((e*(a + b*x)^2)/(c + d*x)^2) + (2*B*a*log(a + b*x))/b - (2*B*
c*log(c + d*x))/d
```

3.267 $\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{f+gx}\,dx$

Optimal. Leaf size=144

$$-\frac{2B\log\left(-\frac{g(a+bx)}{bf-ag}\right)\log(f+gx)}{g}+\frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\log(f+gx)}{g}+\frac{2B\log\left(-\frac{g(c+dx)}{df-cg}\right)\log(f+gx)}{g}-\frac{2B}{}$$

```
[Out] -2*B*ln(-g*(b*x+a)/(-a*g+b*f))*ln(g*x+f)/g+(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))*
ln(g*x+f)/g+2*B*ln(-g*(d*x+c)/(-c*g+d*f))*ln(g*x+f)/g-2*B*polylog(2,b*(g*x+
f)/(-a*g+b*f))/g+2*B*polylog(2,d*(g*x+f)/(-c*g+d*f))/g
```

Rubi [A]
time = 0.09, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {2546, 2441, 2440, 2438}

$$-\frac{2B\text{PolyLog}\left(2,\frac{b(f+gx)}{bf-ag}\right)}{g}+\frac{2B\text{PolyLog}\left(2,\frac{d(f+gx)}{df-cg}\right)}{g}+\frac{\log(f+gx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{g}-\frac{2B\log(f+gx)\log\left(-\frac{g(a+bx)}{bf-ag}\right)}{g}+\frac{2B\log(f+gx)\log\left(-\frac{g(c+dx)}{df-cg}\right)}{g}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(f + g*x),x]
```

```
[Out] (-2*B*Log[-((g*(a + b*x))/(b*f - a*g))]*Log[f + g*x])/g + ((A + B*Log[(e*(a
 + b*x)^2)/(c + d*x)^2])*Log[f + g*x])/g + (2*B*Log[-((g*(c + d*x))/(d*f -
c*g))]*Log[f + g*x])/g - (2*B*PolyLog[2, (b*(f + g*x))/(b*f - a*g)])/g + (2
*B*PolyLog[2, (d*(f + g*x))/(d*f - c*g)])/g
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
```

```
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2546

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))/((f_.) + (g_.)*(x_)), x_Symbol] :> Simp[Log[f + g*x]*((A + B*Log[
e*((a + b*x)^n/(c + d*x)^n)])/g), x] + (-Dist[b*B*(n/g), Int[Log[f + g*x]/(
a + b*x), x], x] + Dist[B*d*(n/g), Int[Log[f + g*x]/(c + d*x), x], x]) /; F
reeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[n + mn, 0] && NeQ[b*c - a*d,
 0]
```

Rubi steps

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{f+gx}\,dx = \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\log(f+gx)}{g} - \frac{B\int \frac{(c+dx)^2\left(-\frac{2de(a+bx)^2}{(c+dx)^3}+\frac{2be(a+bx)}{(c+dx)^2}\right)\log(f+gx)}{e(a+bx)^2}}{g}$$

$$= \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\log(f+gx)}{g} - \frac{B\int \frac{(c+dx)^2\left(-\frac{2de(a+bx)^2}{(c+dx)^3}+\frac{2be(a+bx)}{(c+dx)^2}\right)\log(f+gx)}{(a+bx)^2}}{eg}$$

$$= \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\log(f+gx)}{g} - \frac{B\int \left(\frac{2be\log(f+gx)}{a+bx}-\frac{2de\log(f+gx)}{c+dx}\right)dx}{eg}$$

$$= \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\log(f+gx)}{g} - \frac{(2bB)\int \frac{\log(f+gx)}{a+bx}\,dx}{g} + \frac{(2Bd)\int \frac{\log(f+g}{c+dx}}{g}$$

$$= -\frac{2B\log\left(-\frac{g(a+bx)}{bf-ag}\right)\log(f+gx)}{g} + \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\log(f+gx)}{g} + \frac{2B}{}$$

$$= -\frac{2B\log\left(-\frac{g(a+bx)}{bf-ag}\right)\log(f+gx)}{g} + \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\log(f+gx)}{g} + \frac{2B}{}$$

$$= -\frac{2B\log\left(-\frac{g(a+bx)}{bf-ag}\right)\log(f+gx)}{g} + \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\log(f+gx)}{g} + \frac{2B}{}$$

Mathematica [A]
time = 0.04, size = 119, normalized size = 0.83

$$\frac{\left(A-2B\log\left(\frac{g(a+bx)}{-bf+ag}\right)+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+2B\log\left(\frac{g(c+dx)}{-df+cg}\right)\right)\log(f+gx)-2B\text{Li}_2\left(\frac{b(f+gx)}{bf-ag}\right)+2B\text{Li}_2\left(\frac{d(f+gx)}{df-cg}\right)}{g}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(f + g*x),x]
```

```
[Out] ((A - 2*B*Log[(g*(a + b*x))/(-(b*f) + a*g)] + B*Log[(e*(a + b*x)^2)/(c + d*
x)^2] + 2*B*Log[(g*(c + d*x))/(-(d*f) + c*g)])*Log[f + g*x] - 2*B*PolyLog[2
, (b*(f + g*x))/(b*f - a*g)] + 2*B*PolyLog[2, (d*(f + g*x))/(d*f - c*g)])/g
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1168 vs. $2(144) = 288$.
time = 3.53, size = 1169, normalized size = 8.12 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/(g*x+f),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d*(d*A/g*ln(1/(d*x+c))-d*A/g*ln(c*g/(d*x+c)-f/(d*x+c)*d-g)+d*B/g*ln(1/(d
*x+c))*ln(e*(a/(d*x+c)*d-b*c/(d*x+c)+b)^2/d^2)-2*d^2*B/g*dilog(((a*d-b*c)/(
d*x+c)+b)/b)/(a*d-b*c)*a+2*d*B/g*dilog(((a*d-b*c)/(d*x+c)+b)/b)/(a*d-b*c)*b
*c-2*d^2*B/g*ln(1/(d*x+c))*ln(((a*d-b*c)/(d*x+c)+b)/b)/(a*d-b*c)*a+2*d*B/g*
ln(1/(d*x+c))*ln(((a*d-b*c)/(d*x+c)+b)/b)/(a*d-b*c)*b*c-d*B*ln((c*g-d*f)/(d
*x+c)-g)/(c*g-d*f)*ln(e*(a/(d*x+c)*d-b*c/(d*x+c)+b)^2/d^2)*c+d^2*B/g*ln((c*
g-d*f)/(d*x+c)-g)/(c*g-d*f)*ln(e*(a/(d*x+c)*d-b*c/(d*x+c)+b)^2/d^2)*f+2*d^2
*B/(c*g-d*f)*dilog(((a*d-b*c)*((c*g-d*f)/(d*x+c)-g)+a*d*g-b*d*f)/(a*d*g-b*d
*f))/(a*d-b*c)*a*c-2*d^3*B/g/(c*g-d*f)*dilog(((a*d-b*c)*((c*g-d*f)/(d*x+c)-
g)+a*d*g-b*d*f)/(a*d*g-b*d*f))/(a*d-b*c)*a*f-2*d*B/(c*g-d*f)*dilog(((a*d-b*
c)*((c*g-d*f)/(d*x+c)-g)+a*d*g-b*d*f)/(a*d*g-b*d*f))/(a*d-b*c)*b*c^2+2*d^2*
B/g/(c*g-d*f)*dilog(((a*d-b*c)*((c*g-d*f)/(d*x+c)-g)+a*d*g-b*d*f)/(a*d*g-b*
d*f))/(a*d-b*c)*b*c*f+2*d^2*B/(c*g-d*f)*ln((c*g-d*f)/(d*x+c)-g)*ln(((a*d-b*
c)*((c*g-d*f)/(d*x+c)-g)+a*d*g-b*d*f)/(a*d*g-b*d*f))/(a*d-b*c)*a*c-2*d^3*B/
g/(c*g-d*f)*ln((c*g-d*f)/(d*x+c)-g)*ln(((a*d-b*c)*((c*g-d*f)/(d*x+c)-g)+a*d
*g-b*d*f)/(a*d*g-b*d*f))/(a*d-b*c)*a*f-2*d*B/(c*g-d*f)*ln((c*g-d*f)/(d*x+c)
-g)*ln(((a*d-b*c)*((c*g-d*f)/(d*x+c)-g)+a*d*g-b*d*f)/(a*d*g-b*d*f))/(a*d-b*
c)*b*c^2+2*d^2*B/g/(c*g-d*f)*ln((c*g-d*f)/(d*x+c)-g)*ln(((a*d-b*c)*((c*g-d*
f)/(d*x+c)-g)+a*d*g-b*d*f)/(a*d*g-b*d*f))/(a*d-b*c)*b*c*f)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(g*x+f),x, algorithm="maxima")
```

```
[Out] -B*integrate(-(2*log(b*x + a) - 2*log(d*x + c) + 1)/(g*x + f), x) + A*log(g
*x + f)/g
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(g*x+f),x, algorithm="fricas")
```

```
[Out] integral((B*log((b^2*x^2 + 2*a*b*x + a^2)*e/(d^2*x^2 + 2*c*d*x + c^2)) + A)
/(g*x + f), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(b*x+a)**2/(d*x+c)**2))/(g*x+f),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(g*x+f),x, algorithm="giac")
```

```
[Out] integrate((B*log((b*x + a)^2*e/(d*x + c)^2) + A)/(g*x + f), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\,\ln\left(\frac{e\,{\left(a+b\,x\right)}^2}{{\left(c+d\,x\right)}^2}\right)}{f+g\,x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))/(f + g*x),x)
```

```
[Out] int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))/(f + g*x), x)
```

3.268 $\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f+gx)^2}\,dx$

Optimal. Leaf size=90

$$\frac{(a+bx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(bf-ag)(f+gx)}+\frac{2B(bc-ad)\log\left(\frac{f+gx}{c+dx}\right)}{(bf-ag)(df-cg)}$$

```
[Out] (b*x+a)*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/(-a*g+b*f)/(g*x+f)+2*B*(-a*d+b*c)*l
n((g*x+f)/(d*x+c))/(-a*g+b*f)/(-c*g+d*f)
```

Rubi [A]
time = 0.06, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2554, 2351, 31}

$$\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{(f+gx)(bf-ag)}+\frac{2B(bc-ad)\log\left(\frac{f+gx}{c+dx}\right)}{(bf-ag)(df-cg)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(f + g*x)^2,x]
```

```
[Out] ((a + b*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]))/((b*f - a*g)*(f + g*x)
) + (2*B*(b*c - a*d)*Log[(f + g*x)/(c + d*x)])/((b*f - a*g)*(d*f - c*g))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2554

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[b*c - a*d, Su
bst[Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m +
2)), x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n
}, x] && EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegerQ[m] &
```

```
& IGtQ[p, 0]
```

Rubi steps

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f+gx)^2}\,dx = -\frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{g(f+gx)} + \frac{B\int \frac{2(bc-ad)}{(a+bx)(c+dx)(f+gx)}\,dx}{g}$$
$$= -\frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{g(f+gx)} + \frac{(2B(bc-ad))\int \frac{1}{(a+bx)(c+dx)(f+gx)}\,dx}{g}$$
$$= -\frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{g(f+gx)} + \frac{(2B(bc-ad))\int \left(\frac{b^2}{(bc-ad)(bf-ag)(a+bx)} + \frac{d^2}{(bc-ad)(-df+cg}\right.}{g}$$
$$= \frac{2bB\log(a+bx)}{g(bf-ag)} - \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{g(f+gx)} - \frac{2Bd\log(c+dx)}{g(df-cg)} + \frac{2B(bc-ad)\,\mathrm{lo}}{(bf-ag)(a}$$

Mathematica [A]
time = 0.09, size = 108, normalized size = 1.20

$$\frac{-\frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{f+gx} + \frac{2B(b(df-cg)\log(a+bx)+(-bdf+adg)\log(c+dx)+(bc-ad)g\log(f+gx))}{(bf-ag)(df-cg)}}{g}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(f + g*x)^2,x]
```

```
[Out] (-((A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(f + g*x)) + (2*B*(b*(d*f - c*g
)*Log[a + b*x] + (-(b*d*f) + a*d*g)*Log[c + d*x] + (b*c - a*d)*g*Log[f + g*
x]))/((b*f - a*g)*(d*f - c*g)))/g
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 298 vs. 2(90) = 180.
time = 0.42, size = 299, normalized size = 3.32

method	result
risch	$-\frac{B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)}{g(gx+f)} - \frac{2B\ln(gx+f)ad\,g^2x-2B\ln(gx+f)bc\,g^2x+2B\ln(-bx-a)bc\,g^2x-2B\ln(-bx-a)bdf\,gx-2B\ln(-}{}$
derivativedivides	$-\frac{-\frac{d^2A}{\left(\frac{cg-df}{dx+c}-g\right)(cg-df)} + \frac{-\frac{bBd\ln\left(\frac{e\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^2}{d^2}\right)}{ag-bf} - \frac{Bd(ad-cb)\ln\left(\frac{e\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^2}{d^2}\right)}{(ag-bf)(dx+c)}}{\frac{cg}{dx+c}-\frac{fd}{dx+c}-g} + \frac{2B\,d^2\ln\left(\frac{cg}{dx+c}-\frac{fd}{dx+c}-\right.}{ac\,g^2-adfg-bcfg+bd\,}}{d}$

default	$-\frac{-\frac{d^{2} A}{\left(\frac{cg-df}{dx+c}-g\right)(cg-df)}+\frac{-\frac{bBd\ln\left(\frac{e\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^{2}}{d^{2}}\right)}{ag-bf}-\frac{Bd(ad-cb)\ln\left(\frac{e\left(\frac{ad}{dx+c}-\frac{bc}{dx+c}+b\right)^{2}}{d^{2}}\right)}{(ag-bf)(dx+c)}}{\frac{cg}{dx+c}-\frac{fd}{dx+c}-g}+\frac{2B\,d^{2}\ln\left(\frac{cg}{dx+c}-\frac{fd}{dx+c}-g\right)}{ac\,g^{2}-adfg-bcfg+bd\,f^{2}}}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/(g*x+f)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/d*(-d^2*A/((c*g-d*f)/(d*x+c)-g)/(c*g-d*f)+(-b*B*d/(a*g-b*f)*ln(e*(a/(d*x
+c)*d-b*c/(d*x+c)+b)^2/d^2)-B*d*(a*d-b*c)/(a*g-b*f)/(d*x+c)*ln(e*(a/(d*x+c)
*d-b*c/(d*x+c)+b)^2/d^2))/(c*g/(d*x+c)-f/(d*x+c)*d-g)+2*B*d^2/(a*c*g^2-a*d*
f*g-b*c*f*g+b*d*f^2)*ln(c*g/(d*x+c)-f/(d*x+c)*d-g)*a-2*B*d/(a*c*g^2-a*d*f*g
-b*c*f*g+b*d*f^2)*ln(c*g/(d*x+c)-f/(d*x+c)*d-g)*b*c)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 195 vs. $2(91) = 182$.
time = 0.28, size = 195, normalized size = 2.17

$$B\left(\frac{2\,b\log\left(bx+a\right)}{bfg-ag^{2}}-\frac{2\,d\log\left(dx+c\right)}{dfg-cg^{2}}+\frac{2\,(bc-ad)\log\left(gx+f\right)}{bdf^{2}+acg^{2}-(bc+ad)fg}-\frac{\log\left(\frac{b^{2}x^{2}e}{d^{2}x^{2}+2\,cdx+c^{2}}+\frac{2\,abxe}{d^{2}x^{2}+2\,cdx+c^{2}}+\frac{a^{2}e}{d^{2}x^{2}+2\,cdx+c^{2}}\right)}{g^{2}x+fg}\right)-\frac{A}{g^{2}x+fg}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(g*x+f)^2,x, algorithm="maxima")
```

```
[Out] B*(2*b*log(b*x + a)/(b*f*g - a*g^2) - 2*d*log(d*x + c)/(d*f*g - c*g^2) + 2*
(b*c - a*d)*log(g*x + f)/(b*d*f^2 + a*c*g^2 - (b*c + a*d)*f*g) - log(b^2*x^
2*e/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*x*e/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e
/(d^2*x^2 + 2*c*d*x + c^2))/(g^2*x + f*g)) - A/(g^2*x + f*g)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 277 vs. $2(91) = 182$.
time = 3.19, size = 277, normalized size = 3.08

$$-\frac{Abdf^{2}+Aacg^{2}-(Abc+Aad)fg-2\,(Bbdf^{2}-Bbcfg+(Bbdfg-Bbcg^{2})x)\log\left(bx+a\right)+2\,(Bbdf^{2}-Badfg+(Bbdfg-Badg^{2})x)\log\left(dx+c\right)-2\,((Bbc-Bad)g^{2}x+(Bbc-Bad)fg)\log\left(gx+f\right)+(Bbdf^{2}+Bacg^{2}-(Bbc+Bad)fg)\log\left(\frac{(b^{2}x^{2}+2\,abx+a^{2})e}{d^{2}x^{2}+2\,cdx+c^{2}}\right)}{bdf^{3}g+acfg^{3}-(bc+ad)f^{2}g^{2}+(bdf^{2}g^{2}+acg^{4}-(bc+ad)fg^{3})x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(g*x+f)^2,x, algorithm="fricas")
```

```
[Out] -(A*b*d*f^2 + A*a*c*g^2 - (A*b*c + A*a*d)*f*g - 2*(B*b*d*f^2 - B*b*c*f*g +
(B*b*d*f*g - B*b*c*g^2)*x)*log(b*x + a) + 2*(B*b*d*f^2 - B*a*d*f*g + (B*b*d
*f*g - B*a*d*g^2)*x)*log(d*x + c) - 2*((B*b*c - B*a*d)*g^2*x + (B*b*c - B*a
*d)*f*g)*log(g*x + f) + (B*b*d*f^2 + B*a*c*g^2 - (B*b*c + B*a*d)*f*g)*log((
b^2*x^2 + 2*a*b*x + a^2)*e/(d^2*x^2 + 2*c*d*x + c^2)))/(b*d*f^3*g + a*c*f*g
^3 - (b*c + a*d)*f^2*g^2 + (b*d*f^2*g^2 + a*c*g^4 - (b*c + a*d)*f*g^3)*x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(b*x+a)**2/(d*x+c)**2))/(g*x+f)**2,x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(g*x+f)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Undef
/Unsigned Inf encountered in limitUndef/Unsigned Inf encountered in limitsa
geVARB*(-(sageVARg*sageVARx+sageVARf)^-1/sageVARg*ln((sageVARb*(-sageVARf+1
/sageVARg/(sageV
```

Mupad [**B**]
time = 5.34, size = 191, normalized size = 2.12

$$\frac{2\,B\,d\,\ln\left(c+d\,x\right)}{c\,g^2-d\,f\,g}-\frac{B\,\ln\left(\frac{e\,a^2+2\,e\,a\,b\,x+e\,b^2\,x^2}{c^2+2\,c\,d\,x+d^2\,x^2}\right)}{x\,g^2+f\,g}-\frac{2\,B\,b\,\ln\left(a+b\,x\right)}{a\,g^2-b\,f\,g}-\frac{A}{x\,g^2+f\,g}-\frac{2\,B\,a\,d\,\ln\left(f+g\,x\right)}{a\,c\,g^2+b\,d\,f^2-a\,d\,f\,g-b\,c\,f\,g}+\frac{2\,B\,b\,c\,\ln\left(f+g\,x\right)}{a\,c\,g^2+b\,d\,f^2-a\,d\,f\,g-b\,c\,f\,g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))/(f + g*x)^2,x)
```

```
[Out] (2*B*d*log(c + d*x))/(c*g^2 - d*f*g) - (B*log((a^2*e + b^2*e*x^2 + 2*a*b*e*
x)/(c^2 + d^2*x^2 + 2*c*d*x)))/(f*g + g^2*x) - (2*B*b*log(a + b*x))/(a*g^2
- b*f*g) - A/(f*g + g^2*x) - (2*B*a*d*log(f + g*x))/(a*c*g^2 + b*d*f^2 - a*
d*f*g - b*c*f*g) + (2*B*b*c*log(f + g*x))/(a*c*g^2 + b*d*f^2 - a*d*f*g - b*
c*f*g)
```

3.269 $\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f+gx)^3}\,dx$

Optimal. Leaf size=175

$$-\frac{B(bc-ad)}{(bf-ag)(df-cg)(f+gx)}+\frac{b^2B\log(a+bx)}{g(bf-ag)^2}-\frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{2g(f+gx)^2}-\frac{Bd^2\log(c+dx)}{g(df-cg)^2}+\frac{B(bc-ad)(2bdf-$$

```
[Out] -B*(-a*d+b*c)/(-a*g+b*f)/(-c*g+d*f)/(g*x+f)+b^2*B*ln(b*x+a)/g/(-a*g+b*f)^2+
1/2*(-A-B*ln(e*(b*x+a)^2/(d*x+c)^2))/g/(g*x+f)^2-B*d^2*ln(d*x+c)/g/(-c*g+d*
f)^2+B*(-a*d+b*c)*(-a*d*g-b*c*g+2*b*d*f)*ln(g*x+f)/(-a*g+b*f)^2/(-c*g+d*f)^
2
```

Rubi [A]
time = 0.13, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {2548, 84}

$$-\frac{B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A}{2g(f+gx)^2}+\frac{b^2B\log(a+bx)}{g(bf-ag)^2}-\frac{B(bc-ad)}{(f+gx)(bf-ag)(df-cg)}+\frac{B(bc-ad)\log(f+gx)(-adg-bcg+2bdf)}{(bf-ag)^2(df-cg)^2}-\frac{Bd^2\log(c+dx)}{g(df-cg)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(f + g*x)^3,x]
```

```
[Out] -((B*(b*c - a*d))/((b*f - a*g)*(d*f - c*g)*(f + g*x))) + (b^2*B*Log[a + b*x
])/(g*(b*f - a*g)^2) - (A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(2*g*(f + g
*x)^2) - (B*d^2*Log[c + d*x])/(g*(d*f - c*g)^2) + (B*(b*c - a*d)*(2*b*d*f -
 b*c*g - a*d*g)*Log[f + g*x])/((b*f - a*g)^2*(d*f - c*g)^2)
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 2548

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Dist[B*n*((b*c
- a*d)/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /;
 FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c -
 a*d, 0] && NeQ[m, -1] && !(EqQ[m, -2] && IntegerQ[n])
```

Rubi steps

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f+gx)^3}\,dx = -\frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{2g(f+gx)^2}+\frac{B\int \frac{2(bc-ad)}{(a+bx)(c+dx)(f+gx)^2}\,dx}{2g}$$
$$= -\frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{2g(f+gx)^2}+\frac{(B(bc-ad))\int \frac{1}{(a+bx)(c+dx)(f+gx)^2}\,dx}{g}$$
$$= -\frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{2g(f+gx)^2}+\frac{(B(bc-ad))\int \left(\frac{b^3}{(bc-ad)(bf-ag)^2(a+bx)}-\frac{d^3}{(bc-ad)(-df+cg}\right.}{}$$
$$= -\frac{B(bc-ad)}{(bf-ag)(df-cg)(f+gx)}+\frac{b^2B\log(a+bx)}{g(bf-ag)^2}-\frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{2g(f+gx)^2}-\frac{B}{}$$

Mathematica [A]
time = 0.35, size = 172, normalized size = 0.98

$$\frac{-\frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f+gx)^2}+2B(bc-ad)\left(\frac{b^2\log(a+bx)}{(bc-ad)(bf-ag)^2}+\frac{\frac{g(-df+cg)}{(bf-ag)(f+gx)}+\frac{d^2\log(c+dx)}{-bc+ad}-\frac{g(-2bdf+bcg+adg)\log(f+gx)}{(bf-ag)^2}}{(df-cg)^2}\right)}{2g}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(f + g*x)^3,x]
```

```
[Out] (-((A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(f + g*x)^2) + 2*B*(b*c - a*d)*
((b^2*Log[a + b*x])/((b*c - a*d)*(b*f - a*g)^2) + ((g*(-(d*f) + c*g))/((b*f
 - a*g)*(f + g*x)) + (d^2*Log[c + d*x])/(-(b*c) + a*d) - (g*(-2*b*d*f + b*c
*g + a*d*g)*Log[f + g*x])/(b*f - a*g)^2)/(d*f - c*g)^2))/(2*g)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1023 vs. $2(176) = 352$.
time = 0.69, size = 1024, normalized size = 5.85 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/(g*x+f)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/d*(-d^3*A*(-1/2*g/(c*g-d*f)^2/(c*g/(d*x+c)-f/(d*x+c)*d-g)^2-1/(c*g-d*f)^
2/(c*g/(d*x+c)-f/(d*x+c)*d-g))+((B*a*d^3*g^2-B*b*c*d^2*g^2)/g^2/(a*g-b*f)/(
d*x+c)^2+b^2*(c*g-d*f)*B*d/(a^2*g^2-2*a*b*f*g+b^2*f^2)/(d*x+c)*ln(e*(a/(d*x
+c)*d-b*c/(d*x+c)+b)^2/d^2)-(B*a*d^3*g^2-B*b*c*d^2*g^2)/g/(a*c*g^2-a*d*f*g-
b*c*f*g+b*d*f^2)/(d*x+c)+1/2*B*d*(a^2*d^2*g-2*a*b*d^2*f-b^2*c^2*g+2*b^2*c*d
*f)/(a^2*g^2-2*a*b*f*g+b^2*f^2)/(d*x+c)^2*ln(e*(a/(d*x+c)*d-b*c/(d*x+c)+b)^
2/d^2)-1/2*b^2*g*B*d/(a^2*g^2-2*a*b*f*g+b^2*f^2)*ln(e*(a/(d*x+c)*d-b*c/(d*x
+c)+b)^2/d^2))/(c*g/(d*x+c)-f/(d*x+c)*d-g)^2-B*d^3/(a^2*c^2*g^4-2*a^2*c*d*f
```

```
*g^3+a^2*d^2*f^2*g^2-2*a*b*c^2*f*g^3+4*a*b*c*d*f^2*g^2-2*a*b*d^2*f^3*g+b^2*
c^2*f^2*g^2-2*b^2*c*d*f^3*g+b^2*d^2*f^4)*ln(c*g/(d*x+c)-f/(d*x+c)*d-g)*a^2*
g+2*B*d^3/(a^2*c^2*g^4-2*a^2*c*d*f*g^3+a^2*d^2*f^2*g^2-2*a*b*c^2*f*g^3+4*a*
b*c*d*f^2*g^2-2*a*b*d^2*f^3*g+b^2*c^2*f^2*g^2-2*b^2*c*d*f^3*g+b^2*d^2*f^4)*
ln(c*g/(d*x+c)-f/(d*x+c)*d-g)*a*b*f+B*d/(a^2*c^2*g^4-2*a^2*c*d*f*g^3+a^2*d^
2*f^2*g^2-2*a*b*c^2*f*g^3+4*a*b*c*d*f^2*g^2-2*a*b*d^2*f^3*g+b^2*c^2*f^2*g^2
-2*b^2*c*d*f^3*g+b^2*d^2*f^4)*ln(c*g/(d*x+c)-f/(d*x+c)*d-g)*b^2*c^2*g-2*B*d
^2/(a^2*c^2*g^4-2*a^2*c*d*f*g^3+a^2*d^2*f^2*g^2-2*a*b*c^2*f*g^3+4*a*b*c*d*f
^2*g^2-2*a*b*d^2*f^3*g+b^2*c^2*f^2*g^2-2*b^2*c*d*f^3*g+b^2*d^2*f^4)*ln(c*g/
(d*x+c)-f/(d*x+c)*d-g)*b^2*c*f)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 408 vs. $2(174) = 348$.
time = 0.31, size = 408, normalized size = 2.33

$$\frac{1}{2}\left(\frac{2\,b^2\log\left(bx+a\right)}{b^2f^2g-2\,abfg^2+a^2g^3}-\frac{2\,d^2\log\left(dx+c\right)}{d^2f^2g-2\,cdfg^2+c^2g^3}+\frac{2\left(2\left(b^2cd-abd^2\right)f-\left(b^2c^2-a^2d^2\right)g\right)\log\left(gx+f\right)}{b^2d^2f^4+a^2c^2g^4-2\left(b^2cd+abd^2\right)f^3g+\left(b^2c^2+4\,abcd+a^2d^2\right)f^2g^2-2\left(abc^2+a^2cd\right)fg^3}-\frac{2\left(bc-ad\right)}{bdf^3+acfg^2-\left(bc+ad\right)f^2g+\left(bdf^2g+acg^3-\left(bc+ad\right)fg^2\right)x}-\frac{\log\left(\frac{b^2x^2e}{d^2x^2+2\,cdx+c^2}+\frac{2\,abxe}{d^2x^2+2\,cdx+c^2}+\frac{a^2e}{d^2x^2+2\,cdx+c^2}\right)}{g^3x^2+2\,fg^2x+f^2g}\right)B-\frac{A}{2\left(g^3x^2+2\,fg^2x+f^2g\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(g*x+f)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*b^2*log(b*x + a)/(b^2*f^2*g - 2*a*b*f*g^2 + a^2*g^3) - 2*d^2*log(d*x
 + c)/(d^2*f^2*g - 2*c*d*f*g^2 + c^2*g^3) + 2*(2*(b^2*c*d - a*b*d^2)*f - (b
^2*c^2 - a^2*d^2)*g)*log(g*x + f)/(b^2*d^2*f^4 + a^2*c^2*g^4 - 2*(b^2*c*d +
 a*b*d^2)*f^3*g + (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*f^2*g^2 - 2*(a*b*c^2 + a^
2*c*d)*f*g^3) - 2*(b*c - a*d)/(b*d*f^3 + a*c*f*g^2 - (b*c + a*d)*f^2*g + (b
*d*f^2*g + a*c*g^3 - (b*c + a*d)*f*g^2)*x) - log(b^2*x^2*e/(d^2*x^2 + 2*c*d
*x + c^2) + 2*a*b*x*e/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x
+ c^2))/(g^3*x^2 + 2*f*g^2*x + f^2*g))*B - 1/2*A/(g^3*x^2 + 2*f*g^2*x + f^2
*g)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1034 vs. $2(174) = 348$.
time = 44.97, size = 1034, normalized size = 5.91

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(g*x+f)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(A*b^2*d^2*f^4 + A*a^2*c^2*g^4 - 2*((A - B)*b^2*c*d + (A + B)*a*b*d^2)
*f^3*g + ((A - 2*B)*b^2*c^2 + 4*A*a*b*c*d + (A + 2*B)*a^2*d^2)*f^2*g^2 - 2*
((A - B)*a*b*c^2 + (A + B)*a^2*c*d)*f*g^3 + 2*((B*b^2*c*d - B*a*b*d^2)*f^2*
g^2 - (B*b^2*c^2 - B*a^2*d^2)*f*g^3 + (B*a*b*c^2 - B*a^2*c*d)*g^4)*x - 2*(B
*b^2*d^2*f^4 - 2*B*b^2*c*d*f^3*g + B*b^2*c^2*f^2*g^2 + (B*b^2*d^2*f^2*g^2 -
 2*B*b^2*c*d*f*g^3 + B*b^2*c^2*g^4)*x^2 + 2*(B*b^2*d^2*f^3*g - 2*B*b^2*c*d*
f^2*g^2 + B*b^2*c^2*f*g^3)*x)*log(b*x + a) + 2*(B*b^2*d^2*f^4 - 2*B*a*b*d^2
```

```
*f^3*g + B*a^2*d^2*f^2*g^2 + (B*b^2*d^2*f^2*g^2 - 2*B*a*b*d^2*f*g^3 + B*a^2
*d^2*g^4)*x^2 + 2*(B*b^2*d^2*f^3*g - 2*B*a*b*d^2*f^2*g^2 + B*a^2*d^2*f*g^3)
*x)*log(d*x + c) - 2*(2*(B*b^2*c*d - B*a*b*d^2)*f^3*g - (B*b^2*c^2 - B*a^2*
d^2)*f^2*g^2 + (2*(B*b^2*c*d - B*a*b*d^2)*f*g^3 - (B*b^2*c^2 - B*a^2*d^2)*g
^4)*x^2 + 2*(2*(B*b^2*c*d - B*a*b*d^2)*f^2*g^2 - (B*b^2*c^2 - B*a^2*d^2)*f*
g^3)*x)*log(g*x + f) + (B*b^2*d^2*f^4 + B*a^2*c^2*g^4 - 2*(B*b^2*c*d + B*a*
b*d^2)*f^3*g + (B*b^2*c^2 + 4*B*a*b*c*d + B*a^2*d^2)*f^2*g^2 - 2*(B*a*b*c^2
 + B*a^2*c*d)*f*g^3)*log((b^2*x^2 + 2*a*b*x + a^2)*e/(d^2*x^2 + 2*c*d*x + c
^2)))/(b^2*d^2*f^6*g + a^2*c^2*f^2*g^5 - 2*(b^2*c*d + a*b*d^2)*f^5*g^2 + (b
^2*c^2 + 4*a*b*c*d + a^2*d^2)*f^4*g^3 - 2*(a*b*c^2 + a^2*c*d)*f^3*g^4 + (b^
2*d^2*f^4*g^3 + a^2*c^2*g^7 - 2*(b^2*c*d + a*b*d^2)*f^3*g^4 + (b^2*c^2 + 4*
a*b*c*d + a^2*d^2)*f^2*g^5 - 2*(a*b*c^2 + a^2*c*d)*f*g^6)*x^2 + 2*(b^2*d^2*
f^5*g^2 + a^2*c^2*f*g^6 - 2*(b^2*c*d + a*b*d^2)*f^4*g^3 + (b^2*c^2 + 4*a*b*
c*d + a^2*d^2)*f^3*g^4 - 2*(a*b*c^2 + a^2*c*d)*f^2*g^5)*x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(b*x+a)**2/(d*x+c)**2))/(g*x+f)**3,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 495 vs. 2(174) = 348.
time = 2.54, size = 495, normalized size = 2.83

$$\frac{Bb^3\log(|bx+a|)}{b^3f^2g-2\,ab^2fg^2+a^2bg^3}-\frac{Bd^3\log(|dx+c|)}{d^3f^2g-2\,cd^2fg^2+c^2dg^3}+\frac{(2\,Bb^2cdf-2\,Babd^2f-Bb^2c^2g+Ba^2d^2g)\log(gx+f)}{b^2d^2f^4-2\,b^2cdf^3g-2\,abd^2f^3g+b^2c^2f^2g^2+4\,abcdf^2g^2+a^2d^2f^2g^2-2\,abc^2fg^3-2\,a^2cdfg^3+a^2c^2g^4}-\frac{B\log\left(\frac{b^2x^2+2\,abx+a^2}{d^2x^2+2\,cdx+c^2}\right)}{2\,(g^3x^2+2\,fg^2x+f^2g)}-\frac{2\,Bbcg^2x-2\,Badg^2x+Abdf^2+Bbdf^2-Abcfg+Bbcfg-Aadfg-3\,Badfg+Aacg^2+Bacg^2}{2\,(bdf^2g^3x^2-bcfg^4x^2-adfg^4x^2+acg^5x^2+2\,bdf^3g^2x-2\,bcf^2g^3x-2\,adf^2g^3x+2\,acfg^4x+bdf^4g-bcf^3g^2-adf^3g^2+acf^2g^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(g*x+f)^3,x, algorithm="giac")
```

```
[Out] B*b^3*log(abs(b*x + a))/(b^3*f^2*g - 2*a*b^2*f*g^2 + a^2*b*g^3) - B*d^3*log
(abs(d*x + c))/(d^3*f^2*g - 2*c*d^2*f*g^2 + c^2*d*g^3) + (2*B*b^2*c*d*f - 2
*B*a*b*d^2*f - B*b^2*c^2*g + B*a^2*d^2*g)*log(g*x + f)/(b^2*d^2*f^4 - 2*b^2
*c*d*f^3*g - 2*a*b*d^2*f^3*g + b^2*c^2*f^2*g^2 + 4*a*b*c*d*f^2*g^2 + a^2*d^
2*f^2*g^2 - 2*a*b*c^2*f*g^3 - 2*a^2*c*d*f*g^3 + a^2*c^2*g^4) - 1/2*B*log((b
^2*x^2 + 2*a*b*x + a^2)/(d^2*x^2 + 2*c*d*x + c^2))/(g^3*x^2 + 2*f*g^2*x + f
^2*g) - 1/2*(2*B*b*c*g^2*x - 2*B*a*d*g^2*x + A*b*d*f^2 + B*b*d*f^2 - A*b*c*
f*g + B*b*c*f*g - A*a*d*f*g - 3*B*a*d*f*g + A*a*c*g^2 + B*a*c*g^2)/(b*d*f^2
*g^3*x^2 - b*c*f*g^4*x^2 - a*d*f*g^4*x^2 + a*c*g^5*x^2 + 2*b*d*f^3*g^2*x -
2*b*c*f^2*g^3*x - 2*a*d*f^2*g^3*x + 2*a*c*f*g^4*x + b*d*f^4*g - b*c*f^3*g^2
 - a*d*f^3*g^2 + a*c*f^2*g^3)
```

Mupad [B]
time = 7.45, size = 412, normalized size = 2.35

$$\frac{\ln\left(f+g\,x\right)\,\left(g\,\left(B\,a^2\,d^2-B\,b^2\,c^2\right)-2\,B\,a\,b\,d^2\,f+2\,B\,b^2\,c\,d\,f\right)}{a^2\,c^2\,g^4-2\,a^2\,c\,d\,f\,g^3+a^2\,d^2\,f^2\,g^2-2\,a\,b\,c^2\,f\,g^3+4\,a\,b\,c\,d\,f^2\,g^2-2\,a\,b\,d^2\,f^3\,g+b^2\,c^2\,f^2\,g^2-2\,b^2\,c\,d\,f^3\,g+b^2\,d^2\,f^4}-\frac{\frac{A\,a\,c\,g^2+A\,b\,d\,f^2-A\,a\,d\,f\,g-A\,b\,c\,f\,g-2\,B\,a\,d\,f\,g+2\,B\,b\,c\,f\,g}{2\,\left(a\,c\,g^2+b\,d\,f^2-a\,d\,f\,g-b\,c\,f\,g\right)}-\frac{x\,\left(B\,a\,d\,g^2-B\,b\,c\,g^2\right)}{a\,c\,g^2+b\,d\,f^2-a\,d\,f\,g-b\,c\,f\,g}}{f^2\,g+2\,f\,g^2\,x+g^3\,x^2}+\frac{B\,b^2\,\ln\left(a+b\,x\right)}{a^2\,g^3-2\,a\,b\,f\,g^2+b^2\,f^2\,g}-\frac{B\,d^2\,\ln\left(c+d\,x\right)}{c^2\,g^3-2\,c\,d\,f\,g^2+d^2\,f^2\,g}-\frac{B\,\ln\left(\frac{e\,{\left(a+b\,x\right)}^2}{{\left(c+d\,x\right)}^2}\right)}{2\,g\,\left(f^2+2\,f\,g\,x+g^2\,x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))/(f + g*x)^3,x)
```

```
[Out] (log(f + g*x)*(g*(B*a^2*d^2 - B*b^2*c^2) - 2*B*a*b*d^2*f + 2*B*b^2*c*d*f))/
(a^2*c^2*g^4 + b^2*d^2*f^4 + a^2*d^2*f^2*g^2 + b^2*c^2*f^2*g^2 - 2*a*b*c^2*
f*g^3 - 2*a*b*d^2*f^3*g - 2*a^2*c*d*f*g^3 - 2*b^2*c*d*f^3*g + 4*a*b*c*d*f^2
*g^2) - ((A*a*c*g^2 + A*b*d*f^2 - A*a*d*f*g - A*b*c*f*g - 2*B*a*d*f*g + 2*B
*b*c*f*g)/(2*(a*c*g^2 + b*d*f^2 - a*d*f*g - b*c*f*g)) - (x*(B*a*d*g^2 - B*b
*c*g^2))/(a*c*g^2 + b*d*f^2 - a*d*f*g - b*c*f*g))/(f^2*g + g^3*x^2 + 2*f*g^
2*x) + (B*b^2*log(a + b*x))/(a^2*g^3 + b^2*f^2*g - 2*a*b*f*g^2) - (B*d^2*lo
g(c + d*x))/(c^2*g^3 + d^2*f^2*g - 2*c*d*f*g^2) - (B*log((e*(a + b*x)^2)/(c
 + d*x)^2))/(2*g*(f^2 + g^2*x^2 + 2*f*g*x))
```

3.270 $\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f+gx)^4}\,dx$

Optimal. Leaf size=277

$$-\frac{B(bc-ad)}{3(bf-ag)(df-cg)(f+gx)^2}-\frac{2B(bc-ad)(2bdf-bcg-adg)}{3(bf-ag)^2(df-cg)^2(f+gx)}+\frac{2b^3B\log(a+bx)}{3g(bf-ag)^3}-\frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{3g(f+gx)^3}$$

```
[Out] -1/3*B*(-a*d+b*c)/(-a*g+b*f)/(-c*g+d*f)/(g*x+f)^2-2/3*B*(-a*d+b*c)*(-a*d*g-
b*c*g+2*b*d*f)/(-a*g+b*f)^2/(-c*g+d*f)^2/(g*x+f)+2/3*b^3*B*ln(b*x+a)/g/(-a*
g+b*f)^3+1/3*(-A-B*ln(e*(b*x+a)^2/(d*x+c)^2))/g/(g*x+f)^3-2/3*B*d^3*ln(d*x+
c)/g/(-c*g+d*f)^3+2/3*B*(-a*d+b*c)*(a^2*d^2*g^2-a*b*d*g*(-c*g+3*d*f)+b^2*(c
^2*g^2-3*c*d*f*g+3*d^2*f^2))*ln(g*x+f)/(-a*g+b*f)^3/(-c*g+d*f)^3
```

Rubi [A]
time = 0.25, antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {2548, 84}

$$\frac{2B(bc-ad)\log(f+gx)\left(a^2d^2g^2-abdg(3df-cg)+b^2(c^2g^2-3cdfg+3d^2f^2)\right)}{3(bf-ag)^3(df-cg)^3}-\frac{B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A}{3g(f+gx)^3}+\frac{2b^3B\log(a+bx)}{3g(bf-ag)^3}-\frac{2B(bc-ad)(-adg-bcg+2bdf)}{3(f+gx)(bf-ag)^2(df-cg)^2}-\frac{B(bc-ad)}{3(f+gx)^2(bf-ag)(df-cg)}-\frac{2Bd^3\log(c+dx)}{3g(df-cg)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(f + g*x)^4,x]
```

```
[Out] -1/3*(B*(b*c - a*d))/((b*f - a*g)*(d*f - c*g)*(f + g*x)^2) - (2*B*(b*c - a*
d)*(2*b*d*f - b*c*g - a*d*g))/(3*(b*f - a*g)^2*(d*f - c*g)^2*(f + g*x)) + (
2*b^3*B*Log[a + b*x])/(3*g*(b*f - a*g)^3) - (A + B*Log[(e*(a + b*x)^2)/(c +
 d*x)^2])/(3*g*(f + g*x)^3) - (2*B*d^3*Log[c + d*x])/(3*g*(d*f - c*g)^3) +
(2*B*(b*c - a*d)*(a^2*d^2*g^2 - a*b*d*g*(3*d*f - c*g) + b^2*(3*d^2*f^2 - 3*
c*d*f*g + c^2*g^2))*Log[f + g*x])/(3*(b*f - a*g)^3*(d*f - c*g)^3)
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 2548

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Dist[B*n*((b*c
- a*d)/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /;
 FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c -
```

```
a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Rubi steps

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f+gx)^4}\,dx = -\frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{3g(f+gx)^3}+\frac{B\int \frac{2(bc-ad)}{(a+bx)(c+dx)(f+gx)^3}\,dx}{3g}$$

$$= -\frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{3g(f+gx)^3}+\frac{(2B(bc-ad))\int \frac{1}{(a+bx)(c+dx)(f+gx)^3}\,dx}{3g}$$

$$= -\frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{3g(f+gx)^3}+\frac{(2B(bc-ad))\int \left(\frac{b^4}{(bc-ad)(bf-ag)^3(a+bx)}+\frac{d^4}{(bc-ad)(-df+cg)}\right.}{}$$

$$= -\frac{B(bc-ad)}{3(bf-ag)(df-cg)(f+gx)^2}-\frac{2B(bc-ad)(2bdf-bcg-adg)}{3(bf-ag)^2(df-cg)^2(f+gx)}+\frac{2b^3B\log}{3g(bf}$$

Mathematica [A]
time = 0.47, size = 263, normalized size = 0.95

$$\frac{-\frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f+gx)^3}+2B(bc-ad)\left(-\frac{g}{2(bf-ag)(df-cg)(f+gx)^2}+\frac{g(-2bdf+bcg+adg)}{(bf-ag)^2(df-cg)^2(f+gx)}+\frac{b^3\log(a+bx)}{(bc-ad)(bf-ag)^3}+\frac{d^3\log(c+dx)}{(bc-ad)(-df+cg)^3}+\frac{g\left(a^2d^2g^2+abdg(-3df+cg)+b^2\left(3d^2f^2-3cdfg+c^2g^2\right)\right)\log(f+gx)}{(bf-ag)^3(df-cg)^3}\right)}{3g}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(f + g*x)^4,x]
```

```
[Out] (-((A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(f + g*x)^3) + 2*B*(b*c - a*d)*
(-1/2*g/((b*f - a*g)*(d*f - c*g)*(f + g*x)^2) + (g*(-2*b*d*f + b*c*g + a*d*
g))/((b*f - a*g)^2*(d*f - c*g)^2*(f + g*x)) + (b^3*Log[a + b*x])/((b*c - a*
d)*(b*f - a*g)^3) + (d^3*Log[c + d*x])/((b*c - a*d)*(-(d*f) + c*g)^3) + (g*
(a^2*d^2*g^2 + a*b*d*g*(-3*d*f + c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^
2))*Log[f + g*x])/((b*f - a*g)^3*(d*f - c*g)^3)))/(3*g)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2562 vs. $2(268) = 536$.
time = 0.88, size = 2563, normalized size = 9.25

method	result	size
risch	Expression too large to display	2293
derivativedivides	Expression too large to display	2563
default	Expression too large to display	2563

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/(g*x+f)^4,x,method=_RETURNVERBOSE)

[Out] -1/d*(d^4*A*(-1/(c*g-d*f)^3/(c*g/(d*x+c)-f/(d*x+c)*d-g)-1/3*g^2/(c*g-d*f)^3
/(c*g/(d*x+c)-f/(d*x+c)*d-g)^3-g/(c*g-d*f)^3/(c*g/(d*x+c)-f/(d*x+c)*d-g)^2)
+((c*g-d*f)*b^3*g*B*d/(a^3*g^3-3*a^2*b*f*g^2+3*a*b^2*f^2*g-b^3*f^3)/(d*x+c)
*ln(e*(a/(d*x+c)*d-b*c/(d*x+c)+b)^2/d^2)-1/3*(2*B*a^2*d^4*g^4-4*B*a*b*d^4*f
*g^3-2*B*b^2*c^2*d^2*g^4+4*B*b^2*c*d^3*f*g^3)/g/(a^2*c^2*g^4-2*a^2*c*d*f*g^
3+a^2*d^2*f^2*g^2-2*a*b*c^2*f*g^3+4*a*b*c*d*f^2*g^2-2*a*b*d^2*f^3*g+b^2*c^2
*f^2*g^2-2*b^2*c*d*f^3*g+b^2*d^2*f^4)/(d*x+c)-1/3*(3*B*a^2*d^4*g^4-B*a*b*c*
d^3*g^4-5*B*a*b*d^4*f*g^3-2*B*b^2*c^2*d^2*g^4+5*B*b^2*c*d^3*f*g^3)/(a^2*g^2
-2*a*b*f*g+b^2*f^2)/g^3/(d*x+c)^3+1/3*(5*B*a^2*d^4*g^4-B*a*b*c*d^3*g^4-9*B*
a*b*d^4*f*g^3-4*B*b^2*c^2*d^2*g^4+9*B*b^2*c*d^3*f*g^3)/(c*g-d*f)/g^2/(a^2*g
^2-2*a*b*f*g+b^2*f^2)/(d*x+c)^2-1/3*B*d*(a^3*d^3*g^2-3*a^2*b*d^3*f*g+3*a*b^
2*d^3*f^2-b^3*c^3*g^2+3*b^3*c^2*d*f*g-3*b^3*c*d^2*f^2)/(a^3*g^3-3*a^2*b*f*g
^2+3*a*b^2*f^2*g-b^3*f^3)/(d*x+c)^3*ln(e*(a/(d*x+c)*d-b*c/(d*x+c)+b)^2/d^2)
-1/3*b^3*g^2*B*d/(a^3*g^3-3*a^2*b*f*g^2+3*a*b^2*f^2*g-b^3*f^3)*ln(e*(a/(d*x
+c)*d-b*c/(d*x+c)+b)^2/d^2)-(c^2*g^2-2*c*d*f*g+d^2*f^2)*b^3*B*d/(a^3*g^3-3*
a^2*b*f*g^2+3*a*b^2*f^2*g-b^3*f^3)/(d*x+c)^2*ln(e*(a/(d*x+c)*d-b*c/(d*x+c)+
b)^2/d^2))/(c*g/(d*x+c)-f/(d*x+c)*d-g)^3+2/3*B*d^4/(a^3*c^3*g^6-3*a^3*c^2*d
*f*g^5+3*a^3*c*d^2*f^2*g^4-a^3*d^3*f^3*g^3-3*a^2*b*c^3*f*g^5+9*a^2*b*c^2*d*
f^2*g^4-9*a^2*b*c*d^2*f^3*g^3+3*a^2*b*d^3*f^4*g^2+3*a*b^2*c^3*f^2*g^4-9*a*b
^2*c^2*d*f^3*g^3+9*a*b^2*c*d^2*f^4*g^2-3*a*b^2*d^3*f^5*g-b^3*c^3*f^3*g^3+3*
b^3*c^2*d*f^4*g^2-3*b^3*c*d^2*f^5*g+b^3*d^3*f^6)*ln(c*g/(d*x+c)-f/(d*x+c)*d
-g)*a^3*g^2-2*B*d^4/(a^3*c^3*g^6-3*a^3*c^2*d*f*g^5+3*a^3*c*d^2*f^2*g^4-a^3*
d^3*f^3*g^3-3*a^2*b*c^3*f*g^5+9*a^2*b*c^2*d*f^2*g^4-9*a^2*b*c*d^2*f^3*g^3+3
*a^2*b*d^3*f^4*g^2+3*a*b^2*c^3*f^2*g^4-9*a*b^2*c^2*d*f^3*g^3+9*a*b^2*c*d^2*
f^4*g^2-3*a*b^2*d^3*f^5*g-b^3*c^3*f^3*g^3+3*b^3*c^2*d*f^4*g^2-3*b^3*c*d^2*f
^5*g+b^3*d^3*f^6)*ln(c*g/(d*x+c)-f/(d*x+c)*d-g)*a^2*b*f*g+2*B*d^4/(a^3*c^3*
g^6-3*a^3*c^2*d*f*g^5+3*a^3*c*d^2*f^2*g^4-a^3*d^3*f^3*g^3-3*a^2*b*c^3*f*g^5
+9*a^2*b*c^2*d*f^2*g^4-9*a^2*b*c*d^2*f^3*g^3+3*a^2*b*d^3*f^4*g^2+3*a*b^2*c^
3*f^2*g^4-9*a*b^2*c^2*d*f^3*g^3+9*a*b^2*c*d^2*f^4*g^2-3*a*b^2*d^3*f^5*g-b^3
*c^3*f^3*g^3+3*b^3*c^2*d*f^4*g^2-3*b^3*c*d^2*f^5*g+b^3*d^3*f^6)*ln(c*g/(d*x
+c)-f/(d*x+c)*d-g)*a*b^2*f^2-2/3*B*d/(a^3*c^3*g^6-3*a^3*c^2*d*f*g^5+3*a^3*c
*d^2*f^2*g^4-a^3*d^3*f^3*g^3-3*a^2*b*c^3*f*g^5+9*a^2*b*c^2*d*f^2*g^4-9*a^2*
b*c*d^2*f^3*g^3+3*a^2*b*d^3*f^4*g^2+3*a*b^2*c^3*f^2*g^4-9*a*b^2*c^2*d*f^3*g
^3+9*a*b^2*c*d^2*f^4*g^2-3*a*b^2*d^3*f^5*g-b^3*c^3*f^3*g^3+3*b^3*c^2*d*f^4*
g^2-3*b^3*c*d^2*f^5*g+b^3*d^3*f^6)*ln(c*g/(d*x+c)-f/(d*x+c)*d-g)*b^3*c^3*g^
2+2*B*d^2/(a^3*c^3*g^6-3*a^3*c^2*d*f*g^5+3*a^3*c*d^2*f^2*g^4-a^3*d^3*f^3*g^
3-3*a^2*b*c^3*f*g^5+9*a^2*b*c^2*d*f^2*g^4-9*a^2*b*c*d^2*f^3*g^3+3*a^2*b*d^3
*f^4*g^2+3*a*b^2*c^3*f^2*g^4-9*a*b^2*c^2*d*f^3*g^3+9*a*b^2*c*d^2*f^4*g^2-3*
a*b^2*d^3*f^5*g-b^3*c^3*f^3*g^3+3*b^3*c^2*d*f^4*g^2-3*b^3*c*d^2*f^5*g+b^3*d
^3*f^6)*ln(c*g/(d*x+c)-f/(d*x+c)*d-g)*b^3*c^2*f*g-2*B*d^3/(a^3*c^3*g^6-3*a^
3*c^2*d*f*g^5+3*a^3*c*d^2*f^2*g^4-a^3*d^3*f^3*g^3-3*a^2*b*c^3*f*g^5+9*a^2*b
*c^2*d*f^2*g^4-9*a^2*b*c*d^2*f^3*g^3+3*a^2*b*d^3*f^4*g^2+3*a*b^2*c^3*f^2*g^
4-9*a*b^2*c^2*d*f^3*g^3+9*a*b^2*c*d^2*f^4*g^2-3*a*b^2*d^3*f^5*g-b^3*c^3*f^3
```

```
*g^3+3*b^3*c^2*d*f^4*g^2-3*b^3*c*d^2*f^5*g+b^3*d^3*f^6)*ln(c*g/(d*x+c)-f/(d
*x+c)*d-g)*b^3*c*f^2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 903 vs. $2(266) = 532$.
time = 0.36, size = 903, normalized size = 3.26

$$\frac{1}{3}\left(\frac{2 b^3 \log(bx+a)}{b^3 f^3 g - 3 a b^2 f^2 g^2 + 3 a^2 b f g^3 - a^3 g^4} - \frac{2 d^3 \log(dx+c)}{d^3 f^3 g - 3 c d^2 f^2 g^2 + 3 c^2 d f g^3 - c^3 g^4} + \frac{2\left(3\left(b^3 c d^2 - a b^2 d^3\right) f^2 - 3\left(b^3 c^2 d - a^2 b d^3\right) f g + \left(b^3 c^3 - a^3 d^3\right) g^2\right) \log(gx+f)}{b^3 d^3 f^6 + a^3 c^3 g^6 - 3\left(b^3 c d^2 + a b^2 d^3\right) f^5 g + 3\left(b^3 c^2 d + 3 a b^2 c d^2 + a^2 b d^3\right) f^4 g^2 - \left(b^3 c^3 + 9 a b^2 c^2 d + 9 a^2 b c d^2 + a^3 d^3\right) f^3 g^3 + 3\left(a b^2 c^3 + 3 a^2 b c^2 d + a^3 c d^2\right) f^2 g^4 - 3\left(a^2 b c^3 + a^3 c^2 d\right) f g^5} - \frac{5\left(b^2 c d - a b d^2\right) f^2 - 3\left(b^2 c^2 - a^2 d^2\right) f g + \left(a b c^2 - a^2 c d\right) g^2 + 2\left(2\left(b^2 c d - a b d^2\right) f g - \left(b^2 c^2 - a^2 d^2\right) g^2\right) x}{b^2 d^2 f^6 + a^2 c^2 f^2 g^4 - 2\left(b^2 c d + a b d^2\right) f^5 g + \left(b^2 c^2 + 4 a b c d + a^2 d^2\right) f^4 g^2 - 2\left(a b c^2 + a^2 c d\right) f^3 g^3 + \left(b^2 d^2 f^4 g^2 + a^2 c^2 g^6 - 2\left(b^2 c d + a b d^2\right) f^3 g^3 + \left(b^2 c^2 + 4 a b c d + a^2 d^2\right) f^2 g^4 - 2\left(a b c^2 + a^2 c d\right) f g^5\right) x^2 + 2\left(b^2 d^2 f^5 g + a^2 c^2 f g^5 - 2\left(b^2 c d + a b d^2\right) f^4 g^2 + \left(b^2 c^2 + 4 a b c d + a^2 d^2\right) f^3 g^3 - 2\left(a b c^2 + a^2 c d\right) f^2 g^4\right) x} - \frac{\log\left(\frac{b^2 x^2 e}{d^2 x^2 + 2 c d x + c^2} + \frac{2 a b x e}{d^2 x^2 + 2 c d x + c^2} + \frac{a^2 e}{d^2 x^2 + 2 c d x + c^2}\right)}{g^4 x^3 + 3 f g^3 x^2 + 3 f^2 g^2 x + f^3 g}\right) B - \frac{A}{3\left(g^4 x^3 + 3 f g^3 x^2 + 3 f^2 g^2 x + f^3 g\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(g*x+f)^4,x, algorithm="maxima")
```

```
[Out] 1/3*(2*b^3*log(b*x + a)/(b^3*f^3*g - 3*a*b^2*f^2*g^2 + 3*a^2*b*f*g^3 - a^3*
g^4) - 2*d^3*log(d*x + c)/(d^3*f^3*g - 3*c*d^2*f^2*g^2 + 3*c^2*d*f*g^3 - c^
3*g^4) + 2*(3*(b^3*c*d^2 - a*b^2*d^3)*f^2 - 3*(b^3*c^2*d - a^2*b*d^3)*f*g +
 (b^3*c^3 - a^3*d^3)*g^2)*log(g*x + f)/(b^3*d^3*f^6 + a^3*c^3*g^6 - 3*(b^3*
c*d^2 + a*b^2*d^3)*f^5*g + 3*(b^3*c^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3)*f^4*g^
2 - (b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*f^3*g^3 + 3*(a*b^2*
c^3 + 3*a^2*b*c^2*d + a^3*c*d^2)*f^2*g^4 - 3*(a^2*b*c^3 + a^3*c^2*d)*f*g^5)
 - (5*(b^2*c*d - a*b*d^2)*f^2 - 3*(b^2*c^2 - a^2*d^2)*f*g + (a*b*c^2 - a^2*
c*d)*g^2 + 2*(2*(b^2*c*d - a*b*d^2)*f*g - (b^2*c^2 - a^2*d^2)*g^2)*x)/(b^2*
d^2*f^6 + a^2*c^2*f^2*g^4 - 2*(b^2*c*d + a*b*d^2)*f^5*g + (b^2*c^2 + 4*a*b*
c*d + a^2*d^2)*f^4*g^2 - 2*(a*b*c^2 + a^2*c*d)*f^3*g^3 + (b^2*d^2*f^4*g^2 +
 a^2*c^2*g^6 - 2*(b^2*c*d + a*b*d^2)*f^3*g^3 + (b^2*c^2 + 4*a*b*c*d + a^2*d
^2)*f^2*g^4 - 2*(a*b*c^2 + a^2*c*d)*f*g^5)*x^2 + 2*(b^2*d^2*f^5*g + a^2*c^2
*f*g^5 - 2*(b^2*c*d + a*b*d^2)*f^4*g^2 + (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*f^
3*g^3 - 2*(a*b*c^2 + a^2*c*d)*f^2*g^4)*x) - log(b^2*x^2*e/(d^2*x^2 + 2*c*d*
x + c^2) + 2*a*b*x*e/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x +
 c^2))/(g^4*x^3 + 3*f*g^3*x^2 + 3*f^2*g^2*x + f^3*g))*B - 1/3*A/(g^4*x^3 +
3*f*g^3*x^2 + 3*f^2*g^2*x + f^3*g)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(g*x+f)^4,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(b*x+a)**2/(d*x+c)**2))/(g*x+f)**4,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1391 vs. $2(266) = 532$.
time = 2.35, size = 1391, normalized size = 5.02

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(g*x+f)^4,x, algorithm="giac")
```

```
[Out] 2/3*B*b^4*log(abs(b*x + a))/(b^4*f^3*g - 3*a*b^3*f^2*g^2 + 3*a^2*b^2*f*g^3
- a^3*b*g^4) - 2/3*B*d^4*log(abs(d*x + c))/(d^4*f^3*g - 3*c*d^3*f^2*g^2 + 3
*c^2*d^2*f*g^3 - c^3*d*g^4) + 2/3*(3*B*b^3*c*d^2*f^2 - 3*B*a*b^2*d^3*f^2 -
3*B*b^3*c^2*d*f*g + 3*B*a^2*b*d^3*f*g + B*b^3*c^3*g^2 - B*a^3*d^3*g^2)*log(
g*x + f)/(b^3*d^3*f^6 - 3*b^3*c*d^2*f^5*g - 3*a*b^2*d^3*f^5*g + 3*b^3*c^2*d
*f^4*g^2 + 9*a*b^2*c*d^2*f^4*g^2 + 3*a^2*b*d^3*f^4*g^2 - b^3*c^3*f^3*g^3 -
9*a*b^2*c^2*d*f^3*g^3 - 9*a^2*b*c*d^2*f^3*g^3 - a^3*d^3*f^3*g^3 + 3*a*b^2*c
^3*f^2*g^4 + 9*a^2*b*c^2*d*f^2*g^4 + 3*a^3*c*d^2*f^2*g^4 - 3*a^2*b*c^3*f*g^
5 - 3*a^3*c^2*d*f*g^5 + a^3*c^3*g^6) - 1/3*B*log((b^2*x^2 + 2*a*b*x + a^2)/
(d^2*x^2 + 2*c*d*x + c^2))/(g^4*x^3 + 3*f*g^3*x^2 + 3*f^2*g^2*x + f^3*g) -
1/3*(4*B*b^2*c*d*f*g^3*x^2 - 4*B*a*b*d^2*f*g^3*x^2 - 2*B*b^2*c^2*g^4*x^2 +
2*B*a^2*d^2*g^4*x^2 + 9*B*b^2*c*d*f^2*g^2*x - 9*B*a*b*d^2*f^2*g^2*x - 5*B*b
^2*c^2*f*g^3*x + 5*B*a^2*d^2*f*g^3*x + B*a*b*c^2*g^4*x - B*a^2*c*d*g^4*x +
A*b^2*d^2*f^4 + B*b^2*d^2*f^4 - 2*A*b^2*c*d*f^3*g + 3*B*b^2*c*d*f^3*g - 2*A
*a*b*d^2*f^3*g - 7*B*a*b*d^2*f^3*g + A*b^2*c^2*f^2*g^2 - 2*B*b^2*c^2*f^2*g^
2 + 4*A*a*b*c*d*f^2*g^2 + 4*B*a*b*c*d*f^2*g^2 + A*a^2*d^2*f^2*g^2 + 4*B*a^2
*d^2*f^2*g^2 - 2*A*a*b*c^2*f*g^3 - B*a*b*c^2*f*g^3 - 2*A*a^2*c*d*f*g^3 - 3*
B*a^2*c*d*f*g^3 + A*a^2*c^2*g^4 + B*a^2*c^2*g^4)/(b^2*d^2*f^4*g^4*x^3 - 2*b
^2*c*d*f^3*g^5*x^3 - 2*a*b*d^2*f^3*g^5*x^3 + b^2*c^2*f^2*g^6*x^3 + 4*a*b*c*
d*f^2*g^6*x^3 + a^2*d^2*f^2*g^6*x^3 - 2*a*b*c^2*f*g^7*x^3 - 2*a^2*c*d*f*g^7
*x^3 + a^2*c^2*g^8*x^3 + 3*b^2*d^2*f^5*g^3*x^2 - 6*b^2*c*d*f^4*g^4*x^2 - 6*
a*b*d^2*f^4*g^4*x^2 + 3*b^2*c^2*f^3*g^5*x^2 + 12*a*b*c*d*f^3*g^5*x^2 + 3*a^
2*d^2*f^3*g^5*x^2 - 6*a*b*c^2*f^2*g^6*x^2 - 6*a^2*c*d*f^2*g^6*x^2 + 3*a^2*c
^2*f*g^7*x^2 + 3*b^2*d^2*f^6*g^2*x - 6*b^2*c*d*f^5*g^3*x - 6*a*b*d^2*f^5*g^
3*x + 3*b^2*c^2*f^4*g^4*x + 12*a*b*c*d*f^4*g^4*x + 3*a^2*d^2*f^4*g^4*x - 6*
a*b*c^2*f^3*g^5*x - 6*a^2*c*d*f^3*g^5*x + 3*a^2*c^2*f^2*g^6*x + b^2*d^2*f^7
*g - 2*b^2*c*d*f^6*g^2 - 2*a*b*d^2*f^6*g^2 + b^2*c^2*f^5*g^3 + 4*a*b*c*d*f^
5*g^3 + a^2*d^2*f^5*g^3 - 2*a*b*c^2*f^4*g^4 - 2*a^2*c*d*f^4*g^4 + a^2*c^2*f
^3*g^5)
```

Mupad [**B**]
time = 11.58, size = 1147, normalized size = 4.14

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))/(f + g*x)^4,x)

[Out] (log(f + g*x)*(g*(6*B*a^2*b*d^3*f - 6*B*b^3*c^2*d*f) - g^2*(2*B*a^3*d^3 - 2*B*b^3*c^3) - 6*B*a*b^2*d^3*f^2 + 6*B*b^3*c*d^2*f^2))/(3*a^3*c^3*g^6 + 3*b^3*d^3*f^6 - 3*a^3*d^3*f^3*g^3 - 3*b^3*c^3*f^3*g^3 - 9*a^2*b*c^3*f*g^5 - 9*a*b^2*d^3*f^5*g - 9*a^3*c^2*d*f*g^5 - 9*b^3*c*d^2*f^5*g + 9*a*b^2*c^3*f^2*g^4 + 9*a^2*b*d^3*f^4*g^2 + 9*a^3*c*d^2*f^2*g^4 + 9*b^3*c^2*d*f^4*g^2 + 27*a*b^2*c*d^2*f^4*g^2 - 27*a*b^2*c^2*d*f^3*g^3 - 27*a^2*b*c*d^2*f^3*g^3 + 27*a^2*b*c^2*d*f^2*g^4) - ((A*a^2*c^2*g^4 + A*b^2*d^2*f^4 + A*a^2*d^2*f^2*g^2 + A*b^2*c^2*f^2*g^2 + 3*B*a^2*d^2*f^2*g^2 - 3*B*b^2*c^2*f^2*g^2 - 2*A*a*b*c^2*f*g^3 - 2*A*a*b*d^2*f^3*g + B*a*b*c^2*f*g^3 - 2*A*a^2*c*d*f*g^3 - 5*B*a*b*d^2*f^3*g - 2*A*b^2*c*d*f^3*g - B*a^2*c*d*f*g^3 + 5*B*b^2*c*d*f^3*g + 4*A*a*b*c*d*f^2*g^2)/(a^2*c^2*g^4 + b^2*d^2*f^4 + a^2*d^2*f^2*g^2 + b^2*c^2*f^2*g^2 - 2*a*b*c^2*f*g^3 - 2*a*b*d^2*f^3*g - 2*a^2*c*d*f*g^3 - 2*b^2*c*d*f^3*g + 4*a*b*c*d*f^2*g^2) + (2*x^2*(B*a^2*d^2*g^4 - B*b^2*c^2*g^4 - 2*B*a*b*d^2*f*g^3 + 2*B*b^2*c*d*f*g^3))/(a^2*c^2*g^4 + b^2*d^2*f^4 + a^2*d^2*f^2*g^2 + b^2*c^2*f^2*g^2 - 2*a*b*c^2*f*g^3 - 2*a*b*d^2*f^3*g - 2*a^2*c*d*f*g^3 - 2*b^2*c*d*f^3*g + 4*a*b*c*d*f^2*g^2) + (x*(5*B*a^2*d^2*f*g^3 - 5*B*b^2*c^2*f*g^3 + B*a*b*c^2*g^4 - B*a^2*c*d*g^4 - 9*B*a*b*d^2*f^2*g^2 + 9*B*b^2*c*d*f^2*g^2))/(a^2*c^2*g^4 + b^2*d^2*f^4 + a^2*d^2*f^2*g^2 + b^2*c^2*f^2*g^2 - 2*a*b*c^2*f*g^3 - 2*a*b*d^2*f^3*g - 2*a^2*c*d*f*g^3 - 2*b^2*c*d*f^3*g + 4*a*b*c*d*f^2*g^2))/(3*f^3*g + 3*g^4*x^3 + 9*f^2*g^2*x + 9*f*g^3*x^2) - (B*log((e*(a + b*x)^2)/(c + d*x)^2))/(3*g*(f^3 + g^3*x^3 + 3*f^2*g*x + 3*f*g^2*x^2)) - (2*B*b^3*log(a + b*x))/(3*a^3*g^4 - 3*b^3*f^3*g + 9*a*b^2*f^2*g^2 - 9*a^2*b*f*g^3) + (2*B*d^3*log(c + d*x))/(3*c^3*g^4 - 3*d^3*f^3*g + 9*c*d^2*f^2*g^2 - 9*c^2*d*f*g^3)

3.271 $\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f+gx)^5}\,dx$

Optimal. Leaf size=381

$$-\frac{B(bc-ad)}{6(bf-ag)(df-cg)(f+gx)^3}-\frac{B(bc-ad)(2bdf-bcg-adg)}{4(bf-ag)^2(df-cg)^2(f+gx)^2}-\frac{B(bc-ad)\left(a^2d^2g^2-abdg(3df-cg)+\right.}{2(bf-ag)^3(df-cg)^3}$$

```
[Out] -1/6*B*(-a*d+b*c)/(-a*g+b*f)/(-c*g+d*f)/(g*x+f)^3-1/4*B*(-a*d+b*c)*(-a*d*g-
b*c*g+2*b*d*f)/(-a*g+b*f)^2/(-c*g+d*f)^2/(g*x+f)^2-1/2*B*(-a*d+b*c)*(a^2*d^
2*g^2-a*b*d*g*(-c*g+3*d*f)+b^2*(c^2*g^2-3*c*d*f*g+3*d^2*f^2))/(-a*g+b*f)^3/
(-c*g+d*f)^3/(g*x+f)+1/2*b^4*B*ln(b*x+a)/g/(-a*g+b*f)^4+1/4*(-A-B*ln(e*(b*x
+a)^2/(d*x+c)^2))/g/(g*x+f)^4-1/2*B*d^4*ln(d*x+c)/g/(-c*g+d*f)^4-1/2*B*(-a*
d+b*c)*(-a*d*g-b*c*g+2*b*d*f)*(2*a*b*d^2*f*g-a^2*d^2*g^2-b^2*(c^2*g^2-2*c*d
*f*g+2*d^2*f^2))*ln(g*x+f)/(-a*g+b*f)^4/(-c*g+d*f)^4
```

Rubi [A]

time = 0.40, antiderivative size = 381, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {2548, 84}

$$-\frac{B(bc-ad)\left(a^2d^2g^2-abdg(3df-cg)+b^2\left(c^2g^2-3cdfg+3d^2f^2\right)\right)}{2(f+gx)(bf-ag)^3(df-cg)^3}-\frac{B(bc-ad)\log(f+gx)(-adg-bcg+2bdf)\left(-a^2d^2g^2+2abd^2fg-\left(b^2\left(c^2g^2-2cdfg+2d^2f^2\right)\right)\right)}{2(bf-ag)^4(df-cg)^4}-\frac{B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A}{4g(f+gx)^4}+\frac{b^4B\log(a+bx)}{2g(bf-ag)^4}-\frac{B(bc-ad)(-adg-bcg+2bdf)}{4(f+gx)^2(bf-ag)^2(df-cg)^2}-\frac{B(bc-ad)}{6(f+gx)^3(bf-ag)(df-cg)}-\frac{Bd^4\log(c+dx)}{2g(df-cg)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(f + g*x)^5,x]
```

```
[Out] -1/6*(B*(b*c - a*d))/((b*f - a*g)*(d*f - c*g)*(f + g*x)^3) - (B*(b*c - a*d)
*(2*b*d*f - b*c*g - a*d*g))/(4*(b*f - a*g)^2*(d*f - c*g)^2*(f + g*x)^2) - (
B*(b*c - a*d)*(a^2*d^2*g^2 - a*b*d*g*(3*d*f - c*g) + b^2*(3*d^2*f^2 - 3*c*d
*f*g + c^2*g^2)))/(2*(b*f - a*g)^3*(d*f - c*g)^3*(f + g*x)) + (b^4*B*Log[a
+ b*x])/(2*g*(b*f - a*g)^4) - (A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(4*g
*(f + g*x)^4) - (B*d^4*Log[c + d*x])/(2*g*(d*f - c*g)^4) - (B*(b*c - a*d)*(
2*b*d*f - b*c*g - a*d*g)*(2*a*b*d^2*f*g - a^2*d^2*g^2 - b^2*(2*d^2*f^2 - 2*
c*d*f*g + c^2*g^2))*Log[f + g*x])/(2*(b*f - a*g)^4*(d*f - c*g)^4)
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 2548

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
```

```
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Dist[B*n*((b*c
- a*d)/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /;
 FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c -
 a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Rubi steps

$$\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f+gx)^5}\,dx = -\frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{4g(f+gx)^4}+\frac{B\int \frac{2(bc-ad)}{(a+bx)(c+dx)(f+gx)^4}\,dx}{4g}$$

$$= -\frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{4g(f+gx)^4}+\frac{(B(bc-ad))\int \frac{1}{(a+bx)(c+dx)(f+gx)^4}\,dx}{2g}$$

$$= -\frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{4g(f+gx)^4}+\frac{(B(bc-ad))\int \left(\frac{b^5}{(bc-ad)(bf-ag)^4(a+bx)}-\frac{d^5}{(bc-ad)(-df+cg)^4}\right.}{}$$

$$= -\frac{B(bc-ad)}{6(bf-ag)(df-cg)(f+gx)^3}-\frac{B(bc-ad)(2bdf-bcg-adg)}{4(bf-ag)^2(df-cg)^2(f+gx)^2}-\frac{B(bc-a}{}$$

Mathematica [A]
time = 0.61, size = 358, normalized size = 0.94

$$\frac{-\frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f+gx)^4}+2B(bc-ad)\left(-\frac{g}{3(bf-ag)(df-cg)(f+gx)^3}+\frac{g(-2bdf+bcg+adg)}{2(bf-ag)^2(df-cg)^2(f+gx)^2}-\frac{g(a^2d^2g^2+abdg(-3df+cg)+b^2(3d^2f^2-3cdfg+c^2g^2))}{(bf-ag)^3(df-cg)^3(f+gx)}+\frac{b^4\log(a+bx)}{(bc-ad)(bf-ag)^4}-\frac{d^4\log(c+dx)}{(bc-ad)(df-cg)^4}-\frac{g(-2bdf+bcg+adg)(-2abd^2fg+a^2d^2g^2+b^2(2d^2f^2-2cdfg+c^2g^2))\log(f+gx)}{(bf-ag)^4(df-cg)^4}\right)}{4g}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(f + g*x)^5,x]
```

```
[Out] (-((A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(f + g*x)^4) + 2*B*(b*c - a*d)*
(-1/3*g/((b*f - a*g)*(d*f - c*g)*(f + g*x)^3) + (g*(-2*b*d*f + b*c*g + a*d*
g))/(2*(b*f - a*g)^2*(d*f - c*g)^2*(f + g*x)^2) - (g*(a^2*d^2*g^2 + a*b*d*g
*(-3*d*f + c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^2)))/((b*f - a*g)^3*(d
*f - c*g)^3*(f + g*x)) + (b^4*Log[a + b*x])/((b*c - a*d)*(b*f - a*g)^4) - (
d^4*Log[c + d*x])/((b*c - a*d)*(d*f - c*g)^4) - (g*(-2*b*d*f + b*c*g + a*d*
g)*(-2*a*b*d^2*f*g + a^2*d^2*g^2 + b^2*(2*d^2*f^2 - 2*c*d*f*g + c^2*g^2))*L
og[f + g*x])/((b*f - a*g)^4*(d*f - c*g)^4)))/(4*g)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 5182 vs. $2(370) = 740$.
time = 1.24, size = 5183, normalized size = 13.60

method	result	size

risch	Expression too large to display	4452
derivativedivides	Expression too large to display	5183
default	Expression too large to display	5183

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/(g*x+f)^5,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1812 vs. $2(368) = 736$.
time = 0.46, size = 1812, normalized size = 4.76

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(g*x+f)^5,x, algorithm="maxima")
```

```
[Out] 1/12*(6*b^4*log(b*x + a)/(b^4*f^4*g - 4*a*b^3*f^3*g^2 + 6*a^2*b^2*f^2*g^3 -
 4*a^3*b*f*g^4 + a^4*g^5) - 6*d^4*log(d*x + c)/(d^4*f^4*g - 4*c*d^3*f^3*g^2
 + 6*c^2*d^2*f^2*g^3 - 4*c^3*d*f*g^4 + c^4*g^5) + 6*(4*(b^4*c*d^3 - a*b^3*d
^4)*f^3 - 6*(b^4*c^2*d^2 - a^2*b^2*d^4)*f^2*g + 4*(b^4*c^3*d - a^3*b*d^4)*f
*g^2 - (b^4*c^4 - a^4*d^4)*g^3)*log(g*x + f)/(b^4*d^4*f^8 + a^4*c^4*g^8 - 4
*(b^4*c*d^3 + a*b^3*d^4)*f^7*g + 2*(3*b^4*c^2*d^2 + 8*a*b^3*c*d^3 + 3*a^2*b
^2*d^4)*f^6*g^2 - 4*(b^4*c^3*d + 6*a*b^3*c^2*d^2 + 6*a^2*b^2*c*d^3 + a^3*b*
d^4)*f^5*g^3 + (b^4*c^4 + 16*a*b^3*c^3*d + 36*a^2*b^2*c^2*d^2 + 16*a^3*b*c*
d^3 + a^4*d^4)*f^4*g^4 - 4*(a*b^3*c^4 + 6*a^2*b^2*c^3*d + 6*a^3*b*c^2*d^2 +
 a^4*c*d^3)*f^3*g^5 + 2*(3*a^2*b^2*c^4 + 8*a^3*b*c^3*d + 3*a^4*c^2*d^2)*f^2
*g^6 - 4*(a^3*b*c^4 + a^4*c^3*d)*f*g^7) - (26*(b^3*c*d^2 - a*b^2*d^3)*f^4 -
 31*(b^3*c^2*d - a^2*b*d^3)*f^3*g + (11*b^3*c^3 + 15*a*b^2*c^2*d - 15*a^2*b
*c*d^2 - 11*a^3*d^3)*f^2*g^2 - 7*(a*b^2*c^3 - a^3*c*d^2)*f*g^3 + 2*(a^2*b*c
^3 - a^3*c^2*d)*g^4 + 6*(3*(b^3*c*d^2 - a*b^2*d^3)*f^2*g^2 - 3*(b^3*c^2*d -
 a^2*b*d^3)*f*g^3 + (b^3*c^3 - a^3*d^3)*g^4)*x^2 + 3*(14*(b^3*c*d^2 - a*b^2
*d^3)*f^3*g - 15*(b^3*c^2*d - a^2*b*d^3)*f^2*g^2 + (5*b^3*c^3 + 3*a*b^2*c^2
*d - 3*a^2*b*c*d^2 - 5*a^3*d^3)*f*g^3 - (a*b^2*c^3 - a^3*c*d^2)*g^4)*x)/(b^
3*d^3*f^9 + a^3*c^3*f^3*g^6 - 3*(b^3*c*d^2 + a*b^2*d^3)*f^8*g + 3*(b^3*c^2*
d + 3*a*b^2*c*d^2 + a^2*b*d^3)*f^7*g^2 - (b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2*b
*c*d^2 + a^3*d^3)*f^6*g^3 + 3*(a*b^2*c^3 + 3*a^2*b*c^2*d + a^3*c*d^2)*f^5*g
^4 - 3*(a^2*b*c^3 + a^3*c^2*d)*f^4*g^5 + (b^3*d^3*f^6*g^3 + a^3*c^3*g^9 - 3
*(b^3*c*d^2 + a*b^2*d^3)*f^5*g^4 + 3*(b^3*c^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3
)*f^4*g^5 - (b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*f^3*g^6 + 3
*(a*b^2*c^3 + 3*a^2*b*c^2*d + a^3*c*d^2)*f^2*g^7 - 3*(a^2*b*c^3 + a^3*c^2*d
)*f*g^8)*x^3 + 3*(b^3*d^3*f^7*g^2 + a^3*c^3*f*g^8 - 3*(b^3*c*d^2 + a*b^2*d^
```

```
3)*f^6*g^3 + 3*(b^3*c^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3)*f^5*g^4 - (b^3*c^3 +
 9*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*f^4*g^5 + 3*(a*b^2*c^3 + 3*a^2*b*
c^2*d + a^3*c*d^2)*f^3*g^6 - 3*(a^2*b*c^3 + a^3*c^2*d)*f^2*g^7)*x^2 + 3*(b^
3*d^3*f^8*g + a^3*c^3*f^2*g^7 - 3*(b^3*c*d^2 + a*b^2*d^3)*f^7*g^2 + 3*(b^3*
c^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3)*f^6*g^3 - (b^3*c^3 + 9*a*b^2*c^2*d + 9*a
^2*b*c*d^2 + a^3*d^3)*f^5*g^4 + 3*(a*b^2*c^3 + 3*a^2*b*c^2*d + a^3*c*d^2)*f
^4*g^5 - 3*(a^2*b*c^3 + a^3*c^2*d)*f^3*g^6)*x) - 3*log(b^2*x^2*e/(d^2*x^2 +
 2*c*d*x + c^2) + 2*a*b*x*e/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*
c*d*x + c^2))/(g^5*x^4 + 4*f*g^4*x^3 + 6*f^2*g^3*x^2 + 4*f^3*g^2*x + f^4*g)
)*B - 1/4*A/(g^5*x^4 + 4*f*g^4*x^3 + 6*f^2*g^3*x^2 + 4*f^3*g^2*x + f^4*g)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(g*x+f)^5,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(b*x+a)**2/(d*x+c)**2))/(g*x+f)**5,x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(g*x+f)^5,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Undef
/Unsigned Inf encountered in limitUndef/Unsigned Inf encountered in limit(-
1/4*sageVARA*sageVARg^3-1/4*sageVARB*sageVARg^3)*(-(sageVARg*sageVARx+sageV
ARf)^-1/sageVARg
```

Mupad [**B**]
time = 17.43, size = 2520, normalized size = 6.61

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))/(f + g*x)^5,x)

[Out] (log(f + g*x)*(g*(6*B*a^2*b^2*d^4*f^2 - 6*B*b^4*c^2*d^2*f^2) - g^2*(4*B*a^3*b*d^4*f - 4*B*b^4*c^3*d*f) + g^3*(B*a^4*d^4 - B*b^4*c^4) - 4*B*a*b^3*d^4*f^3 + 4*B*b^4*c*d^3*f^3))/(2*a^4*c^4*g^8 + 2*b^4*d^4*f^8 + 2*a^4*d^4*f^4*g^4 + 2*b^4*c^4*f^4*g^4 + 12*a^2*b^2*c^4*f^2*g^6 + 12*a^2*b^2*d^4*f^6*g^2 + 12*a^4*c^2*d^2*f^2*g^6 + 12*b^4*c^2*d^2*f^6*g^2 - 8*a^3*b*c^4*f*g^7 - 8*a*b^3*d^4*f^7*g - 8*a^4*c^3*d*f*g^7 - 8*b^4*c*d^3*f^7*g - 8*a*b^3*c^4*f^3*g^5 - 8*a^3*b*d^4*f^5*g^3 - 8*a^4*c*d^3*f^3*g^5 - 8*b^4*c^3*d*f^5*g^3 + 32*a*b^3*c*d^3*f^6*g^2 + 32*a*b^3*c^3*d*f^4*g^4 + 32*a^3*b*c*d^3*f^4*g^4 + 32*a^3*b*c^3*d*f^2*g^6 - 48*a*b^3*c^2*d^2*f^5*g^3 - 48*a^2*b^2*c*d^3*f^5*g^3 - 48*a^2*b^2*c^3*d*f^3*g^5 - 48*a^3*b*c^2*d^2*f^3*g^5 + 72*a^2*b^2*c^2*d^2*f^4*g^4) - ((3*A*a^3*c^3*g^6 + 3*A*b^3*d^3*f^6 - 3*A*a^3*d^3*f^3*g^3 - 3*A*b^3*c^3*f^3*g^3 - 11*B*a^3*d^3*f^3*g^3 + 11*B*b^3*c^3*f^3*g^3 + 9*A*a*b^2*c^3*f^2*g^4 + 9*A*a^2*b*d^3*f^4*g^2 - 7*B*a*b^2*c^3*f^2*g^4 + 9*A*a^3*c*d^2*f^2*g^4 + 31*B*a^2*b*d^3*f^4*g^2 + 9*A*b^3*c^2*d*f^4*g^2 + 7*B*a^3*c*d^2*f^2*g^4 - 31*B*b^3*c^2*d*f^4*g^2 - 9*A*a^2*b*c^3*f*g^5 - 9*A*a*b^2*d^3*f^5*g + 2*B*a^2*b*c^3*f*g^5 - 9*A*a^3*c^2*d*f*g^5 - 26*B*a*b^2*d^3*f^5*g - 9*A*b^3*c*d^2*f^5*g - 2*B*a^3*c^2*d*f*g^5 + 26*B*b^3*c*d^2*f^5*g + 27*A*a*b^2*c*d^2*f^4*g^2 - 27*A*a*b^2*c^2*d*f^3*g^3 - 27*A*a^2*b*c*d^2*f^3*g^3 + 27*A*a^2*b*c^2*d*f^2*g^4 + 15*B*a*b^2*c^2*d*f^3*g^3 - 15*B*a^2*b*c*d^2*f^3*g^3)/(6*(a^3*c^3*g^6 + b^3*d^3*f^6 - a^3*d^3*f^3*g^3 - b^3*c^3*f^3*g^3 - 3*a^2*b*c^3*f*g^5 - 3*a*b^2*d^3*f^5*g - 3*a^3*c^2*d*f*g^5 - 3*b^3*c*d^2*f^5*g + 3*a*b^2*c^3*f^2*g^4 + 3*a^2*b*d^3*f^4*g^2 + 3*a^3*c*d^2*f^2*g^4 + 3*b^3*c^2*d*f^4*g^2 + 9*a*b^2*c*d^2*f^4*g^2 - 9*a*b^2*c^2*d*f^3*g^3 - 9*a^2*b*c*d^2*f^3*g^3 + 9*a^2*b*c^2*d*f^2*g^4)) - (x^2*(B*a*b^2*c^3*g^6 - B*a^3*c*d^2*g^6 + 7*B*a^3*d^3*f*g^5 - 7*B*b^3*c^3*f*g^5 + 20*B*a*b^2*d^3*f^3*g^3 - 21*B*a^2*b*d^3*f^2*g^4 - 20*B*b^3*c*d^2*f^3*g^3 + 21*B*b^3*c^2*d*f^2*g^4 - 3*B*a*b^2*c^2*d*f*g^5 + 3*B*a^2*b*c*d^2*f*g^5))/(2*(a^3*c^3*g^6 + b^3*d^3*f^6 - a^3*d^3*f^3*g^3 - b^3*c^3*f^3*g^3 - 3*a^2*b*c^3*f*g^5 - 3*a*b^2*d^3*f^5*g - 3*a^3*c^2*d*f*g^5 - 3*b^3*c*d^2*f^5*g + 3*a*b^2*c^3*f^2*g^4 + 3*a^2*b*d^3*f^4*g^2 + 3*a^3*c*d^2*f^2*g^4 + 3*b^3*c^2*d*f^4*g^2 + 9*a*b^2*c*d^2*f^4*g^2 - 9*a*b^2*c^2*d*f^3*g^3 - 9*a^2*b*c*d^2*f^3*g^3 + 9*a^2*b*c^2*d*f^2*g^4)) + (x*(B*a^2*b*c^3*g^6 - B*a^3*c^2*d*g^6 - 13*B*a^3*d^3*f^2*g^4 + 13*B*b^3*c^3*f^2*g^4 - 34*B*a*b^2*d^3*f^4*g^2 + 38*B*a^2*b*d^3*f^3*g^3 + 34*B*b^3*c*d^2*f^4*g^2 - 38*B*b^3*c^2*d*f^3*g^3 - 5*B*a*b^2*c^3*f*g^5 + 5*B*a^3*c*d^2*f*g^5 + 12*B*a*b^2*c^2*d*f^2*g^4 - 12*B*a^2*b*c*d^2*f^2*g^4))/(3*(a^3*c^3*g^6 + b^3*d^3*f^6 - a^3*d^3*f^3*g^3 - b^3*c^3*f^3*g^3 - 3*a^2*b*c^3*f*g^5 - 3*a*b^2*d^3*f^5*g - 3*a^3*c^2*d*f*g^5 - 3*b^3*c*d^2*f^5*g + 3*a*b^2*c^3*f^2*g^4 + 3*a^2*b*d^3*f^4*g^2 + 3*a^3*c*d^2*f^2*g^4 + 3*b^3*c^2*d*f^4*g^2 + 9*a*b^2*c*d^2*f^4*g^2 - 9*a*b^2*c^2*d*f^3*g^3 - 9*a^2*b*c*d^2*f^3*g^3 + 9*a^2*b*c^2*d*f^2*g^4)) - (x^3*(B*a^3*d^3*g^6 - B*b^3*c^3*g^6 + 3*B*a*b^2*d^3*f^2*g^4 - 3*B*b^3*c*d^2*f^2*g^4 - 3*B*a^2*b*d^3*f*g^5 + 3*B*b^3*c^2*d*f*g^5))/(a^3*c^3*g^6 +

```
b^3*d^3*f^6 - a^3*d^3*f^3*g^3 - b^3*c^3*f^3*g^3 - 3*a^2*b*c^3*f*g^5 - 3*a*
b^2*d^3*f^5*g - 3*a^3*c^2*d*f*g^5 - 3*b^3*c*d^2*f^5*g + 3*a*b^2*c^3*f^2*g^4
 + 3*a^2*b*d^3*f^4*g^2 + 3*a^3*c*d^2*f^2*g^4 + 3*b^3*c^2*d*f^4*g^2 + 9*a*b^
2*c*d^2*f^4*g^2 - 9*a*b^2*c^2*d*f^3*g^3 - 9*a^2*b*c*d^2*f^3*g^3 + 9*a^2*b*c
^2*d*f^2*g^4))/(2*f^4*g + 2*g^5*x^4 + 8*f^3*g^2*x + 8*f*g^4*x^3 + 12*f^2*g^
3*x^2) + (B*b^4*log(a + b*x))/(2*a^4*g^5 + 2*b^4*f^4*g - 8*a*b^3*f^3*g^2 +
12*a^2*b^2*f^2*g^3 - 8*a^3*b*f*g^4) - (B*log((e*(a + b*x)^2)/(c + d*x)^2))/
(4*g*(f^4 + g^4*x^4 + 4*f^3*g*x + 4*f*g^3*x^3 + 6*f^2*g^2*x^2)) - (B*d^4*lo
g(c + d*x))/(2*c^4*g^5 + 2*d^4*f^4*g - 8*c*d^3*f^3*g^2 + 12*c^2*d^2*f^2*g^3
 - 8*c^3*d*f*g^4)
```

3.272 $\int (f+gx)^3 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx$

Optimal. Leaf size=869

$$\frac{2B^2(bc-ad)^3g^3x}{3b^3d^3}+\frac{B^2(bc-ad)^2g^2(4bdf-3bcg-adg)x}{b^3d^3}+\frac{B^2(bc-ad)^2g^3(c+dx)^2}{3b^2d^4}-\frac{B(bc-ad)g(a^2d^2g^2 \ldots}{}$$

```
[Out] 2/3*B^2*(-a*d+b*c)^3*g^3*x/b^3/d^3+B^2*(-a*d+b*c)^2*g^2*(-a*d*g-3*b*c*g+4*b
*d*f)*x/b^3/d^3+1/3*B^2*(-a*d+b*c)^2*g^3*(d*x+c)^2/b^2/d^4-B*(-a*d+b*c)*g*(
a^2*d^2*g^2-2*a*b*d*g*(-c*g+2*d*f)+b^2*(3*c^2*g^2-8*c*d*f*g+6*d^2*f^2))*(b*
x+a)*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/b^4/d^3-1/2*B*(-a*d+b*c)*g^2*(-a*d*g-3
*b*c*g+4*b*d*f)*(d*x+c)^2*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/b^2/d^4-1/3*B*(-a
*d+b*c)*g^3*(d*x+c)^3*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/b/d^4-1/4*(-a*g+b*f)^
4*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/b^4/g+1/4*(g*x+f)^4*(A+B*ln(e*(b*x+a)^2
/(d*x+c)^2))^2/g-B*(-a*d+b*c)*(-a*d*g-b*c*g+2*b*d*f)*(2*a*b*d^2*f*g-a^2*d^2
*g^2-b^2*(c^2*g^2-2*c*d*f*g+2*d^2*f^2))*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))*ln(
(-a*d+b*c)/b/(d*x+c))/b^4/d^4+2/3*B^2*(-a*d+b*c)^4*g^3*ln((b*x+a)/(d*x+c))/
b^4/d^4+B^2*(-a*d+b*c)^3*g^2*(-a*d*g-3*b*c*g+4*b*d*f)*ln((b*x+a)/(d*x+c))/b
^4/d^4+2/3*B^2*(-a*d+b*c)^4*g^3*ln(d*x+c)/b^4/d^4+B^2*(-a*d+b*c)^3*g^2*(-a*
d*g-3*b*c*g+4*b*d*f)*ln(d*x+c)/b^4/d^4+2*B^2*(-a*d+b*c)^2*g*(a^2*d^2*g^2-2*
a*b*d*g*(-c*g+2*d*f)+b^2*(3*c^2*g^2-8*c*d*f*g+6*d^2*f^2))*ln(d*x+c)/b^4/d^4
-2*B^2*(-a*d+b*c)*(-a*d*g-b*c*g+2*b*d*f)*(2*a*b*d^2*f*g-a^2*d^2*g^2-b^2*(c^
2*g^2-2*c*d*f*g+2*d^2*f^2))*polylog(2,d*(b*x+a)/b/(d*x+c))/b^4/d^4
```

Rubi [A]
time = 1.10, antiderivative size = 869, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.323$, Rules used = {2554, 2398, 2404, 2338, 2356, 46, 2351, 31, 2354, 2438}

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2,x]
```

```
[Out] (2*B^2*(b*c - a*d)^3*g^3*x)/(3*b^3*d^3) + (B^2*(b*c - a*d)^2*g^2*(4*b*d*f -
 3*b*c*g - a*d*g)*x)/(b^3*d^3) + (B^2*(b*c - a*d)^2*g^3*(c + d*x)^2)/(3*b^2
*d^4) - (B*(b*c - a*d)*g*(a^2*d^2*g^2 - 2*a*b*d*g*(2*d*f - c*g) + b^2*(6*d^
2*f^2 - 8*c*d*f*g + 3*c^2*g^2))*(a + b*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d
*x)^2]))/(b^4*d^3) - (B*(b*c - a*d)*g^2*(4*b*d*f - 3*b*c*g - a*d*g)*(c + d*
x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]))/(2*b^2*d^4) - (B*(b*c - a*d)
*g^3*(c + d*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]))/(3*b*d^4) - ((b*
f - a*g)^4*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2)/(4*b^4*g) + ((f + g*
x)^4*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2)/(4*g) - (B*(b*c - a*d)*(2*
```

```
b*d*f - b*c*g - a*d*g)*(2*a*b*d^2*f*g - a^2*d^2*g^2 - b^2*(2*d^2*f^2 - 2*c*
d*f*g + c^2*g^2))*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])*Log[(b*c - a*d)/
(b*(c + d*x))])/(b^4*d^4) + (2*B^2*(b*c - a*d)^4*g^3*Log[(a + b*x)/(c + d*x
)])/(3*b^4*d^4) + (B^2*(b*c - a*d)^3*g^2*(4*b*d*f - 3*b*c*g - a*d*g)*Log[(a
 + b*x)/(c + d*x)])/(b^4*d^4) + (2*B^2*(b*c - a*d)^4*g^3*Log[c + d*x])/(3*b
^4*d^4) + (B^2*(b*c - a*d)^3*g^2*(4*b*d*f - 3*b*c*g - a*d*g)*Log[c + d*x])/
(b^4*d^4) + (2*B^2*(b*c - a*d)^2*g*(a^2*d^2*g^2 - 2*a*b*d*g*(2*d*f - c*g) +
 b^2*(6*d^2*f^2 - 8*c*d*f*g + 3*c^2*g^2))*Log[c + d*x])/(b^4*d^4) - (2*B^2*
(b*c - a*d)*(2*b*d*f - b*c*g - a*d*g)*(2*a*b*d^2*f*g - a^2*d^2*g^2 - b^2*(2
*d^2*f^2 - 2*c*d*f*g + c^2*g^2))*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/(
b^4*d^4)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
```

```
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_)*((
f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(d + e*x)^(q +
 1)*((a + b*Log[c*x^n])^p/((q + 1)*(e*f - d*g))), x] - Dist[b*n*(p/((q + 1)
*(e*f - d*g))), Int[(f + g*x)^(m + 1)*(d + e*x)^(q + 1)*((a + b*Log[c*x^n])
^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, q}, x] && NeQ[e*f
- d*g, 0] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 2404

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2554

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[b*c - a*d, Su
bst[Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m +
2)), x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n
}, x] && EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegerQ[m] &
& IGtQ[p, 0]
```

Rubi steps

$$\int (f+gx)^3 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx = \frac{(f+gx)^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{4g} - \frac{B\int \frac{2(bc-ad)(f+gx)^4\left(A+B\ldots\right.}{(a+bx)(c+\ldots}}{2g}$$

$$= \frac{(f+gx)^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{4g} - \frac{(B(bc-ad))\int \frac{(f+gx)^4}{\ldots}}{g\ldots}$$

$$= \frac{(f+gx)^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{4g} - \frac{(B(bc-ad))\int \left(\frac{g^2(a^2\ldots}{\ldots}\right.}{\ldots}$$

$$= \frac{(f+gx)^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{4g} - \frac{(B(bc-ad)g^3)\int x^2\left(A\ldots\right.}{\ldots}$$

$$= -\frac{AB(bc-ad)g(a^2d^2g^2-abdg(4df-cg)+b^2(6d^2f^2-4cdf\ldots}{b^3d^3}$$

$$= -\frac{AB(bc-ad)g(a^2d^2g^2-abdg(4df-cg)+b^2(6d^2f^2-4cdf\ldots}{b^3d^3}$$

$$= -\frac{AB(bc-ad)g(a^2d^2g^2-abdg(4df-cg)+b^2(6d^2f^2-4cdf\ldots}{b^3d^3}$$

$$= -\frac{2B^2(bc-ad)^2(bc+ad)g^3x}{3b^3d^3} + \frac{B^2(bc-ad)^2g^2(4bdf-bcg-\ldots}{b^3d^3}$$

$$= -\frac{2B^2(bc-ad)^2(bc+ad)g^3x}{3b^3d^3} + \frac{B^2(bc-ad)^2g^2(4bdf-bcg-\ldots}{b^3d^3}$$

$$= -\frac{2B^2(bc-ad)^2(bc+ad)g^3x}{3b^3d^3} + \frac{B^2(bc-ad)^2g^2(4bdf-bcg-\ldots}{b^3d^3}$$

$$= -\frac{2B^2(bc-ad)^2(bc+ad)g^3x}{3b^3d^3} + \frac{B^2(bc-ad)^2g^2(4bdf-bcg-\ldots}{b^3d^3}$$

Mathematica [A]
time = 0.63, size = 746, normalized size = 0.86

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2,x]
```

```
[Out] ((f + g*x)^4*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2 - (2*B*(6*A*b*d*(b*
c - a*d)*g^2*(a^2*d^2*g^2 + a*b*d*g*(-4*d*f + c*g) + b^2*(6*d^2*f^2 - 4*c*d
*f*g + c^2*g^2))*x + 6*B*d*(b*c - a*d)*g^2*(a^2*d^2*g^2 + a*b*d*g*(-4*d*f +
 c*g) + b^2*(6*d^2*f^2 - 4*c*d*f*g + c^2*g^2))*(a + b*x)*Log[(e*(a + b*x)^2
)/(c + d*x)^2] + 3*b^2*d^2*(b*c - a*d)*g^3*(4*b*d*f - b*c*g - a*d*g)*x^2*(A
 + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]) + 2*b^3*d^3*(b*c - a*d)*g^4*x^3*(A +
 B*Log[(e*(a + b*x)^2)/(c + d*x)^2]) + 6*d^4*(b*f - a*g)^4*Log[a + b*x]*(A
+ B*Log[(e*(a + b*x)^2)/(c + d*x)^2]) - 12*B*(b*c - a*d)^2*g^2*(a^2*d^2*g^2
 + a*b*d*g*(-4*d*f + c*g) + b^2*(6*d^2*f^2 - 4*c*d*f*g + c^2*g^2))*Log[c +
d*x] - 6*b^4*(d*f - c*g)^4*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])*Log[c +
 d*x] + 2*B*(b*c - a*d)*g^4*(b*d*(b*c - a*d)*x*(2*b*c + 2*a*d - b*d*x) + 2*
a^3*d^3*Log[a + b*x] - 2*b^3*c^3*Log[c + d*x]) - 6*B*(b*c - a*d)*g^3*(-4*b*
d*f + b*c*g + a*d*g)*(-(a^2*d^2*Log[a + b*x]) + b*(d*(-(b*c) + a*d)*x + b*c
^2*Log[c + d*x])) - 6*B*d^4*(b*f - a*g)^4*(Log[a + b*x]*(Log[a + b*x] - 2*L
og[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]
) + 6*b^4*B*(d*f - c*g)^4*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d
*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(3*b^4*d^4))
/(4*g)
```

Maple [**F**]
time = 0.22, size = 0, normalized size = 0.00

$$\int (gx+f)^3 \left(A + B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^3*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

```
[Out] int((g*x+f)^3*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2188 vs. 2(858) = 1716.
time = 0.42, size = 2188, normalized size = 2.52

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="maxima
")
```

```
[Out] 1/4*A^2*g^3*x^4 + A^2*f*g^2*x^3 + 3/2*A^2*f^2*g*x^2 + 2*(x*log(b^2*x^2*e/(d
^2*x^2 + 2*c*d*x + c^2) + 2*a*b*x*e/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*
x^2 + 2*c*d*x + c^2)) + 2*a*log(b*x + a)/b - 2*c*log(d*x + c)/d)*A*B*f^3 +
3*(x^2*log(b^2*x^2*e/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*x*e/(d^2*x^2 + 2*c*d
```

```
*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) - 2*a^2*log(b*x + a)/b^2 + 2*c
^2*log(d*x + c)/d^2 - 2*(b*c - a*d)*x/(b*d))*A*B*f^2*g + 2*(x^3*log(b^2*x^2
*e/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*x*e/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/
(d^2*x^2 + 2*c*d*x + c^2)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^
3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*A*B*f*g^
2 + 1/6*(3*x^4*log(b^2*x^2*e/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*x*e/(d^2*x^2
 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) - 6*a^4*log(b*x + a)/b
^4 + 6*c^4*log(d*x + c)/d^4 - (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d
 - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3))*A*B*g^3 + A^2*f^3*x
 - 1/3*(6*a^3*c*d^3*g^3 - 3*(8*c*d^3*f*g^2 - c^2*d^2*g^3)*a^2*b + 2*(18*c*d
^3*f^2*g - 6*c^2*d^2*f*g^2 + c^3*d*g^3)*a*b^2 + 2*(6*c*d^3*f^3 - 27*c^2*d^2
*f^2*g + 24*c^3*d*f*g^2 - 7*c^4*g^3)*b^3)*B^2*log(d*x + c)/(b^3*d^4) + 2*(4
*a*b^3*d^4*f^3 - 6*a^2*b^2*d^4*f^2*g + 4*a^3*b*d^4*f*g^2 - a^4*d^4*g^3 - (4
*c*d^3*f^3 - 6*c^2*d^2*f^2*g + 4*c^3*d*f*g^2 - c^4*g^3)*b^4)*(log(b*x + a)*
log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B^2
/(b^4*d^4) + 1/12*(3*B^2*b^4*d^4*g^3*x^4 + 4*(a*b^3*d^4*g^3 + (3*d^4*f*g^2
- c*d^3*g^3)*b^4)*B^2*x^3 - 2*(a^2*b^2*d^4*g^3 - 4*(3*d^4*f*g^2 - c*d^3*g^3
)*a*b^3 - (9*d^4*f^2*g - 12*c*d^3*f*g^2 + 5*c^2*d^2*g^3)*b^4)*B^2*x^2 + 4*(
5*a^2*b^2*c*d^3*g^3 - 2*a^3*b*d^4*g^3 + (18*d^4*f^2*g - 24*c*d^3*f*g^2 + 5*
c^2*d^2*g^3)*a*b^3 + (3*d^4*f^3 - 18*c*d^3*f^2*g + 24*c^2*d^2*f*g^2 - 8*c^3
*d*g^3)*b^4)*B^2*x + 12*(B^2*b^4*d^4*g^3*x^4 + 4*B^2*b^4*d^4*f*g^2*x^3 + 6*
B^2*b^4*d^4*f^2*g*x^2 + 4*B^2*b^4*d^4*f^3*x + (4*a*b^3*d^4*f^3 - 6*a^2*b^2*
d^4*f^2*g + 4*a^3*b*d^4*f*g^2 - a^4*d^4*g^3)*B^2)*log(b*x + a)^2 + 12*(B^2*
b^4*d^4*g^3*x^4 + 4*B^2*b^4*d^4*f*g^2*x^3 + 6*B^2*b^4*d^4*f^2*g*x^2 + 4*B^2
*b^4*d^4*f^3*x + (4*c*d^3*f^3 - 6*c^2*d^2*f^2*g + 4*c^3*d*f*g^2 - c^4*g^3)*
B^2*b^4)*log(d*x + c)^2 + 4*(3*B^2*b^4*d^4*g^3*x^4 + 2*(a*b^3*d^4*g^3 + (6*
d^4*f*g^2 - c*d^3*g^3)*b^4)*B^2*x^3 + 3*(4*a*b^3*d^4*f*g^2 - a^2*b^2*d^4*g^
3 + (6*d^4*f^2*g - 4*c*d^3*f*g^2 + c^2*d^2*g^3)*b^4)*B^2*x^2 + 6*(6*a*b^3*d
^4*f^2*g - 4*a^2*b^2*d^4*f*g^2 + a^3*b*d^4*g^3 + (2*d^4*f^3 - 6*c*d^3*f^2*g
 + 4*c^2*d^2*f*g^2 - c^3*d*g^3)*b^4)*B^2*x + (8*a^4*d^4*g^3 - 2*(12*d^4*f*g
^2 + c*d^3*g^3)*a^3*b + 3*(6*d^4*f^2*g + 4*c*d^3*f*g^2 - c^2*d^2*g^3)*a^2*b
^2 + 6*(2*d^4*f^3 - 6*c*d^3*f^2*g + 4*c^2*d^2*f*g^2 - c^3*d*g^3)*a*b^3)*B^2
)*log(b*x + a) - 4*(3*B^2*b^4*d^4*g^3*x^4 + 2*(a*b^3*d^4*g^3 + (6*d^4*f*g^2
 - c*d^3*g^3)*b^4)*B^2*x^3 + 3*(4*a*b^3*d^4*f*g^2 - a^2*b^2*d^4*g^3 + (6*d^
4*f^2*g - 4*c*d^3*f*g^2 + c^2*d^2*g^3)*b^4)*B^2*x^2 + 6*(6*a*b^3*d^4*f^2*g
- 4*a^2*b^2*d^4*f*g^2 + a^3*b*d^4*g^3 + (2*d^4*f^3 - 6*c*d^3*f^2*g + 4*c^2*
d^2*f*g^2 - c^3*d*g^3)*b^4)*B^2*x + 6*(B^2*b^4*d^4*g^3*x^4 + 4*B^2*b^4*d^4*
f*g^2*x^3 + 6*B^2*b^4*d^4*f^2*g*x^2 + 4*B^2*b^4*d^4*f^3*x + (4*a*b^3*d^4*f^
3 - 6*a^2*b^2*d^4*f^2*g + 4*a^3*b*d^4*f*g^2 - a^4*d^4*g^3)*B^2)*log(b*x + a
))*log(d*x + c))/(b^4*d^4)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="fricas
")
```

```
[Out] integral(A^2*g^3*x^3 + 3*A^2*f*g^2*x^2 + 3*A^2*f^2*g*x + A^2*f^3 + (B^2*g^3
*x^3 + 3*B^2*f*g^2*x^2 + 3*B^2*f^2*g*x + B^2*f^3)*log((b^2*x^2 + 2*a*b*x +
a^2)*e/(d^2*x^2 + 2*c*d*x + c^2))^2 + 2*(A*B*g^3*x^3 + 3*A*B*f*g^2*x^2 + 3*
A*B*f^2*g*x + A*B*f^3)*log((b^2*x^2 + 2*a*b*x + a^2)*e/(d^2*x^2 + 2*c*d*x +
 c^2)), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**3*(A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="giac")
```

```
[Out] integrate((g*x + f)^3*(B*log((b*x + a)^2*e/(d*x + c)^2) + A)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int (f+g\,x)^3\,\left(A+B\,\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^3*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2,x)
```

```
[Out] int((f + g*x)^3*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2, x)
```

3.273 $\int (f+gx)^2 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx$

Optimal. Leaf size=542

$$\frac{4B^2(bc-ad)^2g^2x}{3b^2d^2}-\frac{4B(bc-ad)g(3bdf-2bcg-adg)(a+bx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{3b^3d^2}-\frac{2B(bc-ad)g^2(c+d$$

```
[Out] 4/3*B^2*(-a*d+b*c)^2*g^2*x/b^2/d^2-4/3*B*(-a*d+b*c)*g*(-a*d*g-2*b*c*g+3*b*d
*f)*(b*x+a)*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/b^3/d^2-2/3*B*(-a*d+b*c)*g^2*(d
*x+c)^2*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/b/d^3-1/3*(-a*g+b*f)^3*(A+B*ln(e*(b
*x+a)^2/(d*x+c)^2))^2/b^3/g+1/3*(g*x+f)^3*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2
/g+4/3*B*(-a*d+b*c)*(a^2*d^2*g^2-a*b*d*g*(-c*g+3*d*f)+b^2*(c^2*g^2-3*c*d*f*
g+3*d^2*f^2))*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))*ln((-a*d+b*c)/b/(d*x+c))/b^3/
d^3+4/3*B^2*(-a*d+b*c)^3*g^2*ln((b*x+a)/(d*x+c))/b^3/d^3+4/3*B^2*(-a*d+b*c)
^3*g^2*ln(d*x+c)/b^3/d^3+8/3*B^2*(-a*d+b*c)^2*g*(-a*d*g-2*b*c*g+3*b*d*f)*ln
(d*x+c)/b^3/d^3+8/3*B^2*(-a*d+b*c)*(a^2*d^2*g^2-a*b*d*g*(-c*g+3*d*f)+b^2*(c
^2*g^2-3*c*d*f*g+3*d^2*f^2))*polylog(2,d*(b*x+a)/b/(d*x+c))/b^3/d^3
```

Rubi [A]
time = 0.72, antiderivative size = 542, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.323$, Rules used = {2554, 2398, 2404, 2338, 2356, 46, 2351, 31, 2354, 2438}

$$\frac{8B^2(bc-ad)\left(a^2d^2g^2-abdg(3df-cg)+b^2(c^2g^2-3cdfg+3d^2f^2)\right)\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{3b^3d^3}+\frac{4B(bc-ad)\left(a^2d^2g^2-abdg(3df-cg)+b^2(c^2g^2-3cdfg+3d^2f^2)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{3b^3d^3}-\frac{4Bg(a+bx)(bc-ad)(-adg-2bcg+3bdf)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{3b^3d^2}-\frac{(bf-ag)^3\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{3b^3g}-\frac{2Bg^2(c+dx)^2(bc-ad)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{3bd^3}+\frac{(f+gx)^3\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{3g}+\frac{8B^2g(bc-ad)^2\log(c+dx)(-adg-2bcg+3bdf)}{3b^3d^3}+\frac{4B^2g^2(bc-ad)^3\log\left(\frac{a+bx}{c+dx}\right)}{3b^3d^3}+\frac{4B^2g^2(bc-ad)^3\log(c+dx)}{3b^3d^3}+\frac{4B^2g^2x(bc-ad)^2}{3b^2d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2,x]
```

```
[Out] (4*B^2*(b*c - a*d)^2*g^2*x)/(3*b^2*d^2) - (4*B*(b*c - a*d)*g*(3*b*d*f - 2*b
*c*g - a*d*g)*(a + b*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]))/(3*b^3*d^
2) - (2*B*(b*c - a*d)*g^2*(c + d*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^
2]))/(3*b*d^3) - ((b*f - a*g)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2)
/(3*b^3*g) + ((f + g*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2)/(3*g)
 + (4*B*(b*c - a*d)*(a^2*d^2*g^2 - a*b*d*g*(3*d*f - c*g) + b^2*(3*d^2*f^2 -
 3*c*d*f*g + c^2*g^2))*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])*Log[(b*c -
a*d)/(b*(c + d*x))])/(3*b^3*d^3) + (4*B^2*(b*c - a*d)^3*g^2*Log[(a + b*x)/(
c + d*x)])/(3*b^3*d^3) + (4*B^2*(b*c - a*d)^3*g^2*Log[c + d*x])/(3*b^3*d^3)
 + (8*B^2*(b*c - a*d)^2*g*(3*b*d*f - 2*b*c*g - a*d*g)*Log[c + d*x])/(3*b^3*
d^3) + (8*B^2*(b*c - a*d)*(a^2*d^2*g^2 - a*b*d*g*(3*d*f - c*g) + b^2*(3*d^2
*f^2 - 3*c*d*f*g + c^2*g^2))*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/(3*b^
3*d^3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_)*((
f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(d + e*x)^(q +
 1)*((a + b*Log[c*x^n])^p/((q + 1)*(e*f - d*g))), x] - Dist[b*n*(p/((q + 1)
*(e*f - d*g))), Int[(f + g*x)^(m + 1)*(d + e*x)^(q + 1)*((a + b*Log[c*x^n])
^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, q}, x] && NeQ[e*f
- d*g, 0] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 2404

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2554

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[b*c - a*d, Su
bst[Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m +
2)), x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n
}, x] && EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegerQ[m] &
& IGtQ[p, 0]
```

Rubi steps

$$\int (f+gx)^2 \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx = \frac{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{3g} - \frac{(2B)\int \frac{2(bc-ad)(f+gx)^3 \ldots}{(a+\ldots}}{3\cdots} \ldots$$

$$= \frac{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{3g} - \frac{(4B(bc-ad))\int \frac{(f+g\ldots}{\ldots}}{\ldots} \ldots$$

$$= \frac{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{3g} - \frac{(4B(bc-ad))\int \left(\frac{g^2(\ldots}{\ldots}\right.}{\ldots} \ldots$$

$$= \frac{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{3g} - \frac{(4B(bc-ad)g^2)\int x(\ldots}{\ldots} \ldots$$

$$= -\frac{4AB(bc-ad)g(3bdf-bcg-adg)x}{3b^2d^2} - \frac{2B(bc-ad)g^2x^2(\ldots}{\ldots} \ldots$$

$$= -\frac{4AB(bc-ad)g(3bdf-bcg-adg)x}{3b^2d^2} - \frac{4B^2(bc-ad)g(3b\ldots}{\ldots} \ldots$$

$$= -\frac{4AB(bc-ad)g(3bdf-bcg-adg)x}{3b^2d^2} - \frac{4B^2(bc-ad)g(3b\ldots}{\ldots} \ldots$$

$$= \frac{4B^2(bc-ad)^2g^2x}{3b^2d^2} - \frac{4AB(bc-ad)g(3bdf-bcg-adg)x}{3b^2d^2} + \ldots$$

$$= \frac{4B^2(bc-ad)^2g^2x}{3b^2d^2} - \frac{4AB(bc-ad)g(3bdf-bcg-adg)x}{3b^2d^2} + \ldots$$

$$= \frac{4B^2(bc-ad)^2g^2x}{3b^2d^2} - \frac{4AB(bc-ad)g(3bdf-bcg-adg)x}{3b^2d^2} + \ldots$$

$$= \frac{4B^2(bc-ad)^2g^2x}{3b^2d^2} - \frac{4AB(bc-ad)g(3bdf-bcg-adg)x}{3b^2d^2} + \ldots$$

Mathematica [A]
time = 0.35, size = 497, normalized size = 0.92

$$\frac{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 - \frac{2B\left(2Abd(bc-ad)g^2(3bdf-bcg-adg)x+2Bd(bc-ad)g^2(3bdf-bcg-adg)(a+bx)\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+b^2d^2(bc-ad)g^3x^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)+2d^3(bf-ag)^3\log(a+bx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)+4B(bc-ad)^2g^2(-3bdf+bcg+adg)\log(c+dx)-2b^3(df-cg)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\log(c+dx)-2B(bc-ad)g^3\left(a^2d^2\log(a+bx)-b\left(d(-bc+ad)x+bc^2\log(c+dx)\right)\right)-2Bd^3(bf-ag)^3\left(\log(a+bx)\left(\log(a+bx)-2\log\left(\frac{b(c+dx)}{bc-ad}\right)\right)-2\text{Li}_2\left(\frac{d(a+bx)}{-bc+ad}\right)\right)+2b^3B(df-cg)^3\left(\left(2\log\left(\frac{d(a+bx)}{-bc+ad}\right)-\log(c+dx)\right)\log(c+dx)+2\text{Li}_2\left(\frac{b(c+dx)}{bc-ad}\right)\right)\right)}{b^3d^3}}{3g}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2,x]
```

```
[Out] ((f + g*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2 - (2*B*(2*A*b*d*(b*
c - a*d)*g^2*(3*b*d*f - b*c*g - a*d*g)*x + 2*B*d*(b*c - a*d)*g^2*(3*b*d*f -
 b*c*g - a*d*g)*(a + b*x)*Log[(e*(a + b*x)^2)/(c + d*x)^2] + b^2*d^2*(b*c -
 a*d)*g^3*x^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]) + 2*d^3*(b*f - a*g)^
3*Log[a + b*x]*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]) + 4*B*(b*c - a*d)^2
*g^2*(-3*b*d*f + b*c*g + a*d*g)*Log[c + d*x] - 2*b^3*(d*f - c*g)^3*(A + B*L
og[(e*(a + b*x)^2)/(c + d*x)^2])*Log[c + d*x] - 2*B*(b*c - a*d)*g^3*(a^2*d^
2*Log[a + b*x] - b*(d*(-(b*c) + a*d)*x + b*c^2*Log[c + d*x])) - 2*B*d^3*(b*
f - a*g)^3*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)])
- 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) + 2*b^3*B*(d*f - c*g)^3*((2*L
og[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2
, (b*(c + d*x))/(b*c - a*d)])))/(b^3*d^3))/(3*g)
```

Maple [F]
time = 0.19, size = 0, normalized size = 0.00

$$\int (gx+f)^2 \left(A + B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

```
[Out] int((g*x+f)^2*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1372 vs. 2(526) = 1052.
time = 0.40, size = 1372, normalized size = 2.53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="maxima
")
```

```
[Out] 1/3*A^2*g^2*x^3 + A^2*f*g*x^2 + 2*(x*log(b^2*x^2*e/(d^2*x^2 + 2*c*d*x + c^2
) + 2*a*b*x*e/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2))
+ 2*a*log(b*x + a)/b - 2*c*log(d*x + c)/d)*A*B*f^2 + 2*(x^2*log(b^2*x^2*e/(
d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*x*e/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2
*x^2 + 2*c*d*x + c^2)) - 2*a^2*log(b*x + a)/b^2 + 2*c^2*log(d*x + c)/d^2 -
2*(b*c - a*d)*x/(b*d))*A*B*f*g + 2/3*(x^3*log(b^2*x^2*e/(d^2*x^2 + 2*c*d*x
+ c^2) + 2*a*b*x*e/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c
^2)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^
2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*A*B*g^2 + A^2*f^2*x + 4/3*(2*a
^2*c*d^2*g^2 - (6*c*d^2*f*g - c^2*d*g^2)*a*b - (3*c*d^2*f^2 - 9*c^2*d*f*g +
```

```
4*c^3*g^2)*b^2)*B^2*log(d*x + c)/(b^2*d^3) + 8/3*(3*a*b^2*d^3*f^2 - 3*a^2*
b*d^3*f*g + a^3*d^3*g^2 - (3*c*d^2*f^2 - 3*c^2*d*f*g + c^3*g^2)*b^3)*(log(b
*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*
d)))*B^2/(b^3*d^3) + 1/3*(B^2*b^3*d^3*g^2*x^3 + (2*a*b^2*d^3*g^2 + (3*d^3*f
*g - 2*c*d^2*g^2)*b^3)*B^2*x^2 + (4*(3*d^3*f*g - 2*c*d^2*g^2)*a*b^2 + (3*d^
3*f^2 - 12*c*d^2*f*g + 8*c^2*d*g^2)*b^3)*B^2*x + 4*(B^2*b^3*d^3*g^2*x^3 + 3
*B^2*b^3*d^3*f*g*x^2 + 3*B^2*b^3*d^3*f^2*x + (3*a*b^2*d^3*f^2 - 3*a^2*b*d^3
*f*g + a^3*d^3*g^2)*B^2)*log(b*x + a)^2 + 4*(B^2*b^3*d^3*g^2*x^3 + 3*B^2*b^
3*d^3*f*g*x^2 + 3*B^2*b^3*d^3*f^2*x + (3*c*d^2*f^2 - 3*c^2*d*f*g + c^3*g^2)
*B^2*b^3)*log(d*x + c)^2 + 4*(B^2*b^3*d^3*g^2*x^3 + (a*b^2*d^3*g^2 + (3*d^3
*f*g - c*d^2*g^2)*b^3)*B^2*x^2 + (6*a*b^2*d^3*f*g - 2*a^2*b*d^3*g^2 + (3*d^
3*f^2 - 6*c*d^2*f*g + 2*c^2*d*g^2)*b^3)*B^2*x - (2*a^3*d^3*g^2 - (3*d^3*f*g
 + c*d^2*g^2)*a^2*b - (3*d^3*f^2 - 6*c*d^2*f*g + 2*c^2*d*g^2)*a*b^2)*B^2)*l
og(b*x + a) - 4*(B^2*b^3*d^3*g^2*x^3 + (a*b^2*d^3*g^2 + (3*d^3*f*g - c*d^2*
g^2)*b^3)*B^2*x^2 + (6*a*b^2*d^3*f*g - 2*a^2*b*d^3*g^2 + (3*d^3*f^2 - 6*c*d
^2*f*g + 2*c^2*d*g^2)*b^3)*B^2*x + 2*(B^2*b^3*d^3*g^2*x^3 + 3*B^2*b^3*d^3*f
*g*x^2 + 3*B^2*b^3*d^3*f^2*x + (3*a*b^2*d^3*f^2 - 3*a^2*b*d^3*f*g + a^3*d^3
*g^2)*B^2)*log(b*x + a))*log(d*x + c))/(b^3*d^3)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="fricas
")
```

```
[Out] integral(A^2*g^2*x^2 + 2*A^2*f*g*x + A^2*f^2 + (B^2*g^2*x^2 + 2*B^2*f*g*x +
 B^2*f^2)*log((b^2*x^2 + 2*a*b*x + a^2)*e/(d^2*x^2 + 2*c*d*x + c^2))^2 + 2*
(A*B*g^2*x^2 + 2*A*B*f*g*x + A*B*f^2)*log((b^2*x^2 + 2*a*b*x + a^2)*e/(d^2*
x^2 + 2*c*d*x + c^2)), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2*(A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="giac")
[Out] integrate((g*x + f)^2*(B*log((b*x + a)^2*e/(d*x + c)^2) + A)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int (f+g\,x)^2\,\left(A+B\,\ln\left(\frac{e\,{(a+b\,x)}^2}{{(c+d\,x)}^2}\right)\right)^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^2*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2,x)
[Out] int((f + g*x)^2*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2, x)
```

3.274 $\int (f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx$

Optimal. Leaf size=281

$$-\frac{2B(bc-ad)g(a+bx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{b^2d}-\frac{(bf-ag)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{2b^2g}+\frac{(f+gx)^2\left(A+B\,\mathrm{l}\right.}{2g}$$

```
[Out] -2*B*(-a*d+b*c)*g*(b*x+a)*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/b^2/d-1/2*(-a*g+b
*f)^2*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/b^2/g+1/2*(g*x+f)^2*(A+B*ln(e*(b*x+
a)^2/(d*x+c)^2))^2/g+2*B*(-a*d+b*c)*(-a*d*g-b*c*g+2*b*d*f)*(A+B*ln(e*(b*x+a
)^2/(d*x+c)^2))*ln((-a*d+b*c)/b/(d*x+c))/b^2/d^2+4*B^2*(-a*d+b*c)^2*g*ln(d*
x+c)/b^2/d^2+4*B^2*(-a*d+b*c)*(-a*d*g-b*c*g+2*b*d*f)*polylog(2,d*(b*x+a)/b/
(d*x+c))/b^2/d^2
```

Rubi [A]
time = 0.37, antiderivative size = 281, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {2554, 2398, 2404, 2338, 2351, 31, 2354, 2438}

$$\frac{4B^2(bc-ad)(-adg-bcg+2bdf)\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{b^2d^2}+\frac{2B(bc-ad)(-adg-bcg+2bdf)\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{b^2d^2}-\frac{(bf-ag)^2\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{2b^2g}-\frac{2Bg(a+bx)(bc-ad)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{b^2d}+\frac{(f+gx)^2\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{2g}+\frac{4B^2g(bc-ad)^2\log(c+dx)}{b^2d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2,x]
```

```
[Out] (-2*B*(b*c - a*d)*g*(a + b*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]))/(b^
2*d) - ((b*f - a*g)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2)/(2*b^2*g)
 + ((f + g*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2)/(2*g) + (2*B*(b
*c - a*d)*(2*b*d*f - b*c*g - a*d*g)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]
)*Log[(b*c - a*d)/(b*(c + d*x))])/(b^2*d^2) + (4*B^2*(b*c - a*d)^2*g*Log[c
+ d*x])/(b^2*d^2) + (4*B^2*(b*c - a*d)*(2*b*d*f - b*c*g - a*d*g)*PolyLog[2,
 (d*(a + b*x))/(b*(c + d*x))])/(b^2*d^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
```

```
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_)*((
f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(d + e*x)^(q +
 1)*((a + b*Log[c*x^n])^p/((q + 1)*(e*f - d*g))), x] - Dist[b*n*(p/((q + 1)
*(e*f - d*g))), Int[(f + g*x)^(m + 1)*(d + e*x)^(q + 1)*((a + b*Log[c*x^n])
^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, q}, x] && NeQ[e*f
- d*g, 0] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 2404

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2554

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[b*c - a*d, Su
bst[Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m +
2)), x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n
}, x] && EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegerQ[m] &
& IGtQ[p, 0]
```

Rubi steps

$$\int (f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx = \frac{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{2g} - \frac{B\int \frac{2(bc-ad)(f+gx)^2\left(A+\right.}{(a+bx)(c}}{g}$$

$$= \frac{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{2g} - \frac{(2B(bc-ad))\int \frac{(f+gx}{}}{}$$

$$= \frac{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{2g} - \frac{(2B(bc-ad))\int \left(\frac{g^2(}{}\right.}{}$$

$$= \frac{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{2g} - \frac{(2B(bc-ad)g)\int \left(A\right.}{b}$$

$$= -\frac{2AB(bc-ad)gx}{bd} - \frac{2B(bf-ag)^2\log(a+bx)\left(A+B\log\right.}{b^2g}$$

$$= -\frac{2AB(bc-ad)gx}{bd} - \frac{2B^2(bc-ad)g(a+bx)\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{b^2d}$$

$$= -\frac{2AB(bc-ad)gx}{bd} - \frac{2B^2(bc-ad)g(a+bx)\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{b^2d}$$

$$= -\frac{2AB(bc-ad)gx}{bd} - \frac{2B^2(bc-ad)g(a+bx)\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{b^2d}$$

$$= -\frac{2AB(bc-ad)gx}{bd} - \frac{2B^2(bc-ad)g(a+bx)\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{b^2d}$$

$$= -\frac{2AB(bc-ad)gx}{bd} + \frac{2B^2(bf-ag)^2\log^2(a+bx)}{b^2g} - \frac{2B^2(bc}{}$$

$$= -\frac{2AB(bc-ad)gx}{bd} + \frac{2B^2(bf-ag)^2\log^2(a+bx)}{b^2g} - \frac{2B^2(bc}{}$$

Mathematica [A]
time = 0.19, size = 351, normalized size = 1.25

$$\frac{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 - \frac{4B\left(Abd(bc-ad)g^2x+Bd(bc-ad)g^2(a+bx)\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+d^2(bf-ag)^2\log(a+bx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)-2B(bc-ad)^2g^2\log(c+dx)-b^2(df-cg)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\log(c+dx)-Bd^2(bf-ag)^2\left(\log(a+bx)\left(\log(a+bx)-2\log\left(\frac{b(c+dx)}{bc-ad}\right)\right)-2\text{Li}_2\left(\frac{d(a+bx)}{-bc+ad}\right)\right)+b^2B(df-cg)^2\left(\left(2\log\left(\frac{d(a+bx)}{-bc+ad}\right)-\log(c+dx)\right)\log(c+dx)+2\text{Li}_2\left(\frac{b(c+dx)}{bc-ad}\right)\right)\right)}{b^2d^2}}{2g}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2,x]
```

```
[Out] ((f + g*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2 - (4*B*(A*b*d*(b*c
- a*d)*g^2*x + B*d*(b*c - a*d)*g^2*(a + b*x)*Log[(e*(a + b*x)^2)/(c + d*x)^
2] + d^2*(b*f - a*g)^2*Log[a + b*x]*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]
) - 2*B*(b*c - a*d)^2*g^2*Log[c + d*x] - b^2*(d*f - c*g)^2*(A + B*Log[(e*(a
 + b*x)^2)/(c + d*x)^2])*Log[c + d*x] - B*d^2*(b*f - a*g)^2*(Log[a + b*x]*(
Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x
))/(-(b*c) + a*d)]) + b^2*B*(d*f - c*g)^2*((2*Log[(d*(a + b*x))/(-(b*c) + a
*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]
)))/(b^2*d^2))/(2*g)
```

Maple [F]
time = 0.09, size = 0, normalized size = 0.00

$$\int (gx+f)\left(A+B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

```
[Out] int((g*x+f)*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 753 vs. $2(280) = 560$.
time = 0.39, size = 753, normalized size = 2.68

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="maxima")
```

```
[Out] 1/2*A^2*g*x^2 + 2*(x*log(b^2*x^2*e/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*x*e/(d
^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) + 2*a*log(b*x +
a)/b - 2*c*log(d*x + c)/d)*A*B*f + (x^2*log(b^2*x^2*e/(d^2*x^2 + 2*c*d*x +
c^2) + 2*a*b*x*e/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2
)) - 2*a^2*log(b*x + a)/b^2 + 2*c^2*log(d*x + c)/d^2 - 2*(b*c - a*d)*x/(b*d
))*A*B*g + A^2*f*x - 2*(2*a*c*d*g + (2*c*d*f - 3*c^2*g)*b)*B^2*log(d*x + c)
/(b*d^2) + 4*(2*a*b*d^2*f - a^2*d^2*g - (2*c*d*f - c^2*g)*b^2)*(log(b*x + a
)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B
^2/(b^2*d^2) + 1/2*(B^2*b^2*d^2*g*x^2 + 2*(2*a*b*d^2*g + (d^2*f - 2*c*d*g)*
b^2)*B^2*x + 4*(B^2*b^2*d^2*g*x^2 + 2*B^2*b^2*d^2*f*x + (2*a*b*d^2*f - a^2*
d^2*g)*B^2)*log(b*x + a)^2 + 4*(B^2*b^2*d^2*g*x^2 + 2*B^2*b^2*d^2*f*x + (2*
c*d*f - c^2*g)*B^2*b^2)*log(d*x + c)^2 + 4*(B^2*b^2*d^2*g*x^2 + 2*(a*b*d^2*
g + (d^2*f - c*d*g)*b^2)*B^2*x + (a^2*d^2*g + 2*(d^2*f - c*d*g)*a*b)*B^2)*l
og(b*x + a) - 4*(B^2*b^2*d^2*g*x^2 + 2*(a*b*d^2*g + (d^2*f - c*d*g)*b^2)*B^
```

```
2*x + 2*(B^2*b^2*d^2*g*x^2 + 2*B^2*b^2*d^2*f*x + (2*a*b*d^2*f - a^2*d^2*g)*
B^2)*log(b*x + a))*log(d*x + c))/(b^2*d^2)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="fricas")
[Out] integral(A^2*g*x + A^2*f + (B^2*g*x + B^2*f)*log((b^2*x^2 + 2*a*b*x + a^2)*
e/(d^2*x^2 + 2*c*d*x + c^2))^2 + 2*(A*B*g*x + A*B*f)*log((b^2*x^2 + 2*a*b*x
 + a^2)*e/(d^2*x^2 + 2*c*d*x + c^2)), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2,x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="giac")
[Out] integrate((g*x + f)*(B*log((b*x + a)^2*e/(d*x + c)^2) + A)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int (f+g\,x)\,{\left(A+B\,\ln\left(\frac{e\,{\left(a+b\,x\right)}^2}{{\left(c+d\,x\right)}^2}\right)\right)}^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2,x)
[Out] int((f + g*x)*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2, x)
```

3.275 $\int \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx$

Optimal. Leaf size=129

$$\frac{(a+bx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{b}+\frac{4B(bc-ad)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{bd}+\frac{8B^2(bc-ad)\text{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{bd}$$

```
[Out] (b*x+a)*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/b+4*B*(-a*d+b*c)*(A+B*ln(e*(b*x+a
)^2/(d*x+c)^2))*ln((-a*d+b*c)/b/(d*x+c))/b/d+8*B^2*(-a*d+b*c)*polylog(2,d*(
b*x+a)/b/(d*x+c))/b/d
```

Rubi [A]
time = 0.16, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2536, 2542, 2458, 2378, 2370, 2352}

$$\frac{8B^2(bc-ad)\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{bd}+\frac{4B(bc-ad)\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{bd}+\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2,x]
```

```
[Out] ((a + b*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2)/b + (4*B*(b*c - a*d)
*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])*Log[(b*c - a*d)/(b*(c + d*x))])/(
b*d) + (8*B^2*(b*c - a*d)*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/(b*d)
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2370

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)/(x_))^(q_.)*(
x_)^(m_.), x_Symbol] :> Int[(e + d*x)^q*(a + b*Log[c*x^n])^p, x] /; FreeQ[{
a, b, c, d, e, m, n, p}, x] && EqQ[m, q] && IntegerQ[q]
```

Rule 2378

```
Int[((a_.) + Log[(c_.)*(x_)^(n_)]*(b_.))/((x_)*((d_) + (e_.)*(x_)^(r_.))),
x_Symbol] :> Dist[1/n, Subst[Int[(a + b*Log[c*x])/(x*(d + e*x^(r/n))), x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IntegerQ[r/n]
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2536

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.), x_Symbol] :> Simp[(a + b*x)*((A + B*Log[e*((a + b*x)^n/(c
+ d*x)^n)])^p/b), x] - Dist[B*n*p*((b*c - a*d)/b), Int[(A + B*Log[e*((a + b
*x)^n/(c + d*x)^n)])^(p - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, A,
B, n}, x] && EqQ[n + mn, 0] && NeQ[b*c - a*d, 0] && IGtQ[p, 0]
```

Rule 2542

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))/((f_.) + (g_.)*(x_)), x_Symbol] :> Simp[(-Log[-(b*c - a*d)/(d*(a
+ b*x))])*((A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/g), x] + Dist[B*n*((b*c
 - a*d)/g), Int[Log[-(b*c - a*d)/(d*(a + b*x))]/((a + b*x)*(c + d*x)), x],
x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
 - a*d, 0] && EqQ[b*f - a*g, 0]
```

Rubi steps

$$
\begin{aligned}
\int \left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 dx &= x\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2-(2B)\int \frac{2(bc-ad)x\Big(A+B\log\Big(\ldots}{(a+bx)(c+dx)} \\
&= x\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2-(4B(bc-ad))\int \frac{x\Big(A+B\log\Big(\ldots}{(a+bx)(c+d\ldots} \\
&= x\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2-(4B(bc-ad))\int \Bigg(-\frac{a\Big(A+B\log\Big(\ldots}{(bc-ad)(a\ldots} \\
&= x\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2+(4aB)\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{a+bx}\,dx-\ldots \\
&= \frac{4aB\log(a+bx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{b}+x\Bigg(A+B\log\Bigg(\frac{e(a+bx)^2}{(c+dx)^2}\ldots \\
&= \frac{4aB\log(a+bx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{b}+x\Bigg(A+B\log\Bigg(\frac{e(a+bx)^2}{(c+dx)^2}\ldots \\
&= \frac{4aB\log(a+bx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{b}+x\Bigg(A+B\log\Bigg(\frac{e(a+bx)^2}{(c+dx)^2}\ldots \\
&= \frac{4aB\log(a+bx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{b}+x\Bigg(A+B\log\Bigg(\frac{e(a+bx)^2}{(c+dx)^2}\ldots \\
&= \frac{4aB\log(a+bx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{b}+x\Bigg(A+B\log\Bigg(\frac{e(a+bx)^2}{(c+dx)^2}\ldots \\
&= -\frac{4aB^2\log^2(a+bx)}{b}+\frac{4aB\log(a+bx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{b}+x\Big(\ldots \\
&= -\frac{4aB^2\log^2(a+bx)}{b}+\frac{4aB\log(a+bx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{b}+x\Big(\ldots
\end{aligned}
$$

<u>Mathematica</u> [A]
time = 0.12, size = 220, normalized size = 1.71

$$x\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2+\frac{4B\left(ad\log(a+bx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)-bc\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\log(c+dx)-aBd\left(\log(a+bx)\left(\log(a+bx)-2\log\left(\frac{b(c+dx)}{bc-ad}\right)\right)-2\text{Li}_2\left(\frac{d(a+bx)}{-bc+ad}\right)\right)+bBc\left(\left(2\log\left(\frac{d(a+bx)}{-bc+ad}\right)-\log(c+dx)\right)\log(c+dx)+2\text{Li}_2\left(\frac{b(c+dx)}{bc-ad}\right)\right)\right)}{bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2,x]
```

```
[Out] x*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2 + (4*B*(a*d*Log[a + b*x]*(A +
B*Log[(e*(a + b*x)^2)/(c + d*x)^2]) - b*c*(A + B*Log[(e*(a + b*x)^2)/(c + d
*x)^2])*Log[c + d*x] - a*B*d*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*
x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) + b*B*c*((2
*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog
[2, (b*(c + d*x))/(b*c - a*d)])))/(b*d)
```

Maple [**F**]
time = 0.06, size = 0, normalized size = 0.00

$$\int \left(A+B\ln\left(\frac{e(bx+a)^{2}}{(dx+c)^{2}}\right)\right)^{2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

```
[Out] int((A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="maxima")
```

```
[Out] 2*(2*(a*e*log(b*x + a)/b - c*e*log(d*x + c)/d)*e^(-1) + x*log((b*x + a)^2*e
/(d*x + c)^2))*A*B + A^2*x + B^2*(4*(b*d*x*log(b*x + a)^2 + (b*d*x + b*c)*l
og(d*x + c)^2 - (b*d*x + 2*(b*d*x + a*d)*log(b*x + a))*log(d*x + c))/(b*d)
+ integrate((5*b^2*d*x^2 + a*b*c + (b^2*c + 5*a*b*d)*x + 4*(b^2*d*x^2 + a*b
*c + 2*a^2*d - (b^2*c - 5*a*b*d)*x)*log(b*x + a))/(b^2*d*x^2 + a*b*c + (b^2
*c + a*b*d)*x), x))
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="fricas")
```

```
[Out] integral(B^2*log((b^2*x^2 + 2*a*b*x + a^2)*e/(d^2*x^2 + 2*c*d*x + c^2))^2 +
 2*A*B*log((b^2*x^2 + 2*a*b*x + a^2)*e/(d^2*x^2 + 2*c*d*x + c^2)) + A^2, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="giac")
```

```
[Out] integrate((B*log((b*x + a)^2*e/(d*x + c)^2) + A)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(A+B\,\ln\left(\frac{e\,{\left(a+b\,x\right)}^2}{{\left(c+d\,x\right)}^2}\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2,x)
```

```
[Out] int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2, x)
```

3.276 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{f+gx}\,dx$

Optimal. Leaf size=285

$$-\frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2\log\left(\frac{bc-ad}{b(c+dx)}\right)}{g}+\frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2\log\left(1-\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{g}-\frac{4B\left(A+B\log\left(\right.\right.}{}$$

```
[Out] -(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2*ln((-a*d+b*c)/b/(d*x+c))/g+(A+B*ln(e*(b*
x+a)^2/(d*x+c)^2))^2*ln(1-(-c*g+d*f)*(b*x+a)/(-a*g+b*f)/(d*x+c))/g-4*B*(A+B
*ln(e*(b*x+a)^2/(d*x+c)^2))*polylog(2,d*(b*x+a)/b/(d*x+c))/g+4*B*(A+B*ln(e*
(b*x+a)^2/(d*x+c)^2))*polylog(2,(-c*g+d*f)*(b*x+a)/(-a*g+b*f)/(d*x+c))/g+8*
B^2*polylog(3,d*(b*x+a)/b/(d*x+c))/g-8*B^2*polylog(3,(-c*g+d*f)*(b*x+a)/(-a
*g+b*f)/(d*x+c))/g
```

Rubi [A]

time = 0.34, antiderivative size = 285, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {2554, 2404, 2354, 2421, 6724}

$$\frac{4B\text{PolyLog}\left(2,\frac{(a+bx)(df-cg)}{(c+dx)(bf-ag)}\right)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{g}-\frac{4B\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{g}-\frac{8B^2\text{PolyLog}\left(3,\frac{(a+bx)(df-cg)}{(c+dx)(bf-ag)}\right)}{g}+\frac{8B^2\text{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)}{g}+\frac{\log\left(1-\frac{(a+bx)(df-cg)}{(c+dx)(bf-ag)}\right)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{g}-\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{g}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2/(f + g*x),x]
```

```
[Out] -(((A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2*Log[(b*c - a*d)/(b*(c + d*x))
])/g) + ((A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2*Log[1 - ((d*f - c*g)*(a
 + b*x))/((b*f - a*g)*(c + d*x))])/g - (4*B*(A + B*Log[(e*(a + b*x)^2)/(c +
 d*x)^2])*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/g + (4*B*(A + B*Log[(e*(
a + b*x)^2)/(c + d*x)^2])*PolyLog[2, ((d*f - c*g)*(a + b*x))/((b*f - a*g)*(
c + d*x))])/g + (8*B^2*PolyLog[3, (d*(a + b*x))/(b*(c + d*x))])/g - (8*B^2*
PolyLog[3, ((d*f - c*g)*(a + b*x))/((b*f - a*g)*(c + d*x))])/g
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2404

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
```

```
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2554

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[b*c - a*d, Su
bst[Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m +
2)), x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n
}, x] && EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegerQ[m] &
& IGtQ[p, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{f+gx}\,dx = \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2\log(f+gx)}{g} - \frac{(2B)\int \frac{(c+dx)^2\left(-\frac{2de(a+bx)^2}{(c+dx)^3}+\frac{2be(a+bx)}{(c+dx)^2}\right.}{e(a}}{g}$$

$$= \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2\log(f+gx)}{g} - \frac{(2B)\int \frac{(c+dx)^2\left(-\frac{2de(a+bx)^2}{(c+dx)^3}+\frac{2be(a+bx)}{(c+dx)^2}\right.}{(a+}}{eg}$$

$$= \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2\log(f+gx)}{g} - \frac{(2B)\int \frac{2(bc-ad)e\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(a+bx)(c+dx)}}{eg}$$

$$= \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2\log(f+gx)}{g} - \frac{(4B(bc-ad))\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right.\right.}{(a+bx)(c+}}{g}$$

$$= \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2\log(f+gx)}{g} - \frac{(4B(bc-ad))\int\left(\frac{b\left(A+B\log\left(\frac{e(a+bx}{(c+dx}\right.\right.}{(bc-ad)(}\right.}{}$$

$$= \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2\log(f+gx)}{g} - \frac{(4bB)\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\log(f+g}{a+bx}}{g}$$

$$= \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2\log(f+gx)}{g} - \frac{(4bB)\int\left(\frac{A\log(f+gx)}{a+bx}+\frac{B\log\left(\frac{e(a+bx}{(c+dx)}\right.}{a}\right.}{g}$$

$$= \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2\log(f+gx)}{g} - \frac{(4AbB)\int\frac{\log(f+gx)}{a+bx}\,dx}{g} - \frac{(4bB^2)\int}{}$$

$$= -\frac{4AB\log\left(-\frac{g(a+bx)}{bf-ag}\right)\log(f+gx)}{g} + \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2\log(f+gx}{g}$$

$$= -\frac{4AB\log\left(-\frac{g(a+bx)}{bf-ag}\right)\log(f+gx)}{g} + \frac{4B^2\log\left(-\frac{g(a+bx)}{bf-ag}\right)\left(\log\left((a+bx)^2\right)\right.}{}$$

$$= -\frac{4AB\log\left(-\frac{g(a+bx)}{bf-ag}\right)\log(f+gx)}{g} - \frac{B^2\log^2\left((a+bx)^2\right)\log(f+gx)}{g} - \frac{B}{}$$

$$= -\frac{4AB\log\left(-\frac{g(a+bx)}{bf-ag}\right)\log(f+gx)}{g} - \frac{B^2\log^2\left((a+bx)^2\right)\log(f+gx)}{g} - \frac{B}{}$$

$$= -\frac{4AB\log\left(-\frac{g(a+bx)}{bf-ag}\right)\log(f+gx)}{g} - \frac{B^2\log^2\left((a+bx)^2\right)\log(f+gx)}{g} - \frac{B}{}$$

$$= -\frac{4AB\log\left(-\frac{g(a+bx)}{bf-ag}\right)\log(f+gx)}{g} - \frac{B^2\log^2\left((a+bx)^2\right)\log(f+gx)}{g} - \frac{B}{}$$

Mathematica [F]
time = 1.28, size = 0, normalized size = 0.00

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{f+gx}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2/(f + g*x),x]
```

```
[Out] Integrate[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2/(f + g*x), x]
```

Maple [F]
time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{\left(A+B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)\right)^2}{gx+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/(g*x+f),x)
```

```
[Out] int((A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/(g*x+f),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(g*x+f),x, algorithm="maxima")
```

```
[Out] A^2*log(g*x + f)/g - integrate(-(4*B^2*log(b*x + a)^2 + 2*A*B + B^2 + 4*(A*
B + B^2)*log(b*x + a) - 4*(2*B^2*log(b*x + a) + A*B + B^2)*log(d*x + c))/(g
*x + f), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(g*x+f),x, algorithm="fricas")
```

```
[Out] integral((B^2*log((b^2*x^2 + 2*a*b*x + a^2)*e/(d^2*x^2 + 2*c*d*x + c^2))^2
+ 2*A*B*log((b^2*x^2 + 2*a*b*x + a^2)*e/(d^2*x^2 + 2*c*d*x + c^2)) + A^2)/(
g*x + f), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2/(g*x+f),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(g*x+f),x, algorithm="giac")
```

```
[Out] integrate((B*log((b*x + a)^2*e/(d*x + c)^2) + A)^2/(g*x + f), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+B\,\ln\left(\frac{e\,{\left(a+b\,x\right)}^2}{{\left(c+d\,x\right)}^2}\right)\right)^2}{f+g\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2/(f + g*x),x)
```

```
[Out] int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2/(f + g*x), x)
```

3.277 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(f+gx)^2}\,dx$

Optimal. Leaf size=200

$$\frac{(a+bx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(bf-ag)(f+gx)}+\frac{4B(bc-ad)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\log\left(1-\frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right)}{(bf-ag)(df-cg)}+\frac{8B^2(bc-ad}{(bf-}$$

```
[Out] (b*x+a)*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/(-a*g+b*f)/(g*x+f)+4*B*(-a*d+b*c)
*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))*ln(1-(-c*g+d*f)*(b*x+a)/(-a*g+b*f)/(d*x+c)
)/(-a*g+b*f)/(-c*g+d*f)+8*B^2*(-a*d+b*c)*polylog(2,(-c*g+d*f)*(b*x+a)/(-a*g
+b*f)/(d*x+c))/(-a*g+b*f)/(-c*g+d*f)
```

Rubi [A]
time = 0.12, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {2554, 2355, 2354, 2438}

$$\frac{8B^2(bc-ad)\text{PolyLog}\left(2,\frac{(a+bx)(df-cg)}{(c+dx)(bf-ag)}\right)}{(bf-ag)(df-cg)}+\frac{4B(bc-ad)\log\left(1-\frac{(a+bx)(df-cg)}{(c+dx)(bf-ag)}\right)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{(bf-ag)(df-cg)}+\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{(f+gx)(bf-ag)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2/(f + g*x)^2,x]
```

```
[Out] ((a + b*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2)/((b*f - a*g)*(f + g*
x)) + (4*B*(b*c - a*d)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])*Log[1 - ((d
*f - c*g)*(a + b*x))/((b*f - a*g)*(c + d*x))])/((b*f - a*g)*(d*f - c*g)) +
(8*B^2*(b*c - a*d)*PolyLog[2, ((d*f - c*g)*(a + b*x))/((b*f - a*g)*(c + d*x
))])/((b*f - a*g)*(d*f - c*g))
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2355

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[x*((a + b*Log[c*x^n])^p/(d*(d + e*x))), x] - Dist[b*n*(p/d),
Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n
, p}, x] && GtQ[p, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2554

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[b*c - a*d, Su
bst[Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m +
2)), x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n
}, x] && EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegerQ[m] &
& IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(f+gx)^2}\,dx = -\frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{g(f+gx)} + \frac{(2B)\int \frac{2(bc-ad)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(a+bx)(c+dx)(f+gx)}\,dx}{g}$$

$$= -\frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{g(f+gx)} + \frac{(4B(bc-ad))\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(a+bx)(c+dx)(f+gx)}\,dx}{g}$$

$$= -\frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{g(f+gx)} + \frac{(4B(bc-ad))\int \left(\frac{b^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(bc-ad)(bf-ag)(a+bx)} + \frac{d^2\left(\cdots\right.}{(bc\cdots}\cdots\right.}{g\cdots}$$

$$= -\frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{g(f+gx)} + \frac{(4b^2B)\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{a+bx}\,dx}{g(bf-ag)} - \frac{(4Bd^2)\int \frac{A+\cdots}{\cdots}}{g(df\cdots}$$

$$= \frac{4bB\log(a+bx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{g(bf-ag)} - \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{g(f+gx)} - \frac{4B\cdots}{\cdots}$$

$$= \frac{4bB\log(a+bx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{g(bf-ag)} - \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{g(f+gx)} - \frac{4B\cdots}{\cdots}$$

$$= \frac{4bB\log(a+bx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{g(bf-ag)} - \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{g(f+gx)} - \frac{4B\cdots}{\cdots}$$

$$= \frac{4bB\log(a+bx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{g(bf-ag)} - \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{g(f+gx)} - \frac{4B\cdots}{\cdots}$$

$$= \frac{4bB\log(a+bx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{g(bf-ag)} - \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{g(f+gx)} + \frac{8B^{\cdots}}{\cdots}$$

$$= -\frac{4bB^2\log^2(a+bx)}{g(bf-ag)} + \frac{4bB\log(a+bx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{g(bf-ag)} - \frac{\left(A+B\cdots\right.}{g\cdots}$$

$$= -\frac{4bB^2\log^2(a+bx)}{g(bf-ag)} + \frac{4bB\log(a+bx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{g(bf-ag)} - \frac{\left(A+B\cdots\right.}{g\cdots}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 409 vs. $2(200) = 400$.

time = 0.37, size = 409, normalized size = 2.04

$$\frac{-\frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{f+gx} + \frac{4B\left(b(df-cg)\log(a+bx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right) - d(bf-ag)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\log(c+dx) + (bc-ad)g\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\log(f+gx) - bB(df-cg)\left(\log(a+bx)\left(\log(a+bx)-2\log\left(\frac{b(c+dx)}{bc-ad}\right)\right)-2\text{Li}_2\left(\frac{d(a+bx)}{-bc+ad}\right)\right) + Bd(bf-ag)\left(\left(2\log\left(\frac{d(a+bx)}{-bc+ad}\right)-\log(c+dx)\right)\log(c+dx)+2\text{Li}_2\left(\frac{b(c+dx)}{bc-ad}\right)\right) - 2B(bc-ad)g\left(\left(\log\left(\frac{g(a+bx)}{-bf+ag}\right)-\log\left(\frac{g(c+dx)}{-df+cg}\right)\right)\log(f+gx)+\text{Li}_2\left(\frac{b(f+gx)}{bf-ag}\right)-\text{Li}_2\left(\frac{d(f+gx)}{df-cg}\right)\right)\right)}{(bf-ag)(df-cg)}}{g}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2/(f + g*x)^2,x]
```

```
[Out] (-((A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2/(f + g*x)) + (4*B*(b*(d*f - c
*g)*Log[a + b*x]*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]) - d*(b*f - a*g)*(
A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])*Log[c + d*x] + (b*c - a*d)*g*(A + B
*Log[(e*(a + b*x)^2)/(c + d*x)^2])*Log[f + g*x] - b*B*(d*f - c*g)*(Log[a +
b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a
 + b*x))/(-(b*c) + a*d)]) + B*d*(b*f - a*g)*((2*Log[(d*(a + b*x))/(-(b*c) +
 a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d
)]) - 2*B*(b*c - a*d)*g*((Log[(g*(a + b*x))/(-(b*f) + a*g)] - Log[(g*(c + d
*x))/(-(d*f) + c*g)])*Log[f + g*x] + PolyLog[2, (b*(f + g*x))/(b*f - a*g)]
- PolyLog[2, (d*(f + g*x))/(d*f - c*g)])))/((b*f - a*g)*(d*f - c*g)))/g
```

Maple [F]
time = 0.20, size = 0, normalized size = 0.00

$$\int \frac{\left(A+B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)\right)^2}{(gx+f)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/(g*x+f)^2,x)
```

```
[Out] int((A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/(g*x+f)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(g*x+f)^2,x, algorithm="maxima
")
```

```
[Out] 2*A*B*(2*b*log(b*x + a)/(b*f*g - a*g^2) - 2*d*log(d*x + c)/(d*f*g - c*g^2)
+ 2*(b*c - a*d)*log(g*x + f)/(b*d*f^2 + a*c*g^2 - (b*c + a*d)*f*g) - log(b^
2*x^2*e/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*x*e/(d^2*x^2 + 2*c*d*x + c^2) + a
^2*e/(d^2*x^2 + 2*c*d*x + c^2))/(g^2*x + f*g)) - B^2*(4*log(d*x + c)^2/(g^2
*x + f*g) + integrate(-(d*g*x + 4*(d*g*x + c*g)*log(b*x + a)^2 + c*g + 4*(d
*g*x + c*g)*log(b*x + a) + 4*(d*g*x + 2*d*f - c*g - 2*(d*g*x + c*g)*log(b*x
 + a))*log(d*x + c))/(d*g^3*x^3 + c*f^2*g + (2*d*f*g^2 + c*g^3)*x^2 + (d*f^
2*g + 2*c*f*g^2)*x), x)) - A^2/(g^2*x + f*g)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(g*x+f)^2,x, algorithm="fricas
")
```

```
[Out] integral((B^2*log((b^2*x^2 + 2*a*b*x + a^2)*e/(d^2*x^2 + 2*c*d*x + c^2))^2
+ 2*A*B*log((b^2*x^2 + 2*a*b*x + a^2)*e/(d^2*x^2 + 2*c*d*x + c^2)) + A^2)/(
g^2*x^2 + 2*f*g*x + f^2), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2/(g*x+f)**2,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(g*x+f)^2,x, algorithm="giac")
```

```
[Out] integrate((B*log((b*x + a)^2*e/(d*x + c)^2) + A)^2/(g*x + f)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+B\,\ln\left(\frac{e\,{\left(a+b\,x\right)}^2}{{\left(c+d\,x\right)}^2}\right)\right)^2}{{\left(f+g\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2/(f + g*x)^2,x)
```

```
[Out] int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2/(f + g*x)^2, x)
```

3.278 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(f+gx)^3}\,dx$

Optimal. Leaf size=381

$$\frac{2B(bc-ad)g(a+bx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(bf-ag)^2(df-cg)(f+gx)}+\frac{b^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{2g(bf-ag)^2}-\frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{2g(f+gx)^2}+\frac{4B^2}{(}$$

```
[Out] 2*B*(-a*d+b*c)*g*(b*x+a)*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/(-a*g+b*f)^2/(-c*g
+d*f)/(g*x+f)+1/2*b^2*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/g/(-a*g+b*f)^2-1/2*
(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/g/(g*x+f)^2+4*B^2*(-a*d+b*c)^2*g*ln((g*x+
f)/(d*x+c))/(-a*g+b*f)^2/(-c*g+d*f)^2+2*B*(-a*d+b*c)*(-a*d*g-b*c*g+2*b*d*f)
*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))*ln(1-(-c*g+d*f)*(b*x+a)/(-a*g+b*f)/(d*x+c)
)/(-a*g+b*f)^2/(-c*g+d*f)^2+4*B^2*(-a*d+b*c)*(-a*d*g-b*c*g+2*b*d*f)*polylog
(2,(-c*g+d*f)*(b*x+a)/(-a*g+b*f)/(d*x+c))/(-a*g+b*f)^2/(-c*g+d*f)^2
```

Rubi [A]
time = 0.53, antiderivative size = 381, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.258$, Rules used = {2554, 2398, 2404, 2338, 2351, 31, 2354, 2438}

$$\frac{4B^2(bc-ad)(-adg-bcg+2bdf)\text{PolyLog}\left(2,\frac{(a+bx)(df-cg)}{(c+dx)(bf-ag)}\right)}{(bf-ag)^2(df-cg)^2}+\frac{b^2\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{2g(bf-ag)^2}+\frac{2Bg(a+bx)(bc-ad)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{(f+gx)(bf-ag)^2(df-cg)}+\frac{2B(bc-ad)(-adg-bcg+2bdf)\log\left(1-\frac{(a+bx)(df-cg)}{(c+dx)(bf-ag)}\right)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{(bf-ag)^2(df-cg)^2}-\frac{\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{2g(f+gx)^2}+\frac{4B^2g(bc-ad)^2\log\left(\frac{f+gx}{c+dx}\right)}{(bf-ag)^2(df-cg)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2/(f + g*x)^3,x]
```

```
[Out] (2*B*(b*c - a*d)*g*(a + b*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]))/((b*
f - a*g)^2*(d*f - c*g)*(f + g*x)) + (b^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*
x)^2])^2)/(2*g*(b*f - a*g)^2) - (A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2/
(2*g*(f + g*x)^2) + (4*B^2*(b*c - a*d)^2*g*Log[(f + g*x)/(c + d*x)])/((b*f
- a*g)^2*(d*f - c*g)^2) + (2*B*(b*c - a*d)*(2*b*d*f - b*c*g - a*d*g)*(A + B
*Log[(e*(a + b*x)^2)/(c + d*x)^2])*Log[1 - ((d*f - c*g)*(a + b*x))/((b*f -
a*g)*(c + d*x))])/((b*f - a*g)^2*(d*f - c*g)^2) + (4*B^2*(b*c - a*d)*(2*b*d
*f - b*c*g - a*d*g)*PolyLog[2, ((d*f - c*g)*(a + b*x))/((b*f - a*g)*(c + d*
x))])/((b*f - a*g)^2*(d*f - c*g)^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_)*((
f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(d + e*x)^(q +
 1)*((a + b*Log[c*x^n])^p/((q + 1)*(e*f - d*g))), x] - Dist[b*n*(p/((q + 1)
*(e*f - d*g))), Int[(f + g*x)^(m + 1)*(d + e*x)^(q + 1)*((a + b*Log[c*x^n])
^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, q}, x] && NeQ[e*f
- d*g, 0] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 2404

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2554

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[b*c - a*d, Su
bst[Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m +
2)), x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n
}, x] && EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegerQ[m] &
& IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(f+gx)^3}\,dx = -\frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{2g(f+gx)^2} + \frac{B\int \frac{2(bc-ad)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(a+bx)(c+dx)(f+gx)^2}dx}{g}$$

$$= -\frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{2g(f+gx)^2} + \frac{(2B(bc-ad))\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(a+bx)(c+dx)(f+gx)^2}dx}{g}$$

$$= -\frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{2g(f+gx)^2} + \frac{(2B(bc-ad))\int \left(\frac{b^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(bc-ad)(bf-ag)^2(a+bx)} - \frac{d^3 \cdots}{(b \cdots}\right.}{\cdots}$$

$$= -\frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{2g(f+gx)^2} + \frac{(2b^3B)\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{a+bx}dx}{g(bf-ag)^2} - \frac{(2Bd^3)\int \frac{A+\cdots}{\cdots}}{g(d\cdots}$$

$$= -\frac{2B(bc-ad)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(bf-ag)(df-cg)(f+gx)} + \frac{2b^2B\log(a+bx)\left(A+B\log\left(\frac{e\cdots}{(\cdots}\right.\right.}{g(bf-ag)^2}$$

$$= -\frac{2B(bc-ad)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(bf-ag)(df-cg)(f+gx)} + \frac{2b^2B\log(a+bx)\left(A+B\log\left(\frac{e\cdots}{(\cdots}\right.\right.}{g(bf-ag)^2}$$

$$= -\frac{2B(bc-ad)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(bf-ag)(df-cg)(f+gx)} + \frac{2b^2B\log(a+bx)\left(A+B\log\left(\frac{e\cdots}{(\cdots}\right.\right.}{g(bf-ag)^2}$$

$$= \frac{4bB^2(bc-ad)\log(a+bx)}{(bf-ag)^2(df-cg)} - \frac{2B(bc-ad)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(bf-ag)(df-cg)(f+gx)} + \frac{2b^2\cdots}{\cdots}$$

$$= \frac{4bB^2(bc-ad)\log(a+bx)}{(bf-ag)^2(df-cg)} - \frac{2B(bc-ad)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(bf-ag)(df-cg)(f+gx)} + \frac{2b^2\cdots}{\cdots}$$

$$= \frac{4bB^2(bc-ad)\log(a+bx)}{(bf-ag)^2(df-cg)} - \frac{2b^2B^2\log^2(a+bx)}{g(bf-ag)^2} - \frac{2B(bc-ad)\left(A+B\cdots\right.}{(bf-ag)(df-c\cdots}$$

$$= \frac{4bB^2(bc-ad)\log(a+bx)}{(bf-ag)^2(df-cg)} - \frac{2b^2B^2\log^2(a+bx)}{g(bf-ag)^2} - \frac{2B(bc-ad)\left(A+B\cdots\right.}{(bf-ag)(df-c\cdots}$$

Mathematica [A]

time = 0.93, size = 603, normalized size = 1.58

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2/(f + g*x)^3,x]
```

```
[Out] -1/2*((A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2 + (4*B*(f + g*x)*((b*c - a
*d)*g*(b*f - a*g)*(d*f - c*g)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]) - b^
2*(d*f - c*g)^2*(f + g*x)*Log[a + b*x]*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)
^2]) + d^2*(b*f - a*g)^2*(f + g*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])
*Log[c + d*x] + (b*c - a*d)*g*(-2*b*d*f + b*c*g + a*d*g)*(f + g*x)*(A + B*L
og[(e*(a + b*x)^2)/(c + d*x)^2])*Log[f + g*x] - 2*B*(b*c - a*d)*g*(f + g*x)
*(b*(d*f - c*g)*Log[a + b*x] + (-(b*d*f) + a*d*g)*Log[c + d*x] + (b*c - a*d
)*g*Log[f + g*x]) + b^2*B*(d*f - c*g)^2*(f + g*x)*(Log[a + b*x]*(Log[a + b*
x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c)
 + a*d)]) - B*d^2*(b*f - a*g)^2*(f + g*x)*((2*Log[(d*(a + b*x))/(-(b*c) + a
*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]
) - 2*B*(b*c - a*d)*g*(-2*b*d*f + b*c*g + a*d*g)*(f + g*x)*((Log[(g*(a + b*
x))/(-(b*f) + a*g)] - Log[(g*(c + d*x))/(-(d*f) + c*g)])*Log[f + g*x] + Pol
yLog[2, (b*(f + g*x))/(b*f - a*g)] - PolyLog[2, (d*(f + g*x))/(d*f - c*g)])
))/((b*f - a*g)^2*(d*f - c*g)^2))/(g*(f + g*x)^2)
```

Maple [F]
time = 0.41, size = 0, normalized size = 0.00

$$\int \frac{\left(A+B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)\right)^2}{(gx+f)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/(g*x+f)^3,x)
```

```
[Out] int((A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/(g*x+f)^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(g*x+f)^3,x, algorithm="maxima
")
```

```
[Out] (2*b^2*log(b*x + a)/(b^2*f^2*g - 2*a*b*f*g^2 + a^2*g^3) - 2*d^2*log(d*x + c
)/(d^2*f^2*g - 2*c*d*f*g^2 + c^2*g^3) + 2*(2*(b^2*c*d - a*b*d^2)*f - (b^2*c
^2 - a^2*d^2)*g)*log(g*x + f)/(b^2*d^2*f^4 + a^2*c^2*g^4 - 2*(b^2*c*d + a*b
*d^2)*f^3*g + (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*f^2*g^2 - 2*(a*b*c^2 + a^2*c*
d)*f*g^3) - 2*(b*c - a*d)/(b*d*f^3 + a*c*f*g^2 - (b*c + a*d)*f^2*g + (b*d*f
^2*g + a*c*g^3 - (b*c + a*d)*f*g^2)*x) - log(b^2*x^2*e/(d^2*x^2 + 2*c*d*x +
 c^2) + 2*a*b*x*e/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^
```

```
2))/(g^3*x^2 + 2*f*g^2*x + f^2*g))*A*B - B^2*(2*log(d*x + c)^2/(g^3*x^2 + 2
*f*g^2*x + f^2*g) + integrate(-(d*g*x + 4*(d*g*x + c*g)*log(b*x + a)^2 + c*
g + 4*(d*g*x + c*g)*log(b*x + a) + 4*(d*f - c*g - 2*(d*g*x + c*g)*log(b*x +
 a))*log(d*x + c))/(d*g^4*x^4 + c*f^3*g + (3*d*f*g^3 + c*g^4)*x^3 + 3*(d*f^
2*g^2 + c*f*g^3)*x^2 + (d*f^3*g + 3*c*f^2*g^2)*x), x)) - 1/2*A^2/(g^3*x^2 +
 2*f*g^2*x + f^2*g)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(g*x+f)^3,x, algorithm="fricas
")
```

```
[Out] integral((B^2*log((b^2*x^2 + 2*a*b*x + a^2)*e/(d^2*x^2 + 2*c*d*x + c^2))^2
+ 2*A*B*log((b^2*x^2 + 2*a*b*x + a^2)*e/(d^2*x^2 + 2*c*d*x + c^2)) + A^2)/(
g^3*x^3 + 3*f*g^2*x^2 + 3*f^2*g*x + f^3), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2/(g*x+f)**3,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(g*x+f)^3,x, algorithm="giac")
```

```
[Out] integrate((B*log((b*x + a)^2*e/(d*x + c)^2) + A)^2/(g*x + f)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(A+B\,\ln\left(\frac{e\,{\left(a+b\,x\right)}^2}{{\left(c+d\,x\right)}^2}\right)\right)}^2}{{\left(f+g\,x\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2/(f + g*x)^3,x)
```

```
[Out] int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2/(f + g*x)^3, x)
```

3.279 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(f+gx)^4}\,dx$

Optimal. Leaf size=724

$$\frac{4B^2(bc-ad)^2g^2(c+dx)}{3(bf-ag)^2(df-cg)^3(f+gx)}-\frac{2B(bc-ad)g^2(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{3(bf-ag)(df-cg)^3(f+gx)^2}+\frac{4B(bc-ad)g(3bdf-bcg \cdots}{3(bf- \cdots}$$

```
[Out] 4/3*B^2*(-a*d+b*c)^2*g^2*(d*x+c)/(-a*g+b*f)^2/(-c*g+d*f)^3/(g*x+f)-2/3*B*(-
a*d+b*c)*g^2*(d*x+c)^2*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/(-a*g+b*f)/(-c*g+d*f
)^3/(g*x+f)^2+4/3*B*(-a*d+b*c)*g*(-2*a*d*g-b*c*g+3*b*d*f)*(b*x+a)*(A+B*ln(e
*(b*x+a)^2/(d*x+c)^2))/(-a*g+b*f)^3/(-c*g+d*f)^2/(g*x+f)+1/3*b^3*(A+B*ln(e*
(b*x+a)^2/(d*x+c)^2))^2/g/(-a*g+b*f)^3-1/3*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^
2/g/(g*x+f)^3+4/3*B^2*(-a*d+b*c)^3*g^2*ln((b*x+a)/(d*x+c))/(-a*g+b*f)^3/(-c
*g+d*f)^3-4/3*B^2*(-a*d+b*c)^3*g^2*ln((g*x+f)/(d*x+c))/(-a*g+b*f)^3/(-c*g+d
*f)^3+8/3*B^2*(-a*d+b*c)^2*g*(-2*a*d*g-b*c*g+3*b*d*f)*ln((g*x+f)/(d*x+c))/(
-a*g+b*f)^3/(-c*g+d*f)^3+4/3*B*(-a*d+b*c)*(a^2*d^2*g^2-a*b*d*g*(-c*g+3*d*f)
+b^2*(c^2*g^2-3*c*d*f*g+3*d^2*f^2))*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))*ln(1-(-
c*g+d*f)*(b*x+a)/(-a*g+b*f)/(d*x+c))/(-a*g+b*f)^3/(-c*g+d*f)^3+8/3*B^2*(-a*
d+b*c)*(a^2*d^2*g^2-a*b*d*g*(-c*g+3*d*f)+b^2*(c^2*g^2-3*c*d*f*g+3*d^2*f^2))
*polylog(2,(-c*g+d*f)*(b*x+a)/(-a*g+b*f)/(d*x+c))/(-a*g+b*f)^3/(-c*g+d*f)^3
```

Rubi [A]

time = 1.08, antiderivative size = 724, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.323$, Rules used = {2554, 2398, 2404, 2338, 2356, 46, 2351, 31, 2354, 2438}

$$\frac{8B^2(bc-ad)\left(a^2d^2g^2-abdg(3df-cg)+b^2(c^2g^2-3cdfg+3d^2f^2)\right)\text{PolyLog}\left(2,\frac{(a+bx)(df-cg)}{(c+dx)(bf-ag)}\right)}{3(bf-ag)^3(df-cg)^3}+\frac{4B(bc-ad)\left(a^2d^2g^2-abdg(3df-cg)+b^2(c^2g^2-3cdfg+3d^2f^2)\right)\log\left(1-\frac{(a+bx)(df-cg)}{(c+dx)(bf-ag)}\right)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{3(bf-ag)^3(df-cg)^3}+\frac{b^3\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{3g(bf-ag)^3}-\frac{2Bg^2(c+dx)^2(bc-ad)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{3(f+gx)^2(bf-ag)(df-cg)^3}-\frac{\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}{3g(f+gx)^3}+\frac{4Bg(a+bx)(bc-ad)(-2adg-bcg+3bdf)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{3(f+gx)(bf-ag)^3(df-cg)^2}+\frac{4B^2g^2(c+dx)(bc-ad)^2}{3(f+gx)(bf-ag)^2(df-cg)^3}+\frac{4B^2g^2(bc-ad)^3\log\left(\frac{a+bx}{c+dx}\right)}{3(bf-ag)^3(df-cg)^3}-\frac{4B^2g^2(bc-ad)^3\log\left(\frac{f+gx}{c+dx}\right)}{3(bf-ag)^3(df-cg)^3}+\frac{8B^2g(bc-ad)^2(-2adg-bcg+3bdf)\log\left(\frac{f+gx}{c+dx}\right)}{3(bf-ag)^3(df-cg)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2/(f + g*x)^4,x]
```

```
[Out] (4*B^2*(b*c - a*d)^2*g^2*(c + d*x))/(3*(b*f - a*g)^2*(d*f - c*g)^3*(f + g*x
)) - (2*B*(b*c - a*d)*g^2*(c + d*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^
2]))/(3*(b*f - a*g)*(d*f - c*g)^3*(f + g*x)^2) + (4*B*(b*c - a*d)*g*(3*b*d*
f - b*c*g - 2*a*d*g)*(a + b*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]))/(3
*(b*f - a*g)^3*(d*f - c*g)^2*(f + g*x)) + (b^3*(A + B*Log[(e*(a + b*x)^2)/(
c + d*x)^2])^2)/(3*g*(b*f - a*g)^3) - (A + B*Log[(e*(a + b*x)^2)/(c + d*x)^
2])^2/(3*g*(f + g*x)^3) + (4*B^2*(b*c - a*d)^3*g^2*Log[(a + b*x)/(c + d*x)]
)/(3*(b*f - a*g)^3*(d*f - c*g)^3) - (4*B^2*(b*c - a*d)^3*g^2*Log[(f + g*x)/
(c + d*x)])/(3*(b*f - a*g)^3*(d*f - c*g)^3) + (8*B^2*(b*c - a*d)^2*g*(3*b*d
*f - b*c*g - 2*a*d*g)*Log[(f + g*x)/(c + d*x)])/(3*(b*f - a*g)^3*(d*f - c*g
)^3) + (4*B*(b*c - a*d)*(a^2*d^2*g^2 - a*b*d*g*(3*d*f - c*g) + b^2*(3*d^2*f
```

```
^2 - 3*c*d*f*g + c^2*g^2))*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])*Log[1 -
 ((d*f - c*g)*(a + b*x))/((b*f - a*g)*(c + d*x))])/(3*(b*f - a*g)^3*(d*f -
c*g)^3) + (8*B^2*(b*c - a*d)*(a^2*d^2*g^2 - a*b*d*g*(3*d*f - c*g) + b^2*(3*
d^2*f^2 - 3*c*d*f*g + c^2*g^2))*PolyLog[2, ((d*f - c*g)*(a + b*x))/((b*f -
a*g)*(c + d*x))])/(3*(b*f - a*g)^3*(d*f - c*g)^3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_)*((
f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(d + e*x)^(q +
 1)*((a + b*Log[c*x^n])^p/((q + 1)*(e*f - d*g))), x] - Dist[b*n*(p/((q + 1)
*(e*f - d*g))), Int[(f + g*x)^(m + 1)*(d + e*x)^(q + 1)*((a + b*Log[c*x^n])
^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, q}, x] && NeQ[e*f
- d*g, 0] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 2404

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2554

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[b*c - a*d, Su
bst[Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m +
2)), x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n
}, x] && EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegerQ[m] &
& IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(f+gx)^4}\,dx = -\frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{3g(f+gx)^3}+\frac{(2B)\int\frac{2(bc-ad)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(a+bx)(c+dx)(f+gx)^3}\,dx}{3g}$$

$$= -\frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{3g(f+gx)^3}+\frac{(4B(bc-ad))\int\frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(a+bx)(c+dx)(f+gx)^3}\,dx}{3g}$$

$$= -\frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{3g(f+gx)^3}+\frac{(4B(bc-ad))\int\left(\frac{b^4\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(bc-ad)(bf-ag)^3(a+bx)}+\frac{d^4}{(b}\right.}{}$$

$$= -\frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{3g(f+gx)^3}+\frac{(4b^4B)\int\frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{a+bx}\,dx}{3g(bf-ag)^3}-\frac{(4Bd^4)\int\frac{A+}{}}{3g(}$$

$$= -\frac{2B(bc-ad)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{3(bf-ag)(df-cg)(f+gx)^2}-\frac{4B(bc-ad)(2bdf-bcg-adg)}{3(bf-ag)^2(df-cg}$$

$$= -\frac{2B(bc-ad)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{3(bf-ag)(df-cg)(f+gx)^2}-\frac{4B(bc-ad)(2bdf-bcg-adg)}{3(bf-ag)^2(df-cg}$$

$$= -\frac{2B(bc-ad)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{3(bf-ag)(df-cg)(f+gx)^2}-\frac{4B(bc-ad)(2bdf-bcg-adg)}{3(bf-ag)^2(df-cg}$$

$$= -\frac{4B^2(bc-ad)^2g}{3(bf-ag)^2(df-cg)^2(f+gx)}+\frac{4b^2B^2(bc-ad)\log(a+bx)}{3(bf-ag)^3(df-cg)}+\frac{8bB^2(b}{}$$

$$= -\frac{4B^2(bc-ad)^2g}{3(bf-ag)^2(df-cg)^2(f+gx)}+\frac{4b^2B^2(bc-ad)\log(a+bx)}{3(bf-ag)^3(df-cg)}+\frac{8bB^2(b}{}$$

$$= -\frac{4B^2(bc-ad)^2g}{3(bf-ag)^2(df-cg)^2(f+gx)}+\frac{4b^2B^2(bc-ad)\log(a+bx)}{3(bf-ag)^3(df-cg)}+\frac{8bB^2(b}{}$$

$$= -\frac{4B^2(bc-ad)^2g}{3(bf-ag)^2(df-cg)^2(f+gx)}+\frac{4b^2B^2(bc-ad)\log(a+bx)}{3(bf-ag)^3(df-cg)}+\frac{8bB^2(b}{}$$

Mathematica [A]

time = 1.96, size = 909, normalized size = 1.26

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2/(f + g*x)^4,x]
```

```
[Out] -1/3*((A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2 + (2*B*(f + g*x)*((b*c - a
*d)*g*(b*f - a*g)^2*(d*f - c*g)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])
+ 2*(b*c - a*d)*g*(b*f - a*g)*(-(d*f) + c*g)*(-2*b*d*f + b*c*g + a*d*g)*(f
+ g*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]) - 2*b^3*(d*f - c*g)^3*(f +
g*x)^2*Log[a + b*x]*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]) + 2*d^3*(b*f -
 a*g)^3*(f + g*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])*Log[c + d*x] -
 2*(b*c - a*d)*g*(a^2*d^2*g^2 + a*b*d*g*(-3*d*f + c*g) + b^2*(3*d^2*f^2 - 3
*c*d*f*g + c^2*g^2))*(f + g*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])*L
og[f + g*x] - 4*B*(b*c - a*d)*g*(2*b*d*f - b*c*g - a*d*g)*(f + g*x)^2*(b*(d
*f - c*g)*Log[a + b*x] + (-(b*d*f) + a*d*g)*Log[c + d*x] + (b*c - a*d)*g*Lo
g[f + g*x]) + 2*B*(b*c - a*d)*g*(f + g*x)*((b*c - a*d)*g*(b*f - a*g)*(d*f -
 c*g) - b^2*(d*f - c*g)^2*(f + g*x)*Log[a + b*x] + d^2*(b*f - a*g)^2*(f + g
*x)*Log[c + d*x] + (b*c - a*d)*g*(-2*b*d*f + b*c*g + a*d*g)*(f + g*x)*Log[f
 + g*x]) + 2*b^3*B*(d*f - c*g)^3*(f + g*x)^2*(Log[a + b*x]*(Log[a + b*x] -
2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*
d)]) - 2*B*d^3*(b*f - a*g)^3*(f + g*x)^2*((2*Log[(d*(a + b*x))/(-(b*c) + a*
d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])
 + 4*B*(b*c - a*d)*g*(a^2*d^2*g^2 + a*b*d*g*(-3*d*f + c*g) + b^2*(3*d^2*f^2
 - 3*c*d*f*g + c^2*g^2))*(f + g*x)^2*((Log[(g*(a + b*x))/(-(b*f) + a*g)] -
Log[(g*(c + d*x))/(-(d*f) + c*g)])*Log[f + g*x] + PolyLog[2, (b*(f + g*x))/
(b*f - a*g)] - PolyLog[2, (d*(f + g*x))/(d*f - c*g)])))/((b*f - a*g)^3*(d*f
 - c*g)^3))/(g*(f + g*x)^3)
```

Maple [F]
time = 0.67, size = 0, normalized size = 0.00

$$\int \frac{\left(A+B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)\right)^2}{(gx+f)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/(g*x+f)^4,x)
```

```
[Out] int((A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/(g*x+f)^4,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(g*x+f)^4,x, algorithm="maxima
")
```

```
[Out] 2/3*(2*b^3*log(b*x + a)/(b^3*f^3*g - 3*a*b^2*f^2*g^2 + 3*a^2*b*f*g^3 - a^3*
g^4) - 2*d^3*log(d*x + c)/(d^3*f^3*g - 3*c*d^2*f^2*g^2 + 3*c^2*d*f*g^3 - c^
3*g^4) + 2*(3*(b^3*c*d^2 - a*b^2*d^3)*f^2 - 3*(b^3*c^2*d - a^2*b*d^3)*f*g +
 (b^3*c^3 - a^3*d^3)*g^2)*log(g*x + f)/(b^3*d^3*f^6 + a^3*c^3*g^6 - 3*(b^3*
c*d^2 + a*b^2*d^3)*f^5*g + 3*(b^3*c^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3)*f^4*g^
2 - (b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*f^3*g^3 + 3*(a*b^2*
c^3 + 3*a^2*b*c^2*d + a^3*c*d^2)*f^2*g^4 - 3*(a^2*b*c^3 + a^3*c^2*d)*f*g^5)
 - (5*(b^2*c*d - a*b*d^2)*f^2 - 3*(b^2*c^2 - a^2*d^2)*f*g + (a*b*c^2 - a^2*
c*d)*g^2 + 2*(2*(b^2*c*d - a*b*d^2)*f*g - (b^2*c^2 - a^2*d^2)*g^2)*x)/(b^2*
d^2*f^6 + a^2*c^2*f^2*g^4 - 2*(b^2*c*d + a*b*d^2)*f^5*g + (b^2*c^2 + 4*a*b*
c*d + a^2*d^2)*f^4*g^2 - 2*(a*b*c^2 + a^2*c*d)*f^3*g^3 + (b^2*d^2*f^4*g^2 +
 a^2*c^2*g^6 - 2*(b^2*c*d + a*b*d^2)*f^3*g^3 + (b^2*c^2 + 4*a*b*c*d + a^2*d
^2)*f^2*g^4 - 2*(a*b*c^2 + a^2*c*d)*f*g^5)*x^2 + 2*(b^2*d^2*f^5*g + a^2*c^2
*f*g^5 - 2*(b^2*c*d + a*b*d^2)*f^4*g^2 + (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*f^
3*g^3 - 2*(a*b*c^2 + a^2*c*d)*f^2*g^4)*x) - log(b^2*x^2*e/(d^2*x^2 + 2*c*d*
x + c^2) + 2*a*b*x*e/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x +
 c^2))/(g^4*x^3 + 3*f*g^3*x^2 + 3*f^2*g^2*x + f^3*g))*A*B - 1/3*B^2*(4*log(
d*x + c)^2/(g^4*x^3 + 3*f*g^3*x^2 + 3*f^2*g^2*x + f^3*g) + 3*integrate(-1/3
*(3*d*g*x + 12*(d*g*x + c*g)*log(b*x + a)^2 + 3*c*g + 12*(d*g*x + c*g)*log(
b*x + a) - 4*(d*g*x - 2*d*f + 3*c*g + 6*(d*g*x + c*g)*log(b*x + a))*log(d*x
 + c))/(d*g^5*x^5 + c*f^4*g + (4*d*f*g^4 + c*g^5)*x^4 + 2*(3*d*f^2*g^3 + 2*
c*f*g^4)*x^3 + 2*(2*d*f^3*g^2 + 3*c*f^2*g^3)*x^2 + (d*f^4*g + 4*c*f^3*g^2)*
x), x)) - 1/3*A^2/(g^4*x^3 + 3*f*g^3*x^2 + 3*f^2*g^2*x + f^3*g)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(g*x+f)^4,x, algorithm="fricas
")
```

```
[Out] integral((B^2*log((b^2*x^2 + 2*a*b*x + a^2)*e/(d^2*x^2 + 2*c*d*x + c^2))^2
+ 2*A*B*log((b^2*x^2 + 2*a*b*x + a^2)*e/(d^2*x^2 + 2*c*d*x + c^2)) + A^2)/(
g^4*x^4 + 4*f*g^3*x^3 + 6*f^2*g^2*x^2 + 4*f^3*g*x + f^4), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2/(g*x+f)**4,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(g*x+f)^4,x, algorithm="giac")
[Out] integrate((B*log((b*x + a)^2*e/(d*x + c)^2) + A)^2/(g*x + f)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+B\,\ln\left(\frac{e\,{\left(a+b\,x\right)}^2}{{\left(c+d\,x\right)}^2}\right)\right)^2}{{\left(f+g\,x\right)}^4} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2/(f + g*x)^4,x)
[Out] int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2/(f + g*x)^4, x)
```

3.280 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(f+gx)^5}\,dx$

Optimal. Leaf size=1154

$$-\frac{B^2(bc-ad)^2g^3(c+dx)^2}{3(bf-ag)^2(df-cg)^4(f+gx)^2}-\frac{2B^2(bc-ad)^3g^3(c+dx)}{3(bf-ag)^3(df-cg)^4(f+gx)}+\frac{B^2(bc-ad)^2g^2(4bdf-bcg-3adg)(c}{(bf-ag)^3(df-cg)^4(f+gx)}$$

```
[Out] -1/3*B^2*(-a*d+b*c)^2*g^3*(d*x+c)^2/(-a*g+b*f)^2/(-c*g+d*f)^4/(g*x+f)^2-2/3
*B^2*(-a*d+b*c)^3*g^3*(d*x+c)/(-a*g+b*f)^3/(-c*g+d*f)^4/(g*x+f)+B^2*(-a*d+b
*c)^2*g^2*(-3*a*d*g-b*c*g+4*b*d*f)*(d*x+c)/(-a*g+b*f)^3/(-c*g+d*f)^4/(g*x+f
)+1/3*B*(-a*d+b*c)*g^3*(d*x+c)^3*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/(-a*g+b*f)
/(-c*g+d*f)^4/(g*x+f)^3-1/2*B*(-a*d+b*c)*g^2*(-3*a*d*g-b*c*g+4*b*d*f)*(d*x+
c)^2*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/(-a*g+b*f)^2/(-c*g+d*f)^4/(g*x+f)^2+B*
(-a*d+b*c)*g*(3*a^2*d^2*g^2-2*a*b*d*g*(-c*g+4*d*f)+b^2*(c^2*g^2-4*c*d*f*g+6
*d^2*f^2))*(b*x+a)*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/(-a*g+b*f)^4/(-c*g+d*f)^
3/(g*x+f)+1/4*b^4*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/g/(-a*g+b*f)^4-1/4*(A+B
*ln(e*(b*x+a)^2/(d*x+c)^2))^2/g/(g*x+f)^4-2/3*B^2*(-a*d+b*c)^4*g^3*ln((b*x+
a)/(d*x+c))/(-a*g+b*f)^4/(-c*g+d*f)^4+B^2*(-a*d+b*c)^3*g^2*(-3*a*d*g-b*c*g+
4*b*d*f)*ln((b*x+a)/(d*x+c))/(-a*g+b*f)^4/(-c*g+d*f)^4+2/3*B^2*(-a*d+b*c)^4
*g^3*ln((g*x+f)/(d*x+c))/(-a*g+b*f)^4/(-c*g+d*f)^4-B^2*(-a*d+b*c)^3*g^2*(-3
*a*d*g-b*c*g+4*b*d*f)*ln((g*x+f)/(d*x+c))/(-a*g+b*f)^4/(-c*g+d*f)^4+2*B^2*(
-a*d+b*c)^2*g*(3*a^2*d^2*g^2-2*a*b*d*g*(-c*g+4*d*f)+b^2*(c^2*g^2-4*c*d*f*g+
6*d^2*f^2))*ln((g*x+f)/(d*x+c))/(-a*g+b*f)^4/(-c*g+d*f)^4-B*(-a*d+b*c)*(-a*
d*g-b*c*g+2*b*d*f)*(2*a*b*d^2*f*g-a^2*d^2*g^2-b^2*(c^2*g^2-2*c*d*f*g+2*d^2*
f^2))*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))*ln(1-(-c*g+d*f)*(b*x+a)/(-a*g+b*f)/(d
*x+c))/(-a*g+b*f)^4/(-c*g+d*f)^4-2*B^2*(-a*d+b*c)*(-a*d*g-b*c*g+2*b*d*f)*(2
*a*b*d^2*f*g-a^2*d^2*g^2-b^2*(c^2*g^2-2*c*d*f*g+2*d^2*f^2))*polylog(2,(-c*g
+d*f)*(b*x+a)/(-a*g+b*f)/(d*x+c))/(-a*g+b*f)^4/(-c*g+d*f)^4
```

Rubi [A]

time = 1.66, antiderivative size = 1154, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.323$, Rules used = {2554, 2398, 2404, 2338, 2356, 46, 2351, 31, 2354, 2438}

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2/(f + g*x)^5,x]
```

```
[Out] -1/3*(B^2*(b*c - a*d)^2*g^3*(c + d*x)^2)/((b*f - a*g)^2*(d*f - c*g)^4*(f +
g*x)^2) - (2*B^2*(b*c - a*d)^3*g^3*(c + d*x))/(3*(b*f - a*g)^3*(d*f - c*g)^
4*(f + g*x)) + (B^2*(b*c - a*d)^2*g^2*(4*b*d*f - b*c*g - 3*a*d*g)*(c + d*x)
```

```
)/((b*f - a*g)^3*(d*f - c*g)^4*(f + g*x)) + (B*(b*c - a*d)*g^3*(c + d*x)^3*
(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]))/(3*(b*f - a*g)*(d*f - c*g)^4*(f +
 g*x)^3) - (B*(b*c - a*d)*g^2*(4*b*d*f - b*c*g - 3*a*d*g)*(c + d*x)^2*(A +
B*Log[(e*(a + b*x)^2)/(c + d*x)^2]))/(2*(b*f - a*g)^2*(d*f - c*g)^4*(f + g*
x)^2) + (B*(b*c - a*d)*g*(3*a^2*d^2*g^2 - 2*a*b*d*g*(4*d*f - c*g) + b^2*(6*
d^2*f^2 - 4*c*d*f*g + c^2*g^2))*(a + b*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d
*x)^2]))/((b*f - a*g)^4*(d*f - c*g)^3*(f + g*x)) + (b^4*(A + B*Log[(e*(a +
b*x)^2)/(c + d*x)^2])^2)/(4*g*(b*f - a*g)^4) - (A + B*Log[(e*(a + b*x)^2)/(
c + d*x)^2])^2/(4*g*(f + g*x)^4) - (2*B^2*(b*c - a*d)^4*g^3*Log[(a + b*x)/(
c + d*x)])/(3*(b*f - a*g)^4*(d*f - c*g)^4) + (B^2*(b*c - a*d)^3*g^2*(4*b*d*
f - b*c*g - 3*a*d*g)*Log[(a + b*x)/(c + d*x)])/((b*f - a*g)^4*(d*f - c*g)^4
) + (2*B^2*(b*c - a*d)^4*g^3*Log[(f + g*x)/(c + d*x)])/(3*(b*f - a*g)^4*(d*
f - c*g)^4) - (B^2*(b*c - a*d)^3*g^2*(4*b*d*f - b*c*g - 3*a*d*g)*Log[(f + g
*x)/(c + d*x)])/((b*f - a*g)^4*(d*f - c*g)^4) + (2*B^2*(b*c - a*d)^2*g*(3*a
^2*d^2*g^2 - 2*a*b*d*g*(4*d*f - c*g) + b^2*(6*d^2*f^2 - 4*c*d*f*g + c^2*g^2
))*Log[(f + g*x)/(c + d*x)])/((b*f - a*g)^4*(d*f - c*g)^4) - (B*(b*c - a*d)
*(2*b*d*f - b*c*g - a*d*g)*(2*a*b*d^2*f*g - a^2*d^2*g^2 - b^2*(2*d^2*f^2 -
2*c*d*f*g + c^2*g^2))*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])*Log[1 - ((d*
f - c*g)*(a + b*x))/((b*f - a*g)*(c + d*x))])/((b*f - a*g)^4*(d*f - c*g)^4)
 - (2*B^2*(b*c - a*d)*(2*b*d*f - b*c*g - a*d*g)*(2*a*b*d^2*f*g - a^2*d^2*g^
2 - b^2*(2*d^2*f^2 - 2*c*d*f*g + c^2*g^2))*PolyLog[2, ((d*f - c*g)*(a + b*x
))/((b*f - a*g)*(c + d*x))])/((b*f - a*g)^4*(d*f - c*g)^4)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_)*((
f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(d + e*x)^(q +
 1)*((a + b*Log[c*x^n])^p/((q + 1)*(e*f - d*g))), x] - Dist[b*n*(p/((q + 1)
*(e*f - d*g))), Int[(f + g*x)^(m + 1)*(d + e*x)^(q + 1)*((a + b*Log[c*x^n])
^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, q}, x] && NeQ[e*f
- d*g, 0] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 2404

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2554

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[b*c - a*d, Su
bst[Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m +
2)), x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n
}, x] && EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegerQ[m] &
& IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(f+gx)^5}\,dx = -\frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{4g(f+gx)^4} + \frac{B\int \frac{2(bc-ad)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(a+bx)(c+dx)(f+gx)^4}\,dx}{2g}$$

$$= -\frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{4g(f+gx)^4} + \frac{(B(bc-ad))\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(a+bx)(c+dx)(f+gx)^4}\,dx}{g}$$

$$= -\frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{4g(f+gx)^4} + \frac{(B(bc-ad))\int \left(\frac{b^5\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(bc-ad)(bf-ag)^4(a+bx)} - \frac{d^5\left(A \cdots\right.}{(bc- \cdots} \cdots\right.}{\cdots}$$

$$= -\frac{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{4g(f+gx)^4} + \frac{(b^5B)\int \frac{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{a+bx}\,dx}{g(bf-ag)^4} - \frac{(Bd^5)\int \frac{A+B\log \cdots}{\cdots}}{g(df- \cdots}$$

$$= -\frac{B(bc-ad)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{3(bf-ag)(df-cg)(f+gx)^3} - \frac{B(bc-ad)(2bdf-bcg-adg)\left(A \cdots\right.}{2(bf-ag)^2(df-cg)^2(\cdots}$$

$$= -\frac{B(bc-ad)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{3(bf-ag)(df-cg)(f+gx)^3} - \frac{B(bc-ad)(2bdf-bcg-adg)\left(A \cdots\right.}{2(bf-ag)^2(df-cg)^2(\cdots}$$

$$= -\frac{B(bc-ad)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{3(bf-ag)(df-cg)(f+gx)^3} - \frac{B(bc-ad)(2bdf-bcg-adg)\left(A \cdots\right.}{2(bf-ag)^2(df-cg)^2(\cdots}$$

$$= -\frac{B^2(bc-ad)^2g}{3(bf-ag)^2(df-cg)^2(f+gx)^2} - \frac{5B^2(bc-ad)^2g(2bdf-bcg-adg)}{3(bf-ag)^3(df-cg)^3(f+gx)} + \cdots$$

$$= -\frac{B^2(bc-ad)^2g}{3(bf-ag)^2(df-cg)^2(f+gx)^2} - \frac{5B^2(bc-ad)^2g(2bdf-bcg-adg)}{3(bf-ag)^3(df-cg)^3(f+gx)} + \cdots$$

$$= -\frac{B^2(bc-ad)^2g}{3(bf-ag)^2(df-cg)^2(f+gx)^2} - \frac{5B^2(bc-ad)^2g(2bdf-bcg-adg)}{3(bf-ag)^3(df-cg)^3(f+gx)} + \cdots$$

$$= -\frac{B^2(bc-ad)^2g}{3(bf-ag)^2(df-cg)^2(f+gx)^2} - \frac{5B^2(bc-ad)^2g(2bdf-bcg-adg)}{3(bf-ag)^3(df-cg)^3(f+gx)} + \cdots$$

Mathematica [A]
time = 4.82, size = 1317, normalized size = 1.14

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2/(f + g*x)^5,x]
```

```
[Out] -1/12*(3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2 + (2*B*(f + g*x)*(2*(b*
c - a*d)*g*(b*f - a*g)^3*(d*f - c*g)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)
^2]) - 3*(b*c - a*d)*g*(b*f - a*g)^2*(d*f - c*g)^2*(-2*b*d*f + b*c*g + a*d*
g)*(f + g*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]) + 6*(b*c - a*d)*g*(b*
f - a*g)*(d*f - c*g)*(a^2*d^2*g^2 + a*b*d*g*(-3*d*f + c*g) + b^2*(3*d^2*f^2
 - 3*c*d*f*g + c^2*g^2))*(f + g*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2
]) - 6*b^4*(d*f - c*g)^4*(f + g*x)^3*Log[a + b*x]*(A + B*Log[(e*(a + b*x)^2
)/(c + d*x)^2]) + 6*d^4*(b*f - a*g)^4*(f + g*x)^3*(A + B*Log[(e*(a + b*x)^2
)/(c + d*x)^2])*Log[c + d*x] + 6*(b*c - a*d)*g*(-2*b*d*f + b*c*g + a*d*g)*(
-2*a*b*d^2*f*g + a^2*d^2*g^2 + b^2*(2*d^2*f^2 - 2*c*d*f*g + c^2*g^2))*(f +
g*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])*Log[f + g*x] - 12*B*(b*c -
a*d)*g*(a^2*d^2*g^2 + a*b*d*g*(-3*d*f + c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g +
 c^2*g^2))*(f + g*x)^3*(b*(d*f - c*g)*Log[a + b*x] + (-(b*d*f) + a*d*g)*Log
[c + d*x] + (b*c - a*d)*g*Log[f + g*x]) + 6*B*(b*c - a*d)*g*(2*b*d*f - b*c*
g - a*d*g)*(f + g*x)^2*((b*c - a*d)*g*(b*f - a*g)*(d*f - c*g) - b^2*(d*f -
c*g)^2*(f + g*x)*Log[a + b*x] + d^2*(b*f - a*g)^2*(f + g*x)*Log[c + d*x] +
(b*c - a*d)*g*(-2*b*d*f + b*c*g + a*d*g)*(f + g*x)*Log[f + g*x]) + 2*B*(b*c
 - a*d)*g*(f + g*x)*((b*c - a*d)*g*(b*f - a*g)^2*(d*f - c*g)^2 + 2*(b*c - a
*d)*g*(b*f - a*g)*(-(d*f) + c*g)*(-2*b*d*f + b*c*g + a*d*g)*(f + g*x) - 2*b
^3*(d*f - c*g)^3*(f + g*x)^2*Log[a + b*x] + 2*d^3*(b*f - a*g)^3*(f + g*x)^2
*Log[c + d*x] - 2*(b*c - a*d)*g*(a^2*d^2*g^2 + a*b*d*g*(-3*d*f + c*g) + b^2
*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^2))*(f + g*x)^2*Log[f + g*x]) + 6*b^4*B*(d*
f - c*g)^4*(f + g*x)^3*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b
*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) - 6*B*d^4*(b*f -
a*g)^4*(f + g*x)^3*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Lo
g[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]) - 12*B*(b*c - a*d)*g*
(-2*b*d*f + b*c*g + a*d*g)*(-2*a*b*d^2*f*g + a^2*d^2*g^2 + b^2*(2*d^2*f^2 -
 2*c*d*f*g + c^2*g^2))*(f + g*x)^3*((Log[(g*(a + b*x))/(-(b*f) + a*g)] - Lo
g[(g*(c + d*x))/(-(d*f) + c*g)])*Log[f + g*x] + PolyLog[2, (b*(f + g*x))/(b
*f - a*g)] - PolyLog[2, (d*(f + g*x))/(d*f - c*g)])))/((b*f - a*g)^4*(d*f -
 c*g)^4))/(g*(f + g*x)^4)
```

Maple [F]
time = 1.06, size = 0, normalized size = 0.00

$$\int \frac{\left(A + B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)\right)^2}{(gx+f)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/(g*x+f)^5,x)
```

```
[Out] int((A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/(g*x+f)^5,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(g*x+f)^5,x, algorithm="maxima
")
```

```
[Out] 1/6*(6*b^4*log(b*x + a)/(b^4*f^4*g - 4*a*b^3*f^3*g^2 + 6*a^2*b^2*f^2*g^3 -
4*a^3*b*f*g^4 + a^4*g^5) - 6*d^4*log(d*x + c)/(d^4*f^4*g - 4*c*d^3*f^3*g^2
+ 6*c^2*d^2*f^2*g^3 - 4*c^3*d*f*g^4 + c^4*g^5) + 6*(4*(b^4*c*d^3 - a*b^3*d^
4)*f^3 - 6*(b^4*c^2*d^2 - a^2*b^2*d^4)*f^2*g + 4*(b^4*c^3*d - a^3*b*d^4)*f*
g^2 - (b^4*c^4 - a^4*d^4)*g^3)*log(g*x + f)/(b^4*d^4*f^8 + a^4*c^4*g^8 - 4*
(b^4*c*d^3 + a*b^3*d^4)*f^7*g + 2*(3*b^4*c^2*d^2 + 8*a*b^3*c*d^3 + 3*a^2*b^
2*d^4)*f^6*g^2 - 4*(b^4*c^3*d + 6*a*b^3*c^2*d^2 + 6*a^2*b^2*c*d^3 + a^3*b*d
^4)*f^5*g^3 + (b^4*c^4 + 16*a*b^3*c^3*d + 36*a^2*b^2*c^2*d^2 + 16*a^3*b*c*d
^3 + a^4*d^4)*f^4*g^4 - 4*(a*b^3*c^4 + 6*a^2*b^2*c^3*d + 6*a^3*b*c^2*d^2 +
a^4*c*d^3)*f^3*g^5 + 2*(3*a^2*b^2*c^4 + 8*a^3*b*c^3*d + 3*a^4*c^2*d^2)*f^2*
g^6 - 4*(a^3*b*c^4 + a^4*c^3*d)*f*g^7) - (26*(b^3*c*d^2 - a*b^2*d^3)*f^4 -
31*(b^3*c^2*d - a^2*b*d^3)*f^3*g + (11*b^3*c^3 + 15*a*b^2*c^2*d - 15*a^2*b*
c*d^2 - 11*a^3*d^3)*f^2*g^2 - 7*(a*b^2*c^3 - a^3*c*d^2)*f*g^3 + 2*(a^2*b*c^
3 - a^3*c^2*d)*g^4 + 6*(3*(b^3*c*d^2 - a*b^2*d^3)*f^2*g^2 - 3*(b^3*c^2*d -
a^2*b*d^3)*f*g^3 + (b^3*c^3 - a^3*d^3)*g^4)*x^2 + 3*(14*(b^3*c*d^2 - a*b^2*
d^3)*f^3*g - 15*(b^3*c^2*d - a^2*b*d^3)*f^2*g^2 + (5*b^3*c^3 + 3*a*b^2*c^2*
d - 3*a^2*b*c*d^2 - 5*a^3*d^3)*f*g^3 - (a*b^2*c^3 - a^3*c*d^2)*g^4)*x)/(b^3
*d^3*f^9 + a^3*c^3*f^3*g^6 - 3*(b^3*c*d^2 + a*b^2*d^3)*f^8*g + 3*(b^3*c^2*d
 + 3*a*b^2*c*d^2 + a^2*b*d^3)*f^7*g^2 - (b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2*b*
c*d^2 + a^3*d^3)*f^6*g^3 + 3*(a*b^2*c^3 + 3*a^2*b*c^2*d + a^3*c*d^2)*f^5*g^
4 - 3*(a^2*b*c^3 + a^3*c^2*d)*f^4*g^5 + (b^3*d^3*f^6*g^3 + a^3*c^3*g^9 - 3*
(b^3*c*d^2 + a*b^2*d^3)*f^5*g^4 + 3*(b^3*c^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3)
*f^4*g^5 - (b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*f^3*g^6 + 3*
(a*b^2*c^3 + 3*a^2*b*c^2*d + a^3*c*d^2)*f^2*g^7 - 3*(a^2*b*c^3 + a^3*c^2*d)
*f*g^8)*x^3 + 3*(b^3*d^3*f^7*g^2 + a^3*c^3*f*g^8 - 3*(b^3*c*d^2 + a*b^2*d^3
)*f^6*g^3 + 3*(b^3*c^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3)*f^5*g^4 - (b^3*c^3 +
9*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*f^4*g^5 + 3*(a*b^2*c^3 + 3*a^2*b*c
^2*d + a^3*c*d^2)*f^3*g^6 - 3*(a^2*b*c^3 + a^3*c^2*d)*f^2*g^7)*x^2 + 3*(b^3
*d^3*f^8*g + a^3*c^3*f^2*g^7 - 3*(b^3*c*d^2 + a*b^2*d^3)*f^7*g^2 + 3*(b^3*c
^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3)*f^6*g^3 - (b^3*c^3 + 9*a*b^2*c^2*d + 9*a^
2*b*c*d^2 + a^3*d^3)*f^5*g^4 + 3*(a*b^2*c^3 + 3*a^2*b*c^2*d + a^3*c*d^2)*f^
4*g^5 - 3*(a^2*b*c^3 + a^3*c^2*d)*f^3*g^6)*x) - 3*log(b^2*x^2*e/(d^2*x^2 +
2*c*d*x + c^2) + 2*a*b*x*e/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c
*d*x + c^2))/(g^5*x^4 + 4*f*g^4*x^3 + 6*f^2*g^3*x^2 + 4*f^3*g^2*x + f^4*g))
*A*B - B^2*(log(d*x + c)^2/(g^5*x^4 + 4*f*g^4*x^3 + 6*f^2*g^3*x^2 + 4*f^3*g
^2*x + f^4*g) + integrate(-(d*g*x + 4*(d*g*x + c*g)*log(b*x + a)^2 + c*g +
```

```
4*(d*g*x + c*g)*log(b*x + a) - 2*(d*g*x - d*f + 2*c*g + 4*(d*g*x + c*g)*log
(b*x + a))*log(d*x + c))/(d*g^6*x^6 + c*f^5*g + (5*d*f*g^5 + c*g^6)*x^5 + 5
*(2*d*f^2*g^4 + c*f*g^5)*x^4 + 10*(d*f^3*g^3 + c*f^2*g^4)*x^3 + 5*(d*f^4*g^
2 + 2*c*f^3*g^3)*x^2 + (d*f^5*g + 5*c*f^4*g^2)*x), x)) - 1/4*A^2/(g^5*x^4 +
 4*f*g^4*x^3 + 6*f^2*g^3*x^2 + 4*f^3*g^2*x + f^4*g)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(g*x+f)^5,x, algorithm="fricas
")
[Out] integral((B^2*log((b^2*x^2 + 2*a*b*x + a^2)*e/(d^2*x^2 + 2*c*d*x + c^2))^2
+ 2*A*B*log((b^2*x^2 + 2*a*b*x + a^2)*e/(d^2*x^2 + 2*c*d*x + c^2)) + A^2)/(
g^5*x^5 + 5*f*g^4*x^4 + 10*f^2*g^3*x^3 + 10*f^3*g^2*x^2 + 5*f^4*g*x + f^5),
 x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2/(g*x+f)**5,x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(g*x+f)^5,x, algorithm="giac")
[Out] integrate((B*log((b*x + a)^2*e/(d*x + c)^2) + A)^2/(g*x + f)^5, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+B\,\ln\left(\frac{e\,{\left(a+b\,x\right)}^2}{{\left(c+d\,x\right)}^2}\right)\right)^2}{{\left(f+g\,x\right)}^5} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2/(f + g*x)^5,x)
[Out] int((A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2/(f + g*x)^5, x)
```

3.281 $\int \frac{(f+gx)^2}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx$

Optimal. Leaf size=34

$$\text{Int}\left(\frac{(f+gx)^2}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)},x\right)$$

[Out] Unintegrable((g*x+f)^2/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(f+gx)^2}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx$$

Verification is not applicable to the result.

[In] Int[(f + g*x)^2/(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]),x]

[Out] Defer[Int][(f + g*x)^2/(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]), x]

Rubi steps

$$\begin{aligned}\int \frac{(f+gx)^2}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx &= \int\left(\frac{f^2}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}+\frac{2fgx}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}+\frac{g^2x^2}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\right)d\\ &= f^2\int\frac{1}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx+(2fg)\int\frac{x}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx+g^2\int \frac{}{A+}\end{aligned}$$

Mathematica [A]
time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{(f+gx)^2}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx$$

Verification is not applicable to the result.

[In] Integrate[(f + g*x)^2/(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]),x]

```
[Out] Integrate[(f + g*x)^2/(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]), x]
```

Maple [A]
time = 0.97, size = 0, normalized size = 0.00

$$\int \frac{(gx+f)^2}{A+B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)
[Out] int((g*x+f)^2/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="maxima")
[Out] integrate((g*x + f)^2/(B*log((b*x + a)^2*e/(d*x + c)^2) + A), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="fricas")
[Out] integral((g^2*x^2 + 2*f*g*x + f^2)/(B*log((b^2*x^2 + 2*a*b*x + a^2)*e/(d^2*
x^2 + 2*c*d*x + c^2)) + A), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(f+gx)^2}{A+B\log\left(\frac{a^2e}{c^2+2cdx+d^2x^2}+\frac{2abex}{c^2+2cdx+d^2x^2}+\frac{b^2ex^2}{c^2+2cdx+d^2x^2}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2/(A+B*ln(e*(b*x+a)**2/(d*x+c)**2)),x)
[Out] Integral((f + g*x)**2/(A + B*log(a**2*e/(c**2 + 2*c*d*x + d**2*x**2) + 2*a*
b*e*x/(c**2 + 2*c*d*x + d**2*x**2) + b**2*e*x**2/(c**2 + 2*c*d*x + d**2*x**
2))), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="giac")
[Out] integrate((g*x + f)^2/(B*log((b*x + a)^2*e/(d*x + c)^2) + A), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{(f+g\,x)^2}{A+B\,\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^2/(A + B*log((e*(a + b*x)^2)/(c + d*x)^2)),x)
[Out] int((f + g*x)^2/(A + B*log((e*(a + b*x)^2)/(c + d*x)^2)), x)
```

3.282 $\int \frac{f+gx}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx$

Optimal. Leaf size=32

$$\text{Int}\left(\frac{f+gx}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)},x\right)$$

[Out] Unintegrable((g*x+f)/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{f+gx}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx$$

Verification is not applicable to the result.

[In] Int[(f + g*x)/(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]),x]

[Out] Defer[Int][(f + g*x)/(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]), x]

Rubi steps

$$\begin{aligned}
\int \frac{f+gx}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx &= \int \left(\frac{f}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}+\frac{gx}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\right)dx\\
&= f\int \frac{1}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx+g\int \frac{x}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx
\end{aligned}$$

Mathematica [A]
time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{f+gx}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx$$

Verification is not applicable to the result.

[In] Integrate[(f + g*x)/(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]),x]

[Out] Integrate[(f + g*x)/(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]), x]

Maple [A]
time = 0.83, size = 0, normalized size = 0.00

$$\int \frac{gx+f}{A+B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)
```

```
[Out] int((g*x+f)/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="maxima")
```

```
[Out] integrate((g*x + f)/(B*log((b*x + a)^2*e/(d*x + c)^2) + A), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="fricas")
```

```
[Out] integral((g*x + f)/(B*log((b^2*x^2 + 2*a*b*x + a^2)*e/(d^2*x^2 + 2*c*d*x +
c^2)) + A), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{f+gx}{A+B\log\left(\frac{a^2e}{c^2+2cdx+d^2x^2}+\frac{2abex}{c^2+2cdx+d^2x^2}+\frac{b^2ex^2}{c^2+2cdx+d^2x^2}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)/(A+B*ln(e*(b*x+a)**2/(d*x+c)**2)),x)
```

```
[Out] Integral((f + g*x)/(A + B*log(a**2*e/(c**2 + 2*c*d*x + d**2*x**2) + 2*a*b*e
*x/(c**2 + 2*c*d*x + d**2*x**2) + b**2*e*x**2/(c**2 + 2*c*d*x + d**2*x**2))
), x)
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="giac")
[Out] integrate((g*x + f)/(B*log((b*x + a)^2*e/(d*x + c)^2) + A), x)
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{f+g\,x}{A+B\,\ln\left(\frac{e\,{\left(a+b\,x\right)}^2}{{\left(c+d\,x\right)}^2}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)/(A + B*log((e*(a + b*x)^2)/(c + d*x)^2)),x)
[Out] int((f + g*x)/(A + B*log((e*(a + b*x)^2)/(c + d*x)^2)), x)
```

3.283 $\int \frac{1}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx$

Optimal. Leaf size=26

$$\text{Int}\left(\frac{1}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)},x\right)$$

```
[Out] Unintegrable(1/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)
```

Rubi [A]
time = 0.00, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^(-1),x]
[Out] Defer[Int][(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^(-1), x]
```

Rubi steps

$$\int \frac{1}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx = \int \frac{1}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx$$

Mathematica [A]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{1}{A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^(-1),x]
[Out] Integrate[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^(-1), x]
```

Maple [A]
time = 0.72, size = 0, normalized size = 0.00

$$\int \frac{1}{A+B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)
[Out] int(1/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="maxima")
[Out] integrate(1/(B*log((b*x + a)^2*e/(d*x + c)^2) + A), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="fricas")
[Out] integral(1/(B*log((b^2*x^2 + 2*a*b*x + a^2)*e/(d^2*x^2 + 2*c*d*x + c^2)) +
A), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(A+B*ln(e*(b*x+a)**2/(d*x+c)**2)),x)
[Out] Integral(1/(A + B*log(e*(a + b*x)**2/(c + d*x)**2)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="giac")
[Out] integrate(1/(B*log((b*x + a)^2*e/(d*x + c)^2) + A), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{A + B\,\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(A + B*log((e*(a + b*x)^2)/(c + d*x)^2)),x)
[Out] int(1/(A + B*log((e*(a + b*x)^2)/(c + d*x)^2)), x)
```

3.284 $\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx$

Optimal. Leaf size=34

$$\text{Int}\left(\frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)},x\right)$$

```
[Out] Unintegrable(1/(g*x+f)/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((f + g*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])),x]
[Out] Defer[Int][1/((f + g*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])), x]
```

Rubi steps

$$\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx = \int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx$$

Mathematica [A]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((f + g*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])),x]
[Out] Integrate[1/((f + g*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])), x]
```

Maple [A]
time = 0.74, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(gx+f\right)\left(A+B\ln\left(\frac{e\left(bx+a\right)^{2}}{\left(dx+c\right)^{2}}\right)\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(g*x+f)/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)
```

```
[Out] int(1/(g*x+f)/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="maxima")
```

```
[Out] integrate(1/((g*x + f)*(B*log((b*x + a)^2*e/(d*x + c)^2) + A)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="fricas")
```

```
[Out] integral(1/(A*g*x + A*f + (B*g*x + B*f)*log((b^2*x^2 + 2*a*b*x + a^2)*e/(d^
2*x^2 + 2*c*d*x + c^2))), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)/(A+B*ln(e*(b*x+a)**2/(d*x+c)**2)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="giac")
[Out] integrate(1/((g*x + f)*(B*log((b*x + a)^2*e/(d*x + c)^2) + A)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{(f+g\,x)\,\left(A+B\,\ln\left(\frac{e\,{(a+b\,x)}^2}{{(c+d\,x)}^2}\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((f + g*x)*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))),x)
[Out] int(1/((f + g*x)*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))), x)
```

3.285 $\int \frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx$

Optimal. Leaf size=34

$$\text{Int}\left(\frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)},x\right)$$

```
[Out] Unintegrable(1/(g*x+f)^2/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((f + g*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])),x]
[Out] Defer[Int][1/((f + g*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])), x]
```

Rubi steps

$$\int \frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx = \int \frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx$$

Mathematica [A]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((f + g*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])),x]
[Out] Integrate[1/((f + g*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])), x]
```

Maple [A]
time = 0.88, size = 0, normalized size = 0.00

$$\int \frac{1}{(gx+f)^2 \left(A+B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(g*x+f)^2/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)
[Out] int(1/(g*x+f)^2/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)^2/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="maxima
")
[Out] integrate(1/((g*x + f)^2*(B*log((b*x + a)^2*e/(d*x + c)^2) + A)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)^2/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="fricas
")
[Out] integral(1/(A*g^2*x^2 + 2*A*f*g*x + A*f^2 + (B*g^2*x^2 + 2*B*f*g*x + B*f^2)
*log((b^2*x^2 + 2*a*b*x + a^2)*e/(d^2*x^2 + 2*c*d*x + c^2))), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)**2/(A+B*ln(e*(b*x+a)**2/(d*x+c)**2)),x)
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)^2/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="giac")
[Out] integrate(1/((g*x + f)^2*(B*log((b*x + a)^2*e/(d*x + c)^2) + A)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{(f+g\,x)^2\,\left(A+B\,\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((f + g*x)^2*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))),x)
[Out] int(1/((f + g*x)^2*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))), x)
```

3.286 $\int \frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx$

Optimal. Leaf size=34

$$\text{Int}\left(\frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)},x\right)$$

```
[Out] Unintegrable(1/(g*x+f)^3/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((f + g*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])),x]
[Out] Defer[Int][1/((f + g*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])), x]
```

Rubi steps

$$\int \frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx=\int \frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx$$

Mathematica [A]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((f + g*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])),x]
[Out] Integrate[1/((f + g*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])), x]
```

Maple [A]
time = 0.86, size = 0, normalized size = 0.00

$$\int \frac{1}{(gx+f)^3 \left(A+B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(g*x+f)^3/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)
```

```
[Out] int(1/(g*x+f)^3/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)^3/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="maxima
")
```

```
[Out] integrate(1/((g*x + f)^3*(B*log((b*x + a)^2*e/(d*x + c)^2) + A)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)^3/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="fricas
")
```

```
[Out] integral(1/(A*g^3*x^3 + 3*A*f*g^2*x^2 + 3*A*f^2*g*x + A*f^3 + (B*g^3*x^3 +
3*B*f*g^2*x^2 + 3*B*f^2*g*x + B*f^3)*log((b^2*x^2 + 2*a*b*x + a^2)*e/(d^2*x
^2 + 2*c*d*x + c^2))), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)**3/(A+B*ln(e*(b*x+a)**2/(d*x+c)**2)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)^3/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="giac")
[Out] integrate(1/((g*x + f)^3*(B*log((b*x + a)^2*e/(d*x + c)^2) + A)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{(f+g\,x)^3\,\left(A+B\,\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((f + g*x)^3*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))),x)
[Out] int(1/((f + g*x)^3*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))), x)
```

3.287 $\int \frac{(f+gx)^2}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2} dx$

Optimal. Leaf size=34

$$\text{Int}\left(\frac{(f+gx)^2}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2},x\right)$$

[Out] Unintegrable((g*x+f)^2/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(f+gx)^2}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2} dx$$

Verification is not applicable to the result.

[In] Int[(f + g*x)^2/(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2,x]

[Out] Defer[Int][(f + g*x)^2/(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2, x]

Rubi steps

$$\int \frac{(f+gx)^2}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2} dx = \int \left(\frac{f^2}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2} + \frac{2fgx}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2} + \frac{g^2x^2}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\right) dx$$
$$= f^2\int \frac{1}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2} dx + (2fg)\int \frac{x}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2} dx +$$

Mathematica [A]
time = 0.35, size = 0, normalized size = 0.00

$$\int \frac{(f+gx)^2}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2} dx$$

Verification is not applicable to the result.

```
[In] Integrate[(f + g*x)^2/(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2,x]
[Out] Integrate[(f + g*x)^2/(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2, x]
```

Maple [A]
time = 0.88, size = 0, normalized size = 0.00

$$\int \frac{(gx+f)^2}{\left(A+B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
[Out] int((g*x+f)^2/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="maxima
")
[Out] -1/2*(b*d*g^2*x^4 + a*c*f^2 + (a*d*g^2 + (2*d*f*g + c*g^2)*b)*x^3 + ((2*d*f
*g + c*g^2)*a + (d*f^2 + 2*c*f*g)*b)*x^2 + (b*c*f^2 + (d*f^2 + 2*c*f*g)*a)*
x)/(2*(b*c - a*d)*B^2*log(b*x + a) - 2*(b*c - a*d)*B^2*log(d*x + c) + (b*c
- a*d)*A*B + (b*c - a*d)*B^2) + integrate(1/2*(4*b*d*g^2*x^3 + b*c*f^2 + 3*
(a*d*g^2 + (2*d*f*g + c*g^2)*b)*x^2 + (d*f^2 + 2*c*f*g)*a + 2*((2*d*f*g + c
*g^2)*a + (d*f^2 + 2*c*f*g)*b)*x)/(2*(b*c - a*d)*B^2*log(b*x + a) - 2*(b*c
- a*d)*B^2*log(d*x + c) + (b*c - a*d)*A*B + (b*c - a*d)*B^2), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="fricas
")
[Out] integral((g^2*x^2 + 2*f*g*x + f^2)/(B^2*log((b^2*x^2 + 2*a*b*x + a^2)*e/(d^
2*x^2 + 2*c*d*x + c^2))^2 + 2*A*B*log((b^2*x^2 + 2*a*b*x + a^2)*e/(d^2*x^2
+ 2*c*d*x + c^2)) + A^2), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2/(A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="giac")
```

```
[Out] integrate((g*x + f)^2/(B*log((b*x + a)^2*e/(d*x + c)^2) + A)^2, x)
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{(f+g\,x)^2}{\left(A+B\,\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^2/(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2,x)
```

```
[Out] int((f + g*x)^2/(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2, x)
```

3.288 $\int \frac{f+gx}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$

Optimal. Leaf size=32

$$\mathrm{Int}\left(\frac{f+gx}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2},x\right)$$

```
[Out] Unintegrable((g*x+f)/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{f+gx}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(f + g*x)/(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2,x]
[Out] Defer[Int][(f + g*x)/(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2, x]
```

Rubi steps

$$\begin{aligned}\int \frac{f+gx}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx &= \int \left(\frac{f}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}+\frac{gx}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\right)dx\\ &= f\int \frac{1}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx+g\int \frac{x}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx\end{aligned}$$

Mathematica [A]
time = 0.25, size = 0, normalized size = 0.00

$$\int \frac{f+gx}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(f + g*x)/(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2,x]
```

[Out] Integrate[(f + g*x)/(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2, x]

Maple [A]
time = 0.78, size = 0, normalized size = 0.00

$$\int \frac{gx+f}{\left(A+B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g*x+f)/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)

[Out] int((g*x+f)/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*x+f)/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="maxima")

[Out] -1/2*(b*d*g*x^3 + a*c*f + (a*d*g + (d*f + c*g)*b)*x^2 + (b*c*f + (d*f + c*g
)*a)*x)/(2*(b*c - a*d)*B^2*log(b*x + a) - 2*(b*c - a*d)*B^2*log(d*x + c) +
(b*c - a*d)*A*B + (b*c - a*d)*B^2) + integrate(1/2*(3*b*d*g*x^2 + b*c*f + (
d*f + c*g)*a + 2*(a*d*g + (d*f + c*g)*b)*x)/(2*(b*c - a*d)*B^2*log(b*x + a)
 - 2*(b*c - a*d)*B^2*log(d*x + c) + (b*c - a*d)*A*B + (b*c - a*d)*B^2), x)

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*x+f)/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="fricas")

[Out] integral((g*x + f)/(B^2*log((b^2*x^2 + 2*a*b*x + a^2)*e/(d^2*x^2 + 2*c*d*x
+ c^2))^2 + 2*A*B*log((b^2*x^2 + 2*a*b*x + a^2)*e/(d^2*x^2 + 2*c*d*x + c^2)
) + A^2), x)

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{acf+acgx+adfx+adgx^2+bcfx+bcgx^2+bdfx^2+bdgx^3}{2ABad-2ABbc+(2B^2ad-2B^2bc)\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}-\frac{\int\frac{acg}{A+B\log\left(\frac{a^2e}{c^2+2cdx+d^2x^2}+\frac{2abex}{c^2+2cdx+d^2x^2}+\frac{b^2ex^2}{c^2+2cdx+d^2x^2}\right)}\,dx+\int\frac{adf}{A+B\log\left(\frac{a^2e}{c^2+2cdx+d^2x^2}+\frac{2abex}{c^2+2cdx+d^2x^2}+\frac{b^2ex^2}{c^2+2cdx+d^2x^2}\right)}\,dx+\int\frac{bcf}{A+B\log\left(\frac{a^2e}{c^2+2cdx+d^2x^2}+\frac{2abex}{c^2+2cdx+d^2x^2}+\frac{b^2ex^2}{c^2+2cdx+d^2x^2}\right)}\,dx+\int\frac{2adgx}{A+B\log\left(\frac{a^2e}{c^2+2cdx+d^2x^2}+\frac{2abex}{c^2+2cdx+d^2x^2}+\frac{b^2ex^2}{c^2+2cdx+d^2x^2}\right)}\,dx+\int\frac{2bcgx}{A+B\log\left(\frac{a^2e}{c^2+2cdx+d^2x^2}+\frac{2abex}{c^2+2cdx+d^2x^2}+\frac{b^2ex^2}{c^2+2cdx+d^2x^2}\right)}\,dx+\int\frac{2bdfx}{A+B\log\left(\frac{a^2e}{c^2+2cdx+d^2x^2}+\frac{2abex}{c^2+2cdx+d^2x^2}+\frac{b^2ex^2}{c^2+2cdx+d^2x^2}\right)}\,dx+\int\frac{3bdgx^2}{A+B\log\left(\frac{a^2e}{c^2+2cdx+d^2x^2}+\frac{2abex}{c^2+2cdx+d^2x^2}+\frac{b^2ex^2}{c^2+2cdx+d^2x^2}\right)}\,dx}{2B(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*x+f)/(A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2,x)

```
[Out] (a*c*f + a*c*g*x + a*d*f*x + a*d*g*x**2 + b*c*f*x + b*c*g*x**2 + b*d*f*x**2
 + b*d*g*x**3)/(2*A*B*a*d - 2*A*B*b*c + (2*B**2*a*d - 2*B**2*b*c)*log(e*(a
+ b*x)**2/(c + d*x)**2)) - (Integral(a*c*g/(A + B*log(a**2*e/(c**2 + 2*c*d*
x + d**2*x**2) + 2*a*b*e*x/(c**2 + 2*c*d*x + d**2*x**2) + b**2*e*x**2/(c**2
 + 2*c*d*x + d**2*x**2))), x) + Integral(a*d*f/(A + B*log(a**2*e/(c**2 + 2*
c*d*x + d**2*x**2) + 2*a*b*e*x/(c**2 + 2*c*d*x + d**2*x**2) + b**2*e*x**2/(
c**2 + 2*c*d*x + d**2*x**2))), x) + Integral(b*c*f/(A + B*log(a**2*e/(c**2
+ 2*c*d*x + d**2*x**2) + 2*a*b*e*x/(c**2 + 2*c*d*x + d**2*x**2) + b**2*e*x*
*2/(c**2 + 2*c*d*x + d**2*x**2))), x) + Integral(2*a*d*g*x/(A + B*log(a**2*
e/(c**2 + 2*c*d*x + d**2*x**2) + 2*a*b*e*x/(c**2 + 2*c*d*x + d**2*x**2) + b
**2*e*x**2/(c**2 + 2*c*d*x + d**2*x**2))), x) + Integral(2*b*c*g*x/(A + B*l
og(a**2*e/(c**2 + 2*c*d*x + d**2*x**2) + 2*a*b*e*x/(c**2 + 2*c*d*x + d**2*x
**2) + b**2*e*x**2/(c**2 + 2*c*d*x + d**2*x**2))), x) + Integral(2*b*d*f*x/
(A + B*log(a**2*e/(c**2 + 2*c*d*x + d**2*x**2) + 2*a*b*e*x/(c**2 + 2*c*d*x
+ d**2*x**2) + b**2*e*x**2/(c**2 + 2*c*d*x + d**2*x**2))), x) + Integral(3*
b*d*g*x**2/(A + B*log(a**2*e/(c**2 + 2*c*d*x + d**2*x**2) + 2*a*b*e*x/(c**2
 + 2*c*d*x + d**2*x**2) + b**2*e*x**2/(c**2 + 2*c*d*x + d**2*x**2))), x))/(
2*B*(a*d - b*c))
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="giac")
[Out] integrate((g*x + f)/(B*log((b*x + a)^2*e/(d*x + c)^2) + A)^2, x)
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{f+g\,x}{{\left(A+B\,\ln\left(\frac{e\,{\left(a+b\,x\right)}^2}{{\left(c+d\,x\right)}^2}\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)/(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2,x)
[Out] int((f + g*x)/(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2, x)
```

3.289 $\int \frac{1}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$

Optimal. Leaf size=26

$$\text{Int}\left(\frac{1}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2},x\right)$$

[Out] Unintegrable(1/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)

Rubi [A]
time = 0.00, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$$

Verification is not applicable to the result.

[In] Int[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^(-2),x]

[Out] Defer[Int][(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^(-2), x]

Rubi steps

$$\int \frac{1}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx=\int \frac{1}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$$

Mathematica [A]
time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$$

Verification is not applicable to the result.

[In] Integrate[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^(-2),x]

[Out] Integrate[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^(-2), x]

Maple [A]
time = 0.63, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(A+B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

```
[Out] int(1/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="maxima")
```

```
[Out] -1/2*(b*d*x^2 + a*c + (b*c + a*d)*x)/(2*(b*c - a*d)*B^2*log(b*x + a) - 2*(b
*c - a*d)*B^2*log(d*x + c) + (b*c - a*d)*A*B + (b*c - a*d)*B^2) + integrate
(1/2*(2*b*d*x + b*c + a*d)/(2*(b*c - a*d)*B^2*log(b*x + a) - 2*(b*c - a*d)*
B^2*log(d*x + c) + (b*c - a*d)*A*B + (b*c - a*d)*B^2), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="fricas")
```

```
[Out] integral(1/(B^2*log((b^2*x^2 + 2*a*b*x + a^2)*e/(d^2*x^2 + 2*c*d*x + c^2))^
2 + 2*A*B*log((b^2*x^2 + 2*a*b*x + a^2)*e/(d^2*x^2 + 2*c*d*x + c^2)) + A^2)
, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{ac+adx+bcx+bdx^2}{2ABad-2ABbc+(2B^2ad-2B^2bc)\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}-\frac{\int \frac{ad}{A+B\log\left(\frac{a^2e}{c^2+2cdx+d^2x^2}+\frac{2abex}{c^2+2cdx+d^2x^2}+\frac{b^2ex^2}{c^2+2cdx+d^2x^2}\right)}\,dx+\int \frac{bc}{A+B\log\left(\frac{a^2e}{c^2+2cdx+d^2x^2}+\frac{2abex}{c^2+2cdx+d^2x^2}+\frac{b^2ex^2}{c^2+2cdx+d^2x^2}\right)}\,dx+\int \frac{2bdx}{A+B\log\left(\frac{a^2e}{c^2+2cdx+d^2x^2}+\frac{2abex}{c^2+2cdx+d^2x^2}+\frac{b^2ex^2}{c^2+2cdx+d^2x^2}\right)}\,dx}{2B\,(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2,x)
```

```
[Out] (a*c + a*d*x + b*c*x + b*d*x**2)/(2*A*B*a*d - 2*A*B*b*c + (2*B**2*a*d - 2*B
**2*b*c)*log(e*(a + b*x)**2/(c + d*x)**2)) - (Integral(a*d/(A + B*log(a**2*
e/(c**2 + 2*c*d*x + d**2*x**2) + 2*a*b*e*x/(c**2 + 2*c*d*x + d**2*x**2) + b
**2*e*x**2/(c**2 + 2*c*d*x + d**2*x**2))), x) + Integral(b*c/(A + B*log(a**
2*e/(c**2 + 2*c*d*x + d**2*x**2) + 2*a*b*e*x/(c**2 + 2*c*d*x + d**2*x**2) +
 b**2*e*x**2/(c**2 + 2*c*d*x + d**2*x**2))), x) + Integral(2*b*d*x/(A + B*l
og(a**2*e/(c**2 + 2*c*d*x + d**2*x**2) + 2*a*b*e*x/(c**2 + 2*c*d*x + d**2*x
**2) + b**2*e*x**2/(c**2 + 2*c*d*x + d**2*x**2))), x))/(2*B*(a*d - b*c))
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="giac")
```

```
[Out] integrate((B*log((b*x + a)^2*e/(d*x + c)^2) + A)^(-2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{{\left(A+B\,\ln\left(\frac{e\,{\left(a+b\,x\right)}^2}{{\left(c+d\,x\right)}^2}\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2,x)
```

```
[Out] int(1/(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2, x)
```

3.290 $\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$

Optimal. Leaf size=34

$$\text{Int}\left(\frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2},x\right)$$

```
[Out] Unintegrable(1/(g*x+f)/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((f + g*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2),x]
[Out] Defer[Int][1/((f + g*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2), x]
```

Rubi steps

$$\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx = \int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$$

Mathematica [A]
time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{1}{(f+gx)\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((f + g*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2),x]
[Out] Integrate[1/((f + g*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2), x]
```

Maple [A]
time = 0.73, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(gx+f\right)\left(A+B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(g*x+f)/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

```
[Out] int(1/(g*x+f)/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="maxima
")
```

```
[Out] -1/2*(b*d*x^2 + a*c + (b*c + a*d)*x)/((b*c*f - a*d*f)*A*B + (b*c*f - a*d*f)
*B^2 + ((b*c*g - a*d*g)*A*B + (b*c*g - a*d*g)*B^2)*x + 2*((b*c*g - a*d*g)*B
^2*x + (b*c*f - a*d*f)*B^2)*log(b*x + a) - 2*((b*c*g - a*d*g)*B^2*x + (b*c*
f - a*d*f)*B^2)*log(d*x + c)) + integrate(1/2*(b*d*g*x^2 + 2*b*d*f*x + b*c*
f + (d*f - c*g)*a)/((b*c*f^2 - a*d*f^2)*A*B + (b*c*f^2 - a*d*f^2)*B^2 + ((b
*c*g^2 - a*d*g^2)*A*B + (b*c*g^2 - a*d*g^2)*B^2)*x^2 + 2*((b*c*f*g - a*d*f*
g)*A*B + (b*c*f*g - a*d*f*g)*B^2)*x + 2*((b*c*g^2 - a*d*g^2)*B^2*x^2 + 2*(b
*c*f*g - a*d*f*g)*B^2*x + (b*c*f^2 - a*d*f^2)*B^2)*log(b*x + a) - 2*((b*c*g
^2 - a*d*g^2)*B^2*x^2 + 2*(b*c*f*g - a*d*f*g)*B^2*x + (b*c*f^2 - a*d*f^2)*B
^2)*log(d*x + c)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="fricas
")
```

```
[Out] integral(1/(A^2*g*x + A^2*f + (B^2*g*x + B^2*f)*log((b^2*x^2 + 2*a*b*x + a^
2)*e/(d^2*x^2 + 2*c*d*x + c^2))^2 + 2*(A*B*g*x + A*B*f)*log((b^2*x^2 + 2*a*
b*x + a^2)*e/(d^2*x^2 + 2*c*d*x + c^2))), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)/(A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2,x)
[Out] Timed out
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="giac")
[Out] integrate(1/((g*x + f)*(B*log((b*x + a)^2*e/(d*x + c)^2) + A)^2), x)
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{(f+g\,x)\,\left(A+B\,\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((f + g*x)*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2),x)
[Out] int(1/((f + g*x)*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2), x)
```

3.291 $\int \frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$

Optimal. Leaf size=34

$$\text{Int}\left(\frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2},x\right)$$

```
[Out] Unintegrable(1/(g*x+f)^2/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((f + g*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2),x]
[Out] Defer[Int][1/((f + g*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2), x]
```

Rubi steps

$$\int \frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx = \int \frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$$

Mathematica [A]
time = 0.34, size = 0, normalized size = 0.00

$$\int \frac{1}{(f+gx)^2\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((f + g*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2),x]
[Out] Integrate[1/((f + g*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2), x]
```

Maple [A]
time = 0.76, size = 0, normalized size = 0.00

$$\int \frac{1}{(gx+f)^2 \left(A + B\ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(g*x+f)^2/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

```
[Out] int(1/(g*x+f)^2/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)^2/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="maxi
ma")
```

```
[Out] -1/2*(b*d*x^2 + a*c + (b*c + a*d)*x)/((b*c*f^2 - a*d*f^2)*A*B + (b*c*f^2 -
a*d*f^2)*B^2 + ((b*c*g^2 - a*d*g^2)*A*B + (b*c*g^2 - a*d*g^2)*B^2)*x^2 + 2*
((b*c*f*g - a*d*f*g)*A*B + (b*c*f*g - a*d*f*g)*B^2)*x + 2*((b*c*g^2 - a*d*g
^2)*B^2*x^2 + 2*(b*c*f*g - a*d*f*g)*B^2*x + (b*c*f^2 - a*d*f^2)*B^2)*log(b*
x + a) - 2*((b*c*g^2 - a*d*g^2)*B^2*x^2 + 2*(b*c*f*g - a*d*f*g)*B^2*x + (b*
c*f^2 - a*d*f^2)*B^2)*log(d*x + c)) - integrate(-1/2*(b*c*f + (d*f - 2*c*g)
*a - (a*d*g - (2*d*f - c*g)*b)*x)/(((b*c*g^3 - a*d*g^3)*A*B + (b*c*g^3 - a*
d*g^3)*B^2)*x^3 + (b*c*f^3 - a*d*f^3)*A*B + (b*c*f^3 - a*d*f^3)*B^2 + 3*((b
*c*f*g^2 - a*d*f*g^2)*A*B + (b*c*f*g^2 - a*d*f*g^2)*B^2)*x^2 + 3*((b*c*f^2*
g - a*d*f^2*g)*A*B + (b*c*f^2*g - a*d*f^2*g)*B^2)*x + 2*((b*c*g^3 - a*d*g^3
)*B^2*x^3 + 3*(b*c*f*g^2 - a*d*f*g^2)*B^2*x^2 + 3*(b*c*f^2*g - a*d*f^2*g)*B
^2*x + (b*c*f^3 - a*d*f^3)*B^2)*log(b*x + a) - 2*((b*c*g^3 - a*d*g^3)*B^2*x
^3 + 3*(b*c*f*g^2 - a*d*f*g^2)*B^2*x^2 + 3*(b*c*f^2*g - a*d*f^2*g)*B^2*x +
(b*c*f^3 - a*d*f^3)*B^2)*log(d*x + c)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)^2/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="fric
as")
```

```
[Out] integral(1/(A^2*g^2*x^2 + 2*A^2*f*g*x + A^2*f^2 + (B^2*g^2*x^2 + 2*B^2*f*g*
x + B^2*f^2)*log((b^2*x^2 + 2*a*b*x + a^2)*e/(d^2*x^2 + 2*c*d*x + c^2))^2 +
```

```
 2*(A*B*g^2*x^2 + 2*A*B*f*g*x + A*B*f^2)*log((b^2*x^2 + 2*a*b*x + a^2)*e/(d
^2*x^2 + 2*c*d*x + c^2))), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)**2/(A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2,x)
[Out] Timed out
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)^2/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="giac
")
[Out] integrate(1/((g*x + f)^2*(B*log((b*x + a)^2*e/(d*x + c)^2) + A)^2), x)
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{(f+g\,x)^2\,\left(A+B\,\ln\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((f + g*x)^2*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2),x)
[Out] int(1/((f + g*x)^2*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2), x)
```

3.292 $\int \frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$

Optimal. Leaf size=34

$$\text{Int}\left(\frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2},x\right)$$

[Out] Unintegrable(1/(g*x+f)^3/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$$

Verification is not applicable to the result.

[In] Int[1/((f + g*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2),x]

[Out] Defer[Int][1/((f + g*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2), x]

Rubi steps

$$\int \frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx = \int \frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$$

Mathematica [A]
time = 0.35, size = 0, normalized size = 0.00

$$\int \frac{1}{(f+gx)^3\left(A+B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}\,dx$$

Verification is not applicable to the result.

[In] Integrate[1/((f + g*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2),x]

[Out] Integrate[1/((f + g*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2), x]

Maple [A]
time = 0.88, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(gx+f\right)^{3} \left(A+B\ln\left(\frac{e\left(bx+a\right)^{2}}{\left(dx+c\right)^{2}}\right)\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(g*x+f)^3/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

```
[Out] int(1/(g*x+f)^3/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)^3/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="maxi
ma")
```

```
[Out] -1/2*(b*d*x^2 + a*c + (b*c + a*d)*x)/(((b*c*g^3 - a*d*g^3)*A*B + (b*c*g^3 -
 a*d*g^3)*B^2)*x^3 + (b*c*f^3 - a*d*f^3)*A*B + (b*c*f^3 - a*d*f^3)*B^2 + 3*
((b*c*f*g^2 - a*d*f*g^2)*A*B + (b*c*f*g^2 - a*d*f*g^2)*B^2)*x^2 + 3*((b*c*f
^2*g - a*d*f^2*g)*A*B + (b*c*f^2*g - a*d*f^2*g)*B^2)*x + 2*((b*c*g^3 - a*d*
g^3)*B^2*x^3 + 3*(b*c*f*g^2 - a*d*f*g^2)*B^2*x^2 + 3*(b*c*f^2*g - a*d*f^2*g
)*B^2*x + (b*c*f^3 - a*d*f^3)*B^2)*log(b*x + a) - 2*((b*c*g^3 - a*d*g^3)*B^
2*x^3 + 3*(b*c*f*g^2 - a*d*f*g^2)*B^2*x^2 + 3*(b*c*f^2*g - a*d*f^2*g)*B^2*x
 + (b*c*f^3 - a*d*f^3)*B^2)*log(d*x + c)) - integrate(1/2*(b*d*g*x^2 - b*c*
f - (d*f - 3*c*g)*a + 2*(a*d*g - (d*f - c*g)*b)*x)/(((b*c*g^4 - a*d*g^4)*A*
B + (b*c*g^4 - a*d*g^4)*B^2)*x^4 + 4*((b*c*f*g^3 - a*d*f*g^3)*A*B + (b*c*f*
g^3 - a*d*f*g^3)*B^2)*x^3 + (b*c*f^4 - a*d*f^4)*A*B + (b*c*f^4 - a*d*f^4)*B
^2 + 6*((b*c*f^2*g^2 - a*d*f^2*g^2)*A*B + (b*c*f^2*g^2 - a*d*f^2*g^2)*B^2)*
x^2 + 4*((b*c*f^3*g - a*d*f^3*g)*A*B + (b*c*f^3*g - a*d*f^3*g)*B^2)*x + 2*(
(b*c*g^4 - a*d*g^4)*B^2*x^4 + 4*(b*c*f*g^3 - a*d*f*g^3)*B^2*x^3 + 6*(b*c*f^
2*g^2 - a*d*f^2*g^2)*B^2*x^2 + 4*(b*c*f^3*g - a*d*f^3*g)*B^2*x + (b*c*f^4 -
 a*d*f^4)*B^2)*log(b*x + a) - 2*((b*c*g^4 - a*d*g^4)*B^2*x^4 + 4*(b*c*f*g^3
 - a*d*f*g^3)*B^2*x^3 + 6*(b*c*f^2*g^2 - a*d*f^2*g^2)*B^2*x^2 + 4*(b*c*f^3*
g - a*d*f^3*g)*B^2*x + (b*c*f^4 - a*d*f^4)*B^2)*log(d*x + c)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)^3/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="fric
as")
```

```
[Out] integral(1/(A^2*g^3*x^3 + 3*A^2*f*g^2*x^2 + 3*A^2*f^2*g*x + A^2*f^3 + (B^2*
g^3*x^3 + 3*B^2*f*g^2*x^2 + 3*B^2*f^2*g*x + B^2*f^3)*log((b^2*x^2 + 2*a*b*x
 + a^2)*e/(d^2*x^2 + 2*c*d*x + c^2))^2 + 2*(A*B*g^3*x^3 + 3*A*B*f*g^2*x^2 +
 3*A*B*f^2*g*x + A*B*f^3)*log((b^2*x^2 + 2*a*b*x + a^2)*e/(d^2*x^2 + 2*c*d*
x + c^2))), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)**3/(A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)^3/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="giac
")
```

```
[Out] integrate(1/((g*x + f)^3*(B*log((b*x + a)^2*e/(d*x + c)^2) + A)^2), x)
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{{\left(f+g\,x\right)}^3\,{\left(A+B\,\ln\left(\frac{e\,{\left(a+b\,x\right)}^2}{{\left(c+d\,x\right)}^2}\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((f + g*x)^3*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2),x)
```

```
[Out] int(1/((f + g*x)^3*(A + B*log((e*(a + b*x)^2)/(c + d*x)^2))^2), x)
```

3.293 $\int (g+hx)^4 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right) dx$

Optimal. Leaf size=365

$$\frac{B(bc-ad)h(a^3d^3h^3-a^2bd^2h^2(5dg-ch)+ab^2dh(10d^2g^2-5cdgh+c^2h^2)-b^3(10d^3g^3-10cd^2g^2h+5c^2dgh}{5b^4d^4}$$

```
[Out] 1/5*B*(-a*d+b*c)*h*(a^3*d^3*h^3-a^2*b*d^2*h^2*(-c*h+5*d*g)+a*b^2*d*h*(c^2*h
^2-5*c*d*g*h+10*d^2*g^2)-b^3*(-c^3*h^3+5*c^2*d*g*h^2-10*c*d^2*g^2*h+10*d^3*
g^3))*n*x/b^4/d^4-1/10*B*(-a*d+b*c)*h^2*(a^2*d^2*h^2-a*b*d*h*(-c*h+5*d*g)+b
^2*(c^2*h^2-5*c*d*g*h+10*d^2*g^2))*n*x^2/b^3/d^3-1/15*B*(-a*d+b*c)*h^3*(-a*
d*h-b*c*h+5*b*d*g)*n*x^3/b^2/d^2-1/20*B*(-a*d+b*c)*h^4*n*x^4/b/d-1/5*B*(-a*
h+b*g)^5*n*ln(b*x+a)/b^5/h+1/5*B*(-c*h+d*g)^5*n*ln(d*x+c)/d^5/h+1/5*(h*x+g)
^5*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/h
```

Rubi [A]

time = 0.36, antiderivative size = 365, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {2548, 84}

$$-\frac{Bh^2nx^2(bc-ad)\left(a^2d^2h^2-abdh(5dg-ch)+b^2(c^2h^2-5cdgh+10d^2g^2)\right)}{10b^3d^3}+\frac{Bhnx(bc-ad)\left(a^3d^3h^3-a^2bd^2h^2(5dg-ch)+ab^2dh(c^2h^2-5cdgh+10d^2g^2)-(b^3(-c^3h^3+5c^2dgh^2-10cd^2g^2h+10d^3g^3))\right)}{5b^4d^4}+\frac{(g+hx)^5\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{5h}-\frac{Bn(bg-ah)^5\log(a+bx)}{5b^5h}-\frac{Bh^3nx^3(bc-ad)(-adh-bch+5bdg)}{15b^2d^2}-\frac{Bh^4nx^4(bc-ad)}{20bd}+\frac{Bn(dg-ch)^5\log(c+dx)}{5d^5h}$$

Antiderivative was successfully verified.

```
[In] Int[(g + h*x)^4*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]),x]
```

```
[Out] (B*(b*c - a*d)*h*(a^3*d^3*h^3 - a^2*b*d^2*h^2*(5*d*g - c*h) + a*b^2*d*h*(10
*d^2*g^2 - 5*c*d*g*h + c^2*h^2) - b^3*(10*d^3*g^3 - 10*c*d^2*g^2*h + 5*c^2*
d*g*h^2 - c^3*h^3))*n*x)/(5*b^4*d^4) - (B*(b*c - a*d)*h^2*(a^2*d^2*h^2 - a*
b*d*h*(5*d*g - c*h) + b^2*(10*d^2*g^2 - 5*c*d*g*h + c^2*h^2))*n*x^2)/(10*b^
3*d^3) - (B*(b*c - a*d)*h^3*(5*b*d*g - b*c*h - a*d*h)*n*x^3)/(15*b^2*d^2) -
 (B*(b*c - a*d)*h^4*n*x^4)/(20*b*d) - (B*(b*g - a*h)^5*n*Log[a + b*x])/(5*b
^5*h) + (B*(d*g - c*h)^5*n*Log[c + d*x])/(5*d^5*h) + ((g + h*x)^5*(A + B*Lo
g[(e*(a + b*x)^n)/(c + d*x)^n]))/(5*h)
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 2548

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Dist[B*n*((b*c
- a*d)/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /;
 FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c -
```

```
a*d, 0] && NeQ[m, -1] && !(EqQ[m, -2] && IntegerQ[n])
```

Rubi steps

$$\int (g+hx)^4 \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right)\,dx = \int \left(A(g+hx)^4+B(g+hx)^4\log \left(e(a+bx)^n(c+dx\right.\right.$$

$$= \frac{A(g+hx)^5}{5h}+B\int (g+hx)^4\log \left(e(a+bx)^n(c+dx\right.$$

$$= \frac{A(g+hx)^5}{5h}+\frac{B(g+hx)^5\log \left(e(a+bx)^n(c+dx)^{-n}\right.}{5h}$$

$$= \frac{A(g+hx)^5}{5h}+\frac{B(g+hx)^5\log \left(e(a+bx)^n(c+dx)^{-n}\right.}{5h}$$

$$= \frac{B(bc-ad)h(a^3d^3h^3-a^2bd^2h^2(5dg-ch)+ab^2dh(1}{}$$

Mathematica [**A**]
time = 0.33, size = 452, normalized size = 1.24

$$\frac{12aBd^5(5b^4g^4-10ab^3g^3h+10a^2b^2g^2h^2-5a^3bgh^3+a^4h^4)\,n\log(a+bx)-12b^5Bc(5d^4g^4-10cd^3g^3h+10c^2d^2g^2h^2-5c^3dgh^3+c^4h^4)\,n\log(c+dx)+bdx(12Ab^4d^4(5g^4+10g^3hx+10g^2h^2x^2+5gh^3x^3+h^4x^4)+B(bc-ad)hn(12a^3d^3h^3-6a^2bd^2h^2(10dg-2ch+dhx)+2ab^2dh(6c^2h^2-3cdh(10g+hx)+d^2(60g^2+15ghx+2h^2x^2))-b^3(-12c^3h^3+6c^2dh^2(10g+hx)-2cd^2h(60g^2+15ghx+2h^2x^2)+d^3(120g^3+60g^2hx+20gh^2x^2+3h^3x^3))))+12b^4Bd^4(5g^4+10g^3hx+10g^2h^2x^2+5gh^3x^3+h^4x^4)\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{60b^5d^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g + h*x)^4*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]),x]
```

```
[Out] (12*a*B*d^5*(5*b^4*g^4 - 10*a*b^3*g^3*h + 10*a^2*b^2*g^2*h^2 - 5*a^3*b*g*h^
3 + a^4*h^4)*n*Log[a + b*x] - 12*b^5*B*c*(5*d^4*g^4 - 10*c*d^3*g^3*h + 10*c
^2*d^2*g^2*h^2 - 5*c^3*d*g*h^3 + c^4*h^4)*n*Log[c + d*x] + b*d*x*(12*A*b^4*
d^4*(5*g^4 + 10*g^3*h*x + 10*g^2*h^2*x^2 + 5*g*h^3*x^3 + h^4*x^4) + B*(b*c
- a*d)*h*n*(12*a^3*d^3*h^3 - 6*a^2*b*d^2*h^2*(10*d*g - 2*c*h + d*h*x) + 2*a
*b^2*d*h*(6*c^2*h^2 - 3*c*d*h*(10*g + h*x) + d^2*(60*g^2 + 15*g*h*x + 2*h^2
*x^2)) - b^3*(-12*c^3*h^3 + 6*c^2*d*h^2*(10*g + h*x) - 2*c*d^2*h*(60*g^2 +
15*g*h*x + 2*h^2*x^2) + d^3*(120*g^3 + 60*g^2*h*x + 20*g*h^2*x^2 + 3*h^3*x^
3))) + 12*b^4*B*d^4*(5*g^4 + 10*g^3*h*x + 10*g^2*h^2*x^2 + 5*g*h^3*x^3 + h^
4*x^4)*Log[(e*(a + b*x)^n)/(c + d*x)^n]))/(60*b^5*d^5)
```

Maple [**C**] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.64, size = 2612, normalized size = 7.16

method	result	size
risch	Expression too large to display	2612

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)^4*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n))),x,method=_RETURNVERBOSE)

[Out] -1/15/b^2*h^4*B*a^2*n*x^3+1/15*h^4/d^2*B*c^2*n*x^3+1/10/b^3*h^4*B*a^3*n*x^2
-1/10*h^4/d^3*B*c^3*n*x^2-1/5/b^4*h^4*B*a^4*n*x+1/5*h^4/d^4*B*c^4*n*x-1/b^4
*h^3*B*ln(-b*x-a)*a^4*g*n+1/5*A*h^4*x^5+x*A*g^4-1/2*I*h^3*B*Pi*g*x^4*csgn(I
*e)*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)-1/2*I*h^3
*B*Pi*g*x^4*csgn(I*(b*x+a)^n)*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)
^n))-I*h^2*B*Pi*g^2*x^3*csgn(I*(b*x+a)^n)*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a
)^n/((d*x+c)^n))-I*h*B*Pi*g^3*x^2*csgn(I*e)*csgn(I*(b*x+a)^n/((d*x+c)^n))*c
sgn(I*e/((d*x+c)^n)*(b*x+a)^n)-I*h^2*B*Pi*g^2*x^3*csgn(I*e)*csgn(I*(b*x+a)^
n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)+h^3*A*g*x^4+2*h^2*A*g^2*x^3+
2*h*A*g^3*x^2+1/b*B*ln(-b*x-a)*a*g^4*n-1/d*B*ln(d*x+c)*c*g^4*n+B*g^4*x*ln((
b*x+a)^n)+B*ln(e)*g^4*x+1/5/h*B*g^5*ln((b*x+a)^n)+1/5*h^4*B*x^5*ln((b*x+a)^
n)+1/5*h^4*B*ln(e)*x^5-h^3/d^3*B*c^3*g*n*x+1/5/b^5*h^4*B*ln(-b*x-a)*a^5*n-1
/5*h^4/d^5*B*ln(d*x+c)*c^5*n-1/2*I*B*Pi*g^4*x*csgn(I*(b*x+a)^n/((d*x+c)^n))
^3-1/2*I*B*Pi*g^4*x*csgn(I*e)*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+
c)^n)*(b*x+a)^n)+2*h*B*ln(e)*g^3*x^2+2/b^3*h^2*B*ln(-b*x-a)*a^3*g^2*n-2/b^2
*h*B*ln(-b*x-a)*a^2*g^3*n+h^3/d^4*B*ln(d*x+c)*c^4*g*n-2*h^2/d^3*B*ln(d*x+c)
*c^3*g^2*n+2*h/d^2*B*ln(d*x+c)*c^2*g^3*n+1/10*I*h^4*B*Pi*x^5*csgn(I*e)*csgn
(I*e/((d*x+c)^n)*(b*x+a)^n)^2+1/10*I*h^4*B*Pi*x^5*csgn(I*(b*x+a)^n)*csgn(I*
(b*x+a)^n/((d*x+c)^n))^2+1/10*I*h^4*B*Pi*x^5*csgn(I/((d*x+c)^n))*csgn(I*(b*
x+a)^n/((d*x+c)^n))^2-I*h^2*B*Pi*g^2*x^3*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^3-
I*h*B*Pi*g^3*x^2*csgn(I*(b*x+a)^n/((d*x+c)^n))^3-I*h*B*Pi*g^3*x^2*csgn(I*e/
((d*x+c)^n)*(b*x+a)^n)^3+1/10*I*h^4*B*Pi*x^5*csgn(I*(b*x+a)^n/((d*x+c)^n))*
csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2-1/2*I*h^3*B*Pi*g*x^4*csgn(I*(b*x+a)^n/((d
*x+c)^n))^3-1/2*I*h^3*B*Pi*g*x^4*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^3-I*h^2*B*
Pi*g^2*x^3*csgn(I*(b*x+a)^n/((d*x+c)^n))^3+1/2*I*B*Pi*g^4*x*csgn(I/((d*x+c)
^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+1/2*I*B*Pi*g^4*x*csgn(I*e)*csgn(I*e/((
d*x+c)^n)*(b*x+a)^n)^2+1/2*I*B*Pi*g^4*x*csgn(I*(b*x+a)^n)*csgn(I*(b*x+a)^n/
((d*x+c)^n))^2+1/2*I*B*Pi*g^4*x*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*
x+c)^n)*(b*x+a)^n)^2+2*h^2/d^2*B*c^2*g^2*n*x-2*h/d*B*c*g^3*n*x+1/3/b*h^3*B*
a*g*n*x^3-1/3*h^3/d*B*c*g*n*x^3-1/2/b^2*h^3*B*a^2*g*n*x^2+1/b*h^2*B*a*g^2*n
*x^2+1/2*h^3/d^2*B*c^2*g*n*x^2-h^2/d*B*c*g^2*n*x^2+1/b^3*h^3*B*a^3*g*n*x-2/
b^2*h^2*B*a^2*g^2*n*x+2/b*h*B*a*g^3*n*x+I*h*B*Pi*g^3*x^2*csgn(I/((d*x+c)^n)
)*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+1/20/b*h^4*B*a*n*x^4-1/20*h^4/d*B*c*n*x^4
-1/5*(h*x+g)^5*B/h*ln((d*x+c)^n)+h^3*B*ln(e)*g*x^4+2*h^2*B*ln(e)*g^2*x^3-1/
5/h*B*ln(-b*x-a)*g^5*n+1/5/h*B*ln(d*x+c)*g^5*n+h^3*B*g*x^4*ln((b*x+a)^n)+2*
h^2*B*g^2*x^3*ln((b*x+a)^n)+2*h*B*g^3*x^2*ln((b*x+a)^n)+I*h^2*B*Pi*g^2*x^3*
csgn(I*(b*x+a)^n)*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+I*h*B*Pi*g^3*x^2*csgn(I*(
b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2+I*h^2*B*Pi*g^2*x^3*
csgn(I*e)*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2+I*h^2*B*Pi*g^2*x^3*csgn(I/((d*x
+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+I*h*B*Pi*g^3*x^2*csgn(I*e)*csgn(I*e
/((d*x+c)^n)*(b*x+a)^n)^2+I*h*B*Pi*g^3*x^2*csgn(I*(b*x+a)^n)*csgn(I*(b*x+a)
^n/((d*x+c)^n))^2+I*h^2*B*Pi*g^2*x^3*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e
/((d*x+c)^n)*(b*x+a)^n)^2+1/2*I*h^3*B*Pi*g*x^4*csgn(I*(b*x+a)^n/((d*x+c)^n)
```

```
)*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2-1/10*I*h^4*B*Pi*x^5*csgn(I*e)*csgn(I*(b
*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)-1/10*I*h^4*B*Pi*x^5*cs
gn(I*(b*x+a)^n)*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))+1/2*I*h^3
*B*Pi*g*x^4*csgn(I*e)*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2+1/2*I*h^3*B*Pi*g*x^
4*csgn(I*(b*x+a)^n)*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+1/2*I*h^3*B*Pi*g*x^4*cs
gn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))^2-1/2*I*B*Pi*g^4*x*csgn(I*(
b*x+a)^n)*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))-I*h*B*Pi*g^3*x^
2*csgn(I*(b*x+a)^n)*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))-1/2*I
*B*Pi*g^4*x*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^3-1/10*I*h^4*B*Pi*x^5*csgn(I*(b
*x+a)^n/((d*x+c)^n))^3-1/10*I*h^4*B*Pi*x^5*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^
3
```

Maxima [A]
time = 0.31, size = 681, normalized size = 1.87

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^4*(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="maxima
")
```

```
[Out] 1/5*B*h^4*x^5*log((b*x + a)^n*e/(d*x + c)^n) + 1/5*A*h^4*x^5 + B*g*h^3*x^4*
log((b*x + a)^n*e/(d*x + c)^n) + A*g*h^3*x^4 + 2*B*g^2*h^2*x^3*log((b*x + a
)^n*e/(d*x + c)^n) + 2*A*g^2*h^2*x^3 + 2*B*g^3*h*x^2*log((b*x + a)^n*e/(d*x
 + c)^n) + 2*A*g^3*h*x^2 + (a*n*e*log(b*x + a)/b - c*n*e*log(d*x + c)/d)*B*
g^4*e^(-1) - 2*(a^2*n*e*log(b*x + a)/b^2 - c^2*n*e*log(d*x + c)/d^2 + (b*c*
n - a*d*n)*x*e/(b*d))*B*g^3*h*e^(-1) + (2*a^3*n*e*log(b*x + a)/b^3 - 2*c^3*
n*e*log(d*x + c)/d^3 - ((b^2*c*d*n - a*b*d^2*n)*x^2*e - 2*(b^2*c^2*n - a^2*
d^2*n)*x*e)/(b^2*d^2))*B*g^2*h^2*e^(-1) - 1/6*(6*a^4*n*e*log(b*x + a)/b^4 -
 6*c^4*n*e*log(d*x + c)/d^4 + (2*(b^3*c*d^2*n - a*b^2*d^3*n)*x^3*e - 3*(b^3
*c^2*d*n - a^2*b*d^3*n)*x^2*e + 6*(b^3*c^3*n - a^3*d^3*n)*x*e)/(b^3*d^3))*B
*g*h^3*e^(-1) + 1/60*(12*a^5*n*e*log(b*x + a)/b^5 - 12*c^5*n*e*log(d*x + c)
/d^5 - (3*(b^4*c*d^3*n - a*b^3*d^4*n)*x^4*e - 4*(b^4*c^2*d^2*n - a^2*b^2*d^
4*n)*x^3*e + 6*(b^4*c^3*d*n - a^3*b*d^4*n)*x^2*e - 12*(b^4*c^4*n - a^4*d^4*
n)*x*e)/(b^4*d^4))*B*h^4*e^(-1) + B*g^4*x*log((b*x + a)^n*e/(d*x + c)^n) +
A*g^4*x
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 733 vs. $2(352) = 704$.
time = 0.37, size = 733, normalized size = 2.01

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^4*(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="fricas
")
```

```
[Out] 1/60*(12*(A + B)*b^5*d^5*h^4*x^5 + 3*(20*(A + B)*b^5*d^5*g*h^3 - (B*b^5*c*d
^4 - B*a*b^4*d^5)*h^4*n)*x^4 + 4*(30*(A + B)*b^5*d^5*g^2*h^2 - (5*(B*b^5*c*
d^4 - B*a*b^4*d^5)*g*h^3 - (B*b^5*c^2*d^3 - B*a^2*b^3*d^5)*h^4)*n)*x^3 + 6*
(20*(A + B)*b^5*d^5*g^3*h - (10*(B*b^5*c*d^4 - B*a*b^4*d^5)*g^2*h^2 - 5*(B*
b^5*c^2*d^3 - B*a^2*b^3*d^5)*g*h^3 + (B*b^5*c^3*d^2 - B*a^3*b^2*d^5)*h^4)*n
)*x^2 + 12*(5*(A + B)*b^5*d^5*g^4 - (10*(B*b^5*c*d^4 - B*a*b^4*d^5)*g^3*h -
 10*(B*b^5*c^2*d^3 - B*a^2*b^3*d^5)*g^2*h^2 + 5*(B*b^5*c^3*d^2 - B*a^3*b^2*
d^5)*g*h^3 - (B*b^5*c^4*d - B*a^4*b*d^5)*h^4)*n)*x + 12*(B*b^5*d^5*h^4*n*x^
5 + 5*B*b^5*d^5*g*h^3*n*x^4 + 10*B*b^5*d^5*g^2*h^2*n*x^3 + 10*B*b^5*d^5*g^3
*h*n*x^2 + 5*B*b^5*d^5*g^4*n*x + (5*B*a*b^4*d^5*g^4 - 10*B*a^2*b^3*d^5*g^3*
h + 10*B*a^3*b^2*d^5*g^2*h^2 - 5*B*a^4*b*d^5*g*h^3 + B*a^5*d^5*h^4)*n)*log(
b*x + a) - 12*(B*b^5*d^5*h^4*n*x^5 + 5*B*b^5*d^5*g*h^3*n*x^4 + 10*B*b^5*d^5
*g^2*h^2*n*x^3 + 10*B*b^5*d^5*g^3*h*n*x^2 + 5*B*b^5*d^5*g^4*n*x + (5*B*b^5*
c*d^4*g^4 - 10*B*b^5*c^2*d^3*g^3*h + 10*B*b^5*c^3*d^2*g^2*h^2 - 5*B*b^5*c^4
*d*g*h^3 + B*b^5*c^5*h^4)*n)*log(d*x + c))/(b^5*d^5)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)**4*(A+B*ln(e*(b*x+a)**n/((d*x+c)**n))),x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^4*(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**B**]
time = 5.13, size = 1434, normalized size = 3.93

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g + h*x)^4*(A + B*log((e*(a + b*x)^n)/(c + d*x)^n)),x)
```

```
[Out] x*((5*A*b*d*g^4 + 20*A*a*d*g^3*h + 20*A*b*c*g^3*h + 30*A*a*c*g^2*h^2 + 10*B
*a*d*g^3*h*n - 10*B*b*c*g^3*h*n)/(5*b*d) - ((5*a*d + 5*b*c)*((20*A*a*c*g*h^
```

```
3 + 20*A*b*d*g^3*h + 30*A*a*d*g^2*h^2 + 30*A*b*c*g^2*h^2 + 10*B*a*d*g^2*h^2
*n - 10*B*b*c*g^2*h^2*n)/(5*b*d) + ((5*a*d + 5*b*c)*((((5*A*a*d*h^4 + 5*A*b
*c*h^4 + 20*A*b*d*g*h^3 + B*a*d*h^4*n - B*b*c*h^4*n)/(5*b*d) - (A*h^4*(5*a*
d + 5*b*c))/(5*b*d))*(5*a*d + 5*b*c))/(5*b*d) - (5*A*a*c*h^4 + 20*A*a*d*g*h
^3 + 20*A*b*c*g*h^3 + 30*A*b*d*g^2*h^2 + 5*B*a*d*g*h^3*n - 5*B*b*c*g*h^3*n)
/(5*b*d) + (A*a*c*h^4)/(b*d)))/(5*b*d) - (a*c*((5*A*a*d*h^4 + 5*A*b*c*h^4 +
 20*A*b*d*g*h^3 + B*a*d*h^4*n - B*b*c*h^4*n)/(5*b*d) - (A*h^4*(5*a*d + 5*b*
c))/(5*b*d)))/(b*d)))/(5*b*d) + (a*c*((((5*A*a*d*h^4 + 5*A*b*c*h^4 + 20*A*b
*d*g*h^3 + B*a*d*h^4*n - B*b*c*h^4*n)/(5*b*d) - (A*h^4*(5*a*d + 5*b*c))/(5*
b*d))*(5*a*d + 5*b*c))/(5*b*d) - (5*A*a*c*h^4 + 20*A*a*d*g*h^3 + 20*A*b*c*g
*h^3 + 30*A*b*d*g^2*h^2 + 5*B*a*d*g*h^3*n - 5*B*b*c*g*h^3*n)/(5*b*d) + (A*a
*c*h^4)/(b*d)))/(b*d)) + log((e*(a + b*x)^n)/(c + d*x)^n)*((B*h^4*x^5)/5 +
B*g^4*x + 2*B*g^2*h^2*x^3 + 2*B*g^3*h*x^2 + B*g*h^3*x^4) + x^4*((5*A*a*d*h^
4 + 5*A*b*c*h^4 + 20*A*b*d*g*h^3 + B*a*d*h^4*n - B*b*c*h^4*n)/(20*b*d) - (A
*h^4*(5*a*d + 5*b*c))/(20*b*d)) - x^3*((((5*A*a*d*h^4 + 5*A*b*c*h^4 + 20*A*
b*d*g*h^3 + B*a*d*h^4*n - B*b*c*h^4*n)/(5*b*d) - (A*h^4*(5*a*d + 5*b*c))/(5
*b*d))*(5*a*d + 5*b*c))/(15*b*d) - (5*A*a*c*h^4 + 20*A*a*d*g*h^3 + 20*A*b*c
*g*h^3 + 30*A*b*d*g^2*h^2 + 5*B*a*d*g*h^3*n - 5*B*b*c*g*h^3*n)/(15*b*d) + (
A*a*c*h^4)/(3*b*d)) + x^2*((20*A*a*c*g*h^3 + 20*A*b*d*g^3*h + 30*A*a*d*g^2*
h^2 + 30*A*b*c*g^2*h^2 + 10*B*a*d*g^2*h^2*n - 10*B*b*c*g^2*h^2*n)/(10*b*d)
+ ((5*a*d + 5*b*c)*((((5*A*a*d*h^4 + 5*A*b*c*h^4 + 20*A*b*d*g*h^3 + B*a*d*h
^4*n - B*b*c*h^4*n)/(5*b*d) - (A*h^4*(5*a*d + 5*b*c))/(5*b*d))*(5*a*d + 5*b
*c))/(5*b*d) - (5*A*a*c*h^4 + 20*A*a*d*g*h^3 + 20*A*b*c*g*h^3 + 30*A*b*d*g^
2*h^2 + 5*B*a*d*g*h^3*n - 5*B*b*c*g*h^3*n)/(5*b*d) + (A*a*c*h^4)/(b*d)))/(1
0*b*d) - (a*c*((5*A*a*d*h^4 + 5*A*b*c*h^4 + 20*A*b*d*g*h^3 + B*a*d*h^4*n -
B*b*c*h^4*n)/(5*b*d) - (A*h^4*(5*a*d + 5*b*c))/(5*b*d)))/(2*b*d)) + (A*h^4*
x^5)/5 + (log(a + b*x)*((B*a^5*h^4*n)/5 + B*a*b^4*g^4*n + 2*B*a^3*b^2*g^2*h
^2*n - B*a^4*b*g*h^3*n - 2*B*a^2*b^3*g^3*h*n))/b^5 - (log(c + d*x)*(B*c^5*h
^4*n + 5*B*c*d^4*g^4*n + 10*B*c^3*d^2*g^2*h^2*n - 5*B*c^4*d*g*h^3*n - 10*B*
c^2*d^3*g^3*h*n))/(5*d^5)
```

3.294 $\int (g+hx)^3 \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right) \, dx$

Optimal. Leaf size=236

$$-\frac{B(bc-ad)h(a^2d^2h^2-abdh(4dg-ch)+b^2(6d^2g^2-4cdgh+c^2h^2))\,nx}{4b^3d^3}-\frac{B(bc-ad)h^2(4bdg-bch-adh)}{8b^2d^2}$$

```
[Out] -1/4*B*(-a*d+b*c)*h*(a^2*d^2*h^2-a*b*d*h*(-c*h+4*d*g)+b^2*(c^2*h^2-4*c*d*g*
h+6*d^2*g^2))*n*x/b^3/d^3-1/8*B*(-a*d+b*c)*h^2*(-a*d*h-b*c*h+4*b*d*g)*n*x^2
/b^2/d^2-1/12*B*(-a*d+b*c)*h^3*n*x^3/b/d-1/4*B*(-a*h+b*g)^4*n*ln(b*x+a)/b^4
/h+1/4*B*(-c*h+d*g)^4*n*ln(d*x+c)/d^4/h+1/4*(h*x+g)^4*(A+B*ln(e*(b*x+a)^n/(
(d*x+c)^n)))/h
```

Rubi [A]

time = 0.22, antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {2548, 84}

$$-\frac{Bhnx(bc-ad)\left(a^2d^2h^2-abdh(4dg-ch)+b^2\left(c^2h^2-4cdgh+6d^2g^2\right)\right)}{4b^3d^3}+\frac{(g+hx)^4\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{4h}-\frac{Bn(bg-ah)^4\log(a+bx)}{4b^4h}-\frac{Bh^2nx^2(bc-ad)(-adh-bch+4bdg)}{8b^2d^2}-\frac{Bh^3nx^3(bc-ad)}{12bd}+\frac{Bn(dg-ch)^4\log(c+dx)}{4d^4h}$$

Antiderivative was successfully verified.

```
[In] Int[(g + h*x)^3*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]),x]
```

```
[Out] -1/4*(B*(b*c - a*d)*h*(a^2*d^2*h^2 - a*b*d*h*(4*d*g - c*h) + b^2*(6*d^2*g^2
 - 4*c*d*g*h + c^2*h^2))*n*x)/(b^3*d^3) - (B*(b*c - a*d)*h^2*(4*b*d*g - b*c
*h - a*d*h)*n*x^2)/(8*b^2*d^2) - (B*(b*c - a*d)*h^3*n*x^3)/(12*b*d) - (B*(b
*g - a*h)^4*n*Log[a + b*x])/(4*b^4*h) + (B*(d*g - c*h)^4*n*Log[c + d*x])/(4
*d^4*h) + ((g + h*x)^4*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]))/(4*h)
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 2548

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Dist[B*n*((b*c
- a*d)/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /;
 FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c -
 a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Rubi steps

$$\int (g+hx)^3 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\,dx = \int \left(A(g+hx)^3+B(g+hx)^3\log\left(e(a+bx)^n(c+dx\right.\right.$$
$$= \frac{A(g+hx)^4}{4h}+B\int (g+hx)^3\log\left(e(a+bx)^n(c+dx\right.$$
$$= \frac{A(g+hx)^4}{4h}+\frac{B(g+hx)^4\log\left(e(a+bx)^n(c+dx)^{-n}\right.}{4h}$$
$$= \frac{A(g+hx)^4}{4h}+\frac{B(g+hx)^4\log\left(e(a+bx)^n(c+dx)^{-n}\right.}{4h}$$
$$= -\frac{B(bc-ad)h(a^2d^2h^2-abdh(4dg-ch)+b^2(6d^2g^2}{4b^3d^3}$$

Mathematica [A]
time = 0.22, size = 303, normalized size = 1.28

$$\frac{-6aBd^4(-4b^3g^3+6ab^2g^2h-4a^2bgh^2+a^3h^3)\,n\log(a+bx)+6b^4Bc(-4d^3g^3+6cd^2g^2h-4c^2dgh^2+c^3h^3)\,n\log(c+dx)+bdx(6Ab^3d^3(4g^3+6g^2hx+4gh^2x^2+h^3x^3)-B(bc-ad)hn(6a^2d^2h^2-3abdh(8dg-2ch+dhx)+b^2(6c^2h^2-3cdh(8g+hx)+2d^2(18g^2+6ghx+h^2x^2))))+6b^3Bd^3(4g^3+6g^2hx+4gh^2x^2+h^3x^3)\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{24b^4d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g + h*x)^3*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]),x]
```

```
[Out] (-6*a*B*d^4*(-4*b^3*g^3 + 6*a*b^2*g^2*h - 4*a^2*b*g*h^2 + a^3*h^3)*n*Log[a
+ b*x] + 6*b^4*B*c*(-4*d^3*g^3 + 6*c*d^2*g^2*h - 4*c^2*d*g*h^2 + c^3*h^3)*n
*Log[c + d*x] + b*d*x*(6*A*b^3*d^3*(4*g^3 + 6*g^2*h*x + 4*g*h^2*x^2 + h^3*x
^3) - B*(b*c - a*d)*h*n*(6*a^2*d^2*h^2 - 3*a*b*d*h*(8*d*g - 2*c*h + d*h*x)
+ b^2*(6*c^2*h^2 - 3*c*d*h*(8*g + h*x) + 2*d^2*(18*g^2 + 6*g*h*x + h^2*x^2)
)) + 6*b^3*B*d^3*(4*g^3 + 6*g^2*h*x + 4*g*h^2*x^2 + h^3*x^3)*Log[(e*(a + b*
x)^n)/(c + d*x)^n]))/(24*b^4*d^4)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.58, size = 2000, normalized size = 8.47

method	result	size
risch	Expression too large to display	2000

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)^3*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n))),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*A*h^3*x^4+x*A*g^3+h^2*B*g*x^3*ln((b*x+a)^n)+3/2*h*B*g^2*x^2*ln((b*x+a)^
n)+h^2*B*ln(e)*g*x^3+3/2*h*B*ln(e)*g^2*x^2-1/4/h*B*ln(b*x+a)*g^4*n+1/4/h*B*
ln(-d*x-c)*g^4*n-1/2*I*B*Pi*g^3*x*csgn(I*e)*csgn(I*(b*x+a)^n/((d*x+c)^n))*c
sgn(I*e/((d*x+c)^n)*(b*x+a)^n)-1/8*I*h^3*B*Pi*x^4*csgn(I*e)*csgn(I*(b*x+a)^
```

```
n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)+3/4*I*h*B*Pi*g^2*x^2*csgn(I*
e)*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2+3/4*I*h*B*Pi*g^2*x^2*csgn(I*(b*x+a)^n/
((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2+3/4*I*h*B*Pi*g^2*x^2*csgn(I*
(b*x+a)^n)*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+3/4*I*h*B*Pi*g^2*x^2*csgn(I/((d*
x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))^2-1/8*I*h^3*B*Pi*x^4*csgn(I*(b*x+a)^
n)*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))+1/2*I*h^2*B*Pi*g*x^3*c
sgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+1/2*I*h^2*B*Pi*g*x^3*csg
n(I*(b*x+a)^n)*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+1/2*I*h^2*B*Pi*g*x^3*csgn(I*
e)*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2+1/2*I*h^2*B*Pi*g*x^3*csgn(I*(b*x+a)^n/
((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2-1/2*I*B*Pi*g^3*x*csgn(I*(b*x
+a)^n)*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))+1/12/b*h^3*B*a*n*x
^3-1/12*h^3/d*B*c*n*x^3-1/8/b^2*h^3*B*a^2*n*x^2+1/8*h^3/d^2*B*c^2*n*x^2+1/4
/b^3*h^3*B*a^3*n*x-1/4*h^3/d^3*B*c^3*n*x+1/b^3*h^2*B*ln(b*x+a)*a^3*g*n-3/2/
b^2*h*B*ln(b*x+a)*a^2*g^2*n-h^2/d^3*B*ln(-d*x-c)*c^3*g*n+3/2*h/d^2*B*ln(-d*
x-c)*c^2*g^2*n+1/2*I*B*Pi*g^3*x*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*
x+c)^n)*(b*x+a)^n)^2+1/8*I*h^3*B*Pi*x^4*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(
I*e/((d*x+c)^n)*(b*x+a)^n)^2-1/2*I*h^2*B*Pi*g*x^3*csgn(I*(b*x+a)^n/((d*x+c)
^n))^3-3/4*I*h*B*Pi*g^2*x^2*csgn(I*(b*x+a)^n/((d*x+c)^n))^3-3/4*I*h*B*Pi*g^
2*x^2*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^3+1/8*I*h^3*B*Pi*x^4*csgn(I/((d*x+c)^
n))*csgn(I*(b*x+a)^n/((d*x+c)^n))^2-1/2*I*h^2*B*Pi*g*x^3*csgn(I*e/((d*x+c)^
n)*(b*x+a)^n)^3+1/8*I*h^3*B*Pi*x^4*csgn(I*e)*csgn(I*e/((d*x+c)^n)*(b*x+a)^n
)^2+1/8*I*h^3*B*Pi*x^4*csgn(I*(b*x+a)^n)*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+1/
2*I*B*Pi*g^3*x*csgn(I*e)*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2+1/2*I*B*Pi*g^3*x
*csgn(I*(b*x+a)^n)*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+1/2*I*B*Pi*g^3*x*csgn(I/
((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+B*g^3*x*ln((b*x+a)^n)+B*ln(e)*
g^3*x+1/4*h^3*B*ln(e)*x^4+1/4*h^3*B*x^4*ln((b*x+a)^n)+1/4/h*B*g^4*ln((b*x+a
)^n)-1/2*I*h^2*B*Pi*g*x^3*csgn(I*(b*x+a)^n)*csgn(I/((d*x+c)^n))*csgn(I*(b*x
+a)^n/((d*x+c)^n))-3/4*I*h*B*Pi*g^2*x^2*csgn(I*(b*x+a)^n)*csgn(I/((d*x+c)^n
))*csgn(I*(b*x+a)^n/((d*x+c)^n))-1/2*I*h^2*B*Pi*g*x^3*csgn(I*e)*csgn(I*(b*x
+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)-3/4*I*h*B*Pi*g^2*x^2*csg
n(I*e)*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)+1/2/b*
h^2*B*a*g*n*x^2-1/2*h^2/d*B*c*g*n*x^2-1/b^2*h^2*B*a^2*g*n*x+3/2/b*h*B*a*g^2
*n*x+h^2/d^2*B*c^2*g*n*x-3/2*h/d*B*c*g^2*n*x+h^2*A*g*x^3+3/2*h*A*g^2*x^2-1/
4/b^4*h^3*B*ln(b*x+a)*a^4*n+1/4*h^3/d^4*B*ln(-d*x-c)*c^4*n+1/b*B*ln(b*x+a)*
a*g^3*n-1/d*B*ln(-d*x-c)*c*g^3*n-1/2*I*B*Pi*g^3*x*csgn(I*(b*x+a)^n/((d*x+c)
^n))^3-1/2*I*B*Pi*g^3*x*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^3-1/8*I*h^3*B*Pi*x^
4*csgn(I*(b*x+a)^n/((d*x+c)^n))^3-1/8*I*h^3*B*Pi*x^4*csgn(I*e/((d*x+c)^n)*(
b*x+a)^n)^3-1/4*(h*x+g)^4*B/h*ln((d*x+c)^n)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 475 vs. $2(225) = 450$.
time = 0.29, size = 475, normalized size = 2.01

$$\frac{1}{4}Bh^3x^4\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+\frac{1}{4}Ah^3x^4+Bgh^2x^3\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+Agh^2x^3+\frac{3}{2}Bg^2hx^2\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+\frac{3}{2}Ag^2hx^2+\left(\frac{ane\log(bx+a)}{b}-\frac{cne\log(dx+c)}{d}\right)Bg^3e^{(-1)}-\frac{3}{2}\left(\frac{a^2ne\log(bx+a)}{b^2}-\frac{c^2ne\log(dx+c)}{d^2}+\frac{(bcn-adn)xe}{bd}\right)Bg^2he^{(-1)}+\frac{1}{2}\left(\frac{2a^3ne\log(bx+a)}{b^3}-\frac{2c^3ne\log(dx+c)}{d^3}-\frac{(b^2cdn-abd^2n)x^2e-2(b^2c^2n-a^2d^2n)xe}{b^2d^2}\right)Bgh^2e^{(-1)}-\frac{1}{24}\left(\frac{6a^4ne\log(bx+a)}{b^4}-\frac{6c^4ne\log(dx+c)}{d^4}+\frac{2(b^3cd^2n-ab^2d^3n)x^3e-3(b^3c^2dn-a^2bd^3n)x^2e+6(b^3c^3n-a^3d^3n)xe}{b^3d^3}\right)Bh^3e^{(-1)}+Bg^3x\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+Ag^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^3*(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="maxima
")
```

```
[Out] 1/4*B*h^3*x^4*log((b*x + a)^n*e/(d*x + c)^n) + 1/4*A*h^3*x^4 + B*g*h^2*x^3*
log((b*x + a)^n*e/(d*x + c)^n) + A*g*h^2*x^3 + 3/2*B*g^2*h*x^2*log((b*x + a
)^n*e/(d*x + c)^n) + 3/2*A*g^2*h*x^2 + (a*n*e*log(b*x + a)/b - c*n*e*log(d*
x + c)/d)*B*g^3*e^(-1) - 3/2*(a^2*n*e*log(b*x + a)/b^2 - c^2*n*e*log(d*x +
c)/d^2 + (b*c*n - a*d*n)*x*e/(b*d))*B*g^2*h*e^(-1) + 1/2*(2*a^3*n*e*log(b*x
 + a)/b^3 - 2*c^3*n*e*log(d*x + c)/d^3 - ((b^2*c*d*n - a*b*d^2*n)*x^2*e - 2
*(b^2*c^2*n - a^2*d^2*n)*x*e)/(b^2*d^2))*B*g*h^2*e^(-1) - 1/24*(6*a^4*n*e*l
og(b*x + a)/b^4 - 6*c^4*n*e*log(d*x + c)/d^4 + (2*(b^3*c*d^2*n - a*b^2*d^3*
n)*x^3*e - 3*(b^3*c^2*d*n - a^2*b*d^3*n)*x^2*e + 6*(b^3*c^3*n - a^3*d^3*n)*
x*e)/(b^3*d^3))*B*h^3*e^(-1) + B*g^3*x*log((b*x + a)^n*e/(d*x + c)^n) + A*g
^3*x
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 515 vs. $2(225) = 450$.
time = 0.35, size = 515, normalized size = 2.18

$$\frac{6\,(A+B)b^4d^4h^3x^4 + 2\,(12\,(A+B)b^4d^4gh^2 - (Bb^4cd^3 - Bab^3d^4)h^3n)x^3 + 3\,(12\,(A+B)b^4d^4g^2h - (4\,(Bb^4cd^3 - Bab^3d^4)gh^2 - (Bb^4c^2d^2 - Ba^2b^2d^4)h^3)n)x^2 + 6\,(4\,(A+B)b^4d^4g^3 - (6\,(Bb^4cd^3 - Bab^3d^4)g^2h - 4\,(Bb^4c^2d^2 - Ba^2b^2d^4)gh^2 + (Bb^4c^3d - Ba^3bd^4)h^3)n)x + 6\,(Bb^4d^4h^3nx^4 + 4\,Bb^4d^4gh^2nx^3 + 6\,Bb^4d^4g^2hnx^2 + 4\,Bb^4d^4g^3nx + (4\,Bab^3d^4g^3 - 6\,Ba^2b^2d^4g^2h + 4\,Ba^3bd^4gh^2 - Ba^4d^4h^3)n)\log(bx+a) - 6\,(Bb^4d^4h^3nx^4 + 4\,Bb^4d^4gh^2nx^3 + 6\,Bb^4d^4g^2hnx^2 + 4\,Bb^4d^4g^3nx + (4\,Bb^4cd^3g^3 - 6\,Bb^4c^2d^2g^2h + 4\,Bb^4c^3dgh^2 - Bb^4c^4h^3)n)\log(dx+c)}{24\,b^4d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^3*(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="fricas
")
```

```
[Out] 1/24*(6*(A + B)*b^4*d^4*h^3*x^4 + 2*(12*(A + B)*b^4*d^4*g*h^2 - (B*b^4*c*d^
3 - B*a*b^3*d^4)*h^3*n)*x^3 + 3*(12*(A + B)*b^4*d^4*g^2*h - (4*(B*b^4*c*d^3
 - B*a*b^3*d^4)*g*h^2 - (B*b^4*c^2*d^2 - B*a^2*b^2*d^4)*h^3)*n)*x^2 + 6*(4*
(A + B)*b^4*d^4*g^3 - (6*(B*b^4*c*d^3 - B*a*b^3*d^4)*g^2*h - 4*(B*b^4*c^2*d
^2 - B*a^2*b^2*d^4)*g*h^2 + (B*b^4*c^3*d - B*a^3*b*d^4)*h^3)*n)*x + 6*(B*b^
4*d^4*h^3*n*x^4 + 4*B*b^4*d^4*g*h^2*n*x^3 + 6*B*b^4*d^4*g^2*h*n*x^2 + 4*B*b
^4*d^4*g^3*n*x + (4*B*a*b^3*d^4*g^3 - 6*B*a^2*b^2*d^4*g^2*h + 4*B*a^3*b*d^4
*g*h^2 - B*a^4*d^4*h^3)*n)*log(b*x + a) - 6*(B*b^4*d^4*h^3*n*x^4 + 4*B*b^4*
d^4*g*h^2*n*x^3 + 6*B*b^4*d^4*g^2*h*n*x^2 + 4*B*b^4*d^4*g^3*n*x + (4*B*b^4*
c*d^3*g^3 - 6*B*b^4*c^2*d^2*g^2*h + 4*B*b^4*c^3*d*g*h^2 - B*b^4*c^4*h^3)*n)
*log(d*x + c))/(b^4*d^4)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)**3*(A+B*ln(e*(b*x+a)**n/((d*x+c)**n))),x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^3*(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B]
time = 4.76, size = 767, normalized size = 3.25

$$x\left(\frac{4Abdg^3+12Aacgh^2+12Aadg^2h+12Abcg^2h+6Badg^2hn-6Bbcg^2hn}{4bd}+\frac{(4ad+4bc)\left(\frac{\left(\frac{4Aadh^3+4Abch^3+12Abdgh^2+Badh^3n-Bbch^3n}{4bd}-\frac{Ah^3(4ad+4bc)}{4bd}\right)(4ad+4bc)}{4bd}-\frac{4Aach^3+12Aadgh^2+12Abcgh^2+12Abdg^2h+4Badgh^2n-4Bbcgh^2n}{4bd}+\frac{Aach^3}{bd}\right)}{4bd}-\frac{ac\left(\frac{4Aadh^3+4Abch^3+12Abdgh^2+Badh^3n-Bbch^3n}{4bd}-\frac{Ah^3(4ad+4bc)}{4bd}\right)}{bd}\right)+\ln\left(\frac{e(a+bx)^n}{(c+dx)^n}\right)\left(Bg^3x+\frac{3Bg^2hx^2}{2}+Bgh^2x^3+\frac{Bh^3x^4}{4}\right)-x^2\left(\frac{\left(\frac{4Aadh^3+4Abch^3+12Abdgh^2+Badh^3n-Bbch^3n}{4bd}-\frac{Ah^3(4ad+4bc)}{4bd}\right)(4ad+4bc)}{8bd}-\frac{4Aach^3+12Aadgh^2+12Abcgh^2+12Abdg^2h+4Badgh^2n-4Bbcgh^2n}{8bd}+\frac{Aach^3}{2bd}\right)+x^3\left(\frac{4Aadh^3+4Abch^3+12Abdgh^2+Badh^3n-Bbch^3n}{12bd}-\frac{Ah^3(4ad+4bc)}{12bd}\right)+\frac{Ah^3x^4}{4}-\frac{\ln(a+bx)\left(Bna^4h^3-4Bna^3bgh^2+6Bna^2b^2g^2h-4Bnab^3g^3\right)}{4b^4}+\frac{\ln(c+dx)\left(Bnc^4h^3-4Bnc^3dgh^2+6Bnc^2d^2g^2h-4Bncd^3g^3\right)}{4d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g + h*x)^3*(A + B*log((e*(a + b*x)^n)/(c + d*x)^n)),x)
```

```
[Out] x*((4*A*b*d*g^3 + 12*A*a*c*g*h^2 + 12*A*a*d*g^2*h + 12*A*b*c*g^2*h + 6*B*a*
d*g^2*h*n - 6*B*b*c*g^2*h*n)/(4*b*d) + ((4*a*d + 4*b*c)*((((4*A*a*d*h^3 + 4
*A*b*c*h^3 + 12*A*b*d*g*h^2 + B*a*d*h^3*n - B*b*c*h^3*n)/(4*b*d) - (A*h^3*(
4*a*d + 4*b*c))/(4*b*d))*(4*a*d + 4*b*c))/(4*b*d) - (4*A*a*c*h^3 + 12*A*a*d
*g*h^2 + 12*A*b*c*g*h^2 + 12*A*b*d*g^2*h + 4*B*a*d*g*h^2*n - 4*B*b*c*g*h^2*
n)/(4*b*d) + (A*a*c*h^3)/(b*d)))/(4*b*d) - (a*c*((4*A*a*d*h^3 + 4*A*b*c*h^3
 + 12*A*b*d*g*h^2 + B*a*d*h^3*n - B*b*c*h^3*n)/(4*b*d) - (A*h^3*(4*a*d + 4*
b*c))/(4*b*d)))/(b*d)) + log((e*(a + b*x)^n)/(c + d*x)^n)*((B*h^3*x^4)/4 +
B*g^3*x + (3*B*g^2*h*x^2)/2 + B*g*h^2*x^3) - x^2*((((4*A*a*d*h^3 + 4*A*b*c*
h^3 + 12*A*b*d*g*h^2 + B*a*d*h^3*n - B*b*c*h^3*n)/(4*b*d) - (A*h^3*(4*a*d +
 4*b*c))/(4*b*d))*(4*a*d + 4*b*c))/(8*b*d) - (4*A*a*c*h^3 + 12*A*a*d*g*h^2
+ 12*A*b*c*g*h^2 + 12*A*b*d*g^2*h + 4*B*a*d*g*h^2*n - 4*B*b*c*g*h^2*n)/(8*b
*d) + (A*a*c*h^3)/(2*b*d)) + x^3*((4*A*a*d*h^3 + 4*A*b*c*h^3 + 12*A*b*d*g*h
^2 + B*a*d*h^3*n - B*b*c*h^3*n)/(12*b*d) - (A*h^3*(4*a*d + 4*b*c))/(12*b*d)
) + (A*h^3*x^4)/4 - (log(a + b*x)*(B*a^4*h^3*n - 4*B*a*b^3*g^3*n - 4*B*a^3*
b*g*h^2*n + 6*B*a^2*b^2*g^2*h*n))/(4*b^4) + (log(c + d*x)*(B*c^4*h^3*n - 4*
B*c*d^3*g^3*n - 4*B*c^3*d*g*h^2*n + 6*B*c^2*d^2*g^2*h*n))/(4*d^4)
```

3.295 $\int (g+hx)^2 \left(A + B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right) dx$

Optimal. Leaf size=158

$$-\frac{B(bc-ad)h(3bdg-bch-adh)nx}{3b^2d^2}-\frac{B(bc-ad)h^2nx^2}{6bd}-\frac{B(bg-ah)^3n\log(a+bx)}{3b^3h}+\frac{B(dg-ch)^3n\log(c-}{3d^3h}$$

```
[Out] -1/3*B*(-a*d+b*c)*h*(-a*d*h-b*c*h+3*b*d*g)*n*x/b^2/d^2-1/6*B*(-a*d+b*c)*h^2
*n*x^2/b/d-1/3*B*(-a*h+b*g)^3*n*ln(b*x+a)/b^3/h+1/3*B*(-c*h+d*g)^3*n*ln(d*x
+c)/d^3/h+1/3*(h*x+g)^3*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/h
```

Rubi [A]
time = 0.11, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {2548, 84}

$$\frac{(g+hx)^3\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{3h}-\frac{Bn(bg-ah)^3\log(a+bx)}{3b^3h}-\frac{Bhnx(bc-ad)(-adh-bch+3bdg)}{3b^2d^2}-\frac{Bh^2nx^2(bc-ad)}{6bd}+\frac{Bn(dg-ch)^3\log(c+dx)}{3d^3h}$$

Antiderivative was successfully verified.

```
[In] Int[(g + h*x)^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]),x]
```

```
[Out] -1/3*(B*(b*c - a*d)*h*(3*b*d*g - b*c*h - a*d*h)*n*x)/(b^2*d^2) - (B*(b*c -
a*d)*h^2*n*x^2)/(6*b*d) - (B*(b*g - a*h)^3*n*Log[a + b*x])/(3*b^3*h) + (B*(
d*g - c*h)^3*n*Log[c + d*x])/(3*d^3*h) + ((g + h*x)^3*(A + B*Log[(e*(a + b*
x)^n)/(c + d*x)^n]))/(3*h)
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 2548

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Dist[B*n*((b*c
- a*d)/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /;
 FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c -
 a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Rubi steps

$$\begin{aligned}\int (g+hx)^2 \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right)\,dx &= \int \left(A(g+hx)^2+B(g+hx)^2\log \left(e(a+bx)^n(c+dx)\right.\right. \\ &= \frac{A(g+hx)^3}{3h}+B\int (g+hx)^2\log \left(e(a+bx)^n(c+dx)^{-}\right. \\ &= \frac{A(g+hx)^3}{3h}+\frac{B(g+hx)^3\log \left(e(a+bx)^n(c+dx)^{-n}\right)}{3h} \\ &= \frac{A(g+hx)^3}{3h}+\frac{B(g+hx)^3\log \left(e(a+bx)^n(c+dx)^{-n}\right)}{3h} \\ &= -\frac{B(bc-ad)h(3bdg-bch-adh)nx}{3b^2d^2}-\frac{B(bc-ad)h^2}{6bd}\end{aligned}$$

Mathematica [**A**]
time = 0.14, size = 195, normalized size = 1.23

$$\frac{2aBd^3(3b^2g^2-3abgh+a^2h^2)\,n\log(a+bx)-2b^3Bc(3d^2g^2-3cdgh+c^2h^2)\,n\log(c+dx)+bdx(B(bc-ad)hn(-6bdg+2bch+2adh-bdhx)+2Ab^2d^2(3g^2+3ghx+h^2x^2)+2b^2Bd^2(3g^2+3ghx+h^2x^2)\log\left(e(a+bx)^n(c+dx)^{-n}\right))}{6b^3d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g + h*x)^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]),x]
```

```
[Out] (2*a*B*d^3*(3*b^2*g^2 - 3*a*b*g*h + a^2*h^2)*n*Log[a + b*x] - 2*b^3*B*c*(3*
d^2*g^2 - 3*c*d*g*h + c^2*h^2)*n*Log[c + d*x] + b*d*x*(B*(b*c - a*d)*h*n*(-
6*b*d*g + 2*b*c*h + 2*a*d*h - b*d*h*x) + 2*A*b^2*d^2*(3*g^2 + 3*g*h*x + h^2
*x^2) + 2*b^2*B*d^2*(3*g^2 + 3*g*h*x + h^2*x^2)*Log[(e*(a + b*x)^n)/(c + d*
x)^n]))/(6*b^3*d^3)
```

Maple [**C**] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.38, size = 1425, normalized size = 9.02

method	result	size
risch	Expression too large to display	1425

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n))),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*h^2*B*x^3*ln((b*x+a)^n)+1/3*h^2*B*ln(e)*x^3+1/3/h*B*g^3*ln((b*x+a)^n)+B
*g^2*x*ln((b*x+a)^n)+B*ln(e)*g^2*x+1/3*A*h^2*x^3+x*A*g^2+1/b*h*B*a*g*n*x-h/
d*B*c*g*n*x+1/2*I*h*B*Pi*g*x^2*csgn(I*(b*x+a)^n)*csgn(I*(b*x+a)^n/((d*x+c)^
n))^2-1/6*I*h^2*B*Pi*x^3*csgn(I*(b*x+a)^n)*csgn(I/((d*x+c)^n))*csgn(I*(b*x+
a)^n/((d*x+c)^n))-1/6*I*h^2*B*Pi*x^3*csgn(I*e)*csgn(I*(b*x+a)^n/((d*x+c)^n)
)*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)+1/2*I*h*B*Pi*g*x^2*csgn(I/((d*x+c)^n))*cs
```

```
gn(I*(b*x+a)^n/((d*x+c)^n))^2+1/2*I*h*B*Pi*g*x^2*csgn(I*e)*csgn(I*e/((d*x+c
)^n)*(b*x+a)^n)^2+1/2*I*h*B*Pi*g*x^2*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e
/((d*x+c)^n)*(b*x+a)^n)^2-1/2*I*B*Pi*g^2*x*csgn(I*(b*x+a)^n)*csgn(I/((d*x+c
)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))-1/2*I*B*Pi*g^2*x*csgn(I*e)*csgn(I*(b*x+
a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)-1/2*I*h*B*Pi*g*x^2*csgn(I
*(b*x+a)^n)*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))-1/2*I*h*B*Pi*
g*x^2*csgn(I*e)*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^
n)+h*A*g*x^2-1/d*B*ln(d*x+c)*c*g^2*n+1/b*B*ln(-b*x-a)*a*g^2*n-1/3*h^2/d^3*B
*ln(d*x+c)*c^3*n+1/3/b^3*h^2*B*ln(-b*x-a)*a^3*n-1/2*I*B*Pi*g^2*x*csgn(I*(b*
x+a)^n/((d*x+c)^n))^3-1/6*I*h^2*B*Pi*x^3*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^3-
1/6*I*h^2*B*Pi*x^3*csgn(I*(b*x+a)^n/((d*x+c)^n))^3-1/2*I*B*Pi*g^2*x*csgn(I*
e/((d*x+c)^n)*(b*x+a)^n)^3-1/3*(h*x+g)^3*B/h*ln((d*x+c)^n)+h*B*g*x^2*ln((b*
x+a)^n)+h*B*ln(e)*g*x^2+1/3/h*B*ln(d*x+c)*g^3*n-1/3/h*B*ln(-b*x-a)*g^3*n+1/
6/b*h^2*B*a*n*x^2-1/6*h^2/d*B*c*n*x^2-1/3/b^2*h^2*B*a^2*n*x+1/3*h^2/d^2*B*c
^2*n*x+h/d^2*B*ln(d*x+c)*c^2*g*n-1/b^2*h*B*ln(-b*x-a)*a^2*g*n+1/2*I*B*Pi*g^
2*x*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+1/2*I*B*Pi*g^2*x*cs
gn(I*e)*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2+1/2*I*B*Pi*g^2*x*csgn(I*(b*x+a)^n
/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2+1/2*I*B*Pi*g^2*x*csgn(I*(b*
x+a)^n)*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+1/6*I*h^2*B*Pi*x^3*csgn(I/((d*x+c)^
n))*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+1/6*I*h^2*B*Pi*x^3*csgn(I*e)*csgn(I*e/(
(d*x+c)^n)*(b*x+a)^n)^2+1/6*I*h^2*B*Pi*x^3*csgn(I*(b*x+a)^n/((d*x+c)^n))*cs
gn(I*e/((d*x+c)^n)*(b*x+a)^n)^2+1/6*I*h^2*B*Pi*x^3*csgn(I*(b*x+a)^n)*csgn(I
*(b*x+a)^n/((d*x+c)^n))^2-1/2*I*h*B*Pi*g*x^2*csgn(I*e/((d*x+c)^n)*(b*x+a)^n
)^3-1/2*I*h*B*Pi*g*x^2*csgn(I*(b*x+a)^n/((d*x+c)^n))^3
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 300 vs. $2(149) = 298$.
time = 0.29, size = 300, normalized size = 1.90

$$\frac{1}{3}\,Bh^{2}x^{3}\log\left(\frac{(bx+a)^{n}e}{(dx+c)^{n}}\right)+\frac{1}{3}\,Ah^{2}x^{3}+Bghx^{2}\log\left(\frac{(bx+a)^{n}e}{(dx+c)^{n}}\right)+Aghx^{2}+\left(\frac{ane\log\left(bx+a\right)}{b}-\frac{cne\log\left(dx+c\right)}{d}\right)Bg^{2}e^{(-1)}-\left(\frac{a^{2}ne\log\left(bx+a\right)}{b^{2}}-\frac{c^{2}ne\log\left(dx+c\right)}{d^{2}}+\frac{(bcn-adn)xe}{bd}\right)Bghe^{(-1)}+\frac{1}{6}\left(\frac{2\,a^{3}ne\log\left(bx+a\right)}{b^{3}}-\frac{2\,c^{3}ne\log\left(dx+c\right)}{d^{3}}-\frac{(b^{2}cdn-abd^{2}n)x^{2}e-2\,(b^{2}c^{2}n-a^{2}d^{2}n)xe}{b^{2}d^{2}}\right)Bh^{2}e^{(-1)}+Bg^{2}x\log\left(\frac{(bx+a)^{n}e}{(dx+c)^{n}}\right)+Ag^{2}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^2*(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="maxima
")
```

```
[Out] 1/3*B*h^2*x^3*log((b*x + a)^n*e/(d*x + c)^n) + 1/3*A*h^2*x^3 + B*g*h*x^2*lo
g((b*x + a)^n*e/(d*x + c)^n) + A*g*h*x^2 + (a*n*e*log(b*x + a)/b - c*n*e*lo
g(d*x + c)/d)*B*g^2*e^(-1) - (a^2*n*e*log(b*x + a)/b^2 - c^2*n*e*log(d*x +
c)/d^2 + (b*c*n - a*d*n)*x*e/(b*d))*B*g*h*e^(-1) + 1/6*(2*a^3*n*e*log(b*x +
 a)/b^3 - 2*c^3*n*e*log(d*x + c)/d^3 - ((b^2*c*d*n - a*b*d^2*n)*x^2*e - 2*(
b^2*c^2*n - a^2*d^2*n)*x*e)/(b^2*d^2))*B*h^2*e^(-1) + B*g^2*x*log((b*x + a)
^n*e/(d*x + c)^n) + A*g^2*x
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 325 vs. $2(149) = 298$.

time = 0.37, size = 325, normalized size = 2.06

$$\frac{2\,(A+B)b^3d^3h^2x^3+(6\,(A+B)b^3d^3gh-(Bb^3cd^2-Bab^2d^3)h^2n)x^2+2\,(3\,(A+B)b^3d^3g^2-(3\,(Bb^3cd^2-Bab^2d^3)gh-(Bb^3c^2d-Ba^2bd^3)h^2)n)x+2\,(Bb^3d^3h^2nx^3+3\,Bb^3d^3ghnx^2+3\,Bb^3d^3g^2nx+(3\,Bab^2d^3g^2-3\,Ba^2bd^3gh+Ba^3d^3h^2)n)\log(bx+a)-2\,(Bb^3d^3h^2nx^3+3\,Bb^3d^3ghnx^2+3\,Bb^3d^3g^2nx+(3\,Bb^3cd^2g^2-3\,Bb^3c^2dgh+Bb^3c^3h^2)n)\log(dx+c)}{6\,b^3d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^2*(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="fricas
")
```

```
[Out] 1/6*(2*(A + B)*b^3*d^3*h^2*x^3 + (6*(A + B)*b^3*d^3*g*h - (B*b^3*c*d^2 - B*
a*b^2*d^3)*h^2*n)*x^2 + 2*(3*(A + B)*b^3*d^3*g^2 - (3*(B*b^3*c*d^2 - B*a*b^
2*d^3)*g*h - (B*b^3*c^2*d - B*a^2*b*d^3)*h^2)*n)*x + 2*(B*b^3*d^3*h^2*n*x^3
 + 3*B*b^3*d^3*g*h*n*x^2 + 3*B*b^3*d^3*g^2*n*x + (3*B*a*b^2*d^3*g^2 - 3*B*a
^2*b*d^3*g*h + B*a^3*d^3*h^2)*n)*log(b*x + a) - 2*(B*b^3*d^3*h^2*n*x^3 + 3*
B*b^3*d^3*g*h*n*x^2 + 3*B*b^3*d^3*g^2*n*x + (3*B*b^3*c*d^2*g^2 - 3*B*b^3*c^
2*d*g*h + B*b^3*c^3*h^2)*n)*log(d*x + c))/(b^3*d^3)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)**2*(A+B*ln(e*(b*x+a)**n/((d*x+c)**n))),x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^2*(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**B**]
time = 4.47, size = 372, normalized size = 2.35

$$x^2\left(\frac{3\,A\,a\,d\,h^2+3\,A\,b\,c\,h^2+6\,A\,b\,d\,g\,h+B\,a\,d\,h^2\,n-B\,b\,c\,h^2\,n}{6\,b\,d}-\frac{A\,h^2\,(3\,a\,d+3\,b\,c)}{6\,b\,d}\right)+\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\left(B\,g^2\,x+B\,g\,h\,x^2+\frac{B\,h^2\,x^3}{3}\right)-x\left(\frac{(3\,a\,d+3\,b\,c)\left(\frac{3\,A\,a\,d\,h^2+3\,A\,b\,c\,h^2+6\,A\,b\,d\,g\,h+B\,a\,d\,h^2\,n-B\,b\,c\,h^2\,n}{3\,b\,d}-\frac{A\,h^2\,(3\,a\,d+3\,b\,c)}{3\,b\,d}\right)}{3\,b\,d}-\frac{3\,A\,a\,c\,h^2+3\,A\,b\,d\,g^2+6\,A\,a\,d\,g\,h+6\,A\,b\,c\,g\,h+3\,B\,a\,d\,g\,h\,n-3\,B\,b\,c\,g\,h\,n}{3\,b\,d}+\frac{A\,a\,c\,h^2}{b\,d}\right)+\frac{A\,h^2\,x^3}{3}+\frac{\ln(a+b\,x)\,(B\,n\,a^3\,h^2-3\,B\,n\,a^2\,b\,g\,h+3\,B\,n\,a\,b^2\,g^2)}{3\,b^3}-\frac{\ln(c+d\,x)\,(B\,n\,c^3\,h^2-3\,B\,n\,c^2\,d\,g\,h+3\,B\,n\,c\,d^2\,g^2)}{3\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g + h*x)^2*(A + B*log((e*(a + b*x)^n)/(c + d*x)^n)),x)
```

```
[Out] x^2*((3*A*a*d*h^2 + 3*A*b*c*h^2 + 6*A*b*d*g*h + B*a*d*h^2*n - B*b*c*h^2*n)/
(6*b*d) - (A*h^2*(3*a*d + 3*b*c))/(6*b*d)) + log((e*(a + b*x)^n)/(c + d*x)^
```

```
n)*((B*h^2*x^3)/3 + B*g^2*x + B*g*h*x^2) - x*(((3*a*d + 3*b*c)*((3*A*a*d*h^
2 + 3*A*b*c*h^2 + 6*A*b*d*g*h + B*a*d*h^2*n - B*b*c*h^2*n)/(3*b*d) - (A*h^2
*(3*a*d + 3*b*c))/(3*b*d)))/(3*b*d) - (3*A*a*c*h^2 + 3*A*b*d*g^2 + 6*A*a*d*
g*h + 6*A*b*c*g*h + 3*B*a*d*g*h*n - 3*B*b*c*g*h*n)/(3*b*d) + (A*a*c*h^2)/(b
*d)) + (A*h^2*x^3)/3 + (log(a + b*x)*(B*a^3*h^2*n + 3*B*a*b^2*g^2*n - 3*B*a
^2*b*g*h*n))/(3*b^3) - (log(c + d*x)*(B*c^3*h^2*n + 3*B*c*d^2*g^2*n - 3*B*c
^2*d*g*h*n))/(3*d^3)
```

3.296 $\int (g+hx) (A+B\log (e(a+bx)^n(c+dx)^{-n})) \, dx$

Optimal. Leaf size=116

$$-\frac{B(bc-ad)hnx}{2bd}-\frac{B(bg-ah)^2n\log(a+bx)}{2b^2h}+\frac{B(dg-ch)^2n\log(c+dx)}{2d^2h}+\frac{(g+hx)^2\left(A+B\log\left(e(a+bx)^n(\ldots\right.\right.}{2h}$$

```
[Out] -1/2*B*(-a*d+b*c)*h*n*x/b/d-1/2*B*(-a*h+b*g)^2*n*ln(b*x+a)/b^2/h+1/2*B*(-c*
h+d*g)^2*n*ln(d*x+c)/d^2/h+1/2*(h*x+g)^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/
h
```

Rubi [A]

time = 0.07, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {2548, 84}

$$\frac{(g+hx)^2\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{2h}-\frac{Bn(bg-ah)^2\log(a+bx)}{2b^2h}-\frac{Bhnx(bc-ad)}{2bd}+\frac{Bn(dg-ch)^2\log(c+dx)}{2d^2h}$$

Antiderivative was successfully verified.

```
[In] Int[(g + h*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]),x]
```

```
[Out] -1/2*(B*(b*c - a*d)*h*n*x)/(b*d) - (B*(b*g - a*h)^2*n*Log[a + b*x])/(2*b^2*
h) + (B*(d*g - c*h)^2*n*Log[c + d*x])/(2*d^2*h) + ((g + h*x)^2*(A + B*Log[(
e*(a + b*x)^n)/(c + d*x)^n]))/(2*h)
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 2548

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Dist[B*n*((b*c
- a*d)/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /;
 FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c -
 a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Rubi steps

$$\int (g+hx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\,dx = \int \left(A(g+hx)+B(g+hx)\log\left(e(a+bx)^n(c+dx)^{-\right.$$
$$= \frac{A(g+hx)^2}{2h} + B\int (g+hx)\log\left(e(a+bx)^n(c+dx)^{-\right.$$
$$= \frac{A(g+hx)^2}{2h} + \frac{B(g+hx)^2\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{2h}$$
$$= \frac{A(g+hx)^2}{2h} + \frac{B(g+hx)^2\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{2h}$$
$$= -\frac{B(bc-ad)hnx}{2bd} + \frac{A(g+hx)^2}{2h} - \frac{B(bg-ah)^2n\log}{2b^2h}$$

Mathematica [A]
time = 0.09, size = 112, normalized size = 0.97

$$\frac{aBd^2(2bg-ah)n\log(a+bx)+b(bBc(-2dg+ch)n\log(c+dx)+dx(B(-bc+ad)hn+Abd(2g+hx)+bBd(2g+hx)\log\left(e(a+bx)^n(c+dx)^{-n}\right)))}{2b^2d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g + h*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]),x]
```

```
[Out] (a*B*d^2*(2*b*g - a*h)*n*Log[a + b*x] + b*(b*B*c*(-2*d*g + c*h)*n*Log[c + d
*x] + d*x*(B*(-(b*c) + a*d)*h*n + A*b*d*(2*g + h*x) + b*B*d*(2*g + h*x)*Log
[(e*(a + b*x)^n)/(c + d*x)^n])))/(2*b^2*d^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.30, size = 839, normalized size = 7.23

method	result
risch	$\frac{Ah\,x^2}{2}+xAg-\frac{iB\pi h\,x^2\mathrm{csgn}\left(i(bx+a)^n(dx+c)^{-n}\right)^3}{4}-\frac{iB\pi h\,x^2\mathrm{csgn}\left(ie(dx+c)^{-n}(bx+a)^n\right)^3}{4}-\frac{iB\pi gx\mathrm{csgn}\left(i(bx+a)^n(dx+\right.}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n))),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*A*h*x^2+x*A*g-1/4*I*B*Pi*h*x^2*csgn(I*(b*x+a)^n)*csgn(I/((d*x+c)^n))*cs
gn(I*(b*x+a)^n/((d*x+c)^n))-1/4*I*B*Pi*h*x^2*csgn(I*e)*csgn(I*(b*x+a)^n/((d
*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)-1/2*I*B*Pi*g*x*csgn(I*(b*x+a)^n)*
csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))-1/2*I*B*Pi*g*x*csgn(I*e)*
csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)-1/2/b^2*B*ln(
b*x+a)*a^2*h*n+1/b*B*ln(b*x+a)*a*g*n+1/2/d^2*B*ln(-d*x-c)*c^2*h*n-1/d*B*ln(
-d*x-c)*c*g*n-1/4*I*B*Pi*h*x^2*csgn(I*(b*x+a)^n/((d*x+c)^n))^3-1/4*I*B*Pi*h
```

```
*x^2*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^3-1/2*I*B*Pi*g*x*csgn(I*(b*x+a)^n/((d*
x+c)^n))^3-1/2*I*B*Pi*g*x*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^3+1/2*ln((b*x+a)^
n)*x^2*B*h+ln((b*x+a)^n)*x*B*g+1/2*B*ln(e)*h*x^2+B*ln(e)*g*x-1/2*B*x*(h*x+2
*g)*ln((d*x+c)^n)+1/2/b*B*a*h*n*x-1/2/d*B*c*h*n*x+1/4*I*B*Pi*h*x^2*csgn(I*(
b*x+a)^n)*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+1/4*I*B*Pi*h*x^2*csgn(I*(b*x+a)^n
/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2+1/4*I*B*Pi*h*x^2*csgn(I*e)*
csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2+1/2*I*B*Pi*g*x*csgn(I/((d*x+c)^n))*csgn(I
*(b*x+a)^n/((d*x+c)^n))^2+1/2*I*B*Pi*g*x*csgn(I*(b*x+a)^n)*csgn(I*(b*x+a)^n
/((d*x+c)^n))^2+1/2*I*B*Pi*g*x*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x
+c)^n)*(b*x+a)^n)^2+1/2*I*B*Pi*g*x*csgn(I*e)*csgn(I*e/((d*x+c)^n)*(b*x+a)^n
)^2+1/4*I*B*Pi*h*x^2*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))^2
```

Maxima [A]
time = 0.27, size = 158, normalized size = 1.36

$$\frac{1}{2}\,Bhx^2\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+\frac{1}{2}\,Ahx^2+\left(\frac{ane\log\left(bx+a\right)}{b}-\frac{cne\log\left(dx+c\right)}{d}\right)Bge^{(-1)}-\frac{1}{2}\left(\frac{a^2ne\log\left(bx+a\right)}{b^2}-\frac{c^2ne\log\left(dx+c\right)}{d^2}+\frac{(bcn-adn)xe}{bd}\right)Bhe^{(-1)}+Bgx\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+Agx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="maxima")
```

```
[Out] 1/2*B*h*x^2*log((b*x + a)^n*e/(d*x + c)^n) + 1/2*A*h*x^2 + (a*n*e*log(b*x +
 a)/b - c*n*e*log(d*x + c)/d)*B*g*e^(-1) - 1/2*(a^2*n*e*log(b*x + a)/b^2 -
c^2*n*e*log(d*x + c)/d^2 + (b*c*n - a*d*n)*x*e/(b*d))*B*h*e^(-1) + B*g*x*lo
g((b*x + a)^n*e/(d*x + c)^n) + A*g*x
```

Fricas [A]
time = 0.39, size = 169, normalized size = 1.46

$$\frac{(A+B)b^2d^2hx^2+(2\,(A+B)b^2d^2g-(Bb^2cd-Babd^2)hn)x+(Bb^2d^2hnx^2+2\,Bb^2d^2gnx+(2\,Babd^2g-Ba^2d^2h)n)\log\left(bx+a\right)-(Bb^2d^2hnx^2+2\,Bb^2d^2gnx+(2\,Bb^2cdg-Bb^2c^2h)n)\log\left(dx+c\right)}{2\,b^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="fricas")
```

```
[Out] 1/2*((A + B)*b^2*d^2*h*x^2 + (2*(A + B)*b^2*d^2*g - (B*b^2*c*d - B*a*b*d^2)
*h*n)*x + (B*b^2*d^2*h*n*x^2 + 2*B*b^2*d^2*g*n*x + (2*B*a*b*d^2*g - B*a^2*d
^2*h)*n)*log(b*x + a) - (B*b^2*d^2*h*n*x^2 + 2*B*b^2*d^2*g*n*x + (2*B*b^2*c
*d*g - B*b^2*c^2*h)*n)*log(d*x + c))/(b^2*d^2)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(A+B*ln(e*(b*x+a)**n/((d*x+c)**n))),x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [A]
time = 12.00, size = 149, normalized size = 1.28

$$\frac{1}{2}\,(Ah+Bh)x^2+\frac{1}{2}\,(Bhnx^2+2\,Bgnx)\log\left(bx+a\right)-\frac{1}{2}\,(Bhnx^2+2\,Bgnx)\log\left(dx+c\right)-\frac{(Bbchn-Badhn-2\,Abdg-2\,Bbdg)x}{2\,bd}+\frac{(2\,Babgn-Ba^2hn)\log\left(bx+a\right)}{2\,b^2}-\frac{(2\,Bcdgn-Bc^2hn)\log\left(-dx-c\right)}{2\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="giac")
```

```
[Out] 1/2*(A*h + B*h)*x^2 + 1/2*(B*h*n*x^2 + 2*B*g*n*x)*log(b*x + a) - 1/2*(B*h*n
*x^2 + 2*B*g*n*x)*log(d*x + c) - 1/2*(B*b*c*h*n - B*a*d*h*n - 2*A*b*d*g - 2
*B*b*d*g)*x/(b*d) + 1/2*(2*B*a*b*g*n - B*a^2*h*n)*log(b*x + a)/b^2 - 1/2*(2
*B*c*d*g*n - B*c^2*h*n)*log(-d*x - c)/d^2
```

Mupad [B]
time = 4.39, size = 154, normalized size = 1.33

$$\ln\left(\frac{e\,{\left(a+b\,x\right)}^n}{{\left(c+d\,x\right)}^n}\right)\,\left(\frac{B\,h\,x^2}{2}+B\,g\,x\right)+x\,\left(\frac{2\,A\,a\,d\,h+2\,A\,b\,c\,h+2\,A\,b\,d\,g+B\,a\,d\,h\,n-B\,b\,c\,h\,n}{2\,b\,d}-\frac{A\,h\,\left(2\,a\,d+2\,b\,c\right)}{2\,b\,d}\right)-\frac{\ln\left(a+b\,x\right)\,\left(B\,a^2\,h\,n-2\,B\,a\,b\,g\,n\right)}{2\,b^2}+\frac{\ln\left(c+d\,x\right)\,\left(B\,c^2\,h\,n-2\,B\,c\,d\,g\,n\right)}{2\,d^2}+\frac{A\,h\,x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g + h*x)*(A + B*log((e*(a + b*x)^n)/(c + d*x)^n)),x)
```

```
[Out] log((e*(a + b*x)^n)/(c + d*x)^n)*(B*g*x + (B*h*x^2)/2) + x*((2*A*a*d*h + 2*
A*b*c*h + 2*A*b*d*g + B*a*d*h*n - B*b*c*h*n)/(2*b*d) - (A*h*(2*a*d + 2*b*c)
)/(2*b*d)) - (log(a + b*x)*(B*a^2*h*n - 2*B*a*b*g*n))/(2*b^2) + (log(c + d*
x)*(B*c^2*h*n - 2*B*c*d*g*n))/(2*d^2) + (A*h*x^2)/2
```

3.297 $\int (A+B\log(e(a+bx)^n(c+dx)^{-n}))\,dx$

Optimal. Leaf size=57

$$Ax-\frac{B(bc-ad)n\log(c+dx)}{bd}+\frac{B(a+bx)\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{b}$$

```
[Out] A*x-B*(-a*d+b*c)*n*ln(d*x+c)/b/d+B*(b*x+a)*ln(e*(b*x+a)^n/((d*x+c)^n))/b
```

Rubi [A]

time = 0.02, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2536, 31}

$$\frac{B(a+bx)\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{b}-\frac{Bn(bc-ad)\log(c+dx)}{bd}+Ax$$

Antiderivative was successfully verified.

```
[In] Int[A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n],x]
```

```
[Out] A*x - (B*(b*c - a*d)*n*Log[c + d*x])/(b*d) + (B*(a + b*x)*Log[(e*(a + b*x)^
n)/(c + d*x)^n])/b
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2536

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.), x_Symbol] :> Simp[(a + b*x)*((A + B*Log[e*((a + b*x)^n/(c
+ d*x)^n)])^p/b), x] - Dist[B*n*p*((b*c - a*d)/b), Int[(A + B*Log[e*((a + b
*x)^n/(c + d*x)^n)])^(p - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, A,
B, n}, x] && EqQ[n + mn, 0] && NeQ[b*c - a*d, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\,dx &= Ax+B\int \log\left(e(a+bx)^n(c+dx)^{-n}\right)\,dx\\ &= Ax+\frac{B(a+bx)\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{b}-\frac{(B(bc-ad)n)\int}{b}\\ &= Ax-\frac{B(bc-ad)n\log(c+dx)}{bd}+\frac{B(a+bx)\log\left(e(a+bx)^n(c+\right.}{b}\end{aligned}$$

Mathematica [**A**]
time = 0.01, size = 57, normalized size = 1.00

$$Ax - \frac{B(bc-ad)n\log(c+dx)}{bd} + \frac{B(a+bx)\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n],x]
```

```
[Out] A*x - (B*(b*c - a*d)*n*Log[c + d*x])/(b*d) + (B*(a + b*x)*Log[(e*(a + b*x)^
n)/(c + d*x)^n])/b
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 122 vs. $2(57) = 114$.
time = 0.18, size = 123, normalized size = 2.16

method	result
default	$Ax + B\ln\left(e(bx+a)^n(dx+c)^{-n}\right)x - \frac{Bnc\ln(dx+c)a}{ad-cb} + \frac{Bn\,c^2\ln(dx+c)b}{(ad-cb)d} + \frac{Bn\,a^2\ln(bx+a)d}{(ad-cb)b} - \frac{Bna\ln(bx+a)c}{ad-cb}$
risch	$Ax - Bx\ln\left((dx+c)^n\right) - \frac{iB\pi x\mathrm{csgn}\left(i(bx+a)^n(dx+c)^{-n}\right)^3}{2} + \frac{iB\pi x\,\mathrm{csgn}(i(bx+a)^n)\mathrm{csgn}\left(i(bx+a)^n(dx+c)^{-n}\right)^2}{2} + \frac{iB\cdots}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)),x,method=_RETURNVERBOSE)
```

```
[Out] A*x+B*ln(e*(b*x+a)^n/((d*x+c)^n))*x-B*n*c/(a*d-b*c)*ln(d*x+c)*a+B*n*c^2/(a*
d-b*c)/d*ln(d*x+c)*b+B*n*a^2/(a*d-b*c)/b*ln(b*x+a)*d-B*n*a/(a*d-b*c)*ln(b*x
+a)*c
```

Maxima [**A**]
time = 0.27, size = 61, normalized size = 1.07

$$\left(\frac{ane\log(bx+a)}{b} - \frac{cne\log(dx+c)}{d}\right)Be^{(-1)} + Bx\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + Ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(A+B*log(e*(b*x+a)^n/((d*x+c)^n)),x, algorithm="maxima")
```

```
[Out] (a*n*e*log(b*x + a)/b - c*n*e*log(d*x + c)/d)*B*e^(-1) + B*x*log((b*x + a)^
n*e/(d*x + c)^n) + A*x
```

Fricas [**A**]
time = 0.39, size = 54, normalized size = 0.95

$$\frac{(A+B)bdx + (Bbdnx + Badn)\log(bx+a) - (Bbdnx + Bbcn)\log(dx+c)}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(A+B*log(e*(b*x+a)^n/((d*x+c)^n)),x, algorithm="fricas")
```

```
[Out] ((A + B)*b*d*x + (B*b*d*n*x + B*a*d*n)*log(b*x + a) - (B*b*d*n*x + B*b*c*n)
*log(d*x + c))/(b*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(A+B*ln(e*(b*x+a)**n/((d*x+c)**n)),x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [A]
time = 2.03, size = 55, normalized size = 0.96

$$\left(nx\log\left(bx+a\right)-nx\log\left(dx+c\right)+\frac{an\log\left(bx+a\right)}{b}-\frac{cn\log\left(-dx-c\right)}{d}+x\right)B+Ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(A+B*log(e*(b*x+a)^n/((d*x+c)^n)),x, algorithm="giac")
```

```
[Out] (n*x*log(b*x + a) - n*x*log(d*x + c) + a*n*log(b*x + a)/b - c*n*log(-d*x -
c)/d + x)*B + A*x
```

Mupad [B]
time = 4.11, size = 53, normalized size = 0.93

$$A\,x+B\,x\,\ln\left(\frac{e\,{\left(a+b\,x\right)}^n}{{\left(c+d\,x\right)}^n}\right)+\frac{B\,a\,n\,\ln\left(a+b\,x\right)}{b}-\frac{B\,c\,n\,\ln\left(c+d\,x\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(A + B*log((e*(a + b*x)^n)/(c + d*x)^n),x)
```

```
[Out] A*x + B*x*log((e*(a + b*x)^n)/(c + d*x)^n) + (B*a*n*log(a + b*x))/b - (B*c*
n*log(c + d*x))/d
```

3.298 $\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{g+hx}\,dx$

Optimal. Leaf size=148

$$-\frac{Bn\log\left(-\frac{h(a+bx)}{bg-ah}\right)\log(g+hx)}{h}+\frac{Bn\log\left(-\frac{h(c+dx)}{dg-ch}\right)\log(g+hx)}{h}+\frac{(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right))\log\text{(}}{h}$$

```
[Out] -B*n*ln(-h*(b*x+a)/(-a*h+b*g))*ln(h*x+g)/h+B*n*ln(-h*(d*x+c)/(-c*h+d*g))*ln
(h*x+g)/h+(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))*ln(h*x+g)/h-B*n*polylog(2,b*(h*
x+g)/(-a*h+b*g))/h+B*n*polylog(2,d*(h*x+g)/(-c*h+d*g))/h
```

Rubi [A]

time = 0.10, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {2546, 2441, 2440, 2438}

$$-\frac{Bn\text{PolyLog}\left(2,\frac{b(g+hx)}{bg-ah}\right)}{h}+\frac{Bn\text{PolyLog}\left(2,\frac{d(g+hx)}{dg-ch}\right)}{h}+\frac{\log(g+hx)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{h}-\frac{Bn\log(g+hx)\log\left(-\frac{h(a+bx)}{bg-ah}\right)}{h}+\frac{Bn\log(g+hx)\log\left(-\frac{h(c+dx)}{dg-ch}\right)}{h}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(g + h*x),x]
```

```
[Out] -((B*n*Log[-((h*(a + b*x))/(b*g - a*h))]*Log[g + h*x])/h) + (B*n*Log[-((h*(
c + d*x))/(d*g - c*h))]*Log[g + h*x])/h + ((A + B*Log[(e*(a + b*x)^n)/(c +
d*x)^n])*Log[g + h*x])/h - (B*n*PolyLog[2, (b*(g + h*x))/(b*g - a*h)])/h +
(B*n*PolyLog[2, (d*(g + h*x))/(d*g - c*h)])/h
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2546

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))/((f_.) + (g_.)*(x_)), x_Symbol] :> Simp[Log[f + g*x]*((A + B*Log[
e*((a + b*x)^n/(c + d*x)^n)])/g), x] + (-Dist[b*B*(n/g), Int[Log[f + g*x]/(
a + b*x), x], x] + Dist[B*d*(n/g), Int[Log[f + g*x]/(c + d*x), x], x]) /; F
reeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[n + mn, 0] && NeQ[b*c - a*d,
 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{g+hx}\,dx &= \int \left(\frac{A}{g+hx}+\frac{B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{g+hx}\right)dx\\
&= \frac{A\log(g+hx)}{h}+B\int \frac{\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{g+hx}\,dx\\
&= \frac{A\log(g+hx)}{h}+\frac{B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\log(g+hx)}{h}-\frac{(bB}{\ }\\
&= \frac{A\log(g+hx)}{h}-\frac{Bn\log\left(-\frac{h(a+bx)}{bg-ah}\right)\log(g+hx)}{h}+\frac{Bn\log\left(-\frac{h(c+}{dg-}\right.}{\ }\\
&= \frac{A\log(g+hx)}{h}-\frac{Bn\log\left(-\frac{h(a+bx)}{bg-ah}\right)\log(g+hx)}{h}+\frac{Bn\log\left(-\frac{h(c+}{dg-}\right.}{\ }\\
&= \frac{A\log(g+hx)}{h}-\frac{Bn\log\left(-\frac{h(a+bx)}{bg-ah}\right)\log(g+hx)}{h}+\frac{Bn\log\left(-\frac{h(c+}{dg-}\right.}{\ }
\end{aligned}
$$

Mathematica [A]
time = 0.06, size = 150, normalized size = 1.01

$$\frac{(A+B(-n\log(a+bx)+n\log(c+dx)+\log\left(e(a+bx)^n(c+dx)^{-n}\right)))\log(g+hx)+Bn\left(\log(a+bx)\log\left(\frac{b(g+hx)}{bg-ah}\right)+\mathrm{Li}_2\left(\frac{h(a+bx)}{-bg+ah}\right)\right)-Bn\left(\log(c+dx)\log\left(\frac{d(g+hx)}{dg-ch}\right)+\mathrm{Li}_2\left(\frac{h(c+dx)}{-dg+ch}\right)\right)}{h}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(g + h*x),x]
```

```
[Out] ((A + B*(-(n*Log[a + b*x]) + n*Log[c + d*x] + Log[(e*(a + b*x)^n)/(c + d*x)
^n]))*Log[g + h*x] + B*n*(Log[a + b*x]*Log[(b*(g + h*x))/(b*g - a*h)] + Pol
yLog[2, (h*(a + b*x))/(-(b*g) + a*h)]) - B*n*(Log[c + d*x]*Log[(d*(g + h*x)
)/(d*g - c*h)] + PolyLog[2, (h*(c + d*x))/(-(d*g) + c*h)]))/h
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.28, size = 597, normalized size = 4.03

method	result
risch	$-\frac{i\ln(hx+g)B\pi\,\mathrm{csgn}(ie)\mathrm{csgn}\left(i(bx+a)^n(dx+c)^{-n}\right)\mathrm{csgn}\left(ie(dx+c)^{-n}(bx+a)^n\right)}{2h}+\frac{i\ln(hx+g)B\pi\,\mathrm{csgn}\left(i(dx+c)^{-n}\right)\mathrm{csgn}\left(i(bx+a)\right.}{2h}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/(h*x+g),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*I*ln(h*x+g)/h*B*Pi*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^3+1/2*I*ln(h*x+g)/h
*B*Pi*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+1/2*I*ln(h*x+g)/h
*B*Pi*csgn(I*(b*x+a)^n)*csgn(I*(b*x+a)^n/((d*x+c)^n))^2-1/2*I*ln(h*x+g)/h*B
*Pi*csgn(I*(b*x+a)^n)*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))-1/2
*I*ln(h*x+g)/h*B*Pi*csgn(I*e)*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+
c)^n)*(b*x+a)^n)+1/2*I*ln(h*x+g)/h*B*Pi*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(
I*e/((d*x+c)^n)*(b*x+a)^n)^2+1/2*I*ln(h*x+g)/h*B*Pi*csgn(I*e)*csgn(I*e/((d*
x+c)^n)*(b*x+a)^n)^2-1/2*I*ln(h*x+g)/h*B*Pi*csgn(I*(b*x+a)^n/((d*x+c)^n))^3
+ln(h*x+g)/h*B*ln(e)+A*ln(h*x+g)/h+B*ln(h*x+g)/h*ln((b*x+a)^n)-B/h*n*dilog(
((h*x+g)*b+a*h-b*g)/(a*h-b*g))-B/h*n*ln(h*x+g)*ln(((h*x+g)*b+a*h-b*g)/(a*h-
b*g))-B*ln(h*x+g)/h*ln((d*x+c)^n)+B/h*n*dilog((d*(h*x+g)+c*h-d*g)/(c*h-d*g)
)+B/h*n*ln(h*x+g)*ln((d*(h*x+g)+c*h-d*g)/(c*h-d*g))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(h*x+g),x, algorithm="maxima")
```

```
[Out] -B*integrate(-(log((b*x + a)^n) - log((d*x + c)^n) + 1)/(h*x + g), x) + A*l
og(h*x + g)/h
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(h*x+g),x, algorithm="fricas")
```

```
[Out] integral((B*log((b*x + a)^n*e/(d*x + c)^n) + A)/(h*x + g), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))/(h*x+g),x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(h*x+g),x, algorithm="giac")
```

```
[Out] integrate((B*log((b*x + a)^n*e/(d*x + c)^n) + A)/(h*x + g), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\,\ln\left(\frac{e\,{\left(a+b\,x\right)}^n}{{\left(c+d\,x\right)}^n}\right)}{g+h\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))/(g + h*x),x)
```

```
[Out] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))/(g + h*x), x)
```

3.299 $\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(g+hx)^2}\,dx$

Optimal. Leaf size=120

$$\frac{bBn\log(a+bx)}{h(bg-ah)}-\frac{Bdn\log(c+dx)}{h(dg-ch)}-\frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{h(g+hx)}+\frac{B(bc-ad)n\log(g+hx)}{(bg-ah)(dg-ch)}$$

```
[Out] b*B*n*ln(b*x+a)/h/(-a*h+b*g)-B*d*n*ln(d*x+c)/h/(-c*h+d*g)+(-A-B*ln(e*(b*x+a
)^n/((d*x+c)^n)))/h/(h*x+g)+B*(-a*d+b*c)*n*ln(h*x+g)/(-a*h+b*g)/(-c*h+d*g)
```

Rubi [A]
time = 0.08, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {2548, 84}

$$-\frac{B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A}{h(g+hx)}+\frac{Bn(bc-ad)\log(g+hx)}{(bg-ah)(dg-ch)}+\frac{bBn\log(a+bx)}{h(bg-ah)}-\frac{Bdn\log(c+dx)}{h(dg-ch)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(g + h*x)^2,x]
```

```
[Out] (b*B*n*Log[a + b*x])/(h*(b*g - a*h)) - (B*d*n*Log[c + d*x])/(h*(d*g - c*h))
 - (A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(h*(g + h*x)) + (B*(b*c - a*d)*
n*Log[g + h*x])/((b*g - a*h)*(d*g - c*h))
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 2548

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Dist[B*n*((b*c
- a*d)/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /;
 FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c -
 a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Rubi steps

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(g+hx)^2}\,dx = \int \left(\frac{A}{(g+hx)^2}+\frac{B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(g+hx)^2}\right)dx$$
$$= -\frac{A}{h(g+hx)} + B\int \frac{\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(g+hx)^2}\,dx$$
$$= -\frac{A}{h(g+hx)} + \frac{B(a+bx)\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(bg-ah)(g+hx)} - \frac{(B(bc-a$$
$$= -\frac{A}{h(g+hx)} + \frac{B(a+bx)\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(bg-ah)(g+hx)} - \frac{(Bd(bc-}{(bg-}$$
$$= -\frac{A}{h(g+hx)} - \frac{B(bc-ad)n\log(c+dx)}{(bg-ah)(dg-ch)} + \frac{B(a+bx)\log\left(e(a+b}{(bg-ah)(g-}$$

Mathematica [A]
time = 0.16, size = 155, normalized size = 1.29

$$-\frac{A}{h(g+hx)}+\frac{bBn\log(a+bx)}{bgh-ah^2}+\frac{Bdn\log(c+dx)}{h(-dg+ch)}-\frac{B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{h(g+hx)}+\frac{bBcn\log(g+hx)}{(bg-ah)(dg-ch)}-\frac{aBdn\log(g+hx)}{(bg-ah)(dg-ch)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(g + h*x)^2,x]
```

```
[Out] -(A/(h*(g + h*x))) + (b*B*n*Log[a + b*x])/(b*g*h - a*h^2) + (B*d*n*Log[c +
d*x])/(h*(-(d*g) + c*h)) - (B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(h*(g + h*x
)) + (b*B*c*n*Log[g + h*x])/((b*g - a*h)*(d*g - c*h)) - (a*B*d*n*Log[g + h*
x])/((b*g - a*h)*(d*g - c*h))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.37, size = 1796, normalized size = 14.97

method	result	size
risch	Expression too large to display	1796

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/(h*x+g)^2,x,method=_RETURNVERBOSE)
```

```
[Out] B/h/(h*x+g)*ln((d*x+c)^n)-1/2*(I*B*Pi*a*c*h^2*csgn(I*(b*x+a)^n/((d*x+c)^n))
*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2+I*B*Pi*a*d*h*csgn(I*(b*x+a)^n/((d*x+c)^n
))^3*g+I*B*Pi*b*d*g^2*csgn(I*e)*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2-I*B*Pi*a*
c*h^2*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^3+2*A*a*c*h^2+2*A*b*d*g^2+I*B*Pi*b*d*
g^2*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+I*B*Pi*b*d*g^2*csgn
(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2-I*B*Pi*b*c*h*cs
gn(I*(b*x+a)^n)*csgn(I*(b*x+a)^n/((d*x+c)^n))^2*g-I*B*Pi*a*d*h*csgn(I*e)*cs
```

```
gn(I*e/((d*x+c)^n)*(b*x+a)^n)^2*g-I*B*Pi*b*c*h*csgn(I/((d*x+c)^n))*csgn(I*(
b*x+a)^n/((d*x+c)^n))^2*g+I*B*Pi*a*c*h^2*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)
^n/((d*x+c)^n))^2+I*B*Pi*a*d*h*csgn(I*e)*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn
(I*e/((d*x+c)^n)*(b*x+a)^n)*g+I*B*Pi*a*d*h*csgn(I*(b*x+a)^n)*csgn(I/((d*x+c
)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))*g-2*B*ln(e)*a*d*h*g-2*B*ln(e)*b*c*h*g-2
*B*a*d*g*h*ln((b*x+a)^n)-2*B*b*c*g*h*ln((b*x+a)^n)+I*B*Pi*a*d*h*csgn(I*e/((
d*x+c)^n)*(b*x+a)^n)^3*g+I*B*Pi*b*c*h*csgn(I*e)*csgn(I*(b*x+a)^n/((d*x+c)^n
))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)*g+I*B*Pi*b*c*h*csgn(I*(b*x+a)^n)*csgn(I/
((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))*g-I*B*Pi*a*c*h^2*csgn(I*e)*csgn(
I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)-I*B*Pi*a*c*h^2*csg
n(I*(b*x+a)^n)*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))-I*B*Pi*a*d
*h*csgn(I*(b*x+a)^n)*csgn(I*(b*x+a)^n/((d*x+c)^n))^2*g-I*B*Pi*a*c*h^2*csgn(
I*(b*x+a)^n/((d*x+c)^n))^3+I*B*Pi*a*c*h^2*csgn(I*e)*csgn(I*e/((d*x+c)^n)*(b
*x+a)^n)^2-2*A*a*d*h*g-2*A*b*c*h*g+2*B*ln(e)*a*c*h^2+2*B*ln(e)*b*d*g^2+I*B*
Pi*b*c*h*csgn(I*(b*x+a)^n/((d*x+c)^n))^3*g+I*B*Pi*b*c*h*csgn(I*e/((d*x+c)^n
)*(b*x+a)^n)^3*g+I*B*Pi*a*c*h^2*csgn(I*(b*x+a)^n)*csgn(I*(b*x+a)^n/((d*x+c)
^n))^2+2*B*a*c*h^2*ln((b*x+a)^n)+2*B*b*d*g^2*ln((b*x+a)^n)+2*B*ln(-d*x-c)*b
*d*g^2*n-2*B*ln(-b*x-a)*b*d*g^2*n-I*B*Pi*b*d*g^2*csgn(I*(b*x+a)^n/((d*x+c)^
n))^3-I*B*Pi*b*d*g^2*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^3-I*B*Pi*b*c*h*csgn(I*
(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2*g-I*B*Pi*b*d*g^2*c
sgn(I*e)*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)-I*B*
Pi*b*d*g^2*csgn(I*(b*x+a)^n)*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^
n))-I*B*Pi*a*d*h*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))^2*g-I*B*
Pi*a*d*h*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2*g-
I*B*Pi*b*c*h*csgn(I*e)*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2*g+I*B*Pi*b*d*g^2*c
sgn(I*(b*x+a)^n)*csgn(I*(b*x+a)^n/((d*x+c)^n))^2-2*B*ln(-d*x-c)*a*d*h^2*n*x
+2*B*ln(-b*x-a)*b*c*h^2*n*x+2*B*ln(h*x+g)*a*d*h^2*n*x-2*B*ln(h*x+g)*b*c*h^2
*n*x-2*B*ln(-d*x-c)*a*d*g*h*n+2*B*ln(-b*x-a)*b*c*g*h*n+2*B*ln(h*x+g)*a*d*g*
h*n-2*B*ln(h*x+g)*b*c*g*h*n+2*B*ln(-d*x-c)*b*d*g*h*n*x-2*B*ln(-b*x-a)*b*d*g
*h*n*x)/(h*x+g)/(a*c*h^2-a*d*g*h-b*c*g*h+b*d*g^2)/h
```

Maxima [A]
time = 0.27, size = 153, normalized size = 1.28

$$\left(\frac{bne\log\left(bx+a\right)}{bgh-ah^2}-\frac{dne\log\left(dx+c\right)}{dgh-ch^2}-\frac{(bcn-adn)e\log\left(hx+g\right)}{(dgh-ch^2)a-(dg^2-cgh)b}\right)Be^{(-1)}-\frac{B\log\left(\frac{(bx+a)^ne}{(dx+c)^n}\right)}{h^2x+gh}-\frac{A}{h^2x+gh}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(h*x+g)^2,x, algorithm="maxima
")
```

```
[Out] (b*n*e*log(b*x + a)/(b*g*h - a*h^2) - d*n*e*log(d*x + c)/(d*g*h - c*h^2) -
(b*c*n - a*d*n)*e*log(h*x + g)/((d*g*h - c*h^2)*a - (d*g^2 - c*g*h)*b))*B*e
^(-1) - B*log((b*x + a)^n*e/(d*x + c)^n)/(h^2*x + g*h) - A/(h^2*x + g*h)
```

Fricas [A]
time = 3.33, size = 227, normalized size = 1.89

$$-\frac{(A+B)bdg^2+(A+B)ach^2-((A+B)bc+(A+B)ad)gh-((Bbdgh-Bbch^2)nx+(Badgh-Bach^2)n)\log(bx+a)+((Bbdgh-Badh^2)nx+(Bbcgh-Bach^2)n)\log(dx+c)-((Bbc-Bad)h^2nx+(Bbc-Bad)ghn)\log(hx+g)}{bdg^3h+acgh^3-(bc+ad)g^2h^2+(bdg^2h^2+ach^4-(bc+ad)gh^3)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(h*x+g)^2,x, algorithm="fricas
")
[Out] -((A + B)*b*d*g^2 + (A + B)*a*c*h^2 - ((A + B)*b*c + (A + B)*a*d)*g*h - ((B
*b*d*g*h - B*b*c*h^2)*n*x + (B*a*d*g*h - B*a*c*h^2)*n)*log(b*x + a) + ((B*b
*d*g*h - B*a*d*h^2)*n*x + (B*b*c*g*h - B*a*c*h^2)*n)*log(d*x + c) - ((B*b*c
 - B*a*d)*h^2*n*x + (B*b*c - B*a*d)*g*h*n)*log(h*x + g))/(b*d*g^3*h + a*c*g
*h^3 - (b*c + a*d)*g^2*h^2 + (b*d*g^2*h^2 + a*c*h^4 - (b*c + a*d)*g*h^3)*x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))/(h*x+g)**2,x)
[Out] Timed out
```

Giac [A]
time = 4.62, size = 166, normalized size = 1.38

$$\frac{Bb^2n\log(|-bx-a|)}{b^2gh-abh^2}-\frac{Bd^2n\log(|dx+c|)}{d^2gh-cdh^2}-\frac{Bn\log(bx+a)}{h^2x+gh}+\frac{Bn\log(dx+c)}{h^2x+gh}+\frac{(Bbcn-Badn)\log(hx+g)}{bdg^2-bcgh-adgh+ach^2}-\frac{A+B}{h^2x+gh}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(h*x+g)^2,x, algorithm="giac")
[Out] B*b^2*n*log(abs(-b*x - a))/(b^2*g*h - a*b*h^2) - B*d^2*n*log(abs(d*x + c))/
(d^2*g*h - c*d*h^2) - B*n*log(b*x + a)/(h^2*x + g*h) + B*n*log(d*x + c)/(h^
2*x + g*h) + (B*b*c*n - B*a*d*n)*log(h*x + g)/(b*d*g^2 - b*c*g*h - a*d*g*h
+ a*c*h^2) - (A + B)/(h^2*x + g*h)
```

Mupad [B]
time = 4.72, size = 141, normalized size = 1.18

$$\frac{B\,d\,n\,\ln(c+d\,x)}{c\,h^2-d\,g\,h}-\frac{\ln(g+h\,x)\,\left(B\,a\,d\,n-B\,b\,c\,n\right)}{a\,c\,h^2+b\,d\,g^2-a\,d\,g\,h-b\,c\,g\,h}-\frac{B\,\ln\left(\frac{e\,{(a+b\,x)}^n}{{(c+d\,x)}^n}\right)}{h\,\left(g+h\,x\right)}-\frac{B\,b\,n\,\ln(a+b\,x)}{a\,h^2-b\,g\,h}-\frac{A}{x\,h^2+g\,h}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))/(g + h*x)^2,x)
[Out] (B*d*n*log(c + d*x))/(c*h^2 - d*g*h) - (log(g + h*x)*(B*a*d*n - B*b*c*n))/(
a*c*h^2 + b*d*g^2 - a*d*g*h - b*c*g*h) - (B*log((e*(a + b*x)^n)/(c + d*x)^n
))/(h*(g + h*x)) - (B*b*n*log(a + b*x))/(a*h^2 - b*g*h) - A/(g*h + h^2*x)
```

3.300 $\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(g+hx)^3}\,dx$

Optimal. Leaf size=191

$$-\frac{B(bc-ad)n}{2(bg-ah)(dg-ch)(g+hx)}+\frac{b^2Bn\log(a+bx)}{2h(bg-ah)^2}-\frac{Bd^2n\log(c+dx)}{2h(dg-ch)^2}-\frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{2h(g+hx)^2}+$$

```
[Out] -1/2*B*(-a*d+b*c)*n/(-a*h+b*g)/(-c*h+d*g)/(h*x+g)+1/2*b^2*B*n*ln(b*x+a)/h/(
-a*h+b*g)^2-1/2*B*d^2*n*ln(d*x+c)/h/(-c*h+d*g)^2+1/2*(-A-B*ln(e*(b*x+a)^n/(
(d*x+c)^n)))/h/(h*x+g)^2+1/2*B*(-a*d+b*c)*(-a*d*h-b*c*h+2*b*d*g)*n*ln(h*x+g
)/(-a*h+b*g)^2/(-c*h+d*g)^2
```

Rubi [A]
time = 0.15, antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {2548, 84}

$$-\frac{B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A}{2h(g+hx)^2}+\frac{b^2Bn\log(a+bx)}{2h(bg-ah)^2}-\frac{Bn(bc-ad)}{2(g+hx)(bg-ah)(dg-ch)}+\frac{Bn(bc-ad)\log(g+hx)(-adh-bch+2bdg)}{2(bg-ah)^2(dg-ch)^2}-\frac{Bd^2n\log(c+dx)}{2h(dg-ch)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(g + h*x)^3,x]
```

```
[Out] -1/2*(B*(b*c - a*d)*n)/((b*g - a*h)*(d*g - c*h)*(g + h*x)) + (b^2*B*n*Log[a
 + b*x])/(2*h*(b*g - a*h)^2) - (B*d^2*n*Log[c + d*x])/(2*h*(d*g - c*h)^2) -
 (A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(2*h*(g + h*x)^2) + (B*(b*c - a*d
)*(2*b*d*g - b*c*h - a*d*h)*n*Log[g + h*x])/(2*(b*g - a*h)^2*(d*g - c*h)^2)
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 2548

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Dist[B*n*((b*c
- a*d)/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /;
 FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c -
 a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)}{(g+hx)^3}\,dx &= \int \left(\frac{A}{(g+hx)^3}+\frac{B\log \left(e(a+bx)^n(c+dx)^{-n}\right)}{(g+hx)^3}\right)dx\\ &= -\frac{A}{2h(g+hx)^2}+B\int \frac{\log \left(e(a+bx)^n(c+dx)^{-n}\right)}{(g+hx)^3}\,dx\\ &= -\frac{A}{2h(g+hx)^2}-\frac{B\log \left(e(a+bx)^n(c+dx)^{-n}\right)}{2h(g+hx)^2}+\frac{(B(bc-ad)n)\int}{}\\ &= -\frac{A}{2h(g+hx)^2}-\frac{B\log \left(e(a+bx)^n(c+dx)^{-n}\right)}{2h(g+hx)^2}+\frac{(B(bc-ad)n)\int}{}\\ &= -\frac{A}{2h(g+hx)^2}-\frac{B(bc-ad)n}{2(bg-ah)(dg-ch)(g+hx)}+\frac{b^2Bn\log(a+bx}{2h(bg-ah)^2}\end{aligned}$$

Mathematica [A]
time = 0.25, size = 238, normalized size = 1.25

$$\frac{1}{2}\left(\frac{B(-bc+ad)n}{(bg-ah)(dg-ch)(g+hx)}+\frac{b^2Bn\log(a+bx)}{h(bg-ah)^2}-\frac{Bn\log(a+bx)}{h(g+hx)^2}-\frac{Bd^2n\log(c+dx)}{h(dg-ch)^2}+\frac{Bn\log(c+dx)}{h(g+hx)^2}-\frac{A+B(-n\log(a+bx)+n\log(c+dx)+\log\left(e(a+bx)^n(c+dx)^{-n}\right))}{h(g+hx)^2}-\frac{B(bc-ad)(-2bdg+bch+adh)n\log(g+hx)}{(bg-ah)^2(dg-ch)^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(g + h*x)^3,x]
```

```
[Out] ((B*(-(b*c) + a*d)*n)/((b*g - a*h)*(d*g - c*h)*(g + h*x)) + (b^2*B*n*Log[a
+ b*x])/(h*(b*g - a*h)^2) - (B*n*Log[a + b*x])/(h*(g + h*x)^2) - (B*d^2*n*L
og[c + d*x])/(h*(d*g - c*h)^2) + (B*n*Log[c + d*x])/(h*(g + h*x)^2) - (A +
B*(-(n*Log[a + b*x]) + n*Log[c + d*x] + Log[(e*(a + b*x)^n)/(c + d*x)^n]))/
(h*(g + h*x)^2) - (B*(b*c - a*d)*(-2*b*d*g + b*c*h + a*d*h)*n*Log[g + h*x])
/((b*g - a*h)^2*(d*g - c*h)^2))/2
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.70, size = 4925, normalized size = 25.79

method	result	size
risch	Expression too large to display	4925

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/(h*x+g)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*B/h/(h*x+g)^2*ln((d*x+c)^n)-1/4*(-2*I*B*Pi*a*b*c^2*g*h^3*csgn(I*e)*csgn
(I*e/((d*x+c)^n)*(b*x+a)^n)^2-2*I*B*Pi*a*b*c^2*g*h^3*csgn(I*(b*x+a)^n)*csgn
(I*(b*x+a)^n/((d*x+c)^n))^2-2*I*B*Pi*a*b*c^2*g*h^3*csgn(I/((d*x+c)^n))*csgn
(I*(b*x+a)^n/((d*x+c)^n))^2-4*B*a^2*c*d*g*h^3*ln((b*x+a)^n)-4*B*a*b*c^2*g*h
^3*ln((b*x+a)^n)-4*B*a*b*d^2*g^3*h*ln((b*x+a)^n)-4*B*b^2*c*d*g^3*h*ln((b*x+
```

a)^n)+2*B*ln(e)*a^2*d^2*g^2*h^2+2*B*ln(e)*b^2*c^2*g^2*h^2+I*B*Pi*a^2*d^2*g^2*h^2*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2+I*B*Pi*b^2*c^2*g^2*h^2*csgn(I*e)*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2-4*A*a^2*c*d*g*h^3-4*A*a*b*c^2*g*h^3-4*A*a*b*d^2*g^3*h-4*A*b^2*c*d*g^3*h-4*B*ln(e)*a*b*c^2*g*h^3-4*B*ln(e)*a*b*d^2*g^3*h-2*B*a^2*c*d*h^4*n*x+2*B*a^2*d^2*g*h^3*n*x+2*B*a*b*c^2*h^4*n*x-2*B*b^2*c^2*g*h^3*n*x+8*A*a*b*c*d*g^2*h^2-I*B*Pi*a^2*d^2*g^2*h^2*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^3-I*B*Pi*b^2*c^2*g^2*h^2*csgn(I*(b*x+a)^n/((d*x+c)^n))^3-2*I*B*Pi*a*b*d^2*g^3*h*csgn(I*e)*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2-2*I*B*Pi*a*b*d^2*g^3*h*csgn(I*(b*x+a)^n)*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+2*I*B*Pi*a*b*d^2*g^3*h*csgn(I*(b*x+a)^n/((d*x+c)^n))^3-2*I*B*Pi*a*b*d^2*g^3*h*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))^2-2*I*B*Pi*a*b*d^2*g^3*h*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2-4*I*B*Pi*a*b*c*d*g^2*h^2*csgn(I*(b*x+a)^n)*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))+2*A*a^2*d^2*g^2*h^2+2*A*b^2*c^2*g^2*h^2+I*B*Pi*b^2*c^2*g^2*h^2*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+I*B*Pi*b^2*c^2*g^2*h^2*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2-I*B*Pi*a^2*c^2*h^4*csgn(I*(b*x+a)^n)*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))+2*I*B*Pi*a*b*d^2*g^3*h*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^3+I*B*Pi*a^2*d^2*g^2*h^2*csgn(I*(b*x+a)^n)*csgn(I*(b*x+a)^n/((d*x+c)^n))^2-2*B*a^2*c*d*g*h^3*n+2*B*a*b*c^2*g*h^3*n-2*B*a*b*d^2*g^3*h*n+2*B*b^2*c*d*g^3*h*n+2*I*B*Pi*a*b*c^2*g*h^3*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^3-2*I*B*Pi*b^2*c*d*g^3*h*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2+2*I*B*Pi*a*b*c^2*g*h^3*csgn(I*(b*x+a)^n)*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))+2*I*B*Pi*a*b*d^2*g^3*h*csgn(I*e)*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)-4*I*B*Pi*a*b*c*d*g^2*h^2*csgn(I*e)*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)+2*I*B*Pi*a^2*c*d*g*h^3*csgn(I*(b*x+a)^n)*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))+8*B*ln(e)*a*b*c*d*g^2*h^2+2*I*B*Pi*b^2*c*d*g^3*h*csgn(I*(b*x+a)^n/((d*x+c)^n))^3+2*I*B*Pi*b^2*c*d*g^3*h*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^3-I*B*Pi*a^2*c^2*h^4*csgn(I*e)*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)-8*B*ln(-d*x-c)*a*b*d^2*g^2*h^2*n*x+8*B*ln(-h*x-g)*a*b*d^2*g^2*h^2*n*x-8*B*ln(-h*x-g)*b^2*c*d*g^2*h^2*n*x-4*B*ln(-d*x-c)*a*b*d^2*g*h^3*n*x^2+4*B*ln(-h*x-g)*a*b*d^2*g*h^3*n*x^2-4*B*ln(-h*x-g)*b^2*c*d*g*h^3*n*x^2+4*B*ln(b*x+a)*b^2*c*d*g*h^3*n*x^2+8*B*ln(b*x+a)*b^2*c*d*g^2*h^2*n*x+2*I*B*Pi*b^2*c*d*g^3*h*csgn(I*e)*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)+2*I*B*Pi*b^2*c*d*g^3*h*csgn(I*(b*x+a)^n)*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))+I*B*Pi*a^2*c^2*h^4*csgn(I*e)*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2+I*B*Pi*a^2*c^2*h^4*csgn(I*(b*x+a)^n)*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+I*B*Pi*a^2*c^2*h^4*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+I*B*Pi*a^2*c^2*h^4*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2+2*B*a^2*c^2*h^4*ln((b*x+a)^n)+2*B*b^2*d^2*g^4*ln((b*x+a)^n)+4*I*B*Pi*a*b*c*d*g^2*h^2*csgn(I*e)*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2-I*B*Pi*b^2*d^2*g^4*csgn(I*(b*x+a)^n)*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))+I*B*Pi*a^2*d^2*g^2*h^2*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+I*B*Pi*b^2*c^2*g^2*h^2*csgn(I*(b*x+a)^n)*csgn(I*(b*x+a)^n/((d*x+c)^n))^2-2*I*B*Pi*b^2*c*d*

```
g^3*h*csgn(I*e)*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2-2*I*B*Pi*b^2*c*d*g^3*h*cs
gn(I*(b*x+a)^n)*csgn(I*(b*x+a)^n/((d*x+c)^n))^2-2*I*B*Pi*b^2*c*d*g^3*h*csgn
(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))^2-4*I*B*Pi*a*b*c*d*g^2*h^2*cs
gn(I*(b*x+a)^n/((d*x+c)^n))^3-2*I*B*Pi*a^2*c*d*g*h^3*csgn(I*e)*csgn(I*e/((d
*x+c)^n)*(b*x+a)^n)^2-2*I*B*Pi*a*b*c^2*g*h^3*csgn(I*(b*x+a)^n/((d*x+c)^n))*
csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2-I*B*Pi*b^2*c^2*g^2*h^2*csgn(I*e)*csgn(I*(
b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)+2*I*B*Pi*a*b*c^2*g*h^
3*csgn(I*e)*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)-4
*B*ln(e)*b^2*c*d*g^3*h-4*B*ln(e)*a^2*c*d*g*h^3-I*B*Pi*a^2*c^2*h^4*csgn(I*(b
*x+a)^n/((d*x+c)^n))^3-I*B*Pi*a^2*c^2*h^4*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^3
-I*B*Pi*b^2*d^2*g^4*csgn(I*(b*x+a)^n/((d*x+c)^n))^3+2*B*ln(-d*x-c)*b^2*d^2*
g^2*h^2*n*x^2-2*B*ln(b*x+a)*b^2*d^2*g^2*h^2*n*x^2+4*B*ln(-d*x-c)*a^2*d^2*g*
h^3*n*x+4*B*ln(-d*x-c)*b^2*d^2*g^3*h*n*x-4*B*ln(-h*x-g)*a^2*d^2*g*h^3*n*x+4
*B*ln(-h*x-g)*b^2*c^2*g*h^3*n*x-4*B*ln(b*x+a)*b^2*c^2*g*h^3*n*x-4*B*ln(b*x+
a)*b^2*d^2*g^3*h*n*x-4*B*ln(-d*x-c)*a*b*d^2*g^3*h*n-4*I*B*Pi*a*b*c*d*g^2*h^
2*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^3-I*B*Pi*a^2*d^2*g^2*h^2*csgn(I*(b*x+a)^n
/((d*x+c)^n))^3+2*B*a^2*d^2*g^2*h^2*ln((b*x+a)^...
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 382 vs. $2(182) = 364$.
time = 0.30, size = 382, normalized size = 2.00

$$\frac{1}{2}\left(\frac{b^2ne\log(bx+a)}{b^2g^2h-2\,abgh^2+a^2h^3}-\frac{d^2ne\log(dx+c)}{d^2g^2h-2\,cdgh^2+c^2h^3}-\frac{(2\,abd^2gn-a^2d^2hn-(2\,cdgn-c^2hn)b^2)e\log(hx+g)}{(d^2g^2h^2-2\,cdgh^3+c^2h^4)a^2-2\,(d^2g^3h-2\,cdg^2h^2+c^2gh^3)ab+(d^2g^4-2\,cdg^3h+c^2g^2h^2)b^2}+\frac{(bcn-adn)e}{(dg^2h-cgh^2)a-(dg^3-cg^2h)b+((dgh^2-ch^3)a-(dg^2h-cgh^2)b)x}\right)Be^{(-1)}-\frac{B\log\left(\frac{(bx+a)^ne}{(dx+c)^n}\right)}{2\,(h^3x^2+2\,gh^2x+g^2h)}-\frac{A}{2\,(h^3x^2+2\,gh^2x+g^2h)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(h*x+g)^3,x, algorithm="maxima
")
```

```
[Out] 1/2*(b^2*n*e*log(b*x + a)/(b^2*g^2*h - 2*a*b*g*h^2 + a^2*h^3) - d^2*n*e*log
(d*x + c)/(d^2*g^2*h - 2*c*d*g*h^2 + c^2*h^3) - (2*a*b*d^2*g*n - a^2*d^2*h*
n - (2*c*d*g*n - c^2*h*n)*b^2)*e*log(h*x + g)/((d^2*g^2*h^2 - 2*c*d*g*h^3 +
 c^2*h^4)*a^2 - 2*(d^2*g^3*h - 2*c*d*g^2*h^2 + c^2*g*h^3)*a*b + (d^2*g^4 -
2*c*d*g^3*h + c^2*g^2*h^2)*b^2) + (b*c*n - a*d*n)*e/((d*g^2*h - c*g*h^2)*a
- (d*g^3 - c*g^2*h)*b + ((d*g*h^2 - c*h^3)*a - (d*g^2*h - c*g*h^2)*b)*x))*B
*e^(-1) - 1/2*B*log((b*x + a)^n*e/(d*x + c)^n)/(h^3*x^2 + 2*g*h^2*x + g^2*h
) - 1/2*A/(h^3*x^2 + 2*g*h^2*x + g^2*h)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1046 vs. $2(182) = 364$.
time = 45.46, size = 1046, normalized size = 5.48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(h*x+g)^3,x, algorithm="fricas
")
```

```
[Out] -1/2*((A + B)*b^2*d^2*g^4 + (A + B)*a^2*c^2*h^4 - 2*((A + B)*b^2*c*d + (A +
 B)*a*b*d^2)*g^3*h + ((A + B)*b^2*c^2 + 4*(A + B)*a*b*c*d + (A + B)*a^2*d^2
)*g^2*h^2 - 2*((A + B)*a*b*c^2 + (A + B)*a^2*c*d)*g*h^3 + ((B*b^2*c*d - B*a
*b*d^2)*g^2*h^2 - (B*b^2*c^2 - B*a^2*d^2)*g*h^3 + (B*a*b*c^2 - B*a^2*c*d)*h
^4)*n*x + ((B*b^2*c*d - B*a*b*d^2)*g^3*h - (B*b^2*c^2 - B*a^2*d^2)*g^2*h^2
+ (B*a*b*c^2 - B*a^2*c*d)*g*h^3)*n - ((B*b^2*d^2*g^2*h^2 - 2*B*b^2*c*d*g*h^
3 + B*b^2*c^2*h^4)*n*x^2 + 2*(B*b^2*d^2*g^3*h - 2*B*b^2*c*d*g^2*h^2 + B*b^2
*c^2*g*h^3)*n*x + (2*B*a*b*d^2*g^3*h - B*a^2*c^2*h^4 - (4*B*a*b*c*d + B*a^2
*d^2)*g^2*h^2 + 2*(B*a*b*c^2 + B*a^2*c*d)*g*h^3)*n)*log(b*x + a) + ((B*b^2*
d^2*g^2*h^2 - 2*B*a*b*d^2*g*h^3 + B*a^2*d^2*h^4)*n*x^2 + 2*(B*b^2*d^2*g^3*h
 - 2*B*a*b*d^2*g^2*h^2 + B*a^2*d^2*g*h^3)*n*x + (2*B*b^2*c*d*g^3*h - B*a^2*
c^2*h^4 - (B*b^2*c^2 + 4*B*a*b*c*d)*g^2*h^2 + 2*(B*a*b*c^2 + B*a^2*c*d)*g*h
^3)*n)*log(d*x + c) - ((2*(B*b^2*c*d - B*a*b*d^2)*g*h^3 - (B*b^2*c^2 - B*a^
2*d^2)*h^4)*n*x^2 + 2*(2*(B*b^2*c*d - B*a*b*d^2)*g^2*h^2 - (B*b^2*c^2 - B*a
^2*d^2)*g*h^3)*n*x + (2*(B*b^2*c*d - B*a*b*d^2)*g^3*h - (B*b^2*c^2 - B*a^2*
d^2)*g^2*h^2)*n)*log(h*x + g))/(b^2*d^2*g^6*h + a^2*c^2*g^2*h^5 - 2*(b^2*c*
d + a*b*d^2)*g^5*h^2 + (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*g^4*h^3 - 2*(a*b*c^2
 + a^2*c*d)*g^3*h^4 + (b^2*d^2*g^4*h^3 + a^2*c^2*h^7 - 2*(b^2*c*d + a*b*d^2
)*g^3*h^4 + (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*g^2*h^5 - 2*(a*b*c^2 + a^2*c*d)
*g*h^6)*x^2 + 2*(b^2*d^2*g^5*h^2 + a^2*c^2*g*h^6 - 2*(b^2*c*d + a*b*d^2)*g^
4*h^3 + (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*g^3*h^4 - 2*(a*b*c^2 + a^2*c*d)*g^2
*h^5)*x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))/(h*x+g)**3,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 523 vs. 2(182) = 364.
time = 2.72, size = 523, normalized size = 2.74

$$\frac{Bb^3n\log(|bx+a|)}{2(b^3g^2h-2ab^2gh^2+a^2bh^3)}-\frac{Bd^3n\log(|dx+c|)}{2(d^3g^2h-2cd^2gh^2+c^2dh^3)}-\frac{Bn\log(bx+a)}{2(h^3x^2+2gh^2x+g^2h)}+\frac{Bn\log(dx+c)}{2(h^3x^2+2gh^2x+g^2h)}+\frac{(2Bb^2cdgn-2Babd^2gn-Bb^2c^2hn+Ba^2d^2hn)\log(hx+g)}{2(b^2d^2g^4-2b^2cdg^3h-2abd^2g^3h+b^2c^2g^2h^2+4abcdg^2h^2+a^2d^2g^2h^2-2abc^2gh^3-2a^2cdgh^3+a^2c^2h^4)}-\frac{Bbch^2nx-Badh^2nx+Bbcghn-Badghn+Abdg^2+Bbdg^2-Abcgh-Bbcgh-Aadgh-Badgh+Aach^2+Bach^2}{2(bdg^2h^3x^2-bcgh^4x^2-adgh^4x^2+ach^5x^2+2bdg^3h^2x-2bcg^2h^3x-2adg^2h^3x+2acgh^4x+bdg^4h-bcg^3h^2-adg^3h^2+acg^2h^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(h*x+g)^3,x, algorithm="giac")
```

```
[Out] 1/2*B*b^3*n*log(abs(b*x + a))/(b^3*g^2*h - 2*a*b^2*g*h^2 + a^2*b*h^3) - 1/2
*B*d^3*n*log(abs(d*x + c))/(d^3*g^2*h - 2*c*d^2*g*h^2 + c^2*d*h^3) - 1/2*B*
n*log(b*x + a)/(h^3*x^2 + 2*g*h^2*x + g^2*h) + 1/2*B*n*log(d*x + c)/(h^3*x^
2 + 2*g*h^2*x + g^2*h) + 1/2*(2*B*b^2*c*d*g*n - 2*B*a*b*d^2*g*n - B*b^2*c^2
```

```
*h*n + B*a^2*d^2*h*n)*log(h*x + g)/(b^2*d^2*g^4 - 2*b^2*c*d*g^3*h - 2*a*b*d
^2*g^3*h + b^2*c^2*g^2*h^2 + 4*a*b*c*d*g^2*h^2 + a^2*d^2*g^2*h^2 - 2*a*b*c^
2*g*h^3 - 2*a^2*c*d*g*h^3 + a^2*c^2*h^4) - 1/2*(B*b*c*h^2*n*x - B*a*d*h^2*n
*x + B*b*c*g*h*n - B*a*d*g*h*n + A*b*d*g^2 + B*b*d*g^2 - A*b*c*g*h - B*b*c*
g*h - A*a*d*g*h - B*a*d*g*h + A*a*c*h^2 + B*a*c*h^2)/(b*d*g^2*h^3*x^2 - b*c
*g*h^4*x^2 - a*d*g*h^4*x^2 + a*c*h^5*x^2 + 2*b*d*g^3*h^2*x - 2*b*c*g^2*h^3*
x - 2*a*d*g^2*h^3*x + 2*a*c*g*h^4*x + b*d*g^4*h - b*c*g^3*h^2 - a*d*g^3*h^2
 + a*c*g^2*h^3)
```

Mupad [B]
time = 6.35, size = 431, normalized size = 2.26

$$\frac{\ln(g+hx)\,\left(h\,(B\,a^2\,d^2\,n-B\,b^2\,c^2\,n)-2\,B\,a\,b\,d^2\,g\,n+2\,B\,b^2\,c\,d\,g\,n\right)}{2\,a^2\,c^2\,h^4-4\,a^2\,c\,d\,g\,h^3+2\,a^2\,d^2\,g^2\,h^2-4\,a\,b\,c^2\,g\,h^3+8\,a\,b\,c\,d\,g^2\,h^2-4\,a\,b\,d^2\,g^3\,h+2\,b^2\,c^2\,g^2\,h^2-4\,b^2\,c\,d\,g^3\,h+2\,b^2\,d^2\,g^4}-\frac{\frac{A\,a\,c\,h^2+A\,b\,d\,g^2-A\,a\,d\,g\,h-A\,b\,c\,g\,h-B\,a\,d\,g\,h\,n+B\,b\,c\,g\,h\,n}{a\,c\,h^2+b\,d\,g^2-a\,d\,g\,h-b\,c\,g\,h}-\frac{x\,(B\,a\,d\,h^2\,n-B\,b\,c\,h^2\,n)}{a\,c\,h^2+b\,d\,g^2-a\,d\,g\,h-b\,c\,g\,h}}{2\,g^2\,h+4\,g\,h^2\,x+2\,h^3\,x^2}-\frac{B\,\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)}{2\,h\,(g^2+2\,g\,h\,x+h^2\,x^2)}+\frac{B\,b^2\,n\,\ln(a+b\,x)}{2\,a^2\,h^3-4\,a\,b\,g\,h^2+2\,b^2\,g^2\,h}-\frac{B\,d^2\,n\,\ln(c+d\,x)}{2\,c^2\,h^3-4\,c\,d\,g\,h^2+2\,d^2\,g^2\,h}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))/(g + h*x)^3,x)
```

```
[Out] (log(g + h*x)*(h*(B*a^2*d^2*n - B*b^2*c^2*n) - 2*B*a*b*d^2*g*n + 2*B*b^2*c*
d*g*n))/(2*a^2*c^2*h^4 + 2*b^2*d^2*g^4 + 2*a^2*d^2*g^2*h^2 + 2*b^2*c^2*g^2*
h^2 - 4*a*b*c^2*g*h^3 - 4*a*b*d^2*g^3*h - 4*a^2*c*d*g*h^3 - 4*b^2*c*d*g^3*h
 + 8*a*b*c*d*g^2*h^2) - ((A*a*c*h^2 + A*b*d*g^2 - A*a*d*g*h - A*b*c*g*h - B
*a*d*g*h*n + B*b*c*g*h*n)/(a*c*h^2 + b*d*g^2 - a*d*g*h - b*c*g*h) - (x*(B*a
*d*h^2*n - B*b*c*h^2*n))/(a*c*h^2 + b*d*g^2 - a*d*g*h - b*c*g*h))/(2*g^2*h
+ 2*h^3*x^2 + 4*g*h^2*x) - (B*log((e*(a + b*x)^n)/(c + d*x)^n))/(2*h*(g^2 +
 h^2*x^2 + 2*g*h*x)) + (B*b^2*n*log(a + b*x))/(2*a^2*h^3 + 2*b^2*g^2*h - 4*
a*b*g*h^2) - (B*d^2*n*log(c + d*x))/(2*c^2*h^3 + 2*d^2*g^2*h - 4*c*d*g*h^2)
```

3.301 $\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(g+hx)^4}\,dx$

Optimal. Leaf size=284

$$-\frac{B(bc-ad)n}{6(bg-ah)(dg-ch)(g+hx)^2}-\frac{B(bc-ad)(2bdg-bch-adh)n}{3(bg-ah)^2(dg-ch)^2(g+hx)}+\frac{b^3Bn\log(a+bx)}{3h(bg-ah)^3}-\frac{Bd^3n\log(c+dx)}{3h(dg-ch)^3}-$$

```
[Out] -1/6*B*(-a*d+b*c)*n/(-a*h+b*g)/(-c*h+d*g)/(h*x+g)^2-1/3*B*(-a*d+b*c)*(-a*d*
h-b*c*h+2*b*d*g)*n/(-a*h+b*g)^2/(-c*h+d*g)^2/(h*x+g)+1/3*b^3*B*n*ln(b*x+a)/
h/(-a*h+b*g)^3-1/3*B*d^3*n*ln(d*x+c)/h/(-c*h+d*g)^3+1/3*(-A-B*ln(e*(b*x+a)^
n/((d*x+c)^n)))/h/(h*x+g)^3+1/3*B*(-a*d+b*c)*(a^2*d^2*h^2-a*b*d*h*(-c*h+3*d
*g)+b^2*(c^2*h^2-3*c*d*g*h+3*d^2*g^2))*n*ln(h*x+g)/(-a*h+b*g)^3/(-c*h+d*g)^
3
```

Rubi [A]
time = 0.29, antiderivative size = 284, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {2548, 84}

$$\frac{Bn(bc-ad)\log(g+hx)\left(a^2d^2h^2-abdh(3dg-ch)+b^2\left(c^2h^2-3cdgh+3d^2g^2\right)\right)}{3(bg-ah)^3(dg-ch)^3}-\frac{B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A}{3h(g+hx)^3}+\frac{b^3Bn\log(a+bx)}{3h(bg-ah)^3}-\frac{Bn(bc-ad)(-adh-bch+2bdg)}{3(g+hx)(bg-ah)^2(dg-ch)^2}-\frac{Bn(bc-ad)}{6(g+hx)^2(bg-ah)(dg-ch)}-\frac{Bd^3n\log(c+dx)}{3h(dg-ch)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(g + h*x)^4,x]
```

```
[Out] -1/6*(B*(b*c - a*d)*n)/((b*g - a*h)*(d*g - c*h)*(g + h*x)^2) - (B*(b*c - a*
d)*(2*b*d*g - b*c*h - a*d*h)*n)/(3*(b*g - a*h)^2*(d*g - c*h)^2*(g + h*x)) +
 (b^3*B*n*Log[a + b*x])/(3*h*(b*g - a*h)^3) - (B*d^3*n*Log[c + d*x])/(3*h*(
d*g - c*h)^3) - (A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(3*h*(g + h*x)^3)
+ (B*(b*c - a*d)*(a^2*d^2*h^2 - a*b*d*h*(3*d*g - c*h) + b^2*(3*d^2*g^2 - 3*
c*d*g*h + c^2*h^2))*n*Log[g + h*x])/(3*(b*g - a*h)^3*(d*g - c*h)^3)
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 2548

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Dist[B*n*((b*c
- a*d)/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /;
 FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c -
 a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Rubi steps

$$
\begin{aligned}
\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(g+hx)^4}\,dx &= \int \left(\frac{A}{(g+hx)^4}+\frac{B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(g+hx)^4}\right) dx \\
&= -\frac{A}{3h(g+hx)^3}+B\int \frac{\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(g+hx)^4}\,dx \\
&= -\frac{A}{3h(g+hx)^3}-\frac{B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{3h(g+hx)^3}+\frac{(B(bc-ad)n)\int
\\
&= -\frac{A}{3h(g+hx)^3}-\frac{B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{3h(g+hx)^3}+\frac{(B(bc-ad)n)\int
\\
&= -\frac{A}{3h(g+hx)^3}-\frac{B(bc-ad)n}{6(bg-ah)(dg-ch)(g+hx)^2}-\frac{B(bc-ad)(2bc}{3(bg-ah)^2(d
\end{aligned}
$$

Mathematica [A]
time = 0.42, size = 330, normalized size = 1.16

$$\frac{1}{6}\left(\frac{B(-bc+ad)n}{(bg-ah)(dg-ch)(g+hx)^2}+\frac{2B(bc-ad)(-2bdg+bch+adh)n}{(bg-ah)^2(dg-ch)^2(g+hx)}-\frac{2b^3Bn\log(a+bx)}{h(-bg+ah)^3}-\frac{2Bn\log(a+bx)}{h(g+hx)^3}+\frac{2Bd^3n\log(c+dx)}{h(-dg+ch)^3}+\frac{2Bn\log(c+dx)}{h(g+hx)^3}-\frac{2(A-Bn\log(a+bx)+Bn\log(c+dx)+B\log\left(e(a+bx)^n(c+dx)^{-n}\right))}{h(g+hx)^3}+\frac{2B(bc-ad)\left(a^2d^2h^2+abdh(-3dg+ch)+b^2(3d^2g^2-3cdgh+c^2h^2)\right)n\log(g+hx)}{(bg-ah)^3(dg-ch)^3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(g + h*x)^4,x]
```

```
[Out] ((B*(-(b*c) + a*d)*n)/((b*g - a*h)*(d*g - c*h)*(g + h*x)^2) + (2*B*(b*c - a
*d)*(-2*b*d*g + b*c*h + a*d*h)*n)/((b*g - a*h)^2*(d*g - c*h)^2*(g + h*x)) -
 (2*b^3*B*n*Log[a + b*x])/(h*(-(b*g) + a*h)^3) - (2*B*n*Log[a + b*x])/(h*(g
 + h*x)^3) + (2*B*d^3*n*Log[c + d*x])/(h*(-(d*g) + c*h)^3) + (2*B*n*Log[c +
 d*x])/(h*(g + h*x)^3) - (2*(A - B*n*Log[a + b*x] + B*n*Log[c + d*x] + B*Lo
g[(e*(a + b*x)^n)/(c + d*x)^n]))/(h*(g + h*x)^3) + (2*B*(b*c - a*d)*(a^2*d^
2*h^2 + a*b*d*h*(-3*d*g + c*h) + b^2*(3*d^2*g^2 - 3*c*d*g*h + c^2*h^2))*n*L
og[g + h*x])/((b*g - a*h)^3*(d*g - c*h)^3))/6
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.74, size = 9645, normalized size = 33.96

method	result	size
risch	Expression too large to display	9645

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/(h*x+g)^4,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 914 vs. $2(273) = 546$.
time = 0.38, size = 914, normalized size = 3.22

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(h*x+g)^4,x, algorithm="maxima
")
```

```
[Out] 1/6*(2*b^3*n*e*log(b*x + a)/(b^3*g^3*h - 3*a*b^2*g^2*h^2 + 3*a^2*b*g*h^3 -
a^3*h^4) - 2*d^3*n*e*log(d*x + c)/(d^3*g^3*h - 3*c*d^2*g^2*h^2 + 3*c^2*d*g*
h^3 - c^3*h^4) + 2*(3*a*b^2*d^3*g^2*n - 3*a^2*b*d^3*g*h*n + a^3*d^3*h^2*n -
 (3*c*d^2*g^2*n - 3*c^2*d*g*h*n + c^3*h^2*n)*b^3)*e*log(h*x + g)/((d^3*g^3*
h^3 - 3*c*d^2*g^2*h^4 + 3*c^2*d*g*h^5 - c^3*h^6)*a^3 - 3*(d^3*g^4*h^2 - 3*c
*d^2*g^3*h^3 + 3*c^2*d*g^2*h^4 - c^3*g*h^5)*a^2*b + 3*(d^3*g^5*h - 3*c*d^2*
g^4*h^2 + 3*c^2*d*g^3*h^3 - c^3*g^2*h^4)*a*b^2 - (d^3*g^6 - 3*c*d^2*g^5*h +
 3*c^2*d*g^4*h^2 - c^3*g^3*h^3)*b^3) + (2*(2*a*b*d^2*g*h*n - a^2*d^2*h^2*n
- (2*c*d*g*h*n - c^2*h^2*n)*b^2)*x*e - ((3*d^2*g*h*n - c*d*h^2*n)*a^2 - (5*
d^2*g^2*n - c^2*h^2*n)*a*b + (5*c*d*g^2*n - 3*c^2*g*h*n)*b^2)*e)/((d^2*g^4*
h^2 - 2*c*d*g^3*h^3 + c^2*g^2*h^4)*a^2 - 2*(d^2*g^5*h - 2*c*d*g^4*h^2 + c^2
*g^3*h^3)*a*b + (d^2*g^6 - 2*c*d*g^5*h + c^2*g^4*h^2)*b^2 + ((d^2*g^2*h^4 -
 2*c*d*g*h^5 + c^2*h^6)*a^2 - 2*(d^2*g^3*h^3 - 2*c*d*g^2*h^4 + c^2*g*h^5)*a
*b + (d^2*g^4*h^2 - 2*c*d*g^3*h^3 + c^2*g^2*h^4)*b^2)*x^2 + 2*((d^2*g^3*h^3
 - 2*c*d*g^2*h^4 + c^2*g*h^5)*a^2 - 2*(d^2*g^4*h^2 - 2*c*d*g^3*h^3 + c^2*g^
2*h^4)*a*b + (d^2*g^5*h - 2*c*d*g^4*h^2 + c^2*g^3*h^3)*b^2)*x))*B*e^(-1) -
1/3*B*log((b*x + a)^n*e/(d*x + c)^n)/(h^4*x^3 + 3*g*h^3*x^2 + 3*g^2*h^2*x +
 g^3*h) - 1/3*A/(h^4*x^3 + 3*g*h^3*x^2 + 3*g^2*h^2*x + g^3*h)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(h*x+g)^4,x, algorithm="fricas
")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))/(h*x+g)**4,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1512 vs. $2(273) = 546$.
time = 3.25, size = 1512, normalized size = 5.32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(h*x+g)^4,x, algorithm="giac")
```

```
[Out] 1/3*B*b^4*n*log(abs(b*x + a))/(b^4*g^3*h - 3*a*b^3*g^2*h^2 + 3*a^2*b^2*g*h^
3 - a^3*b*h^4) - 1/3*B*d^4*n*log(abs(d*x + c))/(d^4*g^3*h - 3*c*d^3*g^2*h^2
 + 3*c^2*d^2*g*h^3 - c^3*d*h^4) - 1/3*B*n*log(b*x + a)/(h^4*x^3 + 3*g*h^3*x
^2 + 3*g^2*h^2*x + g^3*h) + 1/3*B*n*log(d*x + c)/(h^4*x^3 + 3*g*h^3*x^2 + 3
*g^2*h^2*x + g^3*h) + 1/3*(3*B*b^3*c*d^2*g^2*n - 3*B*a*b^2*d^3*g^2*n - 3*B*
b^3*c^2*d*g*h*n + 3*B*a^2*b*d^3*g*h*n + B*b^3*c^3*h^2*n - B*a^3*d^3*h^2*n)*
log(h*x + g)/(b^3*d^3*g^6 - 3*b^3*c*d^2*g^5*h - 3*a*b^2*d^3*g^5*h + 3*b^3*c
^2*d*g^4*h^2 + 9*a*b^2*c*d^2*g^4*h^2 + 3*a^2*b*d^3*g^4*h^2 - b^3*c^3*g^3*h^
3 - 9*a*b^2*c^2*d*g^3*h^3 - 9*a^2*b*c*d^2*g^3*h^3 - a^3*d^3*g^3*h^3 + 3*a*b
^2*c^3*g^2*h^4 + 9*a^2*b*c^2*d*g^2*h^4 + 3*a^3*c*d^2*g^2*h^4 - 3*a^2*b*c^3*
g*h^5 - 3*a^3*c^2*d*g*h^5 + a^3*c^3*h^6) - 1/6*(4*B*b^2*c*d*g*h^3*n*x^2 - 4
*B*a*b*d^2*g*h^3*n*x^2 - 2*B*b^2*c^2*h^4*n*x^2 + 2*B*a^2*d^2*h^4*n*x^2 + 9*
B*b^2*c*d*g^2*h^2*n*x - 9*B*a*b*d^2*g^2*h^2*n*x - 5*B*b^2*c^2*g*h^3*n*x + 5
*B*a^2*d^2*g*h^3*n*x + B*a*b*c^2*h^4*n*x - B*a^2*c*d*h^4*n*x + 5*B*b^2*c*d*
g^3*h*n - 5*B*a*b*d^2*g^3*h*n - 3*B*b^2*c^2*g^2*h^2*n + 3*B*a^2*d^2*g^2*h^2
*n + B*a*b*c^2*g*h^3*n - B*a^2*c*d*g*h^3*n + 2*A*b^2*d^2*g^4 + 2*B*b^2*d^2*
g^4 - 4*A*b^2*c*d*g^3*h - 4*B*b^2*c*d*g^3*h - 4*A*a*b*d^2*g^3*h - 4*B*a*b*d
^2*g^3*h + 2*A*b^2*c^2*g^2*h^2 + 2*B*b^2*c^2*g^2*h^2 + 8*A*a*b*c*d*g^2*h^2
+ 8*B*a*b*c*d*g^2*h^2 + 2*A*a^2*d^2*g^2*h^2 + 2*B*a^2*d^2*g^2*h^2 - 4*A*a*b
*c^2*g*h^3 - 4*B*a*b*c^2*g*h^3 - 4*A*a^2*c*d*g*h^3 - 4*B*a^2*c*d*g*h^3 + 2*
A*a^2*c^2*h^4 + 2*B*a^2*c^2*h^4)/(b^2*d^2*g^4*h^4*x^3 - 2*b^2*c*d*g^3*h^5*x
^3 - 2*a*b*d^2*g^3*h^5*x^3 + b^2*c^2*g^2*h^6*x^3 + 4*a*b*c*d*g^2*h^6*x^3 +
a^2*d^2*g^2*h^6*x^3 - 2*a*b*c^2*g*h^7*x^3 - 2*a^2*c*d*g*h^7*x^3 + a^2*c^2*h
^8*x^3 + 3*b^2*d^2*g^5*h^3*x^2 - 6*b^2*c*d*g^4*h^4*x^2 - 6*a*b*d^2*g^4*h^4*
x^2 + 3*b^2*c^2*g^3*h^5*x^2 + 12*a*b*c*d*g^3*h^5*x^2 + 3*a^2*d^2*g^3*h^5*x^
2 - 6*a*b*c^2*g^2*h^6*x^2 - 6*a^2*c*d*g^2*h^6*x^2 + 3*a^2*c^2*g*h^7*x^2 + 3
*b^2*d^2*g^6*h^2*x - 6*b^2*c*d*g^5*h^3*x - 6*a*b*d^2*g^5*h^3*x + 3*b^2*c^2*
g^4*h^4*x + 12*a*b*c*d*g^4*h^4*x + 3*a^2*d^2*g^4*h^4*x - 6*a*b*c^2*g^3*h^5*
x - 6*a^2*c*d*g^3*h^5*x + 3*a^2*c^2*g^2*h^6*x + b^2*d^2*g^7*h - 2*b^2*c*d*g
^6*h^2 - 2*a*b*d^2*g^6*h^2 + b^2*c^2*g^5*h^3 + 4*a*b*c*d*g^5*h^3 + a^2*d^2*
g^5*h^3 - 2*a*b*c^2*g^4*h^4 - 2*a^2*c*d*g^4*h^4 + a^2*c^2*g^3*h^5)
```

Mupad [B]

time = 9.26, size = 1183, normalized size = 4.17

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))/(g + h*x)^4,x)
```

```
[Out] (B*d^3*n*log(c + d*x))/(3*c^3*h^4 - 3*d^3*g^3*h + 9*c*d^2*g^2*h^2 - 9*c^2*d
*g*h^3) - (log(g + h*x)*(h^2*(B*a^3*d^3*n - B*b^3*c^3*n) - h*(3*B*a^2*b*d^3
*g*n - 3*B*b^3*c^2*d*g*n) + 3*B*a*b^2*d^3*g^2*n - 3*B*b^3*c*d^2*g^2*n))/(3*
a^3*c^3*h^6 + 3*b^3*d^3*g^6 - 3*a^3*d^3*g^3*h^3 - 3*b^3*c^3*g^3*h^3 - 9*a^2
*b*c^3*g*h^5 - 9*a*b^2*d^3*g^5*h - 9*a^3*c^2*d*g*h^5 - 9*b^3*c*d^2*g^5*h +
9*a*b^2*c^3*g^2*h^4 + 9*a^2*b*d^3*g^4*h^2 + 9*a^3*c*d^2*g^2*h^4 + 9*b^3*c^2
*d*g^4*h^2 + 27*a*b^2*c*d^2*g^4*h^2 - 27*a*b^2*c^2*d*g^3*h^3 - 27*a^2*b*c*d
^2*g^3*h^3 + 27*a^2*b*c^2*d*g^2*h^4) - (B*log((e*(a + b*x)^n)/(c + d*x)^n))
/(3*h*(g^3 + h^3*x^3 + 3*g^2*h*x + 3*g*h^2*x^2)) - (B*b^3*n*log(a + b*x))/(
3*a^3*h^4 - 3*b^3*g^3*h + 9*a*b^2*g^2*h^2 - 9*a^2*b*g*h^3) - ((2*A*a^2*c^2*
h^4 + 2*A*b^2*d^2*g^4 + 2*A*a^2*d^2*g^2*h^2 + 2*A*b^2*c^2*g^2*h^2 + 3*B*a^2
*d^2*g^2*h^2*n - 3*B*b^2*c^2*g^2*h^2*n - 4*A*a*b*c^2*g*h^3 - 4*A*a*b*d^2*g^
3*h - 4*A*a^2*c*d*g*h^3 - 4*A*b^2*c*d*g^3*h + 8*A*a*b*c*d*g^2*h^2 + B*a*b*c
^2*g*h^3*n - 5*B*a*b*d^2*g^3*h*n - B*a^2*c*d*g*h^3*n + 5*B*b^2*c*d*g^3*h*n)
/(2*(a^2*c^2*h^4 + b^2*d^2*g^4 + a^2*d^2*g^2*h^2 + b^2*c^2*g^2*h^2 - 2*a*b*
c^2*g*h^3 - 2*a*b*d^2*g^3*h - 2*a^2*c*d*g*h^3 - 2*b^2*c*d*g^3*h + 4*a*b*c*d
*g^2*h^2)) + (x*(B*a*b*c^2*h^4*n - B*a^2*c*d*h^4*n + 5*B*a^2*d^2*g*h^3*n -
5*B*b^2*c^2*g*h^3*n - 9*B*a*b*d^2*g^2*h^2*n + 9*B*b^2*c*d*g^2*h^2*n))/(2*(a
^2*c^2*h^4 + b^2*d^2*g^4 + a^2*d^2*g^2*h^2 + b^2*c^2*g^2*h^2 - 2*a*b*c^2*g*
h^3 - 2*a*b*d^2*g^3*h - 2*a^2*c*d*g*h^3 - 2*b^2*c*d*g^3*h + 4*a*b*c*d*g^2*h
^2)) + (x^2*(B*a^2*d^2*h^4*n - B*b^2*c^2*h^4*n - 2*B*a*b*d^2*g*h^3*n + 2*B*
b^2*c*d*g*h^3*n))/(a^2*c^2*h^4 + b^2*d^2*g^4 + a^2*d^2*g^2*h^2 + b^2*c^2*g^
2*h^2 - 2*a*b*c^2*g*h^3 - 2*a*b*d^2*g^3*h - 2*a^2*c*d*g*h^3 - 2*b^2*c*d*g^3
*h + 4*a*b*c*d*g^2*h^2))/(3*g^3*h + 3*h^4*x^3 + 9*g^2*h^2*x + 9*g*h^3*x^2)
```

3.302 $\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(g+hx)^5}\,dx$

Optimal. Leaf size=389

$$-\frac{B(bc-ad)n}{12(bg-ah)(dg-ch)(g+hx)^3}-\frac{B(bc-ad)(2bdg-bch-adh)n}{8(bg-ah)^2(dg-ch)^2(g+hx)^2}-\frac{B(bc-ad)\left(a^2d^2h^2-abdh(3dg-ch)+\right.}{4(bg-ah)^3(dg-ch}$$

```
[Out] -1/12*B*(-a*d+b*c)*n/(-a*h+b*g)/(-c*h+d*g)/(h*x+g)^3-1/8*B*(-a*d+b*c)*(-a*d
*h-b*c*h+2*b*d*g)*n/(-a*h+b*g)^2/(-c*h+d*g)^2/(h*x+g)^2-1/4*B*(-a*d+b*c)*(a
^2*d^2*h^2-a*b*d*h*(-c*h+3*d*g)+b^2*(c^2*h^2-3*c*d*g*h+3*d^2*g^2))*n/(-a*h+
b*g)^3/(-c*h+d*g)^3/(h*x+g)+1/4*b^4*B*n*ln(b*x+a)/h/(-a*h+b*g)^4-1/4*B*d^4*
n*ln(d*x+c)/h/(-c*h+d*g)^4+1/4*(-A-B*ln(e*(b*x+a)^n/((d*x+c)^n)))/h/(h*x+g)
^4-1/4*B*(-a*d+b*c)*(-a*d*h-b*c*h+2*b*d*g)*(2*a*b*d^2*g*h-a^2*d^2*h^2-b^2*(
c^2*h^2-2*c*d*g*h+2*d^2*g^2))*n*ln(h*x+g)/(-a*h+b*g)^4/(-c*h+d*g)^4
```

Rubi [A]

time = 0.46, antiderivative size = 389, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {2548, 84}

$$-\frac{Bn(bc-ad)\left(a^2d^2h^2-abdh(3dg-ch)+b^2(c^2h^2-3cdgh+3d^2g^2)\right)}{4(g+hx)(bg-ah)^3(dg-ch)^3}-\frac{Bn(bc-ad)\log(g+hx)(-adh-bch+2bdg)\left(-a^2d^2h^2+2abd^2gh-(b^2(c^2h^2-2cdgh+2d^2g^2))\right)}{4(bg-ah)^4(dg-ch)^4}-\frac{B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A}{4h(g+hx)^4}+\frac{b^4Bn\log(a+bx)}{4h(bg-ah)^4}-\frac{Bn(bc-ad)(-adh-bch+2bdg)}{8(g+hx)^2(bg-ah)^2(dg-ch)^2}-\frac{Bn(bc-ad)}{12(g+hx)^3(bg-ah)(dg-ch)}-\frac{Bd^4n\log(c+dx)}{4h(dg-ch)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(g + h*x)^5,x]
```

```
[Out] -1/12*(B*(b*c - a*d)*n)/((b*g - a*h)*(d*g - c*h)*(g + h*x)^3) - (B*(b*c - a
*d)*(2*b*d*g - b*c*h - a*d*h)*n)/(8*(b*g - a*h)^2*(d*g - c*h)^2*(g + h*x)^2
) - (B*(b*c - a*d)*(a^2*d^2*h^2 - a*b*d*h*(3*d*g - c*h) + b^2*(3*d^2*g^2 -
3*c*d*g*h + c^2*h^2))*n)/(4*(b*g - a*h)^3*(d*g - c*h)^3*(g + h*x)) + (b^4*B
*n*Log[a + b*x])/(4*h*(b*g - a*h)^4) - (B*d^4*n*Log[c + d*x])/(4*h*(d*g - c
*h)^4) - (A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(4*h*(g + h*x)^4) - (B*(b
*c - a*d)*(2*b*d*g - b*c*h - a*d*h)*(2*a*b*d^2*g*h - a^2*d^2*h^2 - b^2*(2*d
^2*g^2 - 2*c*d*g*h + c^2*h^2))*n*Log[g + h*x])/(4*(b*g - a*h)^4*(d*g - c*h)
^4)
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 2548

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Dist[B*n*((b*c
```

```
- a*d)/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /;
 FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c -
 a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Rubi steps

$$\begin{aligned}
\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(g+hx)^5}\,dx &= \int \left(\frac{A}{(g+hx)^5}+\frac{B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(g+hx)^5}\right)dx \\
&= -\frac{A}{4h(g+hx)^4}+B\int \frac{\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(g+hx)^5}\,dx \\
&= -\frac{A}{4h(g+hx)^4}-\frac{B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{4h(g+hx)^4}+\frac{(B(bc-ad)n)}{} \\
&= -\frac{A}{4h(g+hx)^4}-\frac{B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{4h(g+hx)^4}+\frac{(B(bc-ad)n)}{} \\
&= -\frac{A}{4h(g+hx)^4}-\frac{B(bc-ad)n}{12(bg-ah)(dg-ch)(g+hx)^3}-\frac{B(bc-ad)(}{8(bg-ah)^2}
\end{aligned}$$

Mathematica [A]
time = 0.41, size = 455, normalized size = 1.17

$$\frac{1}{24}\left(\frac{2B(-bc+ad)n}{(bg-ah)(dg-ch)(g+hx)^3}+\frac{3B(bc-ad)(-2bdg+bch+adh)n}{(bg-ah)^2(dg-ch)^2(g+hx)^2}-\frac{6B(bc-ad)\left(a^2d^2h^2+abdh(-3dg+ch)+b^2(3d^2g^2-3cdgh+c^2h^2)\right)n}{(bg-ah)^3(dg-ch)^3(g+hx)}+\frac{6b^4Bn\log(a+bx)}{h(bg-ah)^4}-\frac{6Bn\log(a+bx)}{h(g+hx)^4}-\frac{6Bd^4n\log(c+dx)}{h(dg-ch)^4}+\frac{6Bn\log(c+dx)}{h(g+hx)^4}-\frac{6(A-Bn\log(a+bx)+Bn\log(c+dx)+B\log\left(e(a+bx)^n(c+dx)^{-n}\right))}{h(g+hx)^4}+\frac{6B(-4ab^3d^4g^3+6a^2b^2d^4g^2h-4a^3bd^4gh^2+a^4d^4h^3+b^4c(4d^3g^3-6cd^2g^2h+4c^2dgh^2-c^3h^3))\,n\log(g+hx)}{(bg-ah)^4(dg-ch)^4}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(g + h*x)^5,x]
```

```
[Out] ((2*B*(-(b*c) + a*d)*n)/((b*g - a*h)*(d*g - c*h)*(g + h*x)^3) + (3*B*(b*c -
 a*d)*(-2*b*d*g + b*c*h + a*d*h)*n)/((b*g - a*h)^2*(d*g - c*h)^2*(g + h*x)^
2) - (6*B*(b*c - a*d)*(a^2*d^2*h^2 + a*b*d*h*(-3*d*g + c*h) + b^2*(3*d^2*g^
2 - 3*c*d*g*h + c^2*h^2))*n)/((b*g - a*h)^3*(d*g - c*h)^3*(g + h*x)) + (6*b
^4*B*n*Log[a + b*x])/(h*(b*g - a*h)^4) - (6*B*n*Log[a + b*x])/(h*(g + h*x)^
4) - (6*B*d^4*n*Log[c + d*x])/(h*(d*g - c*h)^4) + (6*B*n*Log[c + d*x])/(h*(
g + h*x)^4) - (6*(A - B*n*Log[a + b*x] + B*n*Log[c + d*x] + B*Log[(e*(a + b
*x)^n)/(c + d*x)^n]))/(h*(g + h*x)^4) + (6*B*(-4*a*b^3*d^4*g^3 + 6*a^2*b^2*
d^4*g^2*h - 4*a^3*b*d^4*g*h^2 + a^4*d^4*h^3 + b^4*c*(4*d^3*g^3 - 6*c*d^2*g^
2*h + 4*c^2*d*g*h^2 - c^3*h^3))*n*Log[g + h*x])/((b*g - a*h)^4*(d*g - c*h)^
4))/24
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 1.10, size = 16077, normalized size = 41.33

method	result	size
risch	Expression too large to display	16077

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/(h*x+g)^5,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1888 vs. $2(376) = 752$.
time = 0.51, size = 1888, normalized size = 4.85

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(h*x+g)^5,x, algorithm="maxima
")
```

```
[Out] 1/24*(6*b^4*n*e*log(b*x + a)/(b^4*g^4*h - 4*a*b^3*g^3*h^2 + 6*a^2*b^2*g^2*h
^3 - 4*a^3*b*g*h^4 + a^4*h^5) - 6*d^4*n*e*log(d*x + c)/(d^4*g^4*h - 4*c*d^3
*g^3*h^2 + 6*c^2*d^2*g^2*h^3 - 4*c^3*d*g*h^4 + c^4*h^5) - 6*(4*a*b^3*d^4*g^
3*n - 6*a^2*b^2*d^4*g^2*h*n + 4*a^3*b*d^4*g*h^2*n - a^4*d^4*h^3*n - (4*c*d^
3*g^3*n - 6*c^2*d^2*g^2*h*n + 4*c^3*d*g*h^2*n - c^4*h^3*n)*b^4)*e*log(h*x +
 g)/((d^4*g^4*h^4 - 4*c*d^3*g^3*h^5 + 6*c^2*d^2*g^2*h^6 - 4*c^3*d*g*h^7 + c
^4*h^8)*a^4 - 4*(d^4*g^5*h^3 - 4*c*d^3*g^4*h^4 + 6*c^2*d^2*g^3*h^5 - 4*c^3*
d*g^2*h^6 + c^4*g*h^7)*a^3*b + 6*(d^4*g^6*h^2 - 4*c*d^3*g^5*h^3 + 6*c^2*d^2
*g^4*h^4 - 4*c^3*d*g^3*h^5 + c^4*g^2*h^6)*a^2*b^2 - 4*(d^4*g^7*h - 4*c*d^3*
g^6*h^2 + 6*c^2*d^2*g^5*h^3 - 4*c^3*d*g^4*h^4 + c^4*g^3*h^5)*a*b^3 + (d^4*g
^8 - 4*c*d^3*g^7*h + 6*c^2*d^2*g^6*h^2 - 4*c^3*d*g^5*h^3 + c^4*g^4*h^4)*b^4
) - (6*(3*a*b^2*d^3*g^2*h^2*n - 3*a^2*b*d^3*g*h^3*n + a^3*d^3*h^4*n - (3*c*
d^2*g^2*h^2*n - 3*c^2*d*g*h^3*n + c^3*h^4*n)*b^3)*x^2*e + 3*((5*d^3*g*h^3*n
 - c*d^2*h^4*n)*a^3 - 3*(5*d^3*g^2*h^2*n - c*d^2*g*h^3*n)*a^2*b + (14*d^3*g
^3*h*n - 3*c^2*d*g*h^3*n + c^3*h^4*n)*a*b^2 - (14*c*d^2*g^3*h*n - 15*c^2*d*
g^2*h^2*n + 5*c^3*g*h^3*n)*b^3)*x*e + ((11*d^3*g^2*h^2*n - 7*c*d^2*g*h^3*n
+ 2*c^2*d*h^4*n)*a^3 - (31*d^3*g^3*h*n - 15*c*d^2*g^2*h^2*n + 2*c^3*h^4*n)*
a^2*b + (26*d^3*g^4*n - 15*c^2*d*g^2*h^2*n + 7*c^3*g*h^3*n)*a*b^2 - (26*c*d
^2*g^4*n - 31*c^2*d*g^3*h*n + 11*c^3*g^2*h^2*n)*b^3)*e)/((d^3*g^6*h^3 - 3*c
*d^2*g^5*h^4 + 3*c^2*d*g^4*h^5 - c^3*g^3*h^6)*a^3 - 3*(d^3*g^7*h^2 - 3*c*d^
2*g^6*h^3 + 3*c^2*d*g^5*h^4 - c^3*g^4*h^5)*a^2*b + 3*(d^3*g^8*h - 3*c*d^2*g
^7*h^2 + 3*c^2*d*g^6*h^3 - c^3*g^5*h^4)*a*b^2 - (d^3*g^9 - 3*c*d^2*g^8*h +
3*c^2*d*g^7*h^2 - c^3*g^6*h^3)*b^3 + ((d^3*g^3*h^6 - 3*c*d^2*g^2*h^7 + 3*c^
2*d*g*h^8 - c^3*h^9)*a^3 - 3*(d^3*g^4*h^5 - 3*c*d^2*g^3*h^6 + 3*c^2*d*g^2*h
^7 - c^3*g*h^8)*a^2*b + 3*(d^3*g^5*h^4 - 3*c*d^2*g^4*h^5 + 3*c^2*d*g^3*h^6
- c^3*g^2*h^7)*a*b^2 - (d^3*g^6*h^3 - 3*c*d^2*g^5*h^4 + 3*c^2*d*g^4*h^5 - c
```

```
^3*g^3*h^6)*b^3)*x^3 + 3*((d^3*g^4*h^5 - 3*c*d^2*g^3*h^6 + 3*c^2*d*g^2*h^7
- c^3*g*h^8)*a^3 - 3*(d^3*g^5*h^4 - 3*c*d^2*g^4*h^5 + 3*c^2*d*g^3*h^6 - c^3
*g^2*h^7)*a^2*b + 3*(d^3*g^6*h^3 - 3*c*d^2*g^5*h^4 + 3*c^2*d*g^4*h^5 - c^3*
g^3*h^6)*a*b^2 - (d^3*g^7*h^2 - 3*c*d^2*g^6*h^3 + 3*c^2*d*g^5*h^4 - c^3*g^4
*h^5)*b^3)*x^2 + 3*((d^3*g^5*h^4 - 3*c*d^2*g^4*h^5 + 3*c^2*d*g^3*h^6 - c^3*
g^2*h^7)*a^3 - 3*(d^3*g^6*h^3 - 3*c*d^2*g^5*h^4 + 3*c^2*d*g^4*h^5 - c^3*g^3
*h^6)*a^2*b + 3*(d^3*g^7*h^2 - 3*c*d^2*g^6*h^3 + 3*c^2*d*g^5*h^4 - c^3*g^4*
h^5)*a*b^2 - (d^3*g^8*h - 3*c*d^2*g^7*h^2 + 3*c^2*d*g^6*h^3 - c^3*g^5*h^4)*
b^3)*x))*B*e^(-1) - 1/4*B*log((b*x + a)^n*e/(d*x + c)^n)/(h^5*x^4 + 4*g*h^4
*x^3 + 6*g^2*h^3*x^2 + 4*g^3*h^2*x + g^4*h) - 1/4*A/(h^5*x^4 + 4*g*h^4*x^3
+ 6*g^2*h^3*x^2 + 4*g^3*h^2*x + g^4*h)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(h*x+g)^5,x, algorithm="fricas
")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))/(h*x+g)**5,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 3293 vs. 2(376) = 752.
time = 7.23, size = 3293, normalized size = 8.47

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(h*x+g)^5,x, algorithm="giac")
```

```
[Out] 1/4*B*b^5*n*log(abs(b*x + a))/(b^5*g^4*h - 4*a*b^4*g^3*h^2 + 6*a^2*b^3*g^2*
h^3 - 4*a^3*b^2*g*h^4 + a^4*b*h^5) - 1/4*B*d^5*n*log(abs(d*x + c))/(d^5*g^4
*h - 4*c*d^4*g^3*h^2 + 6*c^2*d^3*g^2*h^3 - 4*c^3*d^2*g*h^4 + c^4*d*h^5) - 1
/4*B*n*log(b*x + a)/(h^5*x^4 + 4*g*h^4*x^3 + 6*g^2*h^3*x^2 + 4*g^3*h^2*x +
```

g^4*h) + 1/4*B*n*log(d*x + c)/(h^5*x^4 + 4*g*h^4*x^3 + 6*g^2*h^3*x^2 + 4*g^3*h^2*x + g^4*h) + 1/4*(4*B*b^4*c*d^3*g^3*n - 4*B*a*b^3*d^4*g^3*n - 6*B*b^4*c^2*d^2*g^2*h*n + 6*B*a^2*b^2*d^4*g^2*h*n + 4*B*b^4*c^3*d*g*h^2*n - 4*B*a^3*b*d^4*g*h^2*n - B*b^4*c^4*h^3*n + B*a^4*d^4*h^3*n)*log(h*x + g)/(b^4*d^4*g^8 - 4*b^4*c*d^3*g^7*h - 4*a*b^3*d^4*g^7*h + 6*b^4*c^2*d^2*g^6*h^2 + 16*a*b^3*c*d^3*g^6*h^2 + 6*a^2*b^2*d^4*g^6*h^2 - 4*b^4*c^3*d*g^5*h^3 - 24*a*b^3*c^2*d^2*g^5*h^3 - 24*a^2*b^2*c*d^3*g^5*h^3 - 4*a^3*b*d^4*g^5*h^3 + b^4*c^4*g^4*h^4 + 16*a*b^3*c^3*d*g^4*h^4 + 36*a^2*b^2*c^2*d^2*g^4*h^4 + 16*a^3*b*c*d^3*g^4*h^4 + a^4*d^4*g^4*h^4 - 4*a*b^3*c^4*g^3*h^5 - 24*a^2*b^2*c^3*d*g^3*h^5 - 24*a^3*b*c^2*d^2*g^3*h^5 - 4*a^4*c*d^3*g^3*h^5 + 6*a^2*b^2*c^4*g^2*h^6 + 16*a^3*b*c^3*d*g^2*h^6 + 6*a^4*c^2*d^2*g^2*h^6 - 4*a^3*b*c^4*g*h^7 - 4*a^4*c^3*d*g*h^7 + a^4*c^4*h^8) - 1/24*(18*B*b^3*c*d^2*g^2*h^4*n*x^3 - 18*B*a*b^2*d^3*g^2*h^4*n*x^3 - 18*B*b^3*c^2*d*g*h^5*n*x^3 + 18*B*a^2*b*d^3*g*h^5*n*x^3 + 6*B*b^3*c^3*h^6*n*x^3 - 6*B*a^3*d^3*h^6*n*x^3 + 60*B*b^3*c*d^2*g^3*h^3*n*x^2 - 60*B*a*b^2*d^3*g^3*h^3*n*x^2 - 63*B*b^3*c^2*d*g^2*h^4*n*x^2 + 63*B*a^2*b*d^3*g^2*h^4*n*x^2 + 21*B*b^3*c^3*g*h^5*n*x^2 + 9*B*a*b^2*c^2*d*g*h^5*n*x^2 - 9*B*a^2*b*c*d^2*g*h^5*n*x^2 - 21*B*a^3*d^3*g*h^5*n*x^2 - 3*B*a*b^2*c^3*h^6*n*x^2 + 3*B*a^3*c*d^2*h^6*n*x^2 + 68*B*b^3*c*d^2*g^4*h^2*n*x - 68*B*a*b^2*d^3*g^4*h^2*n*x - 76*B*b^3*c^2*d*g^3*h^3*n*x + 76*B*a^2*b*d^3*g^3*h^3*n*x + 26*B*b^3*c^3*g^2*h^4*n*x + 24*B*a*b^2*c^2*d*g^2*h^4*n*x - 24*B*a^2*b*c*d^2*g^2*h^4*n*x - 26*B*a^3*d^3*g^2*h^4*n*x - 10*B*a*b^2*c^3*g*h^5*n*x + 10*B*a^3*c*d^2*g*h^5*n*x + 2*B*a^2*b*c^3*h^6*n*x - 2*B*a^3*c^2*d*h^6*n*x + 26*B*b^3*c*d^2*g^5*h*n - 26*B*a*b^2*d^3*g^5*h*n - 31*B*b^3*c^2*d*g^4*h^2*n + 31*B*a^2*b*d^3*g^4*h^2*n + 11*B*b^3*c^3*g^3*h^3*n + 15*B*a*b^2*c^2*d*g^3*h^3*n - 15*B*a^2*b*c*d^2*g^3*h^3*n - 11*B*a^3*d^3*g^3*h^3*n - 7*B*a*b^2*c^3*g^2*h^4*n + 7*B*a^3*c*d^2*g^2*h^4*n + 2*B*a^2*b*c^3*g*h^5*n - 2*B*a^3*c^2*d*g*h^5*n + 6*A*b^3*d^3*g^6 + 6*B*b^3*d^3*g^6 - 18*A*b^3*c*d^2*g^5*h - 18*B*b^3*c*d^2*g^5*h - 18*A*a*b^2*d^3*g^5*h - 18*B*a*b^2*d^3*g^5*h + 18*A*b^3*c^2*d*g^4*h^2 + 18*B*b^3*c^2*d*g^4*h^2 + 54*A*a*b^2*c*d^2*g^4*h^2 + 54*B*a*b^2*c*d^2*g^4*h^2 + 18*A*a^2*b*d^3*g^4*h^2 + 18*B*a^2*b*d^3*g^4*h^2 - 6*A*b^3*c^3*g^3*h^3 - 6*B*b^3*c^3*g^3*h^3 - 54*A*a*b^2*c^2*d*g^3*h^3 - 54*B*a*b^2*c^2*d*g^3*h^3 - 54*A*a^2*b*c*d^2*g^3*h^3 - 54*B*a^2*b*c*d^2*g^3*h^3 - 6*A*a^3*d^3*g^3*h^3 - 6*B*a^3*d^3*g^3*h^3 + 18*A*a*b^2*c^3*g^2*h^4 + 18*B*a*b^2*c^3*g^2*h^4 + 54*A*a^2*b*c^2*d*g^2*h^4 + 54*B*a^2*b*c^2*d*g^2*h^4 + 18*A*a^3*c*d^2*g^2*h^4 + 18*B*a^3*c*d^2*g^2*h^4 - 18*A*a^2*b*c^3*g*h^5 - 18*B*a^2*b*c^3*g*h^5 - 18*A*a^3*c^2*d*g*h^5 - 18*B*a^3*c^2*d*g*h^5 + 6*A*a^3*c^3*h^6 + 6*B*a^3*c^3*h^6)/(b^3*d^3*g^6*h^5*x^4 - 3*b^3*c*d^2*g^5*h^6*x^4 - 3*a*b^2*d^3*g^5*h^6*x^4 + 3*b^3*c^2*d*g^4*h^7*x^4 + 9*a*b^2*c*d^2*g^4*h^7*x^4 + 3*a^2*b*d^3*g^4*h^7*x^4 - b^3*c^3*g^3*h^8*x^4 - 9*a*b^2*c^2*d*g^3*h^8*x^4 - 9*a^2*b*c*d^2*g^3*h^8*x^4 - a^3*d^3*g^3*h^8*x^4 + 3*a*b^2*c^3*g^2*h^9*x^4 + 9*a^2*b*c^2*d*g^2*h^9*x^4 + 3*a^3*c*d^2*g^2*h^9*x^4 - 3*a^2*b*c^3*g*h^10*x^4 - 3*a^3*c^2*d*g*h^10*x^4 + a^3*c^3*h^11*x^4 + 4*b^3*d^3*g^7*h^4*x^3 - 12*b^3*c*d^2*g^6*h^5*x^3 - 12*a*b^2*d^3*g^6*h^5*x^3 + 12*b^3*c^2*d*g^5*h^6*x^3 + 36*a*b^2*c*d^2*g^5*h^6*x^3 + 12*a^2*b*d^3*g^5*h^6*x^3 - 4*b^3*c^3*g^4*h^7*x^3 - 36*a*b^2*c^2*d*g^4*h^7*x^3 - 36*a^2*b*c*d^2*g^4*h^7*x^3 - 4

```
*a^3*d^3*g^4*h^7*x^3 + 12*a*b^2*c^3*g^3*h^8*x^3 + 36*a^2*b*c^2*d*g^3*h^8*x^
3 + 12*a^3*c*d^2*g^3*h^8*x^3 - 12*a^2*b*c^3*g^2*h^9*x^3 - 12*a^3*c^2*d*g^2*
h^9*x^3 + 4*a^3*c^3*g*h^10*x^3 + 6*b^3*d^3*g^8*h^3*x^2 - 18*b^3*c*d^2*g^7*h
^4*x^2 - 18*a*b^2*d^3*g^7*h^4*x^2 + 18*b^3*c^2*d*g^6*h^5*x^2 + 54*a*b^2*c*d
^2*g^6*h^5*x^2 + 18*a^2*b*d^3*g^6*h^5*x^2 - 6*b^3*c^3*g^5*h^6*x^2 - 54*a*b^
2*c^2*d*g^5*h^6*x^2 - 54*a^2*b*c*d^2*g^5*h^6*x^2 - 6*a^3*d^3*g^5*h^6*x^2 +
18*a*b^2*c^3*g^4*h^7*x^2 + 54*a^2*b*c^2*d*g^4*h^7*x^2 + 18*a^3*c*d^2*g^4*h^
7*x^2 - 18*a^2*b*c^3*g^3*h^8*x^2 - 18*a^3*c^2*d*g^3*h^8*x^2 + 6*a^3*c^3*g^2
*h^9*x^2 + 4*b^3*d^3*g^9*h^2*x - 12*b^3*c*d^2*g^8*h^3*x - 12*a*b^2*d^3*g^8*
h^3*x + 12*b^3*c^2*d*g^7*h^4*x + 36*a*b^2*c*d^2*g^7*h^4*x + 12*a^2*b*d^3*g^
7*h^4*x - 4*b^3*c^3*g^6*h^5*x - 36*a*b^2*c^2*d*g^6*h^5*x - 36*a^2*b*c*d^2*g
^6*h^5*x - 4*a^3*d^3*g^6*h^5*x + 12*a*b^2*c^3*g^5*h^6*x + 36*a^2*b*c^2*d*g^
5*h^6*x + 12*a^3*c*d^2*g^5*h^6*x - 12*a^2*b*c^3*g^4*h^7*x - 12*a^3*c^2*d*g^
4*h^7*x + 4*a^3*c^3*g^3*h^8*x + b^3*d^3*g^10*h - 3*b^3*c*d^2*g^9*h^2 - 3*a*
b^2*d^3*g^9*h^2 + 3*b^3*c^2*d*g^8*h^3 + 9*a*b^2*c*d^2*g^8*h^3 + 3*a^2*b*d^3
*g^8*h^3 - b^3*c^3*g^7*h^4 - 9*a*b^2*c^2*d*g^7*h^4 - 9*a^2*b*c*d^2*g^7*h^4
- a^3*d^3*g^7*h^4 + 3*a*b^2*c^3*g^6*h^5 + 9*a^2*b*c^2*d*g^6*h^5 + 3*a^3*c*d
^2*g^6*h^5 - 3*a^2*b*c^3*g^5*h^6 - 3*a^3*c^2*d*...
```

Mupad [B]
time = 14.28, size = 2570, normalized size = 6.61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))/(g + h*x)^5,x)
```

```
[Out] ((x*(13*B*a^3*d^3*g^2*h^4*n - 13*B*b^3*c^3*g^2*h^4*n - B*a^2*b*c^3*h^6*n +
B*a^3*c^2*d*h^6*n + 5*B*a*b^2*c^3*g*h^5*n - 5*B*a^3*c*d^2*g*h^5*n + 34*B*a*
b^2*d^3*g^4*h^2*n - 38*B*a^2*b*d^3*g^3*h^3*n - 34*B*b^3*c*d^2*g^4*h^2*n + 3
8*B*b^3*c^2*d*g^3*h^3*n - 12*B*a*b^2*c^2*d*g^2*h^4*n + 12*B*a^2*b*c*d^2*g^2
*h^4*n))/(3*(a^3*c^3*h^6 + b^3*d^3*g^6 - a^3*d^3*g^3*h^3 - b^3*c^3*g^3*h^3
- 3*a^2*b*c^3*g*h^5 - 3*a*b^2*d^3*g^5*h - 3*a^3*c^2*d*g*h^5 - 3*b^3*c*d^2*g
^5*h + 3*a*b^2*c^3*g^2*h^4 + 3*a^2*b*d^3*g^4*h^2 + 3*a^3*c*d^2*g^2*h^4 + 3*
b^3*c^2*d*g^4*h^2 + 9*a*b^2*c*d^2*g^4*h^2 - 9*a*b^2*c^2*d*g^3*h^3 - 9*a^2*b
*c*d^2*g^3*h^3 + 9*a^2*b*c^2*d*g^2*h^4)) - (6*A*a^3*c^3*h^6 + 6*A*b^3*d^3*g
^6 - 6*A*a^3*d^3*g^3*h^3 - 6*A*b^3*c^3*g^3*h^3 + 18*A*a*b^2*c^3*g^2*h^4 + 1
8*A*a^2*b*d^3*g^4*h^2 + 18*A*a^3*c*d^2*g^2*h^4 + 18*A*b^3*c^2*d*g^4*h^2 - 1
1*B*a^3*d^3*g^3*h^3*n + 11*B*b^3*c^3*g^3*h^3*n - 18*A*a^2*b*c^3*g*h^5 - 18*
A*a*b^2*d^3*g^5*h - 18*A*a^3*c^2*d*g*h^5 - 18*A*b^3*c*d^2*g^5*h + 2*B*a^2*b
*c^3*g*h^5*n - 26*B*a*b^2*d^3*g^5*h*n - 2*B*a^3*c^2*d*g*h^5*n + 26*B*b^3*c*
d^2*g^5*h*n + 54*A*a*b^2*c*d^2*g^4*h^2 - 54*A*a*b^2*c^2*d*g^3*h^3 - 54*A*a^
2*b*c*d^2*g^3*h^3 + 54*A*a^2*b*c^2*d*g^2*h^4 - 7*B*a*b^2*c^3*g^2*h^4*n + 31
*B*a^2*b*d^3*g^4*h^2*n + 7*B*a^3*c*d^2*g^2*h^4*n - 31*B*b^3*c^2*d*g^4*h^2*n
 + 15*B*a*b^2*c^2*d*g^3*h^3*n - 15*B*a^2*b*c*d^2*g^3*h^3*n)/(6*(a^3*c^3*h^6
```

+ b^3*d^3*g^6 - a^3*d^3*g^3*h^3 - b^3*c^3*g^3*h^3 - 3*a^2*b*c^3*g*h^5 - 3*a*b^2*d^3*g^5*h - 3*a^3*c^2*d*g*h^5 - 3*b^3*c*d^2*g^5*h + 3*a*b^2*c^3*g^2*h^4 + 3*a^2*b*d^3*g^4*h^2 + 3*a^3*c*d^2*g^2*h^4 + 3*b^3*c^2*d*g^4*h^2 + 9*a*b^2*c*d^2*g^4*h^2 - 9*a*b^2*c^2*d*g^3*h^3 - 9*a^2*b*c*d^2*g^3*h^3 + 9*a^2*b*c^2*d*g^2*h^4)) + (x^3*(B*a^3*d^3*h^6*n - B*b^3*c^3*h^6*n - 3*B*a^2*b*d^3*g*h^5*n + 3*B*b^3*c^2*d*g*h^5*n + 3*B*a*b^2*d^3*g^2*h^4*n - 3*B*b^3*c*d^2*g^2*h^4*n))/(a^3*c^3*h^6 + b^3*d^3*g^6 - a^3*d^3*g^3*h^3 - b^3*c^3*g^3*h^3 - 3*a^2*b*c^3*g*h^5 - 3*a*b^2*d^3*g^5*h - 3*a^3*c^2*d*g*h^5 - 3*b^3*c*d^2*g^5*h + 3*a*b^2*c^3*g^2*h^4 + 3*a^2*b*d^3*g^4*h^2 + 3*a^3*c*d^2*g^2*h^4 + 3*b^3*c^2*d*g^4*h^2 + 9*a*b^2*c*d^2*g^4*h^2 - 9*a*b^2*c^2*d*g^3*h^3 - 9*a^2*b*c*d^2*g^3*h^3 + 9*a^2*b*c^2*d*g^2*h^4) + (x^2*(B*a*b^2*c^3*h^6*n - B*a^3*c*d^2*h^6*n + 7*B*a^3*d^3*g*h^5*n - 7*B*b^3*c^3*g*h^5*n + 20*B*a*b^2*d^3*g^3*h^3*n - 21*B*a^2*b*d^3*g^2*h^4*n - 20*B*b^3*c*d^2*g^3*h^3*n + 21*B*b^3*c^2*d*g^2*h^4*n - 3*B*a*b^2*c^2*d*g*h^5*n + 3*B*a^2*b*c*d^2*g*h^5*n))/(2*(a^3*c^3*h^6 + b^3*d^3*g^6 - a^3*d^3*g^3*h^3 - b^3*c^3*g^3*h^3 - 3*a^2*b*c^3*g*h^5 - 3*a*b^2*d^3*g^5*h - 3*a^3*c^2*d*g*h^5 - 3*b^3*c*d^2*g^5*h + 3*a*b^2*c^3*g^2*h^4 + 3*a^2*b*d^3*g^4*h^2 + 3*a^3*c*d^2*g^2*h^4 + 3*b^3*c^2*d*g^4*h^2 + 9*a*b^2*c*d^2*g^4*h^2 - 9*a*b^2*c^2*d*g^3*h^3 - 9*a^2*b*c*d^2*g^3*h^3 + 9*a^2*b*c^2*d*g^2*h^4)))/(4*g^4*h + 4*h^5*x^4 + 16*g^3*h^2*x + 16*g*h^4*x^3 + 24*g^2*h^3*x^2) + (log(g + h*x)*(h*(6*B*a^2*b^2*d^4*g^2*n - 6*B*b^4*c^2*d^2*g^2*n) - h^2*(4*B*a^3*b*d^4*g*n - 4*B*b^4*c^3*d*g*n) + h^3*(B*a^4*d^4*n - B*b^4*c^4*n) - 4*B*a*b^3*d^4*g^3*n + 4*B*b^4*c*d^3*g^3*n))/(4*a^4*c^4*h^8 + 4*b^4*d^4*g^8 + 4*a^4*d^4*g^4*h^4 + 4*b^4*c^4*g^4*h^4 + 24*a^2*b^2*c^4*g^2*h^6 + 24*a^2*b^2*d^4*g^6*h^2 + 24*a^4*c^2*d^2*g^2*h^6 + 24*b^4*c^2*d^2*g^6*h^2 - 16*a^3*b*c^4*g*h^7 - 16*a*b^3*d^4*g^7*h - 16*a^4*c^3*d*g*h^7 - 16*b^4*c*d^3*g^7*h - 16*a*b^3*c^4*g^3*h^5 - 16*a^3*b*d^4*g^5*h^3 - 16*a^4*c*d^3*g^3*h^5 - 16*b^4*c^3*d*g^5*h^3 + 64*a*b^3*c*d^3*g^6*h^2 + 64*a*b^3*c^3*d*g^4*h^4 + 64*a^3*b*c*d^3*g^4*h^4 + 64*a^3*b*c^3*d*g^2*h^6 - 96*a*b^3*c^2*d^2*g^5*h^3 - 96*a^2*b^2*c*d^3*g^5*h^3 - 96*a^2*b^2*c^3*d*g^3*h^5 - 96*a^3*b*c^2*d^2*g^3*h^5 + 144*a^2*b^2*c^2*d^2*g^4*h^4) - (B*log((e*(a + b*x)^n)/(c + d*x)^n))/(4*h*(g^4 + h^4*x^4 + 4*g^3*h*x + 4*g*h^3*x^3 + 6*g^2*h^2*x^2)) + (B*b^4*n*log(a + b*x))/(4*a^4*h^5 + 4*b^4*g^4*h - 16*a*b^3*g^3*h^2 + 24*a^2*b^2*g^2*h^3 - 16*a^3*b*g*h^4) - (B*d^4*n*log(c + d*x))/(4*c^4*h^5 + 4*d^4*g^4*h - 16*c*d^3*g^3*h^2 + 24*c^2*d^2*g^2*h^3 - 16*c^3*d*g*h^4)

3.303 $\int (g+hx)^2 \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2 \, dx$

Optimal. Leaf size=570

$$\frac{B^2(bc-ad)^2h^2n^2x}{3b^2d^2}+\frac{B^2(bc-ad)^3h^2n^2\log\left(\frac{a+bx}{c+dx}\right)}{3b^3d^3}+\frac{B^2(bc-ad)^3h^2n^2\log(c+dx)}{3b^3d^3}+\frac{2B^2(bc-ad)^2h(3bdg-$$

```
[Out] 1/3*B^2*(-a*d+b*c)^2*h^2*n^2*x/b^2/d^2+1/3*B^2*(-a*d+b*c)^3*h^2*n^2*ln((b*x
+a)/(d*x+c))/b^3/d^3+1/3*B^2*(-a*d+b*c)^3*h^2*n^2*ln(d*x+c)/b^3/d^3+2/3*B^2
*(-a*d+b*c)^2*h*(-a*d*h-2*b*c*h+3*b*d*g)*n^2*ln(d*x+c)/b^3/d^3-2/3*B*(-a*d+
b*c)*h*(-a*d*h-2*b*c*h+3*b*d*g)*n*(b*x+a)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))
/b^3/d^2-1/3*B*(-a*d+b*c)*h^2*n*(d*x+c)^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))
/b/d^3+2/3*B*(-a*d+b*c)*(a^2*d^2*h^2-a*b*d*h*(-c*h+3*d*g)+b^2*(c^2*h^2-3*c*
d*g*h+3*d^2*g^2))*n*ln((-a*d+b*c)/b/(d*x+c))*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n
)))/b^3/d^3-1/3*(-a*h+b*g)^3*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/b^3/h+1/3*
(h*x+g)^3*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/h+2/3*B^2*(-a*d+b*c)*(a^2*d^2
*h^2-a*b*d*h*(-c*h+3*d*g)+b^2*(c^2*h^2-3*c*d*g*h+3*d^2*g^2))*n^2*polylog(2,
d*(b*x+a)/b/(d*x+c))/b^3/d^3
```

Rubi [A]
time = 0.79, antiderivative size = 570, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2573, 2553, 2398, 2404, 2338, 2356, 46, 2351, 31, 2354, 2438}

$$\frac{2B^2n^2(bc-ad)\left(a^2d^2h^2-abdh(3dg-ch)+b^2(c^2h^2-3cdgh+3d^2g^2)\right)\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{3b^3d^3}+\frac{2Bn(bc-ad)\left(a^2d^2h^2-abdh(3dg-ch)+b^2(c^2h^2-3cdgh+3d^2g^2)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{3b^3d^3}-\frac{2Bhn(a+bx)(bc-ad)(-adh-2bch+3bdg)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{3b^3d^2}-\frac{(bg-ah)^3\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^2}{3b^3h}-\frac{Bh^2n(c+dx)^2(bc-ad)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{3bd^3}+\frac{(g+hx)^3\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^2}{3h}+\frac{2B^2hn^2(bc-ad)^2\log(c+dx)(-adh-2bch+3bdg)}{3b^3d^3}+\frac{B^2h^2n^2(bc-ad)^3\log\left(\frac{a+bx}{c+dx}\right)}{3b^3d^3}+\frac{B^2h^2n^2(bc-ad)^3\log(c+dx)}{3b^3d^3}+\frac{B^2h^2n^2x(bc-ad)^2}{3b^2d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(g + h*x)^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2,x]
```

```
[Out] (B^2*(b*c - a*d)^2*h^2*n^2*x)/(3*b^2*d^2) + (B^2*(b*c - a*d)^3*h^2*n^2*Log[
(a + b*x)/(c + d*x)])/(3*b^3*d^3) + (B^2*(b*c - a*d)^3*h^2*n^2*Log[c + d*x]
)/(3*b^3*d^3) + (2*B^2*(b*c - a*d)^2*h*(3*b*d*g - 2*b*c*h - a*d*h)*n^2*Log[
c + d*x])/(3*b^3*d^3) - (2*B*(b*c - a*d)*h*(3*b*d*g - 2*b*c*h - a*d*h)*n*(a
 + b*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]))/(3*b^3*d^2) - (B*(b*c - a
*d)*h^2*n*(c + d*x)^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]))/(3*b*d^3) +
 (2*B*(b*c - a*d)*(a^2*d^2*h^2 - a*b*d*h*(3*d*g - c*h) + b^2*(3*d^2*g^2 - 3
*c*d*g*h + c^2*h^2))*n*Log[(b*c - a*d)/(b*(c + d*x))]*(A + B*Log[(e*(a + b*
x)^n)/(c + d*x)^n]))/(3*b^3*d^3) - ((b*g - a*h)^3*(A + B*Log[(e*(a + b*x)^n
)/(c + d*x)^n])^2)/(3*b^3*h) + ((g + h*x)^3*(A + B*Log[(e*(a + b*x)^n)/(c +
 d*x)^n])^2)/(3*h) + (2*B^2*(b*c - a*d)*(a^2*d^2*h^2 - a*b*d*h*(3*d*g - c*h
) + b^2*(3*d^2*g^2 - 3*c*d*g*h + c^2*h^2))*n^2*PolyLog[2, (d*(a + b*x))/(b*
(c + d*x))])/(3*b^3*d^3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_)*((
f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(d + e*x)^(q +
 1)*((a + b*Log[c*x^n])^p/((q + 1)*(e*f - d*g))), x] - Dist[b*n*(p/((q + 1)
*(e*f - d*g))), Int[(f + g*x)^(m + 1)*(d + e*x)^(q + 1)*((a + b*Log[c*x^n])
^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, q}, x] && NeQ[e*f
- d*g, 0] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 2404

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2553

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[b*c - a*d, Subst[
Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)),
 x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x
] && NeQ[b*c - a*d, 0] && IntegerQ[m] && IGtQ[p, 0]
```

Rule 2573

```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

Rubi steps

$$\int (g+hx)^2 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2\, dx = \int \left(A^2(g+hx)^2+2AB(g+hx)^2\log\left(e(a+bx)^n(c+\right.\right.$$

$$= \frac{A^2(g+hx)^3}{3h}+(2AB)\int (g+hx)^2\log\left(e(a+bx)^n(c\right.$$

$$= \frac{A^2(g+hx)^3}{3h}+\frac{2AB(g+hx)^3\log\left(e(a+bx)^n(c+dx\right.}{3h}$$

$$= \frac{A^2(g+hx)^3}{3h}+\frac{2AB(g+hx)^3\log\left(e(a+bx)^n(c+dx\right.}{3h}$$

$$= -\frac{2AB(bc-ad)h(3bdg-bch-adh)nx}{3b^2d^2}-\frac{AB(bc-}{3b}$$

$$= -\frac{2AB(bc-ad)h(3bdg-bch-adh)nx}{3b^2d^2}-\frac{AB(bc-}{3b}$$

$$= -\frac{2AB(bc-ad)h(3bdg-bch-adh)nx}{3b^2d^2}-\frac{AB(bc-}{3b}$$

$$= -\frac{2AB(bc-ad)h(3bdg-bch-adh)nx}{3b^2d^2}+\frac{B^2(bc-a}{3b^2}$$

$$= -\frac{2AB(bc-ad)h(3bdg-bch-adh)nx}{3b^2d^2}+\frac{B^2(bc-a}{3b^2}$$

$$= -\frac{2AB(bc-ad)h(3bdg-bch-adh)nx}{3b^2d^2}+\frac{B^2(bc-a}{3b^2}$$

Mathematica [A]
time = 0.97, size = 906, normalized size = 1.59

Antiderivative was successfully verified.

```
[In] Integrate[(g + h*x)^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2,x]
```

```
[Out] (-(a*B^2*d^3*(3*b^2*g^2 - 3*a*b*g*h + a^2*h^2)*n^2*Log[a + b*x]^2) + B*n*Lo
g[a + b*x]*(2*b^3*B*c*(3*d^2*g^2 - 3*c*d*g*h + c^2*h^2)*n*Log[c + d*x] + 2*
B*(3*a*b^2*d^3*g^2 - 3*a^2*b*d^3*g*h + a^3*d^3*h^2 - b^3*c*(3*d^2*g^2 - 3*c
*d*g*h + c^2*h^2))*n*Log[(b*(c + d*x))/(b*c - a*d)] + a*d*(2*A*d^2*(3*b^2*g
^2 - 3*a*b*g*h + a^2*h^2) + B*(-3*a^2*d^2*h^2 + a*b*d*h*(6*d*g + c*h) + 2*b
^2*(3*d^2*g^2 - 3*c*d*g*h + c^2*h^2))*n + 2*B*d^2*(3*b^2*g^2 - 3*a*b*g*h +
```

```
a^2*h^2)*Log[(e*(a + b*x)^n)/(c + d*x)^n])) + b*(-(b^2*B^2*c*(3*d^2*g^2 - 3
*c*d*g*h + c^2*h^2)*n^2*Log[c + d*x]^2) + B*n*Log[c + d*x]*(-2*A*b^2*c*(3*d
^2*g^2 - 3*c*d*g*h + c^2*h^2) + B*(2*a^2*c*d^2*h^2 - 3*b^2*c^2*h*(-2*d*g +
c*h) + a*b*d*(-6*d^2*g^2 - 6*c*d*g*h + c^2*h^2))*n - 2*b^2*B*c*(3*d^2*g^2 -
 3*c*d*g*h + c^2*h^2)*Log[(e*(a + b*x)^n)/(c + d*x)^n]) + d*(a^2*B*d^2*h^2*
n*(-2*A + B*n)*x + a*b*B*n*(A*d^2*(-6*g^2 + 6*g*h*x + h^2*x^2) - 2*B*n*(3*d
^2*g^2 + c^2*h^2 + c*d*h*(-3*g + h*x))) + b^2*x*(B^2*c^2*h^2*n^2 + A^2*d^2*
(3*g^2 + 3*g*h*x + h^2*x^2) + A*B*c*h*n*(2*c*h - d*(6*g + h*x))) + B*(-2*a^
2*B*d^2*h^2*n*x + a*b*B*d^2*n*(-6*g^2 + 6*g*h*x + h^2*x^2) + b^2*x*(B*c*h*n
*(-6*d*g + 2*c*h - d*h*x) + 2*A*d^2*(3*g^2 + 3*g*h*x + h^2*x^2)))*Log[(e*(a
 + b*x)^n)/(c + d*x)^n] + b^2*B^2*d^2*x*(3*g^2 + 3*g*h*x + h^2*x^2)*Log[(e*
(a + b*x)^n)/(c + d*x)^n]^2)) + 2*B^2*(3*a*b^2*d^3*g^2 - 3*a^2*b*d^3*g*h +
a^3*d^3*h^2 - b^3*c*(3*d^2*g^2 - 3*c*d*g*h + c^2*h^2))*n^2*PolyLog[2, (d*(a
 + b*x))/(-(b*c) + a*d)])/(3*b^3*d^3)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 4.21, size = 23167, normalized size = 40.64

method	result	size
risch	Expression too large to display	23167

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1606 vs. 2(554) = 1108.
time = 0.75, size = 1606, normalized size = 2.82

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^2*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2,x, algorithm="maxi
ma")
```

```
[Out] 2/3*A*B*h^2*x^3*log((b*x + a)^n*e/(d*x + c)^n) + 1/3*A^2*h^2*x^3 + 2*A*B*g*
h*x^2*log((b*x + a)^n*e/(d*x + c)^n) + A^2*g*h*x^2 + 2*(a*n*e*log(b*x + a)/
b - c*n*e*log(d*x + c)/d)*A*B*g^2*e^(-1) - 2*(a^2*n*e*log(b*x + a)/b^2 - c^
2*n*e*log(d*x + c)/d^2 + (b*c*n - a*d*n)*x*e/(b*d))*A*B*g*h*e^(-1) + 1/3*(2
*a^3*n*e*log(b*x + a)/b^3 - 2*c^3*n*e*log(d*x + c)/d^3 - ((b^2*c*d*n - a*b*
d^2*n)*x^2*e - 2*(b^2*c^2*n - a^2*d^2*n)*x*e)/(b^2*d^2))*A*B*h^2*e^(-1) + 2
*A*B*g^2*x*log((b*x + a)^n*e/(d*x + c)^n) + A^2*g^2*x + 1/3*(2*a^2*c*d^2*h^
2*n^2 - (6*c*d^2*g*h*n^2 - c^2*d*h^2*n^2)*a*b + (6*(n^2 + n)*c^2*d*g*h - (3
*n^2 + 2*n)*c^3*h^2 - 6*c*d^2*g^2*n)*b^2)*B^2*log(d*x + c)/(b^2*d^3) + 2/3*
```

```
(3*a*b^2*d^3*g^2*n^2 - 3*a^2*b*d^3*g*h*n^2 + a^3*d^3*h^2*n^2 - (3*c*d^2*g^2
*n^2 - 3*c^2*d*g*h*n^2 + c^3*h^2*n^2)*b^3)*(log(b*x + a)*log((b*d*x + a*d)/
(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B^2/(b^3*d^3) + 1/3*(
B^2*b^3*d^3*h^2*x^3 + 2*(3*c*d^2*g^2*n^2 - 3*c^2*d*g*h*n^2 + c^3*h^2*n^2)*B
^2*b^3*log(b*x + a)*log(d*x + c) - (3*c*d^2*g^2*n^2 - 3*c^2*d*g*h*n^2 + c^3
*h^2*n^2)*B^2*b^3*log(d*x + c)^2 + (a*b^2*d^3*h^2*n - (c*d^2*h^2*n - 3*d^3*
g*h)*b^3)*B^2*x^2 - (3*a*b^2*d^3*g^2*n^2 - 3*a^2*b*d^3*g*h*n^2 + a^3*d^3*h^
2*n^2)*B^2*log(b*x + a)^2 + ((n^2 - 2*n)*a^2*b*d^3*h^2 - 2*(c*d^2*h^2*n^2 -
 3*d^3*g*h*n)*a*b^2 + ((n^2 + 2*n)*c^2*d*h^2 - 6*c*d^2*g*h*n + 3*d^3*g^2)*b
^3)*B^2*x - ((3*n^2 - 2*n)*a^3*d^3*h^2 - (c*d^2*h^2*n^2 + 6*(n^2 - n)*d^3*g
*h)*a^2*b + 2*(3*c*d^2*g*h*n^2 - c^2*d*h^2*n^2 - 3*d^3*g^2*n)*a*b^2)*B^2*lo
g(b*x + a) + (B^2*b^3*d^3*h^2*x^3 + 3*B^2*b^3*d^3*g*h*x^2 + 3*B^2*b^3*d^3*g
^2*x)*log((b*x + a)^n)^2 + (B^2*b^3*d^3*h^2*x^3 + 3*B^2*b^3*d^3*g*h*x^2 + 3
*B^2*b^3*d^3*g^2*x)*log((d*x + c)^n)^2 + (2*B^2*b^3*d^3*h^2*x^3 - 2*(3*c*d^
2*g^2*n - 3*c^2*d*g*h*n + c^3*h^2*n)*B^2*b^3*log(d*x + c) + (a*b^2*d^3*h^2*
n - (c*d^2*h^2*n - 6*d^3*g*h)*b^3)*B^2*x^2 + 2*(3*a*b^2*d^3*g*h*n - a^2*b*d
^3*h^2*n - (3*c*d^2*g*h*n - c^2*d*h^2*n - 3*d^3*g^2)*b^3)*B^2*x + 2*(3*a*b^
2*d^3*g^2*n - 3*a^2*b*d^3*g*h*n + a^3*d^3*h^2*n)*B^2*log(b*x + a))*log((b*x
 + a)^n) - (2*B^2*b^3*d^3*h^2*x^3 - 2*(3*c*d^2*g^2*n - 3*c^2*d*g*h*n + c^3*
h^2*n)*B^2*b^3*log(d*x + c) + (a*b^2*d^3*h^2*n - (c*d^2*h^2*n - 6*d^3*g*h)*
b^3)*B^2*x^2 + 2*(3*a*b^2*d^3*g*h*n - a^2*b*d^3*h^2*n - (3*c*d^2*g*h*n - c^
2*d*h^2*n - 3*d^3*g^2)*b^3)*B^2*x + 2*(3*a*b^2*d^3*g^2*n - 3*a^2*b*d^3*g*h*
n + a^3*d^3*h^2*n)*B^2*log(b*x + a) + 2*(B^2*b^3*d^3*h^2*x^3 + 3*B^2*b^3*d^
3*g*h*x^2 + 3*B^2*b^3*d^3*g^2*x)*log((b*x + a)^n))*log((d*x + c)^n))/(b^3*d
^3)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^2*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2,x, algorithm="fric
as")
```

```
[Out] integral(A^2*h^2*x^2 + 2*A^2*g*h*x + A^2*g^2 + (B^2*h^2*x^2 + 2*B^2*g*h*x +
 B^2*g^2)*log((b*x + a)^n*e/(d*x + c)^n)^2 + 2*(A*B*h^2*x^2 + 2*A*B*g*h*x +
 A*B*g^2)*log((b*x + a)^n*e/(d*x + c)^n), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)**2*(A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**2,x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^2*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2,x, algorithm="giac
")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int (g+h\,x)^2\,\left(A+B\,\ln\left(\frac{e\,{(a+b\,x)}^n}{{(c+d\,x)}^n}\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g + h*x)^2*(A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^2,x)
```

```
[Out] int((g + h*x)^2*(A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^2, x)
```

3.304 $\int (g+hx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2\,dx$

Optimal. Leaf size=294

$$\frac{B^2(bc-ad)^2hn^2\log(c+dx)}{b^2d^2}-\frac{B(bc-ad)hn(a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{b^2d}+\frac{B(bc-ad)(2bdg$$

```
[Out] B^2*(-a*d+b*c)^2*h*n^2*ln(d*x+c)/b^2/d^2-B*(-a*d+b*c)*h*n*(b*x+a)*(A+B*ln(e
*(b*x+a)^n/((d*x+c)^n)))/b^2/d+B*(-a*d+b*c)*(-a*d*h-b*c*h+2*b*d*g)*n*ln((-a
*d+b*c)/b/(d*x+c))*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/b^2/d^2-1/2*(-a*h+b*g)
^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/b^2/h+1/2*(h*x+g)^2*(A+B*ln(e*(b*x+a
)^n/((d*x+c)^n)))^2/h+B^2*(-a*d+b*c)*(-a*d*h-b*c*h+2*b*d*g)*n^2*polylog(2,d
*(b*x+a)/b/(d*x+c))/b^2/d^2
```

Rubi [A]
time = 0.39, antiderivative size = 294, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.290$, Rules used = {2573, 2553, 2398, 2404, 2338, 2351, 31, 2354, 2438}

$$\frac{B^2n^2(bc-ad)(-adh-bch+2bdg)\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{b^2d^2}+\frac{Bn(bc-ad)(-adh-bch+2bdg)\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{b^2d^2}-\frac{(bg-ah)^2\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^2}{2b^2h}-\frac{Bhn(a+bx)(bc-ad)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{b^2d}+\frac{(g+hx)^2\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^2}{2h}+\frac{B^2hn^2(bc-ad)^2\log(c+dx)}{b^2d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(g + h*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2,x]
```

```
[Out] (B^2*(b*c - a*d)^2*h*n^2*Log[c + d*x])/(b^2*d^2) - (B*(b*c - a*d)*h*n*(a +
b*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]))/(b^2*d) + (B*(b*c - a*d)*(2*
b*d*g - b*c*h - a*d*h)*n*Log[(b*c - a*d)/(b*(c + d*x))]*(A + B*Log[(e*(a +
b*x)^n)/(c + d*x)^n]))/(b^2*d^2) - ((b*g - a*h)^2*(A + B*Log[(e*(a + b*x)^n
)/(c + d*x)^n])^2)/(2*b^2*h) + ((g + h*x)^2*(A + B*Log[(e*(a + b*x)^n)/(c +
 d*x)^n])^2)/(2*h) + (B^2*(b*c - a*d)*(2*b*d*g - b*c*h - a*d*h)*n^2*PolyLog
[2, (d*(a + b*x))/(b*(c + d*x))])/(b^2*d^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
```

```
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_)*((
f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(d + e*x)^(q +
 1)*((a + b*Log[c*x^n])^p/((q + 1)*(e*f - d*g))), x] - Dist[b*n*(p/((q + 1)
*(e*f - d*g))), Int[(f + g*x)^(m + 1)*(d + e*x)^(q + 1)*((a + b*Log[c*x^n])
^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, q}, x] && NeQ[e*f
- d*g, 0] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 2404

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2553

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[b*c - a*d, Subst[
Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)),
 x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x
] && NeQ[b*c - a*d, 0] && IntegerQ[m] && IGtQ[p, 0]
```

Rule 2573

```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

Rubi steps

$$\int (g+hx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2\,dx = \int \left(A^2(g+hx)+2AB(g+hx)\log\left(e(a+bx)^n(c+dx\right.\right.$$

$$= \frac{A^2(g+hx)^2}{2h} + (2AB)\int (g+hx)\log\left(e(a+bx)^n(c+\right.$$

$$= \frac{A^2(g+hx)^2}{2h} + \frac{AB(g+hx)^2\log\left(e(a+bx)^n(c+dx)^{-}\right.}{h}$$

$$= \frac{A^2(g+hx)^2}{2h} + \frac{AB(g+hx)^2\log\left(e(a+bx)^n(c+dx)^{-}\right.}{h}$$

$$= -\frac{AB(bc-ad)hnx}{bd} + \frac{A^2(g+hx)^2}{2h} - \frac{AB(bg-ah)^2n}{b^2h}$$

$$= -\frac{AB(bc-ad)hnx}{bd} + \frac{A^2(g+hx)^2}{2h} - \frac{AB(bg-ah)^2n}{b^2h}$$

$$= -\frac{AB(bc-ad)hnx}{bd} + \frac{A^2(g+hx)^2}{2h} - \frac{AB(bg-ah)^2n}{b^2h}$$

$$= -\frac{AB(bc-ad)hnx}{bd} + \frac{A^2(g+hx)^2}{2h} - \frac{AB(bg-ah)^2n}{b^2h}$$

$$= -\frac{AB(bc-ad)hnx}{bd} + \frac{A^2(g+hx)^2}{2h} - \frac{AB(bg-ah)^2n}{b^2h}$$

$$= -\frac{AB(bc-ad)hnx}{bd} + \frac{A^2(g+hx)^2}{2h} - \frac{AB(bg-ah)^2n}{b^2h}$$

Mathematica [A]
time = 0.50, size = 472, normalized size = 1.61

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[(g + h*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2,x]
```

```
[Out] (a*B^2*d^2*(-2*b*g + a*h)*n^2*Log[a + b*x]^2 - 2*B*n*Log[a + b*x]*(b^2*B*c*
(-2*d*g + c*h)*n*Log[c + d*x] - B*(b*c - a*d)*(-2*b*d*g + b*c*h + a*d*h)*n*
Log[(b*(c + d*x))/(b*c - a*d)] + a*d*(A*(-2*b*d*g + a*d*h) + B*(-2*b*d*g +
b*c*h - a*d*h)*n + B*d*(-2*b*g + a*h)*Log[(e*(a + b*x)^n)/(c + d*x)^n])) +
b*(b*B^2*c*(-2*d*g + c*h)*n^2*Log[c + d*x]^2 + 2*B*n*Log[c + d*x]*(A*b*c*(-
2*d*g + c*h) + B*(b*c^2*h - a*d*(2*d*g + c*h))*n + b*B*c*(-2*d*g + c*h)*Log
```

```
[(e*(a + b*x)^n)/(c + d*x)^n]) + d*(A*b*x*(2*A*d*g - 2*B*c*h*n + A*d*h*x) +
 2*a*B*n*(-2*A*d*g - 2*B*d*g*n + B*c*h*n + A*d*h*x) + 2*B*(a*B*d*n*(-2*g +
h*x) + b*x*(2*A*d*g - B*c*h*n + A*d*h*x))*Log[(e*(a + b*x)^n)/(c + d*x)^n]
+ b*B^2*d*x*(2*g + h*x)*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2)) + 2*B^2*(b*c -
 a*d)*(-2*b*d*g + b*c*h + a*d*h)*n^2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d
)])/(2*b^2*d^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 2.19, size = 11007, normalized size = 37.44

method	result	size
risch	Expression too large to display	11007

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 874 vs. $2(293) = 586$.
time = 0.79, size = 874, normalized size = 2.97

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2,x, algorithm="maxima
")
```

```
[Out] A*B*h*x^2*log((b*x + a)^n*e/(d*x + c)^n) + 1/2*A^2*h*x^2 + 2*(a*n*e*log(b*x
 + a)/b - c*n*e*log(d*x + c)/d)*A*B*g*e^(-1) - (a^2*n*e*log(b*x + a)/b^2 -
c^2*n*e*log(d*x + c)/d^2 + (b*c*n - a*d*n)*x*e/(b*d))*A*B*h*e^(-1) + 2*A*B*
g*x*log((b*x + a)^n*e/(d*x + c)^n) + A^2*g*x - (a*c*d*h*n^2 - ((n^2 + n)*c^
2*h - 2*c*d*g*n)*b)*B^2*log(d*x + c)/(b*d^2) + (2*a*b*d^2*g*n^2 - a^2*d^2*h
*n^2 - (2*c*d*g*n^2 - c^2*h*n^2)*b^2)*(log(b*x + a)*log((b*d*x + a*d)/(b*c
- a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B^2/(b^2*d^2) + 1/2*(B^2*b
^2*d^2*h*x^2 + 2*(2*c*d*g*n^2 - c^2*h*n^2)*B^2*b^2*log(b*x + a)*log(d*x + c
) - (2*c*d*g*n^2 - c^2*h*n^2)*B^2*b^2*log(d*x + c)^2 - (2*a*b*d^2*g*n^2 - a
^2*d^2*h*n^2)*B^2*log(b*x + a)^2 + 2*(a*b*d^2*h*n - (c*d*h*n - d^2*g)*b^2)*
B^2*x + 2*((n^2 - n)*a^2*d^2*h - (c*d*h*n^2 - 2*d^2*g*n)*a*b)*B^2*log(b*x +
 a) + (B^2*b^2*d^2*h*x^2 + 2*B^2*b^2*d^2*g*x)*log((b*x + a)^n)^2 + (B^2*b^2
*d^2*h*x^2 + 2*B^2*b^2*d^2*g*x)*log((d*x + c)^n)^2 + 2*(B^2*b^2*d^2*h*x^2 -
 (2*c*d*g*n - c^2*h*n)*B^2*b^2*log(d*x + c) + (a*b*d^2*h*n - (c*d*h*n - 2*d
^2*g)*b^2)*B^2*x + (2*a*b*d^2*g*n - a^2*d^2*h*n)*B^2*log(b*x + a))*log((b*x
 + a)^n) - 2*(B^2*b^2*d^2*h*x^2 - (2*c*d*g*n - c^2*h*n)*B^2*b^2*log(d*x + c
) + (a*b*d^2*h*n - (c*d*h*n - 2*d^2*g)*b^2)*B^2*x + (2*a*b*d^2*g*n - a^2*d^
```

```
2*h*n)*B^2*log(b*x + a) + (B^2*b^2*d^2*h*x^2 + 2*B^2*b^2*d^2*g*x)*log((b*x
+ a)^n))*log((d*x + c)^n))/(b^2*d^2)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2,x, algorithm="fricas
")
```

```
[Out] integral(A^2*h*x + A^2*g + (B^2*h*x + B^2*g)*log((b*x + a)^n*e/(d*x + c)^n)
^2 + 2*(A*B*h*x + A*B*g)*log((b*x + a)^n*e/(d*x + c)^n), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**2,x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2,x, algorithm="giac")
```

```
[Out] integrate((h*x + g)*(B*log((b*x + a)^n*e/(d*x + c)^n) + A)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (g+h\,x)\,\left(A+B\,\ln\left(\frac{e\,{\left(a+b\,x\right)}^n}{{\left(c+d\,x\right)}^n}\right)\right)^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g + h*x)*(A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^2,x)
```

```
[Out] int((g + h*x)*(A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^2, x)
```

3.305 $\int (A+B\log(e(a+bx)^n(c+dx)^{-n}))^2\,dx$

Optimal. Leaf size=137

$$\frac{2B(bc-ad)n\log\left(\frac{bc-ad}{b(c+dx)}\right)(A+B\log(e(a+bx)^n(c+dx)^{-n}))}{bd}+\frac{(a+bx)(A+B\log(e(a+bx)^n(c+dx)^{-n}$$

```
[Out] 2*B*(-a*d+b*c)*n*ln((-a*d+b*c)/b/(d*x+c))*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))
/b/d+(b*x+a)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/b+2*B^2*(-a*d+b*c)*n^2*pol
ylog(2,d*(b*x+a)/b/(d*x+c))/b/d
```

Rubi [A]

time = 0.17, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {2536, 2542, 2458, 2378, 2370, 2352}

$$\frac{2B^2n^2(bc-ad)\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{bd}+\frac{2Bn(bc-ad)\log\left(\frac{bc-ad}{b(c+dx)}\right)(B\log(e(a+bx)^n(c+dx)^{-n})+A)}{bd}+\frac{(a+bx)(B\log(e(a+bx)^n(c+dx)^{-n})+A)^2}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2,x]
```

```
[Out] (2*B*(b*c - a*d)*n*Log[(b*c - a*d)/(b*(c + d*x))]*(A + B*Log[(e*(a + b*x)^n
)/(c + d*x)^n]))/(b*d) + ((a + b*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]
)^2)/b + (2*B^2*(b*c - a*d)*n^2*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/(b
*d)
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2370

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)/(x_))^(q_.)*(
x_)^(m_.), x_Symbol] :> Int[(e + d*x)^q*(a + b*Log[c*x^n])^p, x] /; FreeQ[{
a, b, c, d, e, m, n, p}, x] && EqQ[m, q] && IntegerQ[q]
```

Rule 2378

```
Int[((a_.) + Log[(c_.)*(x_)^(n_)]*(b_.))/((x_)*((d_) + (e_.)*(x_)^(r_.))),
x_Symbol] :> Dist[1/n, Subst[Int[(a + b*Log[c*x])/(x*(d + e*x^(r/n))), x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IntegerQ[r/n]
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
```

```
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2536

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.), x_Symbol] :> Simp[(a + b*x)*((A + B*Log[e*((a + b*x)^n/(c
+ d*x)^n)])^p/b), x] - Dist[B*n*p*((b*c - a*d)/b), Int[(A + B*Log[e*((a + b
*x)^n/(c + d*x)^n)])^(p - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, A,
B, n}, x] && EqQ[n + mn, 0] && NeQ[b*c - a*d, 0] && IGtQ[p, 0]
```

Rule 2542

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))/((f_.) + (g_.)*(x_)), x_Symbol] :> Simp[(-Log[-(b*c - a*d)/(d*(a
+ b*x))])*((A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/g), x] + Dist[B*n*((b*c
 - a*d)/g), Int[Log[-(b*c - a*d)/(d*(a + b*x))]/((a + b*x)*(c + d*x)), x],
x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
 - a*d, 0] && EqQ[b*f - a*g, 0]
```

Rubi steps

$$\int \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2\,dx=\int \left(A^2+2AB\log\left(e(a+bx)^n(c+dx)^{-n}\right)+B^2\log^2\left(e(a+b\right.\right.$$

$$=A^2x+(2AB)\int \log\left(e(a+bx)^n(c+dx)^{-n}\right)\,dx+B^2\int \log^2$$

$$=A^2x+\frac{2AB(a+bx)\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{b}+\frac{B^2(a+bx}{}$$

$$=A^2x-\frac{2AB(bc-ad)n\log(c+dx)}{bd}+\frac{2AB(a+bx)\log\left(e(a+\right.}{b}$$

$$=A^2x-\frac{2AB(bc-ad)n\log(c+dx)}{bd}+\frac{2AB(a+bx)\log\left(e(a+\right.}{b}$$

$$=A^2x-\frac{2AB(bc-ad)n\log(c+dx)}{bd}+\frac{2AB(a+bx)\log\left(e(a+\right.}{b}$$

$$=A^2x-\frac{2AB(bc-ad)n\log(c+dx)}{bd}+\frac{2AB(a+bx)\log\left(e(a+\right.}{b}$$

$$=A^2x-\frac{2AB(bc-ad)n\log(c+dx)}{bd}+\frac{2AB(a+bx)\log\left(e(a+\right.}{b}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 327 vs. 2(137) = 274.
time = 0.15, size = 327, normalized size = 2.39

$$-\frac{2aABdn+2aB^2dn^2-A^2bdx+aB^2dn^2\log^2(a+bx)+2AbBcn\log(c+dx)+2aB^2dn^2\log(c+dx)+bB^2cn^2\log^2(c+dx)+2aB^2dn\log\left(e(a+bx)^n(c+dx)^{-n}\right)-2AbBdx\log\left(e(a+bx)^n(c+dx)^{-n}\right)+2bB^2cn\log(c+dx)\log\left(e(a+bx)^n(c+dx)^{-n}\right)-bB^2dx\log^2\left(e(a+bx)^n(c+dx)^{-n}\right)-2Bn\log(a+bx)\left(bBcn\log(c+dx)+B(-bc+ad)n\log\left(\frac{b(c+dx)}{bc-ad}\right)+ad\left(A+Bn+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\right)+2B^2(bc-ad)n^2\text{Li}_2\left(\frac{d(a+bx)}{-bc+ad}\right)}{bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2,x]
```

```
[Out] -((2*a*A*B*d*n + 2*a*B^2*d*n^2 - A^2*b*d*x + a*B^2*d*n^2*Log[a + b*x]^2 + 2
*A*b*B*c*n*Log[c + d*x] + 2*a*B^2*d*n^2*Log[c + d*x] + b*B^2*c*n^2*Log[c +
d*x]^2 + 2*a*B^2*d*n*Log[(e*(a + b*x)^n)/(c + d*x)^n] - 2*A*b*B*d*x*Log[(e*
(a + b*x)^n)/(c + d*x)^n] + 2*b*B^2*c*n*Log[c + d*x]*Log[(e*(a + b*x)^n)/(c
 + d*x)^n] - b*B^2*d*x*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2 - 2*B*n*Log[a + b
*x]*(b*B*c*n*Log[c + d*x] + B*(-(b*c) + a*d)*n*Log[(b*(c + d*x))/(b*c - a*d
)] + a*d*(A + B*n + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])) + 2*B^2*(b*c - a*d
)*n^2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)])/(b*d))
```

Mathematica [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.10, size = 4749, normalized size = 34.66

method	result	size
risch	Expression too large to display	4749

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2,x,method=_RETURNVERBOSE)
```

```
[Out] I*B^2*c*n/d*ln(d*x+c)*Pi*csgn(I*(b*x+a)^n/((d*x+c)^n))^3+1/2*B^2*Pi^2*x*csg
n(I*e)*csgn(I*(b*x+a)^n)*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))*
csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2-1/2*B^2*Pi^2*x*csgn(I*e)*csgn(I*(b*x+a)^n
)*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))^2*csgn(I*e/((d*x+c)^n)*
(b*x+a)^n)-I*x*ln((b*x+a)^n)*B^2*Pi*csgn(I*e)*csgn(I*(b*x+a)^n/((d*x+c)^n))
*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)+1/2*B^2*Pi^2*x*csgn(I*(b*x+a)^n)*csgn(I/((
d*x+c)^n))^2*csgn(I*(b*x+a)^n/((d*x+c)^n))^3-B^2*Pi^2*x*csgn(I*(b*x+a)^n)*c
sgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))^4-1/2*B^2*Pi^2*x*csgn(I*(b
*x+a)^n)*csgn(I*(b*x+a)^n/((d*x+c)^n))^3*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2-
I*B^2*ln(e)*Pi*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^3*x-I*A*B*Pi*csgn(I*(b*x+a)^
n/((d*x+c)^n))^3*x-1/4*B^2*Pi^2*x*csgn(I*e)^2*csgn(I*(b*x+a)^n/((d*x+c)^n))
^2*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2+1/2*B^2*Pi^2*x*csgn(I*e)^2*csgn(I*(b*x
+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^3-1/2*B^2*Pi^2*x*csgn(I*
e)*csgn(I*(b*x+a)^n/((d*x+c)^n))^4*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)-n^2*B^2*
c/d*ln(d*x+c)^2-1/2*B^2*Pi^2*x*csgn(I*e)*csgn(I*(b*x+a)^n)*csgn(I*(b*x+a)^n
/((d*x+c)^n))^2*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2+I/b*B^2*ln(b*x+a)*Pi*a*n*
csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))^2-I*B^2*c*n/d*ln(d*x+c)*P
i*csgn(I*e)*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2-I*B^2*c*n/d*ln(d*x+c)*Pi*csgn
(I*(b*x+a)^n)*csgn(I*(b*x+a)^n/((d*x+c)^n))^2-I*B^2*c*n/d*ln(d*x+c)*Pi*csgn
(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+x*A^2-I/b*B^2*ln(b*x+a)*Pi*
a*n*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^3-I*A*B*Pi*csgn(I*(b*x+a)^n)*csgn(I/((d
*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))*x+B^2*x*ln((b*x+a)^n)^2-1/4*B^2*Pi^
2*x*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^6-1/2*B^2*Pi^2*x*csgn(I*(b*x+a)^n/((d*x
+c)^n))^3*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^3-1/4*B^2*Pi^2*x*csgn(I*(b*x+a)^n
/((d*x+c)^n))^2*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^4+I*x*ln((b*x+a)^n)*B^2*Pi*
csgn(I*e)*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2+I*x*ln((b*x+a)^n)*B^2*Pi*csgn(I
*(b*x+a)^n)*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+I*x*ln((b*x+a)^n)*B^2*Pi*csgn(I
/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))^2-I/b*B^2*ln(b*x+a)*Pi*a*n*csgn
(I*(b*x+a)^n)*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))-2*B^2*a*n/b
*ln((b*x+a)^n)+I*B^2*c*n/d*ln(d*x+c)*Pi*csgn(I*(b*x+a)^n)*csgn(I/((d*x+c)^n
))*csgn(I*(b*x+a)^n/((d*x+c)^n))+I*B^2*c*n/d*ln(d*x+c)*Pi*csgn(I*e)*csgn(I*
(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)-I/b*B^2*ln(b*x+a)*Pi
*a*n*csgn(I*e)*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n
)-2*n^2*B^2*c/d+2*B*x*ln((b*x+a)^n)*A+B^2*a/b*ln((b*x+a)^n)^2+2*x*ln((b*x+a
)^n)*B^2*ln(e)+2*A*B*ln(e)*x+1/2*B^2*Pi^2*x*csgn(I*e)*csgn(I*e/((d*x+c)^n)*
(b*x+a)^n)^5-1/4*B^2*Pi^2*x*csgn(I*(b*x+a)^n)^2*csgn(I*(b*x+a)^n/((d*x+c)^n
))^4+1/2*B^2*Pi^2*x*csgn(I*(b*x+a)^n)*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/
```

```
((d*x+c)^n))^2*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2-1/2*B^2*Pi^2*x*csgn(I*(b*x
+a)^n)*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^
n)*(b*x+a)^n)^3+I*B^2*ln(e)*Pi*csgn(I*(b*x+a)^n)*csgn(I*(b*x+a)^n/((d*x+c)^
n))^2*x+I*B^2*ln(e)*Pi*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))^2*
x-1/4*B^2*Pi^2*x*csgn(I*(b*x+a)^n/((d*x+c)^n))^6+2*n^2*B^2*c/d*dilog((b*(d*
x+c)+a*d-c*b)/(a*d-b*c))+I*B^2*ln(e)*Pi*csgn(I*e)*csgn(I*e/((d*x+c)^n)*(b*x
+a)^n)^2*x+1/2*B^2*Pi^2*x*csgn(I*e)*csgn(I*(b*x+a)^n)*csgn(I*(b*x+a)^n/((d*
x+c)^n))^3*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)+(-2*B^2*ln((b*x+a)^n)*x-B*(-I*B*
Pi*b*d*x*csgn(I*e)*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+
a)^n)+I*B*Pi*b*d*x*csgn(I*e)*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2-I*B*Pi*b*d*x
*csgn(I*(b*x+a)^n)*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))+I*B*Pi
*b*d*x*csgn(I*(b*x+a)^n)*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+I*B*Pi*b*d*x*csgn(
I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))^2-I*B*Pi*b*d*x*csgn(I*(b*x+a)^
n/((d*x+c)^n))^3+I*B*Pi*b*d*x*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+
c)^n)*(b*x+a)^n)^2-I*B*Pi*b*d*x*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^3+2*B*ln(e)
*b*d*x-2*B*ln(d*x+c)*b*c*n+2*B*a*d*n*ln(b*x+a)+2*A*b*d*x)/b/d)*ln((d*x+c)^n
)-1/4*B^2*Pi^2*x*csgn(I*(b*x+a)^n)^2*csgn(I/((d*x+c)^n))^2*csgn(I*(b*x+a)^n
/((d*x+c)^n))^2+1/2*B^2*Pi^2*x*csgn(I*(b*x+a)^n)^2*csgn(I/((d*x+c)^n))*csgn
(I*(b*x+a)^n/((d*x+c)^n))^3+2/b*B^2*ln(b*x+a)*ln(e)*a*n-2*B^2*c*n/d*ln(d*x+
c)*ln(e)+1/2*B^2*Pi^2*x*csgn(I*e)*csgn(I*(b*x+a)^n/((d*x+c)^n))^2*csgn(I*e/
((d*x+c)^n)*(b*x+a)^n)^3-B^2*Pi^2*x*csgn(I*e)*csgn(I*(b*x+a)^n/((d*x+c)^n))
*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^4-2/d*n*B^2*ln((b*x+a)^n)*c*ln(d*x+c)+1/2*
B^2*Pi^2*x*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^5-
1/4*B^2*Pi^2*x*csgn(I*e)^2*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^4+2*B^2*a*n^2/b+
1/2*B^2*Pi^2*x*csgn(I*(b*x+a)^n/((d*x+c)^n))^4*csgn(I*e/((d*x+c)^n)*(b*x+a)
^n)^2+2*n^2*B^2/b*a*ln(b*(d*x+c)+a*d-c*b)+I*A*B*Pi*csgn(I*(b*x+a)^n)*csgn(I
*(b*x+a)^n/((d*x+c)^n))^2*x+I*A*B*Pi*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/(
(d*x+c)^n))^2*x+I*A*B*Pi*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)
*(b*x+a)^n)^2*x+1/2*B^2*Pi^2*x*csgn(I*e)*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)
^n/((d*x+c)^n))^3*csgn(I*e/((d*x+c)^n)*(b*x+a)^...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2,x, algorithm="maxima")
```

```
[Out] 2*(a*n*e*log(b*x + a)/b - c*n*e*log(d*x + c)/d)*A*B*e^(-1) + 2*A*B*x*log((b
*x + a)^n*e/(d*x + c)^n) + A^2*x + B^2*((2*b*c*n^2*log(b*x + a)*log(d*x + c
) - b*c*n^2*log(d*x + c)^2 + b*d*x*log((b*x + a)^n)^2 + b*d*x*log((d*x + c)
^n)^2 + 2*(a*d*n*log(b*x + a) - b*c*n*log(d*x + c) + b*d*x)*log((b*x + a)^n
) - 2*(a*d*n*log(b*x + a) - b*c*n*log(d*x + c) + b*d*x*log((b*x + a)^n) + b
*d*x)*log((d*x + c)^n))/(b*d) - integrate(-(b^2*d*x^2 + a*b*c + (a*b*d*(2*n
```

```
 + 1) - b^2*c*(2*n - 1))*x - 2*(b^2*c*n^2*x + 2*a*b*c*n^2 - a^2*d*n^2)*log(
b*x + a))/(b^2*d*x^2 + a*b*c + (b^2*c + a*b*d)*x), x))
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2,x, algorithm="fricas")
```

```
[Out] integral(B^2*log((b*x + a)^n*e/(d*x + c)^n)^2 + 2*A*B*log((b*x + a)^n*e/(d*
x + c)^n) + A^2, x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**2,x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2,x, algorithm="giac")
```

```
[Out] integrate((B*log((b*x + a)^n*e/(d*x + c)^n) + A)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(A+B\,\ln\left(\frac{e\,{\left(a+b\,x\right)}^n}{{\left(c+d\,x\right)}^n}\right)\right)}^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^2,x)
```

```
[Out] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^2, x)
```

3.306 $\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{g+hx}\,dx$

Optimal. Leaf size=301

$$-\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{h}+\frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2\log\left(1-\frac{(dg-ch)(a}{(bg-ah)(c}\right.}{h}$$

```
[Out] -ln((-a*d+b*c)/b/(d*x+c))*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/h+(A+B*ln(e*(
b*x+a)^n/((d*x+c)^n)))^2*ln(1-(-c*h+d*g)*(b*x+a)/(-a*h+b*g)/(d*x+c))/h-2*B*
n*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))*polylog(2,d*(b*x+a)/b/(d*x+c))/h+2*B*n*
(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))*polylog(2,(-c*h+d*g)*(b*x+a)/(-a*h+b*g)/(
d*x+c))/h+2*B^2*n^2*polylog(3,d*(b*x+a)/b/(d*x+c))/h-2*B^2*n^2*polylog(3,(-
c*h+d*g)*(b*x+a)/(-a*h+b*g)/(d*x+c))/h
```

Rubi [A]
time = 0.38, antiderivative size = 301, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2573, 2553, 2404, 2354, 2421, 6724}

$$\frac{2Bn\,\text{PolyLog}\left(2,\frac{(a+bx)(dg-ch)}{(c+dx)(bg-ah)}\right)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{h}-\frac{2Bn\,\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{h}-\frac{2B^2n^2\text{PolyLog}\left(3,\frac{(a+bx)(dg-ch)}{(c+dx)(bg-ah)}\right)}{h}+\frac{2B^2n^2\text{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)}{h}+\frac{\log\left(1-\frac{(a+bx)(dg-ch)}{(c+dx)(bg-ah)}\right)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^2}{h}-\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^2}{h}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2/(g + h*x),x]
```

```
[Out] -((Log[(b*c - a*d)/(b*(c + d*x))]*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^
2)/h) + ((A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2*Log[1 - ((d*g - c*h)*(a
 + b*x))/((b*g - a*h)*(c + d*x))])/h - (2*B*n*(A + B*Log[(e*(a + b*x)^n)/(c
 + d*x)^n])*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/h + (2*B*n*(A + B*Log[
(e*(a + b*x)^n)/(c + d*x)^n])*PolyLog[2, ((d*g - c*h)*(a + b*x))/((b*g - a*
h)*(c + d*x))])/h + (2*B^2*n^2*PolyLog[3, (d*(a + b*x))/(b*(c + d*x))])/h -
 (2*B^2*n^2*PolyLog[3, ((d*g - c*h)*(a + b*x))/((b*g - a*h)*(c + d*x))])/h
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2404

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2553

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[b*c - a*d, Subst[
Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)),
 x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x
] && NeQ[b*c - a*d, 0] && IntegerQ[m] && IGtQ[p, 0]
```

Rule 2573

```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(A + B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{g+hx}\,dx = \int \left(\frac{A^2}{g+hx} + \frac{2AB\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{g+hx} + \frac{B^2\log^2\left(e(c\right.}{}\right.$$

$$= \frac{A^2\log(g+hx)}{h} + (2AB)\int \frac{\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{g+hx}\,dx + $$

$$= \frac{A^2\log(g+hx)}{h} + \frac{2AB\log\left(e(a+bx)^n(c+dx)^{-n}\right)\log(g+hx}{h}$$

$$= -\frac{B^2\log\left(\frac{bc-ad}{b(c+dx)}\right)\log^2\left(e(a+bx)^n(c+dx)^{-n}\right)}{h} + \frac{A^2\log(g+h}{h}$$

$$= -\frac{B^2\log\left(\frac{bc-ad}{b(c+dx)}\right)\log^2\left(e(a+bx)^n(c+dx)^{-n}\right)}{h} + \frac{A^2\log(g+h}{h}$$

$$= -\frac{B^2\log\left(\frac{bc-ad}{b(c+dx)}\right)\log^2\left(e(a+bx)^n(c+dx)^{-n}\right)}{h} + \frac{A^2\log(g+h}{h}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 1082 vs. $2(301) = 602$.
time = 0.24, size = 1082, normalized size = 3.59

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2/(g + h*x),x]
```

```
[Out] ((A + B*(-(n*Log[a + b*x]) + n*Log[c + d*x] + Log[(e*(a + b*x)^n)/(c + d*x)
^n]))^2*Log[g + h*x] + 2*B*n*(A + B*(-(n*Log[a + b*x]) + n*Log[c + d*x] + L
og[(e*(a + b*x)^n)/(c + d*x)^n]))*(Log[a + b*x]*Log[(b*(g + h*x))/(b*g - a*
h)] + PolyLog[2, (h*(a + b*x))/(-(b*g) + a*h)]) - 2*A*B*n*(Log[c + d*x]*Log
[(d*(g + h*x))/(d*g - c*h)] + PolyLog[2, (h*(c + d*x))/(-(d*g) + c*h)]) - 2
*B^2*n*(-(n*Log[a + b*x]) + n*Log[c + d*x] + Log[(e*(a + b*x)^n)/(c + d*x)^
n])*(Log[c + d*x]*Log[(d*(g + h*x))/(d*g - c*h)] + PolyLog[2, (h*(c + d*x))
/(-(d*g) + c*h)]) + B^2*n^2*(Log[a + b*x]^2*Log[(b*(g + h*x))/(b*g - a*h)]
+ 2*Log[a + b*x]*PolyLog[2, (h*(a + b*x))/(-(b*g) + a*h)] - 2*PolyLog[3, (h
*(a + b*x))/(-(b*g) + a*h)]) + B^2*n^2*(Log[c + d*x]^2*Log[(d*(g + h*x))/(d
*g - c*h)] + 2*Log[c + d*x]*PolyLog[2, (h*(c + d*x))/(-(d*g) + c*h)] - 2*Po
lyLog[3, (h*(c + d*x))/(-(d*g) + c*h)]) - 2*B^2*n^2*(Log[a + b*x]*Log[c + d
*x]*Log[(b*(g + h*x))/(b*g - a*h)] + (Log[(h*(c + d*x))/(-(d*g) + c*h)]*(-2
*Log[a + b*x] + Log[(h*(c + d*x))/(-(d*g) + c*h)])*(Log[(b*(g + h*x))/(b*g
- a*h)] - Log[(d*(g + h*x))/(d*g - c*h)]))/2 + Log[(h*(c + d*x))/(-(d*g) +
c*h)]*Log[((b*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))]*(-Log[(b*(g + h*
```

```
x))/(b*g - a*h)] + Log[(d*(g + h*x))/(d*g - c*h)]) + (Log[((b*g - a*h)*(c +
 d*x))/((d*g - c*h)*(a + b*x))]^2*(Log[(-(b*c) + a*d)/(d*(a + b*x))] + Log[
(b*(g + h*x))/(b*g - a*h)] - Log[((-(b*c) + a*d)*(g + h*x))/((d*g - c*h)*(a
 + b*x))]))/2 + (Log[c + d*x] - Log[((b*g - a*h)*(c + d*x))/((d*g - c*h)*(a
 + b*x))])*PolyLog[2, (h*(a + b*x))/(-(b*g) + a*h)] + (Log[a + b*x] + Log[(
(b*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))])*PolyLog[2, (h*(c + d*x))/(
-(d*g) + c*h)] + Log[((b*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))]*(Poly
Log[2, (b*(c + d*x))/(d*(a + b*x))] - PolyLog[2, ((b*g - a*h)*(c + d*x))/((
d*g - c*h)*(a + b*x))]) - PolyLog[3, (h*(a + b*x))/(-(b*g) + a*h)] - PolyLo
g[3, (h*(c + d*x))/(-(d*g) + c*h)] - PolyLog[3, (b*(c + d*x))/(d*(a + b*x))
] + PolyLog[3, ((b*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))]))/h
```

Maple [F]
time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{\left(A+B\ln\left(e(bx+a)^{n}\left(dx+c\right)^{-n}\right)\right)^{2}}{hx+g}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(h*x+g),x)
```

```
[Out] int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(h*x+g),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(h*x+g),x, algorithm="maxima
")
```

```
[Out] A^2*log(h*x + g)/h + integrate((B^2*log((b*x + a)^n)^2 + B^2*log((d*x + c)^
n)^2 + 2*A*B + B^2 + 2*(A*B + B^2)*log((b*x + a)^n) - 2*(B^2*log((b*x + a)^
n) + A*B + B^2)*log((d*x + c)^n))/(h*x + g), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(h*x+g),x, algorithm="fricas
")
```

```
[Out] integral((B^2*log((b*x + a)^n*e/(d*x + c)^n)^2 + 2*A*B*log((b*x + a)^n*e/(d
*x + c)^n) + A^2)/(h*x + g), x)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**2/(h*x+g),x)
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(h*x+g),x, algorithm="giac")
[Out] integrate((B*log((b*x + a)^n*e/(d*x + c)^n) + A)^2/(h*x + g), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+B\,\ln\left(\frac{e\,{\left(a+b\,x\right)}^n}{{\left(c+d\,x\right)}^n}\right)\right)^2}{g+h\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^2/(g + h*x),x)
[Out] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^2/(g + h*x), x)
```

3.307 $\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{(g+hx)^2}\,dx$

Optimal. Leaf size=208

$$\frac{(a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{(bg-ah)(g+hx)}+\frac{2B(bc-ad)n(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right))\log\left(1-\frac{(dg}{(bg}\right.}{(bg-ah)(dg-ch)}$$

```
[Out] (b*x+a)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(-a*h+b*g)/(h*x+g)+2*B*(-a*d+b*
c)*n*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))*ln(1-(-c*h+d*g)*(b*x+a)/(-a*h+b*g)/(
d*x+c))/(-a*h+b*g)/(-c*h+d*g)+2*B^2*(-a*d+b*c)*n^2*polylog(2,(-c*h+d*g)*(b*
x+a)/(-a*h+b*g)/(d*x+c))/(-a*h+b*g)/(-c*h+d*g)
```

Rubi [A]

time = 0.14, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {2573, 2553, 2355, 2354, 2438}

$$\frac{2B^2n^2(bc-ad)\text{PolyLog}\left(2,\frac{(a+bx)(dg-ch)}{(c+dx)(bg-ah)}\right)}{(bg-ah)(dg-ch)}+\frac{2Bn(bc-ad)\log\left(1-\frac{(a+bx)(dg-ch)}{(c+dx)(bg-ah)}\right)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{(bg-ah)(dg-ch)}+\frac{(a+bx)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^2}{(g+hx)(bg-ah)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2/(g + h*x)^2,x]
```

```
[Out] ((a + b*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2)/((b*g - a*h)*(g + h*
x)) + (2*B*(b*c - a*d)*n*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])*Log[1 - (
(d*g - c*h)*(a + b*x))/((b*g - a*h)*(c + d*x))])/((b*g - a*h)*(d*g - c*h))
+ (2*B^2*(b*c - a*d)*n^2*PolyLog[2, ((d*g - c*h)*(a + b*x))/((b*g - a*h)*(c
 + d*x))])/((b*g - a*h)*(d*g - c*h))
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2355

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[x*((a + b*Log[c*x^n])^p/(d*(d + e*x))), x] - Dist[b*n*(p/d),
Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n
, p}, x] && GtQ[p, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2553

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[b*c - a*d, Subst[
Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)),
 x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x
] && NeQ[b*c - a*d, 0] && IntegerQ[m] && IGtQ[p, 0]
```

Rule 2573

```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

Rubi steps

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{(g+hx)^2}\,dx = \int \left(\frac{A^2}{(g+hx)^2}+\frac{2AB\log(e(a+bx)^n(c+dx)^{-n})}{(g+hx)^2}+\frac{B^2\log^2(\right.$$

$$= -\frac{A^2}{h(g+hx)}+(2AB)\int \frac{\log(e(a+bx)^n(c+dx)^{-n})}{(g+hx)^2}\,dx+B^2$$

$$= -\frac{A^2}{h(g+hx)}+\frac{2AB(a+bx)\log(e(a+bx)^n(c+dx)^{-n})}{(bg-ah)(g+hx)}+\frac{B^2}{}$$

$$= -\frac{A^2}{h(g+hx)}+\frac{2AB(a+bx)\log(e(a+bx)^n(c+dx)^{-n})}{(bg-ah)(g+hx)}+\frac{B^2}{}$$

$$= -\frac{A^2}{h(g+hx)}-\frac{2AB(bc-ad)n\log(c+dx)}{(bg-ah)(dg-ch)}+\frac{2AB(a+bx)\,\mathrm{lo}}{(bg-}$$

$$= -\frac{A^2}{h(g+hx)}-\frac{2AB(bc-ad)n\log(c+dx)}{(bg-ah)(dg-ch)}+\frac{2AB(a+bx)\,\mathrm{lo}}{(bg-}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 3460 vs. 2(208) = 416.
time = 0.70, size = 3460, normalized size = 16.63

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2/(g + h*x)^2,x]
```

```
[Out] (-(A^2*b*d*g^2) + A^2*b*c*g*h + a*A^2*d*g*h - a*A^2*c*h^2 + 2*A*b*B*d*g^2*n
*Log[a + b*x] - 2*A*b*B*c*g*h*n*Log[a + b*x] + 2*A*b*B*d*g*h*n*x*Log[a + b*
x] - 2*A*b*B*c*h^2*n*x*Log[a + b*x] - b*B^2*d*g^2*n^2*Log[a + b*x]^2 + b*B^
2*c*g*h*n^2*Log[a + b*x]^2 - b*B^2*d*g*h*n^2*x*Log[a + b*x]^2 + b*B^2*c*h^2
*n^2*x*Log[a + b*x]^2 - 2*A*b*B*d*g^2*n*Log[c + d*x] + 2*a*A*B*d*g*h*n*Log[
c + d*x] - 2*A*b*B*d*g*h*n*x*Log[c + d*x] + 2*a*A*B*d*h^2*n*x*Log[c + d*x]
+ 2*b*B^2*d*g^2*n^2*Log[a + b*x]*Log[c + d*x] - 2*a*B^2*d*g*h*n^2*Log[a + b
*x]*Log[c + d*x] + 2*b*B^2*d*g*h*n^2*x*Log[a + b*x]*Log[c + d*x] - 2*a*B^2*
d*h^2*n^2*x*Log[a + b*x]*Log[c + d*x] - b*B^2*d*g^2*n^2*Log[c + d*x]^2 + a*
B^2*d*g*h*n^2*Log[c + d*x]^2 - b*B^2*d*g*h*n^2*x*Log[c + d*x]^2 + a*B^2*d*h
^2*n^2*x*Log[c + d*x]^2 - 2*b*B^2*c*g*h*n^2*Log[a + b*x]*Log[(h*(c + d*x))/
(-(d*g) + c*h)] + 2*a*B^2*d*g*h*n^2*Log[a + b*x]*Log[(h*(c + d*x))/(-(d*g)
+ c*h)] - 2*b*B^2*c*h^2*n^2*x*Log[a + b*x]*Log[(h*(c + d*x))/(-(d*g) + c*h)
] + 2*a*B^2*d*h^2*n^2*x*Log[a + b*x]*Log[(h*(c + d*x))/(-(d*g) + c*h)] + b*
B^2*c*g*h*n^2*Log[(h*(c + d*x))/(-(d*g) + c*h)]^2 - a*B^2*d*g*h*n^2*Log[(h*
(c + d*x))/(-(d*g) + c*h)]^2 + b*B^2*c*h^2*n^2*x*Log[(h*(c + d*x))/(-(d*g)
+ c*h)]^2 - a*B^2*d*h^2*n^2*x*Log[(h*(c + d*x))/(-(d*g) + c*h)]^2 - 2*b*B^2
*c*g*h*n^2*Log[(-(b*c) + a*d)/(d*(a + b*x))]*Log[((b*g - a*h)*(c + d*x))/((
d*g - c*h)*(a + b*x))] + 2*a*B^2*d*g*h*n^2*Log[(-(b*c) + a*d)/(d*(a + b*x))
]*Log[((b*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))] - 2*b*B^2*c*h^2*n^2*
x*Log[(-(b*c) + a*d)/(d*(a + b*x))]*Log[((b*g - a*h)*(c + d*x))/((d*g - c*h
)*(a + b*x))] + 2*a*B^2*d*h^2*n^2*x*Log[(-(b*c) + a*d)/(d*(a + b*x))]*Log[(
(b*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))] - 2*b*B^2*c*g*h*n^2*Log[(h*
(c + d*x))/(-(d*g) + c*h)]*Log[((b*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*
x))] + 2*a*B^2*d*g*h*n^2*Log[(h*(c + d*x))/(-(d*g) + c*h)]*Log[((b*g - a*h)
*(c + d*x))/((d*g - c*h)*(a + b*x))] - 2*b*B^2*c*h^2*n^2*x*Log[(h*(c + d*x)
)/(-(d*g) + c*h)]*Log[((b*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))] + 2*
a*B^2*d*h^2*n^2*x*Log[(h*(c + d*x))/(-(d*g) + c*h)]*Log[((b*g - a*h)*(c + d
*x))/((d*g - c*h)*(a + b*x))] + b*B^2*c*g*h*n^2*Log[((b*g - a*h)*(c + d*x))
/((d*g - c*h)*(a + b*x))]^2 - a*B^2*d*g*h*n^2*Log[((b*g - a*h)*(c + d*x))/(
(d*g - c*h)*(a + b*x))]^2 + b*B^2*c*h^2*n^2*x*Log[((b*g - a*h)*(c + d*x))/(
(d*g - c*h)*(a + b*x))]^2 - a*B^2*d*h^2*n^2*x*Log[((b*g - a*h)*(c + d*x))/(
(d*g - c*h)*(a + b*x))]^2 - 2*A*b*B*d*g^2*Log[(e*(a + b*x)^n)/(c + d*x)^n]
+ 2*A*b*B*c*g*h*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 2*a*A*B*d*g*h*Log[(e*(a
+ b*x)^n)/(c + d*x)^n] - 2*a*A*B*c*h^2*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 2
*b*B^2*d*g^2*n*Log[a + b*x]*Log[(e*(a + b*x)^n)/(c + d*x)^n] - 2*b*B^2*c*g*
h*n*Log[a + b*x]*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 2*b*B^2*d*g*h*n*x*Log[a
 + b*x]*Log[(e*(a + b*x)^n)/(c + d*x)^n] - 2*b*B^2*c*h^2*n*x*Log[a + b*x]*L
og[(e*(a + b*x)^n)/(c + d*x)^n] - 2*b*B^2*d*g^2*n*Log[c + d*x]*Log[(e*(a +
b*x)^n)/(c + d*x)^n] + 2*a*B^2*d*g*h*n*Log[c + d*x]*Log[(e*(a + b*x)^n)/(c
+ d*x)^n] - 2*b*B^2*d*g*h*n*x*Log[c + d*x]*Log[(e*(a + b*x)^n)/(c + d*x)^n]
```

```
 + 2*a*B^2*d*h^2*n*x*Log[c + d*x]*Log[(e*(a + b*x)^n)/(c + d*x)^n] - b*B^2*
d*g^2*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2 + b*B^2*c*g*h*Log[(e*(a + b*x)^n)/
(c + d*x)^n]^2 + a*B^2*d*g*h*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2 - a*B^2*c*h
^2*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2 - 2*A*b*B*d*g^2*n*Log[(b*(g + h*x))/(
b*g - a*h)] + 2*A*b*B*c*g*h*n*Log[(b*(g + h*x))/(b*g - a*h)] - 2*A*b*B*d*g*
h*n*x*Log[(b*(g + h*x))/(b*g - a*h)] + 2*A*b*B*c*h^2*n*x*Log[(b*(g + h*x))/
(b*g - a*h)] + 2*b*B^2*d*g^2*n^2*Log[a + b*x]*Log[(b*(g + h*x))/(b*g - a*h)
] - 2*a*B^2*d*g*h*n^2*Log[a + b*x]*Log[(b*(g + h*x))/(b*g - a*h)] + 2*b*B^2
*d*g*h*n^2*x*Log[a + b*x]*Log[(b*(g + h*x))/(b*g - a*h)] - 2*a*B^2*d*h^2*n^
2*x*Log[a + b*x]*Log[(b*(g + h*x))/(b*g - a*h)] - 2*b*B^2*d*g^2*n^2*Log[(h*
(c + d*x))/(-(d*g) + c*h)]*Log[(b*(g + h*x))/(b*g - a*h)] + 2*b*B^2*c*g*h*n
^2*Log[(h*(c + d*x))/(-(d*g) + c*h)]*Log[(b*(g + h*x))/(b*g - a*h)] - 2*b*B
^2*d*g*h*n^2*x*Log[(h*(c + d*x))/(-(d*g) + c*h)]*Log[(b*(g + h*x))/(b*g - a
*h)] + 2*b*B^2*c*h^2*n^2*x*Log[(h*(c + d*x))/(-(d*g) + c*h)]*Log[(b*(g + h*
x))/(b*g - a*h)] - 2*b*B^2*d*g^2*n*Log[(e*(a + b*x)^n)/(c + d*x)^n]*Log[(b*
(g + h*x))/(b*g - a*h)] + 2*b*B^2*c*g*h*n*Log[(e*(a + b*x)^n)/(c + d*x)^n]*
Log[(b*(g + h*x))/(b*g - a*h)] - 2*b*B^2*d*g*h*n*x*Log[(e*(a + b*x)^n)/(c +
 d*x)^n]*Log[(b*(g + h*x))/(b*g - a*h)] + 2*b*B^2*c*h^2*n*x*Log[(e*(a + b*x
)^n)/(c + d*x)^n]*Log[(b*(g + h*x))/(b*g - a*h)] + 2*A*b*B*d*g^2*n*Log[(d*(
g + h*x))/(d*g - c*h)] - 2*a*A*B*d*g*h*n*Log[(d*(g + h*x))/(d*g - c*h)] + 2
*A*b*B*d*g*h*n*x*Log[(d*(g + h*x))/(d*g - c*h)] - 2*a*A*B*d*h^2*n*x*Log[(d*
(g + h*x))/(d*g - c*h)] - 2*b*B^2*d*g^2*n^2*Log[a + b*x]*Log[(d*(g + h*x))/
(d*g - c*h)] + 2*a*B^2*d*g*h*n^2*Log[a + b*x]*Log[(d*(g + h*x))/(d*g - c*h)
] - 2*b*B^2*d*g*h*n^2*x*Log[a + b*x]*Log[(d*(g + h*x))/(d*g - c*h)] + 2*a*B
^2*d*h^2*n^2*x*Log[a + b*x]*Log[(d*(g + h*x))/(d*g - c*h)] + 2*b*B^2*d*g^2*
n^2*Log[(h*(c + d*x))/(-(d*g) + c*h)]*Log[(d*(g...
```

Maple [**F**]

time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{\left(A+B\ln\left(e(bx+a)^{n}\left(dx+c\right)^{-n}\right)\right)^{2}}{\left(hx+g\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(h*x+g)^2,x)
[Out] int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(h*x+g)^2,x)
```

Maxima [**F**]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(h*x+g)^2,x, algorithm="maxi
ma")
```

```
[Out] 2*(b*n*e*log(b*x + a)/(b*g*h - a*h^2) - d*n*e*log(d*x + c)/(d*g*h - c*h^2)
- (b*c*n - a*d*n)*e*log(h*x + g)/((d*g*h - c*h^2)*a - (d*g^2 - c*g*h)*b))*A
*B*e^(-1) - B^2*(log((d*x + c)^n)^2/(h^2*x + g*h) + integrate(-(d*h*x + (d*
h*x + c*h)*log((b*x + a)^n)^2 + c*h + 2*(d*h*x + c*h)*log((b*x + a)^n) + 2*
(d*h*(n - 1)*x + d*g*n - c*h - (d*h*x + c*h)*log((b*x + a)^n))*log((d*x + c
)^n))/(d*h^3*x^3 + c*g^2*h + (2*d*g*h^2 + c*h^3)*x^2 + (d*g^2*h + 2*c*g*h^2
)*x), x)) - 2*A*B*log((b*x + a)^n*e/(d*x + c)^n)/(h^2*x + g*h) - A^2/(h^2*x
 + g*h)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(h*x+g)^2,x, algorithm="fric
as")
```

```
[Out] integral((B^2*log((b*x + a)^n*e/(d*x + c)^n)^2 + 2*A*B*log((b*x + a)^n*e/(d
*x + c)^n) + A^2)/(h^2*x^2 + 2*g*h*x + g^2), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**2/(h*x+g)**2,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(h*x+g)^2,x, algorithm="giac
")
```

```
[Out] integrate((B*log((b*x + a)^n*e/(d*x + c)^n) + A)^2/(h*x + g)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(A+B\,\ln\left(\frac{e\,{\left(a+b\,x\right)}^n}{{\left(c+d\,x\right)}^n}\right)\right)}^2}{{\left(g+h\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^2/(g + h*x)^2,x)
[Out] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^2/(g + h*x)^2, x)
```

3.308 $\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{(g+hx)^3}\,dx$

Optimal. Leaf size=393

$$\frac{B(bc-ad)hn(a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{(bg-ah)^2(dg-ch)(g+hx)}+\frac{b^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{2h(bg-ah)^2}-\frac{(A+B$$

```
[Out] B*(-a*d+b*c)*h*n*(b*x+a)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/(-a*h+b*g)^2/(-c
*h+d*g)/(h*x+g)+1/2*b^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/h/(-a*h+b*g)^2-
1/2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/h/(h*x+g)^2+B^2*(-a*d+b*c)^2*h*n^2*
ln((h*x+g)/(d*x+c))/(-a*h+b*g)^2/(-c*h+d*g)^2+B*(-a*d+b*c)*(-a*d*h-b*c*h+2*
b*d*g)*n*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))*ln(1-(-c*h+d*g)*(b*x+a)/(-a*h+b*
g)/(d*x+c))/(-a*h+b*g)^2/(-c*h+d*g)^2+B^2*(-a*d+b*c)*(-a*d*h-b*c*h+2*b*d*g)
*n^2*polylog(2,(-c*h+d*g)*(b*x+a)/(-a*h+b*g)/(d*x+c))/(-a*h+b*g)^2/(-c*h+d*
g)^2
```

Rubi [A]
time = 0.57, antiderivative size = 393, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {2573, 2553, 2398, 2404, 2338, 2351, 31, 2354, 2438}

$$\frac{B^2n^2(bc-ad)(-adh-bch+2bdg)\text{PolyLog}\left(2,\frac{(a+bx)(dg-ch)}{(c+dx)(bg-ah)}\right)}{(bg-ah)^2(dg-ch)^2}+\frac{b^2\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^2}{2h(bg-ah)^2}+\frac{Bhn(a+bx)(bc-ad)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{(g+hx)(bg-ah)^2(dg-ch)}+\frac{Bn(bc-ad)(-adh-bch+2bdg)\log\left(1-\frac{(a+bx)(dg-ch)}{(c+dx)(bg-ah)}\right)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{(bg-ah)^2(dg-ch)^2}-\frac{\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^2}{2h(g+hx)^2}+\frac{B^2hn^2(bc-ad)^2\log\left(\frac{g+hx}{c+dx}\right)}{(bg-ah)^2(dg-ch)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2/(g + h*x)^3,x]
```

```
[Out] (B*(b*c - a*d)*h*n*(a + b*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]))/((b*
g - a*h)^2*(d*g - c*h)*(g + h*x)) + (b^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*
x)^n])^2)/(2*h*(b*g - a*h)^2) - (A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2/
(2*h*(g + h*x)^2) + (B^2*(b*c - a*d)^2*h*n^2*Log[(g + h*x)/(c + d*x)])/((b*
g - a*h)^2*(d*g - c*h)^2) + (B*(b*c - a*d)*(2*b*d*g - b*c*h - a*d*h)*n*(A +
 B*Log[(e*(a + b*x)^n)/(c + d*x)^n])*Log[1 - ((d*g - c*h)*(a + b*x))/((b*g
- a*h)*(c + d*x))])/((b*g - a*h)^2*(d*g - c*h)^2) + (B^2*(b*c - a*d)*(2*b*d
*g - b*c*h - a*d*h)*n^2*PolyLog[2, ((d*g - c*h)*(a + b*x))/((b*g - a*h)*(c
+ d*x))])/((b*g - a*h)^2*(d*g - c*h)^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_)*((
f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(d + e*x)^(q +
 1)*((a + b*Log[c*x^n])^p/((q + 1)*(e*f - d*g))), x] - Dist[b*n*(p/((q + 1)
*(e*f - d*g))), Int[(f + g*x)^(m + 1)*(d + e*x)^(q + 1)*((a + b*Log[c*x^n])
^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, q}, x] && NeQ[e*f
- d*g, 0] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 2404

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2553

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[b*c - a*d, Subst[
Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)),
 x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x
] && NeQ[b*c - a*d, 0] && IntegerQ[m] && IGtQ[p, 0]
```

Rule 2573

```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
```

`rQ[n]`

Rubi steps

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{(g+hx)^3}\,dx = \int \left(\frac{A^2}{(g+hx)^3}+\frac{2AB\log(e(a+bx)^n(c+dx)^{-n})}{(g+hx)^3}+\frac{B^2\log^2(e$$

$$= -\frac{A^2}{2h(g+hx)^2}+(2AB)\int \frac{\log(e(a+bx)^n(c+dx)^{-n})}{(g+hx)^3}\,dx+B^2$$

$$= -\frac{A^2}{2h(g+hx)^2}-\frac{AB\log(e(a+bx)^n(c+dx)^{-n})}{h(g+hx)^2}-\frac{B^2\log^2(e(c}{2h}$$

$$= -\frac{A^2}{2h(g+hx)^2}-\frac{AB\log(e(a+bx)^n(c+dx)^{-n})}{h(g+hx)^2}-\frac{B^2\log^2(e(c}{2h}$$

$$= -\frac{A^2}{2h(g+hx)^2}-\frac{AB(bc-ad)n}{(bg-ah)(dg-ch)(g+hx)}+\frac{Ab^2Bn\log(a+}{h(bg-ah)}$$

$$= -\frac{A^2}{2h(g+hx)^2}-\frac{AB(bc-ad)n}{(bg-ah)(dg-ch)(g+hx)}+\frac{Ab^2Bn\log(a+}{h(bg-ah)}$$

$$= -\frac{A^2}{2h(g+hx)^2}-\frac{AB(bc-ad)n}{(bg-ah)(dg-ch)(g+hx)}+\frac{Ab^2Bn\log(a+}{h(bg-ah)}$$

$$= -\frac{A^2}{2h(g+hx)^2}-\frac{AB(bc-ad)n}{(bg-ah)(dg-ch)(g+hx)}+\frac{Ab^2Bn\log(a+}{h(bg-ah)}$$

$$= -\frac{A^2}{2h(g+hx)^2}-\frac{AB(bc-ad)n}{(bg-ah)(dg-ch)(g+hx)}+\frac{Ab^2Bn\log(a+}{h(bg-ah)}$$

$$= -\frac{A^2}{2h(g+hx)^2}-\frac{AB(bc-ad)n}{(bg-ah)(dg-ch)(g+hx)}+\frac{Ab^2Bn\log(a+}{h(bg-ah)}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 15406 vs. $2(393) = 786$.
time = 6.17, size = 15406, normalized size = 39.20

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2/(g + h*x)^3,x]
```

```
[Out] Result too large to show
```

Maple [F]
time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{\left(A+B\ln\left(e(bx+a)^{n}\left(dx+c\right)^{-n}\right)\right)^{2}}{\left(hx+g\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(h*x+g)^3,x)
```

```
[Out] int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(h*x+g)^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(h*x+g)^3,x, algorithm="maxi
ma")
```

```
[Out] (b^2*n*e*log(b*x + a)/(b^2*g^2*h - 2*a*b*g*h^2 + a^2*h^3) - d^2*n*e*log(d*x
 + c)/(d^2*g^2*h - 2*c*d*g*h^2 + c^2*h^3) - (2*a*b*d^2*g*n - a^2*d^2*h*n -
(2*c*d*g*n - c^2*h*n)*b^2)*e*log(h*x + g)/((d^2*g^2*h^2 - 2*c*d*g*h^3 + c^2
*h^4)*a^2 - 2*(d^2*g^3*h - 2*c*d*g^2*h^2 + c^2*g*h^3)*a*b + (d^2*g^4 - 2*c*
d*g^3*h + c^2*g^2*h^2)*b^2) + (b*c*n - a*d*n)*e/((d*g^2*h - c*g*h^2)*a - (d
*g^3 - c*g^2*h)*b + ((d*g*h^2 - c*h^3)*a - (d*g^2*h - c*g*h^2)*b)*x))*A*B*e
^(-1) - 1/2*B^2*(log((d*x + c)^n)^2/(h^3*x^2 + 2*g*h^2*x + g^2*h) + 2*integ
rate(-(d*h*x + (d*h*x + c*h)*log((b*x + a)^n)^2 + c*h + 2*(d*h*x + c*h)*log
((b*x + a)^n) + (d*h*(n - 2)*x + d*g*n - 2*c*h - 2*(d*h*x + c*h)*log((b*x +
 a)^n))*log((d*x + c)^n))/(d*h^4*x^4 + c*g^3*h + (3*d*g*h^3 + c*h^4)*x^3 +
3*(d*g^2*h^2 + c*g*h^3)*x^2 + (d*g^3*h + 3*c*g^2*h^2)*x), x)) - A*B*log((b*
x + a)^n*e/(d*x + c)^n)/(h^3*x^2 + 2*g*h^2*x + g^2*h) - 1/2*A^2/(h^3*x^2 +
2*g*h^2*x + g^2*h)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(h*x+g)^3,x, algorithm="fric
as")
```

```
[Out] integral((B^2*log((b*x + a)^n*e/(d*x + c)^n)^2 + 2*A*B*log((b*x + a)^n*e/(d
*x + c)^n) + A^2)/(h^3*x^3 + 3*g*h^2*x^2 + 3*g^2*h*x + g^3), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**2/(h*x+g)**3,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(h*x+g)^3,x, algorithm="giac
")
```

```
[Out] integrate((B*log((b*x + a)^n*e/(d*x + c)^n) + A)^2/(h*x + g)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+B\,\ln\left(\frac{e\,{\left(a+b\,x\right)}^n}{{\left(c+d\,x\right)}^n}\right)\right)^2}{{\left(g+h\,x\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^2/(g + h*x)^3,x)
```

```
[Out] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^2/(g + h*x)^3, x)
```

3.309 $\int (g+hx)^2 \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3 dx$

Optimal. Leaf size=875

$$-\frac{B^3(bc-ad)^3h^2n^3\log(c+dx)}{b^3d^3}+\frac{B^2(bc-ad)^2h^2n^2(a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{b^3d^2}-\frac{2B^2(bc-}{}$$

```
[Out] -B^3*(-a*d+b*c)^3*h^2*n^3*ln(d*x+c)/b^3/d^3+B^2*(-a*d+b*c)^2*h^2*n^2*(b*x+a
)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/b^3/d^2-2*B^2*(-a*d+b*c)^2*h*(-a*d*h-2*
b*c*h+3*b*d*g)*n^2*ln((-a*d+b*c)/b/(d*x+c))*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)
))/b^3/d^3-B*(-a*d+b*c)*h*(-a*d*h-2*b*c*h+3*b*d*g)*n*(b*x+a)*(A+B*ln(e*(b*x
+a)^n/((d*x+c)^n)))^2/b^3/d^2-1/2*B*(-a*d+b*c)*h^2*n*(d*x+c)^2*(A+B*ln(e*(b
*x+a)^n/((d*x+c)^n)))^2/b/d^3+B*(-a*d+b*c)*(a^2*d^2*h^2-a*b*d*h*(-c*h+3*d*g
)+b^2*(c^2*h^2-3*c*d*g*h+3*d^2*g^2))*n*ln((-a*d+b*c)/b/(d*x+c))*(A+B*ln(e*(
b*x+a)^n/((d*x+c)^n)))^2/b^3/d^3-1/3*(-a*h+b*g)^3*(A+B*ln(e*(b*x+a)^n/((d*x
+c)^n)))^3/b^3/h+1/3*(h*x+g)^3*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/h-B^2*(-
a*d+b*c)^3*h^2*n^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))*ln(1-b*(d*x+c)/d/(b*x+
a))/b^3/d^3-2*B^3*(-a*d+b*c)^2*h*(-a*d*h-2*b*c*h+3*b*d*g)*n^3*polylog(2,d*(
b*x+a)/b/(d*x+c))/b^3/d^3+2*B^2*(-a*d+b*c)*(a^2*d^2*h^2-a*b*d*h*(-c*h+3*d*g
)+b^2*(c^2*h^2-3*c*d*g*h+3*d^2*g^2))*n^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))*
polylog(2,d*(b*x+a)/b/(d*x+c))/b^3/d^3+B^3*(-a*d+b*c)^3*h^2*n^3*polylog(2,b
*(d*x+c)/d/(b*x+a))/b^3/d^3-2*B^3*(-a*d+b*c)*(a^2*d^2*h^2-a*b*d*h*(-c*h+3*d
*g)+b^2*(c^2*h^2-3*c*d*g*h+3*d^2*g^2))*n^3*polylog(3,d*(b*x+a)/b/(d*x+c))/b
^3/d^3
```

Rubi [A]
time = 1.12, antiderivative size = 875, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 16, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.485$, Rules used = {2573, 2553, 2398, 2404, 2339, 30, 2356, 2389, 2379, 2438, 2351, 31, 2355, 2354, 2421, 6724}

Antiderivative was successfully verified.

```
[In] Int[(g + h*x)^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3,x]
```

```
[Out] -((B^3*(b*c - a*d)^3*h^2*n^3*Log[c + d*x])/(b^3*d^3)) + (B^2*(b*c - a*d)^2*
h^2*n^2*(a + b*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]))/(b^3*d^2) - (2*
B^2*(b*c - a*d)^2*h*(3*b*d*g - 2*b*c*h - a*d*h)*n^2*Log[(b*c - a*d)/(b*(c +
 d*x))]*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]))/(b^3*d^3) - (B*(b*c - a*d
)*h*(3*b*d*g - 2*b*c*h - a*d*h)*n*(a + b*x)*(A + B*Log[(e*(a + b*x)^n)/(c +
 d*x)^n])^2)/(b^3*d^2) - (B*(b*c - a*d)*h^2*n*(c + d*x)^2*(A + B*Log[(e*(a
+ b*x)^n)/(c + d*x)^n])^2)/(2*b*d^3) + (B*(b*c - a*d)*(a^2*d^2*h^2 - a*b*d*
h*(3*d*g - c*h) + b^2*(3*d^2*g^2 - 3*c*d*g*h + c^2*h^2))*n*Log[(b*c - a*d)/
(b*(c + d*x))]*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2)/(b^3*d^3) - ((b*
```

```
g - a*h)^3*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3)/(3*b^3*h) + ((g + h*
x)^3*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3)/(3*h) - (B^2*(b*c - a*d)^3
*h^2*n^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])*Log[1 - (b*(c + d*x))/(d*
(a + b*x))])/(b^3*d^3) - (2*B^3*(b*c - a*d)^2*h*(3*b*d*g - 2*b*c*h - a*d*h)
*n^3*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/(b^3*d^3) + (2*B^2*(b*c - a*d
)*(a^2*d^2*h^2 - a*b*d*h*(3*d*g - c*h) + b^2*(3*d^2*g^2 - 3*c*d*g*h + c^2*h
^2))*n^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])*PolyLog[2, (d*(a + b*x))/
(b*(c + d*x))])/(b^3*d^3) + (B^3*(b*c - a*d)^3*h^2*n^3*PolyLog[2, (b*(c + d
*x))/(d*(a + b*x))])/(b^3*d^3) - (2*B^3*(b*c - a*d)*(a^2*d^2*h^2 - a*b*d*h*
(3*d*g - c*h) + b^2*(3*d^2*g^2 - 3*c*d*g*h + c^2*h^2))*n^3*PolyLog[3, (d*(a
 + b*x))/(b*(c + d*x))])/(b^3*d^3)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2355

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[x*((a + b*Log[c*x^n])^p/(d*(d + e*x))), x] - Dist[b*n*(p/d),
Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n
```

```
, p}, x] && GtQ[p, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_)*((
f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(d + e*x)^(q +
 1)*((a + b*Log[c*x^n])^p/((q + 1)*(e*f - d*g))), x] - Dist[b*n*(p/((q + 1)
*(e*f - d*g))), Int[(f + g*x)^(m + 1)*(d + e*x)^(q + 1)*((a + b*Log[c*x^n])
^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, q}, x] && NeQ[e*f
- d*g, 0] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 2404

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
```

```
] && EqQ[d*e, 1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2553

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[b*c - a*d, Subst[
Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)),
 x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x
] && NeQ[b*c - a*d, 0] && IntegerQ[m] && IGtQ[p, 0]
```

Rule 2573

```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (g+hx)^2 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3\,dx = \int \left(A^3(g+hx)^2+3A^2B(g+hx)^2\log\left(e(a+bx)^n(\right.\right.$$

$$= \frac{A^3(g+hx)^3}{3h} + \left(3A^2B\right)\int (g+hx)^2\log\left(e(a+bx)^n\right.$$

$$= \frac{A^3(g+hx)^3}{3h} + \frac{A^2B(g+hx)^3\log\left(e(a+bx)^n(c+\right.}{h}$$

$$= \frac{A^3(g+hx)^3}{3h} + \frac{A^2B(g+hx)^3\log\left(e(a+bx)^n(c+\right.}{h}$$

$$= -\frac{A^2B(bc-ad)h(3bdg-bch-adh)nx}{b^2d^2} - \frac{A^2B(bc}{}$$

$$= -\frac{A^2B(bc-ad)h(3bdg-bch-adh)nx}{b^2d^2} - \frac{A^2B(bc}{}$$

$$= -\frac{A^2B(bc-ad)h(3bdg-bch-adh)nx}{b^2d^2} - \frac{A^2B(bc}{}$$

$$= -\frac{A^2B(bc-ad)h(3bdg-bch-adh)nx}{b^2d^2} + \frac{AB^2(bc}{}$$

$$= -\frac{A^2B(bc-ad)h(3bdg-bch-adh)nx}{b^2d^2} + \frac{AB^2(bc}{}$$

$$= -\frac{A^2B(bc-ad)h(3bdg-bch-adh)nx}{b^2d^2} + \frac{AB^2(bc}{}$$

$$= -\frac{A^2B(bc-ad)h(3bdg-bch-adh)nx}{b^2d^2} + \frac{AB^2(bc}{}$$

$$= -\frac{A^2B(bc-ad)h(3bdg-bch-adh)nx}{b^2d^2} + \frac{AB^2(bc}{}$$

$$= -\frac{A^2B(bc-ad)h(3bdg-bch-adh)nx}{b^2d^2} + \frac{AB^2(bc}{}$$

<u>Mathematica</u> [F]
time = 2.99, size = 0, normalized size = 0.00

$$\int (g+hx)^2 \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(g + h*x)^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3,x]
```

```
[Out] Integrate[(g + h*x)^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3, x]
```

Maple [F]
time = 0.17, size = 0, normalized size = 0.00

$$\int (hx+g)^2 \left(A+B\ln\left(e(bx+a)^n (dx+c)^{-n}\right)\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3,x)
```

```
[Out] int((h*x+g)^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^2*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3,x, algorithm="maxi
ma")
```

```
[Out] A^2*B*h^2*x^3*log((b*x + a)^n*e/(d*x + c)^n) + 1/3*A^3*h^2*x^3 + 3*A^2*B*g*
h*x^2*log((b*x + a)^n*e/(d*x + c)^n) + A^3*g*h*x^2 + 3*(a*n*e*log(b*x + a)/
b - c*n*e*log(d*x + c)/d)*A^2*B*g^2*e^(-1) - 3*(a^2*n*e*log(b*x + a)/b^2 -
c^2*n*e*log(d*x + c)/d^2 + (b*c*n - a*d*n)*x*e/(b*d))*A^2*B*g*h*e^(-1) + 1/
2*(2*a^3*n*e*log(b*x + a)/b^3 - 2*c^3*n*e*log(d*x + c)/d^3 - ((b^2*c*d*n -
a*b*d^2*n)*x^2*e - 2*(b^2*c^2*n - a^2*d^2*n)*x*e)/(b^2*d^2))*A^2*B*h^2*e^(-
1) + 3*A^2*B*g^2*x*log((b*x + a)^n*e/(d*x + c)^n) + A^3*g^2*x - 1/6*(2*(B^3
*b^3*d^3*h^2*x^3 + 3*B^3*b^3*d^3*g*h*x^2 + 3*B^3*b^3*d^3*g^2*x)*log((d*x +
c)^n)^3 + 3*(2*(3*c*d^2*g^2*n - 3*c^2*d*g*h*n + c^3*h^2*n)*B^3*b^3*log(d*x
+ c) - 2*(3*a*b^2*d^3*g^2*n - 3*a^2*b*d^3*g*h*n + a^3*d^3*h^2*n)*B^3*log(b*
x + a) - 2*(A*B^2*b^3*d^3*h^2 + B^3*b^3*d^3*h^2)*x^3 - (6*A*B^2*b^3*d^3*g*h
 + (a*b^2*d^3*h^2*n - (c*d^2*h^2*n - 6*d^3*g*h)*b^3)*B^3)*x^2 - 2*(3*A*B^2*
b^3*d^3*g^2 + (3*a*b^2*d^3*g*h*n - a^2*b*d^3*h^2*n - (3*c*d^2*g*h*n - c^2*d
*h^2*n - 3*d^3*g^2)*b^3)*B^3)*x - 2*(B^3*b^3*d^3*h^2*x^3 + 3*B^3*b^3*d^3*g*
h*x^2 + 3*B^3*b^3*d^3*g^2*x)*log((b*x + a)^n))*log((d*x + c)^n)^2)/(b^3*d^3
) - integrate(-(3*A*B^2*b^3*c*d^2*g^2 + B^3*b^3*c*d^2*g^2 + (3*A*B^2*b^3*d^
3*h^2 + B^3*b^3*d^3*h^2)*x^3 + (B^3*b^3*d^3*h^2*x^3 + B^3*b^3*c*d^2*g^2 + (
2*d^3*g*h + c*d^2*h^2)*B^3*b^3*x^2 + (d^3*g^2 + 2*c*d^2*g*h)*B^3*b^3*x)*log
((b*x + a)^n)^3 + (3*(2*d^3*g*h + c*d^2*h^2)*A*B^2*b^3 + (2*d^3*g*h + c*d^2
*h^2)*B^3*b^3)*x^2 + 3*(A*B^2*b^3*c*d^2*g^2 + B^3*b^3*c*d^2*g^2 + (A*B^2*b^
3*d^3*h^2 + B^3*b^3*d^3*h^2)*x^3 + ((2*d^3*g*h + c*d^2*h^2)*A*B^2*b^3 + (2*
```

```
d^3*g*h + c*d^2*h^2)*B^3*b^3)*x^2 + ((d^3*g^2 + 2*c*d^2*g*h)*A*B^2*b^3 + (d
^3*g^2 + 2*c*d^2*g*h)*B^3*b^3)*x)*log((b*x + a)^n)^2 + (3*(d^3*g^2 + 2*c*d^
2*g*h)*A*B^2*b^3 + (d^3*g^2 + 2*c*d^2*g*h)*B^3*b^3)*x + 3*(2*A*B^2*b^3*c*d^
2*g^2 + B^3*b^3*c*d^2*g^2 + (2*A*B^2*b^3*d^3*h^2 + B^3*b^3*d^3*h^2)*x^3 + (
2*(2*d^3*g*h + c*d^2*h^2)*A*B^2*b^3 + (2*d^3*g*h + c*d^2*h^2)*B^3*b^3)*x^2
+ (2*(d^3*g^2 + 2*c*d^2*g*h)*A*B^2*b^3 + (d^3*g^2 + 2*c*d^2*g*h)*B^3*b^3)*x
)*log((b*x + a)^n) - (6*A*B^2*b^3*c*d^2*g^2 + 3*B^3*b^3*c*d^2*g^2 - 2*(3*c*
d^2*g^2*n^2 - 3*c^2*d*g*h*n^2 + c^3*h^2*n^2)*B^3*b^3*log(d*x + c) + 2*(3*a*
b^2*d^3*g^2*n^2 - 3*a^2*b*d^3*g*h*n^2 + a^3*d^3*h^2*n^2)*B^3*log(b*x + a) +
 (B^3*b^3*d^3*h^2*(2*n + 3) + 2*A*B^2*b^3*d^3*h^2*(n + 3))*x^3 + (6*(d^3*g*
h*(n + 2) + c*d^2*h^2)*A*B^2*b^3 + (a*b^2*d^3*h^2*n^2 - ((n^2 - 3)*c*d^2*h^
2 - 6*d^3*g*h*(n + 1))*b^3)*B^3)*x^2 + 3*(B^3*b^3*d^3*h^2*x^3 + B^3*b^3*c*d
^2*g^2 + (2*d^3*g*h + c*d^2*h^2)*B^3*b^3*x^2 + (d^3*g^2 + 2*c*d^2*g*h)*B^3*
b^3*x)*log((b*x + a)^n)^2 + (6*(d^3*g^2*(n + 1) + 2*c*d^2*g*h)*A*B^2*b^3 +
(6*a*b^2*d^3*g*h*n^2 - 2*a^2*b*d^3*h^2*n^2 + (2*c^2*d*h^2*n^2 - 6*(n^2 - 1)
*c*d^2*g*h + 3*d^3*g^2*(2*n + 1))*b^3)*B^3)*x + 2*(3*A*B^2*b^3*c*d^2*g^2 +
3*B^3*b^3*c*d^2*g^2 + (B^3*b^3*d^3*h^2*(n + 3) + 3*A*B^2*b^3*d^3*h^2)*x^3 +
 3*((2*d^3*g*h + c*d^2*h^2)*A*B^2*b^3 + (d^3*g*h*(n + 2) + c*d^2*h^2)*B^3*b
^3)*x^2 + 3*((d^3*g^2 + 2*c*d^2*g*h)*A*B^2*b^3 + (d^3*g^2*(n + 1) + 2*c*d^2
*g*h)*B^3*b^3)*x)*log((b*x + a)^n))*log((d*x + c)^n))/(b^3*d^3*x + b^3*c*d^
2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^2*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3,x, algorithm="fric
as")
```

```
[Out] integral(A^3*h^2*x^2 + 2*A^3*g*h*x + A^3*g^2 + (B^3*h^2*x^2 + 2*B^3*g*h*x +
 B^3*g^2)*log((b*x + a)^n*e/(d*x + c)^n)^3 + 3*(A*B^2*h^2*x^2 + 2*A*B^2*g*h
*x + A*B^2*g^2)*log((b*x + a)^n*e/(d*x + c)^n)^2 + 3*(A^2*B*h^2*x^2 + 2*A^2
*B*g*h*x + A^2*B*g^2)*log((b*x + a)^n*e/(d*x + c)^n), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)**2*(A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**3,x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^2*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3,x, algorithm="giac
")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int (g+h\,x)^2\,\left(A+B\,\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g + h*x)^2*(A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^3,x)
```

```
[Out] int((g + h*x)^2*(A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^3, x)
```

3.310 $\int (g+hx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3\,dx$

Optimal. Leaf size=466

$$-\frac{3B^2(bc-ad)^2hn^2\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{b^2d^2}-\frac{3B(bc-ad)hn(a+bx)\left(A+B\log\left(\right.\right.}{2b^2d}$$

```
[Out] -3*B^2*(-a*d+b*c)^2*h*n^2*ln((-a*d+b*c)/b/(d*x+c))*(A+B*ln(e*(b*x+a)^n/((d*
x+c)^n)))/b^2/d^2-3/2*B*(-a*d+b*c)*h*n*(b*x+a)*(A+B*ln(e*(b*x+a)^n/((d*x+c)
^n)))^2/b^2/d+3/2*B*(-a*d+b*c)*(-a*d*h-b*c*h+2*b*d*g)*n*ln((-a*d+b*c)/b/(d*
x+c))*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/b^2/d^2-1/2*(-a*h+b*g)^2*(A+B*ln(
e*(b*x+a)^n/((d*x+c)^n)))^3/b^2/h+1/2*(h*x+g)^2*(A+B*ln(e*(b*x+a)^n/((d*x+c
)^n)))^3/h-3*B^3*(-a*d+b*c)^2*h*n^3*polylog(2,d*(b*x+a)/b/(d*x+c))/b^2/d^2+
3*B^2*(-a*d+b*c)*(-a*d*h-b*c*h+2*b*d*g)*n^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)
))*polylog(2,d*(b*x+a)/b/(d*x+c))/b^2/d^2-3*B^3*(-a*d+b*c)*(-a*d*h-b*c*h+2*
b*d*g)*n^3*polylog(3,d*(b*x+a)/b/(d*x+c))/b^2/d^2
```

Rubi [A]
time = 0.54, antiderivative size = 466, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.355$, Rules used = {2573, 2553, 2398, 2404, 2339, 30, 2355, 2354, 2438, 2421, 6724}

$$\frac{3B^2n^2(bc-ad)(-adh-bch+2bdg)\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{b^2d^2}-\frac{3B^3n^3(bc-ad)(-adh-bch+2bdg)\text{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)}{b^2d^2}-\frac{3B^3hn^3(bc-ad)^2\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{b^2d^2}-\frac{3B^2hn^2(bc-ad)^2\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{b^2d^2}+\frac{3Bn(bc-ad)(-adh-bch+2bdg)\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^2}{2b^2d^2}-\frac{(bg-ah)^2\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^3}{2b^2h}-\frac{3Bhn(a+bx)(bc-ad)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^2}{2b^2d}+\frac{(g+hx)^2\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^3}{2h}$$

Antiderivative was successfully verified.

```
[In] Int[(g + h*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3,x]
```

```
[Out] (-3*B^2*(b*c - a*d)^2*h*n^2*Log[(b*c - a*d)/(b*(c + d*x))]*(A + B*Log[(e*(a
 + b*x)^n)/(c + d*x)^n]))/(b^2*d^2) - (3*B*(b*c - a*d)*h*n*(a + b*x)*(A + B
*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2)/(2*b^2*d) + (3*B*(b*c - a*d)*(2*b*d*g
 - b*c*h - a*d*h)*n*Log[(b*c - a*d)/(b*(c + d*x))]*(A + B*Log[(e*(a + b*x)^
n)/(c + d*x)^n])^2)/(2*b^2*d^2) - ((b*g - a*h)^2*(A + B*Log[(e*(a + b*x)^n)
/(c + d*x)^n])^3)/(2*b^2*h) + ((g + h*x)^2*(A + B*Log[(e*(a + b*x)^n)/(c +
d*x)^n])^3)/(2*h) - (3*B^3*(b*c - a*d)^2*h*n^3*PolyLog[2, (d*(a + b*x))/(b*
(c + d*x))])/(b^2*d^2) + (3*B^2*(b*c - a*d)*(2*b*d*g - b*c*h - a*d*h)*n^2*(
A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])*PolyLog[2, (d*(a + b*x))/(b*(c + d*
x))])/(b^2*d^2) - (3*B^3*(b*c - a*d)*(2*b*d*g - b*c*h - a*d*h)*n^3*PolyLog[
3, (d*(a + b*x))/(b*(c + d*x))])/(b^2*d^2)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2355

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[x*((a + b*Log[c*x^n])^p/(d*(d + e*x))), x] - Dist[b*n*(p/d),
Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n
, p}, x] && GtQ[p, 0]
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_)*((
f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(d + e*x)^(q +
 1)*((a + b*Log[c*x^n])^p/((q + 1)*(e*f - d*g))), x] - Dist[b*n*(p/((q + 1)
*(e*f - d*g))), Int[(f + g*x)^(m + 1)*(d + e*x)^(q + 1)*((a + b*Log[c*x^n])
^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, q}, x] && NeQ[e*f
- d*g, 0] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 2404

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2553

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[b*c - a*d, Subst[
Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)),
 x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x
] && NeQ[b*c - a*d, 0] && IntegerQ[m] && IGtQ[p, 0]
```

Rule 2573

```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (g+hx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3\,dx=\int \left(A^3(g+hx)+3A^2B(g+hx)\log\left(e(a+bx)^n(c+d\right.\right.$$

$$=\frac{A^3(g+hx)^2}{2h}+\left(3A^2B\right)\int (g+hx)\log\left(e(a+bx)^n(c\right.$$

$$=\frac{A^3(g+hx)^2}{2h}+\frac{3A^2B(g+hx)^2\log\left(e(a+bx)^n(c+dx\right.}{2h}$$

$$=\frac{A^3(g+hx)^2}{2h}+\frac{3A^2B(g+hx)^2\log\left(e(a+bx)^n(c+dx\right.}{2h}$$

$$=-\frac{3A^2B(bc-ad)hnx}{2bd}+\frac{A^3(g+hx)^2}{2h}-\frac{3A^2B(bg-a}{2}$$

$$=-\frac{3A^2B(bc-ad)hnx}{2bd}+\frac{A^3(g+hx)^2}{2h}-\frac{3A^2B(bg-a}{2}$$

$$=-\frac{3A^2B(bc-ad)hnx}{2bd}+\frac{A^3(g+hx)^2}{2h}-\frac{3A^2B(bg-a}{2}$$

$$=-\frac{3A^2B(bc-ad)hnx}{2bd}+\frac{A^3(g+hx)^2}{2h}-\frac{3A^2B(bg-a}{2}$$

$$=-\frac{3A^2B(bc-ad)hnx}{2bd}+\frac{A^3(g+hx)^2}{2h}-\frac{3A^2B(bg-a}{2}$$

$$=-\frac{3A^2B(bc-ad)hnx}{2bd}+\frac{A^3(g+hx)^2}{2h}-\frac{3A^2B(bg-a}{2}$$

$$=-\frac{3A^2B(bc-ad)hnx}{2bd}+\frac{A^3(g+hx)^2}{2h}-\frac{3A^2B(bg-a}{2}$$

Mathematica [F]
time = 1.61, size = 0, normalized size = 0.00

$$\int (g+hx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(g + h*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3,x]
[Out] Integrate[(g + h*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3, x]
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int (hx+g)\left(A+B\ln\left(e(bx+a)^{n}(dx+c)^{-n}\right)\right)^{3}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3,x)
```

```
[Out] int((h*x+g)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3,x, algorithm="maxima
")
```

```
[Out] 3/2*A^2*B*h*x^2*log((b*x + a)^n*e/(d*x + c)^n) + 1/2*A^3*h*x^2 + 3*(a*n*e*l
og(b*x + a)/b - c*n*e*log(d*x + c)/d)*A^2*B*g*e^(-1) - 3/2*(a^2*n*e*log(b*x
 + a)/b^2 - c^2*n*e*log(d*x + c)/d^2 + (b*c*n - a*d*n)*x*e/(b*d))*A^2*B*h*e
^(-1) + 3*A^2*B*g*x*log((b*x + a)^n*e/(d*x + c)^n) + A^3*g*x - 1/2*((B^3*b^
2*d^2*h*x^2 + 2*B^3*b^2*d^2*g*x)*log((d*x + c)^n)^3 + 3*((2*c*d*g*n - c^2*h
*n)*B^3*b^2*log(d*x + c) - (2*a*b*d^2*g*n - a^2*d^2*h*n)*B^3*log(b*x + a) -
 (A*B^2*b^2*d^2*h + B^3*b^2*d^2*h)*x^2 - (2*A*B^2*b^2*d^2*g + (a*b*d^2*h*n
- (c*d*h*n - 2*d^2*g)*b^2)*B^3)*x - (B^3*b^2*d^2*h*x^2 + 2*B^3*b^2*d^2*g*x)
*log((b*x + a)^n))*log((d*x + c)^n)^2)/(b^2*d^2) - integrate(-(3*A*B^2*b^2*
c*d*g + B^3*b^2*c*d*g + (B^3*b^2*d^2*h*x^2 + B^3*b^2*c*d*g + (d^2*g + c*d*h
)*B^3*b^2*x)*log((b*x + a)^n)^3 + (3*A*B^2*b^2*d^2*h + B^3*b^2*d^2*h)*x^2 +
 3*(A*B^2*b^2*c*d*g + B^3*b^2*c*d*g + (A*B^2*b^2*d^2*h + B^3*b^2*d^2*h)*x^2
 + ((d^2*g + c*d*h)*A*B^2*b^2 + (d^2*g + c*d*h)*B^3*b^2)*x)*log((b*x + a)^n
)^2 + (3*(d^2*g + c*d*h)*A*B^2*b^2 + (d^2*g + c*d*h)*B^3*b^2)*x + 3*(2*A*B^
2*b^2*c*d*g + B^3*b^2*c*d*g + (2*A*B^2*b^2*d^2*h + B^3*b^2*d^2*h)*x^2 + (2*
(d^2*g + c*d*h)*A*B^2*b^2 + (d^2*g + c*d*h)*B^3*b^2)*x)*log((b*x + a)^n) -
3*(2*A*B^2*b^2*c*d*g + B^3*b^2*c*d*g - (2*c*d*g*n^2 - c^2*h*n^2)*B^3*b^2*lo
g(d*x + c) + (2*a*b*d^2*g*n^2 - a^2*d^2*h*n^2)*B^3*log(b*x + a) + (A*B^2*b^
2*d^2*h*(n + 2) + B^3*b^2*d^2*h*(n + 1))*x^2 + (B^3*b^2*d^2*h*x^2 + B^3*b^2
*c*d*g + (d^2*g + c*d*h)*B^3*b^2*x)*log((b*x + a)^n)^2 + (2*(d^2*g*(n + 1)
+ c*d*h)*A*B^2*b^2 + (a*b*d^2*h*n^2 - ((n^2 - 1)*c*d*h - d^2*g*(2*n + 1))*b
^2)*B^3)*x + (2*A*B^2*b^2*c*d*g + 2*B^3*b^2*c*d*g + (B^3*b^2*d^2*h*(n + 2)
+ 2*A*B^2*b^2*d^2*h)*x^2 + 2*((d^2*g + c*d*h)*A*B^2*b^2 + (d^2*g*(n + 1) +
c*d*h)*B^3*b^2)*x)*log((b*x + a)^n))*log((d*x + c)^n))/(b^2*d^2*x + b^2*c*d
), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3,x, algorithm="fricas
")
```

```
[Out] integral(A^3*h*x + A^3*g + (B^3*h*x + B^3*g)*log((b*x + a)^n*e/(d*x + c)^n)
^3 + 3*(A*B^2*h*x + A*B^2*g)*log((b*x + a)^n*e/(d*x + c)^n)^2 + 3*(A^2*B*h*
x + A^2*B*g)*log((b*x + a)^n*e/(d*x + c)^n), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**3,x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (g+h\,x)\,\left(A+B\,\ln\left(\frac{e\,{\left(a+b\,x\right)}^n}{{\left(c+d\,x\right)}^n}\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g + h*x)*(A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^3,x)
```

```
[Out] int((g + h*x)*(A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^3, x)
```

3.311 $\int (A+B\log(e(a+bx)^n(c+dx)^{-n}))^3\,dx$

Optimal. Leaf size=203

$$\frac{3B(bc-ad)n\log\left(\frac{bc-ad}{b(c+dx)}\right)(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{bd}+\frac{(a+bx)\,(A+B\log(e(a+bx)^n(c+dx)^{-}$$

```
[Out] 3*B*(-a*d+b*c)*n*ln((-a*d+b*c)/b/(d*x+c))*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))
^2/b/d+(b*x+a)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/b+6*B^2*(-a*d+b*c)*n^2*(
A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))*polylog(2,d*(b*x+a)/b/(d*x+c))/b/d-6*B^3*(
-a*d+b*c)*n^3*polylog(3,d*(b*x+a)/b/(d*x+c))/b/d
```

Rubi [A]
time = 0.12, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {2536, 2573, 2551, 2354, 2421, 6724}

$$\frac{6B^2n^2(bc-ad)\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)(B\log(e(a+bx)^n(c+dx)^{-n})+A)}{bd}-\frac{6B^3n^3(bc-ad)\text{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)}{bd}+\frac{3Bn(bc-ad)\log\left(\frac{bc-ad}{b(c+dx)}\right)(B\log(e(a+bx)^n(c+dx)^{-n})+A)^2}{bd}+\frac{(a+bx)(B\log(e(a+bx)^n(c+dx)^{-n})+A)^3}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3,x]
```

```
[Out] (3*B*(b*c - a*d)*n*Log[(b*c - a*d)/(b*(c + d*x))]*(A + B*Log[(e*(a + b*x)^n
)/(c + d*x)^n])^2)/(b*d) + ((a + b*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^
n])^3)/b + (6*B^2*(b*c - a*d)*n^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])*
PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/(b*d) - (6*B^3*(b*c - a*d)*n^3*Pol
yLog[3, (d*(a + b*x))/(b*(c + d*x))])/(b*d)
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2536

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.), x_Symbol] :> Simp[(a + b*x)*((A + B*Log[e*((a + b*x)^n/(c
```

```
+ d*x)^n)])^p/b), x] - Dist[B*n*p*((b*c - a*d)/b), Int[(A + B*Log[e*((a + b
*x)^n/(c + d*x)^n)])^(p - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, A,
B, n}, x] && EqQ[n + mn, 0] && NeQ[b*c - a*d, 0] && IGtQ[p, 0]
```

Rule 2551

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(m +
1)*(g/d)^m, Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a + b
*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ[b*c -
 a*d, 0] && IntegersQ[m, p] && EqQ[d*f - c*g, 0] && (GtQ[p, 0] || LtQ[m, -1
])
```

Rule 2573

```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\int \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3\,dx &= \int \left(A^3+3A^2B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+3AB^2\log^2\left(e(a\right.\right. \\
&= A^3x+\left(3A^2B\right)\int \log\left(e(a+bx)^n(c+dx)^{-n}\right)\,dx+\left(3AB^2\right) \\
&= A^3x+\frac{3A^2B(a+bx)\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{b}+\frac{3AB^2(a}{} \\
&= A^3x-\frac{3A^2B(bc-ad)n\log(c+dx)}{bd}+\frac{3A^2B(a+bx)\log\left(e(a\right.}{b} \\
&= A^3x-\frac{3A^2B(bc-ad)n\log(c+dx)}{bd}+\frac{3A^2B(a+bx)\log\left(e(a\right.}{b} \\
&= A^3x-\frac{3A^2B(bc-ad)n\log(c+dx)}{bd}+\frac{3A^2B(a+bx)\log\left(e(a\right.}{b} \\
&= A^3x-\frac{3A^2B(bc-ad)n\log(c+dx)}{bd}+\frac{3A^2B(a+bx)\log\left(e(a\right.}{b} \\
&= A^3x-\frac{3A^2B(bc-ad)n\log(c+dx)}{bd}+\frac{3A^2B(a+bx)\log\left(e(a\right.}{b}
\end{aligned}
$$

Mathematica [A]
time = 0.33, size = 372, normalized size = 1.83

$$
A^3x+\frac{B\left(-3A^2(bc-ad)n\log(c+dx)+3A^2d(a+bx)\log\left(e(a+bx)^n(c+dx)^{-n}\right)+3ABd(a+bx)\log^2\left(e(a+bx)^n(c+dx)^{-n}\right)+B^2d(a+bx)\log^3\left(e(a+bx)^n(c+dx)^{-n}\right)+3AB(bc-ad)n\left(-\log\left(\frac{bc-ad}{bc+bdx}\right)\left(2n\log\left(\frac{d(a+bx)}{-bc+ad}\right)-2\log\left(e(a+bx)^n(c+dx)^{-n}\right)+n\log\left(\frac{bc-ad}{bc+bdx}\right)\right)+2n\mathrm{Li}_2\left(\frac{b(c+dx)}{bc-ad}\right)\right)+3B^2(bc-ad)n\left(\log^2\left(e(a+bx)^n(c+dx)^{-n}\right)\log\left(\frac{bc-ad}{bc+bdx}\right)+2n\log\left(e(a+bx)^n(c+dx)^{-n}\right)\mathrm{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)-2n^2\mathrm{Li}_3\left(\frac{d(a+bx)}{b(c+dx)}\right)\right)\right)}{bd}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3,x]
```

```
[Out] A^3*x + (B*(-3*A^2*(b*c - a*d)*n*Log[c + d*x] + 3*A^2*d*(a + b*x)*Log[(e*(a
 + b*x)^n)/(c + d*x)^n] + 3*A*B*d*(a + b*x)*Log[(e*(a + b*x)^n)/(c + d*x)^n
]^2 + B^2*d*(a + b*x)*Log[(e*(a + b*x)^n)/(c + d*x)^n]^3 + 3*A*B*(b*c - a*d
)*n*(-(Log[(b*c - a*d)/(b*c + b*d*x)]*(2*n*Log[(d*(a + b*x))/(-(b*c) + a*d)
] - 2*Log[(e*(a + b*x)^n)/(c + d*x)^n] + n*Log[(b*c - a*d)/(b*c + b*d*x)]))
 + 2*n*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]) + 3*B^2*(b*c - a*d)*n*(Log[(e
*(a + b*x)^n)/(c + d*x)^n]^2*Log[(b*c - a*d)/(b*c + b*d*x)] + 2*n*Log[(e*(a
 + b*x)^n)/(c + d*x)^n]*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))] - 2*n^2*Pol
yLog[3, (d*(a + b*x))/(b*(c + d*x))])))/(b*d)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$
\int \left(A+B\ln\left(e(bx+a)^n\,(dx+c)^{-n}\right)\right)^3\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3,x)
```

```
[Out] int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3,x, algorithm="maxima")
```

```
[Out] 3*(a*n*e*log(b*x + a)/b - c*n*e*log(d*x + c)/d)*A^2*B*e^(-1) + 3*A^2*B*x*lo
g((b*x + a)^n*e/(d*x + c)^n) + A^3*x - (B^3*b*d*x*log((d*x + c)^n)^3 - 3*(B
^3*a*d*n*log(b*x + a) - B^3*b*c*n*log(d*x + c) + B^3*b*d*x*log((b*x + a)^n)
 + (A*B^2*b*d + B^3*b*d)*x)*log((d*x + c)^n)^2)/(b*d) - integrate(-(3*A*B^2
*b*c + B^3*b*c + (B^3*b*d*x + B^3*b*c)*log((b*x + a)^n)^3 + 3*(A*B^2*b*c +
B^3*b*c + (A*B^2*b*d + B^3*b*d)*x)*log((b*x + a)^n)^2 + (3*A*B^2*b*d + B^3*
b*d)*x + 3*(2*A*B^2*b*c + B^3*b*c + (2*A*B^2*b*d + B^3*b*d)*x)*log((b*x + a
)^n) - 3*(2*B^3*a*d*n^2*log(b*x + a) - 2*B^3*b*c*n^2*log(d*x + c) + 2*A*B^2
*b*c + B^3*b*c + (B^3*b*d*x + B^3*b*c)*log((b*x + a)^n)^2 + (B^3*b*d*(2*n +
 1) + 2*A*B^2*b*d*(n + 1))*x + 2*(A*B^2*b*c + B^3*b*c + (B^3*b*d*(n + 1) +
A*B^2*b*d)*x)*log((b*x + a)^n))*log((d*x + c)^n))/(b*d*x + b*c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3,x, algorithm="fricas")
```

```
[Out] integral(B^3*log((b*x + a)^n*e/(d*x + c)^n)^3 + 3*A*B^2*log((b*x + a)^n*e/(
d*x + c)^n)^2 + 3*A^2*B*log((b*x + a)^n*e/(d*x + c)^n) + A^3, x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**3,x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3,x, algorithm="giac")
[Out] integrate((B*log((b*x + a)^n*e/(d*x + c)^n) + A)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(A+B\,\ln\left(\frac{e\,{\left(a+b\,x\right)}^n}{{\left(c+d\,x\right)}^n}\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^3,x)
[Out] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^3, x)
```

3.312 $\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{g+hx}\,dx$

Optimal. Leaf size=425

$$-\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{h}+\frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3\log\left(1-\frac{(dg-ch)(a+}{(bg-ah)(c+}\right.}{h}$$

```
[Out] -ln((-a*d+b*c)/b/(d*x+c))*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/h+(A+B*ln(e*(
b*x+a)^n/((d*x+c)^n)))^3*ln(1-(-c*h+d*g)*(b*x+a)/(-a*h+b*g)/(d*x+c))/h-3*B*
n*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2*polylog(2,d*(b*x+a)/b/(d*x+c))/h+3*B*
n*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2*polylog(2,(-c*h+d*g)*(b*x+a)/(-a*h+b*
g)/(d*x+c))/h+6*B^2*n^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))*polylog(3,d*(b*x+
a)/b/(d*x+c))/h-6*B^2*n^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))*polylog(3,(-c*h
+d*g)*(b*x+a)/(-a*h+b*g)/(d*x+c))/h-6*B^3*n^3*polylog(4,d*(b*x+a)/b/(d*x+c)
)/h+6*B^3*n^3*polylog(4,(-c*h+d*g)*(b*x+a)/(-a*h+b*g)/(d*x+c))/h
```

Rubi [A]

time = 0.46, antiderivative size = 425, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {2573, 2553, 2404, 2354, 2421, 2430, 6724}

$$-\frac{6B^2n^2\text{PolyLog}\left(3,\frac{(a+bx)(dg-ch)}{(c+dx)(bg-ah)}\right)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{h}+\frac{6B^2n^2\text{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{h}+\frac{3Bn\text{PolyLog}\left(2,\frac{(a+bx)(dg-ch)}{(c+dx)(bg-ah)}\right)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^2}{h}-\frac{3Bn\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^2}{h}+\frac{6B^3n^3\text{PolyLog}\left(4,\frac{(a+bx)(dg-ch)}{(c+dx)(bg-ah)}\right)}{h}-\frac{6B^3n^3\text{PolyLog}\left(4,\frac{d(a+bx)}{b(c+dx)}\right)}{h}+\frac{\log\left(1-\frac{(a+bx)(dg-ch)}{(c+dx)(bg-ah)}\right)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^3}{h}-\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^3}{h}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3/(g + h*x),x]
```

```
[Out] -((Log[(b*c - a*d)/(b*(c + d*x))]*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^
3)/h) + ((A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3*Log[1 - ((d*g - c*h)*(a
 + b*x))/((b*g - a*h)*(c + d*x))])/h - (3*B*n*(A + B*Log[(e*(a + b*x)^n)/(c
 + d*x)^n])^2*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/h + (3*B*n*(A + B*Lo
g[(e*(a + b*x)^n)/(c + d*x)^n])^2*PolyLog[2, ((d*g - c*h)*(a + b*x))/((b*g
- a*h)*(c + d*x))])/h + (6*B^2*n^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])
*PolyLog[3, (d*(a + b*x))/(b*(c + d*x))])/h - (6*B^2*n^2*(A + B*Log[(e*(a +
 b*x)^n)/(c + d*x)^n])*PolyLog[3, ((d*g - c*h)*(a + b*x))/((b*g - a*h)*(c +
 d*x))])/h - (6*B^3*n^3*PolyLog[4, (d*(a + b*x))/(b*(c + d*x))])/h + (6*B^3
*n^3*PolyLog[4, ((d*g - c*h)*(a + b*x))/((b*g - a*h)*(c + d*x))])/h
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2404

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2430

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Dist[b*n*(p/q), Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p - 1
)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 2553

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[b*c - a*d, Subst[
Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)),
 x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x
] && NeQ[b*c - a*d, 0] && IntegerQ[m] && IGtQ[p, 0]
```

Rule 2573

```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{g+hx}\,dx = \int \left(\frac{A^3}{g+hx} + \frac{3A^2B\log(e(a+bx)^n(c+dx)^{-n})}{g+hx} + \frac{3AB^2\log^2(\ldots}{\ldots}\right.$$
$$= \frac{A^3\log(g+hx)}{h} + (3A^2B)\int \frac{\log(e(a+bx)^n(c+dx)^{-n})}{g+hx}\,dx + \ldots$$
$$= \frac{A^3\log(g+hx)}{h} + \frac{3A^2B\log(e(a+bx)^n(c+dx)^{-n})\log(g+hx)\ldots}{h}$$
$$= -\frac{3AB^2\log\left(\frac{bc-ad}{b(c+dx)}\right)\log^2(e(a+bx)^n(c+dx)^{-n})}{h} - \frac{B^3\log\left(\frac{bc\ldots}{b(c\ldots}\right.}{\ldots}$$
$$= -\frac{3AB^2\log\left(\frac{bc-ad}{b(c+dx)}\right)\log^2(e(a+bx)^n(c+dx)^{-n})}{h} - \frac{B^3\log\left(\frac{bc\ldots}{b(c\ldots}\right.}{\ldots}$$
$$= -\frac{3AB^2\log\left(\frac{bc-ad}{b(c+dx)}\right)\log^2(e(a+bx)^n(c+dx)^{-n})}{h} - \frac{B^3\log\left(\frac{bc\ldots}{b(c\ldots}\right.}{\ldots}$$
$$= -\frac{3AB^2\log\left(\frac{bc-ad}{b(c+dx)}\right)\log^2(e(a+bx)^n(c+dx)^{-n})}{h} - \frac{B^3\log\left(\frac{bc\ldots}{b(c\ldots}\right.}{\ldots}$$

Mathematica [F]
time = 0.81, size = 0, normalized size = 0.00

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{g+hx}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3/(g + h*x),x]
```

```
[Out] Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3/(g + h*x), x]
```

Maple [F]
time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{\left(A+B\ln\left(e(bx+a)^n(dx+c)^{-n}\right)\right)^3}{hx+g}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/(h*x+g),x)
```

```
[Out] int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/(h*x+g),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(h*x+g),x, algorithm="maxima
")
```

```
[Out] A^3*log(h*x + g)/h - integrate(-(B^3*log((b*x + a)^n)^3 - B^3*log((d*x + c)
^n)^3 + 3*A^2*B + 3*A*B^2 + B^3 + 3*(A*B^2 + B^3)*log((b*x + a)^n)^2 + 3*(B
^3*log((b*x + a)^n) + A*B^2 + B^3)*log((d*x + c)^n)^2 + 3*(A^2*B + 2*A*B^2
+ B^3)*log((b*x + a)^n) - 3*(B^3*log((b*x + a)^n)^2 + A^2*B + 2*A*B^2 + B^3
 + 2*(A*B^2 + B^3)*log((b*x + a)^n))*log((d*x + c)^n))/(h*x + g), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(h*x+g),x, algorithm="fricas
")
```

```
[Out] integral((B^3*log((b*x + a)^n*e/(d*x + c)^n)^3 + 3*A*B^2*log((b*x + a)^n*e/
(d*x + c)^n)^2 + 3*A^2*B*log((b*x + a)^n*e/(d*x + c)^n) + A^3)/(h*x + g), x
)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**3/(h*x+g),x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(h*x+g),x, algorithm="giac")
```

```
[Out] integrate((B*log((b*x + a)^n*e/(d*x + c)^n) + A)^3/(h*x + g), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+B\,\ln\left(\frac{e\,{\left(a+b\,x\right)}^n}{{\left(c+d\,x\right)}^n}\right)\right)^3}{g+h\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^3/(g + h*x),x)
[Out] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^3/(g + h*x), x)
```

3.313 $\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{(g+hx)^2}\,dx$

Optimal. Leaf size=302

$$\frac{(a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{(bg-ah)(g+hx)}+\frac{3B(bc-ad)n\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2\log\left(1-\right.}{(bg-ah)(dg-ch)}$$

```
[Out] (b*x+a)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/(-a*h+b*g)/(h*x+g)+3*B*(-a*d+b*
c)*n*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2*ln(1-(-c*h+d*g)*(b*x+a)/(-a*h+b*g)
/(d*x+c))/(-a*h+b*g)/(-c*h+d*g)+6*B^2*(-a*d+b*c)*n^2*(A+B*ln(e*(b*x+a)^n/((
d*x+c)^n)))*polylog(2,(-c*h+d*g)*(b*x+a)/(-a*h+b*g)/(d*x+c))/(-a*h+b*g)/(-c
*h+d*g)-6*B^3*(-a*d+b*c)*n^3*polylog(3,(-c*h+d*g)*(b*x+a)/(-a*h+b*g)/(d*x+c
))/(-a*h+b*g)/(-c*h+d*g)
```

Rubi [A]
time = 0.19, antiderivative size = 302, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2573, 2553, 2355, 2354, 2421, 6724}

$$\frac{6B^2n^2(bc-ad)\text{PolyLog}\left(2,\frac{(a+bx)(dg-ch)}{(c+dx)(bg-ah)}\right)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{(bg-ah)(dg-ch)}-\frac{6B^3n^3(bc-ad)\text{PolyLog}\left(3,\frac{(a+bx)(dg-ch)}{(c+dx)(bg-ah)}\right)}{(bg-ah)(dg-ch)}+\frac{3Bn(bc-ad)\log\left(1-\frac{(a+bx)(dg-ch)}{(c+dx)(bg-ah)}\right)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^2}{(bg-ah)(dg-ch)}+\frac{(a+bx)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^3}{(g+hx)(bg-ah)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3/(g + h*x)^2,x]
```

```
[Out] ((a + b*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3)/((b*g - a*h)*(g + h*
x)) + (3*B*(b*c - a*d)*n*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2*Log[1 -
 ((d*g - c*h)*(a + b*x))/((b*g - a*h)*(c + d*x))])/((b*g - a*h)*(d*g - c*h)
) + (6*B^2*(b*c - a*d)*n^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])*PolyLog
[2, ((d*g - c*h)*(a + b*x))/((b*g - a*h)*(c + d*x))])/((b*g - a*h)*(d*g - c
*h)) - (6*B^3*(b*c - a*d)*n^3*PolyLog[3, ((d*g - c*h)*(a + b*x))/((b*g - a*
h)*(c + d*x))])/((b*g - a*h)*(d*g - c*h))
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2355

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[x*((a + b*Log[c*x^n])^p/(d*(d + e*x))), x] - Dist[b*n*(p/d),
Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n
```

```
, p}, x] && GtQ[p, 0]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2553

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[b*c - a*d, Subst[
Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)),
 x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x
] && NeQ[b*c - a*d, 0] && IntegerQ[m] && IGtQ[p, 0]
```

Rule 2573

```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(A + B\log(e(a+bx)^n(c+dx)^{-n}))^3}{(g+hx)^2}\,dx = \int \left(\frac{A^3}{(g+hx)^2} + \frac{3A^2B\log(e(a+bx)^n(c+dx)^{-n})}{(g+hx)^2} + \frac{3AB^2 \text{lo}}{}\right.$$

$$= -\frac{A^3}{h(g+hx)} + (3A^2B)\int \frac{\log(e(a+bx)^n(c+dx)^{-n})}{(g+hx)^2}\,dx + (3$$

$$= -\frac{A^3}{h(g+hx)} + \frac{3A^2B(a+bx)\log(e(a+bx)^n(c+dx)^{-n})}{(bg-ah)(g+hx)} + \frac{3}{}$$

$$= -\frac{A^3}{h(g+hx)} + \frac{3A^2B(a+bx)\log(e(a+bx)^n(c+dx)^{-n})}{(bg-ah)(g+hx)} + \frac{3}{}$$

$$= -\frac{A^3}{h(g+hx)} - \frac{3A^2B(bc-ad)n\log(c+dx)}{(bg-ah)(dg-ch)} + \frac{3A^2B(a+bx)}{(bg}$$

$$= -\frac{A^3}{h(g+hx)} - \frac{3A^2B(bc-ad)n\log(c+dx)}{(bg-ah)(dg-ch)} + \frac{3A^2B(a+bx)}{(bg}$$

Mathematica [F]
time = 1.82, size = 0, normalized size = 0.00

$$\int \frac{(A + B\log(e(a+bx)^n(c+dx)^{-n}))^3}{(g+hx)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3/(g + h*x)^2,x]
```

```
[Out] Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3/(g + h*x)^2, x]
```

Maple [F]
time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{\left(A + B\ln\left(e(bx+a)^n\,(dx+c)^{-n}\right)\right)^3}{(hx+g)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/(h*x+g)^2,x)
```

```
[Out] int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/(h*x+g)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(h*x+g)^2,x, algorithm="maxi
ma")
```

```
[Out] 3*(b*n*e*log(b*x + a)/(b*g*h - a*h^2) - d*n*e*log(d*x + c)/(d*g*h - c*h^2)
- (b*c*n - a*d*n)*e*log(h*x + g)/((d*g*h - c*h^2)*a - (d*g^2 - c*g*h)*b))*A
^2*B*e^(-1) + B^3*log((d*x + c)^n)^3/(h^2*x + g*h) - 3*A^2*B*log((b*x + a)^
n*e/(d*x + c)^n)/(h^2*x + g*h) - A^3/(h^2*x + g*h) + integrate((3*A*B^2*c*h
 + B^3*c*h + (B^3*d*h*x + B^3*c*h)*log((b*x + a)^n)^3 + 3*(A*B^2*c*h + B^3*
c*h + (A*B^2*d*h + B^3*d*h)*x)*log((b*x + a)^n)^2 + 3*(A*B^2*c*h - (d*g*n -
 c*h)*B^3 - (B^3*d*h*(n - 1) - A*B^2*d*h)*x + (B^3*d*h*x + B^3*c*h)*log((b*
x + a)^n))*log((d*x + c)^n)^2 + (3*A*B^2*d*h + B^3*d*h)*x + 3*(2*A*B^2*c*h
+ B^3*c*h + (2*A*B^2*d*h + B^3*d*h)*x)*log((b*x + a)^n) - 3*(2*A*B^2*c*h +
B^3*c*h + (B^3*d*h*x + B^3*c*h)*log((b*x + a)^n)^2 + (2*A*B^2*d*h + B^3*d*h
)*x + 2*(A*B^2*c*h + B^3*c*h + (A*B^2*d*h + B^3*d*h)*x)*log((b*x + a)^n))*l
og((d*x + c)^n))/(d*h^3*x^3 + c*g^2*h + (2*d*g*h^2 + c*h^3)*x^2 + (d*g^2*h
+ 2*c*g*h^2)*x), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(h*x+g)^2,x, algorithm="fric
as")
```

```
[Out] integral((B^3*log((b*x + a)^n*e/(d*x + c)^n)^3 + 3*A*B^2*log((b*x + a)^n*e/
(d*x + c)^n)^2 + 3*A^2*B*log((b*x + a)^n*e/(d*x + c)^n) + A^3)/(h^2*x^2 + 2
*g*h*x + g^2), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**3/(h*x+g)**2,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(h*x+g)^2,x, algorithm="giac
")
```

```
[Out] integrate((B*log((b*x + a)^n*e/(d*x + c)^n) + A)^3/(h*x + g)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+B\,\ln\left(\frac{e\,{\left(a+b\,x\right)}^n}{{\left(c+d\,x\right)}^n}\right)\right)^3}{{\left(g+h\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^3/(g + h*x)^2,x)
```

```
[Out] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^3/(g + h*x)^2, x)
```

3.314 $\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{(g+hx)^3}\,dx$

Optimal. Leaf size=629

$$\frac{3B(bc-ad)hn(a+bx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{2(bg-ah)^2(dg-ch)(g+hx)}+\frac{b^2\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{2h(bg-ah)^2}-\frac{(A+$$

```
[Out] 3/2*B*(-a*d+b*c)*h*n*(b*x+a)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(-a*h+b*g)
^2/(-c*h+d*g)/(h*x+g)+1/2*b^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/h/(-a*h+b
*g)^2-1/2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/h/(h*x+g)^2+3*B^2*(-a*d+b*c)^
2*h*n^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))*ln(1-(-c*h+d*g)*(b*x+a)/(-a*h+b*g
)/(d*x+c))/(-a*h+b*g)^2/(-c*h+d*g)^2+3/2*B*(-a*d+b*c)*(-a*d*h-b*c*h+2*b*d*g
)*n*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2*ln(1-(-c*h+d*g)*(b*x+a)/(-a*h+b*g)/
(d*x+c))/(-a*h+b*g)^2/(-c*h+d*g)^2+3*B^3*(-a*d+b*c)^2*h*n^3*polylog(2,(-c*h
+d*g)*(b*x+a)/(-a*h+b*g)/(d*x+c))/(-a*h+b*g)^2/(-c*h+d*g)^2+3*B^2*(-a*d+b*c
)*(-a*d*h-b*c*h+2*b*d*g)*n^2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))*polylog(2,(-
c*h+d*g)*(b*x+a)/(-a*h+b*g)/(d*x+c))/(-a*h+b*g)^2/(-c*h+d*g)^2-3*B^3*(-a*d+
b*c)*(-a*d*h-b*c*h+2*b*d*g)*n^3*polylog(3,(-c*h+d*g)*(b*x+a)/(-a*h+b*g)/(d*
x+c))/(-a*h+b*g)^2/(-c*h+d*g)^2
```

Rubi [A]

time = 0.77, antiderivative size = 629, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2573, 2553, 2398, 2404, 2339, 30, 2355, 2354, 2438, 2421, 6724}

$$\frac{3B^2n^2(bc-ad)(-adh-bch+2bdg)\text{PolyLog}\left(2,\frac{(a+bx)(dg-ch)}{(c+dx)(bg-ah)}\right)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{(bg-ah)^2(dg-ch)^2}+\frac{3B^3hn^3(bc-ad)^2\text{PolyLog}\left(2,\frac{(a+bx)(dg-ch)}{(c+dx)(bg-ah)}\right)}{(bg-ah)^2(dg-ch)^2}-\frac{3B^3n^3(bc-ad)(-adh-bch+2bdg)\text{PolyLog}\left(3,\frac{(a+bx)(dg-ch)}{(c+dx)(bg-ah)}\right)}{(bg-ah)^2(dg-ch)^2}+\frac{b^2\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^3}{2h(bg-ah)^2}+\frac{3B^2hn^2(bc-ad)^2\log\left(1-\frac{(a+bx)(dg-ch)}{(c+dx)(bg-ah)}\right)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{(bg-ah)^2(dg-ch)^2}+\frac{3Bhn(a+bx)(bc-ad)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^2}{2(g+hx)(bg-ah)^2(dg-ch)}+\frac{3Bn(bc-ad)(-adh-bch+2bdg)\log\left(1-\frac{(a+bx)(dg-ch)}{(c+dx)(bg-ah)}\right)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^2}{2(bg-ah)^2(dg-ch)^2}-\frac{\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^3}{2h(g+hx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3/(g + h*x)^3,x]
```

```
[Out] (3*B*(b*c - a*d)*h*n*(a + b*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2)/
(2*(b*g - a*h)^2*(d*g - c*h)*(g + h*x)) + (b^2*(A + B*Log[(e*(a + b*x)^n)/(
c + d*x)^n])^3)/(2*h*(b*g - a*h)^2) - (A + B*Log[(e*(a + b*x)^n)/(c + d*x)^
n])^3/(2*h*(g + h*x)^2) + (3*B^2*(b*c - a*d)^2*h*n^2*(A + B*Log[(e*(a + b*x
)^n)/(c + d*x)^n])*Log[1 - ((d*g - c*h)*(a + b*x))/((b*g - a*h)*(c + d*x))]
)/((b*g - a*h)^2*(d*g - c*h)^2) + (3*B*(b*c - a*d)*(2*b*d*g - b*c*h - a*d*h
)*n*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2*Log[1 - ((d*g - c*h)*(a + b*
x))/((b*g - a*h)*(c + d*x))])/(2*(b*g - a*h)^2*(d*g - c*h)^2) + (3*B^3*(b*c
 - a*d)^2*h*n^3*PolyLog[2, ((d*g - c*h)*(a + b*x))/((b*g - a*h)*(c + d*x))]
)/((b*g - a*h)^2*(d*g - c*h)^2) + (3*B^2*(b*c - a*d)*(2*b*d*g - b*c*h - a*d
*h)*n^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])*PolyLog[2, ((d*g - c*h)*(a
 + b*x))/((b*g - a*h)*(c + d*x))])/((b*g - a*h)^2*(d*g - c*h)^2) - (3*B^3*(
b*c - a*d)*(2*b*d*g - b*c*h - a*d*h)*n^3*PolyLog[3, ((d*g - c*h)*(a + b*x))
/((b*g - a*h)*(c + d*x))])/((b*g - a*h)^2*(d*g - c*h)^2)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2355

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[x*((a + b*Log[c*x^n])^p/(d*(d + e*x))), x] - Dist[b*n*(p/d),
Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n
, p}, x] && GtQ[p, 0]
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_)*((
f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(d + e*x)^(q +
 1)*((a + b*Log[c*x^n])^p/((q + 1)*(e*f - d*g))), x] - Dist[b*n*(p/((q + 1)
*(e*f - d*g))), Int[(f + g*x)^(m + 1)*(d + e*x)^(q + 1)*((a + b*Log[c*x^n])
^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, q}, x] && NeQ[e*f
- d*g, 0] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 2404

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
```

```
] && EqQ[d*e, 1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2553

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[b*c - a*d, Subst[
Int[(b*f - a*g - (d*f - c*g)*x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)),
 x], x, (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x
] && NeQ[b*c - a*d, 0] && IntegerQ[m] && IGtQ[p, 0]
```

Rule 2573

```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{(g+hx)^3}\,dx = \int \left(\frac{A^3}{(g+hx)^3} + \frac{3A^2B\log(e(a+bx)^n(c+dx)^{-n})}{(g+hx)^3} + \frac{3AB^2\log\right.$$

$$= -\frac{A^3}{2h(g+hx)^2} + (3A^2B)\int \frac{\log(e(a+bx)^n(c+dx)^{-n})}{(g+hx)^3}\,dx +$$

$$= -\frac{A^3}{2h(g+hx)^2} - \frac{3A^2B\log(e(a+bx)^n(c+dx)^{-n})}{2h(g+hx)^2} - \frac{3AB^2\log}{}$$

$$= -\frac{A^3}{2h(g+hx)^2} - \frac{3A^2B\log(e(a+bx)^n(c+dx)^{-n})}{2h(g+hx)^2} - \frac{3AB^2\log}{}$$

$$= -\frac{A^3}{2h(g+hx)^2} - \frac{3A^2B(bc-ad)n}{2(bg-ah)(dg-ch)(g+hx)} + \frac{3A^2b^2Bn\log}{2h(bg-}$$

$$= -\frac{A^3}{2h(g+hx)^2} - \frac{3A^2B(bc-ad)n}{2(bg-ah)(dg-ch)(g+hx)} + \frac{3A^2b^2Bn\log}{2h(bg-}$$

$$= -\frac{A^3}{2h(g+hx)^2} - \frac{3A^2B(bc-ad)n}{2(bg-ah)(dg-ch)(g+hx)} + \frac{3A^2b^2Bn\log}{2h(bg-}$$

$$= -\frac{A^3}{2h(g+hx)^2} - \frac{3A^2B(bc-ad)n}{2(bg-ah)(dg-ch)(g+hx)} + \frac{3A^2b^2Bn\log}{2h(bg-}$$

$$= -\frac{A^3}{2h(g+hx)^2} - \frac{3A^2B(bc-ad)n}{2(bg-ah)(dg-ch)(g+hx)} + \frac{3A^2b^2Bn\log}{2h(bg-}$$

$$= -\frac{A^3}{2h(g+hx)^2} - \frac{3A^2B(bc-ad)n}{2(bg-ah)(dg-ch)(g+hx)} + \frac{3A^2b^2Bn\log}{2h(bg-}$$

Mathematica [F]
time = 3.62, size = 0, normalized size = 0.00

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{(g+hx)^3}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3/(g + h*x)^3,x]
[Out] Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3/(g + h*x)^3, x]
```

Maple [F]
time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{\left(A+B\ln\left(e(bx+a)^{n}\left(dx+c\right)^{-n}\right)\right)^{3}}{\left(hx+g\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/(h*x+g)^3,x)
```

```
[Out] int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/(h*x+g)^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(h*x+g)^3,x, algorithm="maxi
ma")
```

```
[Out] 3/2*(b^2*n*e*log(b*x + a)/(b^2*g^2*h - 2*a*b*g*h^2 + a^2*h^3) - d^2*n*e*log
(d*x + c)/(d^2*g^2*h - 2*c*d*g*h^2 + c^2*h^3) - (2*a*b*d^2*g*n - a^2*d^2*h*
n - (2*c*d*g*n - c^2*h*n)*b^2)*e*log(h*x + g)/((d^2*g^2*h^2 - 2*c*d*g*h^3 +
 c^2*h^4)*a^2 - 2*(d^2*g^3*h - 2*c*d*g^2*h^2 + c^2*g*h^3)*a*b + (d^2*g^4 -
2*c*d*g^3*h + c^2*g^2*h^2)*b^2) + (b*c*n - a*d*n)*e/((d*g^2*h - c*g*h^2)*a
- (d*g^3 - c*g^2*h)*b + ((d*g*h^2 - c*h^3)*a - (d*g^2*h - c*g*h^2)*b)*x))*A
^2*B*e^(-1) + 1/2*B^3*log((d*x + c)^n)^3/(h^3*x^2 + 2*g*h^2*x + g^2*h) - 3/
2*A^2*B*log((b*x + a)^n*e/(d*x + c)^n)/(h^3*x^2 + 2*g*h^2*x + g^2*h) - 1/2*
A^3/(h^3*x^2 + 2*g*h^2*x + g^2*h) + integrate(1/2*(6*A*B^2*c*h + 2*B^3*c*h
+ 2*(B^3*d*h*x + B^3*c*h)*log((b*x + a)^n)^3 + 6*(A*B^2*c*h + B^3*c*h + (A*
B^2*d*h + B^3*d*h)*x)*log((b*x + a)^n)^2 + 3*(2*A*B^2*c*h - (d*g*n - 2*c*h)
*B^3 - (B^3*d*h*(n - 2) - 2*A*B^2*d*h)*x + 2*(B^3*d*h*x + B^3*c*h)*log((b*x
 + a)^n))*log((d*x + c)^n)^2 + 2*(3*A*B^2*d*h + B^3*d*h)*x + 6*(2*A*B^2*c*h
 + B^3*c*h + (2*A*B^2*d*h + B^3*d*h)*x)*log((b*x + a)^n) - 6*(2*A*B^2*c*h +
 B^3*c*h + (B^3*d*h*x + B^3*c*h)*log((b*x + a)^n)^2 + (2*A*B^2*d*h + B^3*d*
h)*x + 2*(A*B^2*c*h + B^3*c*h + (A*B^2*d*h + B^3*d*h)*x)*log((b*x + a)^n))*
log((d*x + c)^n))/(d*h^4*x^4 + c*g^3*h + (3*d*g*h^3 + c*h^4)*x^3 + 3*(d*g^2
*h^2 + c*g*h^3)*x^2 + (d*g^3*h + 3*c*g^2*h^2)*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(h*x+g)^3,x, algorithm="fric
as")
```

```
[Out] integral((B^3*log((b*x + a)^n*e/(d*x + c)^n)^3 + 3*A*B^2*log((b*x + a)^n*e/
(d*x + c)^n)^2 + 3*A^2*B*log((b*x + a)^n*e/(d*x + c)^n) + A^3)/(h^3*x^3 + 3
*g*h^2*x^2 + 3*g^2*h*x + g^3), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**3/(h*x+g)**3,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(h*x+g)^3,x, algorithm="giac
")
```

```
[Out] integrate((B*log((b*x + a)^n*e/(d*x + c)^n) + A)^3/(h*x + g)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+B\,\ln\left(\frac{e\,{\left(a+b\,x\right)}^n}{{\left(c+d\,x\right)}^n}\right)\right)}^3}{{\left(g+h\,x\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^3/(g + h*x)^3,x)
```

```
[Out] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^3/(g + h*x)^3, x)
```

Chapter 4

Appendix

Local contents

4.1 Download section

The following zip files contain the raw integrals used in this test.

Mathematica format Mathematica_syntax.zip

Maple and Mupad format Maple_syntax.zip

Sympy format SYMPY_syntax.zip

Sage math format SAGE_syntax.zip

4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)
```

```
GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A","none"}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A","none"}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
```

```
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
```

```
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.2.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);
```

```
        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A","";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of re
                                convert(leaf_count_result,string)," vs. $2 (",
                                convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
```

```
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well"
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A","";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of o
                               convert(leaf_count_result,string),"$ vs. $2(",
                               convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
```

```
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
```

```
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.2.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
          return True
        else:
          return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_re
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

4.2.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#          Albert Rich to use with Sagemath. This is used to
#          grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#          'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#          issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

    #debug=False
    if debug:
        print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
        return 1
      else:
        return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    else:
      return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.op
  elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstan
    m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
    m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
    return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
    return max(3,expnType(expn.operands()[0]))
  elif is_special_function(expn.operator()): #is_special_function(expn.func)
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(4,m1)   #max(4,m1)
  elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(5,m1)   #max(5,m1)
  elif is_appell_function(expn.operator()):
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(6,m1)   #max(6,m1)
  elif str(expn).find("Integral") != -1: #this will never happen, since it
        #is checked before calling the grading function that is passed.
        #but kept it here.
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
  else:
    return 9

#main function
def grade_antiderivative(result,optimal):


  if debug:
    print ("Enter grade_antiderivative for sagemath")
    print("Enter grade_antiderivative, result=",result)
    print("Enter grade_antiderivative, optimal=",optimal)
    print("type(anti)=",type(result))
    print("type(optimal)=",type(optimal))

  leaf_count_result  = tree_size(result) #leaf_count(result)
  leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

  #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


  expnType_result  = expnType(result)
  expnType_optimal = expnType(optimal)
```

```
    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation ="none"
                else:
                    grade = "B"
                    grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
            else: #result contains complex but optimal is not
                grade = "C"
                grade_annotation ="Result contains complex when optimal does not."
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
    else:
        grade = "C"
        grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


    print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```